SOLUZIONI

1) (i) $\int 8\sqrt{x}\,dx = 8\int x^{1/2}\,dx = \dfrac{8 \cdot x^{3/2}}{3/2} + c$

$$= \boxed{\frac{16}{3}\sqrt{x^3} + c}$$

(ii) $\int \dfrac{3 - x^2}{x^4}\,dx = 3\int x^{-4}\,dx - \int x^{-2}\,dx$

$$= -x^{-3} + x^{-1} = \boxed{\frac{1}{x} - \frac{1}{x^3} + c}$$

(iii) $\int e^x (1 - 2x e^{-x})\,dx$

$$= \int e^x - 2x\,dx = \int e^x\,dx - 2\int x\,dx$$

$$= \boxed{e^x - x^2 + c}$$

(iv) $\int \dfrac{\sin x}{3} - 5\cos x\,dx = \boxed{-\dfrac{1}{3}\cos x - 5\sin x + c}$

(v) $\int tg^2 x\,dx = \int \dfrac{\sin^2 x}{\cos^2 x}\,dx = \int \dfrac{1 - \cos^2 x}{\cos^2 x}\,dx$

$$= \int \frac{1}{\cos^2 x}\,dx - \int dx = \boxed{tg x - x + c}$$

(vi) $\int \cos^2 x \sin x\,dx$

$\cos x = t$

$-\sin x\,dx = dt$

$$= \int -t^2\,dt = -\frac{t^3}{3} + c = \boxed{-\frac{\cos^3 x}{3} + c}$$

(vii) $\int \frac{\log^3 x}{x}\,dx$ $\qquad \log x = t$, $\quad \frac{1}{x}\,dx = dt$

$$\int t^3\,dt = \frac{t^4}{4} + c = \boxed{\frac{\log^4 x}{4} + c}$$

(viii) $\int \frac{x^2}{x^3+2}\,dx = \frac{1}{3}\int \frac{3x^2}{x^3+2}\,dx$

$x^3 = t$

$3x^2\,dx = dt \rightsquigarrow \frac{1}{3}\int \frac{1}{t+2}\,dt$

$$= \frac{1}{3}\log|t+2| + c = \boxed{\frac{1}{3}\log|x^3+2| + c}$$

(ix) $\int x\cos(x^2)\,dx$

$= \frac{1}{2}\int 2x\cos(x^2)\,dx$ $\qquad x^2 = t$, $\quad 2x\,dx = dt$

$$= \frac{1}{2}\int \cos t\,dt = \frac{1}{2}\sin t + c = \boxed{\frac{1}{2}\sin(x^2) + c}$$

(x) $\int \frac{1+e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int \frac{1+e^{\sqrt{x}}}{2\sqrt{x}}\,dx$

$\sqrt{x} = t$

$\frac{1}{2\sqrt{x}}\,dx = dt \rightsquigarrow = 2\int 1+e^t\,dt = 2t + 2e^t + c$

$$= \boxed{2\sqrt{x} + 2e^{\sqrt{x}} + c}$$

(xi) $\displaystyle\int \frac{2\operatorname{arctg} x + 1}{x^2+1}\,dx$ $\qquad \operatorname{arctg} x = t$

$$\frac{1}{1+x^2}\,dx = dt$$

$$\rightsquigarrow \int 2t+1\,dt = t^2+t+c$$

$$= \boxed{\operatorname{arctg}^2 x + \operatorname{arctg} x + c}$$

(xii) $\displaystyle\int (x+2)\sin x\,dx$ (integrazione per parti)

$$= \int x\sin x\,dx + 2\int \sin x\,dx$$

$$= [-x\cos x] - \int(-\cos x)\,dx + 2(-\cos x) + c$$

$$= -x\cos x + \sin x - 2\cos x + c$$

$$= \boxed{\sin x - (x+2)\cos x + c}$$

(xiii) $\displaystyle\int \operatorname{arctg} x\,dx$ (integrazione per parti)

$$= [x\operatorname{arctg} x] - \int \frac{x}{1+x^2}\,dx$$

$$= \boxed{x\operatorname{arctg} x - \frac{1}{2}\log(x^2+1) + c}$$

(xiv) $\displaystyle\int \frac{x}{1+x^2}\,dx = \boxed{\frac{1}{2}\log(x^2+1) + c}$

(XV) $$\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\int \frac{2x}{1+(x^2)^2}\,dx$$

$x^2 = t$
$2x\,dx = dt$

$$= \frac{1}{2}\int \frac{1}{1+t^2}\,dt = \frac{1}{2}\,\mathrm{arctg}(t) + c$$

$$= \boxed{\frac{1}{2}\,\mathrm{arctg}(x^2) + c}$$

(XVi) $$\int \frac{x^4+x^3+6}{x^2+x}\,dx$$

$$\begin{array}{rrr|l} x^4 & + x^3 & + 6 & x^2+x \\ \cline{4-4} -x^4 & - x^3 & & x^2 \\ \hline // & // & 6 & \end{array}$$

$\Rightarrow \quad x^4+x^3+6 = (x^2+x)x^2+6$

$$\Rightarrow \int x^2 + \frac{6}{x^2+x}\,dx = \int x^2\,dx + 6\int \frac{1}{x(x+1)}\,dx$$

$$\int \frac{A}{x} + \frac{B}{x+1}\,dx$$

$A(x+1) + Bx = 1$

$$\Rightarrow \begin{cases} A+B=0 \\ A=1 \end{cases}$$

$$\Rightarrow \int x^2\,dx + 6\int \frac{1}{x}\,dx - 6\int \frac{1}{x+1}\,dx$$

$$\frac{x^3}{3} + 6\log|x| - 6\log|x+1| + c$$

$$= \boxed{\frac{x^3}{3} + 6\log\left|\frac{x}{x+1}\right| + c}$$

(xvii) $\int \frac{x^2-x+1}{x^2-2x+1}\,dx$

$$\begin{array}{r|l} x^2-x+1 & x^2-2x+1 \\ \cline{2-2} -x^2+2x-1 & 1 \\ \hline // \quad x \quad // & \end{array}$$

$x^2-x+1=(x^2-2x+1)+x$

$$= \int 1 + \frac{x}{x^2-2x+1}\,dx = \int dx + \frac{1}{2}\int \frac{2x}{x^2-2x+1}\,dx$$

$$x + \frac{1}{2}\left(\int \frac{2x-2}{x^2-2x+1}\,dx + \int \frac{2}{x^2-2x+1}\,dx\right)$$

$$= x + \frac{1}{2}\log|x^2-2x+1| + \int \frac{1}{(x-1)^2}\,dx$$

$$= \boxed{x + \log|x-1| - \frac{1}{x-1} + c}$$

(xviii) $\int \frac{1}{9x^2+5-6x}\,dx \longrightarrow \Delta<0$

$$\Rightarrow 9x^2+5-6x = (3x-1)^2+4$$

$$= \int \frac{1}{4+(3x-1)^2}\,dx = \frac{1}{4}\int \frac{1}{1+\left(\frac{3x-1}{2}\right)^2}\,dx$$

$$\frac{3x-1}{2} = t$$

$$\frac{3}{2}dx = dt \rightsquigarrow dx = \frac{2}{3}dt$$

$$\Rightarrow \frac{1}{4}\cdot\frac{2}{3}\int \frac{1}{1+t^2}dt = \frac{1}{6}\operatorname{arctg} t + c$$

$$= \boxed{\frac{1}{6}\operatorname{arctg}\left(\frac{3x-1}{2}\right)+c}$$

(xix) $\displaystyle\int \frac{x^2-3x}{x^2-6x+8}\,dx$

$$\begin{array}{r|l} x^2-3x & x^2-6x+8 \\ \hline -x^2+6x-8 & 1 \\ \hline // \quad 3x-8 & \end{array} \Bigg\} \Rightarrow x^2-3x = (x^2-6x+8) + (3x-8)$$

$$= \int 1 + \frac{3x-8}{x^2-6x+8}dx = x + \int\frac{3x-8}{x^2-6x+8}dx$$

$$\int\frac{3x-8}{(x-4)(x-2)}dx = \int\frac{A}{x-4}+\frac{B}{x-2}dx$$

$$A(x-2)+B(x-4) = 3x-8$$

$$(A+B)x+(-2A-4B) = 3x-8 \Rightarrow \begin{cases} A+B=3 \\ -2A-4B=-8 \end{cases}$$

Scansionato con CamScanner

$$\begin{cases} B = 3 - A \\ -2A - 12 + 4A = -8 \end{cases} \quad \begin{cases} B = 3 - A \\ 2A = 4 \end{cases} \quad \begin{cases} B = 1 \\ A = 2 \end{cases}$$

$$\Rightarrow \int \frac{2}{x-4}\,dx + \int \frac{1}{x-2}\,dx = 2\log|x-4| + \log|x-2|$$

$$\Longrightarrow \boxed{x + 2\log|x-4| + \log|x-2| + C}$$

(XX) $$\int \frac{2x+3}{x^3+3x^2-4}\,dx$$

Fattorizziamo x^3+3x^2-4. Osserviamo che $x=1$ è una radice del polinomio. Allora per Ruffini

$$\begin{array}{c|ccc|c} & 1 & 3 & 0 & -4 \\ 1 & & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & // \end{array}$$

$$\Longrightarrow (x^3+3x^2-4) = (x-1)(x^2+4x+4)$$
$$= (x-1)(x+2)^2$$

$$\Rightarrow \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

Troviamo A, B e C

$$\frac{A(x+2)^2+(x+2)(x-1)B+C(x-1)}{(x-1)(x+2)^2}$$

$$= A(x^2+4x+4)+B(x^2+x-2)+C(x-1)$$

$$= \underbrace{(A+B)x^2+(4A+B+C)x+(4A-2B-C)}_{=2x+3}$$

$$\begin{cases} A+B=0 \\ 4A+B+C=2 \\ 4A-2B-C=3 \end{cases} \qquad \begin{cases} B=-A \\ 4A-A+C=2 \\ 4A+2A-C=3 \end{cases}$$

$$\begin{cases} B=-A \\ 3A+C=2 \\ 6A-C=3 \end{cases} \qquad \begin{cases} B=-5/9 \\ C=1/3 \\ A=5/9 \end{cases}$$

$$\Rightarrow \frac{5}{9(x-1)}-\frac{5}{9(x+2)}+\frac{1}{3(x+2)^2}$$

$$\Rightarrow \int \frac{5}{9(x-1)}\,dx-\int \frac{5}{9(x+2)}\,dx+\int \frac{1}{3(x+2)^2}\,dx$$

$$\frac{5}{9}\log|x-1|-\frac{5}{9}\log|x+2|-\frac{1}{3}\cdot\frac{1}{x+2}+c$$

$$= \boxed{\frac{5}{9}\log\left|\frac{x-1}{x+2}\right|-\frac{1}{3x+6}+c}$$

2) (i) $\int_0^1 x^3(x^4+1)^5 dx$

$= \frac{1}{4}\int_0^1 4x^3(x^4+1)^5 dx$ $\qquad x^4 = t$

$4x^3 dx = dt$

in questo caso gli estremi rimangono invariati

$= \frac{1}{4}\int_0^1 (t+1)^5 dt$

$= \frac{1}{24}\left[(t+1)^6\right]_0^1 = \frac{1}{24}\left[2^6 - 1\right] = \frac{63}{24} = \boxed{\frac{21}{8}}$

(ii) $\int_0^1 \frac{x^2}{x^3+1} dx = \frac{1}{3}\int_0^1 \frac{3x^2}{x^3+1} dx$

$= \frac{1}{3}\left[\log|x^3+1|\right]_0^1$

$= \frac{1}{3}(\log 2 - \log 1) = \boxed{\frac{\log 2}{3}}$

(iii) $\int_0^1 \frac{e^x}{1+e^{2x}} dx = \left[\operatorname{arctg}(e^x)\right]_0^1$

$= \operatorname{arctg}(e) - \operatorname{arctg}(1)$

$= \boxed{\operatorname{arctg}(e) - \frac{\pi}{4}}$

$$(iv) \int_{\pi/6}^{\pi/3} 2\,tg\,x\,dx = 2\int_{\pi/6}^{\pi/3} \frac{\sin x}{\cos x}\,dx$$

$$= -2\Big[\log|\cos x|\Big]_{\pi/6}^{\pi/3}$$

$$= -2\left[\log\left(\frac{1}{2}\right) - \log\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= -2\left[\log\left(\frac{1}{\sqrt{3}}\right)\right] = -2\log\left(3^{-1/2}\right)$$

$$= \boxed{\log 3}$$

$$(v) \int_1^3 3x^2 \log x\,dx = \Big[x^3\log x\Big]_1^3 - \int_1^3 x^3\cdot\frac{1}{x}\,dx$$

$$= \Big[x^3\log x\Big]_1^3 - \left[\frac{1}{3}x^3\right]_1^3$$

$$= 27\log 3 - 9 + \frac{1}{3}$$

$$= \boxed{27\log 3 - \frac{26}{3}}$$