Scratch Pad Fuzzy Set

student

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sympy.stats.Normal(name, mean, std)
Create a continuous random variable with a Normal distribution.
The density of the Normal distribution is given by

$$f(x) = \frac{\sqrt{2}e^{-\frac{(\mu-x)^2}{2\sigma^2}}}{2\sqrt{\pi}\sigma} \tag{1}$$

Parameters:
mu : Real number or a list representing the mean or the mean vector
sigma : Real number or a positive definite sqaure matrix,
$\sigma^2 > 0$ the variance
Returns: A Random Symbol.

References
[R575] `http://en.wikipedia.org/wiki/Normal_distribution`
[R576] `http://mathworld.wolfram.com/NormalDistributionFunction.html`
The sympy function

$$f_1 = NormalDistribution(\mu, \sigma) \tag{2}$$

declares the Normal Distribution The following function specifies the random variable z

$$f_2 = \mathrm{f}_1\left(z\right) = \frac{\sqrt{2}}{2\sqrt{\pi}\sigma} e^{-\frac{(-\mu+z)^2}{2\sigma^2}} \tag{3}$$

$$f_3 = \int_{z_i}^{z_f} \mathrm{f}_2\left(z\right) dz = \frac{1}{2}\operatorname{erf}\left(\frac{\sqrt{2}}{2\sigma}\left(-\mu + z_f\right)\right) - \frac{1}{2}\operatorname{erf}\left(\frac{\sqrt{2}}{2\sigma}\left(-\mu + z_i\right)\right) \tag{4}$$

Let $\mu = 0$ and $\sigma = 1$ then

$$f_3 = \int_{-\infty}^{\infty} \mathrm{f}_2\left(z\right) dz = 1 \tag{5}$$

$$\int_{-1}^{1} \mathrm{f}_2\left(z\right) dz = \operatorname{erf}\left(\frac{\sqrt{2}}{2}\right) = 0.682689492137086 \tag{6}$$

$$\int_{-3}^{3} \mathrm{f}_2\left(z\right) dz = \operatorname{erf}\left(\frac{3\sqrt{2}}{2}\right) = 0.99730020393674 \tag{7}$$

$$\int_{-6}^{6} \mathrm{f}_2(z)\, dz = \operatorname{erf}\left(3\sqrt{2}\right) = 0.999999998026825 \tag{8}$$

$$DPM = 317310.507862914 \tag{9}$$

$$DPM = 2699.79606326021 \tag{10}$$

$$DPM = 0.00197317528982666 \tag{11}$$

A set of values of z can be generated randomly by the sympy function $Normal(\mu, \sigma)$. Given $\mu = 5$ and $\sigma = 0.5$ then

$$f_1 = NormalDistribution(5, 0.5) \tag{12}$$

$$f_2 = \frac{1.0\sqrt{2}}{\sqrt{\pi}} e^{-2.0(z-5)^2} \tag{13}$$

$$f_3 = \int_{z_i}^{z_f} \mathrm{f}_2(z)\, dz = 0.353553390593274\sqrt{2}\operatorname{erf}(1.4142135623731 z_f - 7.07106781186548) - 0.353553390593274\sqrt{2}\operatorname{erf}(1.4142135623731 z_i - 7.07106781186548) \tag{14}$$

1 Material

A set of values of z can be generated randomly by the sympy function $Normal(\mu, \sigma)$. Given $\mu = 5$ and $\sigma = 0.5$ then for 10000 pieces we have

$$Rods = random.normal(nmu, nsigma, count) = [9.95149710.3966377410.89338431...9.685125429.6676031411.1437347] \tag{15}$$

The histogram of (15) is shown in Figure 1.

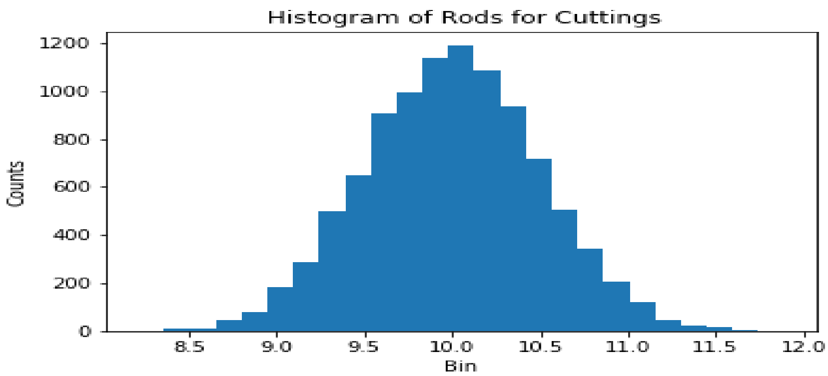


Figure 1: Histogram Rods Variation

Given the specification of ±1, how many are good and what is the yield? Answer number of good rods = 9566, yield = 95.66%. Note that everytime the program is run these values changes. The solution used is to use an algorithm to check each rod and count those that are within tolerance. Try solving it using (14) and compare the results.

2 Equipment

Now consider that these rods are to be cut into halves. The cutter has $\mu = 0.5$ and $\sigma = 0.05$ then for 9566 pieces we have

$$f_1 = NormalDistribution(0.5, 0.05) \tag{16}$$

$$f_2 = \frac{10.0\sqrt{2}}{\sqrt{\pi}} e^{-200.0(z-0.5)^2} \tag{17}$$

$$f_3 = \int_{z_i}^{z_f} \mathrm{f}_2\left(z\right) dz = 0.353553390593274\sqrt{2}\,\mathrm{erf}\left(14.1421356237309z - 7.07106781186547\right) \tag{18}$$

$$Cut_{event} = random.normal(nmu, nsigma, len(GoodRod)) = [0.55064640.538751170.40082747...0.603017850.439647860.38479112] \tag{19}$$

The histogram of (19) is shown in Figure 2.

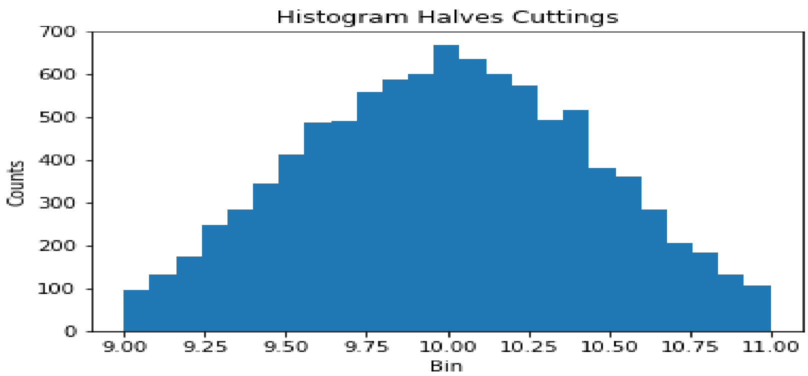


Figure 2: Histogram of Cutter Variation

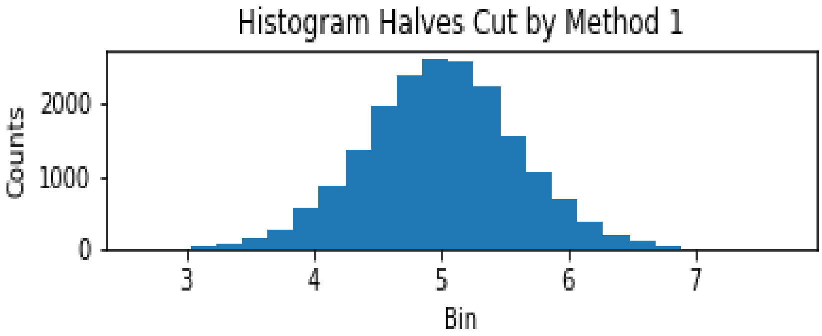
3 Method

There two methods to cut the rod. One method is the cutter is place at a distance of half the rod size exactly 5. Then the rod is referenced at the point of origin and cut is made. The other method is to place the rod at its center for cutter engagement. The formulas for the two methods are different. Determine the formula for each and justify it.

Answers:
1. The cutter with variation, i.e., $5 \pm .25$ is multiplied by the perfect dimension of rod that is 10. Then the answer is substracted from rod with variation. The two answer are the cut products.
2. The rod with variation, i.e., 10 ± 1 is multiplied by the cutter with variation, i.e., $5 \pm .25$. The answer is substracted from the rod with variation. The cut

producst are the two answers.

Determine the average and standard deviation of cut rods for each method and compare them. What is your observation? The histograms of Method 1 and 2 are shown in Figure 3.



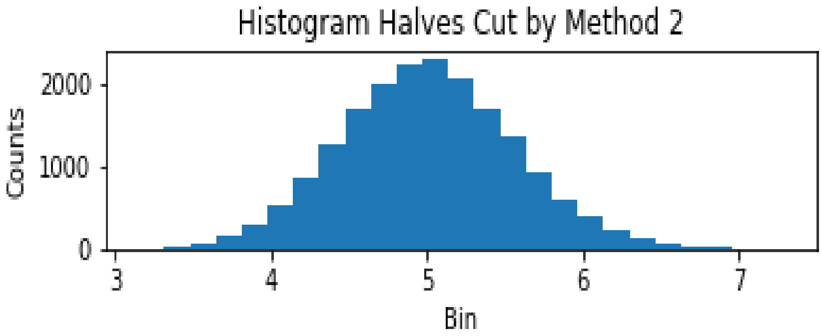


Figure 3: Histogram of Cut Rods

Assuming that the cut specification is $5 \pm .5$, determine the number of good cut and the yield for each method The number of good cut for method 1 is 11630 while for method 2 12131 respectively. The yield for method 1 is 60.7882% while for method 2 is 63.4069% respectively. Why is it method 2 has higher yield?

4 Operator

Assume and operator makes the loading with error of $\mu = 0.4$ and $\sigma = 0.1$. The histograms with operator handling for Method 1 and 2 are shown in Figure 4.

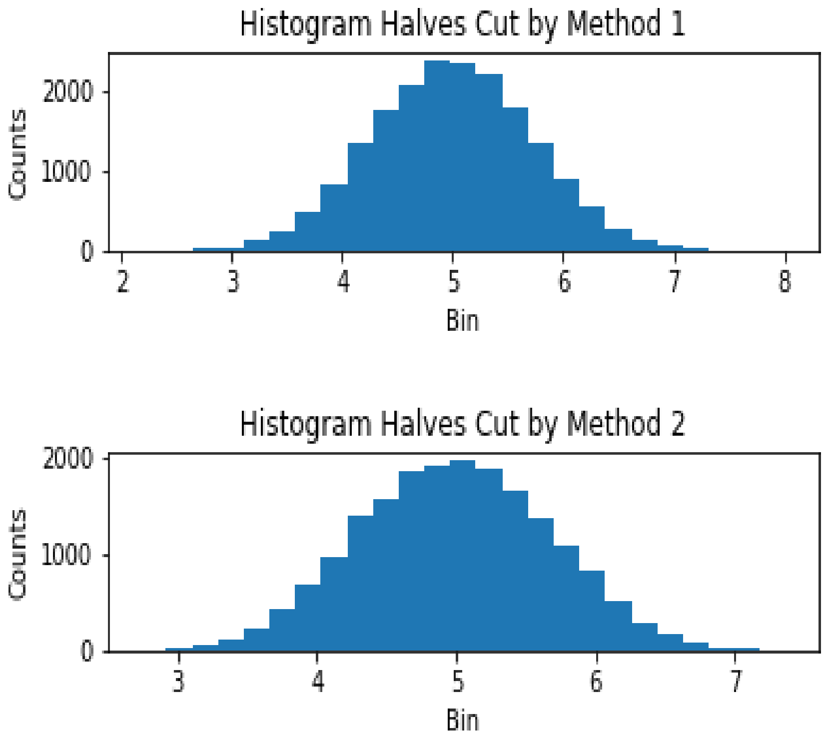


Figure 4: Histogram of Cut Rods with Operator's Handling

The yield for method 1 is 50% while for method 2 is 52% respectively.