FLATIRON SCHOOL

# Polynomials and Regularisation

Jan 7th 2020

# Today's Lesson

## Learning Objectives

- Describe the phases of the CRISP-DM data science process
- Use the CRISP-DM framework to analyze and plan out data science projects

## Activities

- Reflecting on Your Data Science Experience
- Data Science Process Overview
- Revisiting Your Data Science Experience
- Reviewing Your Projects
- Exit Ticket

# Linear Regression Recap

**Task:** Take turns with a neighbor, answering the questions below:

- What is Linear Regression?
- How does it work?
- How do we know if the results are good or bad?
- What requirements should your data meet?

5 minutes

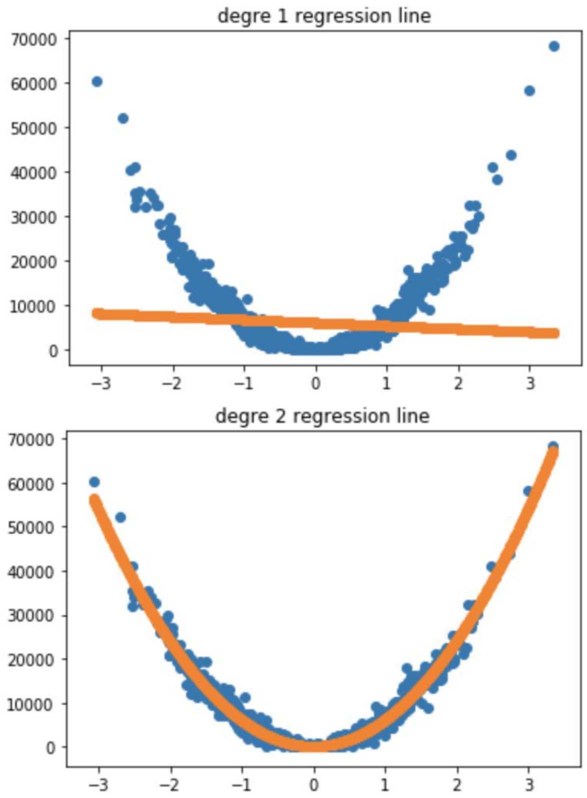
We will then come back to the large group and I'll pick some of you to share your answers with the rest of the class.

# Polynomial Regression

- Problems Linear Regression can’t solve on its own
- Polynomial transformation of predictors
- Interactions
- Feature explosion

# Polynomial Regression

- Some problems just can't be solved with a straight line
- We need models as complex as our problems in order to generate good predictions
- Example: If $y = x^2$ we need to have $x^2$ as a feature
- PolynomialFeatures generates



# Feature Interactions

- PolynomialFeatures generates the n-way interactions for all your predictions

- Example: If you want the interaction of 3 variables together (AxBxC) you will need a degree 3 polynomial transformation $(A+B+C)^3$

| Classifying Polynomials by Degree | | |
|---|---|---|
| **Name** | **Degree** | **Example** |
| Constant | 0 | $-9$ |
| Linear | 1 | $x-4$ |
| Quadratic | 2 | $x^2+3x-1$ |
| Cubic | 3 | $x^3+2x^2+x+1$ |
| Quartic | 4 | $2x^4+x^3+3x^2+4x-1$ |
| Quintic | 5 | $7x^5+x^4-x^3+3x^2+2x-1$ |

**SQUARE OF SUM**

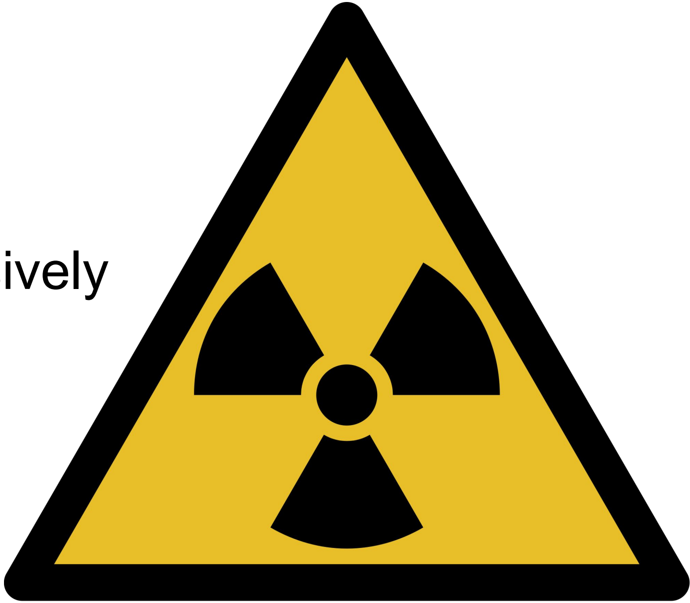
$$(a+b)^2 = a^2 + 2ab + b^2$$

**CUBE OF SUM**

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

# Feature Explosion

- This can get out of hand very quickly
- Example: Just 10 Features with a 3rd degree polynomial would lead to 1000 features
- When the number of features is too high:
  - Coefficients become unstable
  - Chance of multicollinearity increases massively
  - Chance of overfitting explodes
  - Dimensionality becomes a course



# Polynomial Regression Recap

**Task:** Take turns with a neighbor, answering the questions below:

- When is polynomial regression useful?
- When could it be dangerous?
- How do you think you could contain the risks?

5 minutes

We will then come back to the large group and I'll pick some of you to share your answers with the rest of the class.

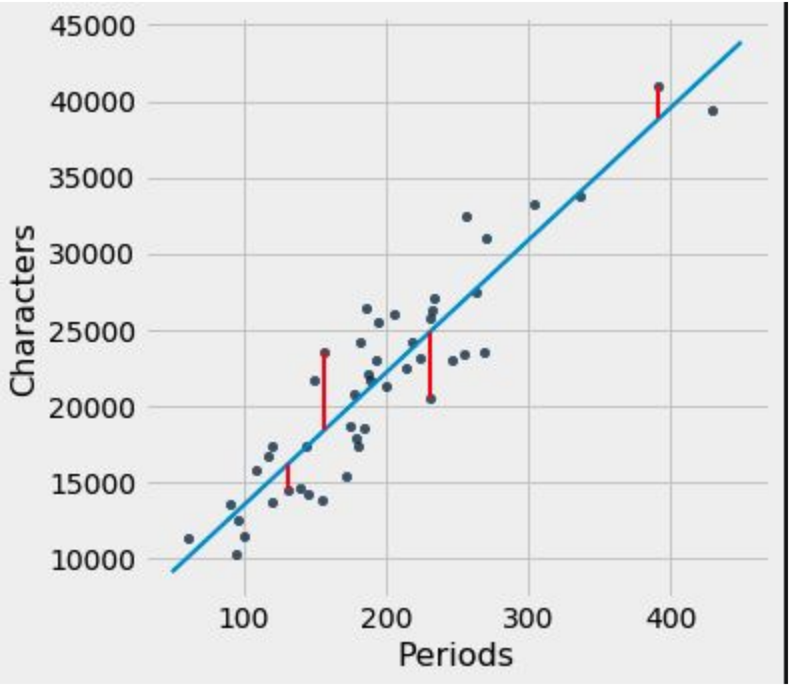
# Regularisation

- MSE
- Complexity penalty
- Scaling
- Ridge Regression
- LASSO Regression
- Elastic net
- Bias / Variance tradeoff

# Mean Squared Error (MSE)

$$\frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

* $n$ is the number of data points
* $Y_i$ represents observed values
* $\hat{Y}_i$ represents predicted values



# Complexity Penalty

- Regularisation in Regression means not only trying to minimise MSE but also the size of the coefficients

$$\sum_{i=1}^{M}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{M}\left(y_i - \sum_{j=0}^{p} w_j \times x_{ij}\right)^2 + \lambda \sum_{j=0}^{p} w_j^2$$

Cost function for ridge regression

- Because the size of the coefficients matter **feature scaling** is required.
- Although you will be scaling all datasets, you will use the **patterns from your training** dataset only
-

# Ridge Regression - L2 Reg

- Penalty based on the squared coefficients.

- This will lead to reducing largest coefficients first

$$\sum_{i=1}^{M} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{M} \left( y_i - \sum_{j=0}^{p} w_j \times x_{ij} \right)^2 + \lambda \sum_{j=0}^{p} w_j^2$$

Cost function for ridge regression

- It will bring irrelevant features' coefficients close to zero but not exactly zero

# LASSO Regression - L1 Reg

- Penalty based on the absolute value of the coefficients.

- This will lead to reducing most irrelevant coefficients first

$$\sum_{i=1}^{M}(y_i - \hat{y_i})^2 = \sum_{i=1}^{M}\left(y_i - \sum_{j=0}^{p} w_j \times x_{ij}\right)^2 + \lambda \sum_{j=0}^{p} |w_j|$$

Cost function for Lasso regression

- It will bring irrelevant features' coefficients exactly to zero. Hence, we can use LASSO as feature selection tool.
- Because of all of the above we call this type of regression LASSO: Least Absolute Shrinkage and Selection Operator

# Elastic net - L2 and L1 Reg

- Combines L2 and L1 penalties in a proportion that you can regulate with a hyperparameter in sklearn

$$(||y - X\beta||^2 + \lambda_2||\beta||^2 + \lambda_1||\beta||_1)$$

- Will bring some coefficients to zero but could also just reduce them if that leads to a better model

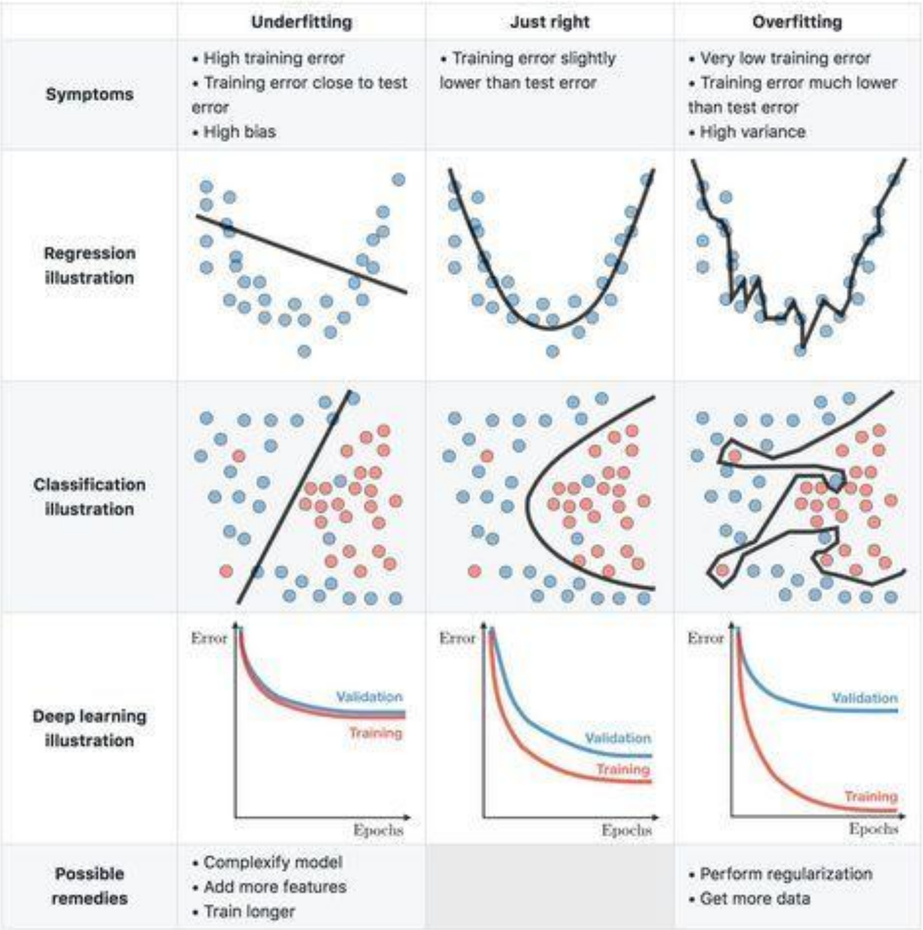
# Information Criteria

- Measure information loss
- Balances the goodness of fit and the complexity of a model
- Akaike Information Criterion $\mathrm{AIC} = 2k - 2\ln(\hat{L})$
- Bayesian Information Criterion $\mathrm{BIC} = \ln(n)k - 2\ln(\widehat{L}).$
- Lower is better

# Solving fitting issues

- **Polynomial transformations** will improving performance on the training dataset
- Convenient when previously underfitting

- **Regularisation** will decrease performance in the training dataset
- Convenient when previously overfitting (as it would likely improve performance on the test dataset)



# Regularisation Recap

**Task:** Take turns with a neighbor, answering the questions below:

- When is regularisation useful?
- What assumptions need to be met by your data before you apply it?
- What negative consequences could it have if done wrong?
- How would you choose between different types of regularisation?

5 minutes

We will then come back to the large group and I'll pick some of you to share your answers with the rest of the class.

# Delivering Value

Your job is not to create high performance models

They pay you to **solve problems**

# Summary + Exit Ticket

Presented by Dan Sanz