①
$$\begin{cases} x_2 + 3x_3 \le 5 \\ x_1 + x_2 + x_3 \le 8 \\ x_1 - 5x_3 \le 4 \\ x_1 \ge 0, x_2 \ge 0, x_3 \ge 0 \end{cases} \xRightarrow{STD} \begin{cases} x_2 + 3x_3 + x_4 = 5 \\ x_1 + x_2 + x_3 + x_5 = 8 \\ x_1 - 5x_3 + x_6 = 4 \\ x_1, x_2, x_3, x_4, x_5, x_6 \ge 0 \end{cases}$$

a) $x^{(1)} = \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}$; $x^{(2)} = \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$; $x^{(3)} = \begin{bmatrix} \frac{7}{2} \\ \frac{9}{2} \\ 0 \end{bmatrix}$ Sono SBA?

x^1

- Verifica ammissibilità $\begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}$

$5 \le 5$ ok

$8 \le 8$ ok

$3 \le$ ok

$3 \ge 0$, $5 \ge 0$, $0 \ge 0$ ok

- SBA?

$5 + x_4 = 5 \Rightarrow x_4 = 0$

$8 + x_5 = 8 \Rightarrow x_5 = 0 \Rightarrow [3, 5, 0, 0, 0, 1]$ SBA!

$3 + x_6 = 4 \Rightarrow x_6 = 1$

\# var $\neq 0$ = \# vincoli

x^2 • Verifico ammissibilità $\begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$

$4 \le 5$

$8 \le 8$

$4 \le 4$

$4 \ge 0$, $4 \ge 0$, $0 \ge 0$

- verifico SBA

$4 + x_4 = 5 \Rightarrow x_4 = 1$

$8 + x_5 = 8 \Rightarrow x_5 = 0$

$4 + x_6 = 4 \Rightarrow x_6 = 0$

$[4, 4, 0, 1, 0, 0]$

\# var $\neq 0$ = \# vincoli $\Rightarrow$ SBA

x^3 • verifico ammissibilità $\begin{bmatrix} \frac{7}{2} \\ \frac{9}{2} \\ 0 \end{bmatrix}$

$4,5 \leq 5$

$8 \leq 8$

$3,5 \leq 4$

$3,5 \geq 0, 4,5 \geq 0, 0 \geq 0$

• verifico SBA

$4,5 + x_4 = 5 \Rightarrow x_4 = \frac{1}{2}$

$8 + x_5 = 8 \Rightarrow x_5 = 0$

$3,5 + x_6 = 4 \Rightarrow x_6 = \frac{1}{2}$

$\left[\frac{7}{2}, \frac{9}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}\right]$ # var tot ≠ # vincoli

$\Rightarrow$ NO SBA

②

$$\min\ 2x_1 - 3x_2 + x_3$$
$$3x_1 + 2x_2 = 12$$
$$4x_1 + 2x_2 - 3x_3 \geq 2$$
$$2x_1 + \frac{1}{4}x_2 - 2x_3 \leq 4$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

std $\Rightarrow$

$$\min\ 2x_1 - 3x_2 + x_3$$
$$3x_1 + 2x_2 = 12$$
$$4x_1 + 2x_2 - 3x_3 - x_4 = 2$$
$$2x_1 + \frac{1}{4}x_2 - 2x_3 + x_5 = 4$$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

a) $x^{(1)} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$, $y^{(2)} = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix}$, $x^{(3)} = \begin{bmatrix} 1 \\ \frac{9}{2} \\ 0 \end{bmatrix}$

• verifica ammissibilità

$12 = 12$ OK	$12 = 12$ OK	$12 = 12$ OK
$10 \geq 2$	$12 \geq 2$	$13 \geq 2$
$4 \leq 4$	$\frac{3}{2} \leq 4$	$\frac{25}{8} \leq 4$
$4 \geq 0,\ 0 \geq 0,\ 2 \geq 0$	$0 \geq 0,\ 6 \geq 0,\ 0 \geq 0$	$1 \geq 0,\ \frac{9}{2} \geq 0,\ 0 \geq 0$

• SBA

$12 = 12$

$10 - x_4 = 2 \Rightarrow x_4 = 8$

$4 + x_5 = 4 \Rightarrow x_5 = 0$

$[4, 0, 2, 8, 0]$

$\#$ var $\neq 0$ = $\#$ vincoli $\Rightarrow$ SBA

$12 = 12$

$12 - x_4 = 2 \Rightarrow x_4 = 10$

$\frac{3}{2} + x_5 = 4 \Rightarrow x_5 = \frac{5}{2}$

$[0, 6, 0, 10, \frac{5}{2}]$

\# vincoli = \# var $\neq 0 \Rightarrow$ SBA

$12 = 12$

$13 - x_4 = 2 \Rightarrow x_4 = 11$

$\frac{25}{8} + x_5 = 4 \Rightarrow x_5 = \frac{7}{8}$

$[1, \frac{9}{2}, 0, 11, \frac{7}{8}]$

no SBA

b) $x^{(1)} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$$\min\ 2x_1 - 3x_2 + x_3$$
$$3x_1 + 2x_2 = 12$$
$$4x_1 + 2x_2 - 3x_3 \geq 2$$
$$2x_1 + \frac{1}{4}x_2 - 2x_3 \leq 4$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

$$\max\ 12y_1 + 2y_2 + 4y_3$$
$$3y_1 + 4y_2 + 2y_3 \leq 2$$
$$2y_1 + 2y_2 + \frac{1}{4}y_3 \leq -3$$
$$-3y_2 - 2y_3 \leq 1$$

$y_2 \geq 0,\ y_3 \leq 0,\ y_1$ libera

$y_1(3x_1 + 2x_2 - 12) = 0 \Rightarrow y_1 0 = 0$ NO

$y_2(4x_1 + 2x_2 - 3x_3 - 2) = 0 \Rightarrow y_2 10 = 0 \Rightarrow y_2 = 0$ ok

$y_3(2x_1 + \frac{1}{4}x_2 - 2x_3 - 4) = 0 \Rightarrow y_3 0 = 0$ NO

$x_1(3y_1 + 4y_2 + 2y_3 - 2) = 0 \Rightarrow 4(3y_1 + 4y_2 + 2y_3 - 2) = 0$ ok

$x_2(2y_1 + 2y_2 + \frac{1}{4}y_3 + 3) = 0 \Rightarrow 0(2y_1 + 2y_2 + \frac{1}{4}y_3 + 3) = 0$ NO.

$x_3(-3y_2 - 2y_3 - 1) = 0 \Rightarrow 2(-3y_2 - 2y_3 - 1) = 0$ ok

$$\begin{cases} 3y_1 + 4y_2 + 2y_3 - 2 = 0 & \Rightarrow y_1 = \frac{2}{3} \\ -3y_2 - 2y_3 - 1 = 0 \Rightarrow -2y_3 = 1 \Rightarrow y_3 = -\frac{1}{2} \\ y_2 = 0 \end{cases}$$

$1 \leq 2$ NO SOL. OTTIMA

$\frac{29}{24} \leq -3$

$$x^{(2)} = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix}$$

$y_1(3x_1 + 2x_2 - 12) = 0 \Rightarrow y_1 \, 0 = 0$ NO

$y_2(4x_1 + 2x_2 - 3x_3 - 2) = 0 \Rightarrow y_2 10 = 0 \Rightarrow$ SI

$y_3(2x_1 + \frac{1}{4}x_2 - 2x_3 - 4) = 0 \Rightarrow -y_3 \frac{5}{2} = 0 \Rightarrow$ SI

$$\begin{cases} 2y_1 + 2y_2 + \frac{1}{4}y_3 + 3 = 0 \\ y_2 = 0 \\ y_3 = 0 \end{cases} \Rightarrow \quad y_1 = -\frac{3}{2}$$

$-3 \leq 2$

$-3 \leq -3$ ok

$0 \leq 1$

$x^{(2)}$ è OTTIMA

$0 \geq 0, \ 0 \leq 0, \ y_1 = -\frac{3}{2}$

3

$$\max \; -4x_1 + 3x_2 - x_3$$
$$x_1 + 3x_2 \geq 10$$
$$x_1 - x_2 + 4x_3 \geq 8$$
$$x_1 \geq 0, \; x_2 \leq 0, \; x_3 \geq 0$$

b) Può esistere una soluzione di base ammissibile con x_2 e x_3 in base

$$\max \; -4x_1 + 3x_2 - x_3$$
$$x_1 + 3x_2 \geq 10$$
$$x_1 - x_2 + 4x_3 \geq 8$$
$$x_1 \geq 0, \; x_2 \leq 0, \; x_3 \geq 0$$

STD $\Rightarrow$

$$\max \; -4x_1 - 3\hat{x}_2 + x_3$$
$$x_1 - 3\hat{x}_2 - x_4 = 10$$
$$x_1 + \hat{x}_2 + 4x_3 + x_5 = 8$$
$$x_1 \geq 0, \; \hat{x}_2 \geq 0, \; x_3 \geq 0$$

$$\begin{bmatrix} 0 \\ x_2 > 0 \\ x_3 > 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{cases} -3\hat{x}_2 = 10 \Rightarrow \hat{x}_2 = -\frac{10}{3} \\ \hat{x}_2 + 4x_3 = 8 \Rightarrow x_3 = \end{cases}$$

$$x_2 = \frac{10}{3}, \quad x_3 = \frac{7}{6}$$

$$x = \begin{bmatrix} 0 \\ \frac{10}{3} \\ \frac{7}{3} \\ 0 \\ 6 \end{bmatrix}$$

c) Può esistere una soluzione ottima del problema con x_1 in base

$$\max -4x_1 - 3\hat{x}_2 + x_3$$

$$x_1 - 3\hat{x}_2 \qquad - x_4 \qquad = 10$$

$$x_1 + \hat{x}_2 + 4x_3 \qquad + x_5 = 8$$

$$x_1 \geq 0, \ \hat{x}_2 \geq 0, \ x_3 \geq 0$$

$$\begin{bmatrix} x_1 > 0 \\ x_2 > 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} x_1 > 0 \\ 0 \\ x_3 > 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} x_1 > 0 \\ 0 \\ 0 \\ x_4 > 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} x_1 > 0 \\ 0 \\ 0 \\ 0 \\ x_5 > 0 \end{bmatrix}$$

$$\begin{cases} x_1 - 3\hat{x}_2 = 10 \Rightarrow x_1 = 10 + 3\hat{x}_2 = 10 - 3x_2 \\ x_1 + \hat{x}_2 = 8 \Rightarrow 10 + 3\hat{x}_2 + \hat{x}_2 = 8 \Rightarrow 4\hat{x}_2 = -2 \Rightarrow \hat{x}_2 = -\frac{1}{2} \end{cases}$$

$$\Rightarrow x_2 = \frac{1}{2} \Rightarrow x_1 = 10 - \frac{3}{2} = \frac{17}{2}$$

$$\begin{bmatrix} \frac{17}{2} \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$