Indian Institute of Technology Bombay

EE 324: Control Systems Lab

Problem Sheet 3 Report

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Contents

1 Question 1

1.1 Part a

We have the transfer function

$$G(s) = \frac{s + 5 + a}{s^2 + 11s + 30}, a \in [-1, 1]$$

The step response of this system for various values of a is plotted using the following commands:

```
s = poly(0, 's');
A = -1:0.01:1;
scf();
for i=1:size(A, 2)
    a = A(i);
    [N, D] = simp(s + 5 + a, s^2 + 11*s + 30);
    sla = syslin('c', N, D);
    t=0:0.05:10;
    plot(t, csim('step', t, sla), color('blue'), 'LineWidth', 2);
    xlabel("$t$", 'fontsize', 3);
    ylabel("$c(t)$", 'fontsize', 3);
    title("$G(s) = \frac{s + 5 + a}{s^2 + 11s + 30}$", "fontsize", 4);
end
xs2png(gcf(), "Q1a.png");
```

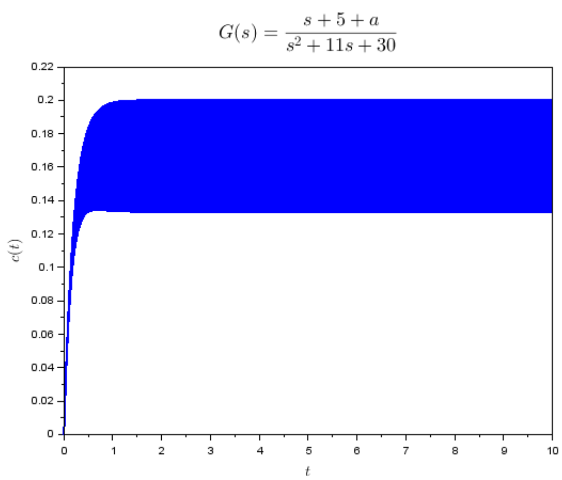

Figure 1: Step Response for various values of a

As we can see from the figure, the form of the response remains the same i.e., the response is monotonically increasing and reaches a steady value.

1.2 Part b

We have the transfer function

$$G(s) = \frac{1}{s^2 - s - 6}$$

The step response of this system is plotted using the following commands:

```
G = 1 / (s^2 - s - 6);
slb = syslin('c', G);
t=0:0.05:10;
```

```
scf();
plot(t, csim('step', t, slb), color('blue'), 'LineWidth', 2);
xlabel("$t$", 'fontsize', 3);
ylabel("$c(t)$", 'fontsize', 3);
title("$G(s) = \frac{1}{s^2 -s -6}$", "fontsize", 4);
xs2png(gcf(), "Q1b1.png");
```

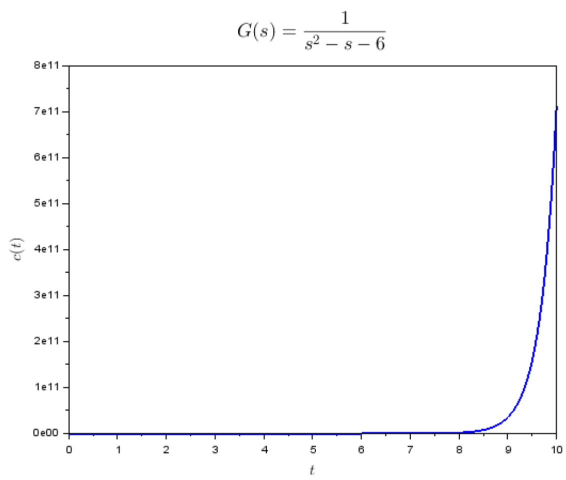

Figure 2: Step Response

As we can see this system has two poles 3 and -2. As one of the poles in the open right half plane, the step response of this system is unbounded.

$$C(s) = \frac{1}{s(s-3)(s+2)} = \frac{-1}{6s} + \frac{1}{10(s+2)} + \frac{1}{15(s-3)}$$

$$\implies c(t) = \frac{-1}{6} + \frac{1}{10}e^{-2t} + \frac{1}{15}e^{3t}$$

After adding a zero to cancel the ORHP pole, we have the new transfer function as:

$$G1(s) = \frac{1}{s+2}$$

Its step response is plotted using the following commands:

```
G1 = 1 / (s + 2);
slbb = syslin('c', G1);
scf();
plot(t, csim('step', t, slbb), color('blue'), 'LineWidth', 2);
xlabel("$t$", 'fontsize', 3);
ylabel("$c(t)$", 'fontsize', 3);
title("$G(s) = \frac{1}{s+2}$", "fontsize", 4);
xs2png(gcf(), "Q1b2.png");
```

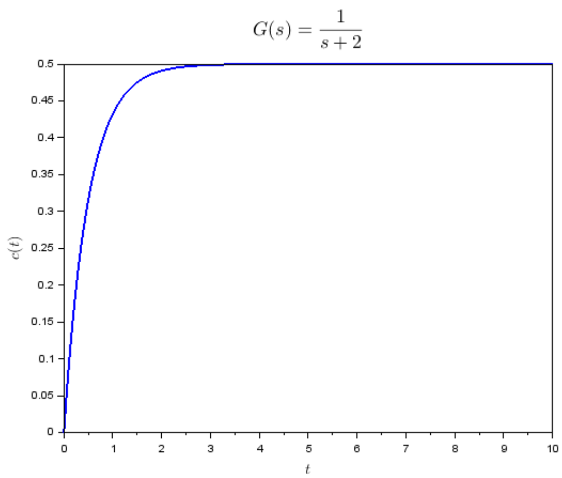

Figure 3: Step response with only OLHP poles

As we can see, the response is that of a typical first order system and it is bounded as all its poles are in OLHP.
On shifting the zero slightly by a small parameter a, and plotting the step response using the following commands we get:

```
A = -0.05:0.01:0.05;
plotcolors = ["scilabred4", "scilab blue2", "scilab green4", "scilab cyan4",...
"scilabmagenta4", "scilabbrown4", "scilabpink4", "black", "royalblue","gold",...
"dodgerblue1"];
scf();
for i=1:size(A, 2)
    a = A(i);
    [N, D] = simp(s - 3 + a, s^2 - s - 6);
    slbbb = syslin('c', N, D);
    t=0:0.01:10;
    xset("thickness",2);plot2d(t, csim('step', t, slbbb), style=[color(plotcolors(i))]);
    xlabel("$t$", 'fontsize', 3);
    ylabel("$c(t)$", 'fontsize', 3);
    title("$G(s) = \frac{s - 3 + a}{s^2 - s - 6}$", "fontsize", 4);
end
h = gca();
h.data_bounds = [0, 0; 10, 0.8];
L = legend(["$a = -0.05$","$a = -0.04$","$a = -0.03$","$a = -0.02$","$a = -0.01$",...
"$a = 0$","$a = 0.01$","$a = 0.02$","$a = 0.03$","$a = 0.04$","$a = 0.05$"]);
L.font_size = 3;
xs2png(gcf(), "Q1b3.png");
```

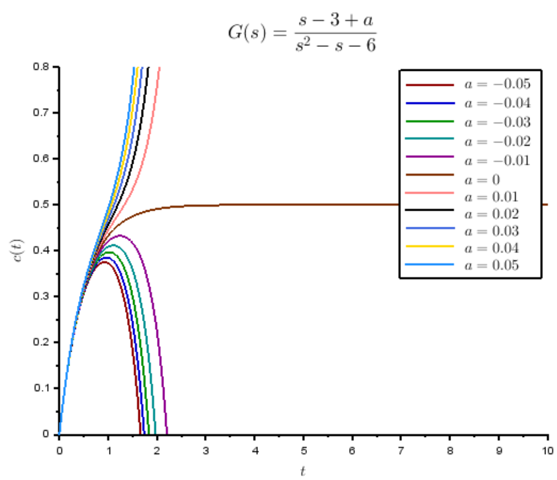

Figure 4: Step response while perturbing the zero

An unstable plant cannot be rendered stable by cancelling unstable poles by adding zeros attempting to cancel the unstable pole. This is because

- We may not know the exact location of the ORHP pole of the plant and we can see from the figure above, the response changes drastically even in slight changes in the zero attempting to cancel the pole.
- Many a times the boundedness or unboundedness of a system depends on the initial conditions of the system. Blindly cancelling out the common poles and zeros may make the system look stable even when it is actually unstable. For example, consider the following system

$$G(s) = \frac{s-1}{s^2+2s-3} = \frac{N(s)}{D(s)}$$

As $s^2+2s-3 = (s-1)(s+3)$, if we cancel out the common terms in $N(s)$ and $D(s)$, we will get

$$G1(s) = \frac{1}{s+3}$$

which looks stable. Now consider the differential equation corresponding to the system $G(s)$

$$\ddot{c} + 2\dot{c} - 3c = \dot{r} - r$$

Taking the Laplace transform of this equation by considering non-zero initial conditionals, we have

$$C(s) = \frac{s-1}{s^2+2s-3}R(s) + \frac{s+2}{s^2+2s-3}c(0) + \frac{1}{s^2+2s-3}\dot{c}(0) - \frac{1}{s^2+2s-3}r(0)$$

Assuming $\dot{c}(0) = 0 = r(0)$ and $c(0) = c_0$, the partial fraction expansion of the term $\frac{s+2}{s^2+2s-3}c_0$ will be

$$\frac{s+2}{s^2+2s-3}c_0 = \frac{c_0}{4}\left(\frac{1}{s+3} + \frac{1}{s-1}\right)$$

which will give the terms

$$\frac{c_0}{4}(e^{-3t} + e^{t})$$

which makes the response unbounded.

2 Question 2

2.1 Part a

We have the system with transfer function

$$G(s) = \frac{85}{s^3 + 7s^2 + 27s + 85}$$

This system has poles $-5, 1 \pm 4j$. The Laplace transform of the step response of this system will be

$$C(s) = \frac{85}{s(s^3 + 7s^2 + 27s + 85)}$$

Using the method of partial fraction, we will have

$$C(s) = \frac{A}{s} + \frac{B}{s+5} + \frac{C(s+1) + 4D}{s^2 + 2s + 17}$$

On solving, we get $A = 1, B = -0.53125, C = -0.46875, D = -0.78125$. As we can see the dominant poles are those that belong to the second order term. Hence, the second order approximation of this system will be

$$G1(s) = \frac{17}{s^2 + 2s + 17}$$

The step response of the system and its approximation are plotted using the following commands:

```
s = poly(0, 's');
G = 85/(s^3+7*s^2+27*s+85);
sl = syslin('c', G);
[z, p, k] = tf2zp(sl);
disp(p);
// ignoring the farthest pole
G_approx = 17 / (s^2 + 2*s + 17);
sl_approx = syslin('c', G_approx);
t = 0:0.01:10;
scf();
plot(t, csim('step', t, sl), 'r', 'LineWidth', 2);
plot(t, csim('step', t, sl_approx), 'b', 'LineWidth', 2);
L = legend(["3rd order system", "2nd order approximation"]);
L.font_size = 2;
xlabel("$t$", 'fontsize', 3);
ylabel("$c(t)$", 'fontsize', 3);
xs2png(gcf(), "Q2a.png");
```

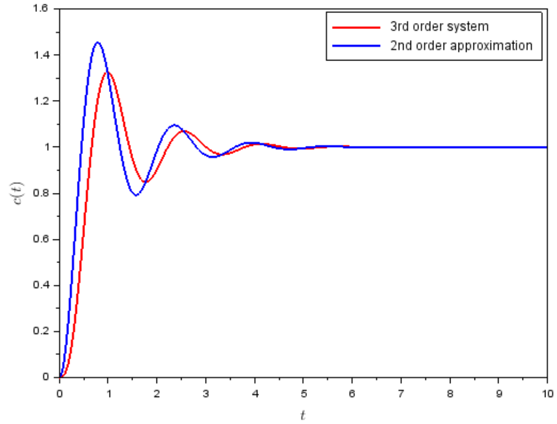

Figure 5: Step response

2.2 Part b

The transfer function of the system given is

$$G(s) = \frac{s + 0.01}{s^3 + 2.02s^2 + 5.04s + 0.1}$$

This system has poles at $-1 \pm 2j$ and at -0.02 and a zero at -0.01. As we know that the residue of a pole closer to a zero will be small, we can neglect the the pole-zero pair which are close and get a second order approximation (preserving the steady-state value) as follows:

$$G1(s) = \frac{0.5}{s^2 + 2s + 5}$$

The step response of both the original system and its approximation are plotted using the following commands:

```
Gb = (s+0.01)/(s^3+(101/50)*s^2+(126/25)*s+0.1);
slb = syslin('c', Gb);
[z, p, k] = tf2zp(slb);
disp(p);
Gbb = 0.5 / (s^2 + 2*s + 5);
slbb = syslin("c", Gbb);
t = 0:0.01:200;
scf();
plot(t, csim('step', t, slb), 'r', 'LineWidth', 2);
plot(t, csim('step', t, slbb), 'b', 'LineWidth', 2);
L = legend(["3rd order system", "2nd order approximation"]);
L.font_size = 2;
xlabel("$t$", 'fontsize', 3);
ylabel("$c(t)$", 'fontsize', 3);
xs2png(gcf(), "Q2b.png");
```

Actually, this system does not have a valid second order approximation as the Laplace transform of the step response has comparable residues:

$$C(s) = \frac{0.1}{s} + \frac{0.1007983}{s + 0.02} + \frac{-0.3995807 - 0.2007983s}{s^2 + 2s + 5}$$

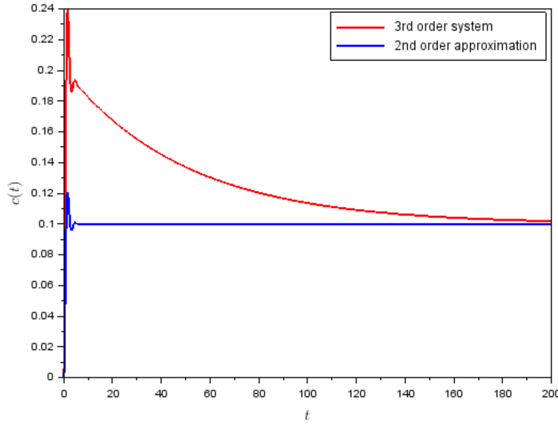

Figure 6: Step responses

3 Question 3

3.1 Part a

We have the system

$$G(s) = \frac{9}{s^2 + 2s + 9}$$

which has poles $-1 \pm j\, 2.8284271$ Now I added a zero at $s = -10$ so the new transfer function will be

$$G1(s) = \frac{9(s + 10)}{s^2 + 2s + 9}$$

The step response of both these systems is plotted using the following code:

```
s = poly(0, 's');
G = 9 / (s^2 + 2*s + 9);
sl = syslin('c', G);
[z, p, k] = tf2zp(sl);
disp("Poles");
disp(p);
G1 = (s + 10) * G;
sl_zero = syslin('c', G1);
t = 0:0.01:10;
scf();
plot(t, csim('step', t, sl), 'r', 'LineWidth', 2);
plot(t, csim('step', t, sl_zero), 'b', 'LineWidth', 2);
L = legend(["$\frac{9}{s^2 + 2s+9}$", "$\frac{9(s+10)}{s^2 + 2s+9}$"]);
L.font_size = 2;
xlabel("$t$", 'fontsize', 3);
ylabel("$c(t)$", 'fontsize', 3);
title("Step Response with and without a zero", "fontsize", 3);
xs2png(gcf(), "Q3a.png");
```

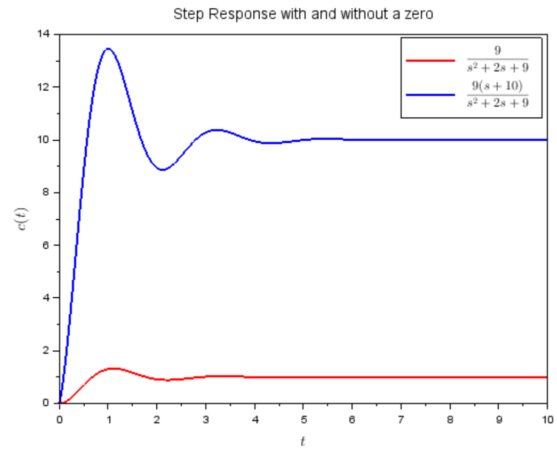

Figure 7: Step responses

The step response of the original system is

$$c(t) = 1 - e^{-t}\left(\cos(2\sqrt{2}t) + \frac{\sqrt{2}}{4}\sin(2\sqrt{2}t)\right)$$

and for that with additional zero is

$$c1(t) = 10 - 10\,e^{-t}\left(\cos(2\sqrt{2}t) + \frac{\sqrt{2}}{40}\sin(2\sqrt{2}t)\right)$$

- The rise time for original transfer function is 0.46s.
- The %OS for the original transfer function is 32.926%
- The rise time for the transfer function with an additional zero at -10 is 0.42s
- The %OS for the transfer function with an additional zero at -10 is 34.5918%

These were found using a custom function that I made:

```
function [rise_time, pos] = TrnPos(t, sl)
    outputs = csim('step', t, sl);
    ss_val = outputs(size(outputs, 2));
    peak_val = max(outputs);
    rise_time_low = 0;
    rise_time_high = 0;
    for i=1:size(outputs, 2)
        if(outputs(i) - (0.1 * ss_val)>= 5*1e-4)
            rise_time_low = t(i);
            break;
            end
    end
    for i=1:size(outputs, 2)
        if(outputs(i) - 0.9 * ss_val>= 5*1e-4)
            rise_time_high = t(i);
            break;
            end
    end
    rise_time = rise_time_high - rise_time_low;
```

```
    pos = (peak_val - ss_val) / ss_val * 100;
endfunction
```

3.2 Part b

The additional poles that I have added are -1 and -10. The corresponding transfer functions are

$$G2(s) = \frac{9}{(s+1)(s^2+2s+9)}$$

$$G3(s) = \frac{9}{(s+10)(s^2+2s+9)}$$

The step response of these transfer functions are plotted using the following commands:

```
// ------------------------------------------
// Part b
Gb = G / (s + 1);
slb = syslin('c', Gb);
Gbb = G / (s + 10);
slbb = syslin('c', Gbb);
t = 0:0.01:20;
scf();
plot(t, csim('step', t, sl), 'r', 'LineWidth', 2);
plot(t, csim('step', t, slb), 'k', 'LineWidth', 2);
plot(t, csim('step', t, slbb), 'b', 'LineWidth', 2);
L = legend(["$\frac{9}{s^2 + 2s+9}$", "$\frac{9}{(s+1)(s^2 + 2s+9)}$",...
"$\frac{9}{(s+10)(s^2 + 2s+9)}$"], 4);
L.font_size = 2;
xlabel("$t$", 'fontsize', 3);
ylabel("$c(t)$", 'fontsize', 3);
title("Step Response with and without additional poles", "fontsize", 3);
xs2png(gcf(), "Q3b.png");
```

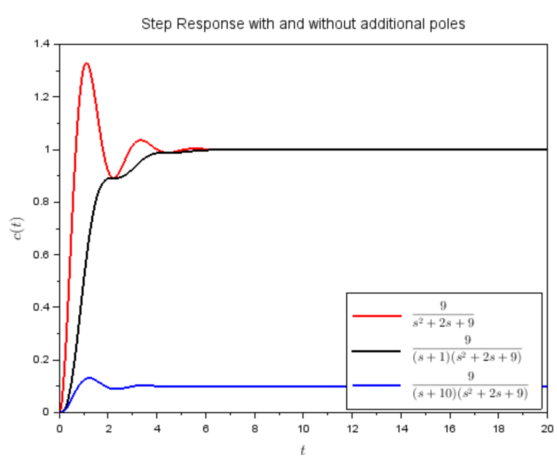

Figure 8: Step Responses with additional poles

The step response of the above two systems is

$$c2(t) = \frac{9}{8}e^{-t} - \frac{9}{8}e^{-t}\ \cos(2\sqrt{2}t)$$

$$c3(t) = \frac{9}{89}e^{-10t} - \frac{9}{89}e^{-t}\Bigg(\cos(2\sqrt{2}t) - \frac{9\sqrt{2}}{4}\sin(2\sqrt{2}t)\Bigg)$$

- The rise time of the system with an additional pole at -1 is 2.19s
- The %OS of the system with an additional pole at -1 is 0%.
- The rise time of the system with an additional pole at -10 is 0.49s
- The %OS of the system with an additional pole at -10 is 31.3447%.

These have been calculated using the same function above.

3.3 Part c

- Addition of zeros effects the amplitude of the response but the form of the response remains same.
- The closer the zero is to the dominant poles, the greater its effect on the transient response. As the zero moves away from the dominant poles, the response approaches that of the two-pole system.
- Additional poles add exponential decay terms to the step response. As the pole is moved farther from the origin in the OLHP, the exponential term cause by the pole will decay to insignificance faster and its effect on the overall step response would be negligible.
- For a system have two complex poles and a real pole, the real pole should be atleast 5 times the real part of the complex poles (dominant poles) for it to have negligible effect.
- The residue of the third pole, in a three-pole system with dominant second-order poles and no zeros, will actually decrease in magnitude as the third pole is moved farther into the left half-plane.

4 Question 4

4.1 Part a

We have the natural frequency $\omega_n = 1$ rad/s. The three cases for the undamped, underdamped and the overdamped responses that I took are

$$G1(s) = \frac{1}{1+s^2}$$

$$G2(s) = \frac{1}{s^2+s+1}, \zeta = 0.5$$

$$G3(s) = \frac{1}{s^2+4s+1}, \zeta = 2$$

The step responses of the above systems were plotted using the following commands:

```
// ——————————————————————
// Part a
s = poly(0, 's');
// undamped
G_un = 1 / (s^2 + 1);
s_un = syslin('c', G_un);
// underdamped
G_under = 1 / (s^2 + s + 1); // zeta = 0.5
s_under = syslin("c", G_under);
// overdamped
```

```
G_over = 1 / (s^2 + 4*s + 1); // zeta = 2
s_over = syslin("c", G_over);
t = 0:0.0001:20;
scf();
plot(t, csim("step", t, s_un), 'r', 'LineWidth', 2);
plot(t, csim("step", t, s_under), 'k', 'LineWidth', 2);
plot(t, csim("step", t, s_over), 'b', 'LineWidth', 2);
L = legend(["$\text{Undamped, } \frac{1}{s^2 + 1}$",...
"$\text{Underdamped, }\frac{1}{s^2 + s + 1}$", "$\text{Overdamped, }\frac{1}{s^2 + 4s + 1}$"]);
L.font_size = 2;
xlabel("$t$", 'fontsize', 3);
ylabel("$c(t)$", 'fontsize', 3);
title("Step Response of undamped, underdamped and overdamped", "fontsize", 3);
xs2png(gcf(), "Q4a.png");
```

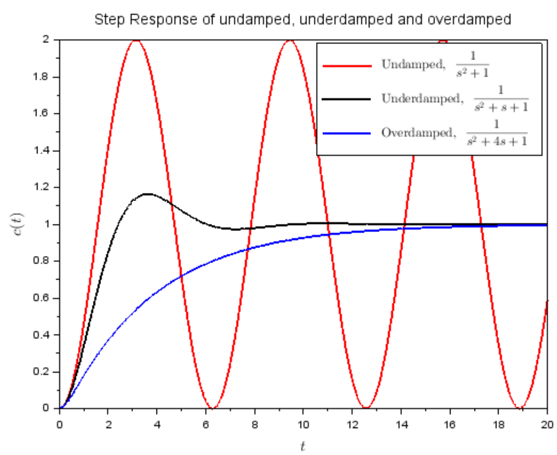

Figure 9: Step responses

System	%OS	Peak Time	Delay Time	Rise Time	2% Settling Time
Undamped	50	3.1416	1.049	1.101	-
Underdamped	16.300	3.6276	1.295	1.6376	8.1154
Overdamped	0	∞	2.8498	8.0806	14.1253

Table 1: Step Response Info

We can observe that

- The %OS decreases as ζ increases. The undamped system has the highest %OS and the overdamped system has the lowest.
- The peak time increases as ζ increases from 0 and the overdamped system has an infinite peak time.
- The delay time also increases as ζ increases from 0.
- The rise time also increases as ζ increases from 0. The undamped system has the steepest rise and the overdamped system has the slowest rise.
- The 2% settling time in undefined in the undamped case as the response will not settle. As ζ increases from 0, the settling time increases.

These values for the underdamped and overdamped case were found using a utility function that we made:

```
function [rise_time, pos, peak_t, delay_t, settling_t] = stepinfo(t, sl)
    outputs = csim('step', t, sl);
    ss_val = outputs(size(outputs, 2));
    peak_val = max(outputs);
    rise_time_low = 0;
    rise_time_high = 0;
    for i=1:size(outputs, 2)
        if(outputs(i) - (0.1 * ss_val)>= 5*1e-4)
            rise_time_low = t(i);
            break;
            end
    end
    for i=1:size(outputs, 2)
        if(outputs(i) - 0.9 * ss_val>= 5*1e-4)
            rise_time_high = t(i);
            break;
            end
    end
    rise_time = rise_time_high - rise_time_low;
    pos = (peak_val - ss_val) / ss_val * 100;
    peak_t = t(find(outputs == max(outputs))(1));
    delay_t = 0
    for i=1:size(outputs, 2)
        if(outputs(i) - 0.5 * ss_val>= 5*1e-4)
            delay_t = t(i);
            break;
            end
    end
    for i=1:size(outputs, 2)
        if(outputs(size(outputs, 2) - i + 1) - 0.98 * ss_val<= 5*1e-4)
            settling_t = t(size(outputs, 2) - i + 1);
            break;
            end
    end
endfunction
```

4.2 Part b

In this case, the natural frequency ω_n of the underdamped system has been varied from 1 rad/s to 9 rad/s in steps of 2 and the step responses were plotted using the following code:

```
wn = 1:2:9;
plotcolors = ["scilabred4", "scilab blue2", "scilab green4", "scilab cyan4",...
"scilabmagenta4"];
scf();
for i=1:size(wn, 2)
    w = wn(i);
    G = w^2 / (s^2 + w * s + w^2);
    sl = syslin("c", G);
    t = 0:0.0001:15;
    plot2d(t, csim('step', t, sl), style=[color(plotcolors(i))]);
end
```

```
L = legend(["$\omega_n = 1$", "$\omega_n = 3$", "$\omega_n = 5$", ...
"$\omega_n = 7$", "$\omega_n = 9$"], 4);
L.font_size = 2;
xlabel("$t$", 'fontsize', 3);
ylabel("$c(t)$", 'fontsize', 3);
title(["Step Response with", "$\zeta = 0.5$", "and varying", "$\omega_n$"], "fontsize", 3);
xs2png(gcf(), "Q4b.png");
```

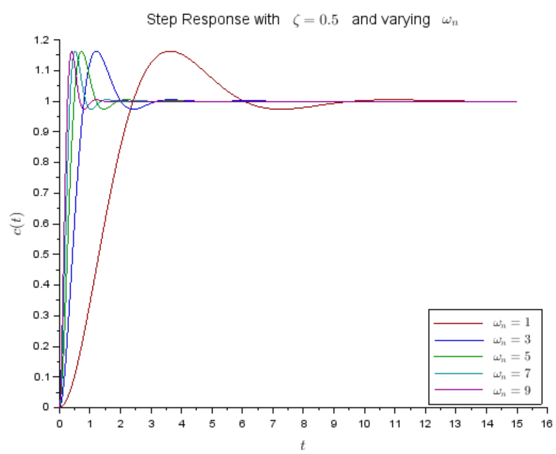

Figure 10: Step responses

$\zeta = 0.5, \omega_n$	%OS	Peak Time	Rise Time	2% Settling Time
1	16.300	3.6276	1.6376	8.1154
3	16.303	1.2092	0.5459	2.7045
5	16.303	0.7255	0.3275	1.6227
7	16.303	0.5182	0.2339	1.1590
9	16.303	0.4031	0.1820	0.9015

Table 2: Step Response Info

These were generated using the same helper function as mentioned above.

- We can observe that the %OS does not change / changes very insignificantly as ω_n is increased. This is because the %OS does not depend on ω_n for an underdamped system.

$$\%\text{OS} = e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$$

- We can see that the peak time decreases as ω_n increases. This is expected because

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

- We can see that the rise time decreases as ω_n is increased. This is because as ω_n is increased, the exponential decay term in the response decays at a faster rate.
- We can see that as ω_n is increased, the 2% settling time is decreased. This is because

$$T_s = \frac{-\ln\left(0.02\sqrt{1-\zeta^2}\right)}{\zeta\omega_n} \approx \frac{4}{\zeta\omega_n}$$