Indian Institute of Technology Bombay

EE 324: Control Systems Lab

Problem Sheet 10 Report

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Contents

1 Question 1

The following matrices were taken:

$$T = \begin{bmatrix} 3 & -5 & 2 \\ 1 & -8 & 7 \\ -3 & -6 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -3 & -8 \\ 0 & 5 & 3 \\ -3 & -5 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 3 & 6 \end{bmatrix}$$

Using these the transfer function

$$G(s) = \frac{1195 - 508s + 34s^2 + 5s^3}{103 - 27s - 3s^2 + s^3}$$

After the linear transformations,

$$T^{-1}AT = \begin{bmatrix} 5.6240602 & 1.7067669 & -0.0676692 \\ -4.7443609 & -27.240602 & 16.406015 \\ -6.7969925 & -39.661654 & 24.616541 \end{bmatrix}$$

$$T^{-1}B = \begin{bmatrix} -0.7218045 \\ -0.6691729 \\ -0.0902256 \end{bmatrix}$$

$$CT = \begin{bmatrix} -12 & -65 & 35 \end{bmatrix}$$

The new transfer function, which we can see is exactly the same!

$$G1(s) = \frac{1195 - 508s + 34s^2 + 5s^3}{103 - 27s - 3s^2 + s^3} = G(s)$$

The eigen values of the system matrix A and the poles of $G(s)$ are found using scilab as follows:

```
The eigen values of the system matrix A are:

 -5.4411649
  4.2205824 + 1.0566238i
  4.2205824 - 1.0566238i

The poles of the system G(s) are:

 -5.4411649
  4.2205824 + 1.0566238i
  4.2205824 - 1.0566238i
```

Using the following proper transfer function

$$G(s) = \frac{(s+5)(s+2)}{(s+3)(s+4)}$$

we will find that the value of $D = 1$. For the following strictly proper transfer function

$$G(s) = \frac{s+5}{(s+3)(s+4)}$$

we will find that the value of $D = 0$. The following code has been used to get the results:

```
clc; clear all;
s = poly(0, 's');
T = [3, -5, 2;
    1, -8, 7;
    -3, -6, 2];
A = [2, -3, -8;
    0, 5, 3;
    -3, -5, -4];
B = [1; 4; 6];
C = [1, 3, 6];
D = 5;
I = eye(3, 3);
G = syslin('c', A, B, C, D);
G_tf = ss2tf(G);
disp(G_tf);
AT = pinv(T) * A * T;
BT = pinv(T) * B;
CT = C * T;
GT = syslin('c', AT, BT, CT, D);
GT_tf = ss2tf(GT);
disp(GT_tf);
eig_val = spec(A);
disp("The eigen values of the system matrix A are: ");
disp(eig_val);
[z, p, k] = ss2zp(G);
disp("The poles of the system G(s) are: ");
disp(p);
g = (s+5) * (s+2) / ((s+3) * (s+4));
G = syslin('c', g);
M = tf2ss(G);
D = M(5);
disp("The value of D for a proper transfer function G(s) is: ");
disp(D);
g1 = (s+5) / ((s+3) * (s+4));
G1 = syslin('c', g1);
M1 = tf2ss(G1);
D1 = M1(5);
disp("The value of D for a strictly proper transfer function G(s) is: ");
disp(D1);
```

2 Question 2

The transfer given is

$$G(s) = \frac{s+3}{s^2+5s+4}$$

The state space realization is as follows:

```
A =

 -1.5384615   0.3076923
  4.3076923  -3.4615385

B =

 -1.1094004
  1.6641006

C =

 -0.9013878   0.

D =

  0.

E =

  1.   0.
  0.   1.
```

Figure 1: State Space realization for $G(s)$

For the transfer function

$$G_1(s) = \frac{s+1}{s^2+5s+4}$$

the state space realization is as follows (after using a similarity transformation):

```
A =

 -4.   0.
  0.  -4.

B =

  1.
  0.

C =

  1.   0.

D =

  0.

E =

  1.   0.
  0.   1.
```

Figure 2: State Space realization for $G_1(s)$

Here, the symbols mean:

$$\begin{aligned} E\dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

The above results were obtained using the following code:

```
clc; clear all;
s = poly(0, 's');
g = (s+3) / (s^2 + 5*s + 4);
G = syslin('c', g);
M = tf2des(G);
disp("A = ");
disp(M("A"));
disp("B = ");
disp(M("B"));
disp("C = ");
disp(M("C"));
disp("D = ");
disp(M("D"));
disp("E = ");
disp(M("E"));
g1 = (s+1) / (s^2 + 5*s + 4);
G1 = syslin('c', g1);
M1 = tf2des(G1);
T = eye(2,2);
disp("A = ");
disp(pinv(T) * M1("A") * T);
disp("B = ");
disp(pinv(T) * [M1("B"); 0]);
disp("C = ");
disp([M1("C"), 0] * T);
disp("D = ");
disp(M1("D"));
disp("E = ");
disp(M1("E") * T);
```

3 Question 3

The following matrices were taken:

```
A =

  1.   0.   0.
  0.   2.   0.
  0.   0.   3.

B =

  1.
  4.
  6.

C =

  2.   3.   5.
```

The eigen values of A and poles of $G(s)$ are as follows:

```
Eigen Values of A are:

  1.
  2.
  3.

Poles of G(s) are:

  3.
  2.
  1.
```

Now we take B such that one of its entry is 0.

```
 A =

   1.   0.   0.
   0.   2.   0.
   0.   0.   3.

 B =

   1.
   0.
   6.

 C =

   2.   3.   5.
```

The eigen values of A and poles of $G(s)$ are as follows:

```
Eigen Values of A are:

  1.
  2.
  3.

Poles of G(s) are:

  3.
  1.
```

Now we take C such that one of its entry is 0.

Please turn over

```
A =

  1.   0.   0.
  0.   2.   0.
  0.   0.   3.

B =

  1.
  4.
  6.

C =

  2.   3.   0.
```

The eigen values of A and poles of $G(s)$ are as follows:

```
Eigen Values of A are:

  1.
  2.
  3.

Poles of G(s) are:

  2.
  1.
```

All the above results were generated using the following code:

```
clc; clear all;
s = poly(0, 's');
//----------------------------------------
disp("No zero entry in B");
A = diag([1, 2, 3], 0);
B = [1; 4; 6];
C = [2, 3, 5];
D = 5;
disp("A = ");
disp(A);
disp("B = ");
disp(B);
disp("C = ");
disp(C);
eig_val = spec(A);
disp("Eigen Values of A are: ");
disp(eig_val);
G = syslin('c', A, B, C, D);
G = ss2tf(G);
[z, p, k] = tf2zp(G);
disp("Poles of G(s) are: ");
disp(p);
//----------------------------------------
disp("2nd entry in B is zero");
A = diag([1, 2, 3], 0);
B1 = [1; 0; 6];
C = [2, 3, 5];
D = 5;
disp("A = ");
```

```
disp(A);
disp("B = ");
disp(B1);
disp("C = ");
disp(C);
eig_val = spec(A);
disp("Eigen Values of A are: ");
disp(eig_val);
G1 = syslin('c', A, B1, C, D);
G1 = ss2tf(G1);
[z, p1, k] = tf2zp(G1);
disp("Poles of G(s) are: ");
disp(p1);
//-------------------------------------
disp("3rd entry in C is zero");
A = diag([1, 2, 3], 0);
B = [1; 4; 6];
C1 = [2, 3, 0];
D = 5;
disp("A = ");
disp(A);
disp("B = ");
disp(B);
disp("C = ");
disp(C1);
eig_val = spec(A);
disp("Eigen Values of A are: ");
disp(eig_val);
G2 = syslin('c', A, B, C1, D);
G2 = ss2tf(G2);
[z, p2, k] = tf2zp(G2);
disp("Poles of G(s) are: ");
disp(p2);
```

4 Question 4

The following examples have been taken:

```
Distinct diagonal entries in A

A =

 -4.  -3.   0.
  0.   5.   3.
  0.   0.   2.

B =

  1.
  4.
  6.

C =

  2.   3.   5.

Eigen Values of A are:

 -4.
  5.
  2.

Poles of G(s) are:

 -4.
  5.
  2.
```

As we can see, when the diagonal entries are repeated, then even for non-zero entries of B and C, pole/zero cancellation can happen The following code was used:

```
clc; clear all;
s = poly(0, 's');
disp("Distinct diagonal entries in A");
A = triu([-4, -3, 0;
    0, 5, 3;
    -3, -5, 2]);
B = [1; 4; 6];
C = [2, 3, 5];
D = 5;
disp("A = ");
disp(A);
disp("B = ");
disp(B);
disp("C = ");
disp(C);
eig_val = spec(A);
disp("Eigen Values of A are: ");
disp(eig_val);
G = syslin('c', A, B, C, D);
G = ss2tf(G);
[z, p, k] = tf2zp(G);
disp("Poles of G(s) are: ");
disp(p);
//--------------------------------
disp("The first two diagonal elements are equal");
A1 = triu([5, -3, 0;
```

```
The first two diagonal elements are equal

A =

  5.  -3.   0.
  0.   5.   3.
  0.   0.   2.

B =

  1.
  4.
  6.

C =

  2.   3.   5.

Eigen Values of A are:

  5.
  5.
  2.

Poles of G(s) are:

  5. + 0.0000003i
  5. - 0.0000003i
```

```
    0, 5, 3;
    -3, -5, 2]);
B = [1; 4; 6];
C = [2, 3, 5];
D = 9;
disp("A = ");
disp(A1);
disp("B = ");
disp(B);
disp("C = ");
disp(C);
eig_val1 = spec(A1);
disp("Eigen Values of A are: ");
disp(eig_val1);
G1 = syslin('c', A1, B, C, D);
G1 = ss2tf(G1);
[z, p1, k] = tf2zp(G1);
disp("Poles of G(s) are: ");
disp(p1);
```