A Study on the Combined Effect of Axle Friction and Rolling Resistance

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Abstract Many textbooks on mechanics for engineering students and engineers consider the concepts of rolling resistance and axle friction separately, expecting readers to combine the given analysis for each of them in determining, for instance, the magnitude of the force needed to move a railroad car. However, this requires a thorough free-body diagram analysis and, since examples are not typically included in the textbooks, students may have difficulty solving such problems. This study represents the solution of the problem in terms of both the dry axle friction and the rolling resistance. It is also suggested as a good synthesis problem that may be considered in teaching the effect of dry friction to engineering students.

Keywords rolling resistance; axle friction

Introduction: the individual effects of dry axle friction and rolling resistance

A bearing supporting an axle rotating with a constant angular velocity, ω, or an axle with an impending motion, and subjected to a load, $\vec{F}_{load}$, is often considered in teaching [1–6] the effect of axle friction (Fig. 1). When the axle is set in motion, due to the effect of friction, it climbs in the bearing until slippage occurs, and therefore the contact line between the axle and the bearing appears to the left of the line of action of $\vec{F}_{load}$ at a distance r_f for the direction of rotation shown in Fig. 2. In Fig. 2, point c represents the side view of the line of contact. The force $\vec{F}_{reac}$ is the resultant of the reaction exerted by the bearing on the axle. $\vec{M}_f$ represents the couple necessary on the axle to balance the opposing rotational action of $\vec{F}_{load}$ and $\vec{F}_{reac}$. If the wheel is rotating with a constant angular velocity, ω, the equilibrium equation, $\Sigma\vec{M} = 0$, results in

$$M_f = r_f F_{reac}$$

where $r_f \cong r\mu_k$ for small values of the angle of friction ϕ_k. [$\phi_k = \tan^{-1}(F_f/N) = \tan^{-1}(\mu_k N/N) = \tan^{-1}(\mu_k)$. Here, μ_k is the coefficient of kinetic friction for the mating surfaces. In the case of impending motion, μ_k is replaced by the coefficient of static friction, μ_s.]

In the same manner, a wheel or a roller which carries a vertical load and rolls over a yielding surface is often considered in the textbooks to explain the concept of 'rolling resistance' (Fig. 3). If a wheel rolls over a yielding surface a resistance to the motion is encountered, because the surface immediately in front of the wheel is being deformed. The reaction of the surface is distributed over the area of contact. The resultant of it passes through a point, e, in the area of contact, as shown in Fig. 3. Since the angular velocity of the wheel is constant, the three forces acting are in

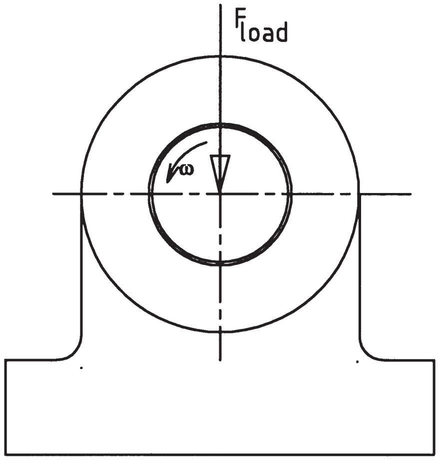

Fig. 1 *A bearing supporting an axle with a load, $F_{load.}$*

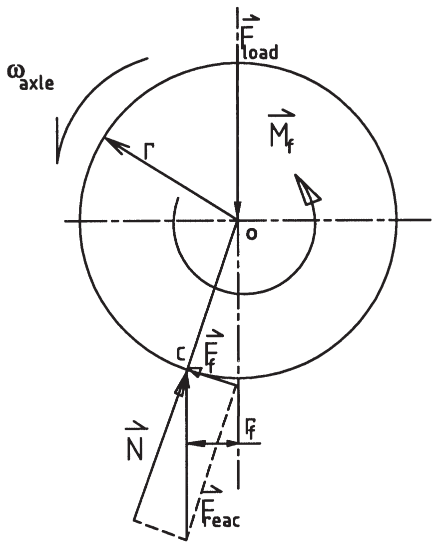

Fig. 2 *Free-body diagram of the part of the axle supported by the bearing.*

equilibrium and hence the reaction, $\vec{F}_{road}$, of the surface on the wheel must pass through the center of the wheel, o. Applying the equation of equilibrium $\Sigma\vec{M}_e = 0$, taking the distance oa $\cong r_{wheel}$ (when the depression of the surface is small compared with the radius of the wheel), the expression for calculating the magnitude of the pushing (or pulling) force, $\vec{F}_p$, which is applied horizontally to the axis of the wheel to move it with a constant angular velocity, is obtained as

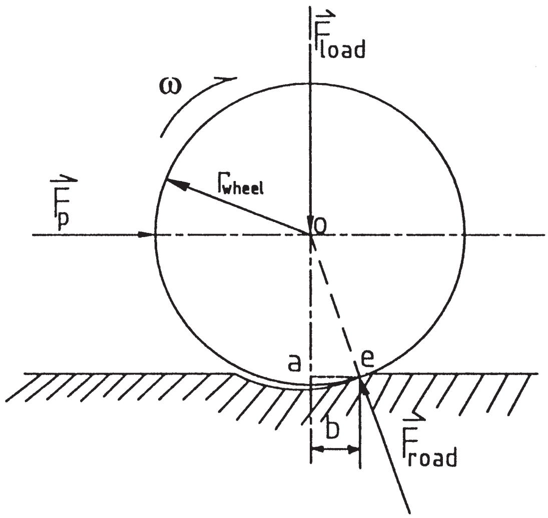

Fig. 3 *Free-body diagram of a rolling wheel.*

TABLE 1 *Coefficients of rolling resistance given by Coulomb and Goodman [6]*

Materials	*b* (mm)
Elm on oak	0.831
Steel on steel	0.178–0.381
Steel on wood	1.52–2.54
Steel on macadam road	1.27–5.08
Steel on soft ground	76.2–127
Pneumatic tires on good road	0.508–0.559
Pneumatic tires on mud road	1.02–1.52
Solid rubber tire on good road	1.02
Solid rubber tire on mud road	2.29–2.79

$$F_p = bF_{load}/r_{wheel}$$

The horizontal component of the reaction, $\vec{F}_{road}$, is equal to $\vec{F}_p$ and is called the rolling friction or rolling resistance. The distance b is commonly called 'the coefficient of rolling resistance'. It is not a dimensionless coefficient. The value of b depends upon several parameters, mainly the material of the road and that of the wheel. The values of b determined by Coulomb and Goodman are given in Table 1 [6].

Combined effect of axle friction and rolling resistance

The analysis providing the formula to be used to calculate the pulling (or pushing) forces to be applied to set a cart (or a railroad car) in motion requires a considera-

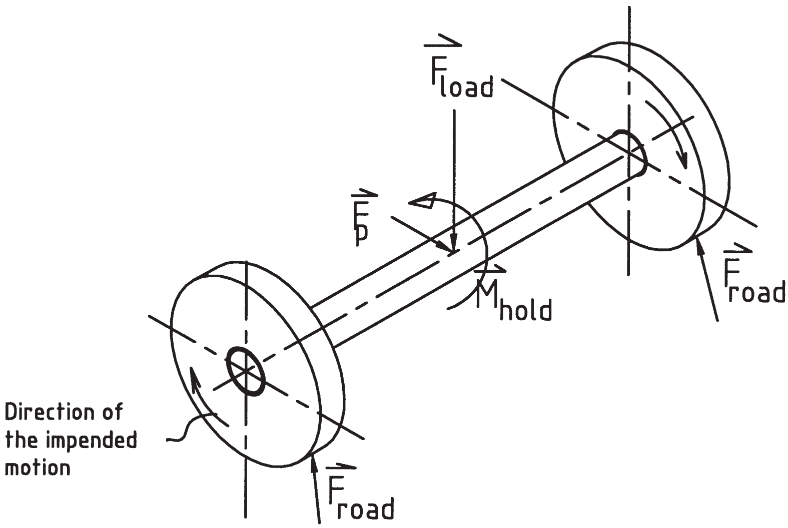

Fig. 4 *The axle and the wheels.*

tion of the combined effects of axle friction and rolling resistance. In giving the method of such a consideration, one of the axles of a railroad car, which is fixed to the chassis, with its wheels symmetrically mounted on its ends and with a vertical load $\vec{F}_{load}$ acting at the middle of it may be considered, as shown in Fig. 4. In this figure, $\vec{F}_p$ represents the horizontal force required on the axle to start the motion and $\vec{M}_{hold}$ indicates the axial couple vector, which is exerted by the chassis of the car through the fastening system of the axle. It prevents the axle from rotating, balancing the moments applied by the friction forces over the mating surfaces between the axle and the hubs of the wheels at both ends of the axle.

In the first part of the analysis, the equilibrium of the axle is considered. In doing this, the free-body diagram of the axle, which represents the schematic view of it on a projection plane perpendicular to the longitudinal axis of the axle, is convenient to consider (Fig. 5). In this figure, $\vec{F}_{axle}$ represents the resultant of $\vec{F}_{load}$ and $\vec{F}_p$:

$$\vec{F}_{axle} = \vec{F}_{load} + \vec{F}_p \tag{1}$$

The reaction force, $\vec{F}_{reac}$, which is applied by the wheel, has its point of application to the left of the line of action of $\vec{F}_{axle}$ relative to an observer looking in the direction of that line, since the axle climbs in the hub of the wheel due to the effect of axle friction as the rotation of the wheel is impending. At that state, the equilibrium equation $\Sigma\vec{F} = 0$ is still applicable and it results in $F_{axle} = F_{reac}$. Hence, $\vec{F}_{axle}$ and $\vec{F}_{reac}$ form a couple. This couple is balanced by the couple $\vec{M}_{hold}$. The second equilibrium equation, $\Sigma\vec{M} = 0$, gives the magnitude of this couple as

$$M_{hold} = r_f F_{axle} \tag{2}$$

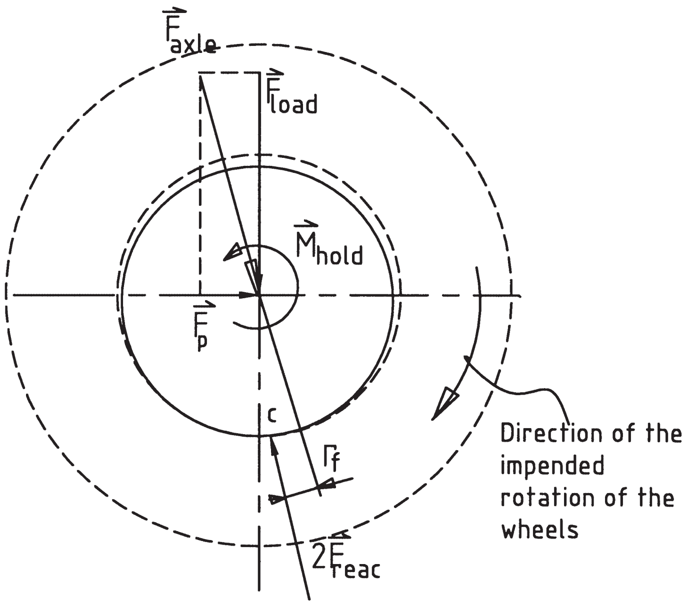

Fig. 5 *Free-body diagram of the axle.*

In the second part of the analysis, the effects of the axle friction and the rolling resistance applied by the road are combined by considering the free-body diagram of one of the wheels (Fig. 6). As can be easily seen from the equilibrium conditions, the x and y components of $\vec{F}_{reac}$ are equal to $\vec{F}_p/2$ and $\vec{F}_{load}/2$, respectively. Therefore, the equilibrium equation, $\Sigma M_e = 0$, results in $F_p = (b - x_c)F_{load}/d$. When the depression of the road is small with respect to $r_o - r_i$, taking $d \cong r_o - r_i$, it may be written as:

$$F_p = (b - x_c)F_{load}/(r_o - r_i) \tag{3}$$

where r_o and r_i are the outer and inner radii of the wheel, respectively. Trigonometric relationships, written by considering the free-body diagram given in Fig. 6, leads to the following expression for x_c:

$$x_c = r_i \sin\alpha \cos[\sin^{-1}(r_f/r_i)] - r_f \cos\alpha \tag{4}$$

where α is the inclination angle of $\vec{F}_{reac}$ (or that of $\vec{F}_{road}$) with respect to the y axis. Substituting eqn 4 into eqn 3, the final expression for calculating the magnitude of the pulling (or pushing) force, $\vec{F}_p$, can be obtained as:

$$F_p = \{b - r_i \sin\alpha \cos[\sin^{-1}(r_f/r_i)] + r_f \cos\alpha\} F_{load}/(r_o - r_i) \tag{5}$$

In the use of this expression, since $\alpha = \tan^{-1}(F_p/F_{load})$, it is necessary to apply an iteration method.

A numerical example, which illustrates the method of calculation, may be given for one axle of a railway car for the following data per ton of load:

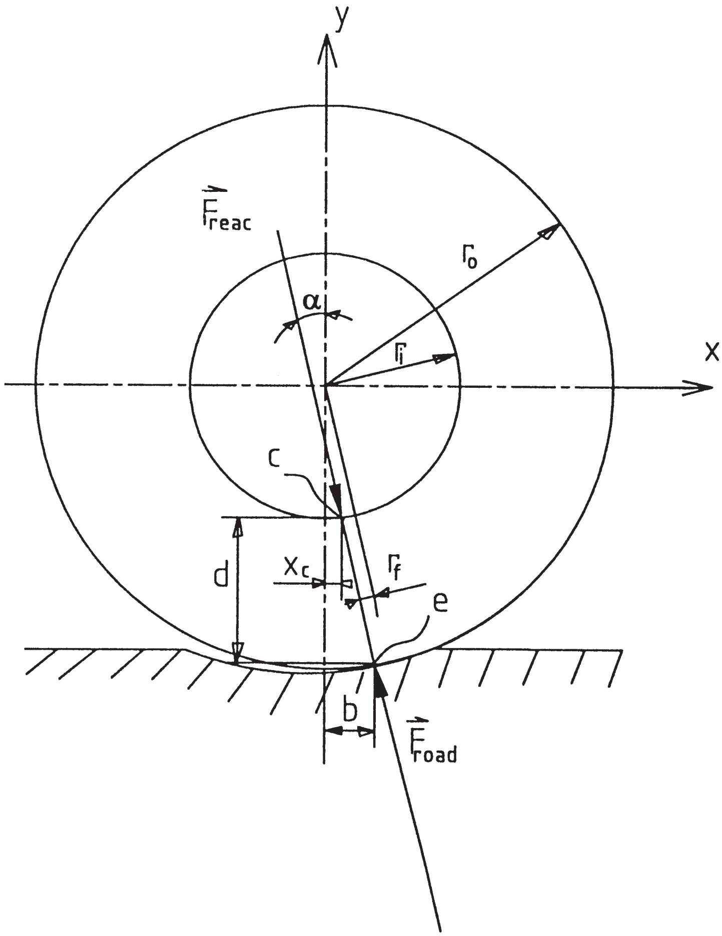

Fig. 6 *Free-body diagram of the wheel.*

wheel material:	steel
wheel diameter:	800 (mm)
axle diameter:	125 (mm)
coefficient of static friction:	0.02
coefficient of rolling resistance:	0.25 (mm)

Substituting these data into eqn 5, a nonlinear single equation in F_p is obtained. This equation can be solved by using the well known Newton–Raphson method, or executing the QBasic program given below. Starting with $(F_p)_0 = 0.1$ as the initial estimate in the Newton–Raphson method, the converged solution is obtained as 36.8 (N/ton load) in three iterations. On the other hand, the QBasic program finds the same answer in nine iterations. The couple which is to be applied by the chassis of the car through the fastening system of the axle for preventing the axle from rotating, M_{hold}, takes its value from eqn 2 as:

$$M_{hold} = r_f F_{axle} = r_i \mu_s \sqrt{F_p^2 + F_{load}^2} = 12.3\,(\text{Nm/ton load})$$

This program determines Fp by using the proposed trial and error method

```
CLS
INPUT "ri="; ri 'inner radius of the wheel
INPUT "r0="; r0 'outer radius of the wheel
INPUT "Fload="; Fload 'axle load in the vertical direction
INPUT "b="; b 'coefficient of rolling resistance
INPUT "mu="; mu 'coefficient of static friction
rf = ri * mu
Fpg# = .1
Fpc# = .1
alpha = ATN(Fpg#/Fload)
beta = ATN(rf/(ri * SQR(−(rf/ri) ^ 2 + 1)))
i = 0
DO
Fpg# = Fpc#
Fpc# = (b − ri * SIN(alpha) * COS(beta) + rf* COS(alpha)) * Fload/(r0 − ri)
alpha = ATN(Fpc#/Fload)
i = i + 1
LOOP UNTIL ABS (Fpc# − Fpg#) < .0001
PRINT "Fp="; Fpc#
PRINT "number of iterations="; i
END
```

Conclusion

The analysis represents a method for solving problems involving the effects of both the dry axle friction and the rolling resistance. It is proposed as a good synthesis problem that may be considered in teaching the effect of dry friction to mechanical engineering students. The expression derived in this analysis (eqn 5) is also represented as the equation to be used by the experimenters investigating the coefficients of rolling resistance by measuring the magnitude of $\vec{F}_p$ for different materials and road conditions.

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