Mining Data Streams (Part 2)

Lecturer: **Andrea Clementi**

University of Rome *Tor Vergata*

Note: These slides are an adaptation, with more details, of the slides by Jure Leskovec, Anand Rajaraman, Jeff Ullman available at http://www.mmds.org

Main Topics

- **Further Tasks on Data Streams:**
  - **(1) *Filtering* a data stream: *Bloom* filters**
    - Select elements with property **x** from Stream
  - **(2) *Counting* distinct elements: *Flajolet-Martin* Algorithm**
    - Number of distinct elements in the last **k** elements of the Stream
  - **(3) *Estimating* moments: *AMS* Algorithm**
    - Estimate Standard Deviation of last ***k*** elements of the Stream

(1) Filtering Data Streams

Filtering Data Streams

- **Stream Model:** Each element of data *Stream* ***X*** is a <u>*tuple*</u> **x**=<key_1,...,key_k>
- **Input:** Given a list of ``good'' key-values (items) ***S*** (*e.g.* a subset of good values for key_1)
- **Task:** Determine and <u>filter</u> those tuples **x** of **X** having its component key_1(x) in ***S***

- **Trivial solution:** Store all **S** in a Hash_Table **T** and, for each element **x, do**: *hash* key_1(x) and verify in **T**.
  - **Big Data:** Suppose there is no enough memory to store all of items in ***S*** in a Hash Table
    - E.g. we might be processing <u>*millions*</u> of filters on the same Stream

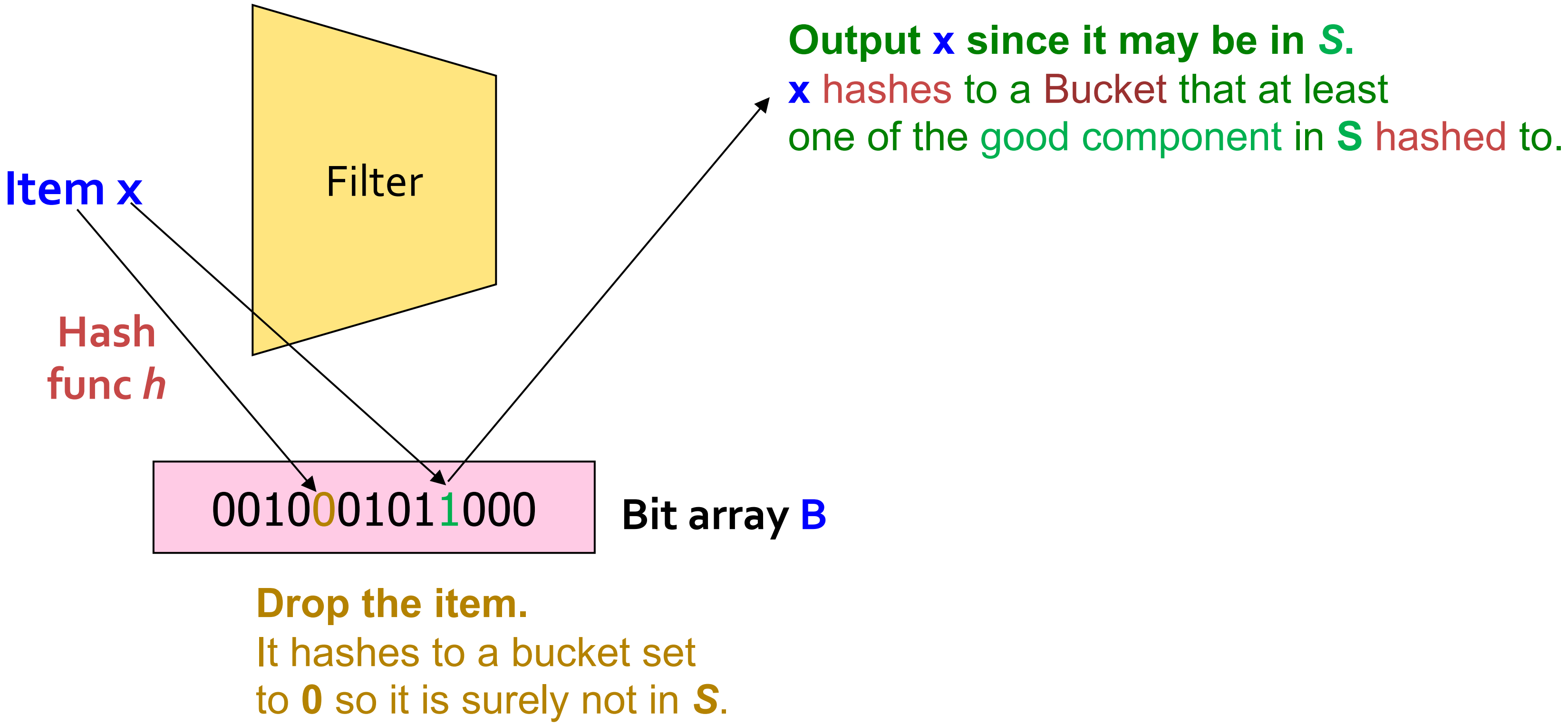
Applications

- **Example: Email spam filtering**
  - We know 1 billion "good" email addresses (key_1)
  - If an email comes from one of these, it is **NOT** *spam*
- **Publish-subscribe systems**
  - You are collecting lots of messages (news articles)
  - People express interest (only) in certain sets of keywords
  - Determine whether each message matches user's interest

First-Cut Algorithm

- **U** = {*all* values for key_1}; ***S*** = {good values for key_1}
- **Phase 1: Pre-processing**
  - **Create** an array **B[1...n]** of ***n*** bits, initially all *0*s
  - **Choose** a hash function ***h:* U → [n]**
  - **For each** component $s \in S$ **compute h**(s)
  - **Set B[h(s)] = 1**
- **Phase 2: Online** (Let **x**=<x_1,...,x_k> the new stream element)
  - **Compute h**(x_1)
  - **If B[h(**x_1**)] = 1** then **accept** stream element **x**

First-Cut Algorithm: Analysis



- Creates false positives but no false negatives
  - If the key-component of **x** is in ***S*** it surely accept it, if not we may still output it

First-Cut Algorithm: Analysis

- **|S| = m (# of good email addresses) e.g. 1 billion**
  |B|= n (size of the hash-array) e.g. 1GB = 8 billion bits

- **THM.** If the email address is in ***S*** ***(i.e.*** $x\_1 \in S$), then it surely hashes to a bucket that has the bit set to **1**, so it always gets through (***no false negatives***)

- Approximately m/n = **1/8** of the bits are set to **1**, so about **1/8** (= Prob[collision]) of the addresses <u>not</u> in ***S*** get through to the output (***false positives***)
  - Actually, less than **1/8**, because more than one address might hash to the same bit

Analysis: Throwing Balls (1)

More accurate analysis for the number of false positives.

Assume |S|= m

The size of the Bucket B is |B|=n

- **OBS:** For some item **u** from **U**, the probability to hit a *full* bucket is exactly the fraction *ERR* of **full** buckets in **B.** To estimate *ERR*, we play a *balls-into-bins* process:

- If we throw ***m*** balls into ***n*** equally likely bins, **what is the probability that a bin gets at least one ball?**

- **In our case:**
  - **Bin** = buckets: 0,1,….,n-1
  - **Balls** = random hash values **h(Key_1(x))** of items in **S**: 1,2,…,m
  - **Pr[one ball hits one bin] = 1/n** (Property of Hash Function **h(..)**)

then…..

Analysis: Throwing Balls (2)

- We have m balls into n bins
- What is the probability that a bin gets **at least one** ball?
- $\mathbf{Pr[Err] = 1 - (1 - 1/n)^m \simeq 1 - e^{-m/n}}$ (using apx: $(1-x) \simeq e^{x}$)

- **Note.** for $m << n$, $\mathbf{Pr[Err] \simeq m/n}$

Analysis: Throwing Balls (3)

- **Fraction of 1s (full Bins) in the array B =**
  = probability of false positive = $1 - e^{-m/n}$

- **Example: 10^9** balls (|S|), **$8 \cdot 10^9$** Bins
  - Fraction of full bins in **B = $1 - e^{-1/8}$ = 0.1175**
    - Compare with our earlier estimate: **1/8 = 0.125**

Bloom Filter

- Consider: $|S| = m$, $|B| = n$
- Use k independent hash functions $h_1, \ldots, h_k$
- **Initialization:**
  - Set **B** to all **0**s
  - Hash each element $s \in S$ using each hash function h_i, set $B[h_i(s)] = 1$ (for each $i = 1,.., k$)

(**note:** we have a single array B!)

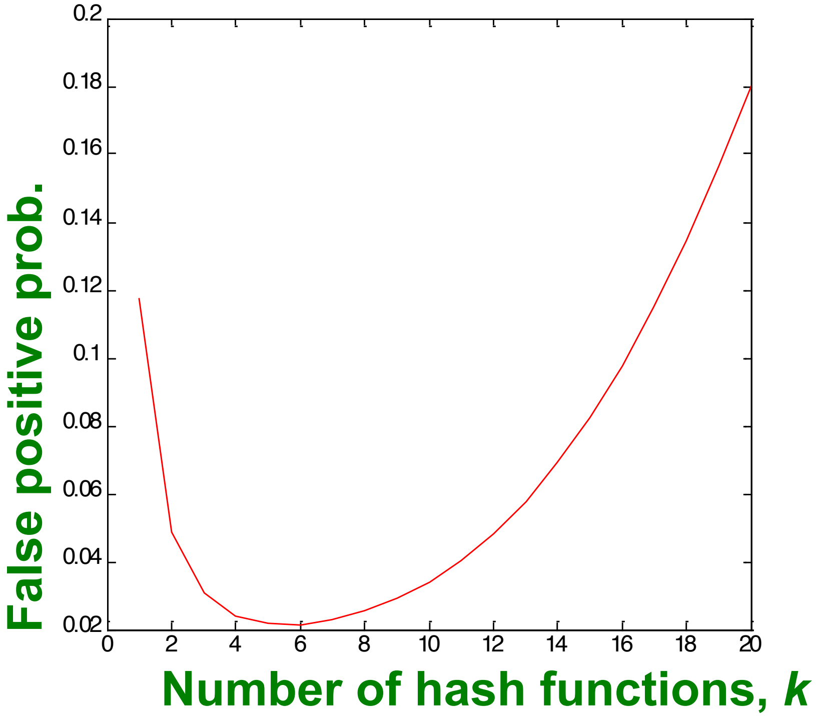
- **Run-time:**
  - When a stream element with key x arrives
    - If $B[h_i(x)] = 1$ for all $i = 1,..., k$ then declare that x is in S
      - That is, x hashes to a bucket set to **1** for every hash function $h_i(x)$
    - Otherwise discard the element x

Bloom Filter -- Analysis

- **What fraction of the bit vector B are 1s?**
  - Throwing $k \cdot m$ balls to n bins
  - So fraction of **1**s in **B** is $(1 - e^{-km/n}) \simeq km/n$
- But we have k <u>mutually-independent</u> hash functions and we only let the element x through **if all** k hash element x to a bucket of value **1**
- So, false **positive probability** = $(1 - e^{-km/n})^k$

Bloom Filter – Analysis (2)

- ***m* = 1 billion, *n* = 8 billion**
  - **k = 1**: $(1 - e^{-1/8})$ = **0.1175**
  - **k = 2**: $(1 - e^{-1/4})^2$ = **0.0493**



- **What happens as we keep increasing *k*?**

- "Optimal" value of ***k***: ***n/m*** **ln(2)**
  - **In our case:** Optimal **k = 8 ln(2) = 5.54 ≈ 6**
    - **Error at k = 6**: $(1 - e^{-1/6})^2$ = **0.0235**

Bloom Filter: Wrap-up

- **Bloom filters guarantee no false negatives, and use limited memory**
  - Great for pre-processing before more expensive checks
- **Suitable for hardware implementation**
  - Hash function computations can be *parallelized*
- Is it better to have **1** big **B** or ***k*** small **B**s?
  - **It is "the same":** $(1 - e^{-km/n})^k$ vs. $(1 - e^{-m/(n/k)})^k$
  - But keeping **1 big B** is simpler

(2) Counting Distinct Elements

Counting Distinct Elements

- **Problem:**
  - Data stream consists of a sequence of elements chosen from a universe set **U** of size ***N***
  - Maintain a count **d** of the *number* of *distinct* elements seen so far in the STREA
- **Obvious approach:** $S \subseteq U \quad (|S| << |U|)$
  Maintain the *set* of elements seen so far
  - That is, keep a ***hash table*** of all the *distinct* elements seen so far

Applications

- **How many different words are found among the Web pages being crawled at a site?**
  - Unusually low or high numbers could indicate artificial pages (spam?)
- **How many different Web pages does each customer request in a week?**
- **How many distinct products have we sold in the last week?**

Using Small Storage

- **Real problem:** What if we do not have *space* (i.e. energy) to maintain the **set** of elements seen so far?
- **d = # of distinct elements seen so far** we only need the number!

1. **Goal:** Compute **d**
2. **Relaxation:** Accept that our **answer** for **d** may have a *little error*, but limit the probability that the error is large

(1)+(2) = Probabilistic Approximation Algorithms

Flajolet-Martin "*Magic*" Approach

- **(Pre-processing) Pick** a hash function ***h*** that maps each of the ***N*** elements of **U** to *at least* $\log_2 N$ bits: $U = [N]$
  $h: [N] \rightarrow \{0,1\}^s$, $s \geq \log N$ (bits)

- **For each** *stream element* ***a*****, Let** (compute h(a)) and:
  - **r(a)** = position of first **1** counting from the right in **h(a)**
    - E.g., say ***h(a) = 01100*** in binary, so ***r(a) = 3***
- **Store** of the STREAM
  - **Sketch R = max { r(a)**, over all the items ***a*** seen so far **}**

- **Magic THM: Return** $\boxed{m = 2^R}$ ***as the Estimated Value*** APX ***for* d *(# of distinct elements seen so far)***

Flajolet-Martin "*Magic*" Approach

Nice **Properties** of the **FM-Algorithm**:

- Repeated occurrences of the same element do not affect the value of **R:** element a will always hash to the same value **h(a)** (so gets the same r(a)).

- Sketch **R** can be easily combined with others: if we have a collection of Sketches R_1, R_2,…, R_k from different Streams and wish to compute **d** in the combined stream, we can simply take
  - $\mathrm{Max}\{R_1, R_2, \dots, R_k\}$ = $\mathrm{MAX}\{\max\{r(a)\}_1, \dots, {}_k\}$

Why It Works: Intuition

- Very very rough and heuristic intuition why **Flajolet-Martin Algorithm** works:
  - ***h(a)*** hashes ***a*** with **equal prob.** to any of the ***N*** slots
  - Then ***h(a)*** is a sequence of *rnd* **$\log_2 N$** bits, where ***2^{-r}*** fraction of all ***a***s have a tail in *h(a)* of ***r*** zeros $\equiv \frac{2^{5-r}}{2^{5}}$
    - About 50% of ***a***s hash to *****0**
    - About 25% of ***a***s hash to ****00**
    - So, if we saw the *longest* tail of ***R=2*** (i.e., item hash ending ***100**) then we have probably seen **about *4*** distinct items so far $4 = 2^R$
  - So, *in expectation*, it takes to hash about **d** = ***2^R*** distinct items before we see one hash with zero-suffix of length ***R*, *i.e.*,**
  - ***2^R*** is the *expected number* of "trials" necessary and sufficient to see a sequence with zero-suffix of length ***R***

Flajolet-Martin "*Magic*" : Formal Analysis

- Assume **h**: $[N] \rightarrow \{0,1\}^s$, where $s \geq \log N$ (bits)
- Since **h** is a perfect hash function:
- **Fact 1.** For any a, it holds $\Pr(r(a) \geq r) = 1/2^r$ $\Leftarrow$ $\frac{2^{s-r}}{2^s}$
- Define *binary r.v.* $X_r = 1$ *iff* $\exists$ a in Stream s.t. $r(a) \geq r$
- **Fact 2.** $\Pr(X_r = 1) = 1 - (1-2^{-r})^d$ and $\Pr[X_r = 0] = (1-2^{-r})^d$
- **Remark:** d = real # of distinct elements seen so far OUR TARGET

Now, for the apx output $m = 2^R$, we can compute the Error Probability: for any $c>0$,

- $\Pr(m > 2^c \cdot d) = \Pr(R > \underbrace{\log d + c}_{r}) = \Pr(X_{\underbrace{\log d + c}_{=r}} = 1) \leq d / 2^{\log d + c} = 2^{-c}$

  (we used ineq. $(1-x)^d \geq 1 - xd$ for "small" $|xd| < 1$)

- $\Pr(m < 2^c \cdot d) = \Pr(X_{\log d + c} = 0) \leq \exp(-2^{c-1})$

  (we used ineq. $(1-x) \leq \exp(-x)$ for "small" $|x| < 1$)

Example: for c=2, we get a *8-apx* with Prob ≥ 2/3

For more detail, see file: Flaj_Martin_Algo_Analysis.pdf

Flajolet-Martin "*Magic*" : Space Complexity

- **Fact.** Space[F-M Algorithm] = O(loglog d), whp.

Proof. It is required to compute and store the quantity R.
From the previous analysis, it holds, whp,

$$\boxed{R \leq \log d + c}$$

Large Deviation and Amplification: Informal

- E[2^R] is actually **infinite**
  - Probability halves when ***R*** $\to$ ***R+1***, but value doubles
- Workaround involves using *many* hash functions h_i and getting many sketches R_i
- How are samples R_i **combined**?

SAMPLES = sketch

  - **Average?** What if one very large value 2^{R_i}?
  - **Median?** All estimates are a *power* of **2**
  - **Solution in Pratice:**
    - Partition your samples into small groups
    - Take the **median** of each group
    - Then take the *average* of the **medians**

OR VICEVERSA?

Improvements of FM approach

- In [Bar-Yossef, Jayram, Kumar, Sivakumar, Trevisan `04], the authors refine the **FM algorithm** to give:
- a **rnd** **(1- eps)-apx algorithm** for **d** using overall memory space:

$$\left[O(eps^{-2} * \log(1/eps)) + 2 \lg \lg n \right]$$

- See the file: Flaj_Martin_Algo_Analysis.pdf

(3) Computing Moments

Generalization: Moments

- Suppose a stream ***I*** has elements chosen from a set *A* of *N* values
- Let m_i be the number of times value *i* occurs in ***I***
- The k^{th} *moment* is

$$\sum_{i \in A} (m_i)^k$$

Special Cases

$$\sum_{i \in A} (m_i)^k$$

- **0th moment =** number of distinct elements
  - The problem just considered
- **1st moment =** $|$***I***$|$
  - Easy to compute
- **2nd moment =** ***surprise number S*** **=**
  a measure of how *uneven* the item **distribution** is

Example: Surprise Number

- **Stream of length 100**
- **11 distinct values**

- Item counts: **10, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9**
  Surprise *S* = 910

- Item counts: **90, 1, 1, 1, 1, 1, 1, 1 ,1, 1, 1**
  Surprise *S* = 8,110

The *AMS* Method: Informal

- *AMS* method works for *all* moments
- Gives an *unbiased* estimate
- **Today: evaluate the 2nd moment *S***
- **1st Step. pick** and **update** a sample of *i.i.d random* variables $\{X_j : j = 1 \ldots k\}$ *defined as follows:*
  - For each variable ***X*** we store ***X.el*** and ***X.val***
    - ***X.el*** corresponds to some item ***i*** *of* ***I*** *(see later...)*
    - ***X.val*** corresponds to the **count** of *future* occurrences of ***i***
  - Note this requires a count in main memory, so number k of ***X***s should be limited

The AMS Algorithm: Details

- How to set ***X.val*** and ***X.el***? (valid for all **X**s)
  - **Input:** stream ***I = I[1,…,L]*** of length ***L*** (we relax this later)
  - **Pick** some **time step *t u.a.r.*** in $1 \leq t < L$
  - ***Set X.el : = i, where i = I[t]***
  - **Compute** counter ***X.val : = c*** as the number of ***i's*** *occurrences* in the *sub-stream* ***I[t,…,L]***
- **Output:** the estimate of the **2nd moment** $\sum_i m_i^2$ is

$$S = f(X) = L \cdot (2 \cdot c - 1)$$

  - **Note**: The algorithm computes *multiple* **X**s: ($X_1, X_2, \ldots X_k$) and the final estimate will be $S = 1/k \sum_j^k f(X_j)$

The AMS Algorithm: Analysis

- Let *r.v.s* step **t**, element **i**, count **c**, and stream length ***L*** defined in the ***AMS*** Algorithm.
- $E(L \cdot (2\,c - 1)) = \sum_{i=1}^{L} L \cdot (2\,c(i) - 1) \cdot (1/L)$
  $= \sum_{i=1}^{L} L \cdot (2\,c(i) - 1) \quad (*)$

To evaluate $(*)$, for any fixed $a \in A$, let $j(1),\ldots j(m_a)$ the occurrences of a in the stream ***l=l[1,...,L]****, then*

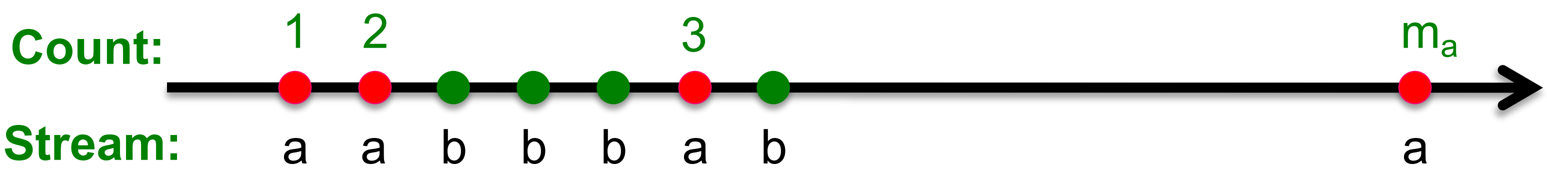
$\mathbf{c}(j(1)) = m_a$, $\mathbf{c}(j(2)) = m_a\text{-}1$, ... , $\mathbf{c}(j(m_a)) = 1$. $(**)$

Rewriting $(*)$ using $(**)$ and grouping w.r.t. each $a \in A$:

$(*) = \sum_{a\in A} \sum_{z=1}^{m\_a} (2j\text{-}1)$ and $\sum_{z=1}^{m\_a} (2j\text{-}1) = (m_a)^2$

So, $\boldsymbol{E(L \cdot (2\,c - 1)) = E(S)} = \sum_i m_i^2$

The AMS Algorithm: Analysis



- **2nd moment is** $S = \sum_i m_i^2$
- c_t = occurrences of item at time t appears from time **t** onwards ($c_1=m_a$, $c_2=m_a-1$, $c_3=m_b$)
- $E(S) = \frac{1}{n}\sum_{t=1}^{n} n(2c_t - 1)$

$$= \frac{1}{n}\sum_i n\,(1 + 3 + 5 + \cdots + 2m_i - 1)$$

m_i … total count of item i in the stream (we are assuming stream has length **n**)

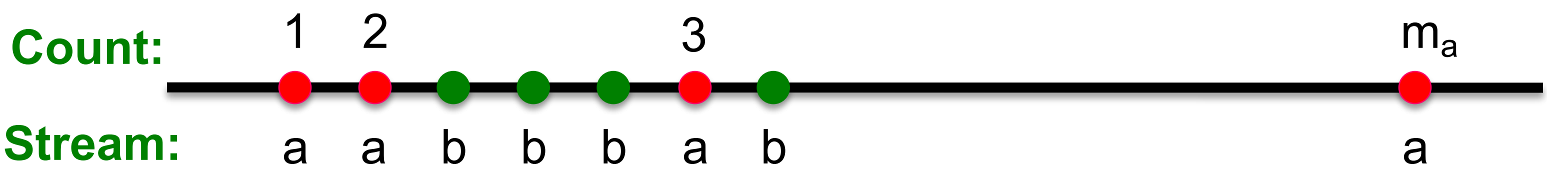
Group times by the value seen

Time t when the last i is seen (c_t=1)

Time t when the penultimate i is seen (c_t=2)

Time t when the first i is seen ($c_t=m_i$)

The AMS Algorithm: Analysis



- $E(S) = \frac{1}{n} \sum_i n\,(1 + 3 + 5 + \cdots + 2m_i - 1)$
  - Little side calculation: $(1 + 3 + 5 + \cdots + 2m_i - 1) = \sum_{i=1}^{m_i} (2i - 1) = 2\frac{m_i(m_i+1)}{2} - m_i = (m\_i)^2$
- **Then** $E(S = f(X)) = \frac{1}{n} \sum_i n\,(m_i)^2$
- **So,** $E(S) = \sum_i (m_i)^2$
- **We have the second moment (in expectation)!**

Higher-Order Moments

- **For estimating k^{th} moment we essentially use the same algorithm but change the estimate:**
  - For **k=2** we used ***n* (2·c – 1)**
  - For **k=3** we use: ***n* (3·c^2 – 3c + 1)** (where **c=X.val**)
- **Why?**
  - **For k=2:** Remember we had $(1 + 3 + 5 + \cdots + 2m_i - 1)$ and we showed terms ***2c-1*** (for **c=1,…,m**) sum to ***m*^{2}**
    - $\sum_{c=1}^{m} 2c - 1 = \sum_{c=1}^{m} c^2 - \sum_{c=1}^{m} (c-1)^2 = m^2$
    - **So:** $\boldsymbol{2c - 1 = c^2 - (c-1)^2}$
  - **For k=3: c^3 - (c-1)^3 = 3c^2 - 3c + 1**
- **Generally:** Estimate $= n\,(c^k - (c-1)^k)$

Combining Samples

- **In practice (to increase confidence):**
  - Compute $f(X) = n(2c - 1)$ for as many *mutually-indep.* variables X as you can fit in memory
  - *Average* them in groups
  - Take *Median* of Averages
- **Problem: Streams never end**
  - We assumed input Stream ***I*** has finite size ***L***:

    I = I[1,…,L]

  - But real streams go on forever, so ***L*** is a variable – the number of inputs seen so far

Streams Never End: Fixups

- **(1)** The variables ***X*** have ***L*** as a factor – keep ***L*** separately; just hold the count in ***X***
- **(2)** Suppose we can only store ***k*** counts. We must throw some ***X***s out as time goes on:
  - **Objective:** Each starting time ***t*** is selected with probability ***k/L***
  - **Solution: (fixed-size sampling!** see previous slides**)**
    - Choose the first ***k*** time (i.e. elements) for ***k*** variables (deterministic)
    - When the L^{th} element arrives (***L*** > ***k***), choose it with Prob = ***k/L***
    - If you choose it, throw one of the previously stored variables **X** out, ***u.a.r.***

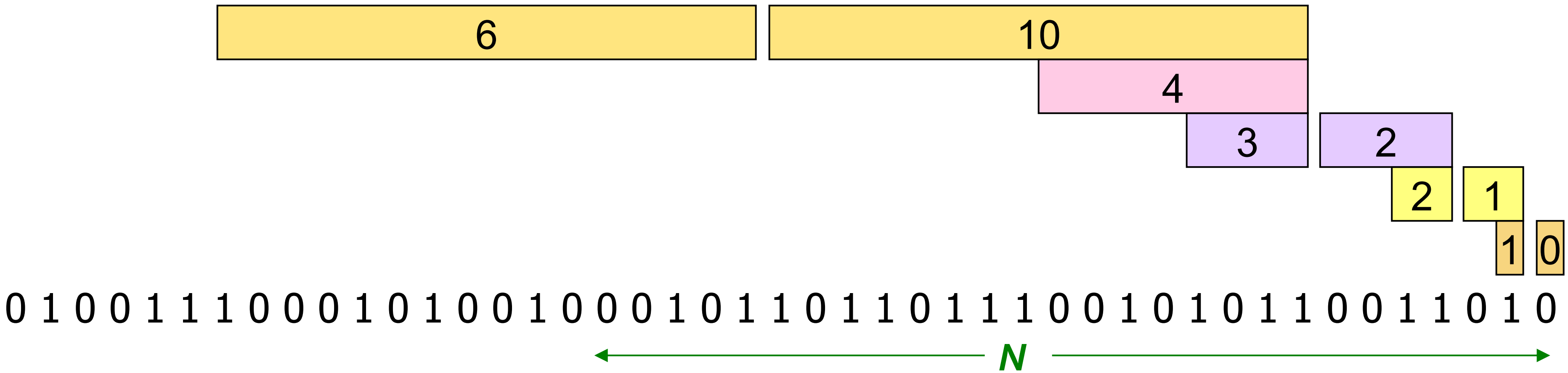
Mining Data Streams: Pattern Matching

- **Pattern Matching Problem:** *Karp-Rabin's Algorithm*
- **Section 4** delle Note di Prezza su Good Notes
- (fai anche Rabin Hashing)

Counting Itemsets

Counting Itemsets

- **New Problem:** **Given a stream, which items appear more than *s* times in the window?**
- **Possible solution:** Think of the stream of baskets as one binary stream per item
  - **1** = item present; **0** = not present
  - Use **DGIM** to estimate counts of **1**s for all items



Extensions

- **In principle, you could count frequent pairs or even larger sets the same way**
  - **One stream per itemset**
- **Drawbacks:**
  - Only approximate
  - **Number of itemsets is way too big**

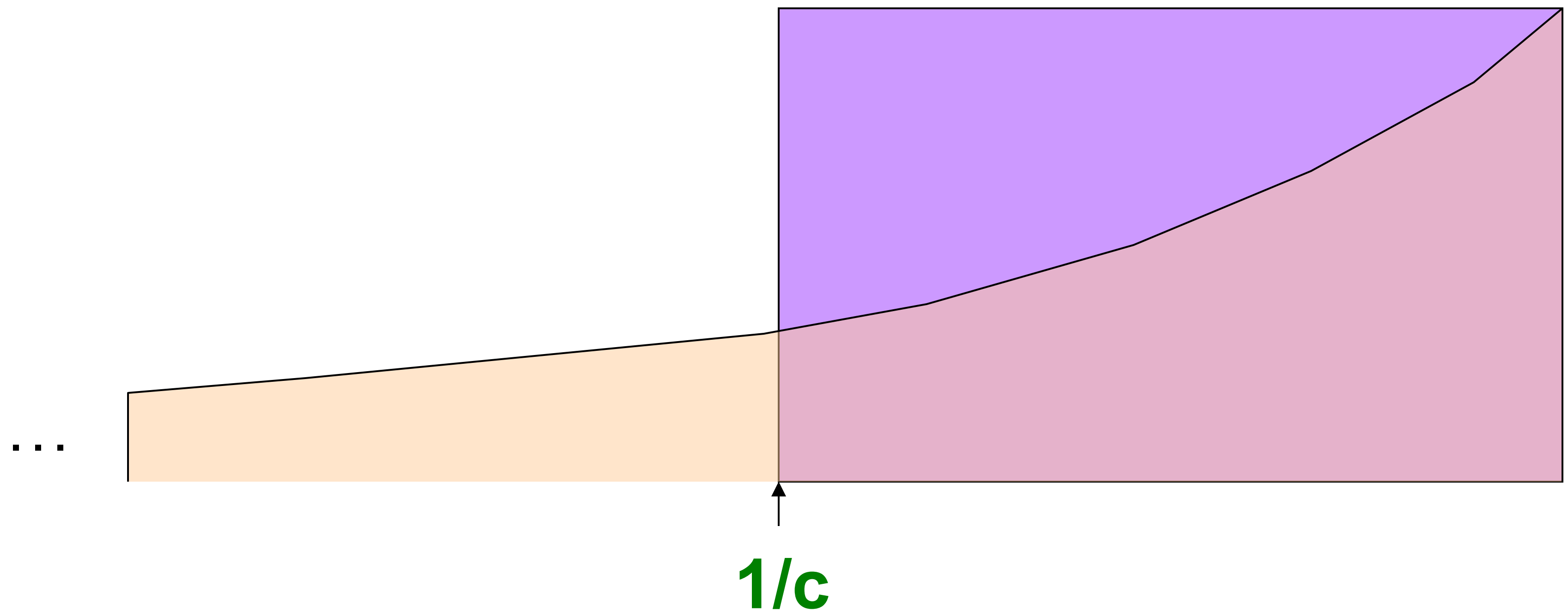
Exponentially Decaying Windows

- **Exponentially decaying windows: A heuristic for selecting likely frequent item(sets)**
  - **What are "currently" most popular movies?**
    - Instead of computing the raw count in last ***N*** elements
    - Compute a **smooth aggregation** over the whole stream
- If stream is $a_1, a_2, \ldots$ and we are taking the sum of the stream, take the answer at time t to be:
  $$= \sum_{i=1}^{t} a_i (1-c)^{t-i}$$
    - **c** is a constant, presumably tiny, like 10^{-6} or 10^{-9}
- **When new a_{t+1} arrives:**
  Multiply current sum by **(1-c)** and add a_{t+1}

Example: Counting Items

- If each a_i is an "item" we can compute the **characteristic function** of each possible item x as an Exponentially Decaying Window
  - That is: $\sum_{i=1}^{t} \delta_i \cdot (1-c)^{t-i}$
    where $\boldsymbol{\delta_i}$**=1** if $\mathbf{a_i}$**=x**, and **0** otherwise
  - Imagine that for each item x we have a binary stream (**1** if x appears, **0** if x does not appear)
  - New item x arrives:
    - Multiply all counts by **(1-c)**
    - Add **+1** to count for element x
- **Call this sum the "weight" of item** x

Sliding Versus Decaying Windows



- **Important property:** Sum over all weights $\sum_t (1-c)^t$ is **1/[1 – (1 – c)] = 1/c**

Example: Counting Items

- **What are "currently" most popular movies?**
- **Suppose we want to find movies of weight > ½**
  - **Important property:** Sum over all weights $\sum_t (1-c)^t$ is 1/[1 – (1 – c)] **= 1/c**
- **Thus:**
  - There cannot be more than **2/c** movies with weight of ½ or more
- So, **2/c** is a limit on the number of movies being counted at any time

Extension to Itemsets

- **Count (some) itemsets in an E.D.W.**
  - **What are currently “hot” itemsets?**
    - **Problem:** Too many itemsets to keep counts of all of them in memory
- **When a basket B comes in:**
  - Multiply all counts by **(1-c)**
  - For uncounted items in **B**, create new count
  - Add **1** to count of any item in **B** and to any **itemset** contained in **B** that is already being counted
  - **Drop counts < ½**
  - Initiate new counts (next slide)

Initiation of New Counts

- Start a count for an itemset $S \subseteq B$ if every proper subset of ***S*** had a count prior to arrival of basket ***B***
  - **Intuitively:** If all subsets of **S** are being counted this means they are "**frequent/hot**" and thus ***S*** has a potential to be "**hot**"
- **Example:**
  - Start counting ***S*={i, j}** iff both **i** and **j** were counted prior to seeing ***B***
  - Start counting ***S*={i, j, k}** iff **{i, j}**, **{i, k}**, and **{j, k}** were all counted prior to seeing ***B***

How many counts do we need?

- Counts for single items **< (2/c)·(avg. number of items in a basket)**
- **Counts for larger itemsets = ??**
- **But we are conservative about starting counts of large sets**
  - If we counted every set we saw, one basket of **20** items would initiate **1M** counts