

MP Board Class 12 Mathematics – MCQs

Final Revision | 50 Questions | 2026 Exam

Chapter 1: Relations & Functions

Q1. If $A = \{1,2\}$, $B = \{3,4\}$, number of relations =

यदि $A=\{1,2\}$, $B=\{3,4\}$, संबंधों की संख्या है—

- A. 4 / चार
- B. 8 / आठ
- C. 16 / सोलह
- D. 2 / दो

Answer: C – 16 / सोलह

Q2. A function must be

फलन होना चाहिए—

- A. One-one / एक-एक
- B. Onto / पूर्ण
- C. Many-one allowed / बहु-एक संभव
- D. Bijective only / केवल एक-एक पूर्ण

Answer: C – Many-one allowed / बहु-एक संभव

Chapter 2: Inverse Trigonometric Functions

Q3. Domain of $\sin^{-1}x$ is

$\sin^{-1}x$ का प्रांत है—

- A. \mathbb{R}
- B. $[-1,1]$
- C. $(-\infty, \infty)$
- D. $(0,1)$

Answer: B – $[-1,1]$

Q4. $\sin^{-1}(0) =$
 $\sin^{-1}(0)$ का मान है—

- A. π
- B. 0
- C. $\pi/2$
- D. $-\pi/2$

Answer: B – 0 / शून्य

Chapter 3: Matrices

Q5. Order of a matrix with 3 rows and 2 columns is
3 पंक्ति व 2 स्तंभ वाले आव्यूह की कोटि—

- A. 2×3
- B. 3×2
- C. 5×5
- D. 1×6

Answer: B – 3×2

Q6. Identity matrix is always
इकाई आव्यूह हमेशा होता है—

- A. Rectangular / आयताकार
- B. Square / वर्गाकार
- C. Zero matrix / शून्य
- D. Singular / एकल

Answer: B – Square / वर्गाकार

Chapter 4: Determinants

Q7. Determinant of identity matrix is
इकाई आव्यूह का निर्धारक—

- A. 0
- B. -1
- C. 1
- D. 2

Answer: C – 1

Q8. If determinant = 0, system has

यदि निर्धारक 0 हो, तो हल—

- A. Unique / अद्वितीय
- B. No solution / कोई हल नहीं
- C. Infinite solutions / अनंत हल
- D. Always consistent / सदैव संगत

Answer: C – Infinite solutions / अनंत हल

Chapter 5: Continuity & Differentiability

Q9. Derivative of constant is

स्थिरांक का अवकलज—

- A. 1
- B. x
- C. 0
- D. ∞

Answer: C – 0 / शून्य

Q10. If f is differentiable, it is

यदि f अवकलनीय है, तो—

- A. Discontinuous
- B. Continuous
- C. Undefined
- D. Constant

Answer: B – Continuous / सतत

Chapter 6: Applications of Derivatives

Q11. Maxima occurs when $f'(x) =$

अधिकतम मान के लिए $f'(x) =$

- A. 1
- B. 0

- C. ∞
- D. -1

Answer: B – 0 / शून्य

Q12. Rate of change is measured by
परिवर्तन की दर मापी जाती है—

- A. Integration
- B. Limit
- C. Differentiation
- D. Matrix

Answer: C – Differentiation / अवकलन

Chapter 7: Integrals

Q13. $\int 0 \, dx =$
 $\int 0 \, dx$ का मान—

- A. 0
- B. x
- C. 1
- D. C

Answer: D – C / स्थिरांक

Q14. Integration is reverse of
समाकलन है—

- A. Limit
- B. Differentiation
- C. Matrix
- D. Determinant

Answer: B – Differentiation / अवकलन

Chapter 8: Applications of Integrals

Q15. Area under curve is found by
वक्र के नीचे का क्षेत्रफल—

- A. Derivative
- B. Limit
- C. Integration
- D. Matrix

Answer: C – Integration / समाकलन

Chapter 9: Differential Equations

Q16. Order of $dy/dx =$

dy/dx का ऋण्ड—

- A. 0
- B. 1
- C. 2
- D. 3

Answer: B – 1 / प्रथम

Chapter 10: Vector Algebra

Q17. Zero vector magnitude is

शून्य सदिश का परिमाण—

- A. 1
- B. ∞
- C. 0
- D. -1

Answer: C – 0 / शून्य

Q18. $i \cdot j =$

$i \cdot j$ का मान—

- A. 1
- B. -1
- C. 0
- D. i

Answer: C – 0 / शून्य

Chapter 11: Three Dimensional Geometry

Q19. Distance formula is based on

दूरी सूत्र आधारित है—

- A. Integration
- B. Pythagoras theorem
- C. Matrix
- D. Vector product

Answer: B – Pythagoras theorem / पाइथागोरस

Chapter 12: Linear Programming

Q20. Feasible region is

साध्य क्षेत्र होता है—

- A. Unbounded
- B. Point
- C. Common shaded region
- D. Line

Answer: C – Common shaded region / साझा छायांकित क्षेत्र

Chapter 13: Probability

Q21. $P(S) =$

नमूना समुच्चय की प्रायिकता—

- A. 0
- B. 1
- C. -1
- D. ∞

Answer: B – 1

Q22. Probability lies between

प्रायिकता का मान—

- A. -1 to 1
- B. 0 to 1
- C. 1 to 2
- D. Any real number

Answer: B – 0 to 1

Q23. $\sin^2\theta + \cos^2\theta =$

- A. 0
- B. 1
- C. 2
- D. θ

Ans: B – 1

Q24. $\det(A) = -\det(-A)$ is true for

- A. Odd order
- B. Even order
- C. All
- D. None

Ans: A – Odd order

Q25. $d/dx (x^n) =$

- A. nx^{n-1}
- B. x^n
- C. n
- D. 0

Ans: A

Q26. $\int e^x dx =$

- A. $e^x + C$
- B. xe^x
- C. 1
- D. 0

Ans: A

Q27. Unit vector magnitude =

- A. 0
- B. 1
- C. 2
- D. -1

Ans: B

Q28. Probability of impossible event =

- A. 1
- B. 0
- C. -1
- D. ∞

Ans: B

Q29. Rank of zero matrix =

- A. 0
- B. 1
- C. 2
- D. Undefined

Ans: A

Q30. $f''(x) < 0$ implies

- A. Minima
- B. Maxima
- C. Point of inflection
- D. Constant

Ans: B

TOP-15 Probable SURE-SHOT MCQs (2026)

1. Relations & Functions

Q1. A function is a relation in which

फलन वह संबंध है जिसमें—

- A. One output for each input
प्रत्येक इनपुट का एक ही आउटपुट
- B. One input for many outputs
एक इनपुट के कई आउटपुट
- C. No output
कोई आउटपुट नहीं
- D. Many outputs allowed
कई आउटपुट मान्य

Answer: A

2. Inverse Trigonometric Functions

Q2. Domain of $\sin^{-1}x$ is

$\sin^{-1}x$ का प्रांत—

- A. \mathbb{R}
- B. $(-\infty, \infty)$
- C. $[-1, 1]$
- D. $(0, 1)$

Answer: C

3. Matrices

Q3. Order of identity matrix is always

इकाई आव्यूह की कोटि होती है—

- A. Rectangular / आयताकार
- B. Square / वर्गाकार
- C. Zero / शून्य
- D. Column / स्तंभ

Answer: **B**

4. Determinants

Q4. If two rows of a determinant are identical, its value is
यदि निर्धारक की दो पंक्तियाँ समान हों—

- A. 1
- B. -1
- C. 0
- D. ∞

Answer: **C**

5. Continuity & Differentiability

Q5. If a function is differentiable, then it is
यदि फलन अवकलनीय है, तो—

- A. Discontinuous / असतत
- B. Continuous / सतत
- C. Constant / स्थिर
- D. Undefined / अपरिभाषित

Answer: **B**

6. Applications of Derivatives

Q6. At maximum or minimum point,
अधिकतम या न्यूनतम बिंदु पर—

- A. $f'(x) = 1$
- B. $f'(x) = -1$
- C. $f'(x) = 0$
- D. $f'(x) = \infty$

Answer: C

7. Integrals

Q7. $\int 0 \, dx =$
 $\int 0 \, dx$ का मान—

- A. 0
- B. x
- C. 1
- D. C

Answer: D

8. Applications of Integrals

Q8. Area under a curve is always
वक्र के नीचे का क्षेत्रफल होता है—

- A. Negative / ऋणात्मक
- B. Positive / धनात्मक
- C. Zero / शून्य
- D. Non-negative / ऋणात्मक नहीं

Answer: D

9. Differential Equations

Q9. Order of differential equation depends on
अवकल समीकरण का क्रम निर्भर करता है—

- A. Power of x
- B. Degree
- C. Highest order derivative
- D. Constant

Answer: C

10. Vector Algebra

Q10. $a \cdot a =$
 $a \cdot a$ का मान—

- A. a
- B. $|a|$
- C. $|a|^2$
- D. 0

Answer: C

11. Three Dimensional Geometry

Q11. Direction ratios of x-axis are

x-अक्ष के दिशा अनुपात—

- A. (0,1,0)
- B. (0,0,1)
- C. (1,1,1)
- D. (1,0,0)

Answer: D

12. Linear Programming

Q12. Optimal solution of LPP occurs at

LPP का सर्वोत्तम हल मिलता है—

- A. Origin
- B. Any interior point
- C. Corner point
- D. Mid-point

Answer: C

13. Probability

Q13. Probability of sure event is

निश्चित घटना की प्रायिकता—

- A. 0
- B. 1
- C. -1
- D. ∞

Answer: B

14. Probability (Complement)

Q14. $P(\bar{A}) =$

पूरक घटना की प्रायिकता—

- A. $P(A)$
- B. $1 + P(A)$
- C. $1 - P(A)$
- D. $P(A)^2$

Answer: C

15. Mixed (Very Important)

Q15. Determinant of identity matrix is

इकाई आव्यूह का निर्धारक—

- A. 0
- B. -1
- C. 1
- D. Depends on order

Answer: C

Assertion–Reason MCQs (30 Questions)

(Most Expected – 2026 Final Revision)

Directions / निर्देश

Choose the correct option:

- A. Both A and R are true and R is the correct explanation of A
- B. Both A and R are true but R is not the correct explanation of A
- C. A is true but R is false
- D. A is false but R is true

1. Relations & Functions

Q1.

Assertion (A): Every function is a relation.

अभिकथन: प्रत्येक फलन एक संबंध है।

Reason (R): A relation assigns one output to one input.

कारण: संबंध प्रत्येक इनपुट को केवल एक आउटपुट देता है।

Answer: C

Q2.

A: A many-one function is allowed.

अभिकथन: बहु-एक फलन मान्य है।

R: One input can have many outputs.

कारण: एक इनपुट के कई आउटपुट हो सकते हैं।

Answer: C

2. Inverse Trigonometric Functions

Q3.

A: Domain of $\sin^{-1}x$ is $[-1,1]$.

R: Sine function lies between -1 and 1 .

Answer: A

Q4.

A: $\tan^{-1}x$ is defined for all real x .

R: $\tan x$ is defined for all real values.

Answer: C

3. Matrices

Q5.

A: Identity matrix is a square matrix.

R: Number of rows equals number of columns.

Answer: A

Q6.

A: Product of matrices is commutative.

R: $AB = BA$ for all matrices A and B .

Answer: D

4. Determinants

Q7.

A: Determinant of a singular matrix is zero.

R: Rows of singular matrix are linearly dependent.

Answer: A

Q8.

- A:** If two rows of a determinant are equal, its value is zero.
R: Interchanging rows changes sign of determinant.

Answer: B

5. Continuity & Differentiability

Q9.

- A:** If a function is differentiable, it is continuous.
R: Differentiability implies continuity.

Answer: A

Q10.

- A:** A continuous function may not be differentiable.
R: $|x|$ is continuous but not differentiable at $x = 0$.

Answer: A

6. Applications of Derivatives

Q11.

- A:** At maxima, $f'(x) = 0$.
R: Slope of tangent becomes zero.

Answer: A

Q12.

- A:** If $f'(x) > 0$, function is increasing.
R: Positive derivative shows increasing nature.

Answer: D

7. Integrals

Q13.

- A:** Integration is inverse of differentiation.
R: $\int f'(x) dx = f(x) + C$.

Answer: A

Q14.

- A:** $\int_0 dx = C$.
R: Integral of zero is a constant.

Answer: A

8. Applications of Integrals

Q15.

- A:** Area under curve cannot be negative.
R: Area represents physical quantity.

Answer: A

Q16.

- A:** Area between curve and x-axis is always finite.
R: Integration always gives finite value.

Answer: C

9. Differential Equations

Q17.

- A:** Order of differential equation depends on highest derivative.
R: Degree depends on power of highest derivative.

Answer: B

Q18.

- A:** General solution contains arbitrary constant.
R: Particular solution has no constant.

Answer: B

10. Vector Algebra

Q19.

- A:** Magnitude of unit vector is 1.
R: Unit vector shows only direction.

Answer: A

Q20.

- A:** $\mathbf{a} \cdot \mathbf{b} = 0$ implies vectors are perpendicular.
R: Dot product of perpendicular vectors is zero.

Answer: A

11. Three Dimensional Geometry

Q21.

- A: Direction ratios of a line are not unique.
R: They can be multiplied by any non-zero constant.

Answer: A

Q22.

- A: Equation of plane has three variables.
R: Plane exists in three-dimensional space.

Answer: A

12. Linear Programming

Q23.

- A: Feasible region is common shaded region.
R: It satisfies all given constraints.

Answer: A

Q24.

- A: Optimal value occurs at corner point.
R: Objective function is linear.

Answer: A

13. Probability

Q25.

- A: Probability of impossible event is zero.
R: Impossible event never occurs.

Answer: A

Q26.

- A: $P(\bar{A}) = 1 - P(A)$.
R: A and \bar{A} are mutually exclusive and exhaustive.

Answer: A

Q27.

- A: Probability can be negative.
R: Probability lies between 0 and 1.

Answer: D

Q28.

A: Sum of probabilities of all outcomes is 1.

R: Sample space contains all possible outcomes.

Answer: A

Mixed Revision (Very High Probability)

Q29.

A: $\det(I) = 1$.

R: Identity matrix has all diagonal elements as 1.

Answer: A

Q30.

A: Rank of zero matrix is zero.

R: All rows of zero matrix are linearly dependent.

Answer: A

Very Short Questions & Answers

Chapter 1: Relations & Functions

Q1. What is a function?

Answer:

English: A relation where each input has exactly one output.

हिंदी: वह संबंध जिसमें प्रत्येक इनपुट का केवल एक आउटपुट हो।

Q2. Define domain of a function.

Answer:

English: Set of all possible inputs.

हिंदी: सभी संभव इनपुट का समुच्चय।

Chapter 2: Inverse Trigonometric Functions

Q3. Write the principal value range of $\sin^{-1}x$.

Answer:

English: $[-\pi/2, \pi/2]$

हिंदी: $[-\pi/2, \pi/2]$

Q4. What is $\sin^{-1}(1)$?

Answer:

English: $\pi/2$

हिंदी: $\pi/2$

Chapter 3: Matrices

Q5. Define identity matrix.

Answer:

English: A square matrix with 1 on diagonal and 0 elsewhere.

हिंदी: विकर्ण पर 1 और अन्य स्थानों पर 0 वाली वर्ग आव्यूह।

Q6. Order of matrix $A = [2 \ 3 \ 4]$?

Answer:

English: 1×3

हिंदी: 1×3

Chapter 4: Determinants

Q7. Determinant of identity matrix of order 2 is?

Answer:

English: 1

हिंदी: 1

Q8. Value of determinant with two identical rows?

Answer:

English: 0

हिंदी: 0

Chapter 5: Continuity & Differentiability

Q9. Define continuity at a point.

Answer:

English: LHL = RHL = f(a)

हिंदी: बाएँ सीमा = दाएँ सीमा = फलन का मान।

Q10. Derivative of constant is?

Answer:

English: 0

हिंदी: 0

Chapter 6: Applications of Derivatives

Q11. What does dy/dx represent?

Answer:

English: Rate of change.

हिंदी: परिवर्तन की दर।

Q12. Maximum or minimum occurs when?

Answer:

English: First derivative is zero.

हिंदी: प्रथम अवकलज शून्य हो।

Chapter 7: Integrals

Q13. $\int 0 \, dx$ equals?

Answer:

English: Constant (C)

हिंदी: स्थिरांक (C)

Q14. $\int x^n \, dx$ formula?

Answer:

English: $x^{n+1}/(n+1) + C$

हिंदी: $x^{n+1}/(n+1) + C$

Chapter 8: Applications of Integrals

Q15. What does definite integral represent?

Answer:

English: Area under curve.

हिंदी: वक्र के नीचे का क्षेत्रफल।

Chapter 9: Differential Equations

Q16. Degree of differential equation $dy/dx = x$?

Answer:

English: 1

हिंदी: 1

Q17. General solution contains?

Answer:

English: Arbitrary constant.

हिंदी: मनचाहा स्थिरांक।

Chapter 10: Vector Algebra

Q18. Magnitude of zero vector is?

Answer:

English: 0

हिंदी: 0

Q19. Unit vector magnitude is?

Answer:

English: 1

हिंदी: 1

Chapter 11: Three Dimensional Geometry

Q20. Distance between identical points?

Answer:

English: 0

हिंदी: 0

Q21. Direction ratios of x-axis?

Answer:

English: (1, 0, 0)

हिंदी: (1, 0, 0)

Chapter 12: Linear Programming

Q22. Objective function is used to?

Answer:

English: Maximize or minimize.

हिंदी: अधिकतम या न्यूनतम करना।

Q23. Feasible region lies in?

Answer:

English: First quadrant.

हिंदी: प्रथम चतुर्थांश।

Chapter 13: Probability

Q24. Probability of impossible event?

Answer:

English: 0

हिंदी: 0

Q25. Maximum value of probability?

Answer:

English: 1

हिंदी: 1

Q26. Formula of conditional probability?

Answer:

English: $P(A|B) = P(A \cap B)/P(B)$

हिंदी: $P(A|B) = P(A \cap B)/P(B)$

Mixed Revision (Highly Repeated)

Q27. Value of $|A| = 0$ indicates?

Answer:

English: No inverse exists.

हिंदी: व्युत्क्रम अस्तित्व में नहीं।

Q28. $\sin^2\theta + \cos^2\theta$ equals?

Answer:

English: 1

हिंदी: 1

Q29. If $f'(x) > 0$, function is?

Answer:

English: Increasing.

हिंदी: वर्धमान।

Q30. Probability of sure event?

Answer:

English: 1

हिंदी: 1

FINAL REVISION: 30 SHORT QUESTIONS (BILINGUAL)

Chapter 1: Relations & Functions

Q1. Check whether the relation $R = \{(1,2), (2,3), (3,4)\}$ is a function.

जाँचिए कि दिया गया संबंध फलन है या नहीं।

Formula (English):

A relation is a function if each element of domain has exactly one image.

Solution (Steps):

$1 \rightarrow 2$ (one image)

$2 \rightarrow 3$ (one image)

$3 \rightarrow 4$ (one image)

Answer: R is a function.

उत्तर (हिंदी):

प्रत्येक इनपुट का केवल एक आउटपुट है।

अतः R एक फलन है।

Q2. Find the domain of $f(x) = \sqrt{x - 3}$.

$f(x)$ का परिभाषा क्षेत्र ज्ञात कीजिए।

Formula:

Expression under square root ≥ 0

Steps:

$$x - 3 \geq 0$$

$$x \geq 3$$

Answer: Domain = $[3, \infty)$

उत्तर: परिभाषा क्षेत्र = $[3, \infty)$

Chapter 2: Inverse Trigonometric Functions

Q3. Find the principal value of $\sin^{-1}(\frac{1}{2})$.

$\sin^{-1}(\frac{1}{2})$ का मुख्य मान ज्ञात कीजिए।

Formula:

$$\sin^{-1}(\sin \theta) = \theta, \text{ where } \theta \in [-\pi/2, \pi/2]$$

Step:

$$\sin(\pi/6) = \frac{1}{2}$$

Answer: $\pi/6$

उत्तर: $\pi/6$

Q4. Evaluate $\tan^{-1}(1) + \tan^{-1}(1)$.

मान ज्ञात कीजिए।

Formula:

$$\tan^{-1}(1) = \pi/4$$

Steps:

$$\pi/4 + \pi/4 = \pi/2$$

Answer: $\pi/2$

उत्तर: $\pi/2$

Chapter 3: Matrices

Q5. Write the order of matrix $A = [2 \ 3 \ 4]$.

आव्यूह की कोटि लिखिए।

Formula:

Order = Number of rows \times Number of columns

Steps:

1 row, 3 columns

Answer: 1×3

उत्तर: 1×3

Q6. Write identity matrix of order 2.

कोटि 2 का इकाई आव्यूह लिखिए।

Formula / Answer:

$$I_2 = (\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix})$$

Chapter 4: Determinants

Q7. Evaluate $|1\ 2; 3\ 4|$.

मान जात कीजिए।

Formula:

$$|a\ b; c\ d| = ad - bc$$

Steps:

$$(1 \times 4) - (2 \times 3) = 4 - 6$$

Answer: -2

उत्तर: -2

Q8. If $|A| = 5$, find $|2A|$.

$|2A|$ जात कीजिए।

Formula:

$$|kA| = k^n |A| \quad (n = \text{order})$$

Steps:

2×2 matrix $\Rightarrow n = 2$

$$|2A| = 2^2 \times 5 = 20$$

Answer: 20

उत्तर: 20

Chapter 5: Continuity & Differentiability

Q9. Differentiate $y = x^2$.

अवकलन कीजिए।

Formula:

$$d/dx(x^n) = nx^{n-1}$$

Step:

$$dy/dx = 2x$$

Answer: $2x$

Q10. Find dy/dx of $y = \sin x$.

अवकलज ज्ञात कीजिए।

Formula:

$$d/dx(\sin x) = \cos x$$

Answer: $\cos x$

Chapter 6: Applications of Derivatives

Q11. Find slope of $y = x^3$ at $x = 1$.

ढाल ज्ञात कीजिए।

Steps:

$$dy/dx = 3x^2$$

$$\text{At } x = 1 \rightarrow 3$$

Answer: 3

उत्तर: 3

Q12. At what point $y = x^2$ is minimum?

न्यूनतम मान कहाँ होगा?

Formula:

$$dy/dx = 0$$

Steps:

$$2x = 0 \rightarrow x = 0$$

Answer: Minimum at $x = 0$

Chapter 7: Integrals

Q13. Evaluate $\int x \, dx$.

समाकलन कीजिए।

Formula:

$$\int x^n \, dx = x^{n+1}/(n+1)$$

Answer: $x^2/2 + C$

Q14. Find $\int \cos x \, dx$.

समाकलन कीजिए।

Answer: $\sin x + C$

Chapter 8: Applications of Integrals

Q15. Find area under $y = x$ from 0 to 1.

क्षेत्रफल ज्ञात कीजिए।

Formula:

$$\text{Area} = \int_0^1 x \, dx$$

Steps:

$$= [x^2/2]_0^1 = 1/2$$

Answer: 1/2 sq unit

Q16. Write formula for area between curves.

सूत्र लिखिए।

Answer:

$$\text{Area} = \int (\text{Upper} - \text{Lower}) \, dx$$

Chapter 9: Differential Equations

Q17. Solve $dy/dx = 2x$.

अवकल समीकरण हल कीजिए।

Steps:

$$dy = 2x \, dx$$

$$\int dy = \int 2x \, dx$$

$$y = x^2 + C$$

Q18. Write general solution of $dy/dx = 0$.

सामान्य हल लिखिए।

Answer: $y = C$

Chapter 10: Vector Algebra

Q19. Find magnitude of $3\mathbf{i} + 4\mathbf{j}$.

परिमाण ज्ञात कीजिए।

Formula:

$$|a| = \sqrt{(a^2 + b^2)}$$

Answer: $\sqrt{9 + 16} = 5$

Q20. Write unit vector along x-axis.

x-अक्ष की इकाई सदिश लिखिए।

Answer: $\hat{\mathbf{i}}$

Chapter 11: 3D Geometry

Q21. Distance between $(0,0,0)$ and $(2,3,6)$.

दूरी ज्ञात कीजिए।

Formula:

$$\sqrt[(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2]$$

Answer: $\sqrt{49} = 7$

Q22. Equation of x-axis.

x-अक्ष का समीकरण।

Answer: $y = 0, z = 0$

Chapter 12: Linear Programming

Q23. What is feasible region?

साध्य क्षेत्र क्या है?

Answer:

Region satisfying all constraints.

उत्तर:

सभी प्रतिबंधों को संतुष्ट करने वाला क्षेत्र।

Q24. Write objective function.

उद्देश्य फलन लिखिए।

Answer: $Z = ax + by$

Chapter 13: Probability

Q25. Probability of sure event.

निश्चित घटना की प्रायिकता।

Answer: 1

Q26. If $P(A)=0.4$, find $P(\text{not } A)$.

पूरक घटना ज्ञात कीजिए।

Formula:

$$P(A') = 1 - P(A)$$

Answer: 0.6

Q27. Define random experiment.

यादृच्छिक प्रयोग की परिभाषा।

Answer:

Experiment with uncertain outcome.

Q28. Write formula of conditional probability.

सशर्त प्रायिकता का सूत्र।

Answer:

$$P(A|B) = P(A \cap B) / P(B)$$

Q29. Mean of binomial distribution.

द्विविपद वितरण का माध्य।

Answer: Mean = np

Q30. Variance of binomial distribution.

प्रसरण।

30 Probable Short Answer Questions (Bilingual)

Chapter 1: Relations & Functions

Q1

EN: Find the domain of ($f(x) = \sqrt{5-2x}$).

HI: ($f(x) = \sqrt{5-2x}$) का परिभाषा क्षेत्र ज्ञात कीजिए।

Solution / उत्तर:

Condition: $(5-2x \geq 0)$

$$\Rightarrow (2x \leq 5)$$

$$\Rightarrow (x \leq \frac{5}{2})$$

Domain: $((-\infty, 5/2])$

Formula: Square root defined for non-negative values

सूत्र: वर्गमूल तभी परिभाषित जब मान ≥ 0

Chapter 2: Inverse Trigonometric Functions

Q2

EN: Evaluate ($\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$).

HI: ($\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$) का मान ज्ञात कीजिए।

Solution:

$$(\tan^{-1}1 = \pi/4)$$

$$(\tan^{-1}2 + \tan^{-1}3 = \pi/2)$$

Answer:

$$(\pi/4 + \pi/2 = \frac{3\pi}{4})$$

Formula:

$$(\tan^{-1}a + \tan^{-1}b = \tan^{-1}\left(\frac{a+b}{1-ab}\right))$$

Chapter 3: Matrices

Q3

EN: Find determinant of ($A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$).

HI: मैट्रिक्स (A) का निर्धारक ज्ञात कीजिए।

Solution:

$$(|A| = (2)(4) - (3)(1) = 8 - 3 = 5)$$

Answer: (5)

Formula:

$$(|A| = ad - bc)$$

Chapter 4: Determinants

Q4

EN: For what value of (k) determinant is zero?

$$(\begin{vmatrix} 1 & 2 \\ 3 & k \end{vmatrix})$$

HI: किस (k) के लिए निर्धारक शून्य होगा?

Solution:

$$(|A| = 1 \cdot k - 6 = 0)$$

$$\Rightarrow (k = 6)$$

Chapter 5: Continuity & Differentiability

Q5

EN: Find ($\frac{d}{dx}(x^3 + 5x)$).

HI: ($x^3 + 5x$) का अवकलन कीजिए।

Solution:

$$(\frac{d}{dx}(x^3) = 3x^2)$$

$$(\frac{d}{dx}(5x) = 5)$$

Answer: ($3x^2 + 5$)

Chapter 6: Applications of Derivatives

Q6

EN: Find slope of tangent to ($y=x^2$) at ($x=2$).

HI: ($x=2$) पर स्पर्शरेखा का ढाल ज्ञात कीजिए।

Solution:

$$(\frac{dy}{dx} = 2x)$$

At ($x=2$): slope = (4)

Chapter 7: Integrals

Q7

EN: Evaluate ($\int 3x^2 dx$).

HI: ($\int 3x^2 dx$) का मान ज्ञात कीजिए।

Solution:

$$(\int x^n dx = \frac{x^{n+1}}{n+1})$$

$$[\int 3x^2 dx = x^3 + C]$$

Chapter 8: Definite Integrals

Q8

EN: Evaluate ($\int_0^1 x dx$).

HI: ($\int_0^1 x dx$) हल कीजिए।

Solution:

$$[\int_0^1 x dx = \left[\frac{x^2}{2} \right]_0^1 = \frac{1}{2}]$$

Chapter 9: Differential Equations

Q9

EN: Solve ($\frac{dy}{dx} = x$).

HI: ($\frac{dy}{dx} = x$) हल कीजिए।

Solution:

```
[  
dy = x dx  
]  
[  
y = \frac{x^2}{2} + C  
]
```

Chapter 10: Vector Algebra

Q10

EN: Find magnitude of ($\vec{a} = 2\hat{i} + 3\hat{j}$).

HI: वेक्टर का परिमाण ज्ञात कीजिए।

Solution:

```
[  
|\vec{a}| = \sqrt{2^2 + 3^2} = \sqrt{13}  
]
```

Chapter 11: 3-D Geometry

Q11

EN: Distance between ((1,2,3)) and ((4,6,3)).

HI: दो बिंदुओं के बीच दूरी ज्ञात कीजिए।

Solution:

```
[  
d = \sqrt{(4-1)^2 + (6-2)^2 + (3-3)^2}  
]  
[  
= \sqrt{9+16} = 5  
]
```

Chapter 12: Linear Programming

Q12

EN: What is a feasible region?

HI: संभव क्षेत्र क्या होता है?

Answer:

EN: Region satisfying all constraints

HI: सभी शर्तों को संतुष्ट करने वाला क्षेत्र

Chapter 13: Probability

Q13

EN: Probability of getting a head in a fair coin.

HI: एक सिक्के में हेड की प्रायिकता।

Answer:

$$[P(H) = \frac{1}{2}]$$

Q14. Differentiation

EN: Find $(\frac{d}{dx})(\sin x)$.

HI: $(\sin x)$ का अवकलन कीजिए।

Formula / सूत्र:

$$[\frac{d}{dx}(\sin x) = \cos x]$$

Answer / उत्तर:

$$[\boxed{\cos x}]$$

Q15. Integration

EN: Evaluate ($\int \cos x dx$).

HI: ($\int \cos x dx$) का मान ज्ञात कीजिए।

Formula / सूत्र:

$$[\int \cos x dx = \sin x + C]$$

Answer:

$$[\boxed{\sin x + C}]$$

Q16. Vector Algebra

EN: Write the formula for unit vector along (\vec{a}).

HI: (\vec{a}) की दिशा में इकाई सदिश का सूत्र लिखिए।

Formula / सूत्र:

$$[\hat{a} = \frac{\vec{a}}{|\vec{a}|}]$$

Answer:

Unit vector = Vector / Magnitude

इकाई सदिश = सदिश / परिमाण

Q17. Matrices

EN: What is the order of a matrix with 3 rows and 2 columns?

HI: 3 पंक्ति और 2 स्तंभ वाले मैट्रिक्स का क्रम क्या है?

Answer / उत्तर:

$$[\boxed{3 \times 2}]$$

Q18. Mean Value Theorem

EN: State the conditions for Mean Value Theorem.

HI: औसत मान प्रमेय की शर्तें लिखिए।

Answer:

EN:

1. Function continuous in $([a,b])$
2. Differentiable in $((a,b))$

HI:

1. फलन $([a,b])$ में सतत हो
 2. $((a,b))$ में अवकलनीय हो
-

Q19. Inverse Trigonometry

EN: Find $(\tan^{-1} 0)$.

HI: $(\tan^{-1} 0)$ का मान ज्ञात कीजिए।

Solution:

$$[\tan 0 = 0]$$

Answer:

$$[\boxed{0}]$$

Q20. Probability

EN: Write the range of probability.

HI: प्रायिकता का परास लिखिए।

Answer / उत्तर:

$$[\boxed{0 \leq P(A) \leq 1}]$$

Q21. Determinants

EN: Find determinant of identity matrix of order 2.

HI: 2 कोटि की इकाई मैट्रिक्स का निर्धारक ज्ञात कीजिए।

Solution:

$$[|I| = 1]$$

Answer:

$$[\boxed{1}]$$

Q22. Vector Algebra

EN: What is the angle between two perpendicular vectors?

HI: दो लम्बवत् सदिशों के बीच कोण कितना होता है?

Answer / उत्तर:

$$[\boxed{90^\circ}]$$

Q23. Differential Equation

EN: Find the degree of the equation

$$[\frac{d^2y}{dx^2} + y = 0]$$

HI: अवकल समीकरण की कोटि ज्ञात कीजिए।

Solution:

Highest order derivative = second order

Answer:

$$[\boxed{2}]$$

Q24. Mensuration (Application of Integrals)

EN: Write formula for volume of a cylinder.

HI: बेलन का आयतन सूत्र लिखिए।

Formula / सूत्र:

$$[V = \pi r^2 h]$$

Q25. Application of Derivatives

EN: Condition for a function to be increasing.

HI: फलन के बढ़ने की शर्त क्या है?

Answer:

$$[\boxed{f'(x) > 0}]$$

Q26. Matrices

EN: What is a null matrix?

HI: शून्य मैट्रिक्स क्या होता है?

Answer:

EN: Matrix with all elements zero

HI: वह मैट्रिक्स जिसके सभी अवयव शून्य हों

Q27. Scalars & Vectors

EN: What is a scalar quantity?

HI: अदिश राशि क्या हैं?

Answer:

EN: Quantity having only magnitude

HI: केवल परिमाण वाली राशि

Q28. Trigonometry

EN: Find maximum value of ($\sin x$).

HI: ($\sin x$) का अधिकतम मान क्या हैं?

Answer:

[
\\boxed{1}
]

Q29. Inverse of Matrix

EN: When does inverse of a matrix exist?

HI: मैट्रिक्स का व्युत्क्रम कब अस्तित्व में होता है?

Answer:

[
\\boxed{|A| \\neq 0}
]

Q30. Probability

EN: Write formula for complementary probability.

HI: पूरक प्रायिकता का सूत्र लिखिए।

Formula / सूत्र:

[

$$P(A') = 1 - P(A)$$

]

Long Questions and Answers

1) Limits & Continuity — L'Hospital type / Indeterminate form

Q (EN): Evaluate ($\lim_{x \rightarrow 0} \frac{\sin(3x) - 3x}{x^3}$).

Q (HI): सीमाएँ — मान निकालिए ($\lim_{x \rightarrow 0} \frac{\sin(3x) - 3x}{x^3}$).

Solution (EN):

Use Taylor expansion or series: ($\sin t = t - \frac{t^3}{6} + o(t^3)$). Put ($t=3x$):
 $(\sin(3x) = 3x - \frac{(3x)^3}{6} + o(x^3) = 3x - \frac{27x^3}{6} + o(x^3) = 3x - \frac{9}{2}x^3 + o(x^3))$.

So numerator = $((3x - \frac{9}{2}x^3 + o(x^3)) - 3x = -\frac{9}{2}x^3 + o(x^3))$. Divide by (x^3) : $(-\frac{9}{2} + o(1))$.

Limit = $(-\frac{9}{2})$.

Solution (HI):

$(\sin t)$ का Taylor: $(t - \frac{t^3}{6} + \dots)$. ($t=3x$) रखें। गणना से उत्तर $(-\frac{9}{2})$ आता है।

Key formula: ($\sin t = t - \frac{t^3}{6} + \dots$)

2) Derivative — Differentiation of inverse trig

Q (EN): If ($y = \tan^{-1}(\frac{2x}{1-x^2})$), show that ($\frac{dy}{dx} = \frac{2}{1+x^2}$).

Q (HI): यदि ($y = \tan^{-1}(\frac{2x}{1-x^2})$), सिद्ध कीजिए
 $(\frac{dy}{dx} = \frac{2}{1+x^2})$.

Solution (EN):

Note identity: ($\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$). So set ($x = \tan\theta \Rightarrow \frac{2x}{1-x^2} = \tan(2\theta)$). Then ($y = \tan^{-1}(\tan 2\theta) = 2\theta$) (take principal branch where valid) $\Rightarrow (y = 2\arctan x)$. Differentiate: ($\frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2} = \frac{2}{1+x^2}$). Wait—this conflicts target; check algebra. Alternative route: Actually ($\tan^{-1}(\frac{2x}{1-x^2}) = 2\arctan x$) only when ($|x|<1$) and branches considered.

Differentiate directly using chain rule: Let ($u = \frac{2x}{1-x^2}$). Then ($\frac{du}{dx} = \frac{2(1-x^2)-2x(-2x)}{(1-x^2)^2} = \frac{2-2x^2+4x^2}{(1-x^2)^2} = \frac{2+2x^2}{(1-x^2)^2} = \frac{2(1+x^2)}{(1-x^2)^2}$). Now ($\frac{dy}{dx} = \frac{1}{u^2} \cdot \frac{du}{dx}$). Compute ($1+u^2 = 1 + \frac{4x^2}{(1-x^2)^2} = \frac{(1-x^2)^2+4x^2}{(1-x^2)^2} = \frac{1-2x^2+x^4+4x^2}{(1-x^2)^2} = \frac{1+2x^2+x^4}{(1-x^2)^2} = \frac{(1+x^2)^2}{(1-x^2)^2}$).

Thus ($\frac{dy}{dx} = \frac{1}{u^2} \cdot \frac{2(1+x^2)}{(1-x^2)^2} \cdot \frac{4x^2}{(1-x^2)^2} = \frac{2(1+x^2)}{(1-x^2)^2} \cdot \frac{4x^2}{(1+x^2)^2} = \frac{8x^2}{(1+x^2)^2}$).

So derivative is ($\frac{8x^2}{(1+x^2)^2}$). (Hence original target ($\frac{1+x^2}{1+x^4}$) is algebraically equal? Check: ($\frac{1+x^2}{1+x^4} = \frac{1+x^2}{(1+x^2)^2 - 2x^2}$) — not same. The correct derivative is ($\frac{8x^2}{(1+x^2)^2}$)).

Solution (HI):

सरलतमः ($u = \frac{2x}{1-x^2}$) लैं, शून्खला और गुणनफल नियम लागू करें। अंतिम उत्तर ($\frac{8x^2}{(1+x^2)^2}$) आता है।

Key formula: ($\frac{d}{dx}(\tan^{-1}u) = \frac{1+u^2}{u'}$).

(Remark: many exam variants ask to show ($y=2\arctan x$) then differentiate.)

3) Application of Derivative — Max/Min & Tangent

Q (EN): For ($f(x)=x^3-3x+1$), find intervals of increase/decrease, local extrema and equation of tangent at point where ($x=1$).

Q (HI): ($f(x)=x^3-3x+1$). वृद्धि/क्षय के अंतराल, स्थानीय अधिकतम/न्यूनतम और ($x=1$) पर स्पर्शरेखा का समीकरण जात कीजिए।

Solution (EN):

($f(x)=3x^2-3=3(x^2-1)=3(x-1)(x+1)$). Critical points at ($x=\pm 1$). Sign: for ($x < -1$), ($f' > 0$) (increasing); ($-1 < x < 1$): ($f' < 0$) (decreasing); ($x > 1$): ($f' > 0$) (increasing). So local max at ($x=-1$), local min at ($x=1$). Values: ($f(-1)=(-1)^3-3(-1)+1=-1+3+1=3$). ($f(1)=1-3+1=-1$). Tangent at ($x=1$): slope ($m=f'(1)=3(1)^2-3=0$). Point ((1,-1)). Equation: ($y+1=0(x-1)$) \Rightarrow ($y=-1$).

Solution (HI):

($f(x)=3(x-1)(x+1)$). क्रमशः वृद्धि/क्षय का वर्णन ऊपर जैसा। ($x=1$) पर न्यूनतम, स्पर्शरेखा क्षैतिज ($y=-1$)।

Key formula: Critical points where ($f(x)=0$); test sign changes for extrema.

4) Rolle's / Mean Value Theorem (MVT)

Q (EN): Verify Rolle's theorem for ($g(x)=x^3-3x$) on ($[-\sqrt{3}, \sqrt{3}]$) and find the c guaranteed by Rolle's theorem.

Q (HI): ($g(x)=x^3-3x$) पर ($[-\sqrt{3}, \sqrt{3}]$) में Rolle की शर्त सत्यापित करें और c खोजिए।

Solution (EN):

Continuous and differentiable everywhere. ($g(-\sqrt{3})=(-\sqrt{3})^3 - 3(-\sqrt{3}) = -3\sqrt{3}$
 $+3\sqrt{3}=0$). Similarly ($g(\sqrt{3})=3\sqrt{3} - 3\sqrt{3}=0$). Hence Rolle applies $\Rightarrow (\exists c \in (-\sqrt{3}, \sqrt{3}))$ with ($g'(c)=0$). ($g'(x)=3x^2-3=3(x^2-1)$). Solve ($3(x^2-1)=0$) $\Rightarrow (x=\pm 1)$. Both ($x=\pm 1$) lie inside interval ($(-\sqrt{3}, \sqrt{3})$) (since ($\sqrt{3} \approx 1.732$)). So ($c=1$) or ($c=-1$).

Solution (HI):

शर्त पूर्ण; ($g' = 3(x^2-1)=0$) पर ($x=\pm 1$)। ये दोनों अंदर आते हैं। अतः ($c=\pm 1$)।

5) Integration — Definite integral using substitution

Q (EN): Evaluate ($\int_0^1 \frac{x^2}{(1+x^3)} dx$).

Q (HI): ($\int_0^1 \frac{x^2}{1+x^3} dx$) का मान ज्ञात कीजिए।

Solution (EN):

Let ($u=1+x^3 \Rightarrow du=3x^2 dx \Rightarrow (x^2 dx = du/3)$). When ($x=0$), ($u=1$); when ($x=1$), ($u=2$).
Integral = ($\int_{u=1}^{u=2} \frac{1}{u^2} \cdot \frac{du}{3} = \frac{1}{3} \ln u \Big|_1^2 = \frac{1}{3} \ln 2$).

Solution (HI):

उपर्युक्त विकल्प (u=1+x^3) करके मिलता है ($\frac{1}{3} \ln 2$).

Key formula: Substitution: ($\int f(g(x))g'(x)dx$).

6) Integration — By parts (long question)

Q (EN): Evaluate ($\int x e^{2x} dx$).

Q (HI): ($\int x e^{2x} dx$) का पूरा हल दीजिए (by parts).

Solution (EN):

Use integration by parts: take ($u=x \Rightarrow (du=dx)$, ($dv=e^{2x} dx \Rightarrow (v=\frac{1}{2}e^{2x})$)).
Then ($\int x e^{2x} dx = uv - \int v du = x \cdot \frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x} dx =$

$$\frac{x e^{2x}}{2} - \frac{1}{2} \cdot \frac{1}{2} e^{2x} + C = \frac{e^{2x}}{2} (x - \frac{1}{2}) + C.$$

Solution (HI):

By parts से उत्तर ($\int e^{2x} dx = \frac{1}{2} e^{2x}$) $\left(x - \frac{1}{2}\right) + C$ ।

Key formula: ($\int u dv = uv - \int v du$).

7) Definite Integrals — Area between curves

Q (EN): Find area enclosed by curves ($y=x^2$) and ($y=2x$).

Q (HI): ($y=x^2$) और ($y=2x$) के बीच बंद क्षेत्रफल ज्ञात कीजिए।

Solution (EN):

Intersection: ($x^2=2x \Rightarrow x(x-2)=0 \Rightarrow x=0,2$). Area = ($\int_0^2 (2x - x^2) dx = [x^2 - \frac{x^3}{3}]_0^2 = (4 - \frac{8}{3}) = \frac{4}{3}$).

Solution (HI):

परिणाम क्षेत्रफल = ($\int_0^2 (2x - x^2) dx$) इकाई²।

8) Differential Equations — First order linear

Q (EN): Solve ($\frac{dy}{dx} + y\tan x = \sin x$), for general solution.

Q (HI): ($\frac{dy}{dx} + y\tan x = \sin x$) हल कीजिए।

Solution (EN):

Standard linear: ($\frac{dy}{dx} + P(x)y = Q(x)$) with ($P=\tan x$). Integrating factor ($\mu(x)=e^{\int \tan x dx}=e^{-\ln|\cos x|}=\sec x$). Multiply equation by ($\sec x$): ($\sec x \frac{dy}{dx} + y\sec x \tan x = \sin x \sec x = \tan x$). Left is ($d(y\sec x)/dx$). So ($d(y\sec x)/dx = \tan x$). Integrate: ($y\sec x = \int \tan x dx = -\ln|\cos x| + C$). Thus ($y = \cos x \left(-\ln|\cos x| + C \right)$).

Solution (HI):

इंटीग्रेटिंग फैक्टर ($\sec x$)। अंतिम: ($y = \cos x \left(C - \ln|\cos x| \right)$)।

Key formula: IF ($e^{\int P dx}$).

9) Matrices — Inverse & consistency of system

Q (EN): For matrix ($A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$), find (A^{-1}). Use it to solve system ($2x+y=5$, $3x+2y=8$.)

Q (HI): ($A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$) का इनवर्स निकालिए और उसी से दिए गए रैखिक समीकरण हल करिए।

Solution (EN):

($\det A = 2 \cdot 1 - 3 \cdot 1 = 2 - 3 = -1$). ($A^{-1} = \frac{1}{\det A} \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$). Write system in matrix form ($\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$). So ($\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} \begin{pmatrix} 5 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \cdot 5 - 1 \cdot 8 \\ 3 \cdot 5 + 2 \cdot 8 \end{pmatrix} = \begin{pmatrix} 10 - 8 \\ 15 + 16 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$).

Solution (HI):

($\det=1$), ($A^{-1} = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$). हल: ($x=2$, $y=1$).

Key formula: ($A^{-1} = \frac{1}{\det A} \operatorname{adj} A$).

10) Determinants — Area & zero determinant interpretation

Q (EN): If area of triangle with vertices ((1,2),(3,5),(4,8)) is zero, show determinant test and evaluate area.

Q (HI): दिए बिंदुओं से बनता त्रिभुज शून्य क्षेत्रफल देता है—determinant से जाँच करिए और क्षेत्रफल ज्ञात कीजिए।

Solution (EN):

Area = ($\frac{1}{2} \left| \begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 1 \\ 4 & 8 & 1 \end{vmatrix} \right|$).

Evaluate determinant: Expand:

Compute determinant of ($\begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 1 \\ 4 & 8 & 1 \end{vmatrix}$). Subtract third column manipulations or compute directly: Using first row expansion:

($1 \cdot \begin{vmatrix} 5 & 1 \\ 8 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 3 & 5 \\ 4 & 8 \end{vmatrix}$) = (1(5-8) - 2(3-4) + 1(24-20)) = (-3 -2(-1)+4 = -3 +2 +4 = 3). Area = ($\frac{1}{2} |3| = \frac{3}{2}$). Not zero—so points are not collinear.

Solution (HI):

डिटर्मिनेंट = 3 \Rightarrow क्षेत्रफल = (3/2)। अतः शून्य नहीं है।

Key formula: Area triangle = ($\frac{1}{2} |\det([...])|$).

11) 3-D Geometry — Equation of plane

Q (EN): Find equation of plane passing through (A(1,0,2)) and perpendicular to vectors ($\vec{u}=(1,1,0)$) and ($\vec{v}=(0,1,1)$).

Q (HI): (A(1,0,2)) से गुजरने वाली और (\vec{u}, \vec{v}) के दोनों के प्रति लम्बवत् विमीय समीकरण निकालें।

Solution (EN):

Normal vector ($\vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \mathbf{i}(1 \cdot 1 - 0 \cdot 0) - \mathbf{j}(1 \cdot 0 - 0 \cdot 1) + \mathbf{k}(1 \cdot 0 - 1 \cdot 1) = \mathbf{i}(1) - \mathbf{j}(0) + \mathbf{k}(-1) = (1, -1, 1)$). Plane: $(x-1) - 1(y-0) + 1(z-2) = 0 \Rightarrow (x-1 - y + z - 2 = 0) \Rightarrow (x - y + z - 3 = 0)$.

Solution (HI):

नॉर्मल $((1, -1, 1))$ । विमीय समीकरण: $(x-y+z-3=0)$ ।

Key formula: plane through $(P_0(x_0, y_0, z_0))$ with normal ($\vec{n}=(a,b,c)$): $(a(x-x_0)+b(y-y_0)+c(z-z_0)=0)$.

12) Vectors — Angle between lines / shortest distance

Q (EN): Find angle between lines through origin with direction vectors ($\vec{a}=(1,2,2)$) and ($\vec{b}=(2,-1,2)$). Also find projection of (\vec{a}) on (\vec{b}).

Q (HI): निर्देश वेक्टर दिए हैं—रेखाओं के बीच कोण और प्रक्षेपण ज्ञात कीजिए।

Solution (EN):

Angle (θ): $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} = \frac{1 \cdot 2 + 2 \cdot (-1) + 2 \cdot 2}{\sqrt{1+4+4} \sqrt{4+1+4}} = \frac{2-2+4}{\sqrt{9} \sqrt{9}} = \frac{4}{9}$. So $\theta = \cos^{-1}(4/9)$. Projection of (\vec{a}) on (\vec{b}): scalar proj = $\frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|} = \frac{4}{9}$. Vector projection = $\left(\frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|^2} \vec{b}\right) = \frac{4}{9} \vec{b} = \frac{4}{9} (2, -1, 2) = \left(\frac{8}{9}, -\frac{4}{9}, \frac{8}{9}\right)$.

Solution (HI):

$(\cos\theta=4/9) \Rightarrow (\theta=\cos^{-1}(4/9))$ । scalar प्रोजेक्शन $(4/3)$ और वेक्टर प्रोजेक्शन ऊपर दिया गया है।

Key formula: ($\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$). Projection formulas as used.

13) Probability — Binomial (long)

Q (EN): A fair coin is tossed 8 times. Find probability of getting exactly 5 heads. Also find expected number of heads.

Q (HI): 8 बार सिक्का उछाला गया—ठीक 5 हेड्स आने की प्रायिकता और अपेक्षित मान क्या होगा?

Solution (EN):

Binomial ($n=8, p=1/2$). $(P(X=5)=\binom{8}{5}(1/2)^5(1/2)^3=\binom{8}{5}/2^8 = \frac{56}{256} = \frac{7}{32})$. Expected value ($E[X]=np=8\cdot\frac{1}{2}=4$).

Solution (HI):

$(P=7/32)$ । अपेक्षित हेड्स = 4।

Key formula: $(P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}); (E[X]=np)$.

14) Probability — Continuous distribution / Normal approximation (concept)

Q (EN): If $(X) \sim (B(n,p))$ with large (n) , write normal approximation and compute approximate probability $(P(X \leq np - 2\sqrt{np(1-p)}))$. (Assume large (n) , use continuity correction conceptually.)

Q (HI): बड़े n के लिए Binomial का Normal approx और $(P(X \leq np - 2\sqrt{np(1-p)}))$ का अनुमान लिखिए (continuity correction का प्रयोग विचारात्मक रूप से करें)।

Solution (EN):

Normal approx: $(X \approx N(\mu=np, \sigma^2=np(1-p)))$. Let $(Z=\frac{X-\mu}{\sigma})$. Then desired approx $(P(Z \leq \frac{np - 2\sqrt{np(1-p)}}{\sqrt{np(1-p)}}) = P(Z \leq -2) \approx \Phi(-2) \approx 0.0228)$. With continuity correction result similar.

Solution (HI):

सामान्यीकरण से $(P \approx \Phi(-2) \approx 0.0228)$ मिलता है।

Key formula: Normal approx to Binomial.

15) Sequences & Series — Convergence and sum (long answer)

Q (EN): Determine whether series ($\sum_{n=1}^{\infty} \frac{1}{2^n}$) converges and find its sum.

Q (HI): ($\sum_{n=1}^{\infty} \frac{n}{2^n}$) का परिणाम ज्ञात कीजिए—क्या यह सम convergence है और यदि हाँ तो योग क्या है?

Solution (EN):

Use known sum: ($\sum_{n=1}^{\infty} nx^n = \frac{x}{(1-x)^2}$) for ($|x|<1$). Put ($x=\frac{1}{2}$): sum = ($\frac{1}{2}/(\frac{1}{2})^2 = \frac{1}{2}/\frac{1}{4} = 2$). So series converges to 2.

Solution (HI):

सूत्र से योग = 2। अतः series converge करती है।

Key formula: ($\sum_{n=1}^{\infty} n x^n = \frac{x}{(1-x)^2}$).

16) Linear Programming — Graphical (long)

Q (EN): Maximize ($Z=3x+4y$) subject to ($x+2y \leq 8$, $3x+y \leq 9$, $x \geq 0$, $y \geq 0$). Find optimal vertex and max (Z).

Q (HI): रैखिक प्रोग्राम: ($Z=3x+4y$) अधिकतम कीजिए जहाँ सीमाएँ दी हैं। ग्राफिक विधि से हल दें।

Solution (EN):

Feasible region vertices: Solve intersections: (A) (0,0). (B) $x=0$, from ($2y \leq 8 \Rightarrow y \leq 4$) so (0,4) is intersection with first. (C) $y=0$, from ($3x \leq 9 \Rightarrow x \leq 3$) so (3,0). Intersection of two lines: solve ($x+2y=8$) and ($3x+y=9$). Multiply second by 2: ($6x+2y=18$). Subtract first: ($5x=10 \Rightarrow x=2$). Then ($1+? \Rightarrow 2+2y=8 \Rightarrow y=3$). So vertex (2,3). Evaluate Z: at (0,0):0; (0,4):(3*0+4*4=16); (3,0):(9); (2,3):(6+12=18). Max Z = 18 at (2,3).

Solution (HI):

सभी वर्टिसों का मूल्यांकन करने पर अधिकतम ($Z=18$) पर ((2,3)) मिलता है।

Key method: Corner point method (graphical).

17) Continuity & Differentiability — Show differentiable at point

Q (EN): Define ($f(x)=\begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x=0 \end{cases}$). Show (f) is differentiable at (0). Find ($f'(0)$).

Q (HI): ($f(x)=x^2 \sin(1/x)$) ($x \neq 0$), और 0 पर 0। दिखाइए कि f 0 पर differentiable है और $(f'(0)=?)$

Solution (EN):

Definition: $(f(0)=\lim_{h \rightarrow 0} \frac{f(h)-f(0)}{h})$ { $h=\lim_{h \rightarrow 0} h^2 \sin(1/h)$ } { $h=\lim_{h \rightarrow 0} h \sin(1/h)$ } Since $(|h \sin(1/h)| \leq |h| \rightarrow 0)$, limit 0. Hence differentiable and $(f'(0)=0)$.

Solution (HI):

लिमिट को bound कर के मिलता है $(f'(0)=0)$.

Key theorem: Squeeze theorem used.

18) Application of Derivative — Curvature / Nature of stationary pt (long)

Q (EN): For $(y=x^4-4x^3+6x^2)$, find and classify stationary points.

Q (HI): $(y=x^4-4x^3+6x^2)$ के stationary points ज्ञात कर के उनका प्रकार बताइए।

Solution (EN):

$(y'=4x^3-12x^2+12x=4x(x^2-3x+3))$. Solve $(y'=0)$: $(x=0)$ or roots of quadratic $(x^2-3x+3=0)$. Discriminant ($\Delta=9-12=-3<0$) \Rightarrow no real roots. So only stationary point $(x=0)$. Second derivative $(y''=12x^2-24x+12=12(x^2-2x+1)=12(x-1)^2)$. At $(x=0)$, $(y''=12(1)=12>0)$ \Rightarrow local minimum at $(x=0)$. Value $(y(0)=0)$.

Solution (HI):

केवल stationary बिन्दु $(x=0)$ । क्योंकि $(y''(0)>0)$, यह स्थानीय न्यूनतम है; मान 0 है।

Key formula: Second derivative test: $(y''(x_0)>0) \Rightarrow$ local min, $(<0) \Rightarrow$ max.

19) Integration — Improper integral (convergence)

Q (EN): Test convergence of $(\int_{-\infty}^{1/\ln x} \frac{1}{x(\ln x)^2} dx)$.

Q (HI): $(\int_{-\infty}^{1/\ln x} \frac{1}{x(\ln x)^2} dx)$ संग्रहीत है या नहीं? जाँच कीजिए।

Solution (EN):

Let $(t=\ln x)$. Then $(dt=dx/x)$. Integral becomes $(\int_{-\infty}^{0} \frac{1}{t^2} dt)$ (since when $x=1$, $t=0$; $x \rightarrow \infty$, $t \rightarrow \infty$). $(\int_{-\infty}^{0} t^{-2} dt)$ improper at 0: $(\int_{-\epsilon}^{0} t^{-2} dt = [-t^{-1}]_{-\epsilon}^{0} = 0 - (-1/\epsilon) = 1/\epsilon)$ which $\rightarrow \infty$ as $(\epsilon \rightarrow 0^+)$.

But careful: original lower limit corresponds to t from 0^+ , so divergence near 0. However evaluate original as $(\int_{-\infty}^{1/\ln x} \frac{1}{x(\ln x)^2} dx = [-1/\ln x]_{-\infty}^{1/\ln x} = (-0) - (-1/0^+))$

infinite. So integral diverges (actually tends to $1/0 \rightarrow \infty$). Wait sign: For large x , $-1/\ln x \rightarrow 0$; at lower $1+$, $-1/\ln x \rightarrow -\infty$, overall infinite. So diverges.

Solution (HI):

substitution से पता चलता है कि यह divergent है (असीमित)।

Key idea: Compare with p-test after substitution.

20) Differential Equations — Variable separable (long)

Q (EN): Solve $(\frac{dy}{dx}) = \frac{x^2}{y^2}$, with initial condition $(y(1)=1)$.

Q (HI): $(\frac{dy}{dx}) = \frac{x^2}{y^2}$ हल कीजिए और $(y(1)=1)$ लागू करें।

Solution (EN):

Separable: $(y^2 dy = x^2 dx)$. Integrate: $(\int y^2 dy = \int x^2 dx) \Rightarrow (\frac{y^3}{3} = \frac{x^3}{3} + C)$. Multiply by 3: $(y^3 = x^3 + C')$. Use $(y(1)=1) \Rightarrow (1 = 1 + C') \Rightarrow (C' = 0)$. So $(y^3 = x^3) \Rightarrow (y=x)$ (real cube root). So solution: $(y=x)$.

Solution (HI):

समीकरण से $(y^3 = x^3 + C')$ और शर्त से $(C' = 0)$ । अतः $(y=x)$.

21) Relations & Functions — Inverse function & monotonicity

Q (EN): Show that function $(f(x)=x^3+3x+1)$ is one-to-one and hence invertible. Find derivative of inverse at $(x=2)$ i.e. $((f^{-1})'(2))$ if $(f(a)=2)$.

Q (HI): $(f(x)=x^3+3x+1)$ एक-एक है इसका प्रमाण दीजिए और यदि $(f(a)=2)$ हो तो $((f^{-1})'(2))$ ज्ञात कीजिए।

Solution (EN):

$(f(x)=3x^2+3=3(x^2+1)>0)$ for all $x \Rightarrow$ strictly increasing \Rightarrow one-to-one. If $(f(a)=2)$, need (a) : solve $(a^3+3a+1=2) \Rightarrow (a^3+3a-1=0)$. Real root is $(a=0)$? Plug 0 gives -1. $(a=1): (1+3-1=3)$. So root between 0 and 1 (call it a). But we can compute $((f^{-1})'(y) = 1 / f'(f^{-1}(y)))$. So $((f^{-1})'(2) = 1/f(a) = 1/(3a^2+3) = 1/[3(a^2+1)])$. If numeric a required, solve cubic numerically; symbolic form suffices.

Solution (HI):

आवश्यक: $(f(x)>0)$ से एक-एकता सिद्ध। $((f^{-1})'(2)=\frac{1}{3a^2+3})$ जहाँ (a) फंक्शन का वह मान है जो $(f(a)=2)$ देता है।

Key formula: $((f^{-1})'(y)=1/f(x))$ where $(x=f^{-1}(y))$.

22) Conic Sections (Parabola) — Tangent & normal

Q (EN): For parabola ($y^2=4ax$), find equation of tangent at point $((at^2, 2at))$ and equation of normal.

Q (HI): $(y^2=4ax)$ पर बिंदु $((at^2, 2at))$ के लिए स्पर्शरेखा और सामान्य की समीकरण खोजिए।

Solution (EN):

Parametric point $((at^2, 2at))$. Tangent: $(ty = x + at^2)$ (standard form: $(ty = x + at^2)$). Normal: $(y = -tx + 2at + at^3)$ — standard normal formula: $(y = -tx + 2at + at^3)$. (One may derive using derivative: $dy/dx = 2a/(y) \Rightarrow at$ param gives slope of tangent = $1/t$, etc.)

Solution (HI):

मानक परिणाम से स्पर्शरेखा: $(ty = x + at^2)$ | सामान्य: $(y = -tx + 2at + at^3)$ ।

Key formulas: Parametric forms & standard tangent/normal.

23) Eigenvalues & Eigenvectors (Matrices) — long

Q (EN): For $(A=\begin{pmatrix} 4 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix})$, find eigenvalues and eigenvectors.

Q (HI): (A) के eigenvalues/eigenvectors निकालिए।

Solution (EN):

Characteristic equation: $(\det(A-\lambda I)=\begin{vmatrix} 4-\lambda & 1 & 2 \\ 1 & 2 & 3-\lambda \end{vmatrix}=(4-\lambda)(3-\lambda)-2 = \lambda^2 - 7\lambda + 10=0)$. Solve: $(\lambda = \frac{7 \pm \sqrt{49-40}}{2} = \frac{7 \pm 3}{2} \Rightarrow (\lambda_1=5, \lambda_2=2))$. For $(\lambda=5)$: solve $((A-5I)\vec{v}=0) \Rightarrow (\begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0) \Rightarrow (-x+y+2z=0) \Rightarrow (y=x)$. Eigenvector e.g. $((1,1)^T)$. For $(\lambda=2)$: $(\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0) \Rightarrow (2x+y+2z=0) \Rightarrow (y=-2x)$. Eigenvector e.g. $((1,-2)^T)$.

Solution (HI):

Eigenvalues 5 और 2; संबंधित eigenvectors $((1,1))$ तथा $((1,-2))$ ।

24) Multivariable (Partial derivatives) — Max/Min constrained by Lagrange multiplier

Q (EN): Find max/min of ($f(x,y)=xy$) subject to ($x^2+y^2=1$).

Q (HI): (xy) का अधिकतम/न्यूनतम ज्ञात कीजिए जहाँ ($x^2+y^2=1$)।

Solution (EN):

Use Lagrange: ($L=xy + \lambda(x^2+y^2-1)$). Partial derivatives: ($L_x = y + 2\lambda x = 0$) (1), ($L_y = x + 2\lambda y = 0$) (2), constraint (3). Multiply (1) by x and (2) by y : ($xy + 2\lambda x^2 = 0$), ($xy + 2\lambda y^2 = 0$). Subtract: ($2\lambda(x^2-y^2) = 0$) \Rightarrow either ($\lambda=0$) or ($x^2=y^2$). If ($\lambda=0$), then $y=0$ and $x=0$ from (1),(2) inconsistent with constraint. So ($x^2=y^2$) \Rightarrow ($y=\pm x$). With constraint ($2x^2=1$) \Rightarrow ($x = \pm \frac{1}{\sqrt{2}}$). If ($y=x$): ($f=x^2 = 1/2$) (max). If ($y=-x$): ($f=-x^2=-1/2$) (min). So max (1/2) at (($1/\sqrt{2}, 1/\sqrt{2}$)) and min (-1/2) at (($1/\sqrt{2}, -1/\sqrt{2}$)) etc with sign permutations.

Solution (HI):

अधिकतम ($1/2$), न्यूनतम ($-1/2$)। बिंदु दिए ऊपर हैं।

Key formula: Lagrange multipliers solve constrained extrema.

25) Integration — Trigonometric integral (long)

Q (EN): Evaluate ($\int \frac{dx}{\sin x + \cos x}$).

Q (HI): ($\int \frac{dx}{\sin x + \cos x}$) का हल दीजिए।

Solution (EN):

Multiply numerator and denominator by ($\sin x - \cos x$): ($\frac{\sin x - \cos x}{\sin^2 x - \cos^2 x} dx = \frac{\sin x - \cos x}{-(\cos 2x)}$). Better trick: set ($t = \tan(x/2)$). Or use substitution: Let ($u = \sin x + \cos x$). Then ($du = (\cos x - \sin x) dx$). So write numerator ($dx = \frac{du}{\cos x - \sin x}$). But ($\cos x - \sin x = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} = \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \frac{\cos 2x}{\cos x + \sin x}$). This becomes messy. Use standard substitution: divide numerator and denominator by ($\cos x$): ($\int \frac{dx}{\cos x(\tan x + 1)}$). Let ($u = \tan x \Rightarrow du = \sec^2 x dx$) not matching. Simpler route: set ($I = \int \frac{dx}{\sin x + \cos x} = \int \frac{dx}{\sin x - \cos x} \frac{\sin^2 x - \cos^2 x}{\sin^2 x - \cos^2 x} dx = \int \frac{dx}{\sin x - \cos x} \frac{-(\cos 2x)}{dx}$). Another standard trick: put ($\sin x + \cos x = \sqrt{2}\sin(x+\pi/4)$). Then integral ($I = \int \frac{dx}{\sqrt{2}\sin(x+\pi/4)} = \frac{1}{\sqrt{2}} \int \frac{dx}{\csc(x+\pi/4)} dx = \frac{1}{\sqrt{2}} \int \frac{dx}{\tan(x+\pi/4)} = \frac{1}{\sqrt{2}} \ln|\tan(x+\pi/4)| + C$). So final: ($I = \frac{1}{\sqrt{2}} \ln|\tan(x+\pi/4)| + C$).

Solution (HI):

($\sin x + \cos x = \sqrt{2}\sin(x+\pi/4)$) प्रयोग कर के उत्तर

($\frac{1}{\sqrt{2}} \ln|\tan(x+\pi/4)| + C$) मिलता है।

Key identity: ($\sin x + \cos x = \sqrt{2}\sin(x+\pi/4)$).

26) Probability — Conditional probability & Bayes (long)

Q (EN): Two boxes: Box A has 2 white, 1 black. Box B has 1 white, 2 black. Choose a box at random then pick a ball at random — probability the ball is white? If observed white, what's probability it came from Box A? (Bayes)

Q (HI): दो डिब्बे सीमा के साथ—प्राथमिक और बायस का प्रयोग कर प्रश्न हल कीजिए।

Solution (EN):

$P(\text{choose A})=P(\text{choose B})=1/2$. $P(\text{white|A})=2/3$. $P(\text{white|B})=1/3$. Total $(P(\text{white})=\frac{1}{2}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2})$. By Bayes: $(P(A|\text{white})=\frac{P(\text{white|A})P(A)}{P(\text{white})} = \frac{(2/3)(1/2)}{1/2} = \frac{2}{3})$.

Solution (HI):

कुल प्रायिकता $(1/2)\cdot(2/3)$ । $(P(A|\text{white})=2/3)$ ।

Key formula: Bayes theorem.

27) Limits — Sandwich theorem / tricky limit

Q (EN): ($\lim_{x \rightarrow 0} \frac{e^x - 1 - x - x^2/2}{x^3}$).

Q (HI): ($\lim_{x \rightarrow 0} \frac{e^x - 1 - x - x^2/2}{x^3}$) ज्ञात कीजिए।

Solution (EN):

Use Taylor: $(e^x = 1 + x + x^2/2 + x^3/6 + o(x^3))$. Numerator $\approx (x^3/6 + o(x^3))$. Divide by $(x^3) \Rightarrow (1/6)$. So limit $(=1/6)$.

Solution (HI):

Taylor से उत्तर $(1/6)$ है।

Key formula: Taylor expansion of (e^x) .

28) Application of Integrals — Volume of revolution

Q (EN): Find volume when region bounded by $(y=x^2)$ and $(y=0)$ between $(x=0)$ and $(x=1)$ is revolved about x-axis.

Q (HI): $(y=x^2)$ के नीचे (0) और (x) के बीच क्षेत्र $(x=0)$ से (1) तक घूर्णन करने पर आयतन ज्ञात कीजिए।

Solution (EN):

Volume by discs: $(V = \pi \int_0^1 (x^2)^2 dx = \pi \int_0^1 x^4 dx = \pi [x^5/5]_0^1 = \pi/5)$.

Solution (HI):

आवश्यक आयतन = $(\pi/5)$ इकाई³।

Key formula: Volume ($= \pi \int y^2 dx$) about x-axis.

29) Differential Equations — Second order linear with constant coefficients (long)

Q (EN): Solve $(y'' - 5y' + 6y = 0)$ with $(y(0)=2, y'(0)=1)$.

Q (HI): $(y'' - 5y' + 6y = 0)$ का सामान्य हल निकालिए और आरंभिक शर्तें लागू कीजिए।

Solution (EN):

Char eqn: $(r^2 - 5r + 6 = 0) \Rightarrow ((r-2)(r-3) = 0) \Rightarrow (r=2,3)$. General solution $(y = C_1 e^{2x} + C_2 e^{3x})$. $(y' = 2C_1 e^{2x} + 3C_2 e^{3x})$. Apply $(x=0)$: $(y(0) = C_1 + C_2 = 2)$. $(y'(0) = 2C_1 + 3C_2 = 1)$. Solve: From first $(C_1 = 2 - C_2)$. Substitute: $(2(2 - C_2) + 3C_2 = 1) \Rightarrow (4 - 2C_2 + 3C_2 = 1) \Rightarrow (4 + C_2 = 1) \Rightarrow (C_2 = -3)$. Then $(C_1 = 2 - (-3) = 5)$. So $(y = 5e^{2x} - 3e^{3x})$.

Solution (HI):

रेखिक समीकरण के हल से $(y = 5e^{2x} - 3e^{3x})$ ।

30) Misc long — Combination of calculus & algebra (challenging)

Q (EN): Let $(f(x) = \ln x - \frac{1}{x})$ for $(x > 0)$. Show that (f) has exactly one real root and approximate it. Also find where (f) is increasing.

Q (HI): $(f(x) = \ln x - 1/x)$ के लिए दिखाइए कि केवल एक वास्तविक मूल है और लगभग मान देने के साथ (f) कहाँ बढ़ती है बताइए।

Solution (EN):

Study $(f(x) = 1/x + 1/x^2 = \frac{x+1}{x^2} > 0)$ for $(x > 0)$. So (f) is strictly increasing on $((0, \infty))$. Limit: $(\lim_{x \rightarrow 0^+} f(x) = -\infty - \infty = -\infty)$. $(\lim_{x \rightarrow \infty} f(x) = \infty)$. By Intermediate Value Theorem there is exactly one root. To approximate solve $(\ln x = 1/x)$. Try $(x=1)$: LHS 0, RHS 1 $\Rightarrow (f(1) = -1 < 0)$. Try $(x=2)$: $(\ln 2 - 1/2 \approx 0.693 - 0.5 = 0.193 > 0)$. So root between 1 and 2. Try $(x=1.5)$: $(\ln 1.5 - 2/3 \approx 0.4055 - 0.6667 = -0.2612)$. Try $(x=1.8)$: $(\ln 1.8 - 0.5556 \approx 0.5878 - 0.5556 = 0.0322)$. Try $(x=1.79)$: $(\ln 1.79 \approx 0.5832 - 0.5587 = 0.0245)$. Try $(x=1.75)$: $(0.5596 - 0.5714 = -0.0118)$. So root $\approx (1.768)$ (approx). (One may refine by Newton's method.) So unique root $\approx (1.769)$. And (f) increasing for all $(x > 0)$.

Solution (HII):

$(f'(x) > 0)$ से f हर जगह वृद्धि करती है, अतः केवल एक शून्य होगा; अनुमानित मान $\approx (1.77)$ मिलता है।

Key methods: Monotonicity (first derivative), IVT, numeric bisection/Newton.