DLI Accelerated Data Science Teaching Kit

Lecture 20.2 - Latent Semantic Indexing (Singular Value Decomposition)

Latent Semantic Indexing (LSI)

Main idea

- map each **document** into some **'concepts'**
- map each **term** into some **'concepts'**

'Concept' : ~ a set of terms, with weights.

For example, DBMS_concept:
"data" (0.8),
"system" (0.5),
"retrieval" (0.6)

Latent Semantic Indexing (LSI)

~ pictorially (before) ~

document-term matrix

	data	system	retrieval	lung	ear
doc1	1	1	1		
doc2	1	1	1		
doc3				1	1
doc4				1	1

Latent Semantic Indexing (LSI)

~ pictorially (after) ~

term-concept matrix

	database concept	medical concept
data	1	
system	1	
retrieval	1	
lung		1
ear		1

… and

document-concept matrix

	database concept	medical concept
doc1	1	
doc2	1	
doc3		1
doc4		1

Latent Semantic Indexing (LSI)

Q: How to search, e.g., for “system”?
A: find the corresponding concept(s); and the corresponding documents

	database concept	medical concept
data	1	
system	1	
retrieval	1	
lung		1
ear		1

	database concept	medical concept
doc1	1	
doc2	1	
doc3		1
doc4		1

Latent Semantic Indexing (LSI)

Works like an **automatically constructed thesaurus**

We may retrieve documents that **DON'T** have the term "system", but they contain almost everything else ("data", "retrieval")

LSI - Discussion

Great idea,

- to derive **‘concepts’** from documents
- to build a **‘thesaurus’** automatically
- to reduce dimensionality (down to few “concepts”)

How does LSI work?

Uses **Singular Value Decomposition** (SVD)

Singular Value Decomposition (SVD)

Motivation

Problem #1

Find "concepts" in matrices

Problem #2

Compression / dimensionality reduction

	bread	lettuce	tomatos	beef	chicken
vegetarians	1	1	1		
	2	2	2		
	1	1	1		
	5	5	5		
meat eaters				2	2
				3	3
				1	1

SVD is a **powerful**, **generalizable** technique.

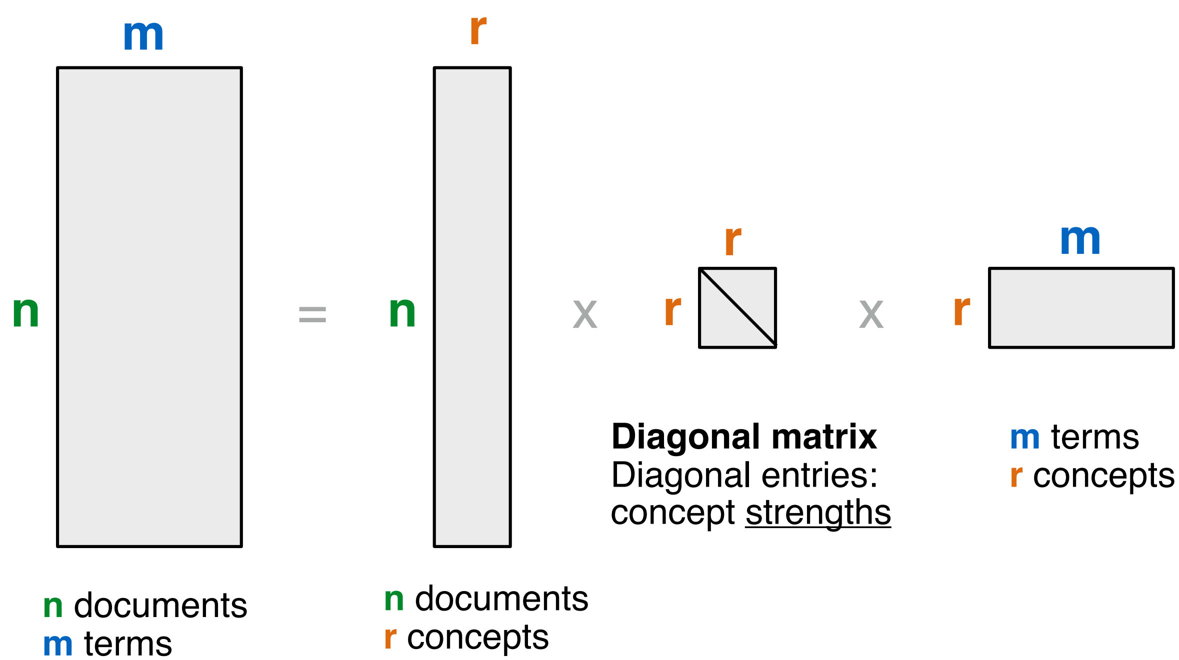
Songs / Movies / Products

Customers

1	1	1		
2	2	2		
1	1	1		
5	5	5		
			2	2
			3	3
			1	1

SVD Definition (pictorially)

$$\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \Lambda_{[r \times r]} (\mathbf{V}_{[m \times r]})^T$$



SVD Definition (in words)

$$\boxed{\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]}\, \Lambda_{[r \times r]}\, (\mathbf{V}_{[m \times r]})^T}$$

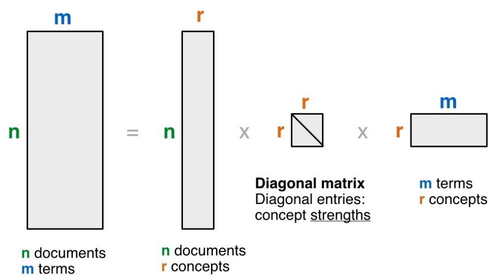
A: n x m matrix
e.g., n documents, m terms

U: n x r matrix
e.g., n documents, r concepts

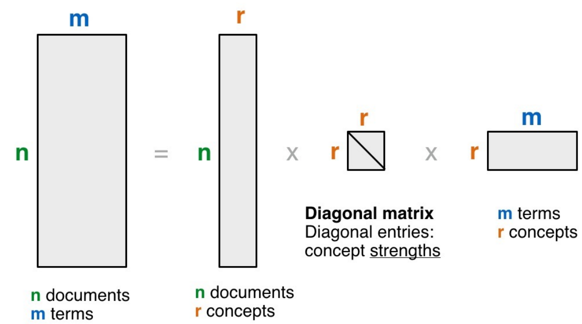
Λ: r x r diagonal matrix
r : rank of the matrix; strength of each ‘concept’

V: m x r matrix

e.g., m terms, r concepts



SVD - Properties



THEOREM [Press+92]:

always possible to decompose matrix **A** into

$\mathbf{A} = \mathbf{U}\,\Lambda\,\mathbf{V}^T$

U, Λ, **V**: **unique**, most of the time

U, **V**: column **orthonormal**

i.e., columns are unit vectors, and orthogonal to each other

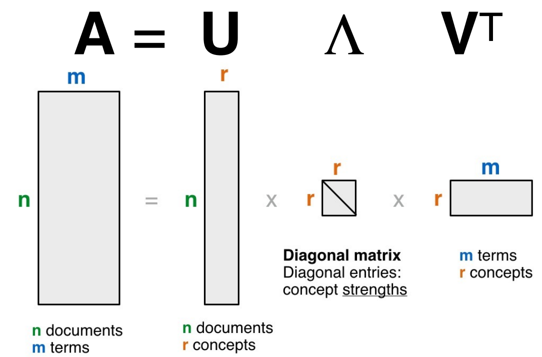
$\mathbf{U}^T\,\mathbf{U} = \mathbf{I}$

$\mathbf{V}^T\,\mathbf{V} = \mathbf{I}$

(**I**: identity matrix)

Λ: **diagonal** matrix with non-negative diagonal entires, sorted in decreasing order

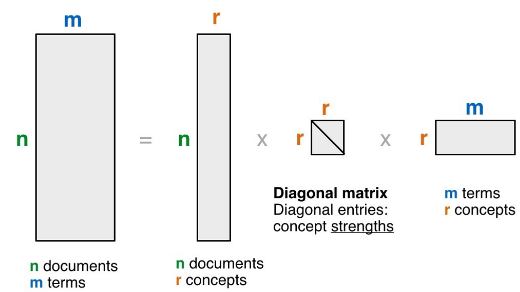
SVD - Example



$$
\begin{array}{cc}
 & \begin{array}{ccccc} \text{data} & \text{info} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\
\begin{array}{c} \text{CS} \\ \text{docs} \\ \\ \\ \text{MD} \\ \text{docs} \\ \\ \end{array} &
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}
\end{array}
=
\begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix}
\times
\begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix}
\times
\begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}
$$

SVD - Example

$$A = U \quad \Lambda \quad V^T$$



$$
\begin{array}{c}
 \\
\text{CS docs} \\
 \\
 \\
 \\
\text{MD docs} \\
 \\
\end{array}
\begin{array}{ccccc}
\text{data} & \text{info} & \text{retrieval} & \text{brain} & \text{lung} \\
\end{array}
$$

$$
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
=
\begin{array}{cc}
\text{CS concept} & \text{MD concept} \\
\left[\begin{array}{cc}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{array}\right] &
\end{array}
\times
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\times
\begin{array}{c}
\begin{array}{ccccc}
\text{data} & \text{info} & \text{retrieval} & \text{brain} & \text{lung}
\end{array} \\
\left[\begin{array}{ccccc}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{array}\right]
\end{array}
\begin{array}{l}
\leftarrow \text{CS concept} \\
\leftarrow \text{MD concept}
\end{array}
$$

“strength” of CS-concept

document-concept similarity matrix

term-concept similarity matrix

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Thank You