# THEORETICAL COMPUTER SCIENCE TUTORING (1)

Maurizio Fiusco

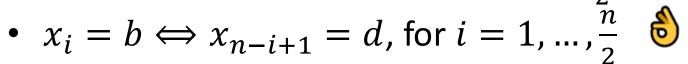




• 
$$x_i \in \{a, b\}$$
, for  $i = 1, ..., \frac{n}{2}$ 

• 
$$x_i \in \{c, d\}$$
, for  $i = \frac{n}{2} + 1, ..., n$ 

• 
$$x_i = a \Leftrightarrow x_{n-i+1} = c$$
, for  $i = 1, \dots, \frac{n}{2}$ 



а	b	а	С	d	С
		l I			

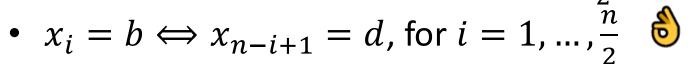




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a a b b d d c c
-----------------





- $x_i \in \{a, b\}$ , for  $i = 1, ..., \frac{n}{2}$
- $x_i \in \{c, d\}$ , for  $i = \frac{n}{2} + 1, ..., n$
- $x_i = a \Leftrightarrow x_{n-i+1} = c$ , for  $i = 1, ..., \frac{n}{2}$
- $x_i = b \iff x_{n-i+1} = d$ , for  $i = 1, \dots, \frac{\overline{n}}{2}$

а	b	а	d	С





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$$x_i \in \{a, b\}$$
, for  $i = 1, ..., \frac{n}{2}$ 

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$$x_i \in \{c, d\}$$
, for  $i = \frac{n}{2} + 1, ..., n$ 

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$$x_i = a \Leftrightarrow x_{n-i+1} = c$$
, for  $i = 1, ..., \frac{n}{2}$ 

• 
$$x_i = b \Leftrightarrow x_{n-i+1} = d$$
, for  $i = 1, \dots, \frac{n}{2}$ 



a	а	b	b	а	d	d	d	С	С
				l				1	l



Let L be the set of strings  $s = \langle x_1 x_2 \dots x_n \rangle$  of even length such that:

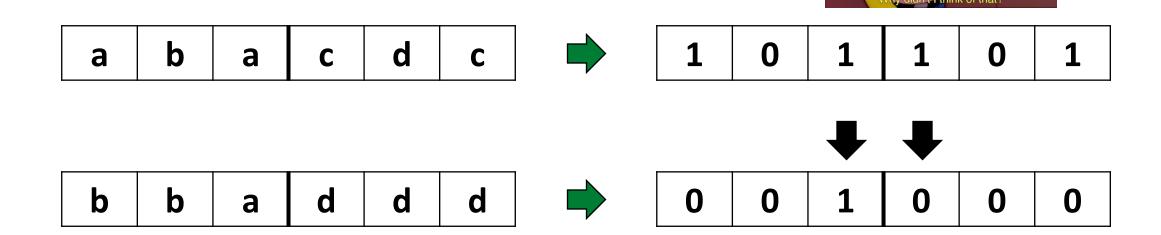
- $x_i \in \{a, b\}$ , for  $i = 1, ..., \frac{n}{2}$
- $x_i \in \{c, d\}$ , for  $i = \frac{n}{2} + 1, ..., n$
- $x_i = a \Leftrightarrow x_{n-i+1} = c$ , for  $i = 1, ..., \frac{n}{2}$
- $x_i = b \Leftrightarrow x_{n-i+1} = d$ , for  $i = 1, ..., \frac{n}{2}$

Define a deterministic Turing Machine that accepts all and only the words contained in  $\boldsymbol{L}$ 



Let  $s = \langle x_1 x_2 ... x_n \rangle \in \{a, b, c, d\}^n$  e  $\sigma = \langle y_1 y_2 ... y_n \rangle \in \{0, 1\}^n$  the binary string associated with s according to the following rules:

- $y_i = 0 \Leftrightarrow x_i = a \ \lor x_i = c$ , per  $1 \le i \le n$
- $y_i = 1 \Leftrightarrow x_i = b \lor x_i = d$ , per  $1 \le i \le n$





I just have to check if the string is palindrome

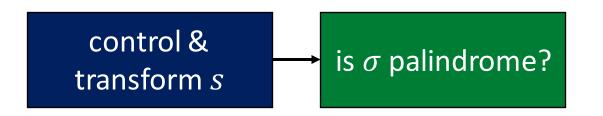
Let  $s = \langle x_1 x_2 \dots x_n \rangle \in \{a, b, c, d\}^n$  e  $\sigma = \langle y_1 y_2 \dots y_n \rangle \in \{0, 1\}^n$  the binary string associated with s according to the following rules:

- $y_i = 0 \Leftrightarrow x_i = a \lor x_i = c$ , for  $1 \le i \le n$
- $y_i = 1 \Leftrightarrow x_i = b \ \lor x_i = d$ , for  $1 \le i \le n$

is  $\sigma$  palindrome?

#### 2 possible ways:

- Transform the string and check if it is a palindrome using the appropriate Turing machine
- Modify the Turing machine that checks if a string is palindrome



**Modified TM** 

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_0$ 



										_		
	а	а	b	d	С	С						
								l			1 ,	i

$$\langle q_0, a, \square, q_a, right \rangle$$

similar if I find b

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



									_	
	a	D	a	C	С					

$$\langle q_a, a, a, q_a, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



								_	 		 
		а	b	d	С	C					

$$\langle q_a, b, b, q_a, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



		а	b	d	С	C					

$$\langle q_a, d, d, q_a, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



							_			_	
		а	b	d	С	С					

$$\langle q_a, c, c, q_a, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



		а	b	d	С	С					

$$\langle q_a, c, c, q_a, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



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		а	b	d	C	С					

$$\langle q_a, \Box, \Box, q_c, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_c$ 



	 								_		
		а	þ	d	С	С					

$$\langle q_c, c, \Box, q_{left}, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_{left}$ 



		а	b	d	С					

$$\langle q_{left}, c, c, q_{left}, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_{left}$ 



		а	b	d	C					

$$\langle q_{left}, d, d, q_{left}, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





<u> </u>									_		
		а	b	d	С						

$$\langle q_{left}, b, b, q_{left}, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_{left}$ 



		а	b	d	С					

$$\langle q_{left}, a, a, q_{left}, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_{left}$ 



 								_	_	 	
		а	b	d	С						

$$\langle q_{left}, \Box, \Box, q_0, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_0$ 



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$$\langle q_0, a, \square, q_a, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_a$ 



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		b	d	С					
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$$\langle q_a, b, b, q_a, right \rangle$$

• • •

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_0$ 



		b	d						

$$\langle q_0, b, \square, q_b, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_b$ 



				_	_					
			d							

$$\langle q_b, d, d, q_b, right \rangle$$

similar if I find a, b or c

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_b$ 



			d						
l l						I	I	l	

$$\langle q_b, \Box, \Box, q_d, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_d$ 



					_				
			d						

$$\langle q_d, d, \square, q_{left}, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





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$$\langle q_{ind}, \Box, \Box, q_0, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_0$ 



$$\langle q_0, \Box, \Box, q_{acc}, stop \rangle$$

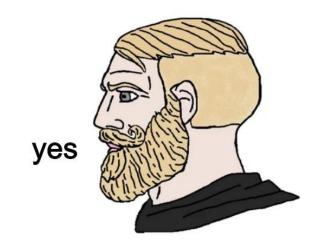
$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





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$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_0$ 



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$$\langle q_0, d, d, q_{rej}, stop \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





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			d	b	a	С	d	d				



$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_0$ 



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$$\langle q_0, c, c, q_{rej}, stop \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





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$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_c$ 



			_								
				b	а	C	d	d			

$$\langle q_c, d, d, q_{rej}, stop \rangle$$

similar if I find a, b or  $\square$ 

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





			b	а	С	d	d			
										4



$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 $q_d$ 



			b	а	С	d	С			

$$\langle q_c, c, c, q_{rej}, stop \rangle$$

similar if I find a, b or  $\square$ 

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





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Design a Turing machine that computes the two functions described below:

• 
$$f(n,k) = \left[\frac{n}{k}\right]$$

• 
$$f(n,k) = \left\lceil \frac{n}{k} \right\rceil$$
  
•  $g(n,k) = \left\lceil \frac{n}{k} \right\rceil$ 



n

f(n,k)

#### Es.

$$n = 15, k = 6$$
  
 $f(15,6) = 3$   
 $g(15,6) = 2$ 

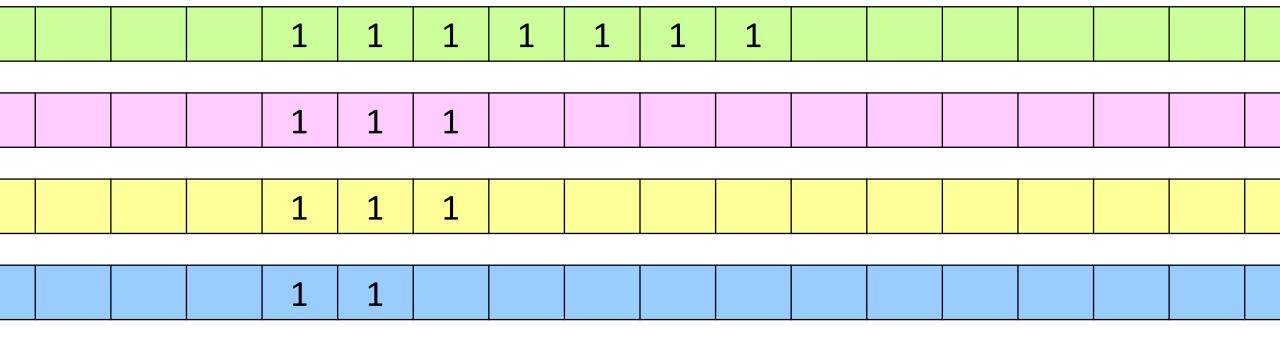


$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

Es.

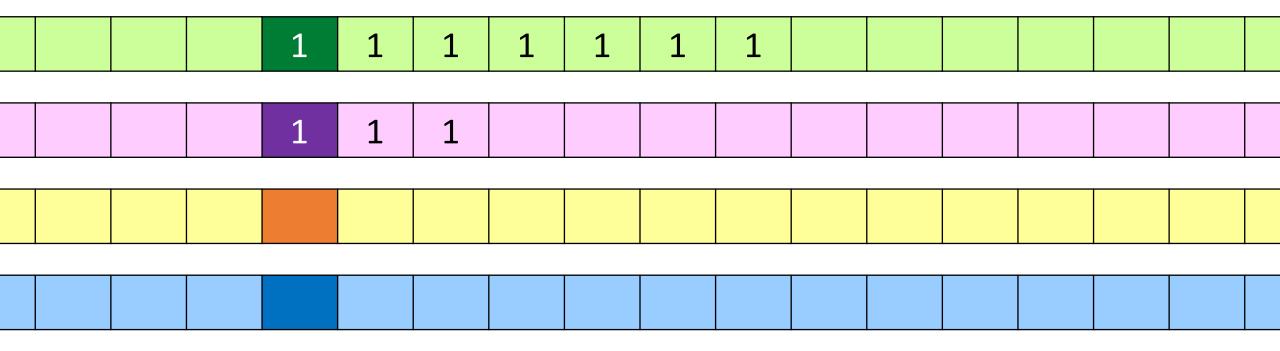
$$n = 11111111, k = 111$$
  
 $f(11111111,111) = 111, g(1111111,111) = 11$ 

 $q_f$ 



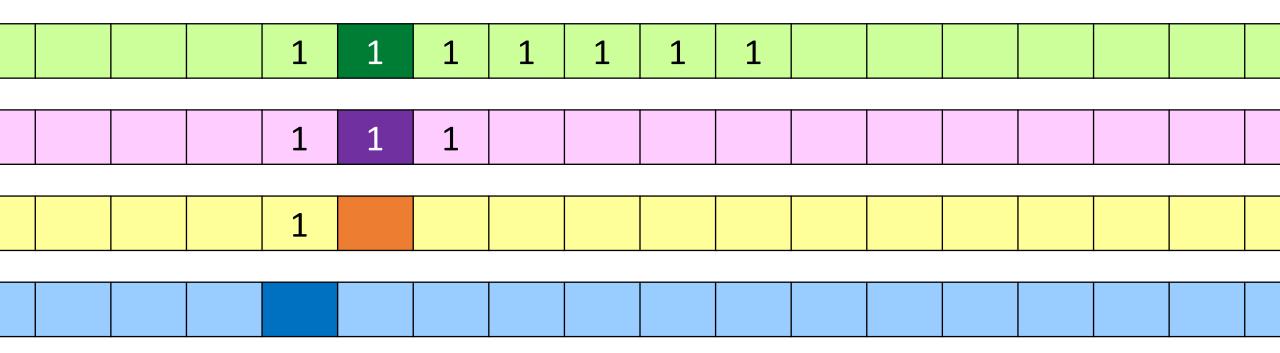
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_0, (1,1,\Box,\Box), (1,1,1,\Box), q_1, (r,r,r,s) \rangle$$



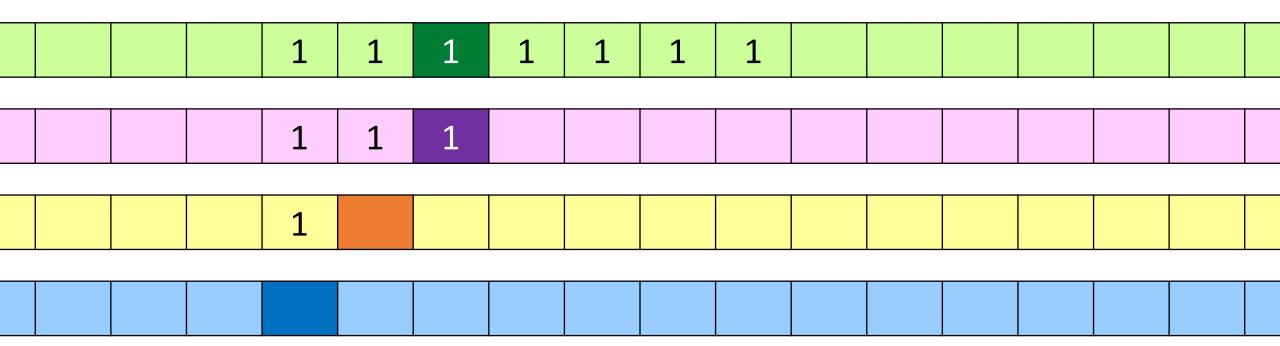
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (1,1,\Box,\Box), (1,1,\Box,\Box), q_1, (r,r,s,s) \rangle$$



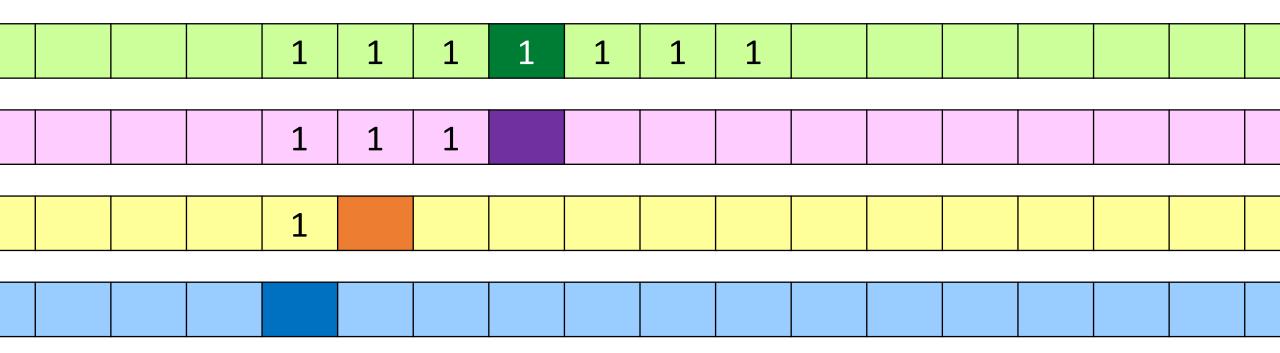
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (1,1,\Box,\Box), (1,1,\Box,\Box), q_1, (r,r,s,s) \rangle$$



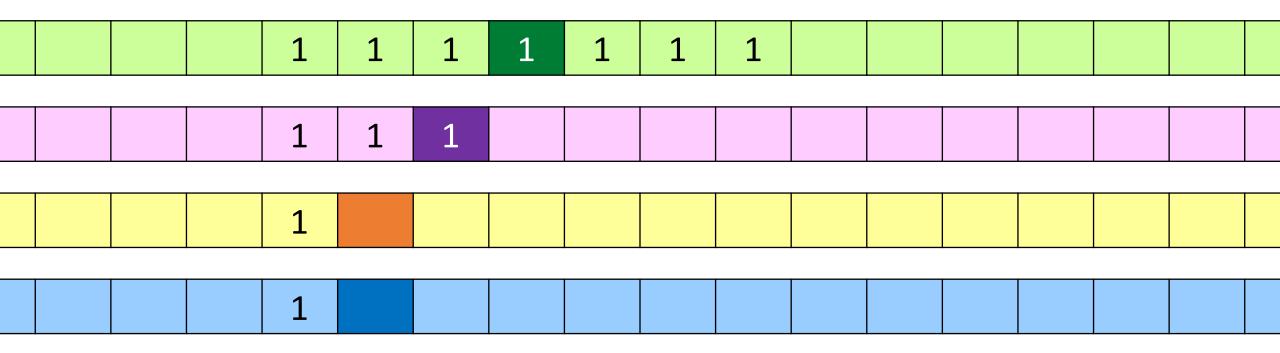
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (1, \square, \square, \square), (1, \square, \square, 1), q_2, (s, l, s, r) \rangle$$



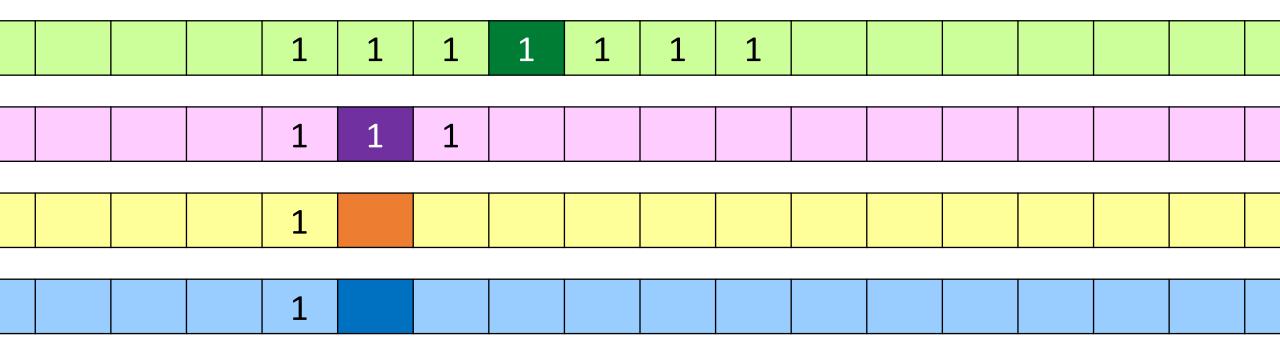
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_2, (1,1,\Box,\Box), (1,1,\Box,\Box), q_2, (s,l,s,s) \rangle$$



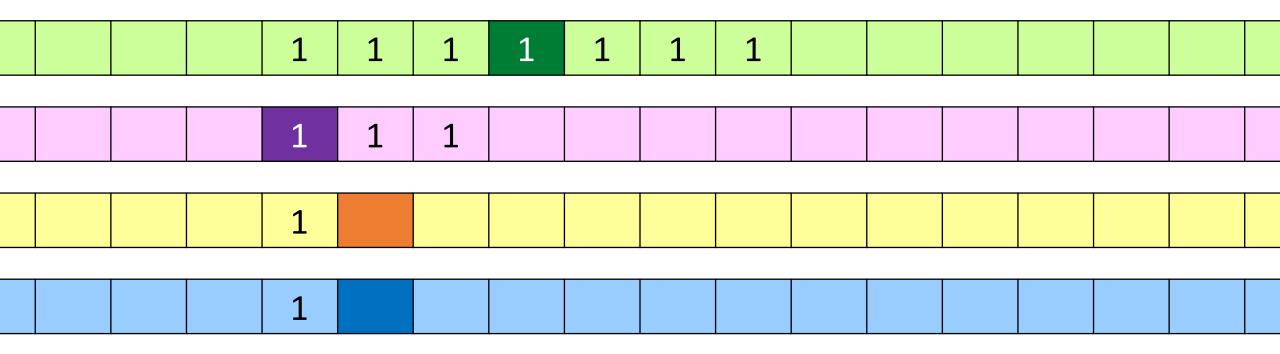
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_2, (1,1,\Box,\Box), (1,1,\Box,\Box), q_2, (s,l,s,s) \rangle$$



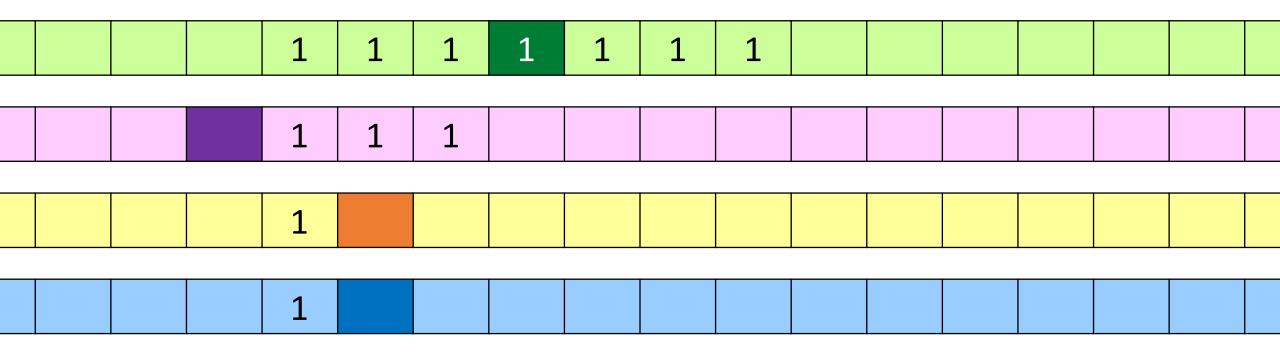
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_2, (1,1,\Box,\Box), (1,1,\Box,\Box), q_2, (s,l,s,s) \rangle$$



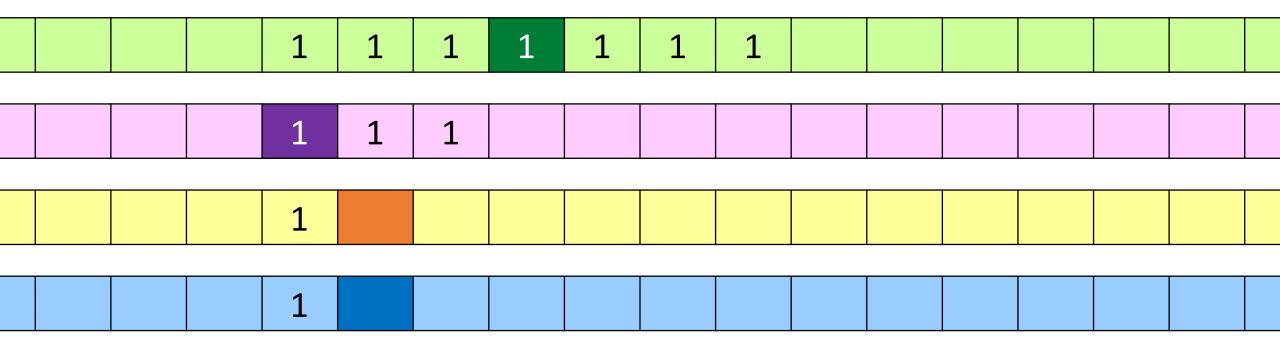
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_2, (1, \square, \square, \square), (1, \square, \square, \square), q_0, (s, r, s, s) \rangle$$



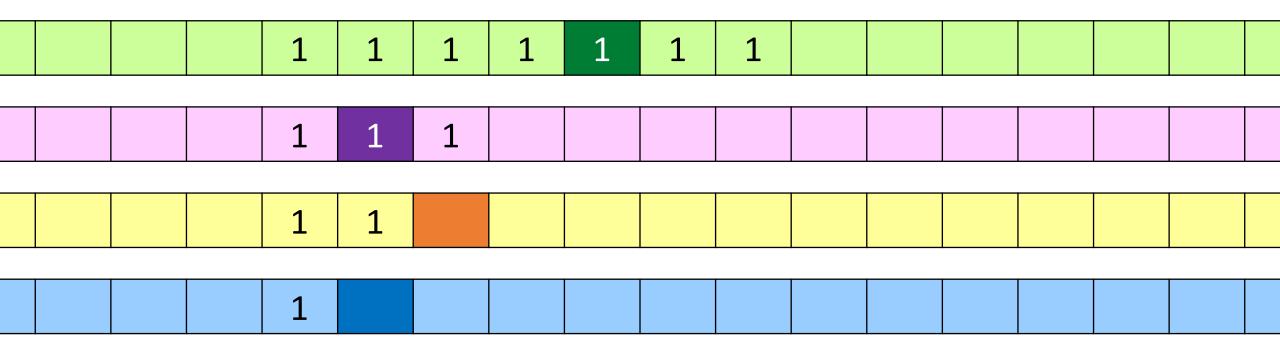
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_0, (1,1,\Box,\Box), (1,1,1,\Box), q_1, (r,r,r,s) \rangle$$



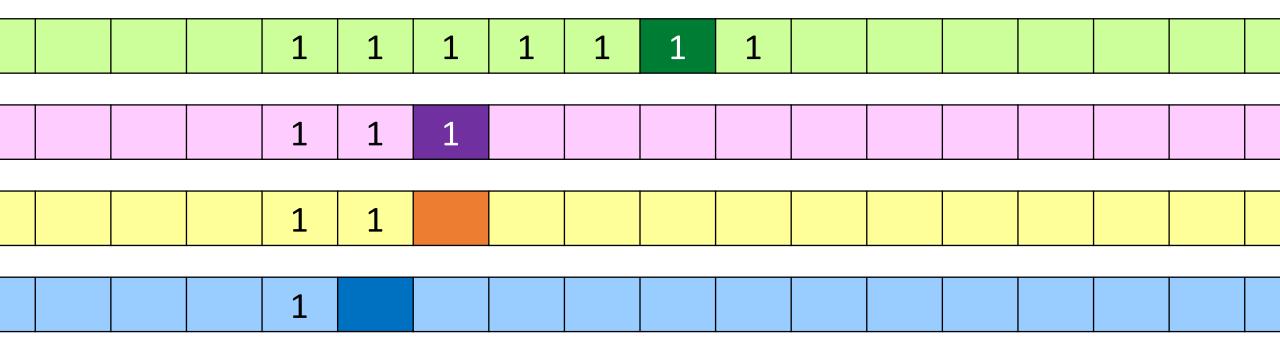
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (1,1,\Box,\Box), (1,1,\Box,\Box), q_1, (r,r,s,s) \rangle$$



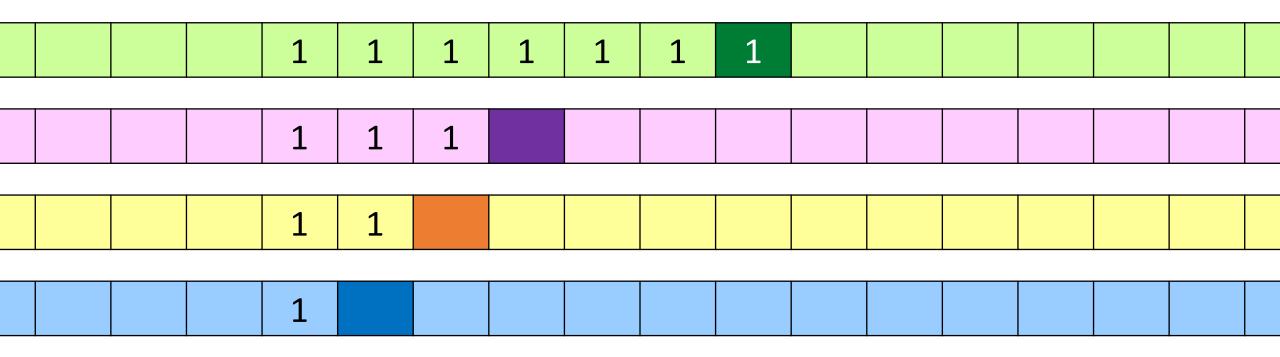
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (1,1,\Box,\Box), (1,1,\Box,\Box), q_1, (r,r,s,s) \rangle$$



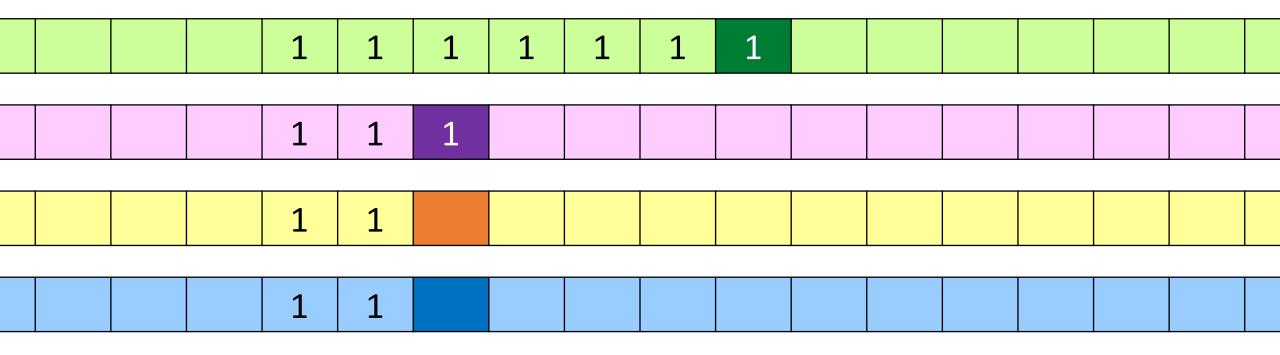
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (1, \square, \square, \square), (1, \square, \square, 1), q_2, (s, l, s, r) \rangle$$



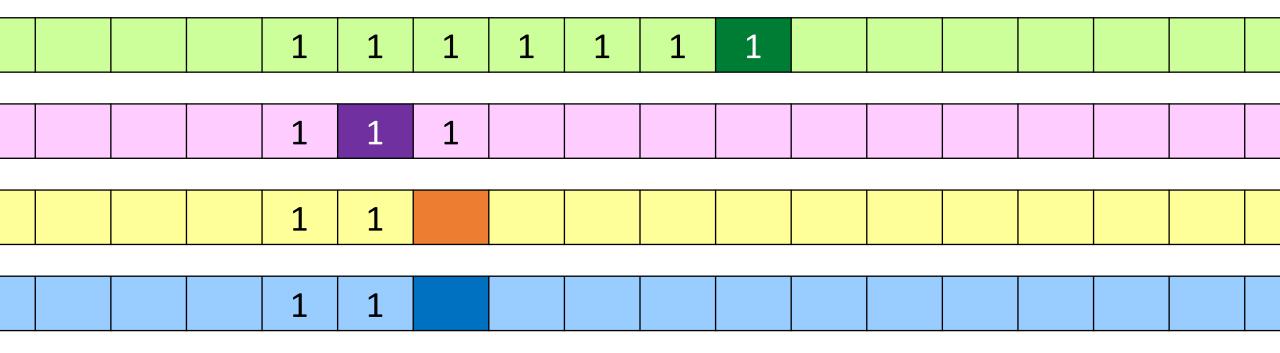
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_2, (1,1,\Box,\Box), (1,1,\Box,\Box), q_2, (s,l,s,s) \rangle$$



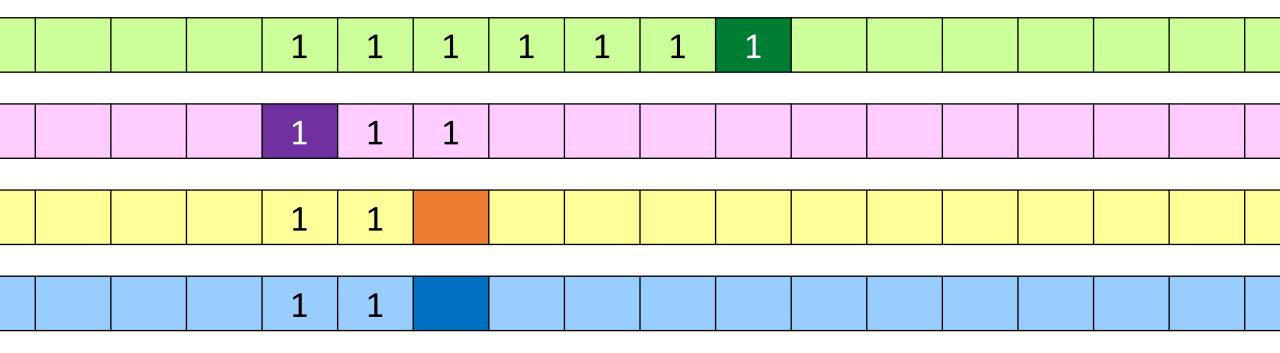
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_2, (1,1,\Box,\Box), (1,1,\Box,\Box), q_2, (s,l,s,s) \rangle$$



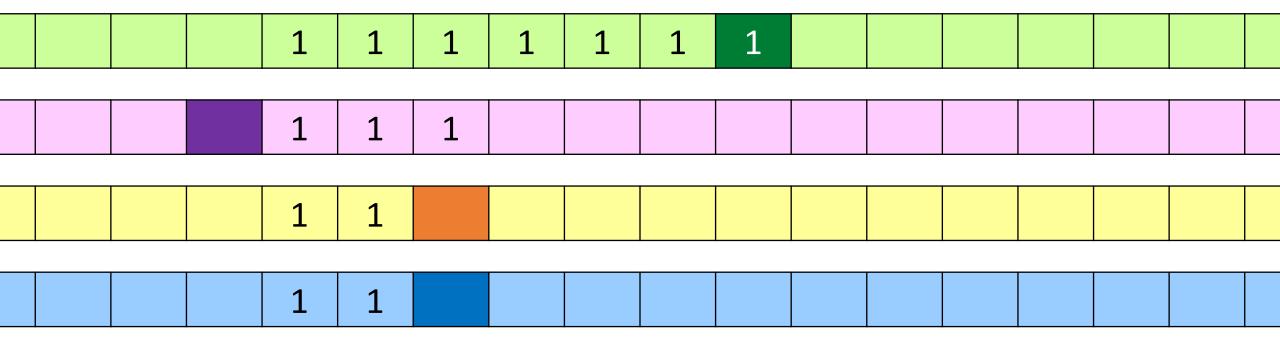
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_2, (1,1,\Box,\Box), (1,1,\Box,\Box), q_2, (s,l,s,s) \rangle$$



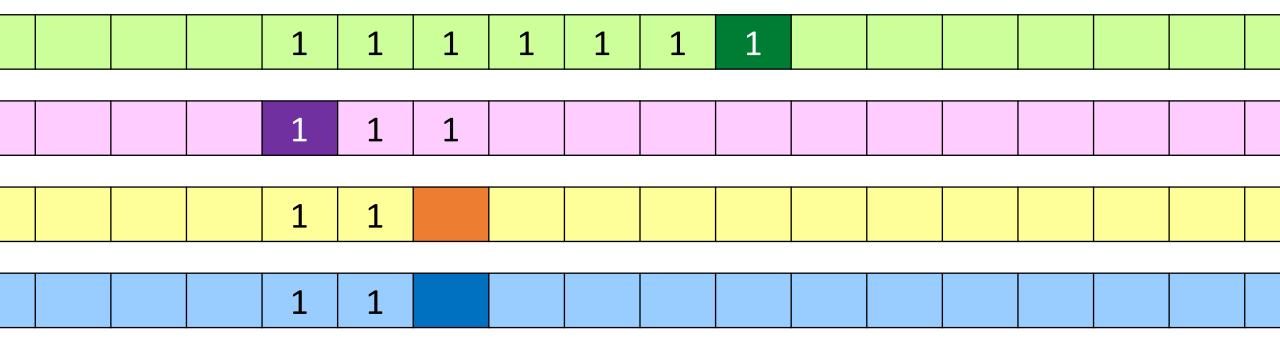
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_2, (1, \square, \square, \square), (1, \square, \square, \square), q_0, (s, r, s, s) \rangle$$



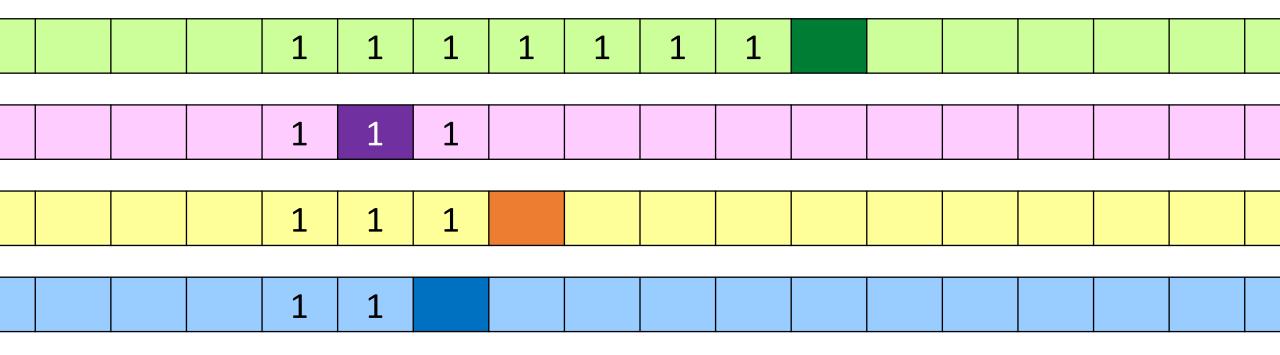
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_0, (1,1,\Box,\Box), (1,1,1,\Box), q_1, (r,r,r,s) \rangle$$



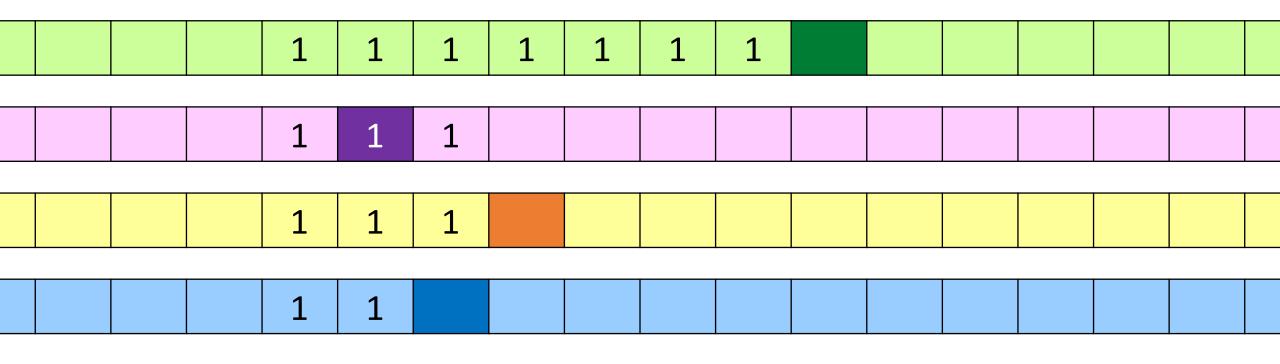
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (\Box, 1, \Box, \Box), (\Box, 1, \Box, \Box), q_f, (s, s, s, s) \rangle$$



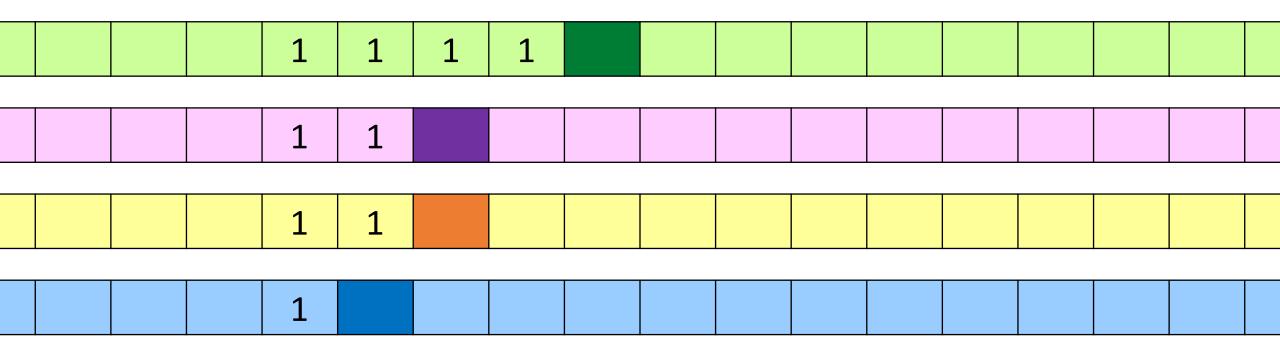
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

 $q_f$ 



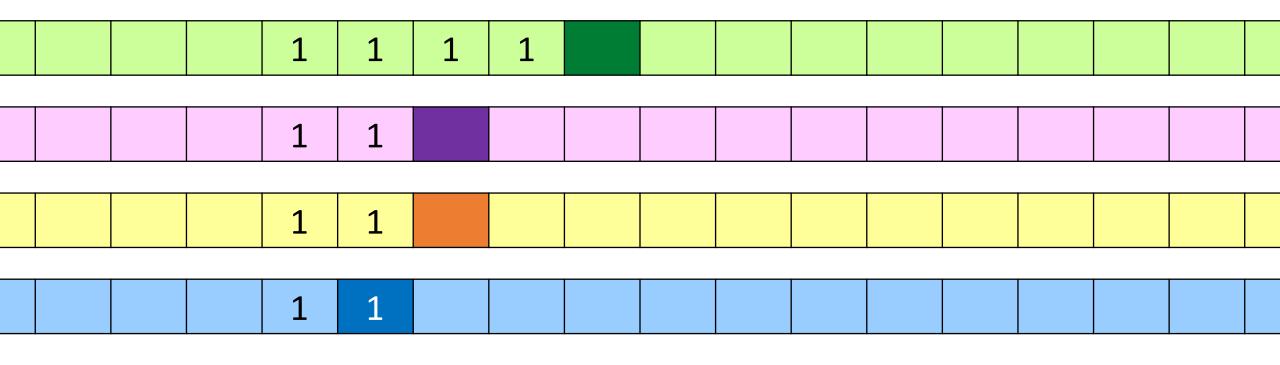
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (\Box, \Box, \Box, \Box), (\Box, \Box, \Box, 1), q_f, (s, s, s, s) \rangle$$



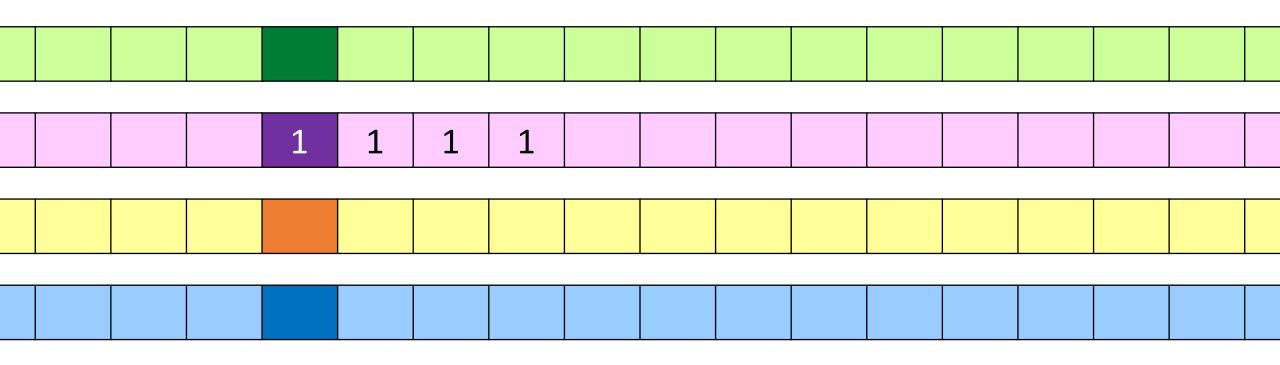
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

 $q_f$ 



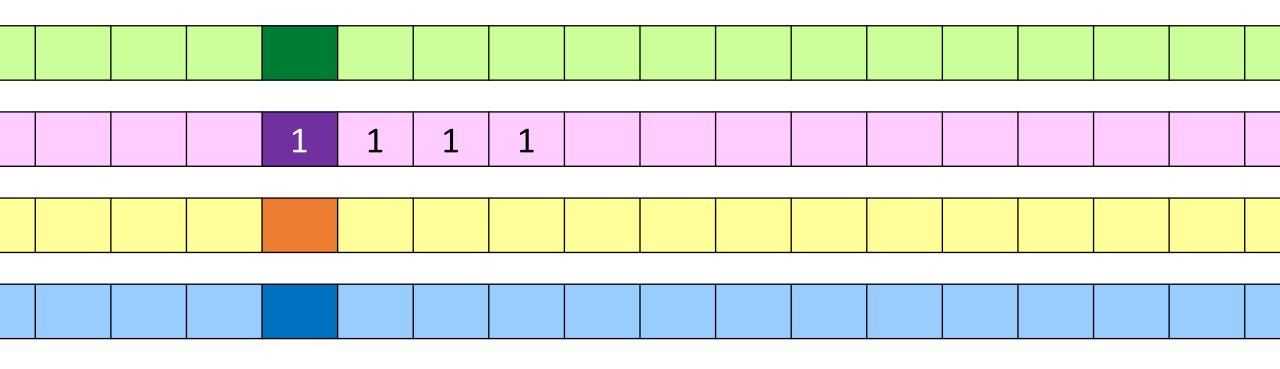
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_0, (\square, 1, \square, \square), (\square, 1, \square, \square), q_f, (s, s, s, s) \rangle$$



$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

 $q_f$ 



Let k be a constant in  $\mathbb{N}$ , and let  $NT_k$  be a non-deterministic Turing machine with a degree of non-determinism equal to k. Define a non-deterministic Turing machine  $NT_2$  with a degree of non-determinism equal to 2 that is equivalent to  $NT_k$