Set Theory Foundation for the Description of SR Flipflop Digital Circuits and Signals

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***Abstract*—The SR (Set/Reset) Flipflop was the basic component of digital logic cirtcuits. There exists certain problem in articulating the behaviour of its circuits and signals characteristics from theoretical perspective. Although the cross couple OR gate with Inverter circuit had long been established, there existed a curiosity whether it can be synthersized out of the analysis of the input and output signals of SR Flipflop Gate. The characteristics of digital signal pulses was studied with the set theory as the working paradigm. The classification of pulses was designed taking into consideration the changes of logic state in an event. The prior and post event were conceptualized that provided meaningful platform of the pulses classification. The relation and equation of set theory were instrumental in the construction of simple latch and SR flipflop circuits. The signal behaviors were shown consistent with the design equations for prior and post events. The operation of set theory on symbolic pulses were consistent.**

***Index Terms*—set theory, prior event, post event, pulse classification, SR flipflop, simple latch, OR Gate, Inverter**

I. Preliminary

Consider the piecewise signal as follows.

$$\mathrm{S}_{\mathrm{ignal}}(t) = \begin{cases} 0 \\ \text{for } (t \geq 0 \wedge t < 5) \vee (t \geq 10 \wedge t < 15) \\ \quad \vee (t \geq 17 \wedge t < 23) \vee (t \geq 24 \wedge t < 34) \\ \quad \vee (t \geq 39 \wedge t < 40) \vee (t \geq 45 \wedge t < 50) \\ \quad \vee (t \geq 55 \wedge t < 57) \vee (t \geq 67 \wedge t < 75) \\ 1 \\ \text{otherwise} \end{cases} \tag{1}$$

The signal(t) could be expressed in term of set of unit time when logic transition occurred. The transition could be either from 0 to 1 or 1 to 0. It depended on the initial logic state. The initial logic state was either 0 or 1 at the first item of the list. For this case, the initial state was 0 or say stream[0]=0. Thereafter stream[5]=1, stream[10]=0, stream[15]=1 and so on. The symbol — represened the transition edge at specified domain unit.

$$stream(|) = [0, 5, 10, 15, 17, 23, 24, 34, 39, 40, 45, 50, 55, 57, 67, 75, 80] \tag{2}$$

The inverse of (2) looked the same as (2) except for stream[0]=1. See Figure 2 "Inverse of Stream" plot. Hence.

$$Inverse(stream(|)) = [1, 5, 10, 15, 17, 23, 24, 34, 39, 40, 45, 50, 55, 57, 67, 75, 80] \tag{3}$$

The signal(t) was plotted in Figure 1. The annotations were Top Pulses, Top Unit, Top Impulse, Rise Pulse, Bottom Impulse, Bottom Unit Pulse, Fall Pulse, and Bottom Pulses.

The stem plot of Figure 1 is shown in Figure 2 with the same annotations.

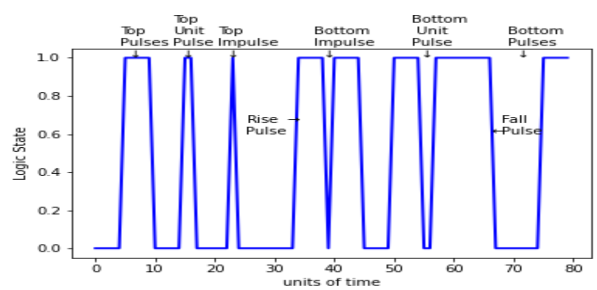

Fig. 1. Classification of Pulses

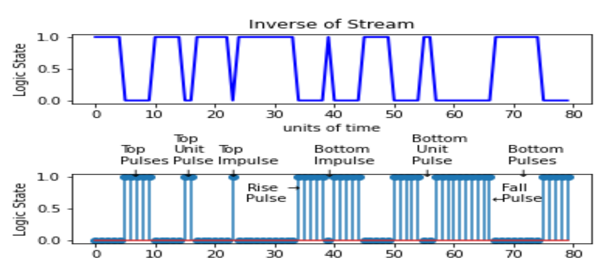

Fig. 2. Inverse and Stem Plot of Figure 1

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A. Definition

The logic level at a given point in time is either low (.) or high (|) called stem. The event was represented by a couple of points in time and defined in Table 1.

TABLE I
EVENT CLASSIFICATION

Item	Event Description	Stem Symbol	Stem Values	Remarks
00	low	..	[0,0]	no change
01	rise	.|	[0,1]	change
02	fall	|.	[1,0]	change
03	high	||	[1,1]	no change

Let a tuple of 3 units of time be defined as follows. Tri-unit = {prior, current, post} = {t-1, t, t+1} = t where the t could be used as the name of the set, e.g. 25 = {24,25,26 }. The anchor was t=25. The prior event of tri-unit was prior = {t-1, t}={24,25} while the post event of it was post = {t,t+1}={25,26}. The pulse clasification and their symbols were shown in Table II.

TABLE II
SYMBOLIC PULSES CLASSIFICATION

Item	Pulse Description	Pulse Symbol	Stem Symbol	Stem Values	Prior Event	Post Event
00	bottom	$\sqcup$	...	[0,0,0]	..	..
01	delay rise	$.\nearrow$	..|	[0,0,1]	..	.|
02	fall	$\searrow$	|..	[1,0,0]	|.	..
03	impulse (-)	$\top$	|.|	[1,0,1]	|.	.|
04	impulse (+)	$\perp$	.|.	[0,1,0]	.|	|.
05	rise	$\nearrow$	.||	[0,1,1]	.|	||
06	delay fall	$\cdot\searrow$	||.	[1,1,0]	||	|.
07	top	$\sqcap$	|||	[1,1,1]	||	||

Let following functions be defined.

$$\mathrm{P_{rior}}\left(P_{ulse}\right) = (t-1,\ t) \tag{4}$$

$$\mathrm{P_{ost}}\left(P_{ulse}\right) = (t,\ t+1) \tag{5}$$

$$\mathrm{P_{ulse}}\left(P_{ulse}\right) = (t-1,\ t,\ t+1) \tag{6}$$

$$\mathrm{P_{ulse}}\left(P_{ulse}\right) = \mathrm{P_{ost}}\left(P_{ulse}\right) \cup \mathrm{P_{rior}}\left(P_{ulse}\right) = (t-1,\ t,\ t\cup 1) \tag{7}$$

For example,$\mathrm{P_{rior}}(\nearrow) = .|$ and $\mathrm{P_{ost}}(\nearrow) = ||$ and $\mathrm{P_{ulse}}(\nearrow) = .||$, the anchor is the common element t.

The inverse function was the compliment of the signal and the overline represent the inverse made.

$$\sqcup = \mathrm{I_{nv}}\left(\sqcap\right) = \overline{\sqcap} \tag{8}$$

$$\perp = \mathrm{I_{nv}}\left(\top\right) = \overline{\top} \tag{9}$$

The definition of pulses in terms of tri-unit of time ensured that the logic state change occurred during the prior event. When there was no change in prior event but a change occurred in post event, the pulse was said to be delayed. The presence of the dot before the symbol indicated the delay.

The top unit pulse was a sequence of the rise event followed by fall event. It was a sequence of pulses, [$\nearrow$,$\cdot\searrow$.

$$\nearrow\searrow = (0,\ 1,\ 1,\ 0) = (\nearrow,\ \cdot\searrow) \tag{10}$$

In like manner bottom unit pulse,

$$\mathrm{I_{nv}}\left(\nearrow\searrow\right) = (1,\ 0,\ 0,\ 1) = (\searrow,\ .\nearrow) \tag{11}$$

The positive impulse was an intersection of rise pulse $\nearrow$ and delayed fall $\cdot\searrow$.

$$\perp = \nearrow \cap \cdot\searrow \tag{12}$$

The negative impulse was a union of $\searrow$ and $.\nearrow$.

$$\top = \searrow \cup .\nearrow \tag{13}$$

The transition pulses were changes in logic state either 0 to 1 or 1 to 0 at prior event. The change at post event without change at prior event was a delayed transition.

$$| = (\nearrow, \searrow, \perp, \top) \tag{14}$$

The plots for the eigth categories of pulses were illustrated in Figure 3.

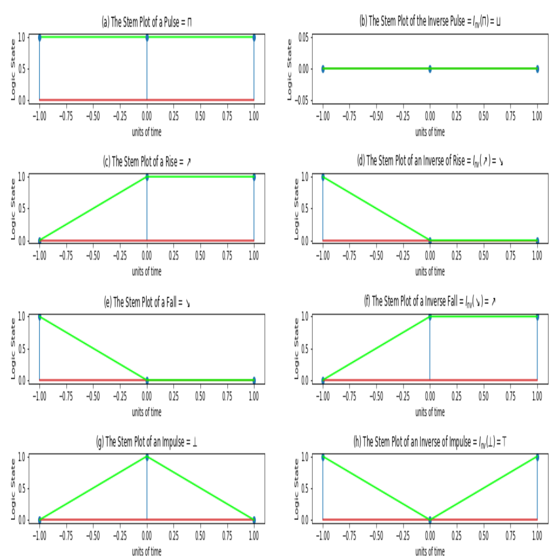

Fig. 3. Type of Pulses Categorized by the Logic State of the Left and Right Adjacent Unit of time

B. Expression

The pulse symbol could be an expression of function. The list of pulses in a signal was expressed as follows.

$$\mathrm{P_{ulse}}\left(Signal\right) = [t_a, t_b, t_c, ..., t_{n-1}, t_n, t_{n+1}, t_k, ...] \tag{15}$$

where $t_{n-1} = 1, t_n = 1, t_{n+1} = 1$

For example,

$$\sqcap(Signal) = [6, 7, 8, 35, 36, 37, 41, 42, 43, 51, 52, 53, 58, 59, 60, 61, 62, 63, 64, 65, 76, 77, 78] \tag{16}$$

For t=35 from (16),

$$\sqcap(Signal)[35] = (Signal[34], Signal[35], Signal[36]) = (1, 1, 1) \tag{17}$$

Another example,

$$\uparrow(Signal)[34] = (Signal[33], Signal[34], Signal[35]) = (0, 1, 1) \tag{18}$$

The union of (17) and (18),

$$\sqcap(\mathrm{S_{ignal}}\,(35)) \cup \uparrow(\mathrm{S_{ignal}}\,(34)) = \{\nearrow, \sqcap\} \tag{19}$$

A signal could be expressed as list of symbolic pulses. Hence, given

A =
[0, # A[0]=undefined
0, # A[1]=⊔
0, # A[2]=. ↗
1, # A[3]=↗
1, # A[4]=⊓
1, # A[5]=· ↘
0, # A[6]=↘
0, # A[7]=⊔
0, # A[8]=. ↗
1, # A[9]=⊥
0, # A[10]=↘
0, # A[11]=⊔
0, # A[12]=. ↗
1, # A[13]=↗
1, # A[14]=· ↘
0, # A[15]=⊤
1, # A[16]=↗
1, # A[17]=· ↘
0, # A[18]=↘
0, # A[19]=. ↗
1, # A[20]=↗
1, # A[21]=· ↘
0, # A[22]=↘
0, # A[23]=⊔
0 # A[24]=undefine]

The sequence of pulses in A were [⊔, . ↗, ↗, ⊓, · ↘, ↘, ⊔, . ↗, ⊥, ↘, ⊔, . ↗, ↗, · ↘, ⊤, ↗, · ↘, ↘, . ↗, ↗, · ↘, ↘, ⊔]

II. Signal Sets

Let's examine the of signal sets behavior in both open and closed systems logic circuit. The open loop system was a feed forward and without a feedback. The closed system has feedback loop.

A. Combinatory Logic: Open Loop System

A given set of signals had a combinations of a number of pulses, impulses, rising and falling edges and their inverses. It could also be said that any signal was a set of time points of stems. A stem represented an impulse. Two consecutive stems represent a rising and a falling pulse. Three consecutive stems makes a pulse. A stem was identified by its location in time domain. A point in time could be classified as a pulse, a rising edge, a falling edge, an impulse and their inverses depending on the changes of its prior and post events. A stem was defined as logic state 1 at a point in time domain.

Consider the followins signals

$$A(t) = \begin{cases} 0 \\ \text{for } (t \geq 0 \wedge t < 4) \vee (t \geq 8 \wedge t < 11) \\ \quad \vee (t \geq 12 \wedge t < 15) \vee (t \geq 20 \wedge t < 22) \\ \quad \vee (t \geq 23 \wedge t < 31) \vee (t \geq 34 \wedge t < 38) \\ \quad \vee (t \geq 48 \wedge t < 50) \\ 1 \\ \text{otherwise} \end{cases} \tag{20}$$

TABLE III
Signal A Pulses

Item	Pulse	Stem Set
00	⊓	[5, 6, 16, 17, 18, 32, 39, 40, 41, 42, 43, 44, 45, 46]
01	⊔	[1, 2, 9, 13, 24, 25, 26, 27, 28, 29, 35, 36]
02	↗	[4, 15, 31, 38]
03	↘	[8, 12, 20, 23, 34, 48]
04	⊥	[11, 22]
05	⊤	[]
06	. ↗	[3, 10, 14, 21, 30, 37]
07	· ↘	[7, 19, 33, 47]
08	|	[4, 8, 11, 12, 15, 20, 22, 23, 31, 34, 38, 48]

$$B(t) = \begin{cases} 1 \\ \text{for } (t \geq 0 \wedge t < 2) \vee (t \geq 7 \wedge t < 9) \vee (t \geq 18 \wedge t < 25) \\ \quad \vee (t \geq 26 \wedge t < 28) \vee (t \geq 30 \wedge t < 36) \\ \quad \vee (t \geq 40 \wedge t < 42) \vee (t \geq 43 \wedge t < 46) \\ 0 \\ \text{otherwise} \end{cases} \tag{21}$$

TABLE IV
Signal B Pulses

Item	Pulse	Stem Set
00	⊓	[19, 20, 21, 22, 23, 31, 32, 33, 34, 44]
01	⊔	[3, 4, 5, 10, 11, 12, 13, 14, 15, 16, 37, 38, 47, 48]
02	↗	[7, 18, 26, 30, 40, 43]
03	↘	[2, 9, 28, 36, 46]
04	⊥	[]
05	⊤	[25, 42]
06	. ↗	[6, 17, 29, 39]
07	· ↘	[1, 8, 24, 27, 35, 41, 45]
08	|	[2, 7, 9, 18, 25, 26, 28, 30, 36, 40, 42, 43, 46]

TABLE V
SIGNAL C = A ∪ B PULSES

Item	Pulse	Stem Set
00	⊓	[5, 6, 7, 16, 17, 18, 19, 20, 21, 22, 23, 31, 32, 33, 34, 39, 40, 41, 42, 43, 44, 45, 46, 51, 52, 53]
01	⊔	[13]
02	↗	[4, 15, 26, 30, 38, 50]
03	↘	[2, 9, 12, 28, 36, 48]
04	⊥	[11]
05	⊤	[25]
06	. ↗	[3, 10, 14, 29, 37, 49]
07	· ↘	[1, 8, 24, 27, 35, 47]
08	|	[2, 4, 9, 11, 12, 15, 25, 26, 28, 30, 36, 38, 48, 50]

TABLE VI
SIGNAL D = A ∩ B PULSES

Item	Pulse	Stem Set
00	⊓	[32, 44]
01	⊔	[1, 2, 3, 4, 5, 9, 10, 11, 12, 13, 14, 15, 16, 24, 25, 26, 27, 28, 29, 35, 36, 37, 38, 47, 48, 49, 50, 51, 52, 53]
02	↗	[18, 31, 40, 43]
03	↘	[8, 20, 23, 34, 46]
04	⊥	[7, 22]
05	⊤	[42]
06	. ↗	[6, 17, 21, 30, 39]
07	· ↘	[19, 33, 41, 45]
08	|	[7, 8, 18, 20, 22, 23, 31, 34, 40, 42, 43, 46]

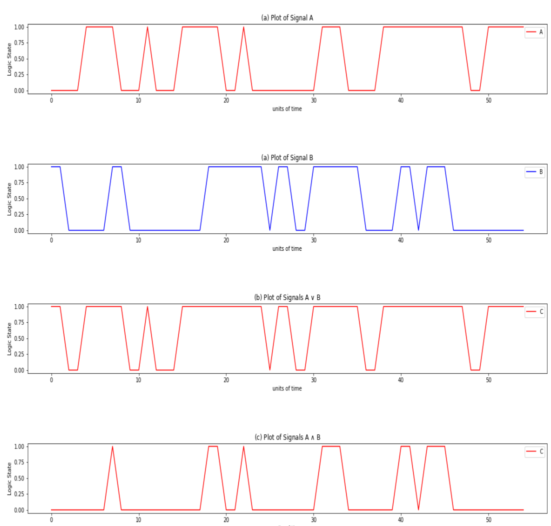

Fig. 4. Signals A and B and their Union and Intersection Operations

B. Simple Latch: a Closed Loop System

A closed loop system could be illustrated by a simple latch. It was a system consisting of an input and an output. Initial, the input is at zero logic state and the output was likewise at zero logic state. When a pulse occured, the output latched to logic state 1 and kept it latched irrespective of changes in logic state at the input thereafter. See Figure 5. Given such signals operating condition, it was desired to derive the simple latch circuit. Its block diagram was shown in Figure 6.

The input signal In and the output signal Out were defined in term of piecewise equation as follows

$$\mathrm{In}\,(t) = \begin{cases} 0 \\ \text{for } (t \geq 0 \wedge t < 3) \vee \\ \quad (t \geq 4 \wedge t < 7) \vee \\ \quad (t \geq 9 \wedge t < 10) \\ 1 \\ \text{otherwise} \end{cases} \tag{22}$$

TABLE VII
SIGNAL IN PULSES

Item	Pulse	Stem Set
00	⊓	[]
01	⊔	[1, 5]
02	↗	[7]
03	↘	[4]
04	⊥	[3]
05	⊤	[]
06	. ↗	[2, 6]
07	· ↘	[8]
08	|	[3, 4, 7]

$$\mathrm{Out}\,(t) = \begin{cases} 0 \\ \text{for } t \geq 0 \wedge t < 3 \\ 1 \\ \text{otherwise} \end{cases} \tag{23}$$

TABLE VIII
SIGNAL OUT PULSES

Item	Pulse	Stem Set
00	⊓	[4, 5, 6, 7, 8, 9, 10, 11, 12, 13]
01	⊔	[1]
02	↗	[3]
03	↘	[]
04	⊥	[]
05	⊤	[]
06	. ↗	[2]
07	· ↘	[]
08	|	[3]

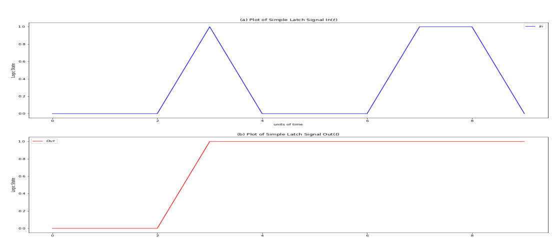

Fig. 5. Simple Latch

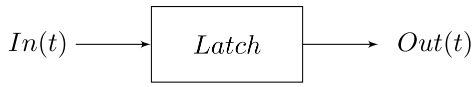

Fig. 6. Block Diagram of Simple Latch

By observation of Tables VII, VIII and Figure 5, the following relationships were noted.

$$\mathrm{In}_{\sqcap}(t) \subset \mathrm{Out}_{\sqcap}(t) \text{ constant pulses} \tag{24}$$

$$\mathrm{In}_{|}(t) \supset \mathrm{Out}_{|}(t) \text{ changing pulses} \tag{25}$$

The circuit could be derived from observed behavior of the relation (24). Since relation had no equivalent real world circuit, it was necessary to determine its equivalent equation as follows.

$$\mathrm{Out}(t) = \mathrm{Out}(t) \cup \mathrm{In}(t) \tag{26}$$

The equation (26) was expressed in terms of set theory. The implementation in hardware circuit of ∪ set operation was the Boolean equation for OR gate. The circuit diagram was shown in Figure 7

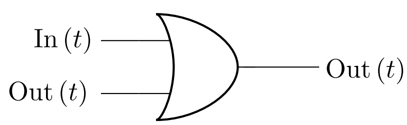

Fig. 7. OR Gate Circuit Diagram for Simple Latch Equation $\mathrm{Out}(t) = \mathrm{Out}(t) \cup \mathrm{In}(t)$

Since the $\mathrm{Out}(t)$ at input terminal of the OR gate was the same $\mathrm{Out}(t)$ at its output terminal, then the two terminal must be connected as shown in Figure 8. Let the Out at the input terminal be tag as In_{fb} keeping in mind $Out = In_{fb}$. Hence, the existing of feed back loop made this circuit closed loop system. The subset relation ($\mathrm{In}_{\sqcap}(t) \subset \mathrm{Out}_{\sqcap}(t)$) that depicted a feedback equation was indicative of memrory feature. In the sequence of $\mathrm{P_{ulse}}(In)$ pulses, the latch happened at $\mathrm{P_{ulse}}(Out)$ when a change in $\mathrm{P_{ulse}}(In)$ occurred as shown in Figure 8.

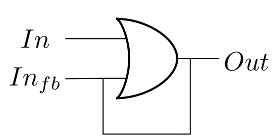

Fig. 8. OR Gate Circuit for Simple Latch Equation ($\mathrm{In}_{\sqcap}(t) \subset \mathrm{Out}_{\sqcap}(t)$)

Note that $\sqcup \subset \sqcup$, $\perp \subset \nearrow$, $\sqcup \subset \sqcap$. The events of In, In_{fb}, and Out were tabulated in IX. The operating equations were tabulated in Table X. There were a sequence of eigth pulses in the events. For each event, there were a set of five equations. The first was the initial prior event condition of In_{fb}. the second equation was the prior event. The third equation was the setting of the initial post condition of In_{fb}. This was the feedback action. The fourth equation was the initial post event condition of In_{fb}. The fifth equation is the whole event equation.

TABLE IX
SIMPLE LATCH EVENT

<table>
<tr><td>Item</td><td>Signals</td><td colspan="10">Stems</td></tr>
<tr><td>00</td><td>In</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td></tr>
<tr><td>01</td><td>Pulses</td><td></td><td>$\sqcup$</td><td>$.\nearrow$</td><td>$\perp$</td><td>$\searrow$</td><td>$.\nearrow$</td><td>$\nearrow$</td><td>$\sqcap$</td><td>$\cdot\searrow$</td><td></td></tr>
<tr><td>02</td><td>Priors</td><td></td><td>..</td><td>..</td><td>.|</td><td>|.</td><td>..</td><td>.|</td><td>||</td><td>||</td><td></td></tr>
<tr><td>03</td><td>Now</td><td></td><td>...</td><td>..|</td><td>.|.</td><td>|..</td><td>..|</td><td>|||</td><td>|||</td><td>||.</td><td></td></tr>
<tr><td>04</td><td>Posts</td><td></td><td>..</td><td>.|</td><td>|.</td><td>..</td><td>.|</td><td>.|</td><td>||</td><td>|.</td><td></td></tr>
<tr><td>05</td><td>In_{fb}</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>06</td><td>Pulses</td><td></td><td>$\sqcup$</td><td>$\sqcup$</td><td>$.\nearrow$</td><td>$\nearrow$</td><td>$\sqcap$</td><td>$\sqcap$</td><td>$\sqcap$</td><td>$\sqcap$</td><td></td></tr>
<tr><td>07</td><td>Priors</td><td></td><td>..</td><td>..</td><td>..</td><td>.|</td><td>||</td><td>||</td><td>||</td><td>||</td><td></td></tr>
<tr><td>08</td><td>Now</td><td></td><td>...</td><td>...</td><td>..|</td><td>.||</td><td>|||</td><td>|||</td><td>|||</td><td>|||</td><td></td></tr>
<tr><td>09</td><td>Posts</td><td></td><td>..</td><td>..</td><td>.|</td><td>||</td><td>||</td><td>||</td><td>||</td><td>||</td><td></td></tr>
<tr><td>10</td><td>Out</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>11</td><td>Pulses</td><td></td><td>$\sqcup$</td><td>$.\nearrow$</td><td>$\nearrow$</td><td>$\sqcap$</td><td>$\sqcap$</td><td>$\sqcap$</td><td>$\sqcap$</td><td>$\sqcap$</td><td></td></tr>
<tr><td>12</td><td>Priors</td><td></td><td>..</td><td>..</td><td>.|</td><td>||</td><td>||</td><td>||</td><td>||</td><td>||</td><td></td></tr>
<tr><td>13</td><td>Now</td><td></td><td>...</td><td>..|</td><td>.||</td><td>|||</td><td>|||</td><td>|||</td><td>|||</td><td>|||</td><td></td></tr>
<tr><td>14</td><td>Posts</td><td></td><td>..</td><td>.|</td><td>||</td><td>||</td><td>||</td><td>||</td><td>||</td><td>||</td><td></td></tr>
</table>

TABLE X
SIMPLE LATCH EQUATION EVENT

<table>
<tr><td>n</td><td>Equations</td><td>Stems</td></tr>
<tr><td>1</td><td>$\mathrm{P_{rior}(In_{fb}(\sqcup)) = P_{ost}(In_{fb}(\sqcup))}$
$\mathrm{P_{rior}(In(\sqcup)) \cup P_{rior}(In_{fb}(\sqcup)) = P_{rior}(Out(\sqcup))}$
$\mathrm{P_{ost}(In_{fb}(\sqcup)) = P_{rior}(Out(\sqcup))}$
$\mathrm{P_{ost}(In(\sqcup)) \cup P_{ost}(In_{fb}(\sqcup)) = P_{ost}(Out(\sqcup))}$
$\mathrm{In(\sqcup) \cup In_{fb}(\sqcup) = Out(\sqcup)}$</td><td>.. = ..
.. ∪ .. = ..
.. = ..
.. ∪ .. = ..
... ∪ ... = ...</td></tr>
<tr><td>2</td><td>$\mathrm{P_{rior}(In_{fb}(\sqcup)) = P_{ost}(In_{fb}(\sqcup))}$
$\mathrm{P_{rior}(In(.\nearrow)) \cup P_{rior}(In_{fb}(\sqcup)) = P_{rior}(Out(.\nearrow))}$
$\mathrm{P_{ost}(In_{fb}(\sqcup)) = P_{rior}(Out(\sqcup))}$
$\mathrm{P_{ost}(In(.\nearrow)) \cup P_{ost}(In_{fb}(\sqcup)) = P_{ost}(Out(.\nearrow))}$
$\mathrm{In(.\nearrow) \cup In_{fb}(\sqcup) = Out(.\nearrow)}$</td><td>.. = ..
.. ∪ .. = ..
.. = ..
.| ∪ .. = .|
..| ∪ ... = ..|</td></tr>
<tr><td>3</td><td>$\mathrm{P_{rior}(In_{fb}(\sqcup)) = P_{ost}(In_{fb}(\sqcup))}$
$\mathrm{P_{rior}(In(\perp)) \cup P_{rior}(In_{fb}(\sqcup)) = P_{rior}(Out(\perp))}$
$\mathrm{P_{ost}(In_{fb}(.\nearrow)) = P_{rior}(Out(\perp))}$
$\mathrm{P_{ost}(In(\perp)) \cup P_{ost}(In_{fb}(.\nearrow)) = P_{ost}(Out(\nearrow))}$
$\mathrm{In(\perp) \cup In_{fb}(.\nearrow) = Out(\nearrow)}$</td><td>.. = ..
.| ∪ .. = .|
.| = .|
|. ∪ .| = ||
.|. ∪ ..| = .||</td></tr>
<tr><td>4</td><td>$\mathrm{P_{rior}(In_{fb}(\nearrow)) = P_{ost}(In_{fb}(.\nearrow))}$
$\mathrm{P_{rior}(In(\searrow)) \cup P_{rior}(In_{fb}(\nearrow)) = P_{rior}(Out(\sqcap))}$
$\mathrm{P_{ost}(In_{fb}(\nearrow)) = P_{rior}(Out(\sqcap))}$
$\mathrm{P_{ost}(In(\searrow)) \cup P_{ost}(In_{fb}(\nearrow)) = P_{ost}(Out(\nearrow))}$
$\mathrm{In(\searrow) \cup In_{fb}(\nearrow) = Out(\sqcap)}$</td><td>.| = .|
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<tr><td>5</td><td>$\mathrm{P_{rior}(In_{fb}(\sqcap)) = P_{ost}(In_{fb}(\nearrow))}$
$\mathrm{P_{rior}(In(.\nearrow)) \cup P_{rior}(In_{fb}(\sqcap)) = P_{rior}(Out(\sqcap))}$
$\mathrm{P_{ost}(In_{fb}(\sqcap)) = P_{rior}(Out(\sqcap))}$
$\mathrm{P_{ost}(In(.\nearrow)) \cup P_{ost}(In_{fb}(\sqcap)) = P_{ost}(Out(\sqcap))}$
$\mathrm{In(.\nearrow) \cup In_{fb}(\sqcap) = Out(\sqcap)}$</td><td>|| = ||
.. ∪ || = ||
|| = ||
.. ∪ || = ||
..| ∪ ||| = |||</td></tr>
<tr><td>6</td><td>$\mathrm{P_{rior}(In_{fb}(\sqcap)) = P_{ost}(In_{fb}(\nearrow))}$
$\mathrm{P_{rior}(In(\nearrow)) \cup P_{rior}(In_{fb}(\sqcap)) = P_{rior}(Out(\sqcap))}$
$\mathrm{P_{ost}(In_{fb}(\sqcap)) = P_{rior}(Out(\sqcap))}$
$\mathrm{P_{ost}(In(\nearrow)) \cup P_{ost}(In_{fb}(\sqcap)) = P_{ost}(Out(\sqcap))}$
$\mathrm{In(\nearrow) \cup In_{fb}(\sqcap) = Out(\sqcap)}$</td><td>|| = ||
.| ∪ || = ||
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.|| ∪ ||| = |||</td></tr>
<tr><td>7</td><td>$\mathrm{P_{rior}(In_{fb}(\sqcap)) = P_{ost}(In_{fb}(\nearrow))}$
$\mathrm{P_{rior}(In(\sqcap)) \cup P_{rior}(In_{fb}(\sqcap)) = P_{rior}(Out(\sqcap))}$
$\mathrm{P_{ost}(In_{fb}(\sqcap)) = P_{rior}(Out(\sqcap))}$
$\mathrm{P_{ost}(In(\sqcap)) \cup P_{ost}(In_{fb}(\sqcap)) = P_{ost}(Out(\sqcap))}$
$\mathrm{In(\nearrow) \cup In_{fb}(\sqcap) = Out(\sqcap)}$</td><td>|| = ||
	∪		=	
	=	
	∪		=	
		∪ ||| = |||</td></tr>
<tr><td>8</td><td>$\mathrm{P_{rior}(In_{fb}(\sqcap)) = P_{ost}(In_{fb}(\nearrow))}$ $\mathrm{P_{rior}(In(\cdot\searrow)) \cup P_{rior}(In_{fb}(\sqcap)) = P_{rior}(Out(\sqcap))}$ $\mathrm{P_{ost}(In_{fb}(\cdot\searrow)) = P_{rior}(Out(\sqcap))}$ $\mathrm{P_{ost}(In(\sqcap)) \cup P_{ost}(In_{fb}(\sqcap)) = P_{ost}(Out(\sqcap))}$ $\mathrm{In(\nearrow) \cup In_{fb}(\sqcap) = Out(\sqcap)}$</td><td>		=	
	∪		=	
	=	
	∪		=	
	. ∪			=			</td></tr>
</table>

Let's consider the event n=3 where the $\mathrm{In}(\perp)$ event occured.

$$\mathrm{P_{rior}}\left(\mathrm{In_{fb}}(\sqcup)\right) = \mathrm{P_{ost}}\left(\mathrm{In_{fb}}(\sqcup)\right) = .. = .. \tag{27}$$

The current prior event of $\mathrm{In}_{\mathrm{fb}}$ was its previous post event. The prior event equation was the feed forward of this feed back system.

$$\mathrm{P}_{\mathrm{rior}}\left(\mathrm{In}\left(\perp\right)\right) \cup \mathrm{P}_{\mathrm{rior}}\left(\mathrm{In}_{\mathrm{fb}}\left(\sqcup\right)\right) = \mathrm{P}_{\mathrm{rior}}\left(\mathrm{Out}\left(\perp\right)\right) = .| \cup .. = .| \tag{28}$$

The $\mathrm{P}_{\mathrm{rior}}\left(\mathrm{In}_{\mathrm{fb}}\left(\sqcup\right)\right) = ..$ could not be changed by the $\mathrm{P}_{\mathrm{rior}}\left(\mathrm{Out}\left(\perp\right)\right) = .|$ feedback since it already happened. However, the $\mathrm{P}_{\mathrm{rior}}\left(\mathrm{Out}\left(\perp\right)\right) = .|$ could define the $\mathrm{P}_{\mathrm{ost}}\left(In_{fb}\right)$ as feed back for next event. Thus, it was set as follows.

$$\mathrm{P}_{\mathrm{ost}}\left(\mathrm{In}_{\mathrm{fb}}\left(.\nearrow\right)\right) = \mathrm{P}_{\mathrm{rior}}\left(\mathrm{Out}\left(\perp\right)\right) = .| = .| \tag{29}$$

Hence, having determine the $\mathrm{P}_{\mathrm{ost}}\left(\mathrm{In}_{\mathrm{fb}}\left(.\nearrow\right)\right)$ and given $\mathrm{P}_{\mathrm{ost}}\left(\mathrm{In}\left(\perp\right)\right)$ the $\mathrm{P}_{\mathrm{ost}}\left(Out\right)$ was determined as follows.

$$\mathrm{P}_{\mathrm{ost}}\left(\mathrm{In}\left(\perp\right)\right) \cup \mathrm{P}_{\mathrm{ost}}\left(\mathrm{In}_{\mathrm{fb}}\left(.\nearrow\right)\right) = \mathrm{P}_{\mathrm{ost}}\left(\mathrm{Out}\left(\nearrow\right)\right) = |. \cup .| = || \tag{30}$$

Taking the union of (28) and (30) we had the following.

$$\mathrm{In}\left(\perp\right) \cup \mathrm{In}_{\mathrm{fb}}\left(.\nearrow\right) = \mathrm{Out}\left(\nearrow\right) = .|. \cup ..| = .|| \tag{31}$$

Although (31) was consistent, the inequality $\mathrm{In}_{\mathrm{fb}}\left(.\nearrow\right) \neq \mathrm{Out}\left(\nearrow\right)$ appeared to contradict the hardware connection in Figure 8 that asserted $In_{fb} = Out$. However, the system was a feed back system whereby the output affect the inputs. The assertion of equality was equation (29). Going to n=4, the $\mathrm{P}_{\mathrm{ost}}\left(\mathrm{In}_{\mathrm{fb}}\left(.\nearrow\right)\right)$ became $\mathrm{P}_{\mathrm{rior}}\left(\mathrm{In}_{\mathrm{fb}}\left(\nearrow\right)\right)$. In like manner, with the falling edge of $\mathrm{In}\left(\searrow\right)$ and the stored memory of $\mathrm{In}_{\mathrm{fb}}\left(\nearrow\right)$, the latch, $\mathrm{Out}\left(\sqcap\right)$ begun at this stage. The latch remained till n=8, at different variation of In. The events n=2, 3,and 4 could be said the transient response of the system.
The space state diagram of simple latch was shown in Figure 9.

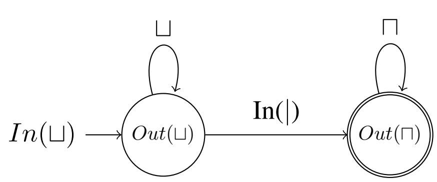

Fig. 9. Space State Diagram for OR Gate Circuit for Simple Latch Equation $\left(\mathrm{In}_{\sqcap}\left(t\right) \subset \mathrm{Out}_{\sqcap}\left(t\right)\right)$

The simple latch could be expressed as follows.

$$Out = \begin{cases} \sqcup \\ \text{for } In \cup Out = \sqcup \\ \\ \sqcap \\ otherwise \end{cases} \tag{32}$$

III. SR Flipflop

Let the signals S and R be generated such that they are disjoint. Let S input set the latched signal QS and let R input reset the latched signal QS. Let the QR be the inverse of QS. Given these condition, the circuit was derived as follows.

$$S = \begin{cases} 0 \\ \text{for } (t \geq 0 \wedge t < 6) \vee (t \geq 9 \wedge t < 13) \\ \quad \vee (t \geq 17 \wedge t < 18) \vee (t \geq 24 \wedge t < 28) \\ \quad \vee (t \geq 30 \wedge t < 60) \vee (t \geq 64 \wedge t < 67) \\ \quad \vee (t \geq 70 \wedge t < 71) \vee (t \geq 74 \wedge t < 78) \\ \quad \vee (t \geq 85 \wedge t < 88) \\ 1 \\ \text{otherwise} \end{cases} \tag{33}$$

TABLE XI
Signal S Pulses

Item	Pulse	Stem Set
00	$\sqcap$	[7, 14, 15, 19, 20, 21, 22, 61, 62, 68, 72, 79, 80, 81, 82, 83]
01	$\sqcup$	[1, 2, 3, 4, 10, 11, 25, 26, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 65, 75, 76, 86]
02	$\nearrow$	[6, 13, 18, 28, 60, 67, 71, 78]
03	$\searrow$	[9, 24, 30, 64, 74, 85]
04	$\perp$	[]
05	$\top$	[17, 70]
06	$.\nearrow$	[5, 12, 27, 59, 66, 77]
07	$\dot{\searrow}$	[8, 16, 23, 29, 63, 69, 73, 84]
08	$\vert$	[6, 9, 13, 17, 18, 24, 28, 30, 60, 64, 67, 70, 71, 74, 78, 85]

$$R = \begin{cases} 0 \\ \text{for } (t \geq 0 \wedge t < 36) \vee (t \geq 39 \wedge t < 42) \\ \quad \vee (t \geq 45 \wedge t < 46) \vee (t \geq 49 \wedge t < 52) \\ \quad \vee (t \geq 54 \wedge t < 90) \vee (t \geq 92 \wedge t < 99) \\ \quad \vee (t \geq 100 \wedge t < 101) \vee (t \geq 102 \wedge t < 104) \\ 1 \\ \text{otherwise} \end{cases} \tag{34}$$

TABLE XII
Signal R Pulses

Item	Pulse	Stem Set
00	$\sqcap$	[37, 43, 47, 105]
01	$\sqcup$	[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 40, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 93, 94, 95, 96, 97]
02	$\nearrow$	[36, 42, 46, 52, 90, 104]
03	$\searrow$	[39, 49, 54, 92, 102]
04	$\perp$	[99, 101]
05	$\top$	[45, 100]
06	$.\nearrow$	[35, 41, 51, 89, 98, 103]
07	$\dot{\searrow}$	[38, 44, 48, 53, 91]
08	$\vert$	[36, 39, 42, 45, 46, 49, 52, 54, 90, 92, 99, 100, 101, 102, 104]

The output signal QS and QR were constructed as follows.

$$QS = \begin{cases} 0 & \text{for } (t \geq 0 \wedge t < 6) \vee (t \geq 37 \wedge t < 60) \\ 1 & \text{otherwise} \end{cases} \tag{35}$$

TABLE XIII
SIGNAL QS PULSES

Item	Pulse	Stem Set
00	⊓	[7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108]
01	⊔	[1, 2, 3, 4, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58]
02	↗	[6, 60]
03	↘	[37]
04	⊥	[]
05	⊤	[]
06	.↗	[5, 59]
07	·↘	[36]
08	|	[6, 37, 60]

$$QR = \begin{cases} 1 \\ \text{for } (t \geq 0 \wedge t < 7) \vee (t \geq 36 \wedge t < 61) \\ \quad \vee (t \geq 90 \wedge t < 92) \vee (t \geq 99 \wedge t < 100) \\ \quad \vee (t \geq 101 \wedge t < 102) \vee (t \geq 104 \wedge t < 110) \\ 0 \\ \text{otherwise} \end{cases} \tag{36}$$

TABLE XIV
SIGNAL QR PULSES

Item	Pulse	Stem Set
00	⊓	[1, 2, 3, 4, 5, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 105, 106, 107, 108]
01	⊔	[8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 93, 94, 95, 96, 97]
02	↗	[36, 90, 104]
03	↘	[7, 61, 92, 102]
04	⊥	[99, 101]
05	⊤	[100]
06	.↗	[35, 89, 98, 103]
07	·↘	[6, 60, 91]
08	|	[7, 36, 61, 90, 92, 99, 100, 101, 102, 104]

The signals S, R, QS and QR were plotted in Figure 10.

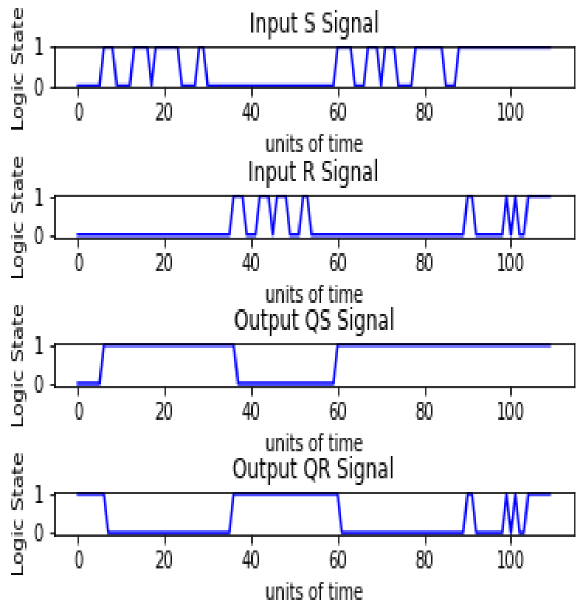

Fig. 10. SR Flipflop Timing Diagram

A. SR Model

By inspection of S and QS in Figure 10, the following was observed.

$$S_{\sqcap} \subset QS_{\sqcap} \tag{37}$$

Transforming (37) into an equation,

$$QS_{\sqcap} = QS_{\sqcap} \cup S_{\sqcap} \tag{38}$$

Likewise inspecting R and QR,

$$R_{\sqcap} \subset QR_{\sqcap} \tag{39}$$

Transforming (39) into an equation,

$$QR_{\sqcap} = QR_{\sqcap} \cup R_{\sqcap} \tag{40}$$

The relationship of QS and QR was established as follows.

$$\mathrm{I_{nv}}\left(QS_{\sqcap}\right) = QR_{\sqcap} = \overline{QS_{\sqcap}} \tag{41}$$

$$\mathrm{I_{nv}}\left(QR_{\sqcap}\right) = QS_{\sqcap} = \overline{QR_{\sqcap}} \tag{42}$$

Substituting (42) in the right hand side of (38) and (41) in the right hand side of (40),

$$QS_{\sqcap} = S_{\sqcap} \cup \overline{QR_{\sqcap}} \tag{43}$$

$$QR_{\sqcap} = R_{\sqcap} \cup \overline{QS_{\sqcap}} \tag{44}$$

B. SR Flipflop Circuit

The equations (43) and (44) were realized through the circuit diagram in Figure 11. The OR gate was the union operation in set theory, the equivalent addition (+) operation in Boolean algebra.

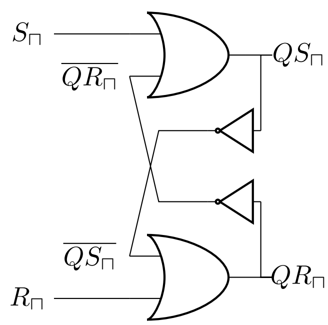

Fig. 11. OR Gate Circuit for SR Flipflop

Note that intersecting (37) with (39),

$$S_{\sqcap} \wedge R_{\sqcap} \subset QS_{\sqcap} \wedge QR_{\sqcap} = null = \phi \tag{45}$$

Therefore, the disjoint feature of S and R must be maintained to keep the SR Flipflop operating in latching function. However, S and R may not be kept disjoint for practical reasonss.

Hence, the set equation of SR flipflop was expressed as follows.

$$\begin{bmatrix} QS \\ QR \end{bmatrix} = \begin{cases} \begin{bmatrix} S \\ R \end{bmatrix} \cup \begin{bmatrix} \overline{QR} \\ \overline{QS} \end{bmatrix} & \text{for } S \cap R = \phi \\ \begin{bmatrix} QS \\ QR \end{bmatrix} & \text{for } S \cup R = \phi \end{cases} \tag{46}$$

C. Model of Cross Coupled OR Gate SR Flipflop

The cross coupled OR Gate SR Flipflop behaved differently when the non-disjoint signals S and R were inputted. Let's consider the following non-disjoint signals.

$$S = \begin{cases} 1 \\ \text{for } (t \geq 0 \wedge t < 10) \vee (t \geq 20 \wedge t < 23) \\ \quad \vee (t \geq 24 \wedge t < 28) \vee (t \geq 65 \wedge t < 66) \\ \quad \vee (t \geq 88 \wedge t < 89) \vee (t \geq 100 \wedge t < 102) \\ \quad \vee (t \geq 105 \wedge t < 106) \vee (t \geq 115 \wedge t < 116) \\ \quad \vee (t \geq 125 \wedge t < 130) \vee (t \geq 131 \wedge t < 141) \\ \quad \vee (t \geq 142 \wedge t < 160) \\ 0 \\ \text{otherwise} \end{cases} \tag{47}$$

TABLE XV
SIGNAL S PULSES

Item	Pulse	Stem Set
00	$\sqcap$	[1, 2, 3, 4, 5, 6, 7, 8, 21, 25, 26, 126, 127, 128, 132, 133, 134, 135, 136, 137, 138, 139, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158]
01	$\sqcup$	[11, 12, 13, 14, 15, 16, 17, 18, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 103, 107, 108, 109, 110, 111, 112, 113, 117, 118, 119, 120, 121, 122, 123]
02	$\nearrow$	[20, 24, 100, 125, 131, 142]
03	$\searrow$	[10, 28, 66, 89, 102, 106, 116]
04	$\perp$	[65, 88, 105, 115]
05	$\top$	[23, 130, 141]
06	$.\nearrow$	[19, 64, 87, 99, 104, 114, 124]
07	$\cdot\searrow$	[9, 22, 27, 101, 129, 140]
08	$\vert$	[10, 20, 23, 24, 28, 65, 66, 88, 89, 100, 102, 105, 106, 115, 116, 125, 130, 131, 141, 142]

$$R = \begin{cases} 1 \\ \text{for } (t \geq 0 \wedge t < 10) \vee (t \geq 35 \wedge t < 36) \\ \quad \vee (t \geq 50 \wedge t < 53) \vee (t \geq 54 \wedge t < 58) \\ \quad \vee (t \geq 75 \wedge t < 78) \vee (t \geq 80 \wedge t < 83) \\ \quad \vee (t \geq 88 \wedge t < 89) \vee (t \geq 115 \wedge t < 116) \\ \quad \vee (t \geq 125 \wedge t < 130) \vee (t \geq 131 \wedge t < 150) \\ \quad \vee (t \geq 151 \wedge t < 160) \\ 0 \\ \text{otherwise} \end{cases} \tag{48}$$

TABLE XVI
SIGNAL R PULSES

Item	Pulse	Stem Set
00	$\sqcap$	[1, 2, 3, 4, 5, 6, 7, 8, 51, 55, 56, 76, 81, 126, 127, 128, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158]
01	$\sqcup$	[11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 117, 118, 119, 120, 121, 122, 123]
02	$\nearrow$	[50, 54, 75, 80, 125, 131, 151]
03	$\searrow$	[10, 36, 58, 78, 83, 89, 116]
04	$\perp$	[35, 88, 115]
05	$\top$	[53, 130, 150]
06	$.\nearrow$	[34, 49, 74, 79, 87, 114, 124]
07	$\cdot\searrow$	[9, 52, 57, 77, 82, 129, 149]
08	$\vert$	[10, 35, 36, 50, 53, 54, 58, 75, 78, 80, 83, 88, 89, 115, 116, 125, 130, 131, 150, 151]

$$QS = \begin{cases} 1 \\ \text{for } (t \geq 0 \wedge t < 10) \vee (t \geq 11 \wedge t < 12) \\ \quad \vee (t \geq 13 \wedge t < 14) \vee (t \geq 15 \wedge t < 16) \\ \quad \vee (t \geq 17 \wedge t < 18) \vee (t \geq 19 \wedge t < 36) \\ \quad \vee (t \geq 37 \wedge t < 38) \vee (t \geq 39 \wedge t < 40) \\ \quad \vee (t \geq 41 \wedge t < 42) \vee (t \geq 43 \wedge t < 44) \\ \quad \vee (t \geq 45 \wedge t < 46) \vee (t \geq 47 \wedge t < 48) \\ \quad \vee (t \geq 49 \wedge t < 50) \vee (t \geq 65 \wedge t < 66) \\ \quad \vee (t \geq 67 \wedge t < 68) \vee (t \geq 69 \wedge t < 70) \\ \quad \vee (t \geq 71 \wedge t < 72) \vee (t \geq 73 \wedge t < 74) \\ \quad \vee (t \geq 75 \wedge t < 76) \vee (t \geq 88 \wedge t < 89) \\ \quad \vee (t \geq 90 \wedge t < 91) \vee (t \geq 92 \wedge t < 93) \\ \quad \vee (t \geq 94 \wedge t < 95) \vee (t \geq 96 \wedge t < 97) \\ \quad \vee (t \geq 98 \wedge t < 99) \vee (t \geq 100 \wedge t < 116) \\ \quad \vee (t \geq 117 \wedge t < 118) \vee (t \geq 119 \wedge t < 120) \\ \quad \vee (t \geq 121 \wedge t < 122) \vee (t \geq 123 \wedge t < 124) \\ \quad \vee (t \geq 125 \wedge t < 130) \vee (t \geq 131 \wedge t < 141) \\ \quad \vee (t \geq 142 \wedge t < 160) \\ 0 \\ \text{otherwise} \end{cases} \tag{49}$$

TABLE XVII
SIGNAL QS PULSES

Item	Pulse	Stem Set
00	⊓	[1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 126, 127, 128, 132, 133, 134, 135, 136, 137, 138, 139, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158]
01	⊔	[51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86]
02	↗	[19, 100, 125, 131, 142]
03	↘	[50, 76]
04	⊥	[11, 13, 15, 17, 37, 39, 41, 43, 45, 47, 49, 65, 67, 69, 71, 73, 75, 88, 90, 92, 94, 96, 98, 117, 119, 121, 123]
05	⊤	[10, 12, 14, 16, 18, 36, 38, 40, 42, 44, 46, 48, 66, 68, 70, 72, 74, 89, 91, 93, 95, 97, 99, 116, 118, 120, 122, 124, 130, 141]
06	. ↗	[64, 87]
07	· ↘	[9, 35, 115, 129, 140]
08	|	[10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 130, 131, 141, 142]

$$QR = \begin{cases} 1 \\ \text{for } (t \geq 0 \wedge t < 10) \vee (t \geq 11 \wedge t < 12) \\ \quad \vee (t \geq 13 \wedge t < 14) \vee (t \geq 15 \wedge t < 16) \\ \quad \vee (t \geq 17 \wedge t < 18) \vee (t \geq 19 \wedge t < 20) \\ \quad \vee (t \geq 35 \wedge t < 36) \vee (t \geq 37 \wedge t < 38) \\ \quad \vee (t \geq 39 \wedge t < 40) \vee (t \geq 41 \wedge t < 42) \\ \quad \vee (t \geq 43 \wedge t < 44) \vee (t \geq 45 \wedge t < 46) \\ \quad \vee (t \geq 47 \wedge t < 48) \vee (t \geq 49 \wedge t < 66) \\ \quad \vee (t \geq 67 \wedge t < 68) \vee (t \geq 69 \wedge t < 70) \\ \quad \vee (t \geq 71 \wedge t < 72) \vee (t \geq 73 \wedge t < 74) \\ \quad \vee (t \geq 75 \wedge t < 89) \vee (t \geq 90 \wedge t < 91) \\ \quad \vee (t \geq 92 \wedge t < 93) \vee (t \geq 94 \wedge t < 95) \\ \quad \vee (t \geq 96 \wedge t < 97) \vee (t \geq 98 \wedge t < 99) \\ \quad \vee (t \geq 100 \wedge t < 101) \vee (t \geq 115 \wedge t < 116) \\ \quad \vee (t \geq 117 \wedge t < 118) \vee (t \geq 119 \wedge t < 120) \\ \quad \vee (t \geq 121 \wedge t < 122) \vee (t \geq 123 \wedge t < 124) \\ \quad \vee (t \geq 125 \wedge t < 130) \vee (t \geq 131 \wedge t < 150) \\ \quad \vee (t \geq 151 \wedge t < 160) \\ 0 \\ \text{otherwise} \end{cases} \tag{50}$$

TABLE XVIII
SIGNAL QR PULSES

Item	Pulse	Stem Set
00	⊓	[1, 2, 3, 4, 5, 6, 7, 8, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 126, 127, 128, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158]
01	⊔	[21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113]
02	↗	[49, 75, 125, 131, 151]
03	↘	[20, 101]
04	⊥	[11, 13, 15, 17, 19, 35, 37, 39, 41, 43, 45, 47, 67, 69, 71, 73, 90, 92, 94, 96, 98, 100, 115, 117, 119, 121, 123]
05	⊤	[10, 12, 14, 16, 18, 36, 38, 40, 42, 44, 46, 48, 66, 68, 70, 72, 74, 89, 91, 93, 95, 97, 99, 116, 118, 120, 122, 124, 130, 150]
06	. ↗	[34, 114]
07	· ↘	[9, 65, 88, 129, 149]
08	|	[10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 130, 131, 150, 151]

Applying the signals from (47) and (48) to OR Gated SR flipflop, the timing diagram behavior was generated as shown in Figure 12. The space state diagram for Figure 12 timing diagram was depicted in Figure 13.

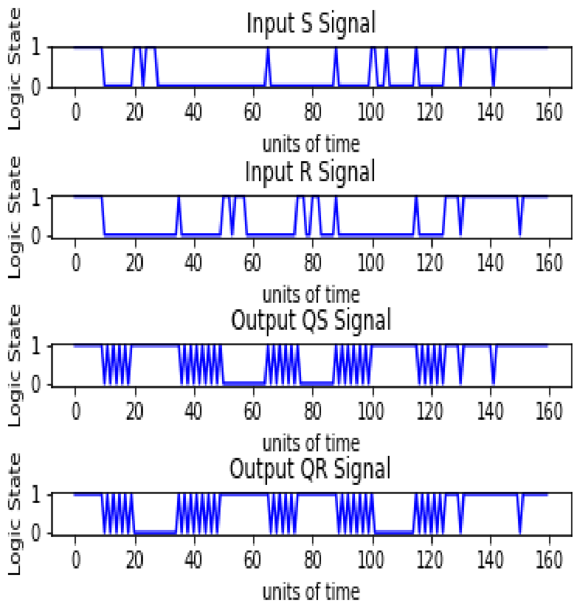

Fig. 12. Non-disjoint S and R Signals SR Flipflop Timing Diagram

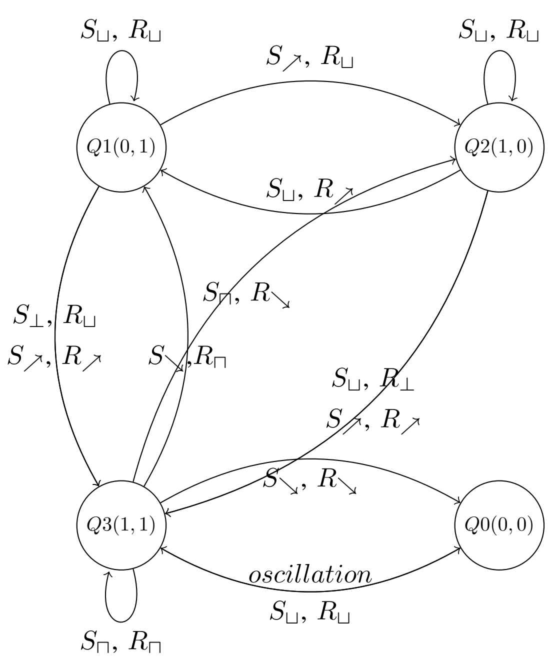

Fig. 13. Space State Diagram SR Flipflop Operation in Figure 12

D. Symbolic Pulses Features

The symbolic pulses were related as follows.

$$\sqcap \supset \nearrow \cup .\nearrow \cup \searrow \cup \cdot\searrow \cup \perp \cup \top \cup | \cup \sqcup \tag{51}$$

Given a set of 5-stem sequence, it could be represented by a sequence of the symbolic pulses as shown in Table 1.

TABLE XIX
COMBINATION OF A THREE OF PULSES IN FOUR TIME POINTS

Items	Stems 0,1,2,3,4	Pulses at Time Points 1	2	3
00	0,0,0,0,0	$\sqcup$,	$\sqcup$,	$\sqcup$
01	1,0,0,0,0	$\searrow$,	$\sqcup$,	$\sqcup$
02	0,1,0,0,0	$\perp$,	$\searrow$,	$\sqcup$
03	1,1,0,0,0	$\cdot\searrow$,	$\searrow$,	$\sqcup$
04	0,0,1,0,0	$.\nearrow$,	$\perp$,	$\searrow$
05	1,0,1,0,0	$\top$,	$\perp$,	$\searrow$
06	0,1,1,0,0	$\nearrow$,	$\cdot\searrow$,	$\searrow$
07	1,1,1,0,0	$\sqcap$,	$\cdot\searrow$,	$\searrow$
08	0,0,0,1,0	$\sqcup$,	$.\nearrow$,	$\perp$
09	1,0,0,1,0	$\searrow$,	$.\nearrow$,	$\perp$
10	0,1,0,1,0	$\perp$,	$\top$,	$\perp$
11	1,1,0,1,0	$\cdot\searrow$,	$\top$,	$\perp$
12	0,0,1,1,0	$.\nearrow$,	$\nearrow$,	$\cdot\searrow$
13	1,0,1,1,0	$\top$,	$\nearrow$,	$\cdot\searrow$
14	0,1,1,1,0	$\nearrow$,	$\sqcap$,	$\cdot\searrow$
15	1,1,1,1,0	$\sqcap$,	$\sqcap$,	$\cdot\searrow$
16	0,0,0,0,1	$\sqcup$,	$\sqcup$,	$.\nearrow$
17	1,0,0,0,1	$\searrow$,	$\sqcup$,	$.\nearrow$
18	0,1,0,0,1	$\perp$,	$\searrow$,	$.\nearrow$
19	1,1,0,0,1	$\cdot\searrow$,	$\searrow$,	$.\nearrow$
20	0,0,1,0,1	$.\nearrow$,	$\perp$,	$\top$
21	1,0,1,0,1	$\top$,	$\perp$,	$\top$
22	0,1,1,0,1	$\nearrow$,	$\cdot\searrow$,	$\top$
23	1,1,1,0,1	$\sqcap$,	$\cdot\searrow$,	$\top$
24	0,0,0,1,1	$\sqcup$,	$.\nearrow$,	$\nearrow$
25	1,0,0,1,1	$\searrow$,	$.\nearrow$,	$\nearrow$
26	0,1,0,1,1	$\perp$,	$\top$,	$\nearrow$
27	1,1,0,1,1	$\cdot\searrow$,	$\top$,	$\nearrow$
28	0,0,1,1,1	$.\nearrow$,	$\nearrow$,	$\sqcap$
28	1,0,1,1,1	$\top$,	$\nearrow$,	$\sqcap$
30	0,1,1,1,1	$\nearrow$,	$\sqcap$,	$\sqcap$
31	1,1,1,1,1	$\sqcap$,	$\sqcap$,	$\sqcap$

TABLE XX
SR FLIPFLOP IN OSCILLATION OPERATION

Item	Signals	Stems									
00	S	1	1 $\sqcap$	1 $\cdot\searrow$	0 $\searrow$	0 $\sqcup$	0 $\sqcup$	0 $\sqcup$	0 $\sqcup$	0 $\sqcup$	0
01	$\overline{QR}$	0	0 $\sqcup$	0 $\sqcup$	0 $.\nearrow$	1 $\perp$	0 $\top$	1 $\perp$	0 $\top$	1 $\perp$	0
02	$S\vee\overline{QR}=QS$	1	1 $\sqcap$	1 $\cdot\searrow$	0 $\top$	1 $\perp$	0 $\top$	1 $\perp$	0 $\top$	1 $\perp$	0
03	R	1	1 $\sqcap$	1 $\cdot\searrow$	0 $\searrow$	0 $\sqcup$	0 $\sqcup$	0 $\sqcup$	0 $\sqcup$	0 $\sqcup$	0
04	$\overline{QS}$	0	0 $\sqcup$	0 $\sqcup$	0 $.\nearrow$	1 $\perp$	0 $\top$	1 $\perp$	0 $\top$	1 $\perp$	0
05	$R\vee\overline{QS}=QR$	1	1 $\sqcap$	1 $\cdot\searrow$	0 $\top$	1 $\perp$	0 $\top$	1 $\perp$	0 $\top$	1 $\perp$	0

At stead state of Q3(1,1),

$$\text{QS}\,(\sqcap) = S(\sqcap) \cup \overline{QR}(\sqcup) \tag{52}$$

$$\text{QR}\,(\sqcap) = R(\sqcap) \cup \overline{QS}(\sqcup) \tag{53}$$

E. Application of Event Property in SR Flip Flop Modeling

As illustrated previously, the pulse consisted of time units tuple (t-1, t, t+1), e.g., (0,1,2). The tuple could be segmented in term of events. Hence prior event was (t-1, t) and the post event (t, t+1). The pulses could be categorized further by in terms of the changes in prior and post events.

TABLE XXI
SYMBOLIC PULSES CLASSIFICATION BY EVENTS

Item	Pulse	Stems			prior Event	post Event
00	$\sqcup$	0	0	0	No change down	No change down
01	$\sqcap$	1	1	1	No change up	No change up
02	$\searrow$	1	0	0	Change down	No change down
03	$\nearrow$	0	1	1	Change up	No change up
04	$.\nearrow$	0	0	1	No change down	Change up
05	$\cdot\searrow$	1	1	0	No change up	Change down
06	$\bot$	0	1	0	Change up	Change down
07	$\top$	1	0	1	Change down	Change up

Note that no change in prior events were $\sqcup$, $\sqcap$, $.\nearrow$, and $\cdot\searrow$. The change in prior events were $\nearrow$, $\searrow$, $\top$, and $\bot$. The no change in post events were $\sqcup$, $\sqcap$, $\nearrow$, and $\searrow$. The change in post events were $.\nearrow$, $\cdot\searrow$, $\bot$, and $\top$.

TABLE XXII
EVENT EQUIVALENT

n	Prior Event Function	Post Event Function	Stem
0	$\mathrm{P_{rior}}\,(\sqcup = ...) =$ $\mathrm{P_{rior}}\,(.\nearrow = ..\vert)$	$\mathrm{P_{ost}}\,(\sqcup = ...) =$ $\mathrm{P_{ost}}\,(\searrow = \vert..)$	..
1	$\mathrm{P_{rior}}\,(\sqcap = \vert\vert\vert) =$ $\mathrm{P_{rior}}\,(\cdot\searrow = \vert\vert.)$	$\mathrm{P_{ost}}\,(\sqcap = \vert\vert\vert) =$ $\mathrm{P_{ost}}\,(\nearrow = .\vert\vert)$	$\vert\vert$
2	$\mathrm{P_{rior}}\,(\nearrow = .\vert\vert) =$ $\mathrm{P_{rior}}\,(\bot = .\vert.)$	$\mathrm{P_{ost}}\,(.\nearrow = ..\vert) =$ $\mathrm{P_{ost}}\,(\top = \vert.\vert)$	$.\vert$
3	$\mathrm{P_{rior}}\,(\searrow = \vert..) =$ $\mathrm{P_{rior}}\,(\top = \vert.\vert)$	$\mathrm{P_{ost}}\,(\cdot\searrow = \vert\vert.) =$ $\mathrm{P_{ost}}\,(\bot = .\vert.)$	$\vert.$

Let's consider the events of pulses of SR Flipflop where $S_{\searrow}$ and $R_{\searrow}$ occurred simultaneously.

$$S(\searrow, \sqcup, \sqcup, \sqcup, \dots) \cup \overline{QR}(.\nearrow, \bot, \top, \bot, \dots) = \mathrm{QS}\,(\top, \bot, \top, \bot, \dots) \tag{54}$$

$$R(\searrow, \sqcup, \sqcup, \sqcup, \dots) \cup \overline{QS}(.\nearrow, \bot, \top, \bot, \dots) = \mathrm{QR}\,(\top, \bot, \top, \bot, \dots) \tag{55}$$

Lets consider the prior event of $S_{\searrow}$ and $R_{\searrow}$.

$$\mathrm{P_{rior}}\,(S(\searrow)) \cup \mathrm{P_{rior}}\left(\overline{QR}(\sqcup)\right) = \mathrm{P_{rior}}\,(\mathrm{QS}\,(\searrow)) = |.\cup..=|. \tag{56}$$

$$\mathrm{P_{rior}}\,(R(\searrow)) \cup \mathrm{P_{rior}}\left(\overline{QS}(\sqcup)\right) = \mathrm{P_{rior}}\,(\mathrm{QR}\,(\searrow)) = |.\cup..=|. \tag{57}$$

The changes in prior events occurred at S, R, QS, and QR. No changes took place at $\overline{QR}$ and $\overline{QS}$. However, the output QS and QR were inverted and fedback as $\overline{QR}$ and $\overline{QS}$. Such inversion had prior event changes. However, the prior events of $\overline{QR}$ and $\overline{QS}$ happened already and could not be change instantaneously. Therefore the prior events of feedbacks should apply only to the post events of $\overline{QR}$ and $\overline{QS}$. Thus,

$$\mathrm{P_{ost}}\left(\overline{QR}(\nearrow)\right) = \mathrm{P_{rior}}\,(\mathrm{I_{nv}}\,(\mathrm{QR}\,(\searrow))) = .|= \overline{|.} \tag{58}$$

$$\mathrm{P_{ost}}\left(\overline{QS}(\nearrow)\right) = \mathrm{P_{rior}}\,(\mathrm{I_{nv}}\,(\mathrm{QS}\,(\searrow))) = .|= \overline{|.} \tag{59}$$

Using (58) and (59), the post event equations were formulated and solved. Therefore, the post events of SR flipflop were expressed as follows.

$$\mathrm{P_{ost}}\,(S(\searrow)) \cup \mathrm{P_{ost}}\left(\overline{QR}(.\nearrow)\right) = \mathrm{P_{ost}}\,(\mathrm{QS}\,(.\nearrow)) = ..\cup.|=.| \tag{60}$$

$$\mathrm{P_{ost}}\,(R(\searrow)) \cup \mathrm{P_{ost}}\left(\overline{QS}(.\nearrow)\right) = \mathrm{P_{ost}}\,(\mathrm{QR}\,(.\nearrow)) = ..\cup.|=.| \tag{61}$$

Taking the union of (56) and (60) and (57) and (61),

$$S(\searrow) \cup \overline{QR}(.\nearrow) = \mathrm{QS}\,(\top) = |..\cup..|=|.| \tag{62}$$

$$R(\searrow) \cup \overline{QS}(.\nearrow) = \mathrm{QR}\,(\top) = |..\cup..|=|.| \tag{63}$$

Hence the equation (62) and (63) satisfied (54) and (55).

Moving on let the post event of (62) and (63) be the new prior event. The equivalent rise pulses were chosen.

$$\mathrm{P_{rior}}\,(S(\sqcup)) \cup \mathrm{P_{rior}}\left(\overline{QR}(\nearrow)\right) = \mathrm{P_{rior}}\,(\mathrm{QS}\,(\nearrow)) = ..\cup.|=.| \tag{64}$$

$$\mathrm{P_{rior}}\,(R(\sqcup)) \cup \mathrm{P_{rior}}\left(\overline{QS}(\nearrow)\right) = \mathrm{P_{rior}}\,(\mathrm{QR}\,(\nearrow)) = ..\cup.|=.| \tag{65}$$

Thereafter, the feedback inversion were established as follows.

$$\mathrm{P_{ost}}\left(\overline{QR}(\cdot\searrow)\right) = \mathrm{P_{rior}}\,(\mathrm{I_{nv}}\,(\mathrm{QR}\,(\nearrow))) = |.= \overline{.|} \tag{66}$$

$$\mathrm{P_{ost}}\left(\overline{QS}(\cdot\searrow)\right) = \mathrm{P_{rior}}\,(\mathrm{I_{nv}}\,(\mathrm{QS}\,(\nearrow))) = |.= \overline{.|} \tag{67}$$

Using (66) and (67), the post event equations were formulated as follows.

$$\mathrm{P_{ost}}\,(S(\sqcup)) \cup \mathrm{P_{ost}}\left(\overline{QR}(\cdot\searrow)\right) = \mathrm{P_{ost}}\,(\mathrm{QS}\,(\cdot\searrow)) = ..\cup|.=|. \tag{68}$$

$$\mathrm{P_{ost}}\,(R(\sqcup)) \cup \mathrm{P_{ost}}\left(\overline{QS}(\cdot\searrow)\right) = \mathrm{P_{ost}}\,(\mathrm{QR}\,(\cdot\searrow)) = ..\cup|.=|. \tag{69}$$

Taking the union of (64) and (68) and (65) and (69),

$$S(\sqcup) \cup \overline{QR}(\bot) = \mathrm{QS}\,(\bot) = ...\cup.|.=.|. \tag{70}$$

$$R(\sqcup) \cup \overline{QS}(\bot) = \mathrm{QR}\,(\bot) = ...\cup.|.=.|. \tag{71}$$

Hence the equation (70) and (71) satisfied (54) and (55)The above computations were illustrated in Table XXI. the prior event equations was initially given and solve for the output prior event. The output prior event was inverted and fed back as post event feedback input. Thereafter the post event equation was completed and solved for the output post event. The union of prior and post equations establisthed the now event equation. For the next event, the previous post event was made the new prior event. Thereafter the process was repeated for the next now event. The event computational method for SR flip flop was tabulated in Table XXIII

TABLE XXIII
THE EVENT COMPUTATIONAL METHOD FOR SR FLIP FLOP

Item	Event Equation	Stem
00	$P_{rior}(\overline{QR}(\sqcup)) = P_{ost}(\overline{QR}(\sqcup))$ $P_{rior}(\overline{QS}(\sqcup)) = P_{ost}(\overline{QS}(\sqcup))$	$.. = ..$ $.. = ..$
01	$P_{rior}(S(\searrow)) \cup P_{rior}(\overline{QR}(\sqcup)) = P_{rior}(QS(\searrow))$ $P_{rior}(R(\searrow)) \cup P_{rior}(\overline{QS}(\sqcup)) = P_{rior}(QR(\searrow))$	$\lvert . \cup .. = \lvert .$ $\lvert . \cup .. = \lvert .$
02	$P_{ost}(\overline{QR}(.\nearrow)) = P_{rior}(I_{nv}(QR(\searrow)))$ $P_{ost}(\overline{QS}(.\nearrow)) = P_{rior}(I_{nv}(QS(\searrow)))$	$.\lvert = \overline{\lvert .}$ $.\lvert = \overline{\lvert .}$
03	$P_{ost}(S(\sqcup)) \cup P_{ost}(\overline{QR}(.\nearrow)) = P_{ost}(QS(\nearrow))$ $P_{ost}(R(\sqcup)) \cup P_{ost}(\overline{QS}(.\nearrow)) = P_{ost}(QR(\nearrow))$	$.. \cup .\lvert = .\lvert$ $.. \cup .\lvert = .\lvert$
04	$S(\searrow) \cup \overline{QR}(.\nearrow) = QS(\top)$ $R(\searrow) \cup \overline{QS}(.\nearrow) = QR(\top)$	$\lvert .. \cup .. \lvert = \lvert . \rvert$ $\lvert .. \cup .. \lvert = \lvert . \rvert$
05	$P_{rior}(\overline{QR}(\nearrow)) = P_{ost}(\overline{QR}(.\nearrow))$ $P_{rior}(\overline{QS}(\nearrow)) = P_{ost}(\overline{QS}(.\nearrow))$	$.\lvert = .\lvert$ $.\lvert = .\lvert$
06	$P_{rior}(S(\sqcup)) \cup P_{rior}(\overline{QR}(\nearrow)) = P_{rior}(QS(\nearrow))$ $P_{rior}(R(\sqcup)) \cup P_{rior}(\overline{QS}(\nearrow)) = P_{rior}(QR(\nearrow))$	$.. \cup .\lvert = .\lvert$ $.. \cup .\lvert = .\lvert$
07	$P_{ost}(\overline{QR}(\cdot\searrow)) = P_{rior}(I_{nv}(QR(\nearrow)))$ $P_{ost}(\overline{QS}(\cdot\searrow)) = P_{rior}(I_{nv}(QS(\nearrow)))$	$\lvert . = \overline{.\lvert}$ $\lvert . = \overline{.\lvert}$
08	$P_{ost}(S(\sqcup)) \cup P_{ost}(\overline{QR}(\cdot\searrow)) = P_{ost}(QS(\cdot\searrow))$ $P_{ost}(R(\sqcup)) \cup P_{ost}(\overline{QS}(\cdot\searrow)) = P_{ost}(QR(\cdot\searrow))$	$.. \cup \lvert . = \lvert .$ $.. \cup \lvert . = \lvert .$
09	$S(\sqcup) \cup \overline{QR}(\bot) = QS(\bot)$ $R(\sqcup) \cup \overline{QS}(\bot) = QR(\bot)$	$... \cup .\lvert . = .\lvert .$ $... \cup .\lvert . = .\lvert .$
10	$P_{rior}(\overline{QR}(\searrow)) = P_{ost}(\overline{QR}(\cdot\searrow))$ $P_{rior}(\overline{QS}(\searrow)) = P_{ost}(\overline{QS}(\cdot\searrow))$	$\lvert . = \lvert .$ $\lvert . = \lvert .$
11	$P_{rior}(S(\sqcup)) \cup P_{rior}(\overline{QR}(\searrow)) = P_{rior}(QS(\searrow))$ $P_{rior}(R(\sqcup)) \cup P_{rior}(\overline{QS}(\searrow)) = P_{rior}(QR(\searrow))$	$.. \cup \lvert . = \lvert .$ $.. \cup \lvert . = \lvert .$
12	$P_{ost}(\overline{QR}(.\nearrow)) = P_{rior}(I_{nv}(QR(\searrow)))$ $P_{ost}(\overline{QS}(.\nearrow)) = P_{rior}(I_{nv}(QS(\searrow)))$	$.\lvert = \overline{\lvert .}$ $.\lvert = \overline{\lvert .}$
13	$P_{ost}(S(\sqcup)) \cup P_{ost}(\overline{QR}(.\nearrow)) = P_{ost}(QS(.\nearrow))$ $P_{ost}(R(\sqcup)) \cup P_{ost}(\overline{QS}(.\nearrow)) = P_{ost}(QR(.\nearrow))$	$.. \cup .\lvert = .\lvert$ $.. \cup .\lvert = .\lvert$
14	$S(\sqcup) \cup \overline{QR}(\top) = QS(\top)$ $R(\sqcup) \cup \overline{QS}(\top) = QR(\top)$	$... \cup \lvert . \rvert = \lvert . \rvert$ $... \cup \lvert . \rvert = \lvert . \rvert$

The initial equation of SR flip flop was illustrate in (46)with restriction that S and R were disjoint sets. Lets consider a disjoint set case as follows.

$$R = \mathrm{I}_{\mathrm{nv}}(S) = \overline{S} \tag{72}$$

Then

$$S \subset QS \tag{73}$$

$$R \subset QR \tag{74}$$

$$QR = \mathrm{I}_{\mathrm{nv}}(QS) = \overline{QS} \tag{75}$$

Substituting, (72) and (75) in (74),

$$\overline{S} \subset \overline{QS} \tag{76}$$

From (73) and (76), the following were determined.

$$QS = S \tag{77}$$

$$QR = \overline{S} \tag{78}$$

For

$$R \cup S = \phi \tag{79}$$

Then,

$$\phi \subset QS = QS \tag{80}$$

$$\phi \subset QR = QR \tag{81}$$

For $S \cap R = \sqcap$,

$$QS = QR = \sqcap \tag{82}$$

Finally, the SR flipflop model was completed as follows.

$$\begin{bmatrix} QS \\ QR \end{bmatrix} = \begin{cases} \begin{bmatrix} S \\ R \end{bmatrix} \cup \begin{bmatrix} \overline{QR} \\ \overline{QS} \end{bmatrix} \\ \text{for } S \cap R = \phi \\ \\ \begin{bmatrix} QS \\ QR \end{bmatrix} \\ \text{for } S \cup R = \phi \\ \\ \begin{bmatrix} S \\ \overline{S} \end{bmatrix} \\ \text{for } R = \overline{S} \\ \\ \begin{bmatrix} \sqcap \\ \sqcap \end{bmatrix} \\ \text{for } S \cap R = \sqcap \\ \\ \begin{bmatrix} \top & \bot & \top & \bot & \dots \\ \top & \bot & \top & \bot & \dots \end{bmatrix} \\ \text{for } S \cap R = \begin{bmatrix} \searrow & \sqcup & \sqcup & \sqcup & \dots \end{bmatrix} \end{cases} \tag{83}$$

IV. CONCLUSION

The set theory could be used to model the SR Flipflop. The classification of pulses were designed to facilitate the formulation of prior and post event functions. The basic set element was the stem and the dot at a point in time. The stem was present if the logic state was 1 and the dot appeared if if the logic state was 0. A pulse was designed to consist of a set of tri-tuple t-1,t,t+1. The number of combination of $2^3 = 8$ was exhausted that yielded 8 categories of pulses. The prior (t-1,t) and post (t,t+1) events were realized from the tri-tuple perspective anchored in t. The classification of pulses were based on the observaton made on the behavior of prior and post events over the 8 combinations of tri-tuple. The delayed category was specifically applied for no change in prior event but with change in post events.

The concept was tested through a process of derivation of circuits for simple latch and SR flipflop. After a number of thought experiments and simulations, the method of calculation was established. The prior function became the front end for handling input to output operation. The post function became the backend for handling output to input feedback operation. The prior event of the output was the post event feedback to the input. The union of the prior and post events led to the the now event equation.

Finally, the thorough model of SR flipflop was formulated under different input conditions in piecewise form. The undefine behavour was included and fully articulated. The memory and the buffer feature were expressed.

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