(2) $L = \{ a^n b^m c^k : k = 2(n+m) \}$ $\quad n \geq 0, \quad m \geq 0$

$a^n b^m c^{2n+2m} = a^n b^m c^n c^n c^m c^m$

$S \to aSc c \mid A \mid \varepsilon$

$A \to bAcc \mid \varepsilon$

$\Downarrow$ $\varepsilon$-transizione

$S \to aScc \mid A \mid acc$

$A \to bAcc \mid bcc$

$\Downarrow$ unitarie

$S \to aScc \mid bAcc \mid acc \mid bcc$

$A \to bAcc \mid bcc$

$\Downarrow$ add $\varepsilon$

$S' \to S \mid \varepsilon$

$S \to aScc \mid bAcc \mid acc \mid bcc$

$A \to bAcc \mid bcc$

$\Downarrow$ CNF

$S' \to Z_0 Z_1 \mid Z_2 Z_1 \mid X_a Z_1 \mid X_b Z_1 \mid \varepsilon$

1 $S \to Z_0 Z_1 \mid Z_2 Z_1 \mid X_a Z_1 \mid X_b Z_1 \mid$

2 $A \to Z_2 Z_1 \mid X_b Z_1$

3 $Z_0 \to aS$

4 $Z_1 \to X_c X_c$

6 $Z_2 \to X_b A$

6 $X_a \to a$

7 $X_b \to b$

8 $X_c \to c$

⇓ GNF

$S' \to aSZ_1 \mid bAZ_1 \mid aZ_1 \mid bZ_1 \mid \varepsilon$

1 $S \to aSZ_1 \mid bAZ_1 \mid aZ_1 \mid bZ_1$

2 $A \to bAZ_1 \mid bZ_1$

3 $Z_0 \to aS$

4 $Z_1 \to cX_c$

6 $Z_2 \to bA$

6 $X_a \to a$

7 $X_b \to b$

8 $X_c \to c$

② $L = \{ a^* b^k c^* a^k b^* \mid k \geq 4 \}$

Sia $n \geq 4$. Scegliamo una stringa $z = b^n a^n$.

Dividiamo $z$ in $uvw$ con $|uv| \leq n$, $|v| \geq 1$.

$uv = b^h$ con $1 \leq h \leq n$, $v = b^\ell$ con $1 \leq \ell \leq h$ e

$u = b^{h-\ell}$. $w = b^{n-h} a^n$. Scegliamo $i = 2$

$$uv^2w = b^{h-\ell} b^\ell b^\ell b^{n-h} a^n = b^{n+\ell} a^n \notin L$$ poiché il le $b$ sono almeno una in più.

③

$$L = \{ a^p b^{p+2q} a^q \mid p, q > 0 \}$$

$a^p b^p | b^q b^q a^q$ (X | Y)

$S \to XY$

$X \to aXb \mid ab$

$Y \to bbYa \mid bbba$

⇓

$S \to XY$

$X \to Z_0 B \mid AB$

$Y \to Z_1 Z_2 \mid Z_1 A$

$Z_0 \to AX$

$Z_1 \to BB$ $Z_2 \to YA$ $A \to a$ $B \to b$

$S \to aXBY \mid aBY$

$X \to aXB \mid aB$

$Y \to bBZ_2 \mid bBA$

$Z_0 \to aX$

$Z_1 \to bB$

$Z_2 \to bBZ_2A \mid bBAA$

$A \to a$

$B \to b$

$Z_2 \to YA \Rightarrow Z_2 \to Z_1Z_2A \mid Z_1AA$

$Z_2 \to BBZ_2A \mid BBAA$

PDA:

$\delta(q_0, a, S) = \{(XBY), (BY)\}$

$\delta(q_0, a, X) = \{(XB), (B)\}$

$\delta(q_0, b, Y) = \{(BZ_2), (BA)\}$

$\delta(q_0, aZ_0) = (X)$

$\delta(q_0, b, Z_1) = (B)$

$\delta(q_0, b, Z_2) = \{(BZ_2A), (BAA)\}$

$\delta(q_0, a, A) = (q_0, \varepsilon)$

$\delta(q_0, b, B) = (q_0, \varepsilon)$

④ $L = \{ a^i b^j c^k \mid i+j \geq 3,\ k \bmod 3 = 0 \}$