Meteor master draft

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1 Free-electron plasma temperature

1 Let's have a simplified description of the relation between electron density and free-electron plasma temperature and intensity described by the solution of phenomenologic and particular model used to low-frequency detection of meteorites first. The input parameters are covered by the electron density n, its total differential gear $\mathrm{d}n$ and the atomic radius R.

$$\mathbf{F}(n) = e^{\frac{-2\left(\left(10|\pi i B \frac{1}{2+2R}|\right)^2 + 10|B \log\left(\frac{1}{2}\mathrm{d}n B \frac{1}{n(1-R)}\right) - \left(\left(\mathrm{d}n B \frac{1}{n(1-R)}\right)^4 - 4\right)^{\frac{1}{2}}|\right)(R-1) + 10\left(|\pi i B \frac{1}{2+2R}|\right)^2 (1-R) + \pi}{B}}$$

The B parameter of the primitive function is given by the ratio of intensity described within the gear

$$B \approx 1 + 0.01 \log_{10}\left(\frac{I}{I_0}\right)$$

2 The addiction between electron density and plasma temperature can be given by the model like

$$f(T^{\min}) = \frac{-2^{\frac{4}{3}}(R-1)n(1-R)\log_{10}(e)\mathbf{F}(n)}{3B^2 n(1 - R(\mathbf{F}(n)))}$$

The varible $T^{\min}$ is the minimal temperature of the impact, which can be both simulated or measured. The computations below cover the dependence between the real temperature of free electrons and the intensity measured.

3 First of all, we need the solution for the minimal temperature and its derivation.

$$\frac{\partial T^{\min}}{\partial \nu, I} = \frac{3^{\frac{2}{3}} q_e^2 N_D^{\frac{2}{3}}}{\epsilon_0 k_B (4\pi)^{\frac{2}{3}} \lambda_{De}^{\frac{6}{5}} f(T^{\min})}$$

Owing to the model description of the minimal temperature, the plasma physics paramateres N_D and λ_{De} can be provided by the known dependences taken from the ionospheric plasma model to get the appropriate initial gears for the real free-electron plasma temperature computation.

4 The partial derivation between the real temperature and the intensity measured and the partial differential gear written above can now both be approached by the numerical solution of the total differential gear. Our approximations are, accordingly, like this

$$\frac{\partial T^{\min}}{\partial \nu, I} \rightarrow \mathrm{d}T^{\min} \wedge \frac{\partial T}{\partial \nu, I} \rightarrow \mathrm{d}T,$$

where T is the free-electron plasma temperature which is recquired for the simulation.

5 The approximations done allow us to define the z parameter based on the total differential temperature gears within the equation

$$z = -\frac{\mathrm{d}T^{\min}}{\mathrm{d}T}$$

6 Finally, the ionospheric and meteoric plasma model is able to be used for the computation of the intensity, which stands up to comparison with the real intentensity measured.

$$I = \log\left(\frac{nEk_BT}{z} e^{-\frac{T^{\min}}{T}}\right)$$

7 Using our approximation, we can now sumarize the total differential gear like this

$$\mathrm{d}T = \frac{e^{I+\frac{T^{\min}}{T}}}{nEk_B}$$

We can see that the input parameters have been extended within the energy of the appropriate atomic spectral line E.

8 At the last step, the approached total differential gear is solved by the system of partial differential equations for the functions of $T = f(T^{min})$, $T = f(I)$ and the ratio $T = f(\mathrm{d}T)$ describing the total differential gear itself.

The routes of the system drawn had been prepared for numerical-integrating tests and are summarized in the following matrix.

$$T = \sum_{i=1}^{k} \begin{pmatrix} \frac{1}{T^{\min}(nEk_B)^{T^{\min}} - \log(T^{\min})} & T^{\min} + \frac{I_1}{nEk_B} & \frac{T^{\min}}{\log(T^{\min} - I_1 nEk_B)} \\ \vdots & \ddots & \vdots \\ \frac{i^{i-1}}{i!}\left(\frac{1}{T^{\min}(nEk_B)^{T^{\min}} - \log(T^{\min})}\right)^i & \frac{i^{i-1}}{i!}\left(\frac{T^{\min}(T^{\min}+I_i)}{nEk_B T^{\min}}\right)^i & \frac{T^{\min}}{\log(T^{\min} - I_i nEk_B)} \\ \vdots & \ddots & \vdots \\ \frac{k^{k-1}}{k!}\left(\frac{1}{T^{\min}(nEk_B)^{T^{\min}} - \log(T^{\min})}\right)^k & \frac{k^{k-1}}{k!}\left(\frac{T^{\min}(T^{\min}+I_k)}{nEk_B T^{\min}}\right)^k & \frac{T^{\min}}{\log(T^{\min} - I_k nEk_B)} \end{pmatrix} + C$$

9 However, having done the numerical solution for the matrix system, we deduce that the addiction to the system described within the second equation is negligible. In fact, owing to the numerical solution, the whole system is able to be simplified by combining the two efficient equations together. After that, we get the resultant numerical-solved shape

$$T = \frac{T^{\min}}{\log(T^{\min} - IEk_B)} \log(I) \log\left(\frac{k!(T^{\min}(Ek_B)^{T^{\min}} - \log(T^{\min}))^k}{k^{k-1}}\right)$$

The numerical simulation has provided results similiar to both atmosheric detection and spectral measurement.

2 Electron density

10 Using the same equation system, we reached the solution of ratio between the electron density and free-electron plasma temperature. Applying the symbolic solution of the system of partial differential equations, we had got the matrix

$$n = \sum_{i=1}^{k} \begin{pmatrix} \frac{1}{3}\left(\frac{9}{16}\frac{1}{\epsilon_0^6 k_B^6 q_e^6 \pi^2} e^{-\frac{60\mathrm{d}T}{4\pi\epsilon_0^3 k_B^3 T^3}}\right)^{\frac{1}{3}} & \cdots & -\frac{1}{3}\left(\frac{9}{16}\frac{1}{\epsilon_0^6 k_B^6 q_e^6 \pi^2} e^{-\frac{60\mathrm{d}T}{4\pi\epsilon_0^3 k_B^3 T^3}}\right)^{\frac{1}{3}} \\ \vdots & \ddots & \vdots \\ \frac{i^{i-1}}{3i!}\frac{1}{3}\left(\frac{9}{16}\frac{1}{\epsilon_0^6 k_B^6 q_e^6 \pi^2} e^{-\frac{60\mathrm{d}T}{4\pi\epsilon_0^3 k_B^3 T^3}}\right)^{\frac{i}{3}} & \cdots & -\frac{i^{i-1}}{3i!}\frac{1}{3}\left(\frac{9}{16}\frac{1}{\epsilon_0^6 k_B^6 q_e^6 \pi^2} e^{-\frac{60\mathrm{d}T}{4\pi\epsilon_0^3 k_B^3 T^3}}\right)^{\frac{i}{3}} \\ \vdots & \ddots & \vdots \\ \frac{k^{k-1}}{3k!}\frac{1}{3}\left(\frac{9}{16}\frac{1}{\epsilon_0^6 k_B^6 q_e^6 \pi^2} e^{-\frac{60\mathrm{d}T}{4\pi\epsilon_0^3 k_B^3 T^3}}\right)^{\frac{k}{3}} & \cdots & -\frac{k^{k-1}}{3k!}\frac{1}{3}\left(\frac{9}{16}\frac{1}{\epsilon_0^6 k_B^6 q_e^6 \pi^2} e^{-\frac{60\mathrm{d}T}{4\pi\epsilon_0^3 k_B^3 T^3}}\right)^{\frac{k}{3}} \end{pmatrix} + C$$

11 The numerical-integrating friendly shape has been reached by fitting the symbolic solutions within the real-measured meteorite detected siultaneously by a spectrographic camera and a very low frequency monitor. From the ionospheric plasma model, there is a known transformation between the free-electron plasma temperature and the electron density:

$$n = \frac{16\epsilon_0^3 k_B^3 \pi^2 \lambda_{De}^{\frac{1}{5}} e^{\frac{10\mathrm{d}T}{N_D}}}{9 q_e^6 N_D^2}$$

The plasma physics parameters have been fitted and numerically combined with the original matrix. The resultant fitted ratio was approached like this

$$n = e^{\frac{-\log(10^{1}2a)}{10}}$$

for the parameter a

$$a = \frac{16\epsilon_0^3 k_B^3 \pi^2 \lambda_D e^{\frac{1}{5}} e^{\frac{10\mathrm{d}T}{N_D}}}{9 q_e^6 N_D^2} \frac{k^{k-1}}{3k!} \left(\frac{16\epsilon_0 k_B q_e e^{-\frac{\mathrm{d}T}{4\pi T^3}}}{9\pi^2}\right)^k$$

12 Finally, our results have stood up to comparison with the time-controlled integrating simulations of the real meteoric plasma geometry. First of all, we draw the relative plasma distribution model based on the dependence between electron density and free-electron plasma temperature $\frac{\mathrm{d}n}{\mathrm{d}T}$. After that, we use the known equations describing the real plasmatic geometry and fit them to calculate both x and y normalized axis.