LAB 6

COMP6925 - Applied Operations Research

INTEGER PROGRAMMING

- Recall that in standard linear programming, our decisions reside in $\mathbb{R}^n$ where n is the number of decision variables
- However, in integer programming, some, if not all, of our decisions reside in a discrete space that can often be modelled as $\mathbb{Z}^m$ where m is the number of discrete decisions that we need to make
- As seen so far in the course, such as in graph problems, using LP yields sensible integer solutions (a property of the type of problem), but this is not always the case

BINARY INTEGER PROGRAMMING

- Of particular interest in the case of binary integer programming
- In BIP, decisions are modelled as yes (1)/ no (0)
- This allows us the flexibility to encode some non-trivial constraints by first stating them in terms of propositional logic
- BIP is a useful technique in MIP (Mixed Integer Programming)
  - In MIP, some decisions are integers while others are real numbers
- If x is a binary decision variable, we can state either
  - $0 \leq x \leq 1,\ x \in \mathbb{Z}$ or
  - $x \in \{0,1\}$

BINARY INTEGER PROGRAMMING

- Suppose that we have a constraint that $x_1 \vee x_2$ must be true
  - How can we state this as a constraint in BIP?

BINARY INTEGER PROGRAMMING

- Suppose that we have a constraint that $x_1 \vee x_2$ must be true
  - How can we state this as a constraint in BIP?
  - Taken another way, $x_1 \vee x_2$ can be re-stated as at least one of x_1 or x_2, which means that
    - $x_1 + x_2 \geq 1$
  - Other such cases exist (see 12.3 of Hillier and Leiberman)

EXAMPLE 1

12.1-2* A young couple, Eve and Steven, want to divide their main household chores (marketing, cooking, dishwashing, and laundering) between them so that each has two tasks but the total time they spend on household duties is kept to a minimum. Their efficiencies on these tasks differ, where the time each would need to perform the task is given by the following table:

	Time Needed per Week			
	Marketing	**Cooking**	**Dishwashing**	**Laundry**
Eve	4.5 hours	7.8 hours	3.6 hours	2.9 hours
Steven	4.9 hours	7.2 hours	4.3 hours	3.1 hours

EXAMPLE 1

- What are the decisions to make here?
  - Treating as BIP:
  - Does Eve do marketing?
  - Does Steve do marketing?
  - Does Eve do cooking?
  - Does Steve do cooking?
  - etc…

EXAMPLE 1

- For the allocation to be fair, both Steve and Eve should have a total of 2 chores
- Moreover, each chore needs to be performed by someone

EXAMPLE 1

- See EXAMPLE1.JL

EXAMPLE 2

12.1-4. The board of directors of General Wheels Co. is considering six large capital investments. Each investment can be made only once. These investments differ in the estimated long-run profit (net present value) that they will generate as well as in the amount of capital required, as shown by the following table (in units of millions of dollars):

	Investment Opportunity					
	1	**2**	**3**	**4**	**5**	**6**
Estimated profit	15	12	16	18	9	11
Capital required	38	33	39	45	23	27

The total amount of capital available for these investments is $100 million. Investment opportunities 1 and 2 are mutually exclusive, and so are 3 and 4. Furthermore, neither 3 nor 4 can be undertaken unless one of the first two opportunities is undertaken. There are no such restrictions on investment opportunities 5 and 6. The objective is to select the combination of capital investments that will maximize the total estimated long-run profit (net present value).

EXAMPLE 1

- See EXAMPLE2.JL