Recursive Rows in Rome

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1 A DEPENDENT CALCULUS WITH ROWS AND RECURSION

1.1 Syntax

$$\text{Term variables} \quad x \; \alpha \qquad \text{labels variables} \quad \ell \in \mathcal{L}$$

$$\begin{array}{llcl}
\text{Terms} & M, N, T & ::= & \mathcal{U} \mid x \\
 & & & \forall \alpha : T.N \mid \lambda x : T.N \mid M\,N \mid \\
 & & & \exists \alpha : T.M \mid (\alpha : T, M) \mid M.1 \mid M.2 \\
 & & & \Pi\ \rho \mid M \mathbin{+\!\!+}_{\mu} N \\
 & & & \Sigma\ \rho \mid M \triangledown_{\mu} N \\
 & & & M \equiv N \mid \mathrm{refl} \mid \mathcal{J}\ H\ M\ N\ P \\
\text{Rows} & \rho & ::= & \{\alpha \mid \overline{\ell \triangleright M}\} \mid \ldots \mid ? \\
\text{Environments} & \Gamma & ::= & \varepsilon \mid \Gamma, \alpha : T
\end{array}$$

Fig. 1. Syntax

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$$\boxed{\vdash \Gamma}$$

$$(\textsc{emp})\ \frac{}{\vdash \varepsilon} \qquad (\textsc{var})\ \frac{\vdash \Gamma \quad \Gamma \vdash M : \sigma}{\vdash \Gamma, x : M}$$

$$\boxed{\Gamma \vdash M : \sigma}$$

$$(\star)\ \frac{}{\Gamma \vdash \star : \mathcal{U}} \qquad (\top_\star)\ \frac{}{\Gamma \vdash \top : \star} \qquad (\top_\mathcal{U})\ \frac{}{\Gamma \vdash \top : \mathcal{U}}$$

$$(Var)\ \frac{\alpha : \sigma \in \Gamma}{\Gamma \vdash \alpha : \sigma} \qquad (\textsc{Nat})\ \frac{}{\Gamma \vdash \mathrm{Nat} : \star} \qquad (\mathrm{Ix})\ \frac{\Gamma \vdash n : \mathrm{Nat}}{\Gamma \vdash \mathrm{Ix}\, n : \star}$$

$$(\Pi)\ \frac{\Gamma \vdash M : \sigma \quad \Gamma, \alpha : M \vdash N : \star}{\Gamma \vdash \Pi\alpha : M.N : \star} \qquad (\Sigma)\ \frac{\Gamma \vdash M : \sigma \quad \Gamma, \alpha : M \vdash N : \star}{\Gamma \vdash \Sigma\alpha : M.N : \star}$$

$$(+)\ \frac{\Gamma \vdash M : \star \quad \Gamma \vdash N : \star}{\Gamma \vdash M + N : \star} \qquad (\equiv)\ \frac{\Gamma \vdash M_1 : N_1 \quad \Gamma \vdash N_1 : \star \quad \Gamma \vdash M_2 : N_2 \quad \Gamma \vdash N_2 : \star}{\Gamma \vdash M_1 \equiv M_2 : \star}$$

$$\boxed{\Gamma \vdash M : N}$$

$$(Var)\ \frac{x : M \in \Gamma}{\Gamma \vdash x : M} \qquad (\mathtt{tt})\ \frac{}{\Gamma \vdash \mathtt{tt} : \top}$$

$$(\mathrm{Zero})\ \frac{}{\Gamma \vdash \mathrm{Zero} : \mathrm{Nat}} \qquad (\mathrm{Suc})\ \frac{\Gamma \vdash n : \mathrm{Nat}}{\Gamma \vdash \mathrm{Suc}\, n : \mathrm{Nat}}$$

$$(\mathrm{FZero})\ \frac{\Gamma \vdash n : \mathrm{Nat}}{\Gamma \vdash \mathrm{FZero} : \mathrm{Ix}\,(\mathrm{Suc}\, n)} \qquad (\mathrm{FSuc})\ \frac{\Gamma \vdash n : \mathrm{Nat} \quad \Gamma \vdash i : \mathrm{Ix}\, n}{\Gamma \vdash \mathrm{FSuc}\, i : \mathrm{Ix}\,(\mathrm{Suc}\, n)}$$

$$(\Pi I)\ \frac{\Gamma \vdash T : \sigma \quad \Gamma, x : T \vdash M : N}{\Gamma \vdash \lambda x : T.M : \Pi(x : T).N} \qquad (\Pi E)\ \frac{\Gamma \vdash M : \Pi(x : T_1).T_2 \quad \Gamma \vdash N : T_1}{\Gamma \vdash M\, N : T_2[N/x]}$$

$$(\Sigma I)\ \frac{\Gamma \vdash M : T_1 \quad \Gamma \vdash N : T_2[M/x]}{\Gamma \vdash (M : T_1, N) : \Sigma(x : T_1).T_2} \qquad (\Sigma E_1)\ \frac{\Gamma \vdash M : \Sigma(x : T_1).T_2}{\Gamma \vdash M.1 : T_1} \qquad (\Sigma E_2)\ \frac{\Gamma \vdash M : \Sigma(x : T_1).T_2}{\Gamma \vdash M.2 : T_2[M.1/x]}$$

$$(\equiv I)\ \frac{\Gamma \vdash M : \sigma}{\Gamma \vdash \mathrm{refl} : M \equiv M} \qquad (\textsc{conv})\ \frac{\Gamma \vdash M : T_1 \quad \Gamma \vdash T_1 = T_2 : \sigma}{\Gamma \vdash M : T_2}$$

$$(\equiv E)\ \frac{\begin{array}{c}\Gamma \vdash P : T_1 \equiv T_2 \\ \Gamma \vdash M : T_1 \\ \Gamma \vdash N : T_1 \\ \Gamma, x : T_1, y : T_1, p : x \equiv y \vdash T : \star \\ \Gamma, z : T_1 \vdash H : T[z/x, z/y, \mathrm{refl}/p]\end{array}}{\Gamma \vdash \mathcal{J}\, H\, M\, N\, P : T[M/x, N/y, P/p]}$$

Fig. 2. Context formation and typing rules for Ix terms

Let the meta-syntax τ denote both sort and term.

$$\boxed{\Gamma \vdash M = N : \sigma}$$

$$(\textsc{e-refl})\,\frac{\Gamma \vdash M : \sigma}{\Gamma \vdash M = M : \sigma} \qquad (\textsc{e-sym})\,\frac{\Gamma \vdash N = M : \sigma}{\Gamma \vdash M = N : \sigma} \qquad (\textsc{e-trans})\,\frac{\Gamma \vdash M = P : \sigma \quad \Gamma \vdash P = N : \sigma}{\Gamma \vdash M = N : \sigma}$$

$$\boxed{\Gamma \vdash M = N : T}$$

$$(\textsc{c-refl})\,\frac{\Gamma \vdash M : T}{\Gamma \vdash M = M : T} \qquad (\textsc{c-sym})\,\frac{\Gamma \vdash N = M : T}{\Gamma \vdash M = N : T} \qquad (\textsc{c-trans})\,\frac{\Gamma \vdash M = P : T \quad \Gamma \vdash P = N : T}{\Gamma \vdash M = N : T}$$

Fig. 3. Definitional equality & computational laws