

## 2.1 Introduction to Functions

**Notebook:** Discrete Mathematics [CM1020]

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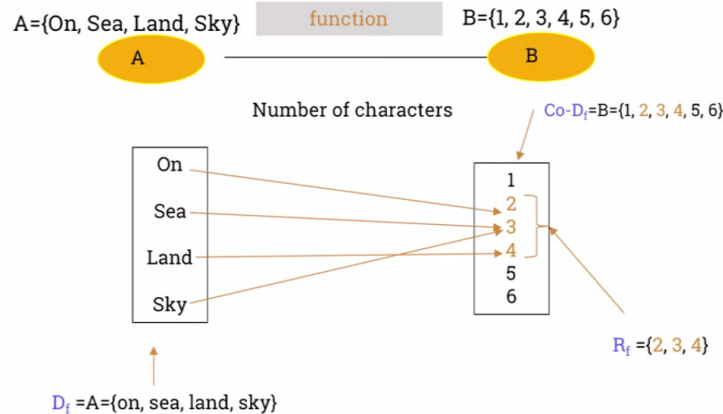
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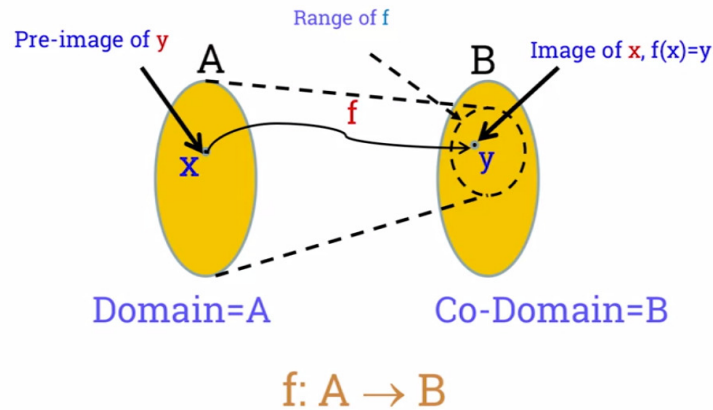
**Tags:** Domain, Exponential, Injective, Linear, Quadratic, Range, Surjective

Cornell Notes	Topic: 2.1 Introduction to functions	Course: BSc Computer Science
		Class: Discrete Mathematics- Lecture
		Date: October 21, 2019
Essential Question:		
What is a function and what are its properties and how do we graph it?		
Questions/Cues:		
<ul style="list-style-type: none"><li>• What is a function?</li><li>• What is the formal definition of a function?</li><li>• What is the domain, co-domain and range of a function?</li><li>• What is a linear function?</li><li>• What is a Quadratic function?</li><li>• What is an Exponential function?</li><li>• What is an Injective function?</li><li>• What is an Surjective function?</li></ul>		
Notes		
<ul style="list-style-type: none"><li>• Function = A rule that relates how one quantity depends on other quantities, a relation between set of inputs and set of outputs, in which each input maps to exactly one output<ul style="list-style-type: none"><li>◦ function maps an element of set to an element in another set</li></ul></li><li>• Formal Def of Function = function <math>f</math> from set <math>A</math> to set <math>B</math> is an assignment of exactly 1 element of <math>B</math> to each element of <math>A</math><ul style="list-style-type: none"><li>◦ <math>f : A \rightarrow B</math></li><li>◦ <math>x \in A : x \rightarrow f(x) = y (y \in B)</math></li></ul></li><li>• <math>x \in A \rightarrow f(x) = y \in B ; f : A \rightarrow B</math><ol style="list-style-type: none"><li>1. A is set of inputs called domain of <math>f</math>, <math>D_f = A</math></li><li>2. B is set of containing outputs called co-domain of <math>f</math>, <math>co-D_f = B</math></li><li>3. set of all outputs called range of <math>f</math>, <math>R_f</math></li><li>4. Y called image of x, x called pre-image of y, <math>f(x) = y</math></li></ol></li></ul>		

# Domain, co-domain and range



## Visualization



- Linear function = of the form  $f(x) = ax + b$ 
  - straight line function
  - passes through point (0, b)
  - a is gradient or slope
    - if a is positive, increasing function,  $x_1 \leq x_2$  then  $f(x_1) \leq f(x_2)$
    - if a is negative, then decreasing function,  $x_1 \leq x_2$  then  $f(x_1) \geq f(x_2)$
- Quadratic function = of the form  $f(x) = ax^2 + bx + c$ 
  - where a, b, c are number with  $a \neq 0$
- Exponential function = of the form  $f(x) = b^x$  where  $b > 0$  and  $b \neq 1$ 
  - variable b called the base of function
  - if  $b > 1$ , then exponential growth
  - if  $0 < b < 1$ , then exponential decay
    - (0,1) is common point for both graphs and x-axis is the horizontal asymptote for both graphs
    - Domain is set of all real numbers and range is  $> 0$

$$b^x b^y = b^{x+y}$$

$$b^{-x} = \frac{1}{b^x}$$

$$\frac{b^x}{b^y} = b^{x-y}$$

$$(b^x)^y = b^{xy}$$

$$(ab)^x = a^x b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

- Injective function (one to one) = any two different inputs lead to two different outputs
  - *for all  $a, b \in A$ , if  $a \neq b$  then  $f(a) \neq f(b)$*
  - Equivalently, *for all  $a, b \in A$ , if  $f(a) = f(b)$  then  $a = b$*
- Surjective function (onto) = every element of the co-domain of  $f$ ,  $B$  has at least one pre-image in domain of  $f$ ,  $A$ 
  - *for all  $y \in B$  there exists  $x \in A$  such that  $y = f(x)$*
  - Equivalently,  $D_f = R_f$

### Summary

In this week, we learned what is function is, with what the domain, co-domain, and range of a function represent. Also we explored the equations and graphs of three common function, Linear, Quadratic and Exponential. Lastly, we looked at the injective and surjective functions.