2.2 More about Functions

Notebook:	Discrete Mathematics [CM1020]		
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Cornell Notes	**Topic:** 2.2 More about functions	Course: BSc Computer Science Class: Discrete Mathematics-Lecture Date: October 28, 2019

Essential Question:

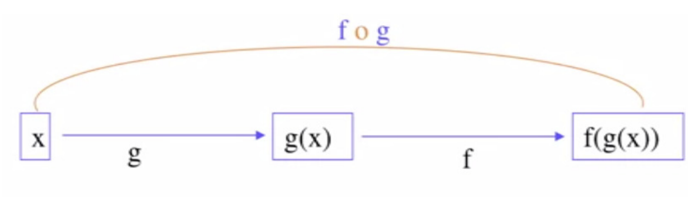
What is function composition, also what does it mean for a function to be bijective(invertible)? Alongside this, what are logarithmic, floor and ceiling functions?

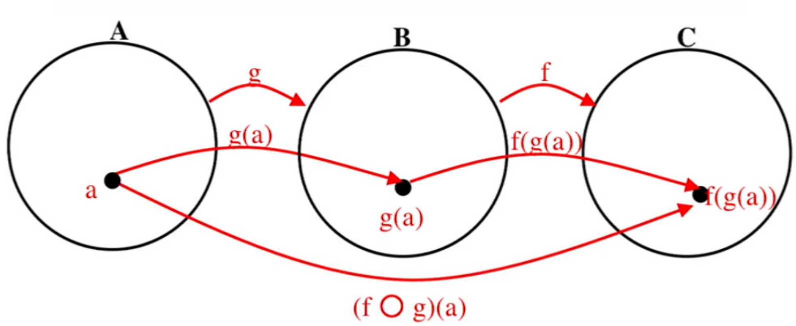
Questions/Cues:

- What is function composition?
- What does it mean when a function is bijective or invertible?
- What is the identity function in terms of function composition?
- What can be said about the graphs of f and its inverse?
- What is logarithmic function and what is its inverse?
- What is the laws of logarithms?
- What is the graph of the logarithmic functions and what are some of it's properties?
- What is the floor function and its respective graph?
- What is the ceiling function and its respective graph?

Notes

- Given 2 functions f and g, $(f\ o\ g)(x) = f(g(x))$





- Function composition is not commutative! $f \circ g \neq g \circ f$
    - If we change order of f and g, we get different function
- Bijective or Invertible = if and only if it's both injective and surjective
    - Injective = one-to-one, for every x there is unique image in the co-domain
    - Surjective = onto, unique pre-image for each element in co-domain, and range = co-domain
- Inverse function = Let $f: A \to B$, if f is bijective (invertible), then inverse function f^{-1} exists and defined $f^{-1}: B \to A$
- $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$

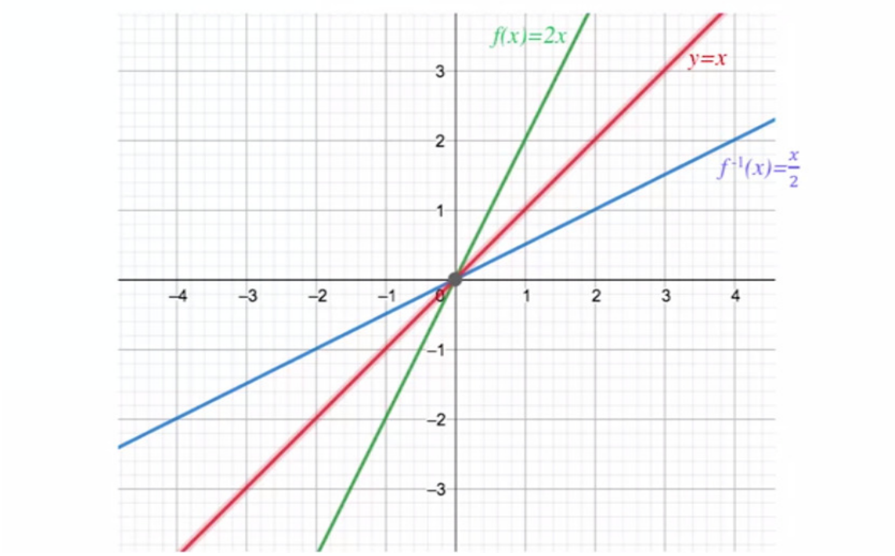
$$f: R \to R \quad \text{with } f(x)=2x$$

$$f^{-1}: R \to R \quad \text{with } f^{-1}(x)=\frac{x}{2}$$

$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f(\tfrac{x}{2}) = 2\,\frac{x}{2} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(2x) = \frac{2x}{2} = x$$

The curves of f and f^{-1} *are symmetric with respect to the straight line* $y = x$.



- Logarithmic function = with base b, b > 0 and b $\neq$ 1 is defined:

$$\log_b x = y \quad \underline{\text{if and only if}} \quad x = b^y$$

$\log_b x$ is the inverse function of the exponential function b^x

$$\log_b m * n = \log_b m + \log_b n$$

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

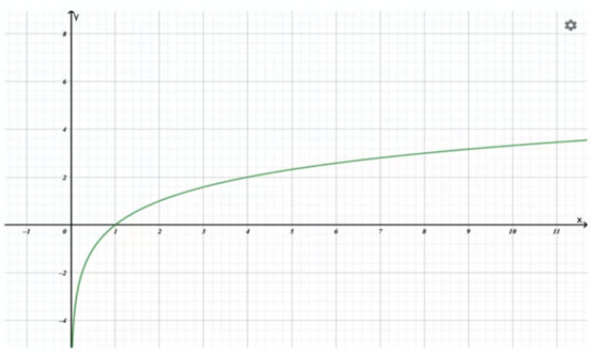
$$\log_b m^n = \text{n}\log_b m$$

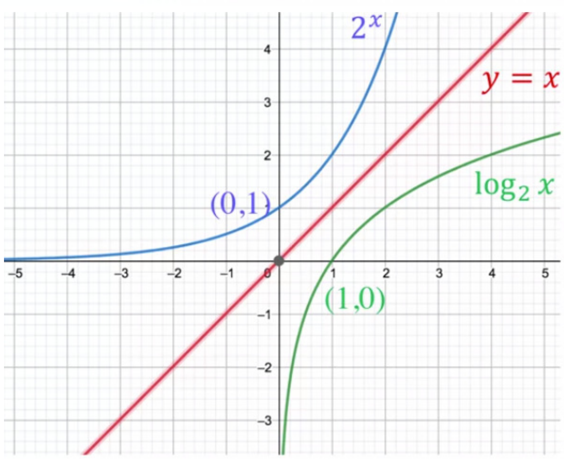
$$\log_b 1 = 0$$

$$\log_b b = 1$$

Consider $\mathbf{f(x) = \log_2 x}$. We will create a table of values for **x** and f(x) and then sketch a graph of **f**.

x	1/8	1/4	1/2	1	2	4	8
$f(x)=\log_2 x$	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**





Log properties:

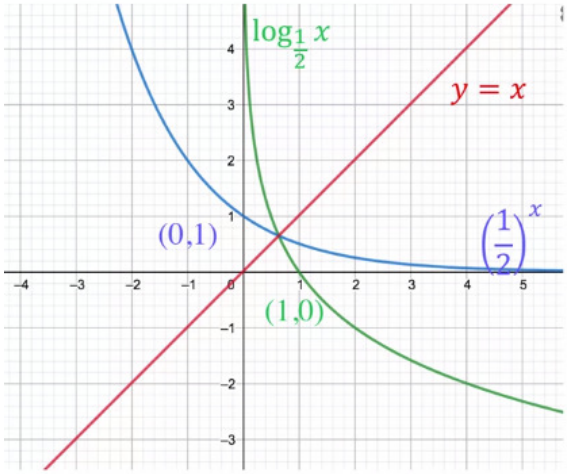
Graph of $\log_2 x$ is symmetric to 2^x with respect to **y=x**

Domain: (0, ∞)

Range: (- ∞, ∞)

x-intercept: (1, 0)

Increasing on: (0, ∞)



Log Properties:

Graph of $\log_{\frac{1}{2}} x$ is symmetric to $\left(\frac{1}{2}\right)^x$ with respect to **y=x**

Domain: (0, ∞)

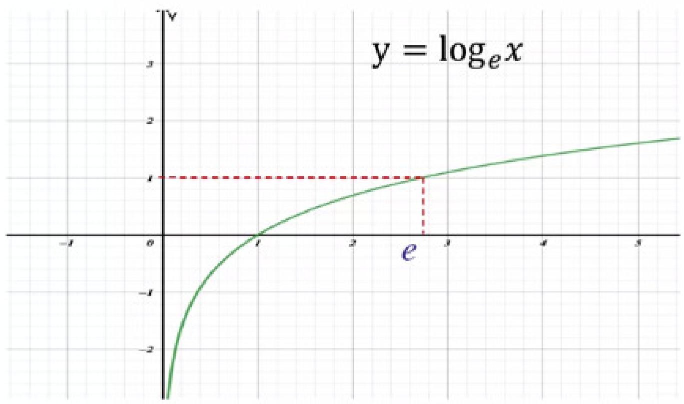
Range: (- ∞, ∞)

x-intercept: (1, 0)

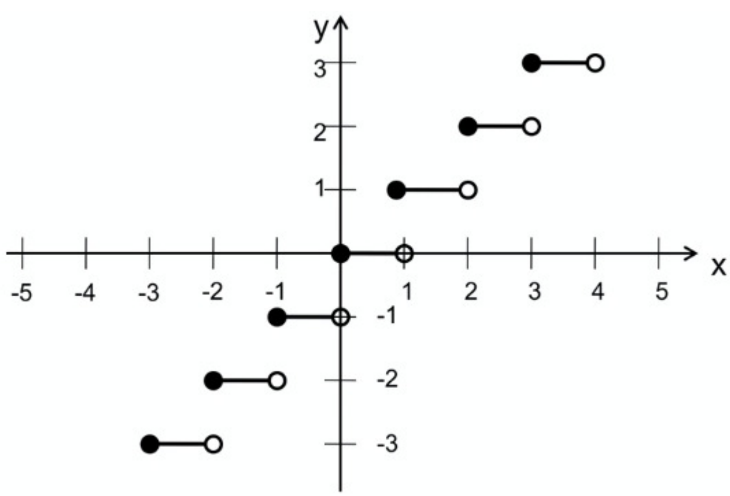
Decreasing on: (0, ∞)

$$\ln x = \log_e x \quad \text{where } e = 2.71828$$

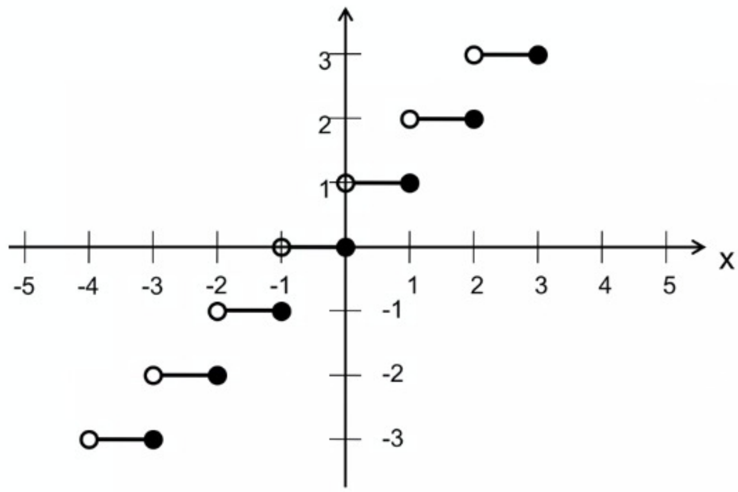
$$\ln e = \log_e e = 1$$



- Floor function = function $R \to Z$ takes real number x as input, returns largest integer that is less than or equal to x, denoted floor(x) = ⌊ x ⌋
  - Floor of any integer is itself



- Ceiling function = function $R \to Z$ takes real x as input, returns smallest integer that is greater than or equal to x, denoted ceiling(x) = ⌈ x ⌉
  - Ceiling of any integer is itself



Let n be an integer and x a real number. Show that:

$$\lfloor x+n \rfloor = \lfloor x \rfloor + n$$

Proof:

- Let $m = \lfloor x \rfloor$
- hence, $\mathbf{m \leq x < m+1}$ (by definition)
- $\mathbf{m+n \leq x+n < m+n+1}$
- this implies that $\lfloor \mathbf{x+n} \rfloor = \mathbf{m + n}$ (by definition)
- hence, $\lfloor \mathbf{x+n} \rfloor = \lfloor \mathbf{x} \rfloor + \mathbf{n}$

Summary

In this week, we learned what function composition is and what is means for a function to be bijective (invertible). Also, we looked at the logarithmic, floor and ceiling functions.