Indian Institute of Technology Bombay

EE 324: Control Systems Lab

Problem Sheet 1 Report

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Contents

1 Question 1

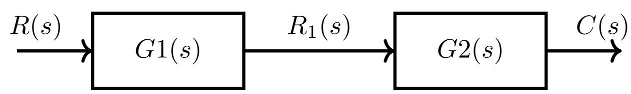
We have two components with transfer functions $G1(s)$ and $G2(s)$ as follows:

$$G1(s) = \frac{10}{s^2 + 2s + 10}$$

$$G2(s) = \frac{5}{s + 5}$$

we have to obtain the transfer functions of the following systems:

1.1 Part a - Cascade System



After the first system, the output $R_1(s)$ will be

$$R_1(s) = G1(s)\, R(s)$$

Now $R_1(s)$ will be the input to the second system $G2(s)$ and hence the output $C(s)$ will be

$$C(s) = G2(s)\, R_1(s) = G1(s)\, G2(s)\, R(s)$$

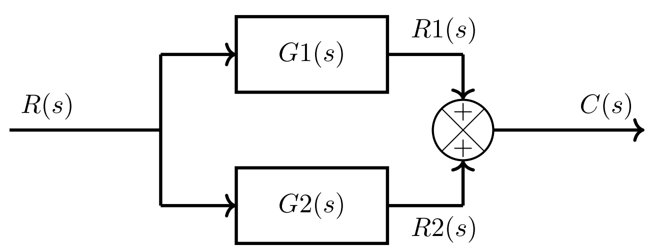
Hence the transfer function of this cascade system is

$$T(s) = G1(s)\, G2(s)$$

By using Scilab, we get

$$\boxed{T(s) = \frac{50}{50 + 20s + 7s^2 + s^3}}$$

1.2 Part b - Parallel System



In this case, the output $C(s)$ will be the sum of the outputs $R1(s)$ and $R2(s)$, i.e.,

$$C(s) = R1(s) + R2(s)$$

$$C(s) = G1(s)\, R(s) + G2(s)\, R(s)$$

$$C(s) = \big(G1(s) + G2(s)\big)\, R(s)$$

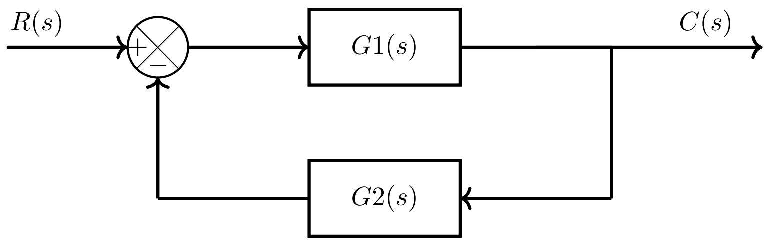
Hence the transfer function $T(s)$ will be

$$T(s) = G1(s) + G2(s)$$

By using Scilab, we get

$$\boxed{T(s) = \frac{100 + 20s + 5s^2}{50 + 20s + 7s^2 + s^3}}$$

1.3 Part c - Feedback (closed loop) System



The relation between input and the output in this case is

$$g1(t) * \big(r(t) - g2(t) * c(t)\big) = c(t)$$

$$\implies g1(t) * r(t) = c(t) + \big(g1(t) * g2(t)\big) * c(t)$$

On taking the Laplace transform of this equation, we will have

$$G1(s)\, R(s) = C(s)\big(1 + G1(s)\, G2(s)\big)$$

$$\implies C(s) = \frac{G1(s)}{1 + G1(s)\, G2(s)}\, R(s)$$

Hence, the transfer function $T(s)$ will be

$$T(s) = \frac{G1(s)}{1 + G1(s)\, G2(s)}$$

By using Scilab, we get

$$\boxed{T(s) = \frac{50 + 10s}{100 + 20s + 7s^2 + s^3}}$$

1.4 Part d - Step Response of $G1(s)$

We need to plot the step response of the system with $G1(s)$ as the transfer function. This has been done using the `csim` and the `plot` commands.

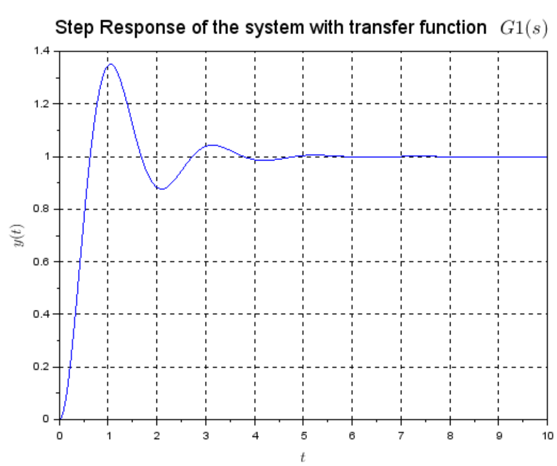


Figure 1: Unit Step Response

2 Problem 2

The poles and zeros of the system have been found using the Scilab command `tf2zp` which returns all the poles, zeros and gains. The poles and zeros were plotted using `plzr` command.

2.1 Part a - Cascade System

$$T(s) = \frac{50}{50 + 20s + 7s^2 + s^3}$$

This system has no zeros as it is evident from the transfer function.

Poles	Zeros
-5	
$-1 + 3i$	-
$-1 - 3i$	

Table 1: Poles and Zeros

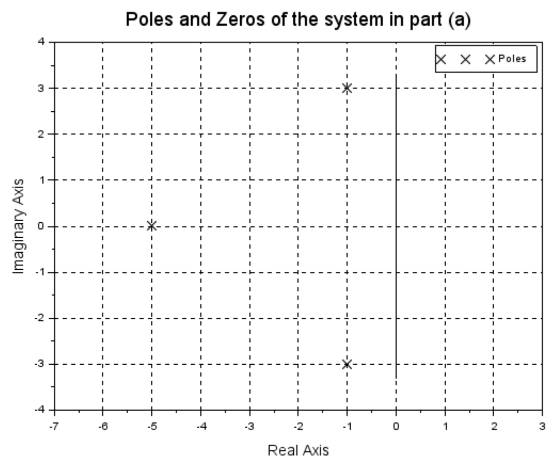


Figure 2: Poles and Zeros of Cascade System

2.2 Part b - Parallel System

$$T(s) = \frac{100 + 20s + 5s^2}{50 + 20s + 7s^2 + s^3}$$

This system will have 2 zeros and 3 poles.

Poles	Zeros
-5	
$-1 + 3i$	$-2 + 4i$
$-1 - 3i$	$-2 - 4i$

Table 2: Poles and Zeros

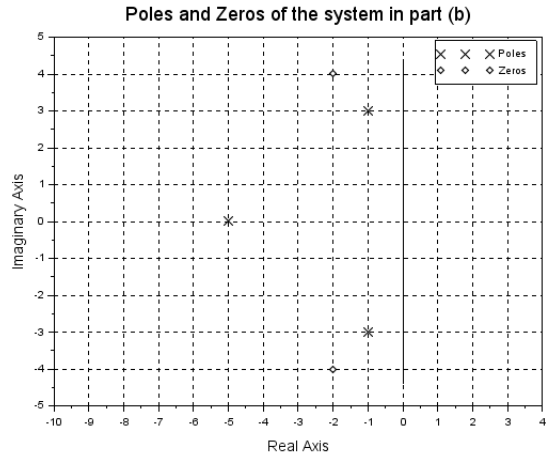


Figure 3: Poles and Zeros of Parallel System

2.3 Part c - Feedback (closed loop) System

$$T(s) = \frac{50 + 10s}{100 + 20s + 7s^2 + s^3}$$

This system will have 1 zero and 3 poles.

Poles	Zeros
-6.3347665	-5
$-0.3326167 + 3.9592004i$	
$-0.3326167 - 3.9592004i$	

Table 3: Poles and Zeros

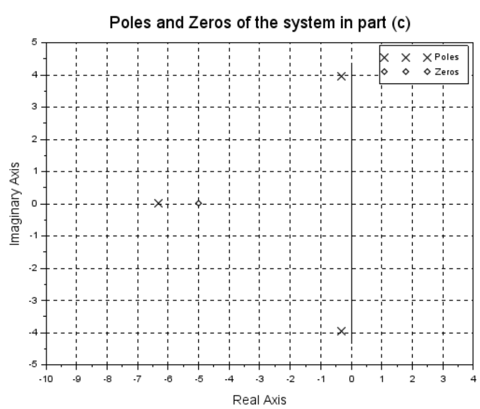


Figure 4: Poles and Zeros of Closed Loop Feedback System

3 Some computations on matrices using Scilab

Consider the following matrix:

$$A = \begin{bmatrix} s & \frac{1}{s} & \frac{s+1}{s-1} \\ 1 & s^3 & 0 \\ 1+s^2 & 2s & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & s & 9 \\ -1 & 0 & 0 \end{bmatrix}$$

We then have

$$A+B = \begin{bmatrix} 1+s & \frac{1+2s}{s} & \frac{2s}{s-1} \\ 2 & s+s^3 & 9 \\ s^2 & 2s & 2 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} \frac{-1-2s^2+s^3}{-s+s^2} & 1+2s & \frac{-9+10s+s^3}{-s+s^2} \\ 1+s^3 & 2+s^4 & 1+9s^3 \\ 2s+s^2 & 2+4s^2 & 2+18s+s^2 \end{bmatrix}$$

$$\det(A) = \frac{-1+3.14\times 10^{-16}s-2s^2-4s^3-3s^4-s^5-s^6}{s}$$

4 Problem 3

The circuit given is as follows:

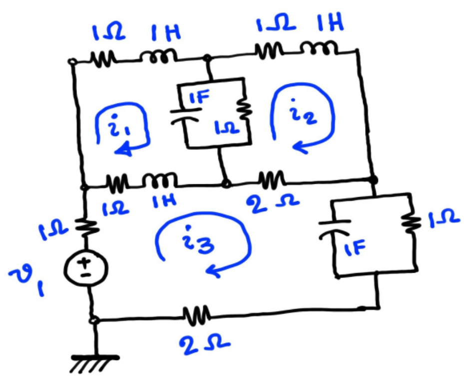


Figure 5: Original Circuit

After converting our analysis to the Laplace domain and reducing all the parallel R-C pair, we will have:

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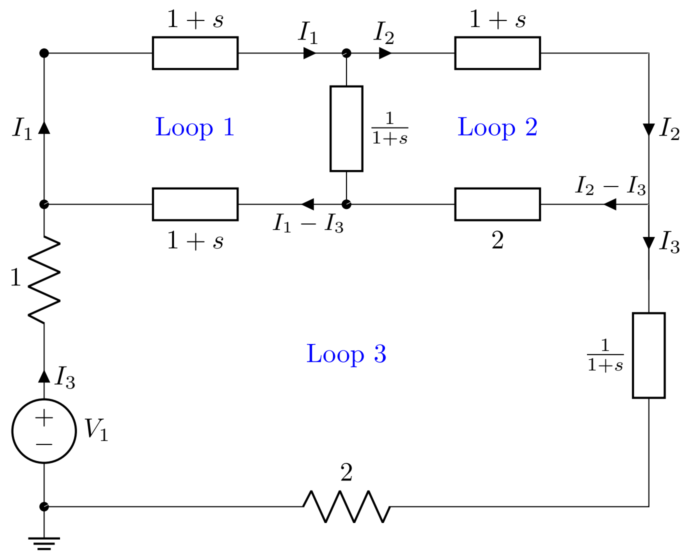

If we use the Kirchhoff's Voltage Law in each of the three loops, we will get -
For Loop 1:

$$I_1\left[1+2\,(s+1)^2\right]+I_2[-1]+I_3\left[-\,(s+1)^2\right]=0 \tag{1}$$

For Loop 2:

$$I_1+I_2[-(s+2)^2]+I_3[2(1+s)]=0 \tag{2}$$

For Loop 3:

$$I_1[-(s+1)]+I_2[-2]+I_3\left[s+6+\frac{1}{s+1}\right]=V_1 \tag{3}$$

By representing the above three equations in matrix form, we will have

$$\begin{bmatrix} 1+2\,(s+1)^2 & -1 & -(s+1)^2 \\ -1 & (s+2)^2 & -2(s+1) \\ -(s+1)^2 & -2(s+1) & s^2+7s+7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_1(s+1) \end{bmatrix}$$

On comparing the above equation with $Z(s)\,I(s)=V(s)$ we have

$$Z(s)=\begin{bmatrix} 1+2\,(s+1)^2 & -1 & -(s+1)^2 \\ -1 & (s+2)^2 & -2(s+1) \\ -(s+1)^2 & -2(s+1) & s^2+7s+7 \end{bmatrix}$$

and

$$V(s)=\begin{bmatrix} 0 \\ 0 \\ V_1(s+1) \end{bmatrix}$$

On solving the above matrix equation for $I(s)$ using Scilab, we get

$$I(s)=Z^{-1}(s)\,V(s)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = V_1 \times \begin{bmatrix} \frac{6+14s+13s^2+6s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \\ \frac{7+16s+13s^2+4s^3}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \\ \frac{11+28s+27s^2+12s^3+2s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \end{bmatrix}$$

Hence, the transfer functions are as follows:

$$\boxed{\frac{I_1(s)}{V_1(s)} = \frac{6 + 14s + 13s^2 + 6s^3 + s^4}{57 + 144s + 147s^2 + 74s^3 + 17s^4 + s^5}}$$

$$\boxed{\frac{I_2(s)}{V_1(s)} = \frac{7 + 16s + 13s^2 + 4s^3}{57 + 144s + 147s^2 + 74s^3 + 17s^4 + s^5}}$$

$$\boxed{\frac{I_3(s)}{V_1(s)} = \frac{11 + 28s + 27s^2 + 12s^3 + 2s^4}{57 + 144s + 147s^2 + 74s^3 + 17s^4 + s^5}}$$

5 Code

```
clear; clc;
// ————————————————————————————————————————
// Question 1 //
s = poly(0, 's');
G1 = 10 / (s^2 + 2*s + 10);
G2 = 5 / (s + 5);
S1 = syslin('c', G1);
S2 = syslin('c', G2);
// ————————————————————————————————————————
// Part a
TA = S1 * S2;
disp("Transfer Function for part a");
disp(TA);
// ————————————————————————————————————————
// Part b
TB = S1 + S2;
disp("Transfer Function for part b");
disp(TB);
// ————————————————————————————————————————
// Part c
TC = (S1) / (1 + (S1 * S2));
disp("Transfer Function for part c");
disp(TC);
// ————————————————————————————————————————
// Part d
t = 0:0.01:10;
scf();
plot(t, csim('step', t, S1));
xgrid(0);
title(["Step Response of the system with transfer function", "$G1(s)$"], 'fontsize', 4);
xlabel("$t$", 'fontsize', 3);
ylabel("$y(t)$", 'fontsize', 3)
// ————————————————————————————————————————
// Question 2 //
// Part a
[za, pa, ga] = tf2zp(TA);
disp('Zeros of Part a');
disp(za);
disp('Poles of Part a');
disp(pa);
```

```
scf();
plzr(TA);
xgrid(0);
title("Poles and Zeros of the system in part (a)", 'fontsize', 4);
xlabel("Real Axis",'fontsize', 3);
ylabel("Imaginary Axis",'fontsize', 3);
// ---------------------------------------------
// Part b
[zb, pb, gb] = tf2zp(TB);
disp('Zeros of Part b');
disp(zb);
disp('Poles of Part b');
disp(pb);
scf();
plzr(TB);
xgrid(0);
title("Poles and Zeros of the system in part (b)", 'fontsize', 4);
xlabel("Real Axis",'fontsize', 3);
ylabel("Imaginary Axis",'fontsize', 3);
// ---------------------------------------------
// Part c
[zc, pc, gc] = tf2zp(TC);
disp('Zeros of Part c');
disp(zc);
disp('Poles of Part c');
disp(pc);
scf();
plzr(TC);
xgrid(0);
title("Poles and Zeros of the system in part (c)", 'fontsize', 4);
xlabel("Real Axis",'fontsize', 3);
ylabel("Imaginary Axis",'fontsize', 3);
// ---------------------------------------------
// Matrices Task //
A = [s 1/s (s+1)/(s-1); 1 s^3 0; 1+s^2 2*s 1];
B = [1 2 1; 1 s 9; -1 0 1];
disp('A+B');
disp(A+B);
disp('A x B');
disp(A * B);
disp("det(A)");
disp(det(A));
// ---------------------------------------------
// Question 3 //
Z = [1+(2*(s+1)^2) -1 -(s+1)^2; -1 (s+2)^2 -2*(s+1); -(s+1)^2 -2*(s+1) (s^2+7*s+7)];
V = [0 0 (1+s)];
T = V * inv(Z);
disp(T);
```