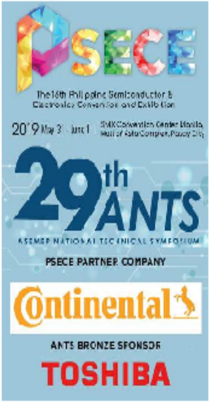

An Introduction to Process Variability Simulation

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1 Abstract

Process of Cutting a number of 10-meter Rods

- Material
- Machine
- Method
- Man

Keywords: variability, Normal Distribution, Gausian, Man, Machine, Method, Material, 4Ms.

2 Introduction: Gaussian Distribution Function

$$f(x) = \frac{\sqrt{2}e^{-\frac{(\mu-x)^2}{2\sigma^2}}}{2\sqrt{\pi}\sigma} \tag{1}$$

where :

μ : Real number or a list representing the mean or the mean vector

σ : Real number or a positive definite square matrix,

$\sigma^2 > 0$ the variance

Returns: A Random Symbol.

3 Introduction: Gaussian Distribution Sympy Function

The sympy function

$$f_1 = NormalDistribution(\mu, \sigma) \tag{2}$$

declares f_1 as the normal distribution function. The random variable of z of it is expressed as follows.

$$f_1(z) = \frac{\sqrt{2} e^{-\frac{(\mu - z)^2}{2\sigma^2}}}{2\sqrt{\pi}\sigma} \tag{3}$$

Integratinng (3), we have the following.

$$f_2(z) = \int_{z_i}^{z_f} f_1(z)\, dz = \frac{1}{2} \operatorname{erf}\left(\frac{\sqrt{2}}{2\sigma}(-\mu + z_f)\right) - \frac{1}{2} \operatorname{erf}\left(\frac{\sqrt{2}}{2\sigma}(-\mu + z_i)\right) \tag{4}$$

4 Introduction: Gaussian Distribution Normalized Function

Let $\mu = 0$ and $\sigma = 1$ then f_2 from $z_i = -\infty$ to $z_f = \infty$ we have normal probability distribution as follows.

$$f_2(z) = \int_{-\infty}^{\infty} f_1(z)\,dz = 1 \tag{5}$$

Probabilities of Multiple of σ

$$\int_{-1}^{1} f_1(z)\,dz = \operatorname{erf}\left(\frac{\sqrt{2}}{2}\right) = 0.682689492137086 \tag{6}$$

$$\int_{-3}^{3} f_1(z)\,dz = \operatorname{erf}\left(\frac{3\sqrt{2}}{2}\right) = 0.99730020393674 \tag{7}$$

$$\int_{-6}^{6} f_1(z)\,dz = \operatorname{erf}\left(3\sqrt{2}\right) = 0.999999998026825 \tag{8}$$

5 Introduction: DPM

$$DPM = 317310.507862914 \tag{9}$$

$$DPM = 2699.79606326021 \tag{10}$$

$$DPM = 0.00197317528982666 \tag{11}$$

6 Simulation of Variables

- The variablilities are quantified in terms of variance.
- The variance are associated with tolerances and/or errors
- Let 10000 be the number of instances.

7 Materials

A set of values of *z* can be generated randomly by the sympy function

$Normal(\mu, \sigma)$. Given $\mu = 10$ and $\sigma = 1$ then for 10000 pieces we have

$$Rods = random.normal(nrodmu, nrodsigma, count) = [9.325653, 7.571058 \dots 9.864124]. \tag{12}$$

Question: Given the specification of ± 1, how many are good and what is the yield?

Answer: The number of good rods $= 6800$, yield $= 68.0\%$.

8 Material Yield

The histogram of (15) is shown in Figure 1.

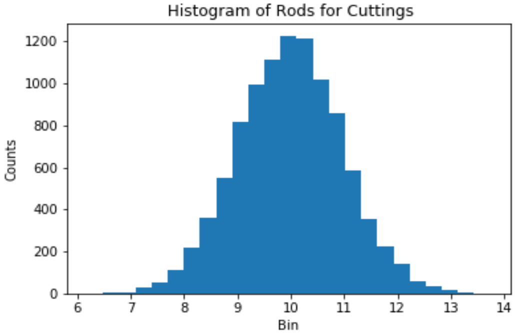

Figure: 1. Histogram Rods Variation

9 Equipment

$$f_1 = NormalDistribution(0.5, 0.1) \tag{13}$$

$$f_1(z) = \frac{5.0\sqrt{2}}{\sqrt{\pi}} e^{-50.0(z-0.5)^2} \tag{14}$$

$$f_2(z) = \int_{z_i}^{z_f} f_1(z)\, dz \tag{15}$$

$$Cut_{event} = random.normal(0.5, 0.1, 6800) = [0.39899838043193736, 0.6053907759591359, \ldots 0.5438583512895652] \tag{16}$$

10 Equipment Variable Histogram

The histogram of (19) is shown in Figure 2.

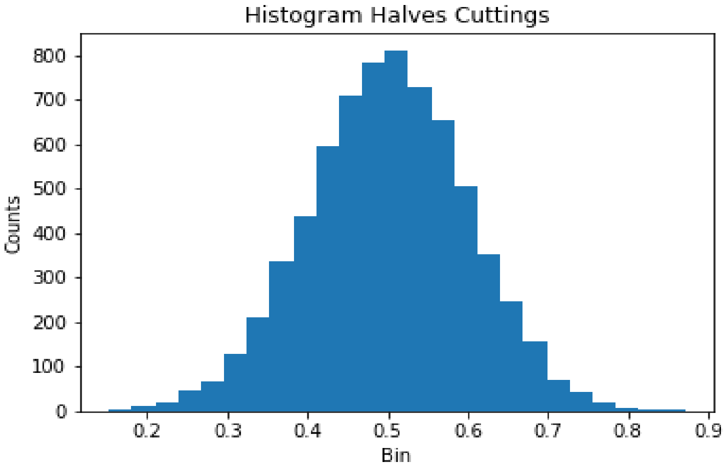

Figure: 2. Histogram of Cutter Variation

11 Method

Method 1

From an end point the cutter is placed 5m away. The rod is placed against that end point.

Method 2

The cutter is located at the center of two end points. The rod is placed between the end ponts. The end points are moved to hold the rod at the center of the cutter.

Question: Determine the formula for each and justify it.

12 Method Variable

Answers:

1. The cutter with variation, i.e., $0.5 \pm .1$ is multiplied by the perfect dimension of rod that is 10. Why perfect 10? Then the answer is substracted from a rod with variation. The two answers are the cut products.

2. The rod with variation, i.e., 10 ± 1 is multiplied by the cutter with variation, i.e., $0.5 \pm .1$. The answer is substracted from the rod with variation. The two cuts are products.

13 Method Histogram

The histograms of Method 1 and 2 are shown in Figure 3.

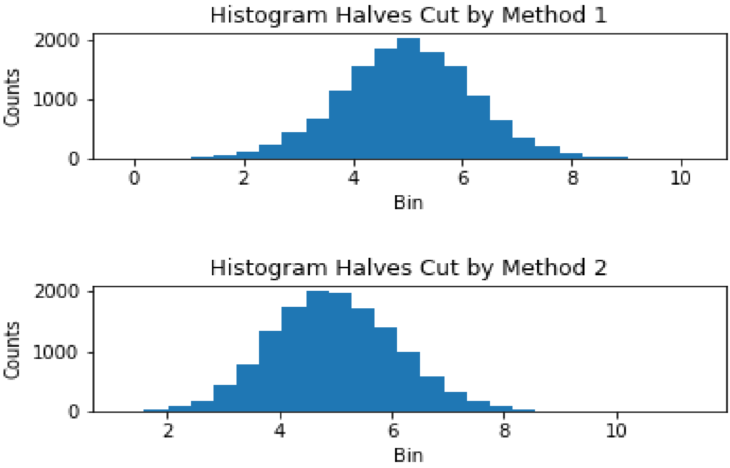

Figure: 3. Histogram of Cut Rods

14 Method Yield

Question:

Assuming that the cut specification is $5 \pm .5$, determine the number of good cut and the yield for each method

Answer:

1. The number of good cut for method 1 is 4427 while for method 2 4713 respectively.

2. The yield for method 1 is 32.5515% while for method 2 is 34.6544% respectively. Why is it method 2 has higher yield?

15 Operator Handling

Assume and operator makes the loading with error of $\mu = 0.4$ and $\sigma = 0.1$.

The histograms with operator handling for Method 1 and 2 are shown in Figure 4.

The yield with operator's handling errors for method 1 is 30% while for method 2 is 32% respectively.

16 Operator Handling Impact

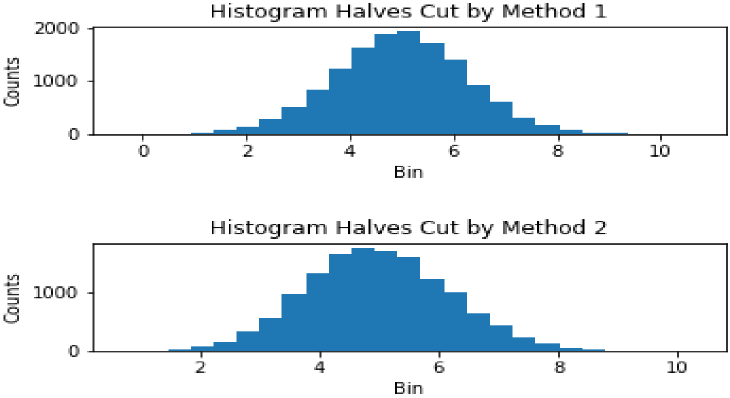

Figure: 4. Histogram of Cut Rods with Operator's Handling

17 Discussion on Sources of Variation

- Material mitigated by tighter tolerance with increased cost
- Machine mitigated by maintenance and continuous improvements
- Method generally difficult to observe and isolate and modelled,
- Man mitigated by error proofing or Poka Yoke.

18 Random Variable Models

- Consider a unit circle where an equilateral triangle is inscribed as shown in Figure 5.

- Suppose a random line is to be drawn across it, determine the probability that the cord is less than the length of the the side of equilateral triangle.

19 Random Lines Across a Circle

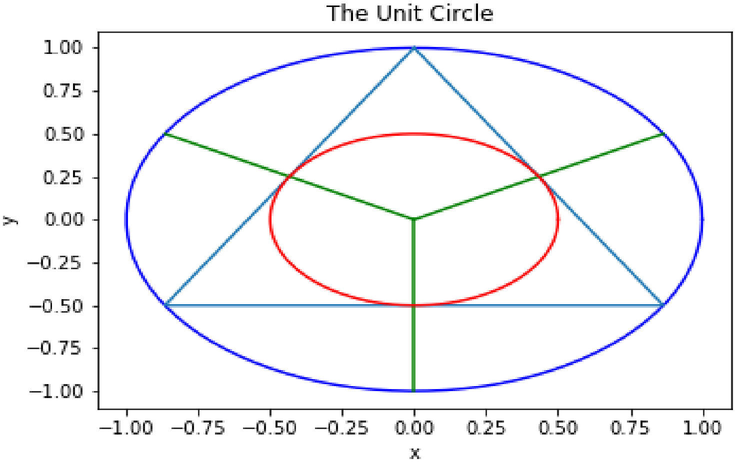

Figure: 5. Equilateral Triangle Inscribe in Unit Circle

20 Matter of Method

There are a number of answer depending on how the random lines are drawn.
1. The line drawn from any two points at the circumference, gives the probability of $\frac{1}{3}$.
2. The line drawn perpendicular to radius from a any point along radius gives the probability of $\frac{1}{2}$
3. The line drawn tangent to any concentric circle gives the probabiity $\frac{1}{4}$.
4. From the length of the cord i.e. 0 to 2, the probability is $\frac{\sqrt{3}}{2}$.

The question is that which of the four probability numbers make sense? In all these methods, does rotation of line matters?

21 Matter of Random Variables

•The random variable in matter of two points at the circumference is the arc of the circle.

•The random variable for line drawn perpendiculare to the radius is the distance of the point where line intersects with radius from the center of the circule.

•The random variable for the line tangent to the concentric circle is the area of the concentric circle.

•The random variable for the length of the cord is the cord itself.
-All the lines segmented by perimeter of the circle are cords. Why is it that are the probabilities of the lines above are different from the last?

22 Sample Problems

Given a set of wirebonders bonding a particular producted, must all the set up be identical exactly?

Generally no however the ranges of parameters must be identical. It means each machine is different and the parameters are used to nullify the differences.

The question is that what are the variables to be considered random? These variables are not readily observable and measurable. These manifest at region of ignorance where it is bounded by upper and lower control limits of SPC charts

23 Conclusion

There are a number of innovation opportunities that may be derived.

• Make the intermittent or not readily observable phenomenon consistently observable

• What is made consistently observable may not be readily measurable. Therefore make it readily measurable.

• The region of ignorance in SPC charts provides the leading clues for such phenomenon.

24 End of Presentation

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