Assignment3

> Computer Vision 2023 Spring

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(**Math**) Nonlinear least-squares. Suppose that $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})) : \mathbb{R}^n \rightarrow \mathbb{R}^m$, $\mathbf{x} \in \mathbb{R}^n, \mathbf{f} \in \mathbb{R}^m$ and some $f_i(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}$ is a (are) non-linear function(s). Then, the problem,

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{f}(\mathbf{x})\|_2^2 = \arg\min_{\mathbf{x}} \frac{1}{2}(\mathbf{f}(\mathbf{x}))^T \mathbf{f}(\mathbf{x})$$

is a nonlinear least-squares problem. In our lecture, we mentioned that Levenberg-Marquardt algorithm is a typical method to solve this problem. In L-M algorithm, for each updating step, at the current $\mathbf{x}$, a local approximation model is constructed as,

$$\begin{aligned} L(\mathbf{h}) &= \frac{1}{2}(\mathbf{f}(\mathbf{x}+\mathbf{h}))^T \mathbf{f}(\mathbf{x}+\mathbf{h}) + \frac{1}{2}\mu\mathbf{h}^T\mathbf{h} \\ &= \frac{1}{2}(\mathbf{f}(\mathbf{x}))^T \mathbf{f}(\mathbf{x}) + \mathbf{h}^T(\mathbf{J}(\mathbf{x}))^T \mathbf{f}(\mathbf{x}) + \frac{1}{2}\mathbf{h}^T(\mathbf{J}(\mathbf{x}))^T \mathbf{J}(\mathbf{x})\mathbf{h} + \frac{1}{2}\mu\mathbf{h}^T\mathbf{h} \end{aligned}$$

where $\mathbf{J}(\mathbf{x})$ is $\mathbf{f}(\mathbf{x})$'s Jacobian matrix, and $\mu > 0$ is the damped coefficient. Please prove that $L(\mathbf{h})$ is a strictly convex function. (Hint: If a function $L(\mathbf{h})$ is differentiable up to at least second order, L is strictly convex if its Hessian matrix is positive definite.)

We have $\mathbf{h} \in \mathbb{R}^n$ and $L(\mathbf{h})$ is differentiable at second order. $L \in \mathbb{R}^n$, therefore

$$\begin{aligned} \nabla L(\mathbf{h}) &= (J(\mathbf{x}))^T f(x) + \frac{1}{2}((J(\mathbf{x}))^T J(\mathbf{x}) + ((J(\mathbf{x}))^T J(\mathbf{x}))^T)\mathbf{h} + \mu\mathbf{h} \\ &= (J(\mathbf{x}))^T f(x) + (J(\mathbf{x}))^T J(\mathbf{x})\mathbf{h} + \mu\mathbf{h} \\ \nabla^2 L(\mathbf{h}) &= (J(\mathbf{x}))^T J(\mathbf{x}) + \mu\mathbf{I} \end{aligned}$$

For, $\forall \mathbf{y} \in \mathbb{R}^n, \mathbf{y} \neq \mathbf{0}$,we have

$$\mathbf{y}^T \nabla^2 L(\mathbf{h})\mathbf{y} = \mathbf{y}^T (J(\mathbf{x}))^T J(\mathbf{x}) y + \mu \mathbf{y}^T \mathbf{y} = \|J(\mathbf{x})\mathbf{y}\|_2^2 + \mu\|\mathbf{y}\|_2^2$$

since $\mu > 0$,

$$\|J(\mathbf{x})\mathbf{y}\|_2^2 \geq 0 \quad \mu\mathbf{y}^T\mathbf{y} > 0 \Rightarrow \mathbf{y}^T\nabla^2 L(\mathbf{h})\mathbf{y} > 0$$

therefore, $\nabla^2 L(\mathbf{h}) > 0$

$L(\mathbf{h})$ is strictly convex.

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(**Programming**) In intelligent retail, one task is to investigate the proportion of each commodity occupying shelves. In this assignment, suppose that you are provided a surveillance video of a shelf and you need to recognize and locate two specific kinds of products, “康师傅鲜虾鱼板面” and “康师傅番茄炖鸡面”, in real time. You are recommended to use YoloVX (an object detection approach) for this task.

The interface may be similar like this,

The test video is given on the course website. Please show your results to TA.

The result test video is file `1953921_supermarketvideo.mp4`.

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(**Experiment**) 3D face scan and editing. Please refer to the files on the course website.

The experiment report is file `1953921_report.pdf`.

The fixed wavefront is file `1953921chenyuanzhe_fixed.obj`.