2. July 1924: A. Einstein translated a paper of S.N. Bose which contained a new derivation of Planck's radiation law based on a statistical treatment of light quanta

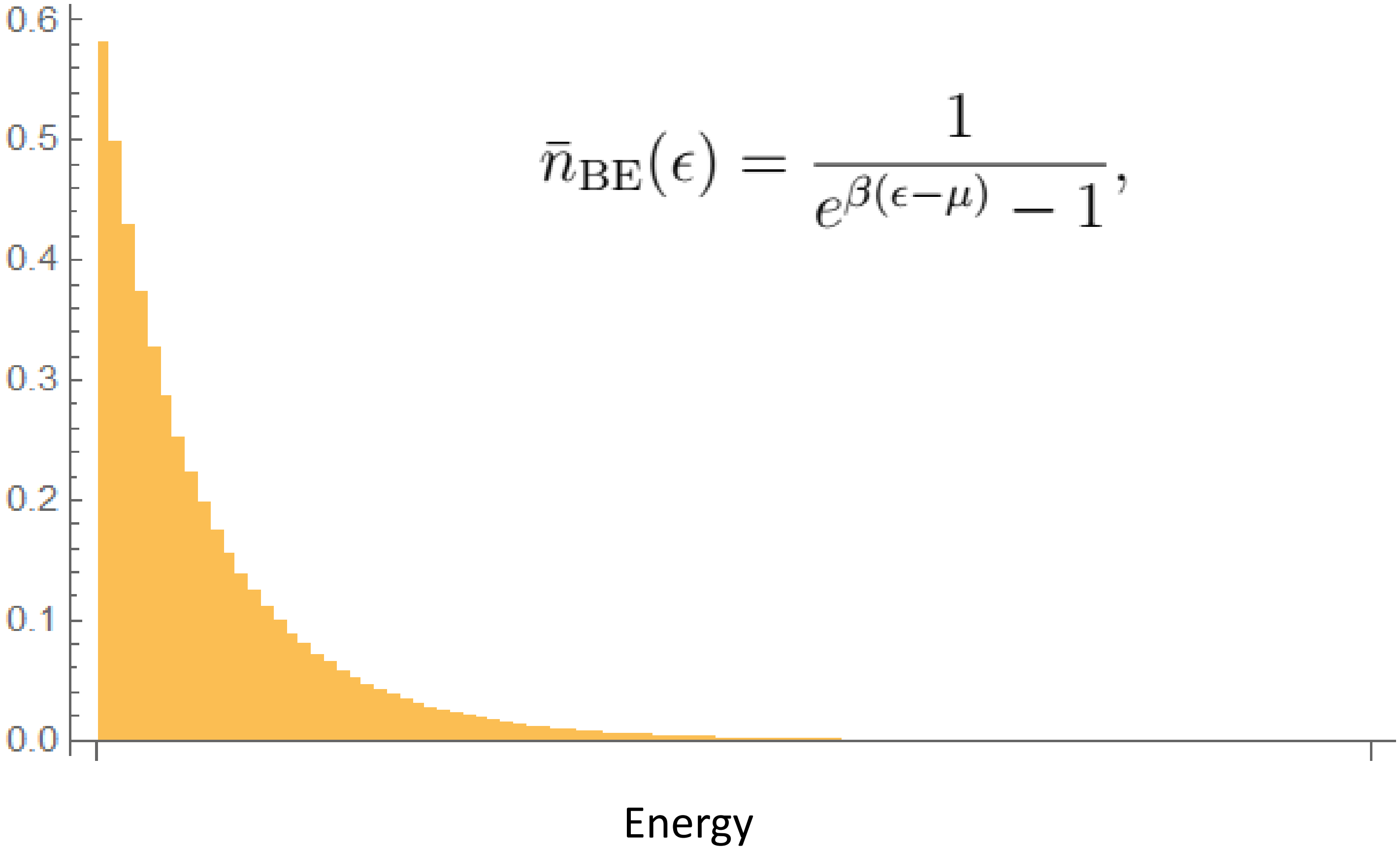
10. July 1924 / 8. Jan 1925: Einstein presented a similar treatment for an ideal gas of indistinguishable particles at the *preussische Akademie der Wissenschaften*. He predicted a new condensation phenomenon.

1938: Pyotr L. Kapitsa (Nobel Prize 1978) discovered the superfluidity of ^{4}He... the first experimental fingerprint of Bose-Einstein condensation in a dense system

5. June 1995: the advent of BEC in trapped ultracold dilute atomic gases...

^{87}Rb	5. June 1995	JILA (E. Cornell et al.)
^{7}Li	July 1995	Rice Univ. (R. Hulet et al.)
^{23}Na	Sept 1995	MIT (W. Ketterle et al.)
^{1}H	24. June 1998	MIT (D. Kleppner et al.)
^{4}He*	12. Feb 2001	ENS (A. Aspect et al.)

$\sim$ 150 groups world-wide are working on BEC in cold atomic gases...



$$\bar{n}_{\mathrm{BE}}(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)} - 1},$$

$$N = \frac{1}{e^{(\epsilon_0-\mu)/k_B T} - 1} + \int \frac{g(\epsilon)}{e^{(\epsilon_i-\mu)/k_B T} - 1} d\epsilon$$

$$N_{\mathrm{ex}} = \int_0^\infty d\epsilon g(\epsilon) f^0(\epsilon).$$

$$N = N_{\mathrm{ex}}(T_{\mathrm{c}}, \mu = 0) = \int_0^\infty d\epsilon g(\epsilon) \frac{1}{e^{\epsilon/kT_{\mathrm{c}}} - 1}.$$

$$N = C_\alpha (kT_{\mathrm{c}})^\alpha \int_0^\infty dx \frac{x^{\alpha-1}}{e^x - 1} = C_\alpha \Gamma(\alpha)\zeta(\alpha)(kT_{\mathrm{c}})^\alpha,$$

$$\int_0^\infty dx \frac{x^{\alpha-1}}{e^x - 1} = \Gamma(\alpha)\zeta(\alpha).$$

$\zeta(\alpha) = \sum_{n=1}^\infty n^{-\alpha}$

Riemann zeta function

$\int_0^\infty dx x^{\alpha-1} e^{-x} = \Gamma(\alpha)$

$$kT_{\mathrm{c}} = \frac{N^{1/\alpha}}{[C_\alpha \Gamma(\alpha)\zeta(\alpha)]^{1/\alpha}}.$$

BEC in a 3D harmonic potential trap

$$V(\mathbf{r}) = \frac{1}{2}(K_1 x^2 + K_2 y^2 + K_3 z^2),$$

$$\omega_i^2 = K_i/m,$$

$$V(\mathbf{r}) = \frac{1}{2}m(\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2).$$

$$\epsilon(n_1, n_2, n_3) = (n_1 + \frac{1}{2})\hbar\omega_1 + (n_2 + \frac{1}{2})\hbar\omega_2 + (n_3 + \frac{1}{2})\hbar\omega_3,$$

$$\epsilon_i = \hbar\omega_i n_i, \qquad \epsilon = \epsilon_1 + \epsilon_2 + \epsilon_3$$

Density of state

$$G(\epsilon) = \frac{1}{\hbar^3\omega_1\omega_2\omega_3}\int_0^{\epsilon} d\epsilon_1 \int_0^{\epsilon-\epsilon_1} d\epsilon_2 \int_0^{\epsilon-\epsilon_1-\epsilon_2} d\epsilon_3 = \frac{\epsilon^3}{6\hbar^3\omega_1\omega_2\omega_3}.$$

$$g(\epsilon) = dG/d\epsilon$$

$$g(\epsilon) = \frac{\epsilon^2}{2\hbar^3\omega_1\omega_2\omega_3}.$$

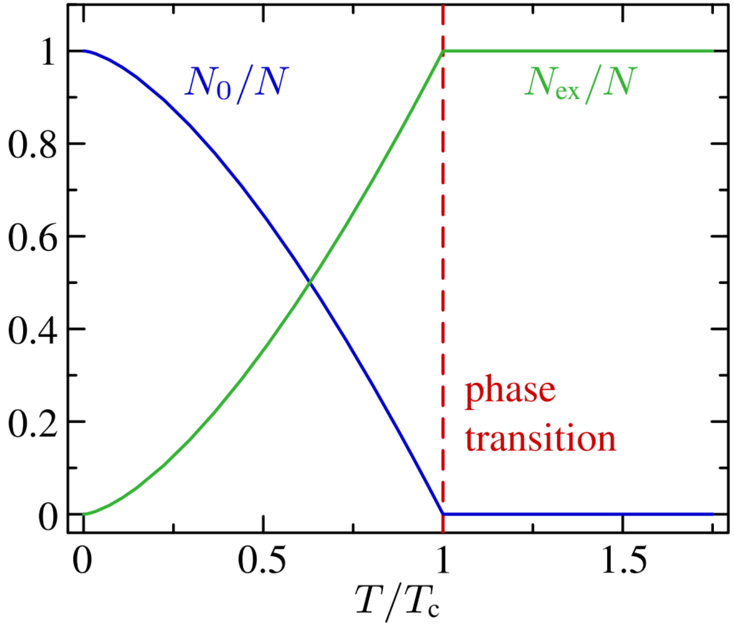
$\alpha = 3,$ 3d harmonic pontential

$$kT_c = \frac{\hbar\bar{\omega}N^{1/3}}{[\zeta(3)]^{1/3}} \approx 0.94\hbar\bar{\omega}N^{1/3}, \qquad \bar{\omega} = (\omega_1\omega_2\omega_3)^{1/3}$$

$$N_{\rm ex}(T) = C_\alpha \int_0^\infty d\epsilon \epsilon^{\alpha-1} \frac{1}{e^{\epsilon/kT} - 1} = C_\alpha \Gamma(\alpha)\zeta(\alpha)(kT)^\alpha.$$

$$N_{\rm ex} = N \left(\frac{T}{T_{\rm c}}\right)^\alpha$$

$$N_0(T) = N - N_{\rm ex}(T) \;\; = N\left[1 - \left(\frac{T}{T_{\rm c}}\right)^\alpha\right]$$



Density profile and velocity distribution

$$n(\mathbf{r}) = N|\phi_0(\mathbf{r})|^2$$

$$\phi_0(\mathbf{r}) = \frac{1}{\pi^{3/4}(a_1 a_2 a_3)^{1/2}} e^{-x^2/2a_1^2} e^{-y^2/2a_2^2} e^{-z^2/2a_3^2}$$

Momentum space, Fourier transform

$$a_i^2 = \frac{\hbar}{m\omega_i}$$

$$\phi_0(\mathbf{p}) = \frac{1}{\pi^{3/4}(c_1 c_2 c_3)^{1/2}} e^{-p_x^2/2c_1^2} e^{-p_y^2/2c_2^2} e^{-p_z^2/2c_3^2}$$

$$c_i = \frac{\hbar}{a_i} = \sqrt{m\hbar\omega_i}$$

$$n(\mathbf{p}) = N|\phi_0(\mathbf{p})|^2 = \frac{N}{\pi^{3/2} c_1 c_2 c_3} e^{-p_x^2/c_1^2} e^{-p_y^2/c_2^2} e^{-p_z^2/c_3^2}$$

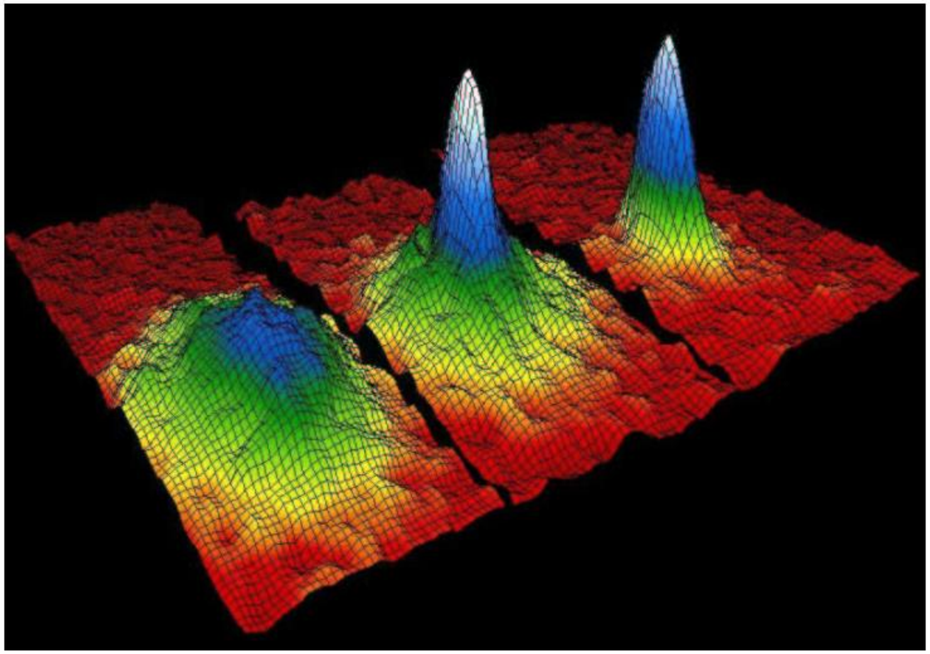
$$c_i = \frac{\hbar}{a_i} = \sqrt{m\hbar\omega_i} \Rightarrow T_i = \hbar\omega_i/2k$$

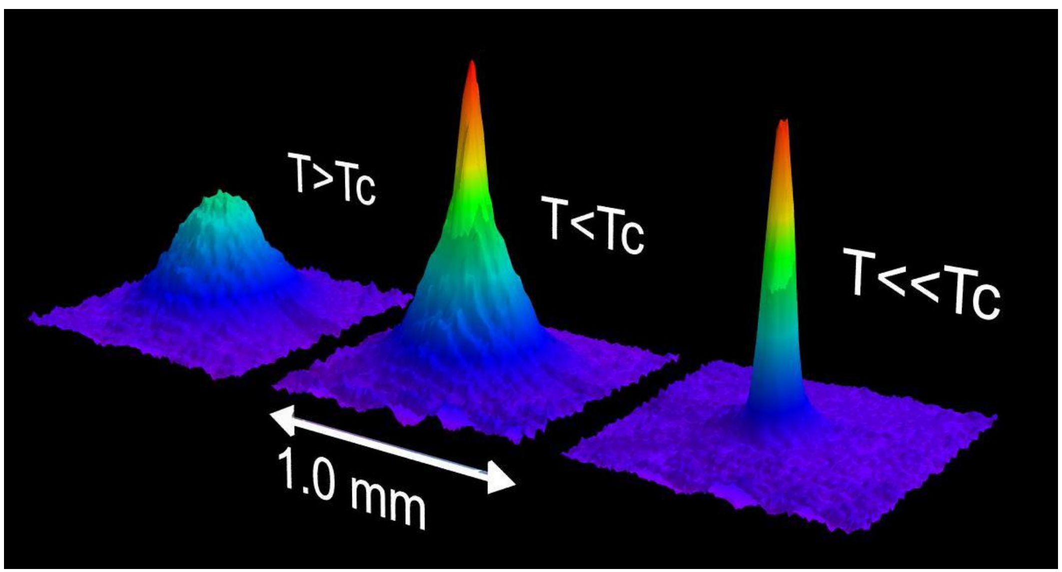
Anisotropy!

$$n(\mathbf{p}) = Ce^{-p^2/2mkT}$$

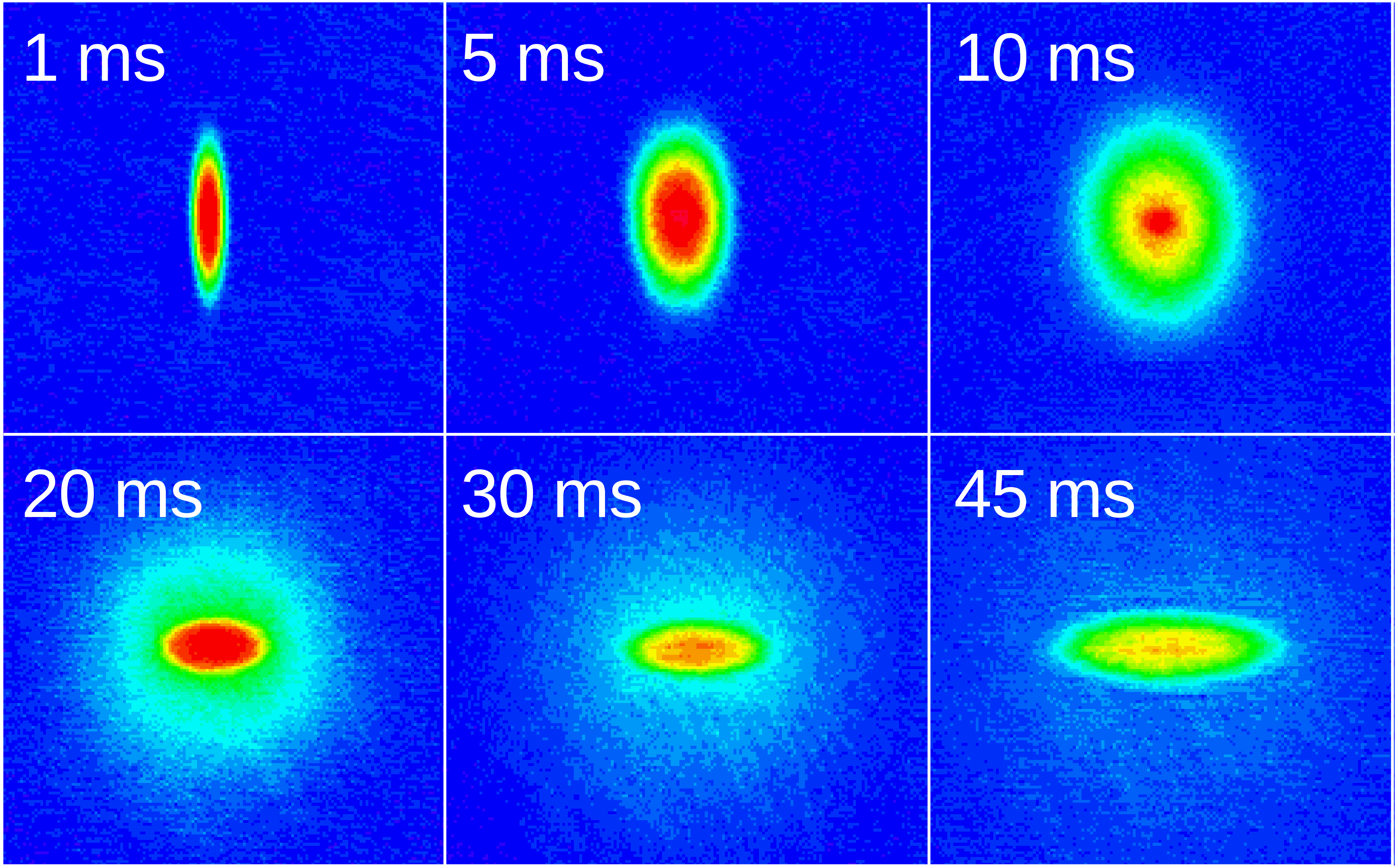
Classical gas, isotropic

Signatures of BEC: Double gauss distribution



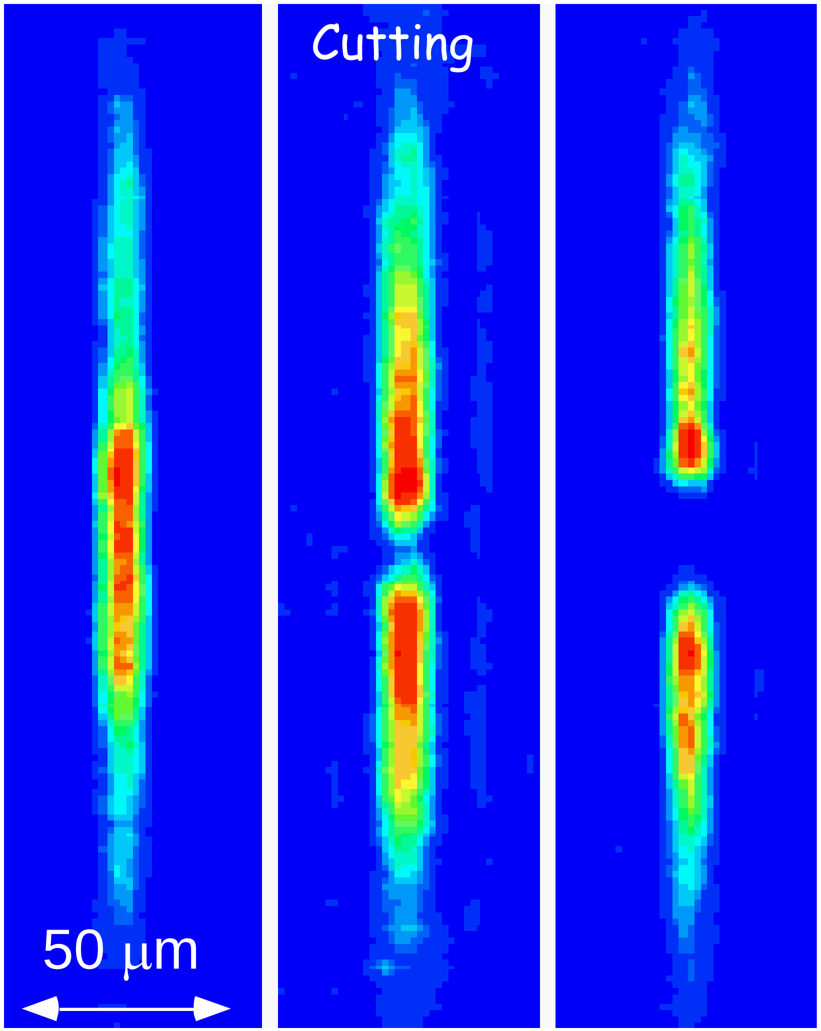


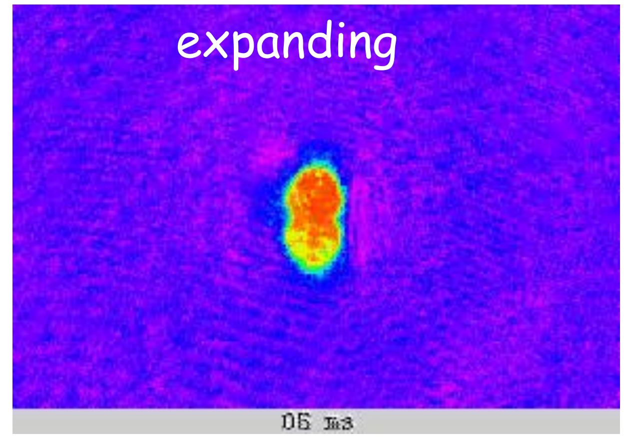
Signatures of BEC: Anisotropic expansion

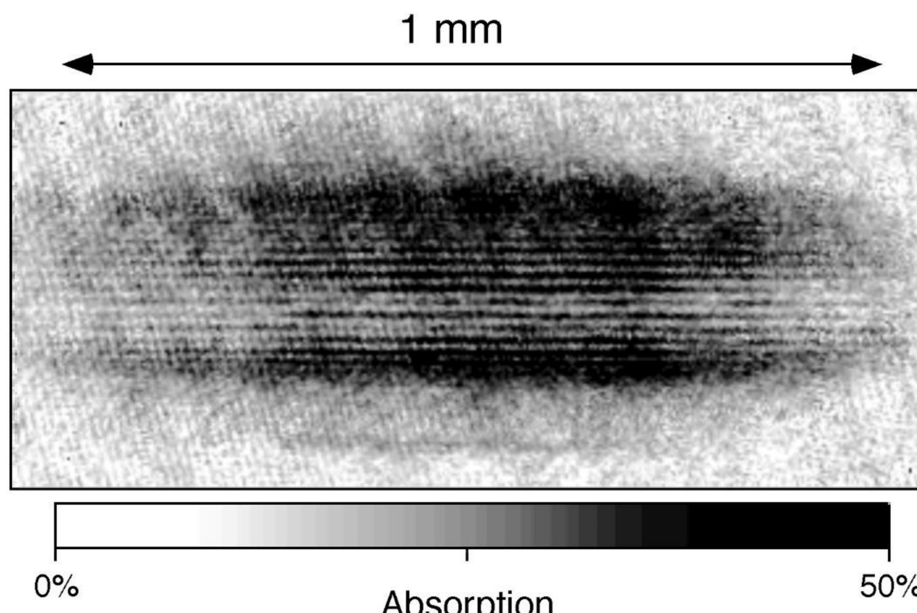


Interference of condensates

Andrews, et al, Science **275**, 589 (1997)



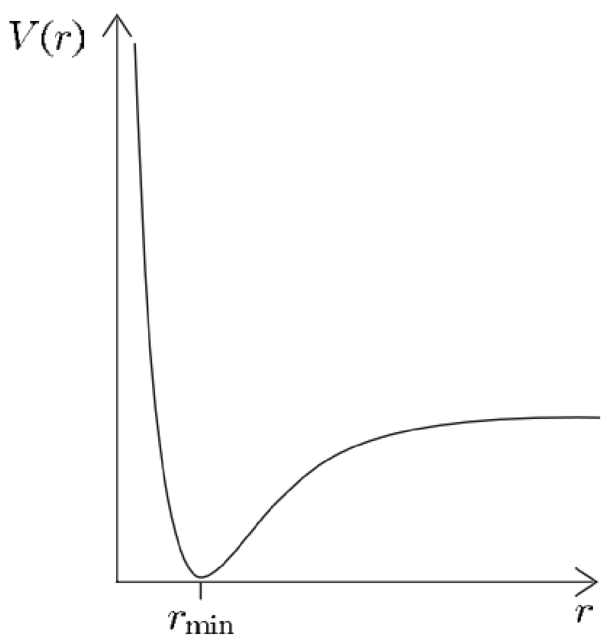




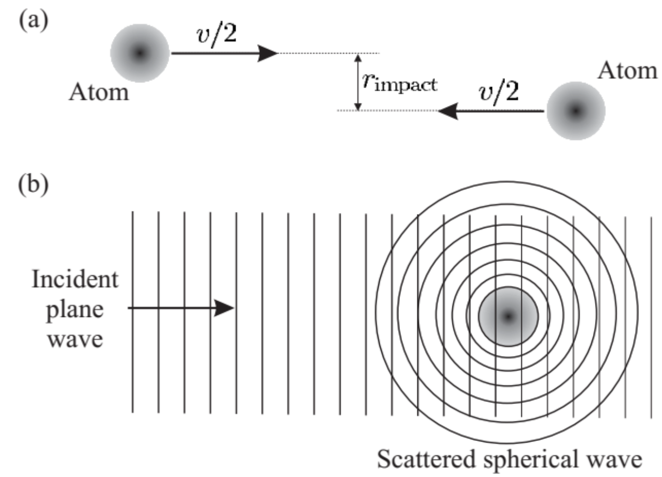
Interatomic potentials and the van der Waals interaction

$$U(r) = -\frac{C_6}{r^6} - \frac{C_8}{r^8} - \frac{C_{10}}{r^{10}} + \cdots .$$

$$U_{\rm ed} = \frac{1}{4\pi\epsilon_0 r^3}\left[\mathbf{d}_1\cdot\mathbf{d}_2 - 3(\mathbf{d}_1\cdot\hat{\mathbf{r}})(\mathbf{d}_2\cdot\hat{\mathbf{r}})\right],$$



$$\psi = e^{ikz} + \psi_{\mathrm{sc}}(\mathbf{r})$$



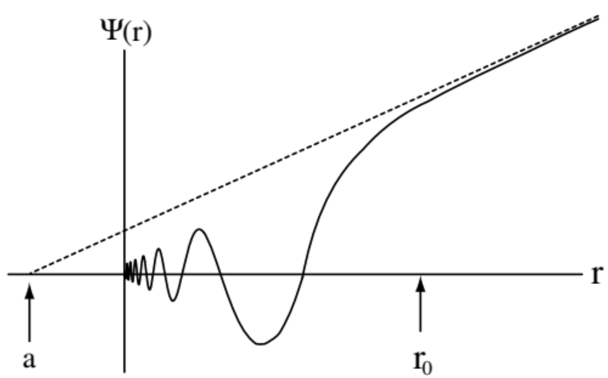
$$\psi = e^{ikz} + f(\theta)\frac{e^{ikr}}{r}.$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2.$$

Low energy scattering: s-wave (l=0)

$$\psi = 1 - \frac{a}{r}.$$

$\sigma = 4\pi a^2$ Classical



$$\hbar l \simeq M' v r_{\text{impact}}$$

相互作用距离尺度

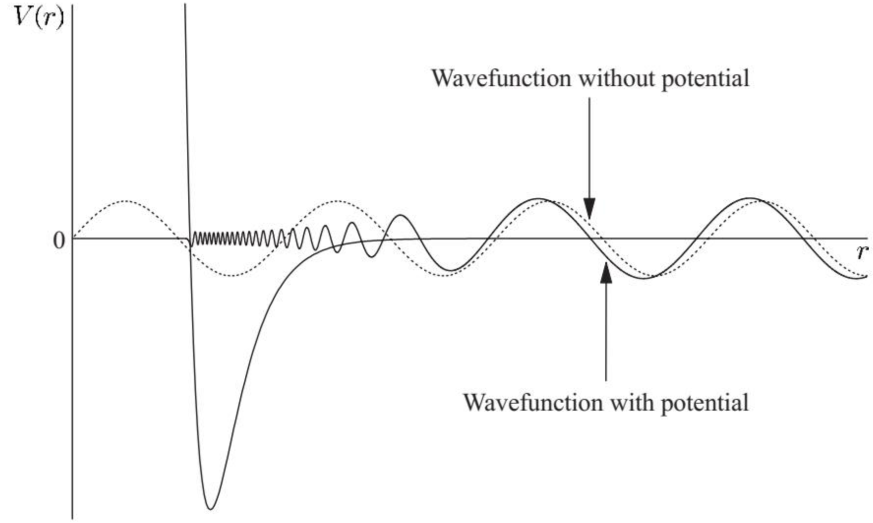
$$\hbar l \lesssim M' v r_{\text{int}} = h r_{\text{int}} / \lambda_{\text{dB}}$$

$$\frac{\lambda_{\text{dB}}}{2\pi} \gg r_{\text{int}} \Longrightarrow l = 0$$

$$\psi = e^{ikz} + f(\theta)\frac{e^{ikr}}{r}.$$

Legendre expansion

$$\psi = \sum_{l=0}^{\infty} A_l P_l(\cos\theta) R_{kl}(r).$$



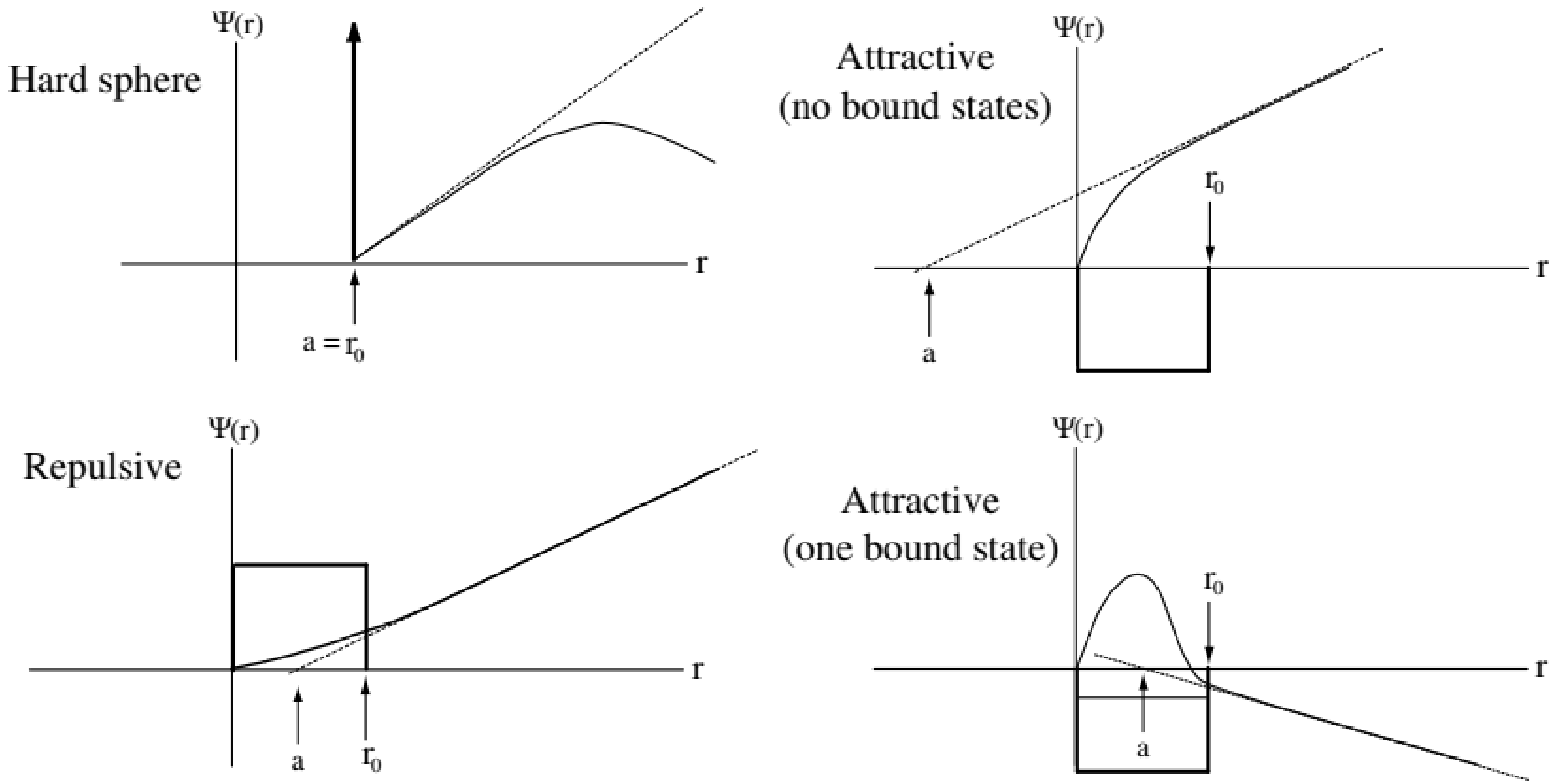
$$R''_{kl}(r) + \frac{2}{r}R'_{kl}(r) + \left[k^2 - \frac{l(l+1)}{r^2} - \frac{2m_{\mathrm{r}}}{\hbar^2}U(r)\right] R_{kl}(r) = 0,$$

$$r \longrightarrow \infty$$

$$R_{kl}(r) \simeq \frac{1}{kr}\sin(kr - \frac{\pi}{2}l + \delta_l).$$

$$\delta_0 = -ka, \quad k \rightarrow 0$$

Scattering Length



Bosons and Fermions

Symmetrize and Anti-symmetrize of the wavefunction

$$\mathbf{r} \rightarrow -\mathbf{r}, \text{ or } r \rightarrow r, \theta \rightarrow \pi - \theta \quad \varphi \rightarrow \pi + \varphi$$

$$\psi = e^{ikz} \pm e^{-ikz} + [f(\theta) \pm f(\pi - \theta)] \frac{e^{ikr}}{r}.$$

$$\frac{d\sigma}{d\Omega} = |f(\theta) \pm f(\pi - \theta)|^2,$$

S-wave cross section

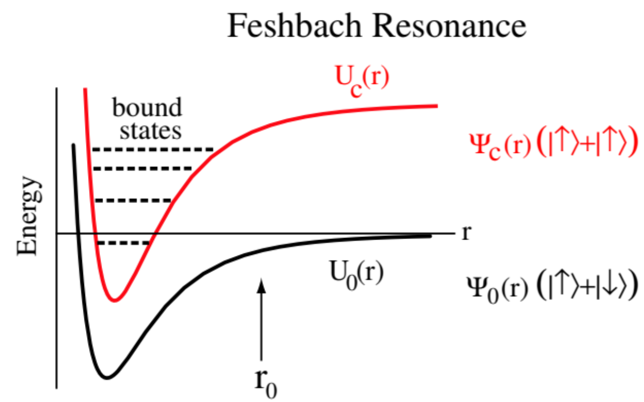
$\sigma = 8\pi a^2$ Boson

$\sigma = 0$ Fermion

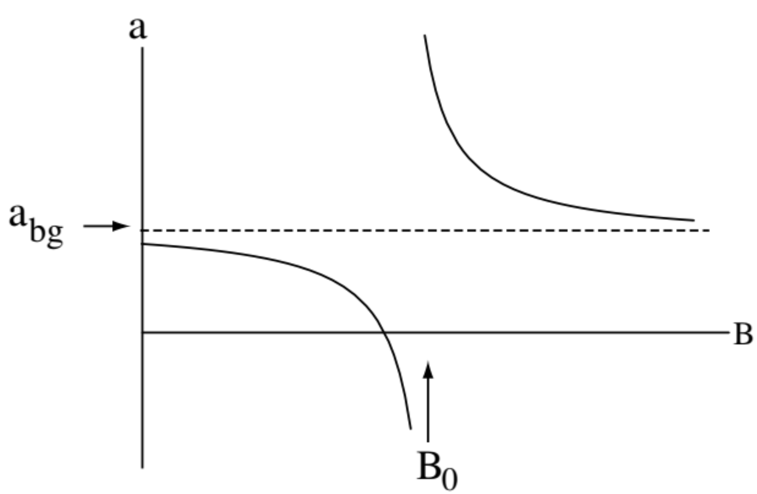
Cold and dilute, but strongly interacting

$$\begin{aligned} ka &\sim 1 \\ kr_0 &\ll 1, \end{aligned}$$

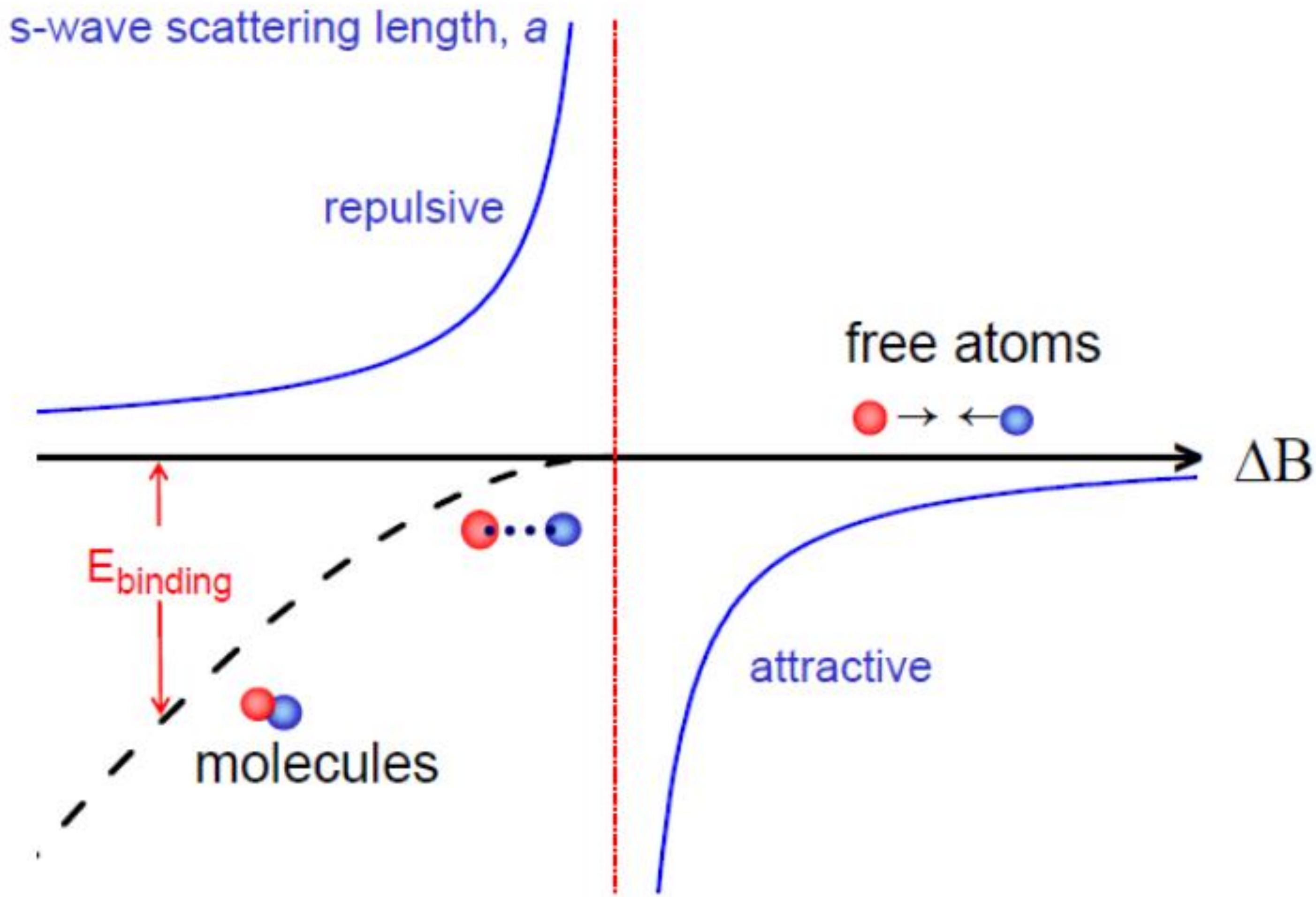
$$a \gg r_0.$$



$$\begin{aligned} a &= a_{bg}\left(1+\frac{\Delta}{E^*_{res}-E}\right) \\ &\approx a_{bg}\left(1+\frac{\Delta}{E^*_{res}}\right), \end{aligned}$$



Magnetic-field Feshbach resonance



$$\hat{H} = \sum_{i=1}^{N}\left(\frac{p_i^2}{2m} + V_{\text{trap}(r_i)}\right) + \sum_{\langle ij\rangle} V_{\text{int}}(r_i - r_j),$$

k r_0<<1, s-wave pseudopotential

$$V_{\text{int}}(r) \approx \underbrace{\frac{4\pi\hbar^2 a}{m}}_{U_0}\delta(r),$$

Second quantization

Bosons: $[\hat{b}_\zeta, \hat{b}^\dagger_{\zeta'}] = \delta_{\zeta,\zeta'}$

$$\hat{\psi}(\mathbf{r}) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\hat{b}_{\mathbf{k}},$$

$$[\hat{\psi}(\mathbf{r}), \hat{\psi}(\mathbf{r}')^\dagger] = \delta(\mathbf{r} - \mathbf{r}').$$

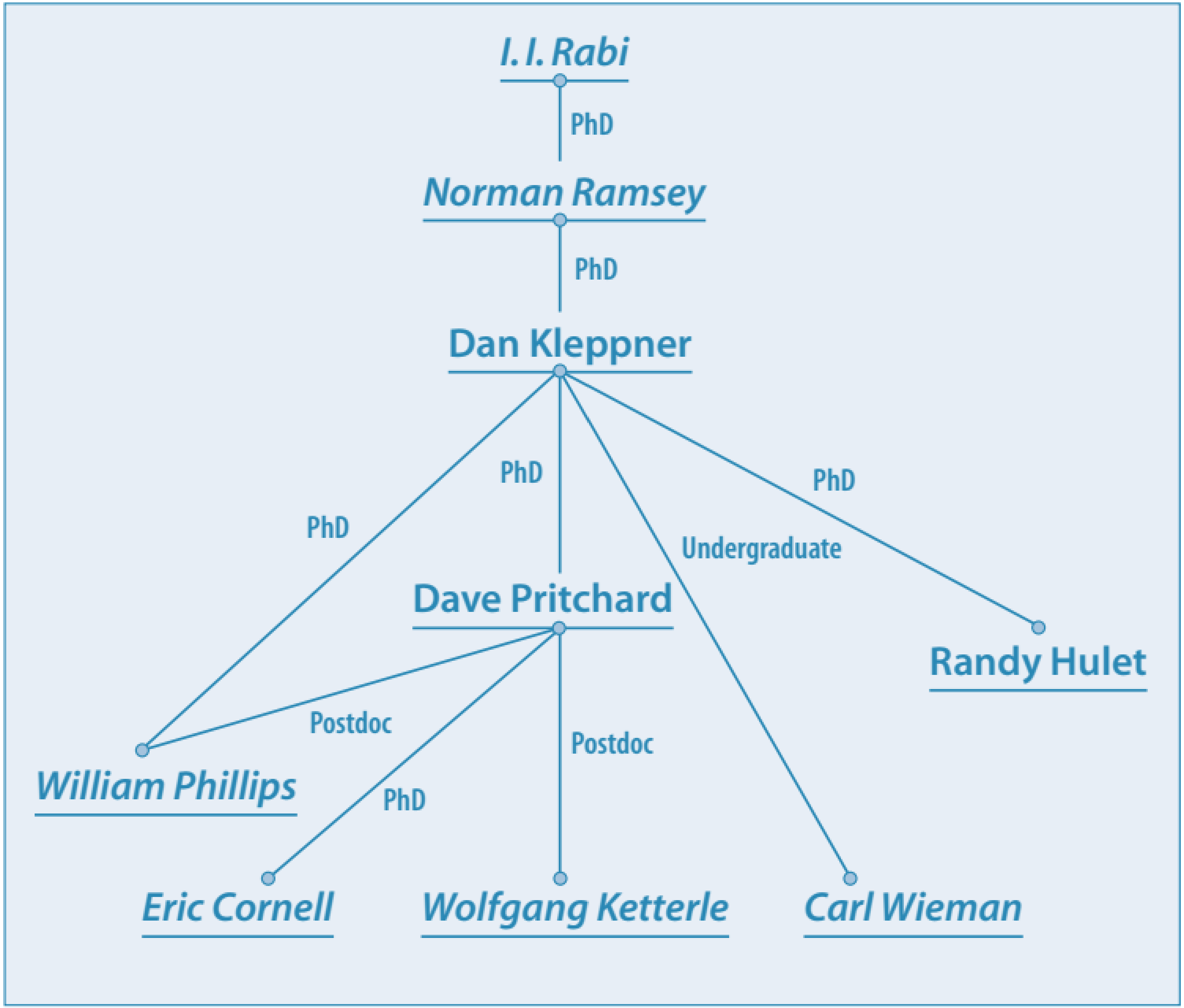
$$\hat{N} = \sum_\zeta \hat{b}^\dagger_\zeta \hat{b}_\zeta = \int d^3\mathbf{r}\hat{\psi}(\mathbf{r})^\dagger\hat{\psi}(\mathbf{r}).$$

annihilation and creation operator at position r

$$\hat{H} = \int d^3\mathbf{r}\ \hat{\psi}(\mathbf{r})^\dagger\left(\frac{\hat{p}^2}{2m} + V_{\text{trap}}(\mathbf{r}) + \frac{U_0}{2}\hat{\psi}(\mathbf{r})^\dagger\hat{\psi}(\mathbf{r})\right)\hat{\psi}(\mathbf{r}).$$

Gross-Pitaevski equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V_{\text{trap}}\psi + U_0|\psi|^2\psi.$$



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