$T(n) = 2T\left(\frac{n}{4}\right) + n\sqrt{n}$

$a = 2$
$b = 4$
$f(n) = n\sqrt{n}$

$n^{\log_4 2}$ vs $n\sqrt{n}$

$\sqrt{n}$ vs $n\sqrt{n}$

$\sqrt{n} = O(n\sqrt{n})$

$n\sqrt{n} \overset{?}{=} \Omega\left(n^{\frac{1}{2}+\frac{1}{2}}\right)$ $\varepsilon = \frac{1}{2}$

$n\sqrt{n} = \Omega(n)$ OK

$T(n) = \Theta(n\sqrt{n})$

$n \log n = \Omega(n)$

$$T(m) = 2T(m-4) + 1$$

$$\sum_{i=1}^{h} 2^i = 2^{h+1} - 1$$

$$= 2^{m-3} - 1$$

$$= \Theta(2^n)$$

② $2T(2T(n-8)+1)+1$

③ $2T(2T(2T(n-12)+1)+1)+1$

$\vdots$

$$T(n) = 2^i T(m-4i) + \sum_{j=1}^{i} 1$$

$m - 4i = 1$

$\Rightarrow \cancel{m} = -1 + 4i$

$\Rightarrow 4i = m - 1$

$$i = \frac{m-1}{4}$$

$$1 = 2^{\frac{n-1}{4}} T(1) + n$$

$=$

$$T(n) \neq 2^{\frac{n-1}{4}} + \Theta(n)$$

$$= \Theta(2^n)$$

$$2T\left(\frac{m}{4}\right) + \sqrt{m}$$

$b = 4$

$a = 2$

$f(n) = \sqrt{m}$

$m^{\log_4 2}$ vs $\sqrt{m}$

$\sqrt{m}$ vs $\sqrt{m}$

$$T(m) = \Theta(\sqrt{m}\log m)$$

$$T(m-1) + \sqrt{m}$$

$$\downarrow$$

$$T(m-1) + \sqrt{m} = T(m-2) + \sqrt{m} + \sqrt{m-1}$$

$$= T(m-3) + \sqrt{m} + \sqrt{m-1} + \sqrt{m-2}$$

$$\vdots$$

$$T(m-i) + \sum_{j=0}^{i} \sqrt{m-j} \qquad i = m-1$$

$m \; (\sqrt{m})$

$\vdots$

$m-1 \; (\sqrt{m})$

$\vdots$

$m-i \; (\sqrt{m})$

$\vdots$

1

$$T(n) = O(n\sqrt{m})$$

m

$\vdots$

$m-1$

$\vdots$

$\frac{m}{2}$ $\left(\sqrt{\frac{m}{2}}\right)$

ogni nodo $> \sqrt{\frac{m}{2}}$

$$\Rightarrow T(n) = \Omega(n\sqrt{m})$$

$$T(m) = T\left(\frac{99}{100} m\right) + m$$

$a = 1$

$b = \frac{99}{100}$

$f(m) = m$

$$m^{\log_{\frac{99}{100}}(1)} \quad vs \quad m$$

$$\underset{\substack{\parallel \\ 1}}{m^0} \; vs \; m$$

$$f(n) = \Omega(1)$$

$$T(m) = \Theta(m)$$

$$T(m-1) + m^3$$

$$T(m-1) + m^3 = T(n-2) + m^3 + (m-1)^3$$

$\vdots$

$$\underset{\substack{\parallel \\ 1}}{T(n-i)} + \sum_{j=0}^{i-1} (m-j)^3 \qquad i = m-1$$

$$\sum_{j=0}^{m-1} (m-j)^2 (m-j)$$

$$\sum_{j=0}^{m-2} (m^2 - 2mj + j^2)(m-j)$$

$$= \sum_{j=0}^{m-2} m^3 - (jm^2 - 2m^2 j) + (2mj^2 + nj^2) - j^3$$

$$= \sum_{j=0}^{m-2} m^3 - \sum_{j=0}^{m-2} 3m^2 j + \sum_{j=0}^{m-2} 3mj^2$$

$$- \sum_{j=0}^{m-2} j^3$$

$$= m^4 - 3m^2 \frac{(m-1)(m-2)}{2} - -$$

$$= - - \Theta(m^4)$$

$$T(n) = 4T\left(\frac{n}{16}\right) + n^2$$

$b = 16 \quad f(n) = n^2$

$a = 4$

$n^{\log_{16} 4}$ vs n^2

$n^{\frac{1}{2}}$ vs n^2

$n^2 = \Omega\left(n^{\frac{1}{2}}\right)$

$\varepsilon = \frac{1}{2} \quad n^2 = \Omega(n)$ ok

$T(n) = \Theta(n^2)$

$$T(m) = T(\sqrt{m}) + 1$$

$$T(m) = T(m^{\frac{1}{2}}) + 1$$

$$m = 2^x \Rightarrow x = \log_2 m$$

$$T(2^x) = T(2^{\frac{x}{2}}) + 1$$

$$R(x) = T(2^x)$$

$$R(x) = R\left(\frac{x}{2}\right) + 1 = O(\log(x))$$

$$R(x) = O(\log(x))$$

$$T(n) = \log(\log(m))$$