Recursive Rows in Rome

AH & JGM

ACM Reference Format:

AH & JGM. 2023. Recursive Rows in Rome. 1, 1 (December 2023), 8 pages. https://doi.org/10.1145/nnnnnnn.nnnnnnn

1 IX: THE INDEX CALCULUS

1.1 Syntax

$$
\begin{array}{llcl}
\text{Sorts} & \sigma & ::= & \star \mid \square \\
\text{Terms} & M, N, T & ::= & \star \mid x \mid \\
& & & \mathbb{N} \mid Z \mid \mathsf{S}\, M \mid \\
& & & \mathrm{case}_{\mathbb{N}}\, M\, N\, T \mid \\
& & & \mathrm{Ix}\, M \mid \mathsf{I}_0 \mid \mathsf{I}_{\mathsf{S}}\ M \mid \\
& & & \mathrm{case}_{\mathrm{Ix}}\, M\, N \mid \mathrm{case}_{\mathrm{Ix}}\, M\, N\, T \mid \\
& & & \{\!\{M_1, \ldots, M_n\}\!\} \mid \lambda\!\!\!\!-() \mid \\
& & & \top \mid \mathtt{tt} \mid \\
& & & \forall \alpha : T.N \mid \lambda x : T.N \mid M\, N \mid \\
& & & \exists \alpha : T.M \mid \langle\!\langle \alpha : T, M \rangle\!\rangle \mid \mathrm{case}_{\exists}\, M\, N \mid \\
& & & M + N \mid \mathrm{left}\, M \mid \mathrm{right}\, M \mid \\
& & & \mathrm{case}_{+}\, M\, N\, T \mid \\
& & & M \equiv N \mid \mathrm{refl}\, T\, M\, N \mid \\
\text{Environments} & \Gamma & ::= & \varepsilon \mid \Gamma, \alpha : T
\end{array}
$$

Fig. 1. Syntax

1.1.1 *Meta-syntax & syntactic sugar.* Let

(1) $\tau \longrightarrow \upsilon$ denote the non-dependent universal quantification $\forall(\_ : \tau).\upsilon$;
(2) $\tau \times \upsilon$ denote the non-dependent existential quantification $\exists(\_ : \tau).\upsilon$;
(3) 0, 1, 2, ... denote object-level natural numbers in the intuitive fashion;
(4) i_n denote the index obtained by n applications of I_{S} to I_0; and

Author's address: AH & JGM.

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https://doi.org/10.1145/nnnnnnn.nnnnnnn

(5) the syntax

$$\{\!\{M_1, ..., M_n\}\!\}$$

denote the large elimination of a known, finite quantity of indices to types $M_1, ..., M_n$, elaborated by the equations:

$$\begin{aligned} \{\!\{M_1\}\!\} &:= \lambda(i : \mathrm{Ix}\ 1).\mathrm{case}_{\mathrm{Ix}}\ i\ M_1 \\ \{\!\{M_1, ..., M_n\}\!\} &:= \lambda(i : \mathrm{Ix}\ n).\mathrm{case}_{\mathrm{Ix}}\ i\ M_1\ \{\!\{M_2, ..., M_n\}\!\} \end{aligned}$$

1.2 Typing

$$\boxed{\vdash \Gamma}$$

$$(\textsc{emp})\ \frac{}{\vdash \varepsilon} \qquad (\textsc{var})\ \frac{\vdash \Gamma \quad \Gamma \vdash M : \sigma}{\vdash \Gamma, x : M}$$

$$\boxed{\Gamma \vdash M : \sigma}$$

$$(\star)\ \frac{\vdash \Gamma}{\Gamma \vdash \star : \square} \qquad (\top)\ \frac{\vdash \Gamma}{\Gamma \vdash \top : \sigma} \qquad (\textsc{Nat})\ \frac{\vdash \Gamma}{\Gamma \vdash \mathbb{N} : \star} \qquad (\textsc{Ix})\ \frac{\Gamma \vdash n : \mathbb{N}}{\Gamma \vdash \mathrm{Ix}\ n : \star}$$

$$(\forall)\ \frac{\Gamma \vdash M : \sigma_1 \quad \Gamma, \alpha : M \vdash N : \sigma_2}{\Gamma \vdash \forall \alpha : M.N : \sigma_2} \qquad (\exists)\ \frac{\Gamma \vdash M : \sigma_1 \quad \Gamma, \alpha : M \vdash N : \sigma_2}{\Gamma \vdash \exists \alpha : M.N : \sigma_2}$$

$$(+)\ \frac{\Gamma \vdash M : \sigma \quad \Gamma \vdash N : \sigma}{\Gamma \vdash M + N : \sigma} \qquad (\equiv)\ \frac{\Gamma \vdash M : \sigma \quad \Gamma \vdash N : \sigma}{\Gamma \vdash M \equiv N : \sigma}$$

Fig. 2. Context and type well-formedness

$$\boxed{\Gamma \vdash M : N}$$

$$(\textsc{Var})\,\frac{\vdash \Gamma \quad x : M \in \Gamma}{\Gamma \vdash x : M} \qquad (\texttt{tt})\,\frac{\vdash \Gamma}{\Gamma \vdash \texttt{tt} : \top}$$

$$(Z)\,\frac{\vdash \Gamma}{\Gamma \vdash Z : \mathbb{N}} \qquad (\mathrm{S})\,\frac{\Gamma \vdash n : \mathbb{N}}{\Gamma \vdash \mathrm{S}\, n : \mathbb{N}} \qquad (\mathbb{N}E)\,\frac{\Gamma \vdash A : \sigma \quad \Gamma \vdash M : \mathbb{N} \quad \Gamma \vdash N : A \quad \Gamma \vdash P : \mathbb{N} \to A}{\Gamma \vdash \mathrm{case}_{\mathbb{N}}\, M\, N\, P : A}$$

$$(\mathsf{I}_0)\,\frac{\Gamma \vdash n : \mathbb{N}}{\Gamma \vdash \mathsf{I}_0 : \mathrm{Ix}\,(\mathrm{S}\, n)} \qquad (\mathsf{I}_{\mathrm{S}})\,\frac{\Gamma \vdash n : \mathbb{N} \quad \Gamma \vdash i : \mathrm{Ix}\, n}{\Gamma \vdash \mathsf{I}_{\mathrm{S}}\, i : \mathrm{Ix}\,(\mathrm{S}\, n)} \qquad (\lambda\!\!\!^{-}())\,\frac{\Gamma \vdash A : \sigma}{\Gamma \vdash \lambda\!\!\!^{-}() : \mathrm{Ix}\, 0 \to A}$$

$$(\mathsf{I}_{\mathrm{S}} E)\,\frac{\Gamma \vdash A : \sigma \quad \Gamma \vdash M : \mathrm{Ix}\,(\mathrm{S}\, n) \quad \Gamma \vdash N : A \quad \Gamma \vdash P : \mathrm{Ix}\, n \to A}{\Gamma \vdash \mathrm{case}_{\mathrm{Ix}}\, M\, N\, P : A}$$

$$(\forall I)\,\frac{\Gamma \vdash T : \sigma \quad \Gamma, x : T \vdash M : N}{\Gamma \vdash \lambda x : T.M : \forall(x : T).N} \qquad (\forall E)\,\frac{\Gamma \vdash M : \forall(x : T_1).T_2 \quad \Gamma \vdash N : T_1}{\Gamma \vdash M\, N : T_2[N/x]}$$

$$(\exists I)\,\frac{\Gamma \vdash T_1 : \sigma \quad \Gamma \vdash M : T_1 \quad \Gamma \vdash N : T_2[M/x]}{\Gamma \vdash \langle\!\langle M : T_1, N \rangle\!\rangle : \exists(x : T_1).T_2}$$

$$(\exists E)\,\frac{\Gamma \vdash A : \sigma \quad \Gamma \vdash M : \exists(x : T_1).T_2 \quad \Gamma \vdash N : \forall(x : T_1).T_2 \to A}{\Gamma \vdash \mathrm{case}_{\exists}\, M\, N : A}$$

$$(+_1 I)\,\frac{\Gamma \vdash M : A}{\Gamma \vdash \mathrm{left}\, M : A + B} \qquad (+_2 I)\,\frac{\Gamma \vdash N : B}{\Gamma \vdash \mathrm{right}\, N : A + B}$$

$$(+E)\,\frac{\Gamma \vdash C : \sigma \quad \Gamma \vdash M : A + B \quad \Gamma \vdash N : A \to C \quad \Gamma \vdash P : B \to C}{\Gamma \vdash \mathrm{case}_{+}\, M\, N\, P : C}$$

$$(\equiv I)\,\frac{\Gamma \vdash M : \sigma}{\Gamma \vdash \mathrm{refl} : M \equiv M} \qquad (\textsc{conv})\,\frac{\Gamma \vdash M : T_1 \quad \Gamma \vdash T_1 = T_2 : \sigma}{\Gamma \vdash M : T_2}$$

$$(\equiv E)\,\frac{\begin{array}{c}\Gamma \vdash P : T_1 \equiv T_2 \\ \Gamma \vdash M : T_1 \\ \Gamma \vdash N : T_1 \\ \Gamma, x : T_1, y : T_1, p : x \equiv y \vdash T : \star \\ \Gamma, z : T_1 \vdash H : T[z/x, z/y, \mathrm{refl}/p]\end{array}}{\Gamma \vdash \mathcal{J}\, H\, M\, N\, P : T[M/x, N/y, P/p]}$$

Fig. 3. Typing Ix terms

$$\boxed{\Gamma \vdash M = N : \sigma}$$

$$(\textsc{e-refl})\,\frac{\Gamma \vdash M : \sigma}{\Gamma \vdash M = M : \sigma} \qquad (\textsc{e-sym})\,\frac{\Gamma \vdash N = M : \sigma}{\Gamma \vdash M = N : \sigma} \qquad (\textsc{e-trans})\,\frac{\Gamma \vdash M = P : \sigma \quad \Gamma \vdash P = N : \sigma}{\Gamma \vdash M = N : \sigma}$$

$$\boxed{\Gamma \vdash M = N : T}$$

$$(\textsc{c-refl})\,\frac{\Gamma \vdash M : T}{\Gamma \vdash M = M : T} \qquad (\textsc{c-sym})\,\frac{\Gamma \vdash N = M : T}{\Gamma \vdash M = N : T} \qquad (\textsc{c-trans})\,\frac{\Gamma \vdash M = P : T \quad \Gamma \vdash P = N : T}{\Gamma \vdash M = N : T}$$

Fig. 4. Definitional equality & computational laws

1.3 Properties

THEOREM 1 (WELL-SORTEDNESS). *if* $\Gamma \vdash M : N$ *then* $\vdash \Gamma$ *and there exists* σ *such that* $\Gamma \vdash N : \sigma$.

1.4 Elaborating Ix to the CoC + `Fin`

Following the above, I believe Ix to a smaller calculus with the syntax below. This is effectively the calculus of constructions with primitive naturals and finite indices.

$$
\begin{array}{llcl}
\text{Sorts} & \sigma & ::= & \star \mid \Box \\
\text{Terms} & M, N, T & ::= & \star \mid x \mid \\
 & & & \mathbb{N} \mid Z \mid \mathrm{S}\, M \mid \\
 & & & \mathrm{case}_{\mathbb{N}}\, M\, N\, T \mid \\
 & & & \mathrm{Ix}\, M \mid \mathsf{I}_0 \mid \mathsf{I}_{\mathrm{S}}\, M \mid \\
 & & & \mathrm{case}_{\mathrm{Ix}}\, M\, N\, T \mid \\
 & & & \forall \alpha : T.N \mid \lambda x : T.N \mid M\, N \mid \\
 & & & M \equiv N \mid \mathrm{refl}\, T\, M\, N \mid \\
\text{Environments} & \Gamma & ::= & \varepsilon \mid \Gamma, \alpha : T
\end{array}
$$

Fig. 5. Syntax

Elaboration is given below. One would also expect a rule for bot elimination—or you could simply encode Ix 0 as $\forall X : \star.X$.

$$
\begin{aligned}
\top &\rightsquigarrow \mathrm{Ix}\, 1 \\
\mathtt{tt} &\rightsquigarrow \mathsf{I}_0 \\
\bot &\rightsquigarrow \mathrm{Ix}\, 0 \\
A \to B &\rightsquigarrow \forall (x : A).B \\
\exists (x : A).B &\rightsquigarrow \forall (C : \Box).(\forall (x : A).B \to C) \to C \\
\langle\!\langle M : T, N \rangle\!\rangle &\rightsquigarrow \lambda (C : \Box).\lambda (f : (\forall (x : A).B \to C)).f\, M\, N \\
A + B &\rightsquigarrow \exists (i : \mathrm{Ix}\, 2).\{\!\{A, B\}\!\}\, i \\
\mathrm{left}\, M &\; \langle\!\langle i_0 : \mathrm{Ix}\, 2, M \rangle\!\rangle \\
\mathrm{right}\, N &\; \langle\!\langle i_1 : \mathrm{Ix}\, 2, N \rangle\!\rangle \\
A \times B &\rightsquigarrow \exists (x : A).B
\end{aligned}
$$

One could also translate naturals away using your favorite functional encoding. (I imagine there are encodings for `Fin`, too.)

2 TRANSLATION FROM Rω

2.1 Untyped Translation

We follow the approach of [Morris and McKinna 2019] and give both typed and untyped translations of Rω types. Figure 7 describe the untyped translation, which is used to show translational soundness of the typed translation (Figure 7).

$$\boxed{[\![\kappa]\!]}$$

$$\begin{aligned}
[\![\star]\!] &= \star \\
[\![\mathsf{L}]\!] &= \top \\
[\![\kappa_1 \to \kappa_2]\!] &= [\![\kappa_1]\!] \to [\![\kappa_2]\!] \\
[\![\mathsf{R}^\kappa]\!] &= \exists(n : \mathbb{N}).\mathrm{Ix}\, n \to [\![\kappa]\!]
\end{aligned}$$

$$\boxed{[\![\Delta \vdash \tau : \kappa]\!]}$$

$$\begin{aligned}
[\![\Delta \vdash \alpha : \kappa]\!] &= \alpha \\
[\![\Delta \vdash \tau_1 \to \tau_2 : \star]\!] &= [\![\tau_1]\!] \to [\![\tau_2]\!] \\
[\![\Delta \vdash \forall\alpha : \kappa.\tau : \star]\!] &= \forall(\alpha : [\![\kappa]\!]).[\![\tau]\!] \\
[\![\Delta \vdash \lambda\alpha : \kappa.\tau : \kappa \to \kappa']\!] &= \forall(\alpha : [\![\kappa]\!]).[\![\tau]\!] \\
[\![\Delta \vdash \pi \Rightarrow \tau : \kappa]\!] &= [\![\pi]\!] \to [\![\tau]\!] \\
[\![\Delta \vdash \tau\, \upsilon : \kappa]\!] &= [\![\tau]\!]\, [\![\upsilon]\!] \\
[\![\Delta \vdash \ell : \mathsf{L}]\!] &= \top \\
[\![\Delta \vdash \lfloor \xi \rfloor : \star]\!] &= \top \\
[\![\Delta \vdash (\xi \triangleright \tau) : \kappa]\!] &= [\![\tau]\!] \\
[\![\Delta \vdash \Pi\rho : \star]\!] &= \mathrm{case}_\exists\, [\![\rho]\!]\, (\lambda n : \mathbb{N}.\lambda P : \mathrm{Ix}\, n \to \star.\forall(i : \mathrm{Ix}\, n).P\, i) \\
[\![\Delta \vdash \Sigma\rho : \star]\!] &= \mathrm{case}_\exists\, [\![\rho]\!]\, (\lambda n : \mathbb{N}.\lambda P : \mathrm{Ix}\, n \to \star.\exists(i : \mathrm{Ix}\, n).P\, i) \\
[\![\Delta \vdash \epsilon : \mathsf{R}^\kappa]\!] &= \langle\!\langle 0 : \mathbb{N}, \not\lambda() \rangle\!\rangle \\
[\![\Delta \vdash \rho \lceil \upsilon \rceil : \mathsf{R}^{\kappa_2}]\!] &= \mathrm{case}_\exists\, [\![\rho]\!]\, (\lambda n : \mathbb{N}.\lambda(P : \mathrm{Ix}\, n \to [\![\kappa_1]\!] \to [\![\kappa_2]\!]).\langle\!\langle n : \mathbb{N}, \lambda(j : \mathrm{Ix}\, n).(P\, j)[\![\tau]\!] \rangle\!\rangle) \\
[\![\Delta \vdash \lceil \tau \rceil\, \rho : \mathsf{R}^{\kappa_2}]\!] &= \mathrm{case}_\exists\, [\![\rho]\!]\, (\lambda n : \mathbb{N}.\lambda(P : \mathrm{Ix}\, n \to [\![\kappa_1]\!]).\langle\!\langle n : \mathbb{N}, \lambda(j : \mathrm{Ix}\, n).[\![\tau]\!](P\, j) \rangle\!\rangle) \\
[\![\Delta \vdash (\xi \triangleright_{\mathsf{R}} \tau) : \mathsf{R}^\kappa]\!] &= \langle\!\langle 1 : \mathbb{N}, \{\!\{ [\![\tau]\!] \}\!\} \rangle\!\rangle
\end{aligned}$$

Fig. 6. Translating kinding derivations to untyped Ix terms

$$\boxed{[\![\Delta \vdash \pi : \kappa]\!]}$$

$$[\![\Delta \vdash \rho_1 \lesssim \rho_2 : \kappa]\!] = \begin{array}{l} \text{case } [\![\rho_1]\!](\lambda n : \mathbb{N}.\lambda(P : \text{Ix } n \to [\![\kappa]\!]). \\ \text{case } [\![\rho_2]\!](\lambda m : \mathbb{N}.\lambda(Q : \text{Ix } m \to [\![\kappa]\!]). \\ \forall(i : \text{Ix } n).\exists(j : \text{Ix } m).P\, i \equiv Q\, j)) \end{array}$$

$$[\![\Delta \vdash \rho_1 \odot \rho_2 \sim \rho_3 : \kappa]\!] = \begin{array}{l} \quad \text{case } [\![\rho_1]\!](\lambda n : \mathbb{N}.\lambda(P : \text{Ix } n \to [\![\kappa]\!]). \\ \quad \text{case } [\![\rho_2]\!](\lambda m : \mathbb{N}.\lambda(Q : \text{Ix } m \to [\![\kappa]\!]). \\ \quad \text{case } [\![\rho_3]\!](\lambda l : \mathbb{N}.\lambda(R : \text{Ix } l \to [\![\kappa]\!]). \\ \quad (\forall(i : \text{Ix } n).\exists(k : \text{Ix } l).P\, i \equiv R\, k) \\ \times (\forall(j : \text{Ix } m).\exists(k : \text{Ix } l).Q\, j \equiv R\, k) \\ \times (\forall(k : \text{Ix } l). \\ \quad (\exists(i : \text{Ix } n).P\, i \equiv R\, k) \\ + (\exists(j : \text{Ix } m).Q\, j \equiv R\, k))))) \end{array}$$

Fig. 7. Translating predicate well-formedness judgments

2.2 Typed translation

There is some subtlety in mechanizing environments. Environments in Rω store kinds, *typing derivations*, and *predicate well-formedness derivations*. If we are to simply translate derivations to untyped syntax, we are losing a bit of information. I am not sure, however, it is possible to translate derivations (in Rω) to derivations (in Ix) without a de facto type checker for Ix. I think we will have to perform the former: let derivations in Rω environments translate to untyped types and sorts in Ix environments. Then, argue as metatheory that $\vdash \Delta \rightsquigarrow \Gamma$ implies $\vdash \Gamma$.

$$\boxed{\vdash \Delta \rightsquigarrow \Gamma}$$

$$(\text{C-}\varepsilon)\,\frac{}{\vdash \epsilon \rightsquigarrow \epsilon} \qquad (\text{C-TVAR})\,\frac{\vdash \Delta \rightsquigarrow \Gamma}{\vdash \Delta, \alpha : \kappa \rightsquigarrow \Gamma, \alpha : [\![\kappa]\!]}$$

$$(\text{C-VAR})\,\frac{\vdash \Delta \rightsquigarrow \Gamma}{\vdash \Delta, x : \tau \rightsquigarrow \Gamma, x : [\![\tau]\!]} \qquad (\text{C-PRED})\,\frac{\vdash \Delta \rightsquigarrow \Gamma}{\vdash \Delta, \pi : \kappa \rightsquigarrow \Gamma, p : [\![\pi]\!]}\,(p\ \text{fresh})$$

$$\boxed{\Delta \vdash M \rightsquigarrow N : \tau}$$

$$(\text{T-VAR})\,\frac{x : \tau \in \Delta}{\Delta \vdash x \rightsquigarrow x : \tau}$$

$$(\text{T-}{\longrightarrow}I)\,\frac{\Delta, x : \tau \vdash M \rightsquigarrow N : \upsilon}{\Delta \vdash \lambda x : \tau.M \rightsquigarrow \lambda x : [\![\tau]\!].N : \tau \longrightarrow \upsilon} \qquad (\text{T-}{\longrightarrow}I)\,\frac{\Delta \vdash M \rightsquigarrow F : \tau \longrightarrow \upsilon \quad \Delta \vdash N \rightsquigarrow E : \tau}{\Delta \vdash M N \rightsquigarrow F E : \upsilon}$$

$$(\text{T-}{\Longrightarrow}I)\,\frac{\Delta, \pi \vdash M \rightsquigarrow N : \tau}{\Delta \vdash M \rightsquigarrow \lambda(p : [\![\pi]\!]).N : \pi \Longrightarrow \tau} \qquad (\text{T-}{\Longrightarrow}E)\,\frac{\Delta \vdash M \rightsquigarrow F : \pi \Longrightarrow \tau \quad \Delta \Vdash \pi \rightsquigarrow E}{\Delta \vdash M \rightsquigarrow F E : \tau}$$

$$(\text{T-}\forall I)\,\frac{\Delta \vdash M \rightsquigarrow N : \tau}{\Delta \vdash \Lambda\alpha : \kappa.M \rightsquigarrow \lambda(\alpha : [\![\kappa]\!]).N : \forall\alpha : \kappa.\tau} \qquad (\text{T-}\forall E)\,\frac{\Delta \vdash M \rightsquigarrow N : \forall\alpha : \kappa.\tau}{\Delta \vdash M[\upsilon] \rightsquigarrow N\,[\![\upsilon]\!] : \tau[\upsilon/\alpha]}$$

$$(\text{T-SING})\,\frac{}{\Delta \vdash \ell \rightsquigarrow \mathtt{tt} : \lfloor \ell \rfloor} \qquad (\text{T-}{\triangleright}\text{I})\,\frac{\Delta \vdash N \rightsquigarrow E : \tau}{\Delta \vdash M \triangleright N \rightsquigarrow E : \ell \triangleright \tau} \qquad (\text{T-}{\triangleright}\text{E})\,\frac{\Delta \vdash M \rightsquigarrow E : \tau \quad \Delta \vdash N \rightsquigarrow \mathtt{tt} : \lfloor \ell \rfloor}{\Delta \vdash M/N \rightsquigarrow E : \ell \triangleright \tau}$$

$$(\text{C-FOO})\,\frac{A}{B} \qquad (\text{C-FOO})\,\frac{A}{B} \qquad (\text{C-FOO})\,\frac{A}{B}$$

Fig. 8. Translation of Rω environments and typing derivations

$$(\text{C-FOO})\,\frac{A}{B}$$

$$\boxed{\Delta \Vdash \pi \rightsquigarrow N}$$

$$(\text{C-FOO})\,\frac{A}{B}$$

$$\boxed{\tau \equiv \upsilon \rightsquigarrow P}$$

$$(\text{C-FOO})\,\frac{A}{B}$$

Fig. 9. Translation of Rω derivations to Ix derivations

2.3 Properties of Translation

Theorem 2 (Translational Soundness (Environments)). *if* $\vdash \Delta \rightsquigarrow \Gamma$ *then* $\vdash \Gamma$.

Theorem 3 (Translational Soundness (Types)). *if* $\Delta \vdash \tau : \kappa$ *and* $\vdash \Delta \rightsquigarrow \Gamma$ *then* $\Gamma \vdash [\![\tau]\!] : [\![\kappa]\!]$.

THEOREM 4 (TRANSLATIONAL SOUNDNESS (TYPE EQUIVALENCE)). *if*

(1) $\Gamma \vdash \tau_1 \rightsquigarrow \upsilon_1 : \kappa_1$*;*
(2) $\Gamma \vdash \tau_2 \rightsquigarrow \upsilon_2 : \kappa_2$*; and*
(3) $\tau_1 \equiv \tau_2 \rightsquigarrow P$*,*

then $[\![\Gamma]\!] \vdash P : \upsilon_1 \equiv \upsilon_2$*.*

THEOREM 5 (TRANSLATIONAL SOUNDNESS (PREDICATES)). *if* $\Gamma \Vdash \pi$ *such that* $\Gamma \Vdash \pi \rightsquigarrow N$ *then* $[\![\Gamma]\!] \vdash N : [\![\pi]\!]$*.*

Finally,

THEOREM 6 (TRANSLATIONAL SOUNDNESS). *if* $\Gamma \vdash M : \tau$ *such that* $\Gamma \vdash M \rightsquigarrow N : \tau$ *then* $[\![\Gamma]\!] \vdash N : [\![\tau]\!]$*.*

3 OPERATIONAL SEMANTICS

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