Tema 5.

Fractali.

Termenul *fractal* a fost generalizat pentru a include obiecte din afara definitiei originale a lui Mandelbrot. Prin *obiect fractal* vom intelege orice
obiect care are proprietatea de autoasemanare (self-similarity in lb. engleza).
Obiectele obtinute in cele ce urmeaza sunt aproximatii ale unui obiect fractal
ideal, fiind obtinute intr-un numar finit de iteratii.

1. Multimea Julia-Fatou : se obtine utilizand un proces iterativ.

   Plecand de la $z_0 \in \mathbf{C}$ se obtin numerele complexe $(z_n)_{n\geq 0}$ astfel :

   $z_{n+1} = z_n^2 + c$, unde $c \in \mathbf{C}$. Un numar complex $x \in \mathbf{C}$ apartine multimii

   Julia-Fatou $\mathbf{J_c}$ daca, plecand cu $z_0 = x$, urmatoarele conditii nu sunt

   indeplinite : $(\exists z \in \mathbf{C})(\lim_{n\to\infty} z_n = z)$ sau $\lim_{n\to\infty} |z_n| = \infty$.
   In programul urmator s-au generat 2 (aproximari ale) multimi Julia-Fatou

   corespunzatoare valorilor c_1 si c_2 pentru $c \in \mathbf{C}$ indicate in figurile de mai jos.
   Cele 2 conditii de mai sus au fost utilizate in program sub forma

   $(\exists n_0 > 0)(z_{n_0} = z_{n_0+1})$ si $(\exists n_0 \geq 0)(\exists M > 0)(|z_{n_0}| > M)$ i.e., intr-un numar finit de
   iteratii n_0, se testeaza daca sirul (z_n) devine constant sau $|z_n|$ depaseste
   M (ales suficient de mare).
   Daca dupa terminarea celor n_0 iteratii nici o conditie nu a fost adevarata atunci
   punctul respectiv apartine multimii Julia-Fatou si a fost colorat cu rosu in figura.

2. Multimea Mandelbrot : se obtine tot printr-un proces iterativ.

   Un numar $c \in \mathbf{C}$ apartine multimii Mandelbrot **M** daca $\lim_{n\to\infty}|z_n| \neq \infty$, unde

   sirul $(z_n)_{n\geq 0}$ este obtinut astfel : $z_0 = 0 + 0i$ iar $z_{n+1} = z_n^2 + c\,, \quad \forall n \geq 0$.

   a. Construiti multimea Mandelbrot bazandu-va pe urmatoarea proprietate a
      acesteia (mai bine zis pe negatia ei): daca numarul complex *c* apartine multimii

      Mandelbrot atunci $|z_n| \leq 2\,, \quad \forall n \geq 0$. In concluzie procesul iterativ se opreste
      daca $|z_n|$ depaseste 2.

   b. Realizati o clasificare a punctelor care nu apartin multimii Mandelbrot,
      colorandu-le cu culori diferite in functie de numarul de iteratii care a
      fost necesar pentru a detecta neapartenenta.

3. In programul urmator sunt generati, recursiv, fractali construiti prin geometria
   „turtle” (in acest stil de grafica imaginile sunt obtinute prin deplasarea unui
   cursor pe ecran, acesta deplasandu-se conform unor comenzi : desenare, rotatie
   catre stanga sau dreapta cu un anumit unghi):

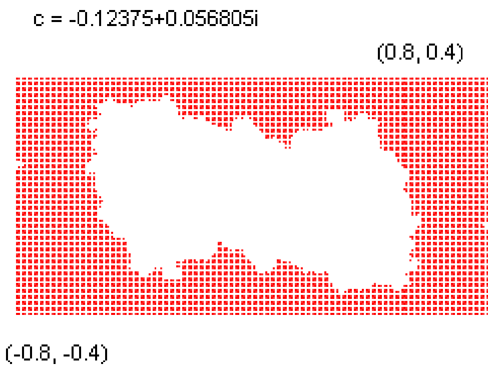
   a. curba lui Koch (fulg de zapada),

   b. arbori binari,

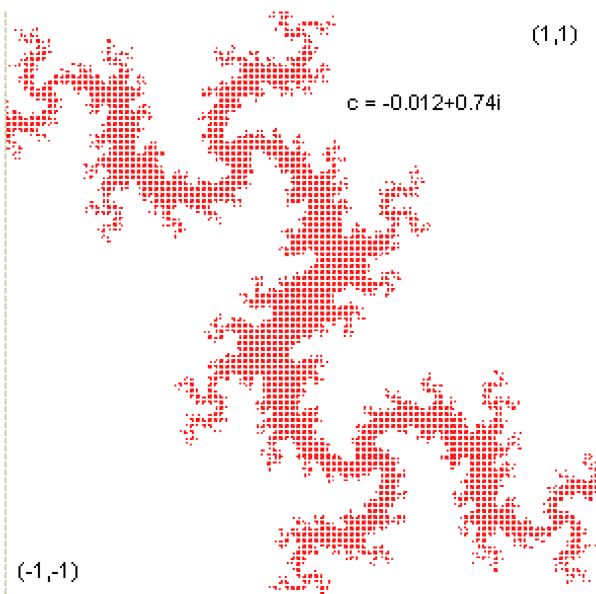
   c. arborele lui Perron,

   d. curba lui Hilbert.

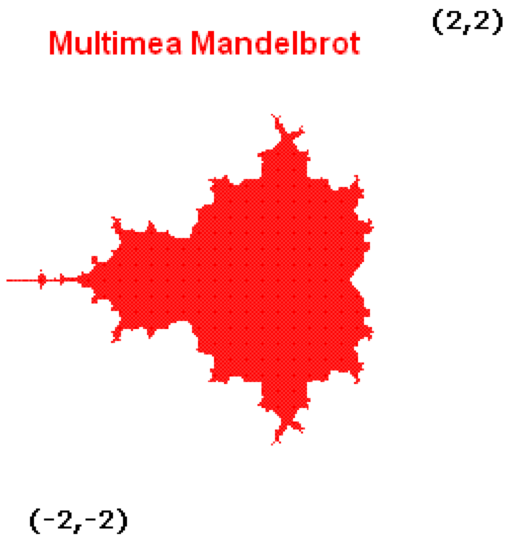
4. Desenati imaginile urmatoare (utilizand geometria „turtle”): 1, 2, 3.

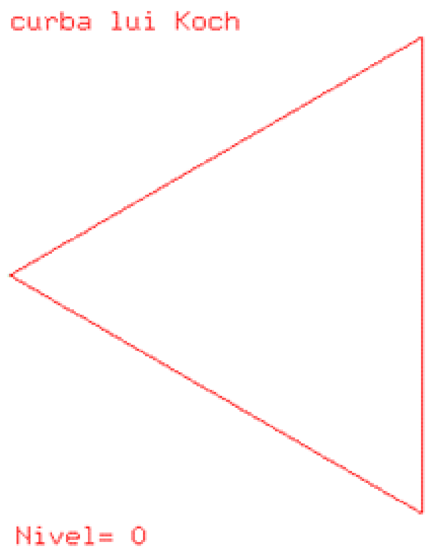


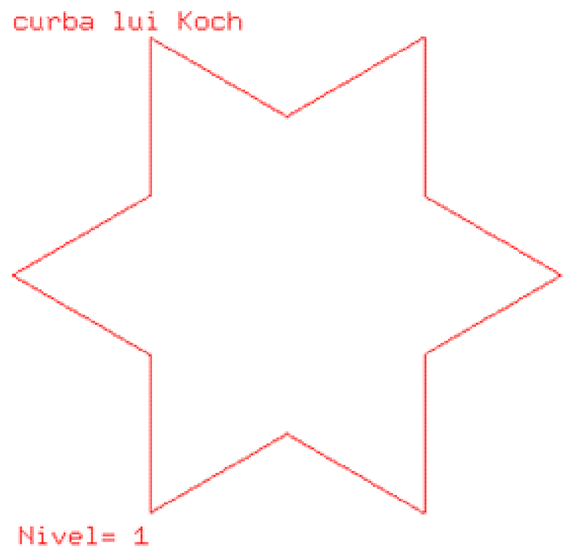
Multime Julia-Fatou

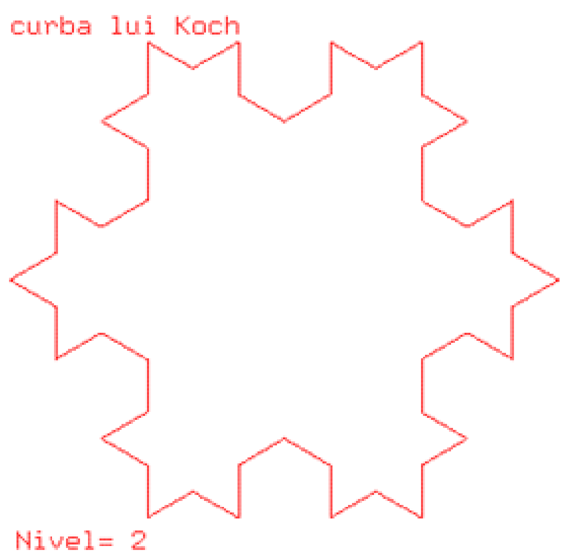


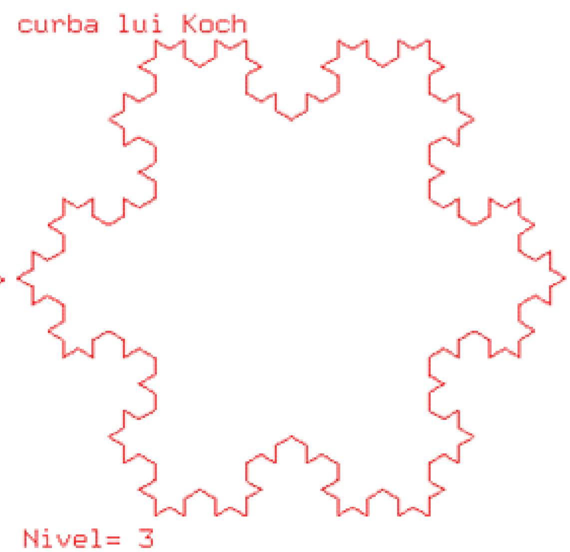
Multime Julia-Fatou

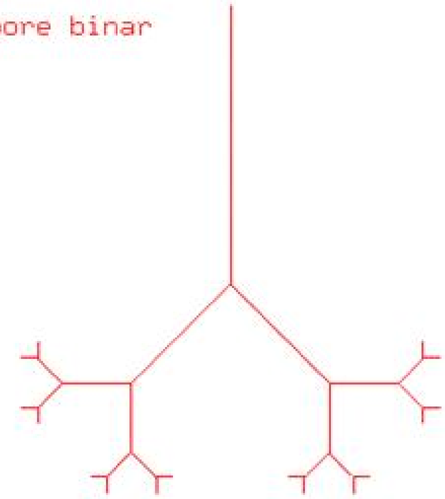

Multimea Mandelbrot
(2,2)
(-2,-2)

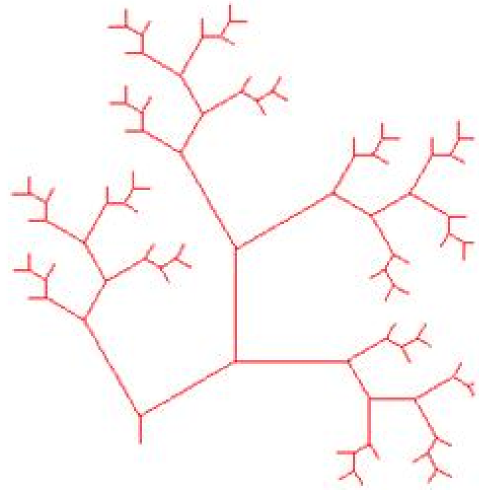

curba lui Koch
Nivel= 0

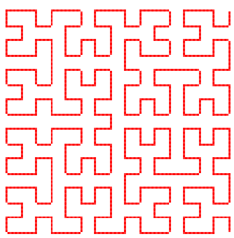

curba lui Koch
Nivel= 1


curba lui Koch
Nivel= 2

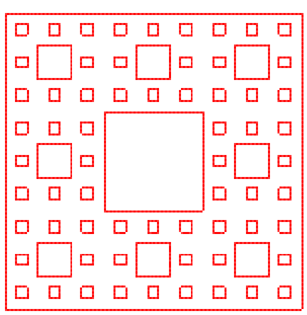

curba lui Koch
Nivel= 3


arbore binar
Nivel= 4

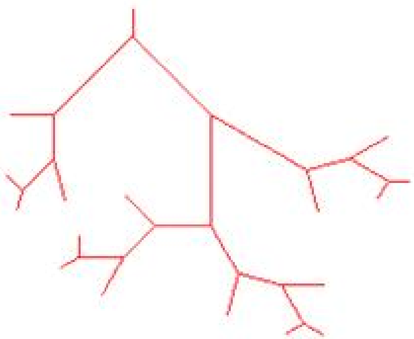

arbore Perron
Nivel= 3


curba Hilbert
Nivel= 4

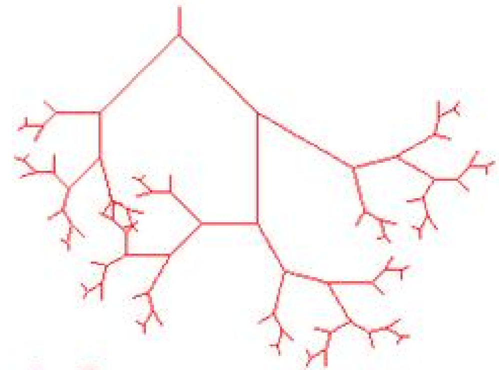
Imaginea 1


Nivel= 2

Imaginea 2

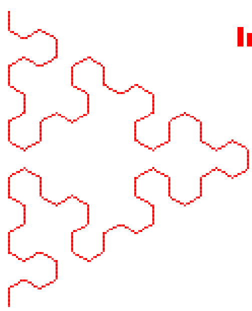


Nivel= 2

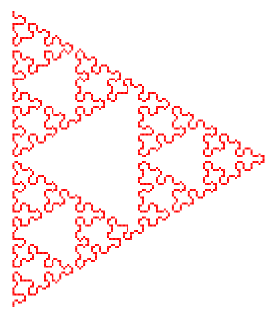


Nivel= 3

Imaginea 3



Nivel= 4



Nivel= 6

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