

UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

Department of Computing

B. Sc. Examination 2015

IS52017C

Algorithms

Duration: 2 hours 15 minutes

Date and time:

There are five questions in this paper. You should answer no more than THREE questions. Full marks will be awarded for complete answers to a total of THREE questions. Each question carries 25 marks. The marks for each part of a question are indicated at the end of the part in [...] brackets.

There are 75 marks available on this paper.

**THIS PAPER MUST NOT BE REMOVED
FROM THE EXAMINATION ROOM**

Question 1

- (a) i. Consider the following Java program:

```
class f1
{

    static void f(int [] a,  int N)
    {
        for (int i=0;i<N-1;i++)
            for (int j=i+1;j<N;j++)
                if (a[i]< a[j])
                {
                    int temp = a[i];
                    a[i]=a[j];
                    a[j]=temp;
                }
    }

    static void print(int [] a,  int N)
    {

        for (int i=0;i<N;i++) System.out.println(a[i]);

    }

    public static void main (String [] args)
    {
        int b[]=new int[100];
        for (int i=0;i<args.length;i++) b[i]=Integer.parseInt(args[i]);
        f(b,args.length);
        print( b, args.length);
    }
}
```

Write down the output resulting from running

```
java f1 3 25 6 2
```

- ii. If instead of $a[i] < a[j]$, the program had $a[j] < a[i]$ the output would be different. Write down the output resulting from running

```
java f1 3 25 6 2
```

iii. Very briefly, explain the purpose of the code:

```
int temp = a[i];
    a[i]=a[j];
    a[j]=temp;
```

[12]

(b) i. If it takes n microseconds to run

```
java f1 3 25 6 2
```

and if it takes m microseconds to run

```
java f1 1 3 25 6 2 67 66 4
```

what is $\frac{n}{m}$ approximately to the nearest integer?

[5]

(c) What is the output of this Scala program?

```
object list21
{

    def map(k:List[Int], f: Int => Int):List[Int]=
    {
        if (k.isEmpty) Nil
        else f(k.head)::map(k.tail,f)
    }

    def main(args:Array[String])
    {
        println(map(List(2,3,4,1), (y:Int) => 2*y));
    }
}
```

[4]

(d) Write a Scala function which adds up the elements of a List of Integers.

[4]

Question 2

- (a) i. A telephone directory of a large town has a million entries. If it takes 1 second to look at each entry, how long in the worst case would it take to find Sebastian Danicic's telephone number if the directory is not sorted?
- (1) Less than one minute
 - (2) Between one minute and two days
 - (3) Between two days and a week
 - (4) More than a week
- ii. Let $x = 2^y$ Which of the following is true:
- (1) $y = \log_e(x)$
 - (2) $y = \log_2(x)$
 - (3) $x = \log_2(y)$
 - (4) None of the above.
- iii. Which of the following functions will produce the largest values (asymptotically) as N gets bigger:
- (1) $f(N) = 30 * N$
 - (2) $f(N) = N^2$
 - (3) $f(N) = N * \log(N)$
 - (4) None of the above.
- iv. A telephone directory of a large town has a million entries. If it takes 1 second to look at each entry, how long in the worst case would it take to find Sebastian Danicic's telephone number if the directory is sorted and we use a binary search?
- (1) Less than one minute
 - (2) Between one minute and two days
 - (3) Between two days and a week
 - (4) More than a week

[12]

(b) Here is a method for performing a binary search

```
static boolean binarySearch(int s, int [] a)
{
    int hi=a.length-1;
    int lo=0;
    while (lo < hi)
    {
        int mid = lo + (hi - lo) / 2;
        if (a[mid] > s) hi=mid;
        else if (a[mid] < s) lo=mid+1;
        else return true;
    }
    return false;
}
```

- i. What must be true of the array a for this method to be able to find elements in the array?
- ii. If we swapped around the two expressions (a[mid] > s) and (a[mid] < s) what would now have to be true for the method to work?

[5]

- (c) The array `words` contains 670000 words of the English language in alphabetic order. Here is a method for searching for a `String` in the dictionary using a linear search.

```
static boolean find(String s)
{
    for (int i=0;i<670000;i++) if (s.equals(words[i])) return true;
    return false;
}
```

- i. Rewrite `binarySearch`, above so it works with `String` rather than `ints`. You will need to write some extra methods to help (or you can use any method in the Java API with which you are familiar). Fully explain your use of any Java API functions that you use.
- ii. How would you call your new method in order to find the word 'rabbit' in the array `words`?

[8]

Question 3

(a) Here is some Java Code for performing *Mergesort*:

```
class Pair
{
    int [] first;
    int [] second;
    Pair(int [] t , int [] u)
    {
        first=t;
        second=u;
    }
}

public class mergeSort1{
    static <answer 1> mergeSort(int[] a)
    {
        if (a.length <=1) return a;
        Pair p = divide(a);
        return merge(mergeSort(<answer 2>),mergeSort(<answer 3>));
    }
    static Pair divide(int [] a)
    {
        int n= a.length/2;
        int b[] = new int[n];
        int c[] = new int[a.length-n];
        for (int i=0;i<n;i++)b[i]=a[i];
        for (int i=0;i<a.length-n;i++)c[i]=a[n+ i];
        return <answer 4>;
    }
    static int [] merge(int [] b, int [] c)
    {
        int a[] = new int[b.length+c.length];
        int i=0,j=0,k=0;
        while (i<b.length && j<c.length)
        {
            if (b[i] <c[j]) {a[k]=<answer 5> ;i++;}
            else {a[k]=c[j];j++;}
            k++;
        }
        while(i<b.length) { a[k]=b[i];i++;k++;}
        while(j<c.length) {a[k]=<answer 6>;j++;k++;}
        return a;
    }
}
```

Fill in the gaps for the six answers.

[12]

(b) What is the time complexity of merge sort? Briefly justify your answer.

[7]

(c) Consider the following program for *Quick Sort*:

```
import java.util.*;
class Pair
{
    ArrayList <Integer> first;
    ArrayList <Integer> second;

    Pair(ArrayList <Integer> t , ArrayList <Integer> u)
    {
        first=t;
        second=u;
    }
}

public class quickSort
{
    static ArrayList <Integer> quickSort(ArrayList <Integer> a)
    {
        if (a.size() <=1) return a;
        Pair p = divide(a);
        ArrayList <Integer> b =quickSort(p.first);
        <answer 1>
        b.addAll(quickSort(p.second));
        return b;
    }
    static Pair divide(ArrayList <Integer> a)
    {
        int pivot=a.get(0);
        ArrayList <Integer> b = new ArrayList <Integer> ();
        ArrayList <Integer> c = new ArrayList <Integer> ();
        for (int i=1;i<a.size();i++)
            if (<answer 2>) b.add(a.get(i)); else c.add(a.get(i));

        return new Pair(b,c);
    }
}
```

Fill in the gaps for the two answers.

[6]

Question 4

- (a) i. Here are two methods for computing x^n :

```
static int powerA(int x, int n)
{
    int total=0;
    while (n>0) {total=total*x;n--;}
    return total;
}
```

```
static int powerB(int x, int n)
{
    if (n==0) return 1;
    int k=n/2;
    int z=powerB(x,k);
    int r=z*z;
    if (n%2==0) return r;
    return x*r;
}
```

Which one of the following is true?

- (1) powerA is linear and powerB has quadratic time-complexity.
 - (2) powerA is exponential and powerB has $\log(N)$ time-complexity.
 - (3) powerA is quadratic and powerB has exponential time-complexity.
 - (4) None of the above.
- ii. What is the time-complexity of this function in terms of N?

```
int f(int N)
{
    int total=0;
    for (int i=0;i<N;i++)
        for (int j=0;j<N;j++)
            for (int k=0;k<N;k++)
                total=total+i+j+k;
    return total;
}
```

- (1) linear
- (2) quadratic
- (3) cubic
- (4) exponential

iii. Here is a correct implementation of the Fibonacci function in Java:

```
static int Fib(int n)
{
    if (n==0 || n==1) return 1;
    return Fib(n-1)+Fib(n-2)
}
```

There are some problems with this implementation. Explain what the problems are.

[12]

(b) Write a linear time complexity method for Fibonacci.

[5]

(c) A *Hash Table* can be represented as an array of arraylists of Strings. A Hash function on Strings is a method which takes a string and returns an integer.

i. What is a collision?

ii. Write a method using the hash function for finding a string in a Hash table.

[8]

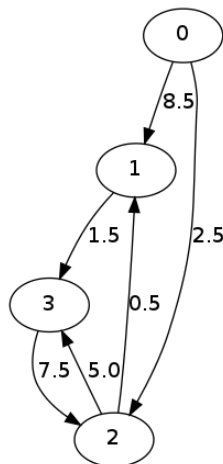
Question 5

- (a) i. Consider the following adjacency matrix of a graph.

```
int [][] a = {{1,0,0,1},{1,0,0,1},{1,1,0,0},{1,0,1,0}}
```

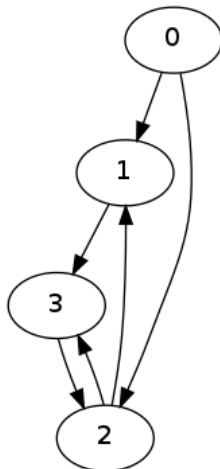
Draw a picture of the graph assuming the vertices are labelled 0-3.

- ii. Consider the following weighted directed graph



Which is the length of the shortest path (in terms of distance) from 0 to 3?

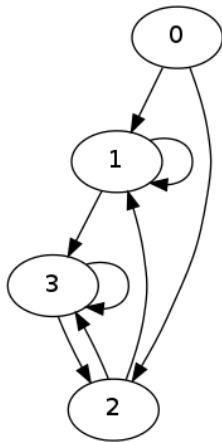
- iii. Consider the following directed graph:



Which one of the following adjacency matrices represents it?

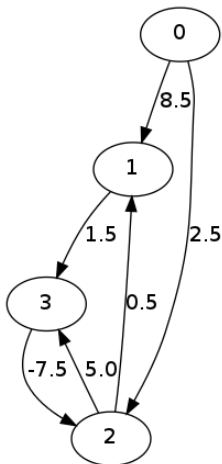
- (1) `int [][] a = {{0,1,1,0},{0,0,0,1},{0,1,0,0},{1,0,1,0}}`
- (2) `int [][] a = {{0,1,1,0},{1,0,0,1},{1,0,0,0},{1,0,1,0}}`
- (3) `int [][] a = {{0,1,1,0},{1,0,0,1},{1,0,0,0},{0,0,1,0}}`
- (4) None of the above.

iv. Consider the following directed graph:



How many paths are there from 0 to 2?

v. Consider the following weighted directed graph

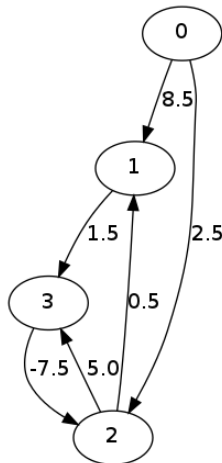


Is there a shortest path (in terms of distance) from 0 to 1?

vi. Consider the following Java program

```
import java.util.HashSet;
class it1
{
    static HashSet <Integer> f (double [][] graph, int i)
    {
        HashSet <Integer> K= new HashSet <Integer>();
        for (int j=0;j<graph.length;j++)
            if (graph[i][j]>0) K.add(j);
        return K;
    }
}
```

If a was the adjacency matrix representing the graph:



What would the output of executing the statement

`System.out.println(f(a,2))?`

[12]

(b) What is the purpose of Dijkstra's Algorithm? Give an example where it could be used in practice.

[5]

(c) Describe Dijkstra's Algorithm either in pseudo-code or Java.

[8]