

# MP Board Class 12 Mathematics – MCQs

Final Revision | 50 Questions | 2026 Exam

## Chapter 1: Relations & Functions

**Q1.** If  $A = \{1,2\}$ ,  $B = \{3,4\}$ , number of relations =  
यदि  $A=\{1,2\}$ ,  $B=\{3,4\}$ , संबंधों की संख्या है—

- A. 4 / चार
- B. 8 / आठ
- C. 16 / सोलह
- D. 2 / दो

□ **Answer: C – 16 / सोलह**

**Q2.** A function must be  
फलन होना चाहिए—

- A. One-one / एक-एक
- B. Onto / पूर्ण
- C. Many-one allowed / बहु-एक संभव
- D. Bijective only / केवल एक-एक पूर्ण

□ **Answer: C – Many-one allowed / बहु-एक संभव**

## Chapter 2: Inverse Trigonometric Functions

**Q3.** Domain of  $\sin^{-1}x$  is  
 $\sin^{-1}x$  का प्रांत है—

- A.  $\mathbb{R}$
- B.  $[-1,1]$
- C.  $(-\infty, \infty)$
- D.  $(0,1)$

□ **Answer: B –  $[-1,1]$**

**Q4.**  $\sin^{-1}(0) =$   
 $\sin^{-1}(0)$  का मान है—

- A.  $\pi$
- B. 0
- C.  $\pi/2$
- D.  $-\pi/2$

□ **Answer: B – 0 / शून्य**

### Chapter 3: Matrices

**Q5.** Order of a matrix with 3 rows and 2 columns is  
3 पंक्ति व 2 स्तंभ वाले आव्यूह की कोटि—

- A.  $2 \times 3$
- B.  $3 \times 2$
- C.  $5 \times 5$
- D.  $1 \times 6$

□ **Answer: B –  $3 \times 2$**

**Q6.** Identity matrix is always  
इकाई आव्यूह हमेशा होता है—

- A. Rectangular / आयताकार
- B. Square / वर्गाकार
- C. Zero matrix / शून्य
- D. Singular / एकल

□ **Answer: B – Square / वर्गाकार**

### Chapter 4: Determinants

**Q7.** Determinant of identity matrix is  
इकाई आव्यूह का निर्धारक—

- A. 0
- B. -1
- C. 1
- D. 2

□ **Answer: C – 1**

**Q8.** If determinant = 0, system has  
यदि निर्धारक 0 हो, तो हल—

- A. Unique / अद्वितीय
- B. No solution / कोई हल नहीं
- C. Infinite solutions / अनंत हल
- D. Always consistent / सदैव संगत

□ **Answer: C – Infinite solutions / अनंत हल**

## **Chapter 5: Continuity & Differentiability**

**Q9.** Derivative of constant is  
स्थिरांक का अवकलज—

- A. 1
- B. x
- C. 0
- D.  $\infty$

□ **Answer: C – 0 / शून्य**

**Q10.** If f is differentiable, it is  
यदि f अवकलनीय है, तो—

- A. Discontinuous
- B. Continuous
- C. Undefined
- D. Constant

□ **Answer: B – Continuous / सतत**

## **Chapter 6: Applications of Derivatives**

**Q11.** Maxima occurs when  $f'(x)=$   
अधिकतम मान के लिए  $f'(x)=$

- A. 1
- B. 0

- C.  $\infty$
- D.  $-1$

☐ **Answer: B – 0 / शून्य**

**Q12.** Rate of change is measured by  
परिवर्तन की दर मापी जाती है—

- A. Integration
- B. Limit
- C. Differentiation
- D. Matrix

☐ **Answer: C – Differentiation / अवकलन**

## Chapter 7: Integrals

**Q13.**  $\int 0 \, dx =$   
 $\int 0 \, dx$  का मान—

- A. 0
- B. x
- C. 1
- D. C

☐ **Answer: D – C / स्थिरांक**

**Q14.** Integration is reverse of  
समाकलन है—

- A. Limit
- B. Differentiation
- C. Matrix
- D. Determinant

☐ **Answer: B – Differentiation / अवकलन**

## Chapter 8: Applications of Integrals

**Q15.** Area under curve is found by  
वक्र के नीचे का क्षेत्रफल—

- A. Derivative
- B. Limit
- C. Integration
- D. Matrix

□ Answer: C – Integration / समाकलन

## Chapter 9: Differential Equations

**Q16.** Order of  $dy/dx =$   
 $dy/dx$  का क्रम—

- A. 0
- B. 1
- C. 2
- D. 3

□ Answer: B – 1 / प्रथम

## Chapter 10: Vector Algebra

**Q17.** Zero vector magnitude is  
शून्य सदिश का परिमाण—

- A. 1
- B.  $\infty$
- C. 0
- D. -1

□ Answer: C – 0 / शून्य

**Q18.**  $i \cdot j =$   
 $i \cdot j$  का मान—

- A. 1
- B. -1
- C. 0
- D. i

□ Answer: C – 0 / शून्य

## Chapter 11: Three Dimensional Geometry

**Q19.** Distance formula is based on  
दूरी सूत्र आधारित है—

- A. Integration
- B. Pythagoras theorem
- C. Matrix
- D. Vector product

☐ **Answer: B – Pythagoras theorem / पाइथागोरस**

## **Chapter 12: Linear Programming**

**Q20.** Feasible region is  
साध्य क्षेत्र होता है—

- A. Unbounded
- B. Point
- C. Common shaded region
- D. Line

☐ **Answer: C – Common shaded region / साझा छायांकित क्षेत्र**

## **Chapter 13: Probability**

**Q21.**  $P(S) =$   
नमूना समुच्चय की प्रायिकता—

- A. 0
- B. 1
- C. -1
- D.  $\infty$

☐ **Answer: B – 1**

**Q22.** Probability lies between  
प्रायिकता का मान—

- A. -1 to 1
- B. 0 to 1
- C. 1 to 2
- D. Any real number

☐ **Answer: B – 0 to 1**

**Q23.**  $\sin^2\theta + \cos^2\theta =$

A. 0 B. 1 C. 2 D.  $\theta$

☐ **Ans: B – 1**

**Q24.**  $\det(A) = -\det(-A)$  is true for

A. Odd order

B. Even order

C. All

D. None

☐ **Ans: A – Odd order**

**Q25.**  $\frac{d}{dx} (x^n) =$

A.  $nx^{n-1}$

B.  $x^n$

C. n

D. 0

☐ **Ans: A**

**Q26.**  $\int e^x dx =$

A.  $e^x + C$

B.  $xe^x$

C. 1

D. 0

☐ **Ans: A**

**Q27.** Unit vector magnitude =

A. 0

B. 1

C. 2

D. -1

☐ **Ans: B**

**Q28.** Probability of impossible event =

A. 1

B. 0

C. -1

D.  $\infty$

☐ **Ans: B**

**Q29.** Rank of zero matrix =

A. 0

B. 1

C. 2

D. Undefined

☐ **Ans: A**

**Q30.**  $f''(x) < 0$  implies

- A. Minima
- B. Maxima
- C. Point of inflection
- D. Constant

□ **Ans: B**

## TOP-15 Probable SURE-SHOT MCQs (2026)

### 1. Relations & Functions

**Q1.** A function is a relation in which

फलन वह संबंध है जिसमें—

- A. One output for each input  
प्रत्येक इनपुट का एक ही आउटपुट
- B. One input for many outputs  
एक इनपुट के कई आउटपुट
- C. No output  
कोई आउटपुट नहीं
- D. Many outputs allowed  
कई आउटपुट मान्य

□ **Answer: A**

### 2. Inverse Trigonometric Functions

**Q2.** Domain of  $\sin^{-1}x$  is

$\sin^{-1}x$  का प्रांत—

- A.  $\mathbb{R}$
- B.  $(-\infty, \infty)$
- C.  $[-1, 1]$
- D.  $(0, 1)$

□ **Answer: C**

### 3. Matrices

**Q3.** Order of identity matrix is always

इकाई आव्यूह की कोटि होती है—



- A. Rectangular / आयताकार
- B. Square / वर्गीकार
- C. Zero / शून्य
- D. Column / स्तंभ

□ Answer: B

#### 4. Determinants

**Q4.** If two rows of a determinant are identical, its value is  
यदि निर्धारक की दो पंक्तियाँ समान हों—

- A. 1
- B.  $-1$
- C. 0
- D.  $\infty$

□ Answer: C

#### 5. Continuity & Differentiability

**Q5.** If a function is differentiable, then it is  
यदि फलन अवकलनीय है, तो—

- A. Discontinuous / असतत
- B. Continuous / सतत
- C. Constant / स्थिर
- D. Undefined / अपरिभाषित

□ Answer: B

#### 6. Applications of Derivatives

**Q6.** At maximum or minimum point,  
अधिकतम या न्यूनतम बिंदु पर—

- A.  $f'(x) = 1$
- B.  $f'(x) = -1$
- C.  $f'(x) = 0$
- D.  $f'(x) = \infty$

☐ Answer: C

## 7. Integrals

Q7.  $\int_0 dx =$   
 $\int_0 dx$  का मान—

- A. 0
- B. x
- C. 1
- D. C

☐ Answer: D

## 8. Applications of Integrals

Q8. Area under a curve is always  
वक्र के नीचे का क्षेत्रफल होता है—

- A. Negative / ऋणात्मक
- B. Positive / धनात्मक
- C. Zero / शून्य
- D. Non-negative / ऋणात्मक नहीं

☐ Answer: D

## 9. Differential Equations

Q9. Order of differential equation depends on  
अवकल समीकरण का क्रम निर्भर करता है—

- A. Power of x
- B. Degree
- C. Highest order derivative
- D. Constant

☐ Answer: C

## 10. Vector Algebra

Q10.  $a \cdot a =$   
 $a \cdot a$  का मान—

- A.  $a$
- B.  $|a|$
- C.  $|a|^2$
- D.  $0$

☐ **Answer: C**

## 11. Three Dimensional Geometry

**Q11.** Direction ratios of x-axis are  
x-अक्ष के दिशा अनुपात—

- A.  $(0,1,0)$
- B.  $(0,0,1)$
- C.  $(1,1,1)$
- D.  $(1,0,0)$

☐ **Answer: D**

## 12. Linear Programming

**Q12.** Optimal solution of LPP occurs at  
LPP का सर्वोत्तम हल मिलता है—

- A. Origin
- B. Any interior point
- C. Corner point
- D. Mid-point

☐ **Answer: C**

## 13. Probability

**Q13.** Probability of sure event is  
निश्चित घटना की प्रायिकता—

- A.  $0$
- B.  $1$
- C.  $-1$
- D.  $\infty$

☐ **Answer: B**

## 14. Probability (Complement)

**Q14.**  $P(\bar{A}) =$

पूरक घटना की प्रायिकता—

- A.  $P(A)$
- B.  $1 + P(A)$
- C.  $1 - P(A)$
- D.  $P(A)^2$

□ **Answer: C**

### 15. Mixed (Very Important)

**Q15.** Determinant of identity matrix is

इकाई आव्यूह का निर्धारक—

- A. 0
- B. -1
- C. 1
- D. Depends on order

□ **Answer: C**

## Assertion–Reason MCQs (30 Questions)

(Most Expected – 2026 Final Revision)

### Directions / निर्देश

Choose the correct option:

- A. Both A and R are true and R is the correct explanation of A
- B. Both A and R are true but R is not the correct explanation of A
- C. A is true but R is false
- D. A is false but R is true

### 1. Relations & Functions

**Q1.**

**Assertion (A):** Every function is a relation.

**अभिकथन:** प्रत्येक फलन एक संबंध है।

**Reason (R):** A relation assigns one output to one input.

**कारण:** संबंध प्रत्येक इनपुट को केवल एक आउटपुट देता है।

☐ **Answer: C**

**Q2.**

**A:** A many-one function is allowed.

**अभिकथन:** बहु-एक फलन मान्य है।

**R:** One input can have many outputs.

**कारण:** एक इनपुट के कई आउटपुट हो सकते हैं।

☐ **Answer: C**

## **2. Inverse Trigonometric Functions**

**Q3.**

**A:** Domain of  $\sin^{-1}x$  is  $[-1,1]$ .

**R:** Sine function lies between  $-1$  and  $1$ .

☐ **Answer: A**

**Q4.**

**A:**  $\tan^{-1}x$  is defined for all real  $x$ .

**R:**  $\tan x$  is defined for all real values.

☐ **Answer: C**

## **3. Matrices**

**Q5.**

**A:** Identity matrix is a square matrix.

**R:** Number of rows equals number of columns.

☐ **Answer: A**

**Q6.**

**A:** Product of matrices is commutative.

**R:**  $AB = BA$  for all matrices  $A$  and  $B$ .

☐ **Answer: D**

## **4. Determinants**

**Q7.**

**A:** Determinant of a singular matrix is zero.

**R:** Rows of singular matrix are linearly dependent.

☐ **Answer: A**

**Q8.**

**A:** If two rows of a determinant are equal, its value is zero.

**R:** Interchanging rows changes sign of determinant.

☐ **Answer: B**

## **5. Continuity & Differentiability**

**Q9.**

**A:** If a function is differentiable, it is continuous.

**R:** Differentiability implies continuity.

☐ **Answer: A**

**Q10.**

**A:** A continuous function may not be differentiable.

**R:**  $|x|$  is continuous but not differentiable at  $x = 0$ .

☐ **Answer: A**

## **6. Applications of Derivatives**

**Q11.**

**A:** At maxima,  $f'(x) = 0$ .

**R:** Slope of tangent becomes zero.

☐ **Answer: A**

**Q12.**

**A:** If  $f'(x) > 0$ , function is decreasing.

**R:** Positive derivative shows increasing nature.

☐ **Answer: D**

## **7. Integrals**

**Q13.**

**A:** Integration is inverse of differentiation.

**R:**  $\int f'(x)dx = f(x) + C$ .

☐ **Answer: A**

**Q14.**

**A:**  $\int 0 dx = C$ .

**R:** Integral of zero is a constant.

☐ **Answer: A**

## **8. Applications of Integrals**

**Q15.**

**A:** Area under curve cannot be negative.

**R:** Area represents physical quantity.

☐ **Answer: A**

**Q16.**

**A:** Area between curve and x-axis is always finite.

**R:** Integration always gives finite value.

☐ **Answer: C**

## **9. Differential Equations**

**Q17.**

**A:** Order of differential equation depends on highest derivative.

**R:** Degree depends on power of highest derivative.

☐ **Answer: B**

**Q18.**

**A:** General solution contains arbitrary constant.

**R:** Particular solution has no constant.

☐ **Answer: B**

## **10. Vector Algebra**

**Q19.**

**A:** Magnitude of unit vector is 1.

**R:** Unit vector shows only direction.

☐ **Answer: A**

**Q20.**

**A:**  $\mathbf{a} \cdot \mathbf{b} = 0$  implies vectors are perpendicular.

**R:** Dot product of perpendicular vectors is zero.

☐ **Answer: A**

## **11. Three Dimensional Geometry**

**Q21.**

**A:** Direction ratios of a line are not unique.

**R:** They can be multiplied by any non-zero constant.

☐ **Answer: A**

**Q22.**

**A:** Equation of plane has three variables.

**R:** Plane exists in three-dimensional space.

☐ **Answer: A**

## **12. Linear Programming**

**Q23.**

**A:** Feasible region is common shaded region.

**R:** It satisfies all given constraints.

☐ **Answer: A**

**Q24.**

**A:** Optimal value occurs at corner point.

**R:** Objective function is linear.

☐ **Answer: A**

## **13. Probability**

**Q25.**

**A:** Probability of impossible event is zero.

**R:** Impossible event never occurs.

☐ **Answer: A**

**Q26.**

**A:**  $P(\bar{A}) = 1 - P(A)$ .

**R:** A and  $\bar{A}$  are mutually exclusive and exhaustive.

☐ **Answer: A**

**Q27.**

**A:** Probability can be negative.

**R:** Probability lies between 0 and 1.

☐ **Answer: D**



**Q28.**

**A:** Sum of probabilities of all outcomes is 1.

**R:** Sample space contains all possible outcomes.

☐ **Answer: A**

### **Mixed Revision (Very High Probability)**

**Q29.**

**A:**  $\det(I) = 1$ .

**R:** Identity matrix has all diagonal elements as 1.

☐ **Answer: A**

**Q30.**

**A:** Rank of zero matrix is zero.

**R:** All rows of zero matrix are linearly dependent.

☐ **Answer: A**

# Very Short Questions & Answers

## Chapter 1: Relations & Functions

**Q1.** What is a function?

**Answer:**

English: A relation where each input has exactly one output.

हिंदी: वह संबंध जिसमें प्रत्येक इनपुट का केवल एक आउटपुट हो।

**Q2.** Define domain of a function.

**Answer:**

English: Set of all possible inputs.

हिंदी: सभी संभव इनपुट का समुच्चय।

## Chapter 2: Inverse Trigonometric Functions

**Q3.** Write the principal value range of  $\sin^{-1}x$ .

**Answer:**

English:  $[-\pi/2, \pi/2]$

हिंदी:  $[-\pi/2, \pi/2]$

**Q4.** What is  $\sin^{-1}(1)$ ?

**Answer:**

English:  $\pi/2$

हिंदी:  $\pi/2$

## Chapter 3: Matrices

**Q5.** Define identity matrix.

**Answer:**

English: A square matrix with 1 on diagonal and 0 elsewhere.

हिंदी: विकर्ण पर 1 और अन्य स्थानों पर 0 वाली वर्ग आव्यूह।

**Q6.** Order of matrix  $A = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$ ?

**Answer:**

English:  $1 \times 3$

हिंदी:  $1 \times 3$

## Chapter 4: Determinants

**Q7.** Determinant of identity matrix of order 2 is?

**Answer:**

English: 1

हिंदी: 1

**Q8.** Value of determinant with two identical rows?

**Answer:**

English: 0

हिंदी: 0

## **Chapter 5: Continuity & Differentiability**

**Q9.** Define continuity at a point.

**Answer:**

English:  $LHL = RHL = f(a)$

हिंदी: बाएँ सीमा = दाएँ सीमा = फलन का मान।

**Q10.** Derivative of constant is?

**Answer:**

English: 0

हिंदी: 0

## **Chapter 6: Applications of Derivatives**

**Q11.** What does  $dy/dx$  represent?

**Answer:**

English: Rate of change.

हिंदी: परिवर्तन की दर।

**Q12.** Maximum or minimum occurs when?

**Answer:**

English: First derivative is zero.

हिंदी: प्रथम अवकलज शून्य हो।

## **Chapter 7: Integrals**

**Q13.**  $\int 0 \, dx$  equals?

**Answer:**

English: Constant (C)

हिंदी: स्थिरांक (C)

**Q14.**  $\int x^n \, dx$  formula?

**Answer:**

English:  $x^{n+1}/(n+1) + C$

हिंदी:  $x^{n+1}/(n+1) + C$

## Chapter 8: Applications of Integrals

**Q15.** What does definite integral represent?

**Answer:**

English: Area under curve.

हिंदी: वक्र के नीचे का क्षेत्रफल।

## Chapter 9: Differential Equations

**Q16.** Degree of differential equation  $dy/dx = x$ ?

**Answer:**

English: 1

हिंदी: 1

**Q17.** General solution contains?

**Answer:**

English: Arbitrary constant.

हिंदी: मनचाहा स्थिरांक।

## Chapter 10: Vector Algebra

**Q18.** Magnitude of zero vector is?

**Answer:**

English: 0

हिंदी: 0

**Q19.** Unit vector magnitude is?

**Answer:**

English: 1

हिंदी: 1

## Chapter 11: Three Dimensional Geometry

**Q20.** Distance between identical points?

**Answer:**

English: 0

हिंदी: 0

**Q21.** Direction ratios of x-axis?

**Answer:**

English: (1, 0, 0)

हिंदी: (1, 0, 0)

## Chapter 12: Linear Programming

**Q22.** Objective function is used to?

**Answer:**

English: Maximize or minimize.

हिंदी: अधिकतम या न्यूनतम करना।

**Q23.** Feasible region lies in?

**Answer:**

English: First quadrant.

हिंदी: प्रथम चतुर्थांश।

## Chapter 13: Probability

**Q24.** Probability of impossible event?

**Answer:**

English: 0

हिंदी: 0

**Q25.** Maximum value of probability?

**Answer:**

English: 1

हिंदी: 1

**Q26.** Formula of conditional probability?

**Answer:**

English:  $P(A|B) = P(A \cap B)/P(B)$

हिंदी:  $P(A|B) = P(A \cap B)/P(B)$

## Mixed Revision (Highly Repeated)

**Q27.** Value of  $|A| = 0$  indicates?

**Answer:**

English: No inverse exists.

हिंदी: व्युत्क्रम अस्तित्व में नहीं।

**Q28.**  $\sin^2\theta + \cos^2\theta$  equals?

**Answer:**

English: 1

हिंदी: 1

**Q29.** If  $f'(x) > 0$ , function is?

**Answer:**

English: Increasing.

हिंदी: वर्धमान।

**Q30.** Probability of sure event?

**Answer:**

English: 1

हिंदी: 1

# FINAL REVISION: 30 SHORT QUESTIONS (BILINGUAL)

## Chapter 1: Relations & Functions

Q1. Check whether the relation  $R = \{(1,2), (2,3), (3,4)\}$  is a function.

जाँचिए कि दिया गया संबंध फलन है या नहीं।

**Formula (English):**

A relation is a function if each element of domain has exactly one image.

**Solution (Steps):**

$1 \rightarrow 2$  (one image)

$2 \rightarrow 3$  (one image)

$3 \rightarrow 4$  (one image)

**Answer:** R is a function.

**उत्तर (हिंदी):**

प्रत्येक इनपुट का केवल एक आउटपुट है।

अतः R एक फलन है।

Q2. Find the domain of  $f(x) = \sqrt{x-3}$ .

$f(x)$  का परिभाषा क्षेत्र ज्ञात कीजिए।

**Formula:**

Expression under square root  $\geq 0$

**Steps:**

$x - 3 \geq 0$

$x \geq 3$

**Answer:** Domain =  $[3, \infty)$

**उत्तर:** परिभाषा क्षेत्र =  $[3, \infty)$

## Chapter 2: Inverse Trigonometric Functions

Q3. Find the principal value of  $\sin^{-1}(1/2)$ .

$\sin^{-1}(1/2)$  का मुख्य मान ज्ञात कीजिए।

**Formula:**

$\sin^{-1}(\sin \theta) = \theta$ , where  $\theta \in [-\pi/2, \pi/2]$

**Step:**

$$\sin(\pi/6) = 1/2$$

**Answer:**  $\pi/6$

**उत्तर:**  $\pi/6$

**Q4. Evaluate  $\tan^{-1}(1) + \tan^{-1}(1)$ .**

मान ज्ञात कीजिए।

**Formula:**

$$\tan^{-1}(1) = \pi/4$$

**Steps:**

$$\pi/4 + \pi/4 = \pi/2$$

**Answer:**  $\pi/2$

**उत्तर:**  $\pi/2$

## Chapter 3: Matrices

**Q5. Write the order of matrix  $A = [2 \ 3 \ 4]$ .**

आव्यूह की कोटि लिखिए।

**Formula:**

Order = Number of rows  $\times$  Number of columns

**Steps:**

1 row, 3 columns

**Answer:**  $1 \times 3$

**उत्तर:**  $1 \times 3$

**Q6. Write identity matrix of order 2.**



कोटि 2 का इकाई आव्यूह लिखिए।

**Formula / Answer:**

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

## Chapter 4: Determinants

**Q7. Evaluate  $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ .**

मान ज्ञात कीजिए।

**Formula:**

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

**Steps:**

$$(1 \times 4) - (2 \times 3) = 4 - 6$$

**Answer:**  $-2$

उत्तर:  $-2$

**Q8. If  $|A| = 5$ , find  $|2A|$ .**

$|2A|$  ज्ञात कीजिए।

**Formula:**

$$|kA| = k^n |A| \quad (n = \text{order})$$

**Steps:**

$$2 \times 2 \text{ matrix} \Rightarrow n = 2$$

$$|2A| = 2^2 \times 5 = 20$$

**Answer:** 20

उत्तर: 20

## Chapter 5: Continuity & Differentiability

**Q9. Differentiate  $y = x^2$ .**

अवकलन कीजिए।

**Formula:**

$$d/dx(x^n) = nx^{n-1}$$

**Step:**

$$dy/dx = 2x$$

**Answer:**  $2x$

**Q10. Find  $dy/dx$  of  $y = \sin x$ .**

**अवकलज ज्ञात कीजिए।**

**Formula:**

$$d/dx(\sin x) = \cos x$$

**Answer:**  $\cos x$

## **Chapter 6: Applications of Derivatives**

**Q11. Find slope of  $y = x^3$  at  $x = 1$ .**

**ढाल ज्ञात कीजिए।**

**Steps:**

$$dy/dx = 3x^2$$

$$\text{At } x = 1 \rightarrow 3$$

**Answer:**  $3$

**उत्तर:**  $3$

**Q12. At what point  $y = x^2$  is minimum?**

**न्यूनतम मान कहाँ होगा?**

**Formula:**

$$dy/dx = 0$$

**Steps:**

$$2x = 0 \rightarrow x = 0$$

**Answer:** Minimum at  $x = 0$

## Chapter 7: Integrals

Q13. Evaluate  $\int x \, dx$ .

समाकलन कीजिए।

Formula:

$$\int x^n \, dx = x^{n+1}/(n+1)$$

Answer:  $x^2/2 + C$

Q14. Find  $\int \cos x \, dx$ .

समाकलन कीजिए।

Answer:  $\sin x + C$

## Chapter 8: Applications of Integrals

Q15. Find area under  $y = x$  from 0 to 1.

क्षेत्रफल ज्ञात कीजिए।

Formula:

$$\text{Area} = \int_0^1 x \, dx$$

Steps:

$$= [x^2/2]_0^1 = 1/2$$

Answer:  $1/2$  sq unit

Q16. Write formula for area between curves.

सूत्र लिखिए।

Answer:

$$\text{Area} = \int (\text{Upper} - \text{Lower}) \, dx$$

## Chapter 9: Differential Equations

Q17. Solve  $dy/dx = 2x$ .

अवकल समीकरण हल कीजिए।

Steps:

$$dy = 2x \, dx$$

$$\int dy = \int 2x \, dx$$

$$y = x^2 + C$$

Q18. Write general solution of  $dy/dx = 0$ .

सामान्य हल लिखिए।

Answer:  $y = C$

## Chapter 10: Vector Algebra

Q19. Find magnitude of  $3i + 4j$ .

परिमाण ज्ञात कीजिए।

Formula:

$$|a| = \sqrt{a^2 + b^2}$$

Answer:  $\sqrt{9 + 16} = 5$

Q20. Write unit vector along x-axis.

x-अक्ष की इकाई सदिश लिखिए।

Answer:  $\hat{i}$

## Chapter 11: 3D Geometry

Q21. Distance between  $(0,0,0)$  and  $(2,3,6)$ .

दूरी ज्ञात कीजिए।

Formula:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Answer:  $\sqrt{49} = 7$

Q22. Equation of x-axis.

**x-अक्ष का समीकरण।**

**Answer:**  $y = 0, z = 0$

## **Chapter 12: Linear Programming**

**Q23. What is feasible region?**

**साध्य क्षेत्र क्या है?**

**Answer:**

Region satisfying all constraints.

**उत्तर:**

सभी प्रतिबंधों को संतुष्ट करने वाला क्षेत्र।

**Q24. Write objective function.**

**उद्देश्य फलन लिखिए।**

**Answer:**  $Z = ax + by$

## **Chapter 13: Probability**

**Q25. Probability of sure event.**

**निश्चित घटना की प्रायिकता।**

**Answer:** 1

**Q26. If  $P(A)=0.4$ , find  $P(\text{not } A)$ .**

**पूरक घटना ज्ञात कीजिए।**

**Formula:**

$$P(A') = 1 - P(A)$$

**Answer:** 0.6

**Q27. Define random experiment.**

**यादृच्छिक प्रयोग की परिभाषा।**

**Answer:**

Experiment with uncertain outcome.

**Q28. Write formula of conditional probability.**

**सशर्त प्रायिकता का सूत्र।**

**Answer:**

$$P(A|B) = P(A \cap B) / P(B)$$

**Q29. Mean of binomial distribution.**

**द्विपद वितरण का माध्य।**

**Answer:** Mean = np

**Q30. Variance of binomial distribution.**

**प्रसरण।**

## 30 Probable Short Answer Questions (Bilingual)

### Chapter 1: Relations & Functions

#### Q1

**EN:** Find the domain of  $(f(x)=\sqrt{5-2x})$ .

**HI:**  $(f(x)=\sqrt{5-2x})$  का परिभाषा क्षेत्र ज्ञात कीजिए।

**Solution / उत्तर:**

Condition:  $(5-2x \geq 0)$

$\Rightarrow (2x \leq 5)$

$\Rightarrow (x \leq \frac{5}{2})$

**Domain:**  $((-\infty, 5/2])$

**Formula:** Square root defined for non-negative values

**सूत्र:** वर्गमूल तभी परिभाषित जब मान  $\geq 0$

---

### Chapter 2: Inverse Trigonometric Functions

#### Q2

**EN:** Evaluate  $(\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3)$ .

**HI:**  $(\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3)$  का मान ज्ञात कीजिए।

**Solution:**

$(\tan^{-1}1 = \pi/4)$

$(\tan^{-1}2 + \tan^{-1}3 = \pi/2)$

**Answer:**

$(\pi/4 + \pi/2 = \frac{3\pi}{4})$

**Formula:**

$(\tan^{-1}a + \tan^{-1}b = \tan^{-1}\left(\frac{a+b}{1-ab}\right))$

---

### Chapter 3: Matrices

### Q3

**EN:** Find determinant of  $(A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix})$ .

**HI:** मैट्रिक्स (A) का निर्धारक ज्ञात कीजिए।

**Solution:**

$$(|A| = (2)(4) - (3)(1) = 8 - 3 = 5)$$

**Answer:** (5)

**Formula:**

$$(|A| = ad - bc)$$

---

## Chapter 4: Determinants

### Q4

**EN:** For what value of (k) determinant is zero?

$$(\begin{vmatrix} 1 & 2 \\ 3 & k \end{vmatrix})$$

**HI:** किस (k) के लिए निर्धारक शून्य होगा?

**Solution:**

$$(|A| = 1 \cdot k - 6 = 0)$$

$$\Rightarrow (k = 6)$$

---

## Chapter 5: Continuity & Differentiability

### Q5

**EN:** Find  $(\frac{d}{dx}(x^3 + 5x))$ .

**HI:**  $(x^3 + 5x)$  का अवकलन कीजिए।

**Solution:**

$$(\frac{d}{dx}(x^3) = 3x^2)$$

$$(\frac{d}{dx}(5x) = 5)$$

**Answer:**  $(3x^2 + 5)$

---



## Chapter 6: Applications of Derivatives

### Q6

**EN:** Find slope of tangent to  $(y=x^2)$  at  $(x=2)$ .

**HI:**  $(x=2)$  पर स्पर्शरेखा का ढाल ज्ञात कीजिए।

**Solution:**

$$\left(\frac{dy}{dx} = 2x\right)$$

At  $(x=2)$ : slope = (4)

---

## Chapter 7: Integrals

### Q7

**EN:** Evaluate  $\left(\int 3x^2 dx\right)$ .

**HI:**  $\left(\int 3x^2 dx\right)$  का मान ज्ञात कीजिए।

**Solution:**

$$\left(\int x^n dx = \frac{x^{n+1}}{n+1}\right)$$

$$\left[\int 3x^2 dx = x^3 + C\right]$$

---

## Chapter 8: Definite Integrals

### Q8

**EN:** Evaluate  $\left(\int_0^1 x dx\right)$ .

**HI:**  $\left(\int_0^1 x dx\right)$  हल कीजिए।

**Solution:**

$$\left[\int_0^1 x dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}\right]$$

---

## Chapter 9: Differential Equations

## Q9

**EN:** Solve  $\left(\frac{dy}{dx}\right)=x$ .

**HI:**  $\left(\frac{dy}{dx}\right)=x$  हल कीजिए।

**Solution:**

$$\left[ \begin{array}{l} dy = x \, dx \end{array} \right]$$

$$\left[ \begin{array}{l} y = \frac{x^2}{2} + C \end{array} \right]$$

---

## Chapter 10: Vector Algebra

### Q10

**EN:** Find magnitude of  $(\vec{a}=2\hat{i}+3\hat{j})$ .

**HI:** वेक्टर का परिमाण ज्ञात कीजिए।

**Solution:**

$$\left[ \begin{array}{l} |\vec{a}| = \sqrt{2^2 + 3^2} = \sqrt{13} \end{array} \right]$$

---

## Chapter 11: 3-D Geometry

### Q11

**EN:** Distance between  $((1,2,3))$  and  $((4,6,3))$ .

**HI:** दो बिंदुओं के बीच दूरी ज्ञात कीजिए।

**Solution:**

$$\left[ \begin{array}{l} d = \sqrt{(4-1)^2 + (6-2)^2 + (3-3)^2} \\ \\ = \sqrt{9+16} = 5 \end{array} \right]$$

---

## Chapter 12: Linear Programming

### Q12

**EN:** What is a feasible region?

**HI:** संभव क्षेत्र क्या होता है?

**Answer:**

**EN:** Region satisfying all constraints

**HI:** सभी शर्तों को संतुष्ट करने वाला क्षेत्र

---

## Chapter 13: Probability

### Q13

**EN:** Probability of getting a head in a fair coin.

**HI:** एक सिक्के में हेड की प्रायिकता।

**Answer:**

[

$$P(H) = \frac{1}{2}$$

]

### Q14. Differentiation

**EN:** Find  $\frac{d}{dx}(\sin x)$ .

**HI:**  $(\sin x)$  का अवकलन कीजिए।

**Formula / सूत्र:**

[

$$\frac{d}{dx}(\sin x) = \cos x$$

]

**Answer / उत्तर:**

[

$$\boxed{\cos x}$$

]

---

### Q15. Integration

**EN:** Evaluate  $(\int \cos x, dx)$ .

**HI:**  $(\int \cos x, dx)$  का मान ज्ञात कीजिए।

**Formula / सूत्र:**

$$\int \cos x, dx = \sin x + C$$

**Answer:**

$$\boxed{\sin x + C}$$

---

### Q16. Vector Algebra

**EN:** Write the formula for unit vector along  $(\vec{a})$ .

**HI:**  $(\vec{a})$  की दिशा में इकाई सदिश का सूत्र लिखिए।

**Formula / सूत्र:**

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

**Answer:**

Unit vector = Vector / Magnitude

इकाई सदिश = सदिश / परिमाण

---

### Q17. Matrices

**EN:** What is the order of a matrix with 3 rows and 2 columns?

**HI:** 3 पंक्ति और 2 स्तंभ वाले मैट्रिक्स का क्रम क्या है?

**Answer / उत्तर:**

$$\boxed{3 \times 2}$$

---

### Q18. Mean Value Theorem

**EN:** State the conditions for Mean Value Theorem.

**HI:** औसत मान प्रमेय की शर्तें लिखिए।

**Answer:**

**EN:**

1. Function continuous in  $[a, b]$
2. Differentiable in  $((a, b))$

**HI:**

1. फलन  $[a, b]$  में सतत हो
  2.  $((a, b))$  में अवकलनीय हो
- 

### Q19. Inverse Trigonometry

**EN:** Find  $(\tan^{-1} 0)$ .

**HI:**  $(\tan^{-1} 0)$  का मान ज्ञात कीजिए।

**Solution:**

[  
 $\tan 0 = 0$   
]

**Answer:**

[  
 $\boxed{0}$   
]

---

### Q20. Probability

**EN:** Write the range of probability.

**HI:** प्रायिकता का परास लिखिए।

**Answer / उत्तर:**

$$\boxed{0 \leq P(A) \leq 1}$$

---

### Q21. Determinants

**EN:** Find determinant of identity matrix of order 2.

**HI:** 2 कोटि की इकाई मैट्रिक्स का निर्धारक ज्ञात कीजिए।

**Solution:**

$$|I| = 1$$

**Answer:**

$$\boxed{1}$$

---

### Q22. Vector Algebra

**EN:** What is the angle between two perpendicular vectors?

**HI:** दो लम्बवत सदिशों के बीच कोण कितना होता है?

**Answer / उत्तर:**

$$\boxed{90^\circ}$$

---

### Q23. Differential Equation

**EN:** Find the degree of the equation

$$\frac{d^2y}{dx^2} + y = 0$$

**HI:** अवकल समीकरण की कोटि ज्ञात कीजिए।

**Solution:**

Highest order derivative = second order

**Answer:**

[  
 $\boxed{2}$   
]

---

**Q24. Mensuration (Application of Integrals)**

**EN:** Write formula for volume of a cylinder.

**HI:** बेलन का आयतन सूत्र लिखिए।

**Formula / सूत्र:**

[  
 $V = \pi r^2 h$   
]

---

**Q25. Application of Derivatives**

**EN:** Condition for a function to be increasing.

**HI:** फलन के बढ़ने की शर्त क्या है?

**Answer:**

[  
 $\boxed{f'(x) > 0}$   
]

---

**Q26. Matrices**

**EN:** What is a null matrix?

**HI:** शून्य मैट्रिक्स क्या होता है?

**Answer:**

**EN:** Matrix with all elements zero

**HI:** वह मैट्रिक्स जिसके सभी अवयव शून्य हों

---

## Q27. Scalars & Vectors

EN: What is a scalar quantity?

HI: अदिश राशि क्या है?

Answer:

EN: Quantity having only magnitude

HI: केवल परिमाण वाली राशि

---

## Q28. Trigonometry

EN: Find maximum value of  $(\sin x)$ .

HI:  $(\sin x)$  का अधिकतम मान क्या है?

Answer:

[  
 $\boxed{1}$   
]

---

## Q29. Inverse of Matrix

EN: When does inverse of a matrix exist?

HI: मैट्रिक्स का व्युत्क्रम कब अस्तित्व में होता है?

Answer:

[  
 $\boxed{|A| \neq 0}$   
]

---

## Q30. Probability

EN: Write formula for complementary probability.

HI: पूरक प्रायिकता का सूत्र लिखिए।

Formula / सूत्र:

[



$$P(A') = 1 - P(A)$$

]

## Long Questions and Answers

### 1) Limits & Continuity — L'Hospital type / Indeterminate form

**Q (EN):** Evaluate  $\lim_{x \rightarrow 0} \frac{\sin(3x) - 3x}{x^3}$ .

**Q (HI):** सीमाएँ — मान निकालिए  $\lim_{x \rightarrow 0} \frac{\sin(3x) - 3x}{x^3}$ .

**Solution (EN):**

Use Taylor expansion or series:  $\sin t = t - \frac{t^3}{6} + o(t^3)$ . Put  $t=3x$ :

$$\sin(3x) = 3x - \frac{(3x)^3}{6} + o(x^3) = 3x - \frac{27x^3}{6} + o(x^3) = 3x - \frac{9}{2}x^3 + o(x^3).$$

So numerator  $= ((3x - \frac{9}{2}x^3 + o(x^3)) - 3x) = -\frac{9}{2}x^3 + o(x^3)$ . Divide by  $(x^3)$ :  $(-\frac{9}{2} + o(1))$ .

**Limit**  $= (-\frac{9}{2})$ .

**Solution (HI):**

$(\sin t)$  का टीaylor:  $(t - \frac{t^3}{6} + \dots)$ .  $(t=3x)$  रखें। गणना से उत्तर  $(-\frac{9}{2})$  आता है।

**Key formula:**  $\sin t = t - \frac{t^3}{6} + \dots$

### 2) Derivative — Differentiation of inverse trig

**Q (EN):** If  $(y = \tan^{-1}(\frac{2x}{1-x^2}))$ , show that  $(\frac{dy}{dx} = \frac{1}{1+x^2})$ .

**Q (HI):** यदि  $(y = \tan^{-1}(\frac{2x}{1-x^2}))$ , सिद्ध कीजिए

$$(\frac{dy}{dx} = \frac{1}{1+x^2}).$$

**Solution (EN):**

Note identity:  $(\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta})$ . So set  $(x = \tan\theta) \Rightarrow (\frac{2x}{1-x^2} = \tan(2\theta))$ . Then  $(y = \tan^{-1}(\frac{2x}{1-x^2})) = 2\theta$  (take principal branch where valid)  $\Rightarrow (y = 2\arctan x)$ . Differentiate:  $(\frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2}) = \frac{2}{1+x^2}$ . Wait—this conflicts target; check algebra. Alternative route: Actually  $(\tan^{-1}(\frac{2x}{1-x^2}) = 2\arctan x)$  only when  $(|x| < 1)$  and branches considered.

Differentiate directly using chain rule: Let  $(u = \frac{2x}{1-x^2})$ . Then  $(\frac{du}{dx} = \frac{2(1-x^2) - 2x(-2x)}{(1-x^2)^2} = \frac{2-2x^2+4x^2}{(1-x^2)^2} = \frac{2+2x^2}{(1-x^2)^2} = \frac{2(1+x^2)}{(1-x^2)^2})$ . Now  $(\frac{dy}{dx} = \frac{1}{1+u^2} \cdot \frac{du}{dx})$ . Compute  $(1+u^2 = 1 + \frac{4x^2}{(1-x^2)^2} = \frac{(1-x^2)^2 + 4x^2}{(1-x^2)^2} = \frac{1-2x^2+x^4+4x^2}{(1-x^2)^2} = \frac{1+2x^2+x^4}{(1-x^2)^2} = \frac{(1+x^2)^2}{(1-x^2)^2})$ .

Thus  $(\frac{dy}{dx} = \frac{1}{(1+x^2)^2/(1-x^2)^2} \cdot \frac{2(1+x^2)}{(1-x^2)^2} = \frac{(1-x^2)^2}{(1+x^2)^2} \cdot \frac{2(1+x^2)}{(1-x^2)^2} = \frac{2}{1+x^2})$ .

So derivative is  $(\frac{2}{1+x^2})$ . (Hence original target  $(\frac{1+x^2}{1+x^4})$  is algebraically equal? Check:  $(\frac{1+x^2}{1+x^4} = \frac{1+x^2}{(1+x^2)^2 - 2x^2})$  — not same. The correct derivative is  $(\frac{2}{1+x^2})$ .)

### Solution (HI):

सरलतम:  $(u = \frac{2x}{1-x^2})$  लें, श्रृंखला और गुणनफल नियम लागू करें। अंतिम उत्तर  $(\frac{2}{1+x^2})$  आता है।

**Key formula:**  $(\frac{d}{dx}(\tan^{-1}u) = \frac{u'}{1+u^2})$ .

(Remark: many exam variants ask to show  $(y=2\arctan x)$  then differentiate.)

## 3) Application of Derivative — Max/Min & Tangent

**Q (EN):** For  $(f(x)=x^3-3x+1)$ , find intervals of increase/decrease, local extrema and equation of tangent at point where  $(x=1)$ .

**Q (HI):**  $(f(x)=x^3-3x+1)$ . वृद्धि/क्षय के अंतराल, स्थानीय अधिकतम/न्यूनतम और  $(x=1)$  पर स्पर्शरेखा का समीकरण ज्ञात कीजिए।

### Solution (EN):

$(f'(x)=3x^2-3=3(x^2-1)=3(x-1)(x+1))$ . Critical points at  $(x=\pm 1)$ . Sign: for  $(x<-1)$ ,  $(f'>0)$  (increasing);  $(-1<x<1)$ :  $(f'<0)$  (decreasing);  $(x>1)$ :  $(f'>0)$  (increasing). So local max at  $(x=-1)$ , local min at  $(x=1)$ . Values:  $(f(-1)=(-1)^3-3(-1)+1=-1+3+1=3)$ .  $(f(1)=1-3+1=-1)$ . Tangent at  $(x=1)$ : slope  $(m=f'(1)=3(1)^2-3=0)$ . Point  $((1,-1))$ . Equation:  $(y+1=0(x-1)) \Rightarrow (y=-1)$ .

### Solution (HI):

$(f'(x)=3(x-1)(x+1))$ . क्रमशः वृद्धि/क्षय का वर्णन ऊपर जैसा।  $(x=1)$  पर न्यूनतम, स्पर्शरेखा क्षैतिज  $(y=-1)$ ।

**Key formula:** Critical points where  $(f'(x)=0)$ ; test sign changes for extrema.

#### 4) Rolle's / Mean Value Theorem (MVT)

**Q (EN):** Verify Rolle's theorem for  $(g(x)=x^3-3x)$  on  $([-\sqrt{3},\sqrt{3}])$  and find the  $c$  guaranteed by Rolle's theorem.

**Q (HI):**  $(g(x)=x^3-3x)$  पर  $([-\sqrt{3},\sqrt{3}])$  में Rolle की शर्तें सत्यापित करें और  $c$  खोजिए।

**Solution (EN):**

Continuous and differentiable everywhere.  $(g(-\sqrt{3})=(-\sqrt{3})^3-3(-\sqrt{3})=-3\sqrt{3}+3\sqrt{3}=0)$ . Similarly  $(g(\sqrt{3})=3\sqrt{3}-3\sqrt{3}=0)$ . Hence Rolle applies  $\Rightarrow (\exists c \in (-\sqrt{3},\sqrt{3}))$  with  $(g'(c)=0)$ .  $(g'(x)=3x^2-3=3(x^2-1))$ . Solve  $(3(x^2-1)=0) \Rightarrow (x=\pm 1)$ . Both  $(x=\pm 1)$  lie inside interval  $([-\sqrt{3},\sqrt{3}])$  (since  $(\sqrt{3}\approx 1.732)$ ). So  $(c=1)$  or  $(c=-1)$ .

**Solution (HI):**

शर्तें पूर्ण;  $(g' = 3(x^2-1)=0)$  पर  $(x=\pm 1)$ । ये दोनों अंदर आते हैं। अतः  $(c=\pm 1)$ ।

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#### 5) Integration — Definite integral using substitution

**Q (EN):** Evaluate  $(\int_0^1 \frac{x^2}{(1+x^3)} dx)$ .

**Q (HI):**  $(\int_0^1 \frac{x^2}{1+x^3} dx)$  का मान ज्ञात कीजिए।

**Solution (EN):**

Let  $(u=1+x^3) \Rightarrow (du=3x^2 dx) \Rightarrow (x^2 dx = du/3)$ . When  $(x=0)$ ,  $(u=1)$ ; when  $(x=1)$ ,  $(u=2)$ .  
Integral  $= (\int_{u=1}^2 \frac{1}{u} \cdot \frac{du}{3}) = \frac{1}{3} \ln u \Big|_1^2 = \frac{1}{3} \ln 2$ .

**Solution (HI):**

उप substitution  $(u=1+x^3)$  करके मिलता है  $(\frac{1}{3} \ln 2)$ ।

**Key formula:** Substitution:  $(\int f(g(x))g'(x)dx)$ .

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#### 6) Integration — By parts (long question)

**Q (EN):** Evaluate  $(\int x e^{2x} dx)$ .

**Q (HI):**  $(\int x e^{2x} dx)$  का पूरा हल दीजिए (by parts).

**Solution (EN):**

Use integration by parts: take  $(u=x) \Rightarrow (du=dx)$ .  $(dv=e^{2x} dx) \Rightarrow (v=\frac{1}{2}e^{2x})$ .  
Then  $(\int x e^{2x} dx = uv - \int v, du = x \cdot \frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x} dx =$

$$\frac{x e^{2x}}{2} - \frac{1}{2} \cdot \frac{1}{2} e^{2x} + C = \frac{e^{2x}}{2} \left( x - \frac{1}{2} \right) + C.$$

**Solution (HI):**

By parts से उत्तर  $\left( \frac{e^{2x}}{2} \right) \big( x - \frac{1}{2} \big) + C$ ।

**Key formula:**  $\int u, dv = uv - \int v, du$ .

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## 7) Definite Integrals — Area between curves

**Q (EN):** Find area enclosed by curves  $(y=x^2)$  and  $(y=2x)$ .

**Q (HI):**  $(y=x^2)$  और  $(y=2x)$  के बीच बंद क्षेत्रफल ज्ञात कीजिए।

**Solution (EN):**

Intersection:  $(x^2=2x) \Rightarrow (x(x-2)=0) \Rightarrow (x=0,2)$ . Area =  $\int_0^2 (2x - x^2) dx = \left[ x^2 - \frac{x^3}{3} \right]_0^2 = \left( 4 - \frac{8}{3} \right) = \frac{12-8}{3} = \frac{4}{3}$ .

**Solution (HI):**

परिणाम क्षेत्रफल =  $\left( \frac{4}{3} \right)$  इकाई<sup>2</sup>।

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## 8) Differential Equations — First order linear

**Q (EN):** Solve  $\left( \frac{dy}{dx} \right) + y \tan x = \sin x$ , for general solution.

**Q (HI):**  $\left( \frac{dy}{dx} \right) + y \tan x = \sin x$  हल कीजिए।

**Solution (EN):**

Standard linear:  $(dy/dx + P(x)y = Q(x))$  with  $(P=\tan x)$ . Integrating factor  $(\mu(x)=e^{\int \tan x dx})=e^{-\ln|\cos x|}=\sec x$ . Multiply equation by  $(\sec x)$ :  $(\sec x, dy/dx + y \sec x \tan x = \sin x \sec x = \tan x)$ . Left is  $(d(y \sec x)/dx)$ . So  $(d(y \sec x)/dx = \tan x)$ . Integrate:  $(y \sec x = \int \tan x dx = -\ln|\cos x| + C)$ . Thus  $(y = \cos x \big( -\ln|\cos x| + C \big))$ .

**Solution (HI):**

इंटीग्रेटिंग फ़ैक्टर  $(\sec x)$ । अंतिम:  $(y = \cos x \big( C - \ln|\cos x| \big))$ ।

**Key formula:** IF  $(e^{\int P dx})$ .

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## 9) Matrices — Inverse & consistency of system

**Q (EN):** For matrix  $(A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix})$ , find  $(A^{-1})$ . Use it to solve system  $(2x+y=5, 3x+2y=8.)$

**Q (HI):**  $(A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix})$  का इन्वर्स निकालिए और उसी से दिए गए रैखिक समीकरण हल करिए।

**Solution (EN):**

$(\det A = 2 \cdot 2 - 1 \cdot 3 = 4 - 3 = 1)$ .  $(A^{-1} = \frac{1}{1} \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix})$ . Write system in matrix form  $(A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix})$ . So  $(\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} \begin{pmatrix} 5 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \cdot 5 - 1 \cdot 8 \\ -3 \cdot 5 + 2 \cdot 8 \end{pmatrix} = \begin{pmatrix} 10 - 8 \\ -15 + 16 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix})$ .

**Solution (HI):**

$(\det=1)$ ,  $(A^{-1} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix})$ . हल:  $(x=2, y=1)$ ।

**Key formula:**  $(A^{-1} = \frac{1}{\det A} \operatorname{adj} A)$ .

## 10) Determinants — Area & zero determinant interpretation

**Q (EN):** If area of triangle with vertices  $((1,2),(3,5),(4,8))$  is zero, show determinant test and evaluate area.

**Q (HI):** दिए बिंदुओं से बनता त्रिभुज शून्य क्षेत्रफल देता है—determinant से जाँच करिए और क्षेत्रफल ज्ञात कीजिए।

**Solution (EN):**

Area =  $(\frac{1}{2} |\begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 1 \\ 4 & 8 & 1 \end{vmatrix}|)$ .

Evaluate determinant: Expand:

Compute determinant of  $(\begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 1 \\ 4 & 8 & 1 \end{vmatrix})$ . Subtract third column manipulations or compute directly: Using first row expansion:

$(1 \cdot \begin{vmatrix} 5 & 1 \\ 8 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 3 & 5 \\ 4 & 8 \end{vmatrix}) = (1(5-8) - 2(3-4) + 1(24-20)) = (-3 - 2(-1) + 4 = -3 + 2 + 4 = 3)$ . Area =  $(\frac{1}{2} |3| = \frac{3}{2})$ . Not zero—so points are not collinear.

**Solution (HI):**

डिटर्मिनेंट = 3  $\Rightarrow$  क्षेत्रफल =  $(3/2)$ । अतः शून्य नहीं है।

**Key formula:** Area triangle =  $(\frac{1}{2} |\det([\dots])|)$ .

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## 11) 3-D Geometry — Equation of plane

**Q (EN):** Find equation of plane passing through  $A(1,0,2)$  and perpendicular to vectors  $(\vec{u}=(1,1,0))$  and  $(\vec{v}=(0,1,1))$ .

**Q (HI):**  $A(1,0,2)$  से गुजरने वाली और  $(\vec{u}, \vec{v})$  के दोनों के प्रति लम्बवत विमीय समीकरण निकालें।

**Solution (EN):**

Normal vector  $(\vec{n} = \vec{u} \times \vec{v}) =$   
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} =$$
$$\mathbf{i}(1 \cdot 1 - 0 \cdot 1) - \mathbf{j}(1 \cdot 1 - 0 \cdot 0) + \mathbf{k}(1 \cdot 1 - 1 \cdot 0) =$$
$$\mathbf{i}(1) - \mathbf{j}(1) + \mathbf{k}(1) = (1, -1, 1).$$
 Plane:  $(1(x-1) - 1(y-0) + 1(z-2)=0) \Rightarrow$   
 $(x - 1 - y + z - 2 = 0) \Rightarrow (x - y + z - 3 = 0).$

**Solution (HI):**

नॉर्मल  $((1, -1, 1))$ । विमीय समीकरण:  $(x - y + z - 3 = 0)$ ।

**Key formula:** plane through  $(P_0(x_0, y_0, z_0))$  with normal  $(\vec{n}=(a,b,c))$ :  $(a(x-x_0)+b(y-y_0)+c(z-z_0)=0).$

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## 12) Vectors — Angle between lines / shortest distance

**Q (EN):** Find angle between lines through origin with direction vectors  $(\vec{a}=(1,2,2))$  and  $(\vec{b}=(2,-1,2))$ . Also find projection of  $(\vec{a})$  on  $(\vec{b})$ .

**Q (HI):** निर्देश वेक्टर दिए हैं—रेखाओं के बीच कोण और प्रक्षेपण ज्ञात कीजिए।

**Solution (EN):**

Angle  $(\theta)$ :  $(\cos \theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} = \frac{1 \cdot 2 + 2 \cdot (-1) + 2 \cdot 2}{\sqrt{1+4+4} \sqrt{4+1+4}} = \frac{2-2+4}{\sqrt{9} \sqrt{9}} = \frac{4}{9}).$  So  $(\theta = \cos^{-1}(4/9)).$  Projection of  $(\vec{a})$  on  $(\vec{b})$ : scalar proj =  $(\frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|} = \frac{4}{3}).$  Vector projection =  $(\left( \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|^2} \right) \vec{b} = \frac{4}{9} \vec{b} = \frac{4}{9} (2, -1, 2) = \left( \frac{8}{9}, -\frac{4}{9}, \frac{8}{9} \right)).$

**Solution (HI):**

$(\cos \theta = 4/9) \Rightarrow (\theta = \cos^{-1}(4/9))$ । scalar प्रोजेक्शन  $(4/3)$  और वेक्टर प्रोजेक्शन ऊपर दिया गया है।

**Key formula:**  $(\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos \theta).$  Projection formulas as used.

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### 13) Probability — Binomial (long)

**Q (EN):** A fair coin is tossed 8 times. Find probability of getting exactly 5 heads. Also find expected number of heads.

**Q (HI):** 8 बार सिक्का उछाला गया—ठीक 5 हेड्स आने की प्रायिकता और अपेक्षित मान क्या होगा?

**Solution (EN):**

Binomial ( $n=8, p=1/2$ ). ( $P(X=5)=\binom{8}{5}(1/2)^5(1/2)^3=\binom{8}{5}/2^8=\frac{56}{256}=\frac{7}{32}$ ). Expected value ( $E[X]=np=8\cdot\frac{1}{2}=4$ ).

**Solution (HI):**

( $P=7/32$ )। अपेक्षित हेड्स = 4।

**Key formula:** ( $P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$ ); ( $E[X]=np$ ).

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### 14) Probability — Continuous distribution / Normal approximation (concept)

**Q (EN):** If  $(X \sim (B(n,p)))$  with large  $(n)$ , write normal approximation and compute approximate probability ( $P(X \leq np - 2\sqrt{np(1-p)})$ ). (Assume large  $(n)$ , use continuity correction conceptually.)

**Q (HI):** बड़े  $n$  के लिए Binomial का Normal approxi और ( $P(X \leq np - 2\sqrt{np(1-p)})$ ) का अनुमान लिखिए (continuity correction का प्रयोग विचारात्मक रूप से करें)।

**Solution (EN):**

Normal approx: ( $X \approx N(\mu=np, \sigma^2=np(1-p))$ ). Let ( $Z=\frac{X-\mu}{\sigma}$ ). Then desired approx ( $P\left(Z \leq \frac{-2\sqrt{np(1-p)}}{\sqrt{np(1-p)}}\right)=P(Z \leq -2) \approx \Phi(-2) \approx 0.0228$ ). With continuity correction result similar.

**Solution (HI):**

सामान्यीकरण से ( $P \approx \Phi(-2) \approx 0.0228$ ) मिलता है।

**Key formula:** Normal approx to Binomial.

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### 15) Sequences & Series — Convergence and sum (long answer)

**Q (EN):** Determine whether series  $(\sum_{n=1}^{\infty} \frac{n}{2^n})$  converges and find its sum.

**Q (HI):**  $(\sum_{n=1}^{\infty} \frac{n}{2^n})$  का परिणाम ज्ञात कीजिए—क्या यह सम convergence है और यदि हाँ तो योग क्या है?

**Solution (EN):**

Use known sum:  $(\sum_{n=1}^{\infty} nx^n = \frac{x}{(1-x)^2})$  for  $(|x|<1)$ . Put  $(x=\frac{1}{2})$ :  $\text{sum} = (\frac{1/2}{(1-1/2)^2} = \frac{1/2}{(1/2)^2} = \frac{1/2}{1/4}=2)$ . So series converges to 2.

**Solution (HI):**

सूत्र से योग = 2 | अतः series converge करती है।

**Key formula:**  $(\sum_{n=1}^{\infty} nx^n = \frac{x}{(1-x)^2})$ .

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## 16) Linear Programming — Graphical (long)

**Q (EN):** Maximize  $(Z=3x+4y)$  subject to  $(x+2y \leq 8, 3x+y \leq 9, x \geq 0, y \geq 0)$ . Find optimal vertex and max  $(Z)$ .

**Q (HI):** रेखिक प्रोग्राम:  $(Z=3x+4y)$  अधिकतम कीजिए जहाँ सीमाएँ दी हैं। ग्राफिक विधि से हल दें।

**Solution (EN):**

Feasible region vertices: Solve intersections: (A) (0,0). (B)  $x=0$ , from  $(2y \leq 8) \Rightarrow (y \leq 4)$  so (0,4) is intersection with first. (C)  $y=0$ , from  $(3x \leq 9) \Rightarrow (x \leq 3)$  so (3,0). Intersection of two lines: solve  $(x+2y=8)$  and  $(3x+y=9)$ . Multiply second by 2:  $(6x+2y=18)$ . Subtract first:  $(5x=10) \Rightarrow (x=2)$ . Then  $(1+?)$  Put back:  $(2+2y=8) \Rightarrow (y=3)$ . So vertex (2,3). Evaluate  $Z$ : at (0,0):0; (0,4): $(3 \cdot 0 + 4 \cdot 4 = 16)$ ; (3,0): $(9)$ ; (2,3): $(6+12=18)$ . Max  $Z = 18$  at (2,3).

**Solution (HI):**

सभी वर्टिसों का मूल्यांकन करने पर अधिकतम  $(Z=18)$  पर  $((2,3))$  मिलता है।

**Key method:** Corner point method (graphical).

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## 17) Continuity & Differentiability — Show differentiable at point

**Q (EN):** Define  $(f(x)=\begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x=0 \end{cases})$ . Show  $(f)$  is differentiable at (0). Find  $(f'(0))$ .



**Q (HI):**  $(f(x)=x^2\sin(1/x))$  ( $x \neq 0$ ), और 0 पर 0। दिखाइए कि  $f$  0 पर differentiable है और  $(f'(0)=?)$  ?

**Solution (EN):**

Definition:  $(f'(0)=\lim_{h \rightarrow 0} \frac{f(h)-f(0)}{h} = \lim_{h \rightarrow 0} \frac{h^2 \sin(1/h)}{h} = \lim_{h \rightarrow 0} h \sin(1/h)$ ). Since  $(|h \sin(1/h)| \leq |h| \rightarrow 0)$ , limit 0. Hence differentiable and  $(f'(0)=0)$ .

**Solution (HI):**

लिमिट को bound कर के मिलता है  $(f'(0)=0)$ ।

**Key theorem:** Squeeze theorem used.

## 18) Application of Derivative — Curvature / Nature of stationary pt (long)

**Q (EN):** For  $(y=x^4-4x^3+6x^2)$ , find and classify stationary points.

**Q (HI):**  $(y=x^4-4x^3+6x^2)$  के stationary points ज्ञात कर के उनका प्रकार बताइए।

**Solution (EN):**

$(y'=4x^3-12x^2+12x=4x(x^2-3x+3))$ . Solve  $(y'=0)$ :  $(x=0)$  or roots of quadratic  $(x^2-3x+3=0)$ . Discriminant  $(\Delta=9-12=-3<0) \Rightarrow$  no real roots. So only stationary point  $(x=0)$ . Second derivative  $(y''=12x^2-24x+12=12(x^2-2x+1)=12(x-1)^2)$ . At  $(x=0)$ ,  $(y''=12(1)=12>0) \Rightarrow$  local minimum at  $(x=0)$ . Value  $(y(0)=0)$ .

**Solution (HI):**

केवल stationary बिन्दु  $(x=0)$ । क्योंकि  $(y''(0)>0)$ , यह स्थानीय न्यूनतम है; मान 0 है।

**Key formula:** Second derivative test:  $(y''(x_0)>0) \Rightarrow$  local min,  $(<0) \Rightarrow$  max.

## 19) Integration — Improper integral (convergence)

**Q (EN):** Test convergence of  $(\int_1^{\infty} \frac{1}{x(\ln x)^2} dx)$ .

**Q (HI):**  $(\int_1^{\infty} \frac{1}{x(\ln x)^2} dx)$  संग्रहीत है या नहीं? जाँच कीजिए।

**Solution (EN):**

Let  $(t=\ln x)$ . Then  $(dt=dx/x)$ . Integral becomes  $(\int_0^{\infty} t^{-2} dt)$  (since when  $x=1$ ,  $t=0$ ;  $x \rightarrow \infty$ ,  $t \rightarrow \infty$ ).  $(\int_0^{\infty} t^{-2} dt)$  improper at 0:  $(\int_{\epsilon}^{\infty} t^{-2} dt = [-t^{-1}]_{\epsilon}^{\infty} = 0 - (-1/\epsilon) = 1/\epsilon)$  which  $\rightarrow \infty$  as  $(\epsilon \rightarrow 0^+)$ . But careful: original lower limit corresponds to  $t$  from  $0^+$ , so divergence near 0. However evaluate original as  $(\int_1^{\infty} 1/(x(\ln x)^2) dx = [-1/\ln x]_1^{\infty} = (-0) - (-1/0^+))$

infinite. So integral diverges (actually tends to  $1/0 \rightarrow \infty$ ). Wait sign: For large  $x$ ,  $-1/\ln x \rightarrow 0$ ; at lower  $1+$ ,  $-1/\ln x \rightarrow -\infty$ , overall infinite. So diverges.

**Solution (HI):**

substitution से पता चलता है कि यह divergent है (असीमित)।

**Key idea:** Compare with p-test after substitution.

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## 20) Differential Equations — Variable separable (long)

**Q (EN):** Solve  $(\frac{dy}{dx} = \frac{x^2}{y^2})$ , with initial condition  $(y(1)=1)$ .

**Q (HI):**  $(\frac{dy}{dx} = \frac{x^2}{y^2})$  हल कीजिए और  $(y(1)=1)$  लागू करें।

**Solution (EN):**

Separable:  $(y^2 dy = x^2 dx)$ . Integrate:  $(\int y^2 dy = \int x^2 dx) \Rightarrow (\frac{y^3}{3} = \frac{x^3}{3} + C)$ . Multiply by 3:  $(y^3 = x^3 + C)$ . Use  $(y(1)=1) \Rightarrow (1 = 1 + C) \Rightarrow (C=0)$ . So  $(y^3 = x^3) \Rightarrow (y=x)$  (real cube root). So solution:  $(y=x)$ .

**Solution (HI):**

समीकरण से  $(y^3=x^3+C)$  और शर्त से  $(C=0)$ । अतः  $(y=x)$ ।

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## 21) Relations & Functions — Inverse function & monotonicity

**Q (EN):** Show that function  $(f(x)=x^3+3x+1)$  is one-to-one and hence invertible. Find derivative of inverse at  $(x=2)$  i.e.  $((f^{-1})'(2))$  if  $(f(a)=2)$ .

**Q (HI):**  $(f(x)=x^3+3x+1)$  एक-एक है इसका प्रमाण दीजिए और यदि  $(f(a)=2)$  हो तो  $((f^{-1})'(2))$  ज्ञात कीजिए।

**Solution (EN):**

$(f'(x)=3x^2+3=3(x^2+1)>0)$  for all  $x \Rightarrow$  strictly increasing  $\Rightarrow$  one-to-one. If  $(f(a)=2)$ , need  $(a)$ : solve  $(a^3+3a+1=2) \Rightarrow (a^3+3a-1=0)$ . Real root is  $(a=0)$ ? Plug 0 gives -1.  $(a=1)$ :  $(1+3-1=3)$ . So root between 0 and 1 (call it  $a$ ). But we can compute  $((f^{-1})'(y) = 1 / f'(f^{-1}(y)))$ . So  $((f^{-1})'(2) = 1/f'(a) = 1/(3a^2+3) = 1/[3(a^2+1)])$ . If numeric  $a$  required, solve cubic numerically; symbolic form suffices.

**Solution (HI):**

आवश्यक:  $(f'(x)>0)$  से एक-एकता सिद्ध।  $((f^{-1})'(2)=\frac{1}{3a^2+3})$  जहाँ  $(a)$  फंक्शन का वह मान है जो  $(f(a)=2)$  देता है।

**Key formula:**  $((f^{-1})'(y) = 1/f'(x))$  where  $(x = f^{-1}(y))$ .

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## 22) Conic Sections (Parabola) — Tangent & normal

**Q (EN):** For parabola  $(y^2=4ax)$ , find equation of tangent at point  $((at^2, 2at))$  and equation of normal.

**Q (HI):**  $(y^2=4ax)$  पर बिंदु  $((at^2, 2at))$  के लिए स्पर्शरेखा और सामान्य की समीकरण खोजिए।

**Solution (EN):**

Parametric point  $((at^2, 2at))$ . Tangent:  $(ty = x + at^2)$  (standard form:  $(ty = x + at^2)$ ). Normal:  $(y = -tx + 2at + at^3)$  — standard normal formula:  $(y = -tx + 2at + at^3)$ . (One may derive using derivative:  $dy/dx = 2a/y \Rightarrow at$  param gives slope of tangent  $= 1/t$ , etc.)

**Solution (HI):**

मानक परिणाम से स्पर्शरेखा:  $(ty = x + at^2)$ । सामान्य:  $(y = -tx + 2at + at^3)$ ।

**Key formulas:** Parametric forms & standard tangent/normal.

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## 23) Eigenvalues & Eigenvectors (Matrices) — long

**Q (EN):** For  $(A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix})$ , find eigenvalues and eigenvectors.

**Q (HI):**  $(A)$  के eigenvalues/eigenvectors निकालिए।

**Solution (EN):**

Characteristic equation:  $(\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{vmatrix} = (4-\lambda)(3-\lambda) - 2 = \lambda^2 - 7\lambda + 10 = 0)$ . Solve:  $(\lambda = \frac{7 \pm \sqrt{49-40}}{2} = \frac{7 \pm 3}{2}) \Rightarrow (\lambda_1 = 5, \lambda_2 = 2)$ . For  $(\lambda = 5)$ : solve  $((A - 5I)\vec{v} = 0) \Rightarrow (\begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0) \Rightarrow (-x + y = 0) \Rightarrow (y = x)$ . Eigenvector e.g.  $((1, 1)^T)$ . For  $(\lambda = 2)$ :  $(\begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0) \Rightarrow (2x + y = 0) \Rightarrow (y = -2x)$ . Eigenvector e.g.  $((1, -2)^T)$ .

**Solution (HI):**

Eigenvalues 5 और 2; संबंधित eigenvectors  $((1, 1))$  तथा  $((1, -2))$ ।

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## 24) Multivariable (Partial derivatives) — Max/Min constrained by Lagrange multiplier

**Q (EN):** Find max/min of  $(f(x,y)=xy)$  subject to  $(x^2+y^2=1)$ .

**Q (HI):**  $(xy)$  का अधिकतम/न्यूनतम ज्ञात कीजिए जहाँ  $(x^2+y^2=1)$ ।

**Solution (EN):**

Use Lagrange:  $(L=xy + \lambda(x^2+y^2-1))$ . Partial derivatives:  $(L_x = y + 2\lambda x = 0)$  (1),  $(L_y = x + 2\lambda y = 0)$  (2), constraint (3). Multiply (1) by  $x$  and (2) by  $y$ :  $(xy + 2\lambda x^2 = 0)$ ,  $(xy + 2\lambda y^2 = 0)$ . Subtract:  $(2\lambda(x^2-y^2)=0) \Rightarrow$  either  $(\lambda=0)$  or  $(x^2=y^2)$ . If  $(\lambda=0)$ , then  $y=0$  and  $x=0$  from (1),(2) inconsistent with constraint. So  $(x^2=y^2) \Rightarrow (y=\pm x)$ . With constraint  $(2x^2=1) \Rightarrow (x=\pm \frac{1}{\sqrt{2}})$ . If  $(y=x)$ :  $(f=x^2 = 1/2)$  (max). If  $(y=-x)$ :  $(f=-x^2=-1/2)$  (min). So max  $(1/2)$  at  $((1/\sqrt{2}, 1/\sqrt{2}))$  and min  $(-1/2)$  at  $((1/\sqrt{2}, -1/\sqrt{2}))$  etc with sign permutations.

**Solution (HI):**

अधिकतम  $(1/2)$ , न्यूनतम  $(-1/2)$ । बिंदु दिए ऊपर हैं।

**Key formula:** Lagrange multipliers solve constrained extrema.

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## 25) Integration — Trigonometric integral (long)

**Q (EN):** Evaluate  $(\int \frac{dx}{\sin x + \cos x})$ .

**Q (HI):**  $(\int \frac{dx}{\sin x + \cos x})$  का हल दीजिए।

**Solution (EN):**

Multiply numerator and denominator by  $(\sin x - \cos x)$ :  $(\frac{\sin x - \cos x}{\sin^2 x - \cos^2 x} dx = \frac{\sin x - \cos x}{-(\cos 2x)} dx)$  Better trick: set  $(t = \tan(x/2))$ . Or use substitution: Let  $(u = \sin x + \cos x)$ . Then  $(du = (\cos x - \sin x) dx)$ . So write numerator  $(dx = \frac{du}{\cos x - \sin x})$ . But  $(\cos x - \sin x = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x}) = \frac{\cos^2 x - \sin^2 x}{\sin x + \cos x} = \frac{\cos 2x}{\sin x + \cos x}$ . This becomes messy. Use standard substitution: divide numerator and denominator by  $(\cos x)$ :  $(\int \frac{dx}{\cos x(\tan x + 1)})$ . Let  $(u = \tan x) \Rightarrow (du = \sec^2 x dx)$  not matching. Simpler route: set  $(I = \int \frac{dx}{\sin x + \cos x} = \int \frac{\sin x - \cos x}{\sin^2 x - \cos^2 x} dx = \int \frac{\sin x - \cos x}{-(\cos 2x)} dx)$ . Another standard trick: put  $(\sin x + \cos x = \sqrt{2}\sin(x+\pi/4))$ . Then integral  $(I = \int \frac{dx}{\sqrt{2}\sin(x+\pi/4)} = \frac{1}{\sqrt{2}} \int \frac{1}{\sin(x+\pi/4)} dx = \frac{1}{\sqrt{2}} \ln|\tan(\frac{x+\pi/4}{2})| + C)$ . So final:  $(I = \frac{1}{\sqrt{2}} \ln|\tan(\frac{x+\pi/4}{2})| + C)$ .

**Solution (HI):**

$(\sin x + \cos x = \sqrt{2}\sin(x+\pi/4))$  प्रयोग कर के उत्तर

$(\frac{1}{\sqrt{2}} \ln|\tan(\frac{x+\pi/4}{2})| + C)$  मिलता है।

**Key identity:**  $(\sin x + \cos x = \sqrt{2}\sin(x+\pi/4))$ .

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## 26) Probability — Conditional probability & Bayes (long)

**Q (EN):** Two boxes: Box A has 2 white, 1 black. Box B has 1 white, 2 black. Choose a box at random then pick a ball at random — probability the ball is white? If observed white, what's probability it came from Box A? (Bayes)

**Q (HI):** दो डिब्बे सीमा के साथ—प्राथमिक और बायस का प्रयोग कर प्रश्न हल कीजिए।

**Solution (EN):**

$P(\text{choose A}) = P(\text{choose B}) = 1/2$ .  $P(\text{white}|\text{A}) = 2/3$ .  $P(\text{white}|\text{B}) = 1/3$ . Total  $P(\text{white}) = \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{2}$ . By Bayes:  $P(\text{A}|\text{white}) = \frac{P(\text{white}|\text{A})P(\text{A})}{P(\text{white})} = \frac{(2/3)(1/2)}{1/2} = 2/3$ .

**Solution (HI):**

कुल प्रायिकता  $(1/2)$ ।  $P(\text{A}|\text{white}) = 2/3$ ।

**Key formula:** Bayes theorem.

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## 27) Limits — Sandwich theorem / tricky limit

**Q (EN):**  $\lim_{x \rightarrow 0} \frac{e^x - 1 - x - x^2/2}{x^3}$ .

**Q (HI):**  $\lim_{x \rightarrow 0} \frac{e^x - 1 - x - x^2/2}{x^3}$  ज्ञात कीजिए।

**Solution (EN):**

Use Taylor:  $(e^x = 1 + x + x^2/2 + x^3/6 + o(x^3))$ . Numerator  $\approx (x^3/6 + o(x^3))$ . Divide by  $(x^3) \Rightarrow (1/6)$ . So limit  $(=1/6)$ .

**Solution (HI):**

Taylor से उत्तर  $(1/6)$  है।

**Key formula:** Taylor expansion of  $(e^x)$ .

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## 28) Application of Integrals — Volume of revolution

**Q (EN):** Find volume when region bounded by  $(y=x^2)$  and  $(y=0)$  between  $(x=0)$  and  $(x=1)$  is revolved about x-axis.

**Q (HI):**  $(y=x^2)$  के नीचे  $(0)$  और  $(x)$  के बीच क्षेत्र  $(x=0)$  से  $(1)$  तक घूर्णन करने पर आयतन ज्ञात कीजिए।

**Solution (EN):**

Volume by discs:  $(V = \pi \int_0^1 (x^2)^2 dx = \pi \int_0^1 x^4 dx = \pi [x^5/5]_0^1 = \pi/5)$ .

**Solution (HI):**

आवश्यक आयतन =  $(\pi/5)$  इकाई<sup>3</sup>।

**Key formula:** Volume  $(= \pi \int y^2 dx)$  about x-axis.

## 29) Differential Equations — Second order linear with constant coefficients (long)

**Q (EN):** Solve  $(y'' - 5y' + 6y = 0)$  with  $(y(0)=2, y'(0)=1)$ .

**Q (HI):**  $(y''-5y'+6y=0)$  का सामान्य हल निकालिए और आरंभिक शर्तें लागू कीजिए।

**Solution (EN):**

Char eqn:  $(r^2-5r+6=0) \Rightarrow ((r-2)(r-3)=0) \Rightarrow (r=2,3)$ . General solution  $(y=C_1 e^{2x}+C_2 e^{3x})$ .  $(y'=2C_1 e^{2x}+3C_2 e^{3x})$ . Apply  $(x=0)$ :  $(y(0)=C_1+C_2=2)$ .  $(y'(0)=2C_1+3C_2=1)$ . Solve: From first  $(C_1=2-C_2)$ . Substitute:  $(2(2-C_2)+3C_2=1) \Rightarrow (4-2C_2+3C_2=1) \Rightarrow (4+C_2=1) \Rightarrow (C_2=-3)$ . Then  $(C_1=2-(-3)=5)$ . So  $(y=5e^{2x}-3e^{3x})$ .

**Solution (HI):**

रैखिक समीकरण के हल से  $(y=5e^{2x}-3e^{3x})$ ।

## 30) Misc long — Combination of calculus & algebra (challenging)

**Q (EN):** Let  $(f(x)=\ln x - \frac{1}{x})$  for  $(x>0)$ . Show that  $(f)$  has exactly one real root and approximate it. Also find where  $(f)$  is increasing.

**Q (HI):**  $(f(x)=\ln x - 1/x)$  के लिए दिखाइए कि केवल एक वास्तविक मूल है और लगभग मान देने के साथ  $(f)$  कहाँ बढ़ती है बताइए।

**Solution (EN):**

Study  $(f(x)=1/x + 1/x^2 = \frac{x+1}{x^2} > 0)$  for  $(x>0)$ . So  $(f)$  is strictly increasing on  $((0, \infty))$ . Limit:  $(\lim_{x \rightarrow 0^+} f(x) = -\infty - \infty = -\infty)$ .  $(\lim_{x \rightarrow \infty} f(x) = \infty)$ . By Intermediate Value Theorem there is exactly one root. To approximate solve  $(\ln x = 1/x)$ . Try  $(x=1)$ : LHS 0, RHS 1  $\Rightarrow (f(1)=-1 < 0)$ . Try  $(x=2)$ :  $(\ln 2 - 1/2 \approx 0.693-0.5=0.193 > 0)$ . So root between 1 and 2. Try  $(x=1.5)$ :  $(\ln 1.5 - 2/3 \approx 0.4055-0.6667=-0.2612)$ . Try  $(x=1.8)$ :  $(\ln 1.8 - 0.5556 \approx 0.5878-0.5556=0.0322)$ . Try  $(x=1.79)$ :  $(\ln 1.79 \approx 0.5832 - 0.5587=0.0245)$ . Try  $(x=1.75)$ :  $(0.5596-0.5714=-0.0118)$ . So root  $\approx (1.768)$  (approx). (One may refine by Newton's method.) So unique root  $\approx (1.769)$ . And  $(f)$  increasing for all  $(x>0)$ .

**Solution (HI):**

( $f(x) > 0$ ) से ( $f$ ) हर जगह वृद्धि करती है, अतः केवल एक शून्य होगा; अनुमानित मान  $\approx (1.77)$  मिलता है।

**Key methods:** Monotonicity (first derivative), IVT, numeric bisection/Newton.