The Speed of Quantum Information Spreading in Chaotic Systems

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Punchline

- We study the speed at which initially local quantum information spreads, i.e., the *information velocity*.
- When this information is entanglement with some reference, this can be defined as the rate of growth of the smallest region whose mutual information with that reference is maximal.
- For a chaotic translationally invariant system after a quench, we claim

$$v_I = \frac{v_E(f)}{1-f} \tag{1}$$

- where
  - v_E is the entanglement velocity
  - f is the ratio of the entropy density immediately after the quench to that in the asymptotic future.
- This claim is supported by a general quantum information argument, spin chain numerics, and a holographic computation.

Outline

- The Setup
- General Quantum Information Argument
- Computations for specific systems
  - Spin Chain
  - Holography
- Overview of results

Setup

- Consider a chaotic quantum system along with a reference system.
- Let $|\psi\rangle$ be a translation-invariant state with fixed energy density ϵ. $|\psi\rangle$ may not be in equilibrium, in which case the entropy density will be increasing.
- We assume that the entanglement entropy of any large region A at a given time t is given to leading order by

$$S(A) = \min \left(s|A|, s\left(f|A| + v_E|\partial A|t\right) \right) \tag{2}$$

- Assume that for a local perturbation W, the Heisenberg operator $W(t)$ approximately has support on a region of size $v_B t$
- Assume that we can think of our Hilbert space as a tensor product of Hilbert spaces on subregions of size ξ, where ξ is the correlation length.
- Assume that v_B is the fastest velocity in our system (at this energy density).
- At a time t_0, we will maximally entangle the reference system to some degrees of freedom localized to a small region.

Quantum Information Argument: Hayden-Preskill

- QI argument is based on a generalization of Hayden-Preskill (2007).
- HP has 3 systems: a reference R, a memory M, and a system S, where $|M| > |S|$.
- R is initially entangled with a small subsystem of S. The rest of S is maximally entangled with M.
- S then evolves by a random unitary
- After evolution, M plus a small subsystem of S is enough to recover entanglement with R.
- Our generalization breaks down into two cases: $f = 1$ (saturated) and $f < 1$ (unsaturated).

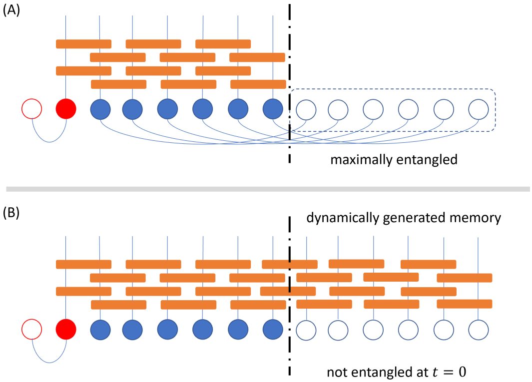

Figure: Our argument vs Hayden Preskill

Quantum Information Argument: $f = 1$

- Consider the case f=0. For simplicity, we will take the total state on the system to be pure.
- We want to find the (half) width $R(t)$ of the smallest region A from which some initially local entanglement can be recovered.
- Because we assumed v_B is the fastest length scale, we must have $R(t) \leq v_B t$.
- On the other hand, if $R(t) < v_B t$, then the information on A is fully scrambled.
- by HP, because A and A^c are maximally entangled, we could recover the entanglement on A^c plus a small subsystem of A.
- This cannot be, so $R(t) = v_B t$.

Quantum Information Argument: $f < 1$

- Now consider $f < 1$. If $v_E < (1-f)v_B$, then entanglement is not generated fast enought to run HP as above.
- The largest system we can run HP on is the one whose entropy has just saturated.
- The saturation time for a region of size R is given by

$$t_{\mathrm{sat}} = \frac{(1-f)R}{v_E(f)} \tag{3}$$

- Inverting the above, we find

$$v_I = \frac{v_E(f)}{1-f} \tag{4}$$

Spin Chain Numerics

- Chain of 22 Spins
- Left most spin initially maximally entangled with reference spin
- System evolved with a chaotic Hamiltonian
- At each time step, use mutual information to diagnose the smallest region which can recover this entanglement
- Small size effects expected, especially for large initial entangling fraction.

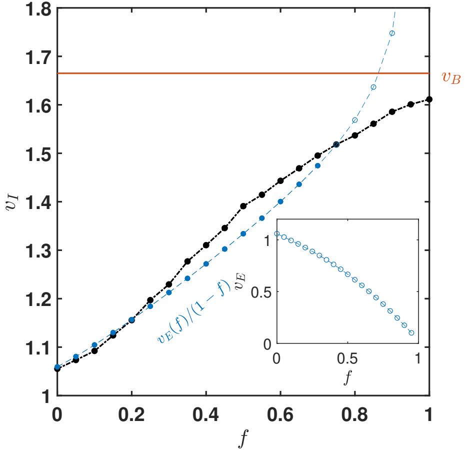

Figure: v_I vs entanglement fraction for a spin chain:

Holographic setup

- We consider either 1 or 2 sided AdS-Vaidya in the thin shell limit.
- We take the null shock to intersect the boundary at $t = 0$.
- We consider a probe particle to fall in the bulk at time $t_0 > 0$.
- The information content of this particle (which we will think of as entanglement with some reference) can be recovered by any entanglement wedge which contains it.
- We compute the information speed by tracking the smallest region whose entanglement wedge includes the particle at each time.
- We only consider strip-like regions

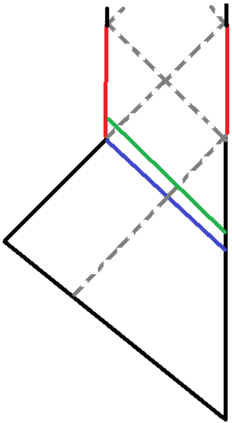

Figure: 1-sided AdS-Vaidya

Entanglement wedges

- There are two families of minimal surfaces, those that cross the shock (O type), and those that don't (E type)
- In the figure on the right:
  - The blue curve sweeps out the widths of the entangling regions whose E-type surfaces just touch the shock.
  - The red curve sweeps out regions whose E-type surfaces barely contain the particle
  - The dashed green curve sweeps out the regions at which the E and O type surfaces have equal area, i.e., each (t, R) pair are such that $t = t_{\text{sat}}(R)$.
- By the formula for saturation time given above, the green curve gives $v_I = v_E(f)/1 - f$ as expected.

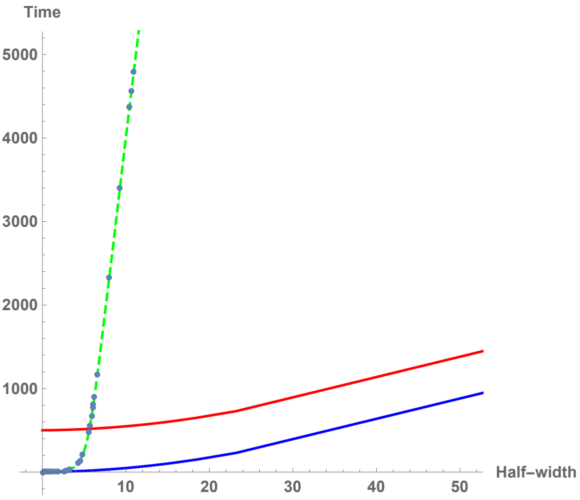

Figure: Various cones

Numerical Results from Holography

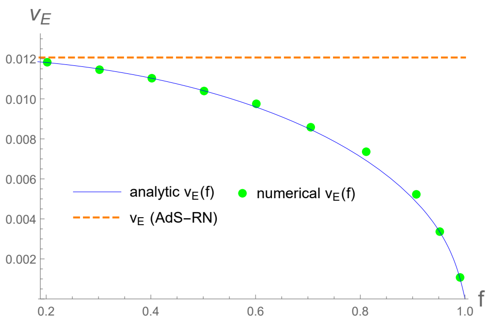

Figure: Numerical results for v_E

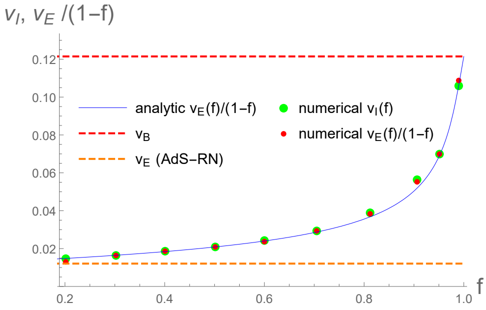

Figure: Numerical results for v_I

Summary of Results:

- Based on our quantum information argument, we seem to have that

$$v_I = \min\left(v_B, \frac{v_E(f)}{1-f}\right) \tag{5}$$

- In all the systems we've checked, we have that $v_E \leq (1-f)v_B$. We have a (not quite rigorous) argument that this is always true, recovering our initial claim.
- numerical results for a spin chain seem to be in relatively good agreement with this expectation. Due to small size effects, we don't expect exact agreement.
- The numerical holographic results also seem to be in good agreement with this expectation.