

# Holographic Complexity and Volume

Work with Stefan Eccles, Ted Jacobson, and Phuc Nguyen  
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# Introduction

We would like to study properties of maximal volume slice. Why?

- Conjectured by Susskind to be dual to circuit complexity of CFT state.
- Regardless of complexity, can capture growth of wormhole for eternal black hole.
- New tools brought to life by Freedman and Headrick, Headrick and Hubeny (max-cut/min-flow).

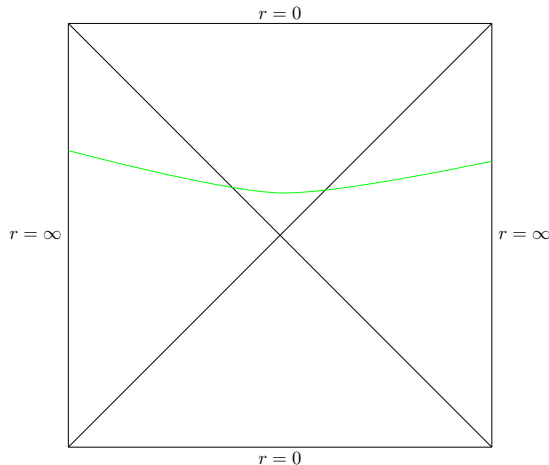


Figure : A maximal spatial slice

# Max-Cut/Min-Flow

We are interested in applying ideas similar to the bit-thread ideas of Freedman and Headrick. This makes use of the min-cut/max-flow theorem, by which you can find a maximum flux in place of a minimal area. In Lorentzian signature, this becomes max-cut/min-flow as detailed by Headrick and Hubeny.

- Let a *flow* be a timelike future directed divergence free vector field  $v$  such that everywhere in spacetime,  $1 \leq |v|$ .
- Consider a Lorentzian manifold with a timelike boundary, and let  $\sigma$  be a cauchy slice of that boundary.
- Then the volume of the maximal slice of the bulk bounded by  $\sigma$  can be found by finding the minimum over all flows of the flux through *any* slice bounded by  $\sigma$ .
- max-cut/min-flow has a *nesting property*, namely that given two boundary cauchy slices  $\sigma_1$  and  $\sigma_2$  such that  $\sigma_1$  is entirely to the past of  $\sigma_2$ , then there is a flow (highly non-unique) flow whose flux computes the maximal volumes bounded by both  $\sigma_1$  and  $\sigma_2$ .
- In fact, we could have taken an entire boundary foliation.

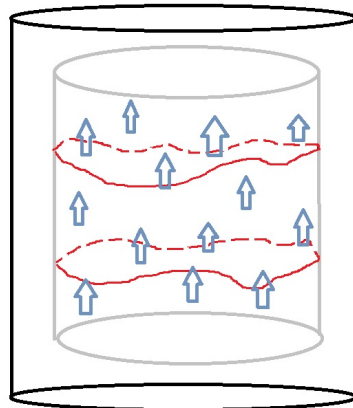


Figure : Volume Flow

# Volume Flow

- The flows defined above are in general highly non-unique.
- However, whenever a boundary foliation induces a bulk foliation by maximal slices, the unit vector field normal to this bulk foliation is an example of such a flow.
- We will focus on this particular flow, which we will call the volume flow.
- Question: Does a foliation of the boundary automatically give a foliation of the bulk by maximal slices?
- In our paper, we answer yes given Einstein's equation and the strong energy condition.

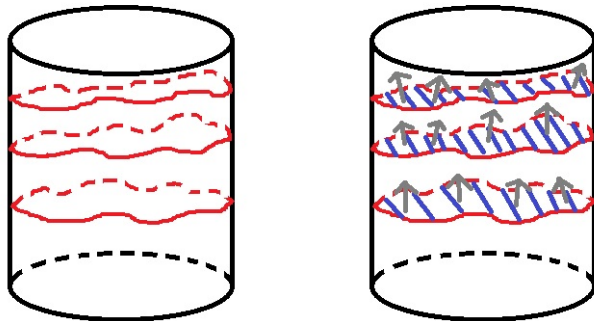


Figure : Foliations

# Insights from Volume Flows

What do we learn from these volume flows?

- In looking at examples, we see that volume seems to flow away from boundary, possibly indicating a flow from UV into IR (of dual theory).
- In volume language, it is clear that the growth of the wormhole is generic:
  - ▶ Given any future horizon, volume flow can have an inbound flux but not outbound flux.
  - ▶ This is of course a direct consequence of the flow being timelike and future directed.
  - ▶ Perhaps reminiscent of the proposed 2nd law of complexity?
- We can also show a monotonicity property for the rate of increase of the maximal volume (in the boundary time).

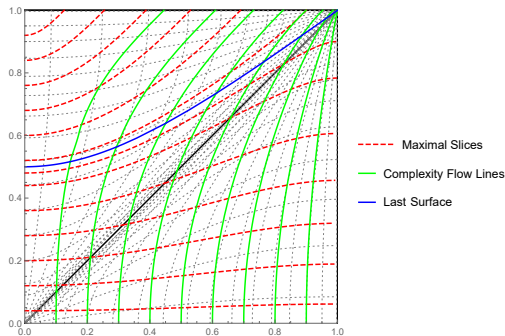


Figure : An example of a volume flow with a future horizon

# A Monotonicity Property

- Consider boundary cauchy slices  $\sigma_1$  and  $\sigma_2$ , and let their union be denoted by  $\sigma_U$ .
- Define  $\sigma_+$  to be the set of points  $x$  in  $\sigma_U$  such that the future of  $x$  only intersects  $\sigma_U$  at  $x$ .
- Likewise define  $\sigma_-$  to be the set of points  $x$  in  $\sigma_U$  such that the past of  $x$  only intersects  $\sigma_U$  at  $x$ .
- Then  $\text{Vol}(\sigma_1) + \text{Vol}(\sigma_2) \leq \text{Vol}(\sigma_+) + \text{Vol}(\sigma_-)$
- We can prove this based on the definition of a maximal slice without reference to flows, but there is a proof of this fact from flows based on their nesting property, which is exactly the Lorentzian analog of the proof of SSA in Freedman and Headrick.
- Applied to a two sided black hole, this implies that the rate of increase of maximal volume (as you increase the  $t_L + t_R$ , where both times increase towards the future) is monotonic.

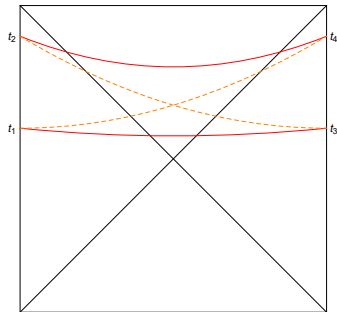


Figure : Maximal slices associated to different boundary slices for two-sided BH.

Complexity = Action or Volume?