# Computational Mathematics

## Module description

This module gives you the mathematical foundations you need to learn how to think abstractly and introduces you to many of the standard mathematical tools and models necessary to understand and design computational systems and algorithms. By taking this module you will learn a wide range of the mathematical concepts and techniques that underpin Computer Science. In particular, you will study number systems, special functions, graphing, linear algebra and basic concept of combinatorics and probability theory.

Learners get to practice all of the above in multiple, weekly exercises and their knowledge is tested through engaging assignments and quizzes.

## Module goals and objectives

Upon successful completion of this module, you will be able to:

1. Manipulate numbers in any base and perform operations with binary numbers;
2. Handle sequence of numbers, use recursive relations, summations and iterations;
3. Manipulate integers using modular arithmetic;
4. Define and plot functions on a plane;
5. Study the behavior of a function using calculus, limits and differentiation;
6. Work in vector spaces and manipulate linear functions using matrix representation and transformations;
7. Use combinatorial techniques to represent and count sample spaces and events, and calculate relative probabilities.

## Textbook and Readings

Specific essential readings for each topic from the following list are included in the Readings page for each topic:

-Croft, A. and R. Davison - Foundation maths (Pearson) – PDF available in Coursera
https://www.dawsonera.com/abstract/9781292095196
-Song Y. Yan- Number theory for computing (Springer)
-Larson, R. - Precalculus with limits (Cengage) -
https://www.dawsonera.com/abstract/9781337516853
-Bone, G., G. Chadha, N. Saunders- A Level physics A for OCR Year 1 and AS Student Book (OUP) PDF available via Coursera
-Vince- Mathematics for computer graphics (Springer) -
https://www.dawsonera.com/abstract/9781846282836
Kuldeep Singh, Linear Algebra: Step by Step Oxford Press (2013

## Module outline

The module consists of ten topics that focus on key areas of the fundamentals of computational mathematics.

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| Topic 1. | **Key concepts:**<br>Number bases, conversion and operations<br>**Learning outcomes:**<br>Represent numbers in different bases<br>Convert from one number base to another<br>Perform basic operations with binary numbers. |

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| Topic 2. | **Key concepts:**<br><br>Sequences and series, recursion, arithmetic and geometric sequences<br><br>**Learning outcomes:**<br><br>Explain the notion of a number sequence and of convergence/divergence of a sequence<br><br>Use the recursion and the induction principle to define the elements of a sequence with application to arithmetic and geometric sequences<br><br>Introduce the notion of series and perform summation of geometric and arithmetic series |
| Topic 3. | **Key concepts:**<br><br>modular arithmetic, congruent integers<br><br>**Learning outcomes:**<br><br>Define congruence modulo an integer<br><br>Classify two integers as congruent<br><br>Perform operations with congruent numbers |

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| Topic 4. | **Key concepts:**<br>Angles, triangles and trigonometric relations<br><br>**Learning outcomes:**<br><br>Work with angles and angle units, convert between degrees and radians.<br><br>Derive and apply basic properties and trigonometric relations in triangles<br><br>Solve basic problems with triangle elements using sine and cosine rule. |
| Topic 5. | **Key concepts:**<br>Functions, Cartesian coordinates, graphs, kinematics.<br><br>**Learning outcomes:**<br><br>Define a function, its domain, codomain and type.<br><br>Locate points on the plane using Cartesian coordinates.<br><br>Plot a function in Cartesian coordinates.<br><br>Derive and plot speed and distance travelled for uniform and uniformly accelerated motion |
| Topic 6. | **Key concepts:**<br>Trigonometric functions<br><br>**Learning outcomes:** |

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| | Extend definition of Sin, Cos and Tan to any angle and derive their properties using the unit circumference in Cartesian coordinates.<br><br>Define and use inverse of trigonometric functions (Arcfunctions)<br><br>Plot trigonometric functions in Cartesian Coordinates |
| Topic 7. | **Key concepts:** Exponential and logarithmic functions<br><br>**Learning outcomes:**<br><br>Define exponential function from extension of integer powers to powers of any real number and derive its basic properties.<br><br>Define logarithm as inverse of exponential function and derive its basic properties.<br><br>Plot exponential and logarithmic functions in the Cartesian plane. |
| Topic 8. | **Key concepts:**<br>Limits and differentiation<br><br>**Learning outcomes:**<br><br>Define and calculate the limit of a function at a point<br><br>Use limits to identify asymptotic behavior of a function |

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| | Formulate and calculate the derivative of a function from first principles<br><br>Use differentiation to characterize the behavior of a function,  to individuate local minima and maxima and turning points in its plot. |
| Topic 9. | **Key concepts:**<br>Algebra, vector and matrices<br><br>**Learning outcomes:**<br><br>Define vector spaces and linear transformations<br><br>Represent linear transformations as matrices and define composition of transformations as product of matrices.<br><br>Solve a matrix equation of the form Mv=w where M is a square matrix and v and w two column vectors |
| Topic 10. | **Key concepts:**<br>Combinatorics and Probability<br><br>**Learning outcomes:** |

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| | Define combinations and permutations and learn how to apply them to counting problems<br><br>Describe the sample space of a given experiment and compute the probability P(x) of an outcome x of the experiment<br><br>Compute the joint probability of two events and determine if they are dependent or independent |

## Activities of this module

The module is comprised of the following elements:

- Lecture videos. In each week the concepts you need to know will be presented through a collection of short video lectures. You may stream these videos for playback within the browser by clicking on their titles or download the videos.
- Practice Quizzes. Topics include practice quizzes, intended for you to assess your understanding of the content. You will be allowed unlimited attempts at each practice quiz. There is no time limit on how long you take to complete each attempt at the quiz. These quizzes do not contribute toward your final score in the class.
- Graded Assignments. There are two graded assignments, each is worth 50% of the final module grade. Each of these assignments is comprised of multiple parts which learners work on during earlier weeks. All assignments will be graded by the project tutors.
- Discussion Prompt. Topics also include discussion prompts. You will see the discussion prompt alongside other items in the lesson. Each prompt provides a space for you to respond. After responding, you can see and comment on your peers' responses. All prompts and responses are also accessible from the general discussion forum and the topic discussion forum.
- Readings. Topics may include several suggested readings. They are good supplementary materials for you to further understand the course topics.

## How to pass this module

The module has two major assessments each worth 50% of your grade:

- Coursework. The coursework consists of several activities. They are detailed in the table below.
- Written examination. Details about the written examination are in the table below. Past papers will be available.

| Activity | Required? | Deadline week | Estimated time per module | % of final grade |
|---|---|---|---|---|
| End of topic quizzes Topics 1-5 | Yes | 1-10 | 1-2 hours per quiz | 25% (5x5%) |
| Written, staff graded coursework | Yes | 11 | Approximately 20 hours | 25% |
| Written examination | Yes | 22 | 2 hours 15 minutes | 50% |