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Numerical Methods (ENUME)
Assignment B: Approximation of functions
Spring Semester 2024

The file `f13.p` contains a MATLAB function of the following syntax:

```
function y = f13(x)
```

implementing a scalar, single-argument mathematical function $f(x) : \mathbb{R} \to \mathbb{R}$.

Task #1

Develop a MATLAB program for estimating the parameters $a_0, ..., a_K, b_1, ..., b_K \in \mathbb{R}$ of the trigonometric polynomial:

$$\hat{f}(x; a_0, ..., a_K, b_1, ..., b_K) = \sum_{k=0}^{K} a_k \cos(kx) + \sum_{k=1}^{K} b_k \sin(kx), \quad K \in \mathbb{N}$$

which minimises the criterion of the least-squares approximation of $f(x)$:

$$J(a_0, ..., a_K, b_1, ..., b_K) = \sum_{n=1}^{N} \left(\hat{f}(x_n; a_0, ..., a_K, b_1, ..., b_K) - f(x_n)\right)^2$$

where $x_1, ..., x_N$ are equidistant numbers from $-x_{\max}$ to $x_{\max}$ with $N = 100$.

Test the developed program for $K = 2, 4, 6$ and $x_{\max} = 0.5, 1.0, 1.5$.

Task #2

Determine the dependence of the mean-square approximation error:

$$e = \sqrt{\frac{1}{N} \sum_{n=1}^{N} \left(\hat{f}(x_n; a_0, ..., a_K, b_1, ..., b_K) - f(x_n)\right)^2}$$

on $K \in [1, 20]$ for several exemplary values of $x_{\max} \in [0.5, 2.0]$.

Task #3

Assuming that $f(x)$ needs to be approximated on the basis of error-corrupted data, modelled as follows:

$$\underline{\tilde{y}_n} = f(x_n) + \underline{\Delta\tilde{y}_n} \quad \text{for} \quad n = 1, ..., N$$

where $\underline{\Delta\tilde{y}_1}, ..., \underline{\Delta\tilde{y}_N}$ are random variables following a zero-mean normal distribution with variance σ_y^2, determine the dependence of the expected value $\bar{e}$ of the mean-square approximation error e on $x_{\max} \in [10^{-2}, 10^1]$ for $K = 4$ and several exemplary values of $\sigma_y^2 \in [10^{-12}, 10^{-2}]$.

In order to estimate $\bar{e}$, perform the approximation $R = 100$ times, each time using a different sequence of pseudorandom numbers $\Delta\tilde{y}_1, ..., \Delta\tilde{y}_N$ to emulate the random errors $\underline{\Delta\tilde{y}_1}, ..., \underline{\Delta\tilde{y}_N}$ (`randn`), and storing the obtained value of the mean-square approximation error as e_r (for $r = 1, ..., R$); estimate $\bar{e}$ according to the formula:

$$\bar{e} = \sqrt{\frac{1}{R}\sum_{r=1}^{R} e_r^2}$$

Task #4

Compare the results obtained using MATLAB's backslash operator (\) applied in two ways:

```
p = Phi \ f13(x);
p = (Phi' * Phi) \ (Phi' * f13(x));
```

where:

- `p` represents the vector of parameters $[a_0, ..., a_K, b_1, ..., b_K]^{\mathrm{T}}$,
- `Phi` represents the matrix of basis functions $\mathbf{\Phi}$ (*cf.* lecture slides #5-10 and #5-11),
- `x` represents the vector $[x_1, ..., x_N]^{\mathrm{T}}$.