

# Recursive Rows in Rome

AH & JGM

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## 1 A DEPENDENT CALCULUS WITH ROWS AND RECURSION

### 1.1 Syntax

	Term variables $x \alpha$	labels variables $\ell \in \mathcal{L}$
Terms	$M, N, T ::= \mathcal{U} \mid x$ $\forall \alpha : T. N \mid \lambda x : T. N \mid M N \mid$ $\exists \alpha : T. M \mid (\alpha : T, M) \mid M.1 \mid M.2$ $\Pi \rho \mid M \#_{\mu} N$ $\Sigma \rho \mid M \nabla_{\mu} N$ $M \equiv N \mid \text{refl} \mid \mathcal{J} H M N P$	
Rows	$\rho ::= \{ \alpha \mid \ell \triangleright M \} \mid \dots \mid ?$	
Environments	$\Gamma ::= \varepsilon \mid \Gamma, \alpha : T$	

Fig. 1. Syntax

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Author's address: AH & JGM.

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$$\begin{array}{c}
\boxed{\vdash \Gamma} \\
\\
(\text{EMP}) \frac{}{\vdash \varepsilon} \quad (\text{VAR}) \frac{\vdash \Gamma \quad \Gamma \vdash M : \sigma}{\vdash \Gamma, x : M} \\
\\
\boxed{\Gamma \vdash M : \sigma} \\
\\
(\star) \frac{}{\Gamma \vdash \star : \mathcal{U}} \quad (\top\star) \frac{}{\Gamma \vdash \top : \star} \quad (\top\mathcal{U}) \frac{}{\Gamma \vdash \top : \mathcal{U}} \\
\\
(\text{Var}) \frac{\alpha : \sigma \in \Gamma}{\Gamma \vdash \alpha : \sigma} \quad (\text{NAT}) \frac{}{\Gamma \vdash \text{Nat} : \star} \quad (\text{Ix}) \frac{\Gamma \vdash n : \text{Nat}}{\Gamma \vdash \text{Ix } n : \star} \\
\\
(\text{II}) \frac{\Gamma \vdash M : \sigma \quad \Gamma, \alpha : M \vdash N : \star}{\Gamma \vdash \Pi \alpha : M. N : \star} \quad (\Sigma) \frac{\Gamma \vdash M : \sigma \quad \Gamma, \alpha : M \vdash N : \star}{\Gamma \vdash \Sigma \alpha : M. N : \star} \\
\\
(+ ) \frac{\Gamma \vdash M : \star \quad \Gamma \vdash N : \star}{\Gamma \vdash M + N : \star} \quad (\equiv) \frac{\Gamma \vdash M_1 : N_1 \quad \Gamma \vdash N_1 : \star \quad \Gamma \vdash M_2 : N_2 \quad \Gamma \vdash N_2 : \star}{\Gamma \vdash M_1 \equiv M_2 : \star} \\
\\
\boxed{\Gamma \vdash M : N} \\
\\
(\text{Var}) \frac{x : M \in \Gamma}{\Gamma \vdash x : M} \quad (\text{tt}) \frac{}{\Gamma \vdash \text{tt} : \top} \\
\\
(\text{Zero}) \frac{}{\Gamma \vdash \text{Zero} : \text{Nat}} \quad (\text{Suc}) \frac{\Gamma \vdash n : \text{Nat}}{\Gamma \vdash \text{Suc } n : \text{Nat}} \\
\\
(\text{FZero}) \frac{\Gamma \vdash n : \text{Nat}}{\Gamma \vdash \text{FZero} : \text{Ix } (\text{Suc } n)} \quad (\text{FSuc}) \frac{\Gamma \vdash n : \text{Nat} \quad \Gamma \vdash i : \text{Ix } n}{\Gamma \vdash \text{FSuc } i : \text{Ix } (\text{Suc } n)} \\
\\
(\text{PII}) \frac{\Gamma \vdash T : \sigma \quad \Gamma, x : T \vdash M : N}{\Gamma \vdash \lambda x : T. M : \Pi(x : T). N} \quad (\text{PIE}) \frac{\Gamma \vdash M : \Pi(x : T_1). T_2 \quad \Gamma \vdash N : T_1}{\Gamma \vdash M N : T_2[N/x]} \\
\\
(\Sigma I) \frac{\Gamma \vdash M : T_1 \quad \Gamma \vdash N : T_2[M/x]}{\Gamma \vdash (M : T_1, N) : \Sigma(x : T_1). T_2} \quad (\Sigma E_1) \frac{\Gamma \vdash M : \Sigma(x : T_1). T_2}{\Gamma \vdash M.1 : T_1} \quad (\Sigma E_2) \frac{\Gamma \vdash M : \Sigma(x : T_1). T_2}{\Gamma \vdash M.2 : T_2[M.1/x]} \\
\\
(\equiv I) \frac{\Gamma \vdash M : \sigma}{\Gamma \vdash \text{refl} : M \equiv M} \quad (\text{CONV}) \frac{\Gamma \vdash M : T_1 \quad \Gamma \vdash T_1 = T_2 : \sigma}{\Gamma \vdash M : T_2} \\
\\
\begin{array}{c}
\Gamma \vdash P : T_1 \equiv T_2 \\
\Gamma \vdash M : T_1 \\
\Gamma \vdash N : T_1 \\
\Gamma, x : T_1, y : T_1, p : x \equiv y \vdash T : \star \\
\Gamma, z : T_1 \vdash H : T[z/x, z/y, \text{refl}/p] \\
(\equiv E) \frac{}{\Gamma \vdash \mathcal{J} H M N P : T[M/x, N/y, P/p]}
\end{array}
\end{array}$$

Fig. 2. Context formation and typing rules for lx terms

Let the meta-syntax  $\tau$  denote both sort and term.

$$\begin{array}{c}
\boxed{\Gamma \vdash M = N : \sigma} \\
\\
\text{(E-REFL)} \frac{\Gamma \vdash M : \sigma}{\Gamma \vdash M = M : \sigma} \quad \text{(E-SYM)} \frac{\Gamma \vdash N = M : \sigma}{\Gamma \vdash M = N : \sigma} \quad \text{(E-TRANS)} \frac{\Gamma \vdash M = P : \sigma \quad \Gamma \vdash P = N : \sigma}{\Gamma \vdash M = N : \sigma} \\
\\
\boxed{\Gamma \vdash M = N : T} \\
\\
\text{(C-REFL)} \frac{\Gamma \vdash M : T}{\Gamma \vdash M = M : T} \quad \text{(C-SYM)} \frac{\Gamma \vdash N = M : T}{\Gamma \vdash M = N : T} \quad \text{(C-TRANS)} \frac{\Gamma \vdash M = P : T \quad \Gamma \vdash P = N : T}{\Gamma \vdash M = N : T}
\end{array}$$

Fig. 3. Definitional equality &amp; computational laws