(1)

$$\begin{cases} x_2 + 3x_3 \le 5 \\ x_1 + x_2 + x_3 \le 8 \\ x_1 - 5x_3 \le 4 \\ x_1 \ge 0, x_2 \ge 0, x_3 \ge 0 \end{cases} \quad \overset{STD}{\Rightarrow} \quad \begin{cases} x_2 + 3x_3 + x_4 = 5 \\ x_1 + x_2 + x_3 + x_5 = 8 \\ x_1 - 5x_3 + x_6 = 4 \\ x_1, x_2, x_3, x_4, x_5, x_6 \ge 0 \end{cases}$$

a) $x^{(1)} = \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}$; $x^{(2)} = \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$; $x^{(3)} = \begin{bmatrix} \frac{7}{2} \\ \frac{9}{2} \\ 0 \end{bmatrix}$ Sono SBA?

x^1

- Verifica ammissibilità $\begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}$

$5 \le 5$ ok

$8 \le 8$ ok

$3 \le$ ok

$3 \ge 0, 5 \ge 0, 0 \ge 0$ ok

- SBA?

$5 + x_4 = 5 \Rightarrow x_4 = 0$

$8 + x_5 = 8 \Rightarrow x_5 = 0 \Rightarrow [3, 5, 0, 0, 0, 1]$

$3 + x_6 = 4 \Rightarrow x_6 = 1$

SBA!

$\#$ var $\neq 0$ = $\#$ vincoli

x^2 • Verifico ammissibilità $\begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$

$4 \le 5$

$8 \le 8$

$4 \le 4$

$4 \ge 0, 4 \ge 0, 0 \ge 0$

- verifico SBA

$4 + x_4 = 5 \Rightarrow x_4 = 1$

$8 + x_5 = 8 \Rightarrow x_5 = 0$

$4 + x_6 = 4 \Rightarrow x_6 = 0$

$[4, 4, 0, 1, 0, 0]$

$\#$ var $\neq 0$ = $\#$ vincoli $\Rightarrow$ SBA

x^3 • verifico ammissibilità $\begin{bmatrix} \frac{7}{2} \\ \frac{9}{2} \\ 0 \end{bmatrix}$

$4,5 \leq 5$

$8 \leq 8$

$3,5 \leq 4$

$3,5 \geq 0,\ 4,5 \geq 0,\ 0 \geq 0$

• verifico SBA

$4,5 + x_4 = 5 \Rightarrow x_4 = \frac{1}{2}$

$8 + x_5 = 8 \Rightarrow x_5 = 0$

$3,5 + x_6 = 4 \Rightarrow x_6 = \frac{1}{2}$

$\left[\frac{7}{2}, \frac{9}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}\right]$ $\#$ vor to $\neq$ $\#$ vincoli

$\Rightarrow$ NO SBA

②

$$\min\ 2x_1 - 3x_2 + x_3$$
$$3x_1 + 2x_2 = 12$$
$$4x_1 + 2x_2 - 3x_3 \geq 2$$
$$2x_1 + \frac{1}{4}x_2 - 2x_3 \leq 4$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

std $\Rightarrow$

$$\min\ 2x_1 - 3x_2 + x_3 \qquad x_1, x_2, x_3, x_4, x_5 \geq 0$$
$$3x_1 + 2x_2 = 12$$
$$4x_1 + 2x_2 - 3x_3 - x_4 = 2$$
$$2x_1 + \frac{1}{4}x_2 - 2x_3 + x_5 = 4$$

a) $x^{(1)} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$, $y^{(2)} = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix}$, $x^{(3)} = \begin{bmatrix} 1 \\ \frac{9}{2} \\ 0 \end{bmatrix}$

• verifica ammissibilità

$12 = 12$ ok	$12 = 12$ ok	$12 = 12$ ok
$10 \geq 2$	$12 \geq 2$	$13 \geq 2$
$4 \leq 4$	$\frac{3}{2} \leq 4$	$\frac{25}{8} \leq 4$
$4 \geq 0,\ 0 \geq 0,\ 2 \geq 0$	$0 \geq 0,\ 6 \geq 0,\ 0 \geq 0$	$1 \geq 0,\ \frac{9}{2} \geq 0,\ 0 \geq 0$

• SBA

$12 = 12$

$10 - x_4 = 2 \Rightarrow x_4 = 8$

$4 + x_5 = 4 \Rightarrow x_5 = 0$

$[4, 0, 2, 8, 0]$

\# var $\neq 0$ = \# vincoli $\Rightarrow$ SBA

$12 = 12$

$12 - x_4 = 2 \Rightarrow x_4 = 10$

$\frac{3}{2} + x_5 = 4 \Rightarrow x_5 = \frac{5}{2}$

$\left[0, 6, 0, 10, \frac{5}{2}\right]$

vincoli = # var $\neq 0 \Rightarrow$ SBA

$12 = 12$

$13 - x_4 = 2 \Rightarrow x_4 = 11$

$\frac{25}{8} + x_5 = 4 \Rightarrow x_5 = \frac{7}{8}$

$\left[1, \frac{9}{2}, 0, 11, \frac{7}{8}\right]$

no SBA

b) $x^{(1)} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$$\begin{aligned} \min\ & 2x_1 - 3x_2 + x_3 \\ & 3x_1 + 2x_2 = 12 \\ & 4x_1 + 2x_2 - 3x_3 \geq 2 \\ & 2x_1 + \frac{1}{4}x_2 - 2x_3 \leq 4 \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 \end{aligned}$$

$$\begin{aligned} \max\ & 12y_1 + 2y_2 + 4y_3 \\ & 3y_1 + 4y_2 + 2y_3 \leq 2 \\ & 2y_1 + 2y_2 + \frac{1}{4}y_3 \leq -3 \\ & -3y_2 - 2y_3 \leq 1 \\ & y_2 \geq 0,\ y_3 \leq 0,\ y_1 \text{ libera} \end{aligned}$$

$y_1(3x_1 + 2x_2 - 12) = 0 \Rightarrow y_1 0 = 0$ NO

$y_2(4x_1 + 2x_2 - 3x_3 - 2) = 0 \Rightarrow y_2 10 = 0 \Rightarrow y_2 = 0$ ok

$y_3(2x_1 + \frac{1}{4}x_2 - 2x_3 - 4) = 0 \Rightarrow y_3 0 = 0$ NO

$x_1(3y_1 + 4y_2 + 2y_3 - 2) = 0 \Rightarrow 4(3y_1 + 4y_2 + 2y_3 - 2) = 0$ ok

$x_2(2y_1 + 2y_2 + \frac{1}{4}y_3 + 3) = 0 \Rightarrow 0(2y_1 + 2y_2 + \frac{1}{4}y_3 + 3) = 0$ NO.

$x_3(-3y_2 - 2y_3 - 1) = 0 \Rightarrow 2(-3y_2 - 2y_3 - 1) = 0$ ok

$$\begin{cases} 3y_1 + 4y_2 + 2y_3 - 2 = 0 & \Rightarrow y_1 = \frac{2}{3} \\ -3y_2 - 2y_3 - 1 = 0 \Rightarrow -2y_3 = 1 \Rightarrow y_3 = -\frac{1}{2} \\ y_2 = 0 \end{cases}$$

$1 \leq 2$ NO SOL. OTTIMA

$\frac{29}{24} \leq -3$

$$x^{(2)} = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix}$$

$y_1(3x_1 + 2x_2 - 12) = 0 \Rightarrow y_1 \, 0 = 0$ NO

$y_2(4x_1 + 2x_2 - 3x_3 - 2) = 0 \Rightarrow y_2 10 = 0 \Rightarrow$ SI

$y_3(2x_1 + \frac{1}{4}x_2 - 2x_3 - 4) = 0 \Rightarrow -y_3 \frac{5}{2} = 0 \Rightarrow$ SI

$$\begin{cases} 2y_1 + 2y_2 + \frac{1}{4}y_3 + 3 = 0 \\ y_2 = 0 \\ y_3 = 0 \end{cases} \Rightarrow y_1 = -\frac{3}{2}$$

$-3 \leq 2$

$-3 \leq -3$ ok

$0 \leq 1$

$x^{(2)}$ è OTTIMA

$0 \geq 0, \; 0 \leq 0, \; y_1 = -\frac{3}{2}$

③ $\max \; -4x_1 + 3x_2 - x_3$

$$x_1 + 3x_2 \geq 10$$
$$x_1 - x_2 + 4x_3 \geq 8$$
$$x_1 \geq 0,\; x_2 \leq 0,\; x_3 \geq 0$$

b) Può esistere una soluzione di base ammissibile con x_2 e x_3 in base

$\max \; -4x_1 + 3x_2 - x_3$ STD $\Rightarrow$ $\max \; -4x_1 - 3\hat{x}_2 + x_3$

$$x_1 + 3x_2 \geq 10 \qquad x_1 - 3\hat{x}_2 - x_4 = 10$$
$$x_1 - x_2 + 4x_3 \geq 8 \qquad x_1 + \hat{x}_2 + 4x_3 + x_5 = 8$$
$$x_1 \geq 0,\; x_2 \leq 0,\; x_3 \geq 0 \qquad x_1 \geq 0,\; \hat{x}_2 \geq 0,\; x_3 \geq 0$$

$$\begin{bmatrix} 0 \\ x_2 > 0 \\ x_3 > 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \begin{cases} -3\hat{x}_2 = 10 \Rightarrow \hat{x}_2 = \frac{-10}{3} \\ \hat{x}_2 + 4x_3 = 8 \Rightarrow x_3 = \end{cases}$$

$$x_2 = \frac{10}{3},\quad x_3 = \frac{7}{6}$$

$$X = \begin{bmatrix} 0 \\ \frac{10}{3} \\ \frac{7}{3} \\ 0 \\ 6 \end{bmatrix}$$

c) Può esistere una soluzione ottima del problema con x_1 in base

$$\max -4x_1 - 3\hat{x}_2 + x_3$$

$$x_1 - 3\hat{x}_2 \quad - x_4 \quad = 10$$

$$x_1 + \hat{x}_2 + 4x_3 \quad + x_5 = 8$$

$$x_1 \geq 0,\ \hat{x}_2 \geq 0,\ x_3 \geq 0$$

$$\begin{bmatrix} x_1 > 0 \\ x_2 > 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} x_1 > 0 \\ 0 \\ x_3 > 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} x_1 > 0 \\ 0 \\ 0 \\ x_4 > 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} x_1 > 0 \\ 0 \\ 0 \\ 0 \\ x_5 > 0 \end{bmatrix}$$

$$\begin{cases} x_1 - 3\hat{x}_2 = 10 \Rightarrow x_1 = 10 + 3\hat{x}_2 = 10 - 3x_2 \\ x_1 + \hat{x}_2 = 8 \Rightarrow 10 + 3\hat{x}_2 + \hat{x}_2 = 8 \Rightarrow 4\hat{x}_2 = -2 \Rightarrow \hat{x}_2 = -\frac{1}{2} \end{cases}$$

$$\Rightarrow x_2 = \frac{1}{2} \Rightarrow x_1 = 10 - \frac{3}{2} = \frac{17}{2}$$

$$\begin{bmatrix} \frac{17}{2} \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\min \ \gamma x_1 - 3x_2 + x_3$$

$$x^{(3)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$3x_1 + 2x_2 \leq 12 - \delta$$

$$4x_1 - x_2 - 3x_3 \geq 2$$

$$x_1 \geq 0, \ x_2 \leq 0, \ x_3 \geq 0$$

$\Downarrow$ DUALE

$$\max \ (12-\delta) y_1 + 2y_2$$

$$3y_1 + 4y_2 \leq \gamma$$

$$2y_1 - y_2 \geq -3$$

$$-3y_2 \leq 1$$

$$y_1 \leq 0, \ y_2 \geq 0$$

$\Rightarrow$ CONDIZIONI COMPLEMENTARIETÀ

$[1, 0, 0]$

$$y_1 (3x_1 + 2x_2 - 12 + \delta) = 0 \Rightarrow y_1 (3 - 12 + \delta) = 0$$

$$y_2 (4x_1 - x_2 - 3x_3 - 2) = 0 \Rightarrow y_2 2 = 0$$

$$y_1 (-9 + \delta) = 0 \quad 2 \Rightarrow \delta = 9 \quad \text{ok}$$

$$y_2 = 0$$

$$\max \; -4x_1 + 3x_2 - x_3$$
$$x_1 + 3x_2 \geq 10$$
$$x_1 - x_2 + 4x_3 \geq 8$$
$$x_1 \geq 0, \; x_2 \leq 0 \; ; \; x_3 \geq 0$$

STD $\Rightarrow$

$$\min \; 4x_1 + 3\bar{x}_2 + x_3$$
$$x_1 - 3\bar{x}_2 \quad - x_4 \quad = 10$$
$$x_1 + \bar{x}_2 + 4x_3 \quad - x_5 = 8$$
$$x_1 \geq 0, \; \bar{x}_2 \geq 0, \; x_3 \geq 0$$
$$x_4 \geq 0 \quad x_5 \geq 0$$

$\Downarrow$ DUALE

$$\max \; 10y_1 + 8y_2$$
$$y_1 + y_2 \leq 4$$
$$-3y_1 + y_2 \leq 3$$
$$4y_2 \leq 1$$
$$-y_1 \leq 0$$
$$-y_2 \leq 0$$
$$y_1, y_2 \in \mathbb{R}$$

$$y^0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

AMMISSIBILITÀ $\Rightarrow$

$0 \leq 4$ OK

$0 \leq 3$

$0 \leq 1$

$0 \leq 0$

$0 \leq 0$

$$\mathcal{S}d = \begin{pmatrix} 4 \\ 3 \\ 1 \\ 0 \\ 6 \end{pmatrix} \Rightarrow \bar{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ x_4 \\ x_5 \end{pmatrix}$$

complementarietà

dal sistema

PRIMALE RISTRETTO

$$\min \quad a_1 + a_2$$

$$-x_4 + a_1 = 10$$

$$-x_5 + a_2 = 8$$

$$x_4, x_5, a_1, a_2 \geq 0$$

	x_4	x_5	a_1	a_2	
z	1	1	0	0	-18
x_4	-1	0	1	0	10
x_5	0	-1	0	1	8

$\neq 0$

la soluzione non va bene

DUALE RISTRETTO

$$\max \quad 10\pi_1 + 8\pi_2$$

$x_4)\ -\pi_1 \leq 0$

$x_5)\ -\pi_2 \leq 0$

$a_1)\ \pi_1 \leq 1$

$a_2)\ \pi_2 \leq 1$

$\pi_1, \pi_2 \in \mathbb{R}$

$$\pi = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$y^{(1)} = y^{(0)} + \theta\pi = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \theta \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \theta \\ \theta \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} 2\theta \\ 3\theta \end{pmatrix} = \begin{pmatrix} 2y_1 \\ 3y_2 \end{pmatrix}$$

$$\max \quad 10\theta + 8\theta$$

$$\left.\begin{aligned}
\theta + \theta &\le 4 &&\Rightarrow \theta \le 2\\
-3\theta + \theta &\le 3 &&\Rightarrow \theta \ge -\tfrac{3}{2}\\
4\theta &\le 1 &&\Rightarrow \theta \le \tfrac{1}{4}\\
-\theta &\le 0 &&\Rightarrow \theta \ge 0\\
-\theta &\le 0 &&\Rightarrow \theta \ge 0
\end{aligned}\right\} \Rightarrow 0 \le \theta \le \frac{1}{4}$$

$$\theta \in \mathbb{R}$$

$$y^1 = \begin{pmatrix} \frac{1}{4} \\ \frac{1}{4} \end{pmatrix}$$