Lecture summary
- Angular momentum basics
- Coupling of angular momenta
    - The hyperfine structure of the hydrogen groundlevel
- Clebsch-Gordan coefficients
- Wigner's 3j symbol
- Rotation d-matrix
- Representation of vector in the spherical coordinates

Homework

1. **Angular momentum coupling**. A system of two angular momenta $j_1\,m_1$ and $j_2\,m_2$ has a total number of $(2\,j_1 + 1)(2\,j_2 + 1)$ states. The two couple to form a combined system of angular momentum J, whose maximum value is $J_{max} = j_1 + j_2$. Find the minimum value for J based on the expectation that the total number of states in the coupled system should equal to that of the original uncoupled system.

2. The **3j symbols** satisfy the following exchange properties:

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = \begin{pmatrix} j_2 & j_3 & j_1 \\ m_2 & m_3 & m_1 \end{pmatrix}$$

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ -m_1 & -m_2 & -m_3 \end{pmatrix} = (-1)^{j_1+j_2+j_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

Express these two types of relations in terms of C-G coefficients.

3. In the table of **Clebsch-Gordan coefficients** (Figure 35.1), find each case of the C-G coefficient being equal to zero, and show that it should be zero based on symmetry properties.

Reading Assignments: n/a