Indian Institute of Technology Bombay

EE 324: Control Systems Lab

Problem Sheet 4 Report

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Contents

1 Question 1

1.1 Part a

The system given is:

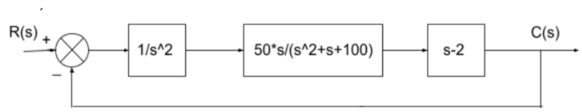

Figure 1: 1a

The transfer function is found using the following code:

```
// Part a
s = poly(0, 's');
G1 = 1/s^2;
G2 = (50 * s) / (s^2 + s + 100);
G3 = s - 2;
G4 = 1;
S1 = syslin('c', G1);
S2 = syslin('c', G2);
S3 = syslin('c', G3, 1);
S4 = syslin('c', G4, 1);
Cs = (S1 * S2 * S3)/.S4;
disp("Transfer function of part a");
disp(Cs);
disp("==================================================");
// ---------------------------------------------------------
```

The transfer function is:

$$T(s) = \frac{-100 + 50s}{-100 + 150s + s^2 + s^3}$$

1.2 Part b

The system given is:

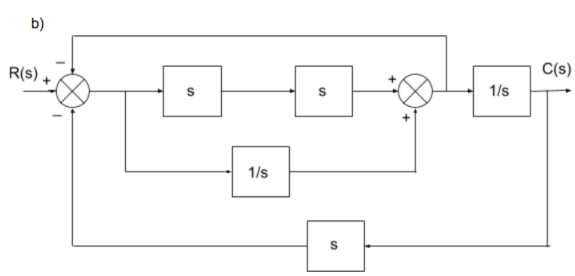

Figure 2: 1b

The transfer function is found using the following code:

```
// Part b
G1 = s;
S1 = syslin('c', G1, 1);
```

```
S2 = S1 * S1;
G2 = 1/s;
S3 = syslin('c', G2);
S4 = S2 + S3;
S5 = S4 /. syslin('c', 1, 1);
Cs = (S5 * syslin('c', G2))/. syslin('c', s, 1);
disp("Transfer function of part b");
disp(Cs);
disp("==================================================");
// ------------------------------------------------------------
```

The transfer function is:

$$T(s) = \frac{1 + s^3}{2s + s^2 + 2s^4}$$

1.3 Part c

The system given is:

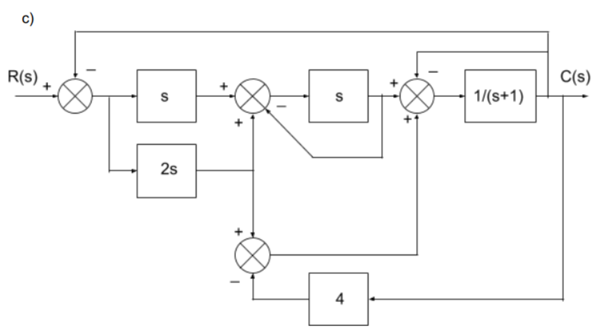

Figure 3: 1c

The transfer function is found using the following code:

```
// Part c
G1 = (3*s^2) / (1 + s) + 2 * s;
S1 = syslin('c', G1);
G2 = 1 / (s + 2);
S2 = syslin('c', G2);
S3 = S2 /. (syslin('c', 4, 1));
S4 = S3 * S1;
Cs = S4 /. syslin('c', 1, 1);
disp("Transfer function of part c");
disp(Cs);
disp("==================================================");
// ------------------------------------------------------------
```

The transfer function is:

$$T(s) = \frac{2s + 5s^2}{6 + 9s + 6s^2}$$

2 Question 2

2.1 Part a

We have a plant with transfer function

$$G(s) = \frac{10}{s(s+2)(s+4)}$$

If a proportionality gain of K is put in the forward path in series and then the feedback loop has been closed with unity negative feedback, then the transfer function of this complex system will be

$$T(s) = \frac{K}{1 + KG(s)} = \frac{10K}{10K + 8s + 6s^2 + s^3}$$

This is calculated using the code given below:

```
/ Question 2
// Part a
s = poly(0, 's');
G = 10 / (s* (s+2) * (s+4));
Gs = syslin('c', G);
K = 10;
Cs = K * Gs /. syslin('c', 1, 1);
disp("Transfer function for K = 10");
disp(Cs);
disp("==================================================");
```

2.2 Part b

The locii of the closed-loop poles has been plotted by varying K from 0 to 100 in the steps of 0.1 using the following code:

```
// Part b
K = 0:0.1:100;
scf();
for i=1:size(K, 2)
    k = K(i);
    Cs = k * Gs /. syslin('c', 1, 1);
    [z, p, _p] = tf2zp(Cs);
    plot(real(p), imag(p), 'b*', 'LineWidth', 2);
end
xlabel("Real Axis", 'fontsize', 3);
ylabel("Imaginary Axis", 'fontsize', 3);
title(["Locus of poles of", "$\frac{10K}{10K + 8s + 6s^2 + s^3}$"], "fontsize", 4);
xs2png(gcf(), "Q2b.png");
// ------------------------------
```

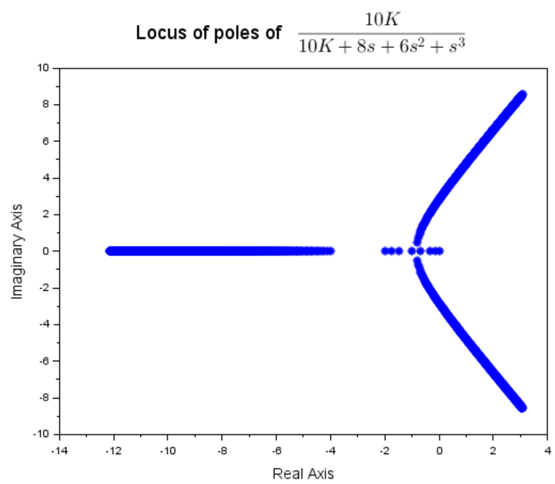

Figure 4: Locii of poles

2.3 Part c

The critical value of K is estimated by conditioning on K such that the smallest value of K for which any one pole falls in the right half plane. We get the critical value as 4.8. This is found using the following code:

```
// Part c
K_critical = -1;
for i=1:size(K, 2)
    k = K(i);
    Cs = k * Gs /. syslin('c', 1, 1);
    [z, p, _p] = tf2zp(Cs);
    rp = real(p);
    if rp(1) > 0 || rp(2) > 0 || rp(3) > 0
        K_critical = k;
        break;
    end
end
disp("Estimated Critical Value of K");
disp(K_critical);
// ------------------------------------
```

2.4 Part d

We have the denominator of the transfer function $T(s)$ as

$$D(s) = s^3 + 6s^2 + 8s + 10K$$

The R-H table will then be

If we assume $K > 0$, then in the first column, all entries except the one corresponding to s will be positive. For that remaining entry to also be positive, we will need:

$$48 - 10K > 0 \implies K < 4.8$$

If we take $K = 4.8$, then we will have a row containing all zeros at s. If we consider the row above s, we will have

$$P(s) = 6s^2 + 48$$

s^3	1	8	0
s^2	6	10K	0
s	$\frac{48-10K}{6}$	0	0
1	$10K$	0	0

Table 1: R-H Table

and its derivative will be

$$P^{'}(s) = 12s$$

so the new R-H table will be

s^3	1	8	0
s^2	6	10K	0
s	12	0	0
1	48	0	0

Table 2: R-H Table

and conce again we have no sign changes in the first column. So the critical value of K is 4.8 which matches with the estimate calculated above.

3 Question 3

3.1 Part a

We have the polynomial

$$P(s) = s^5 + 3s^4 + 5s^3 + 4s^2 + s + 3$$

its routh table is

s^5	1	5	1
s^4	3	4	3
s^3	11/3	0	0
s^2	4	3	0
s	-2.75	0	0
1	3	0	0

Table 3: Routh Table

3.2 Part b

We have the polynomial

$$P(s) = s^5 + 6s^3 + 5s^2 + 8s + 20$$

its routh table is:

s^5	1	6	8
s^4	ϵ	5	20
s^3	$\frac{-5+6\epsilon}{\epsilon}$	$\frac{-20+8\epsilon}{\epsilon}$	0
s^2	$\frac{-25+50\epsilon-8\epsilon^2}{-5+6\epsilon}$	20	0
s	$\frac{-160\epsilon-64\epsilon^2}{-25+50\epsilon-8\epsilon^2}$	0	0
1	20	0	0

Table 4: Routh Table

3.3 Part c

We have the polynomial

$$P(s) = s^5 - 2s^4 + 3s^3 - 6s^2 + 2s - 4$$

its routh table is:

s^5	1	3	2
s^4	-2	-6	-4
s^3	-8	-12	0
s^2	-3	-4	0
s	$\frac{-4}{3}$	0	0
1	-4	0	0

Table 5: Routh Table

3.4 Part d

We have the polynomial

$$P(s) = s^6 + s^5 - 6s^4 + s^2 + s - 6$$

its routh table is:

s^6	1	-6	1	-6
s^5	1	0	1	0
s^4	-6	0	-6	0
s^3	-24	0	0	0
s^2	ϵ	-6	0	0
s	$\frac{-144}{\epsilon}$	0	0	0
1	$\frac{-864}{144}$	0	0	0

Table 6: Routh Table

These tables were generated using the following code:

```
clc; clear;
//--------------------
// Question 3
s = poly(0, 's');
// Part a
Ga = s^5 + 3*s^4 + 5*s^3 + 4*s^2 + s + 3;
disp("Part a");
disp(["Routh Table of", "s^5 + 3*s^4 + 5*s^3 + 4*s^2 + s + 3"]);
disp(routh_t(Ga));
disp("===================================================");
//--------------------
// Part b
Gb = s^5 + 6*s^3 + 5*s^2 + 8*s + 20;
disp("Part b");
disp(["Routh Table of", "s^5 + 6*s^3 + 5*s^2 + 8*s + 20"]);
disp(routh_t(Gb));
disp("===================================================");
//--------------------
// Part c
Gc = s^5 - 2*s^4 + 3*s^3- 6*s^2 + 2*s - 4;
disp("Part c");
disp(["Routh Table of", "s^5 - 2*s^4 + 3*s^3- 6*s^2 + 2*s - 4"]);
disp(routh_t(Gc));
disp("===================================================");
//--------------------
// Part d
Gd = s^6 + s^5 - 6*s^4 + s^2 + s - 6;
disp("Part d");
disp(["Routh Table of", "s^6 + s^5 - 6*s^4 + s^2 + s - 6"]);
disp(routh_t(Gd));
disp("===================================================");
```

4 Question 4

4.1 Part a

We need a degree 6 polynomial which has its entire row corresponding to s^3 as zeros. This means this polynomial should have a 4th degree polynomial as its factor. Keeping this in mind, the following polynomial satisfies the above criterion:

$$P(s) = s^6 + s^5 + s^4 + s^2 + s + 1 = (s^4 + 1)(s^2 + s + 1)$$

s^6	1	1	1	1
s^5	1	0	1	0
s^4	1	0	1	0
s^3	4	0	0	0
s^2	ϵ	1	0	0
s	$\frac{-4}{\epsilon}$	0	0	0
1	1	0	0	0

Table 7: R-H Table

4.2 Part b

We need a degree 8 polynomial which has its entire row corresponding to s^3 as zeros. This means this polynomial should have a 4th degree polynomial as its factor. Keeping this in mind, the following polynomial satisfies the above criterion:

$$P(s) = s^8 + 2s^7 + 3s^6 + 2s^5 + 2s^4 + 2s^3 + 3s^2 + 2s + 1 = (s^4 + 1)(s^2 + s + 1)^2$$

Please see the next page

s^8	1	3	2	3	1
s^7	2	2	2	2	0
s^6	2	1	2	1	0
s^5	1	0	1	0	0
s^4	1	0	1	0	0
s^3	4	0	0	0	0
s^2	ϵ	1	0	0	0
s	$\frac{-4}{\epsilon}$	0	0	0	0
1	0	0	0	0	0

Table 8: R-H Table

4.3 Part c

We need a degree 6 polynomial which has its first entry in the row corresponding to s^3 as zero. If we take a generic degree 6 polynomial and find the entries in the row corresponding to s^3 and set it to zero we get

$$-(a_6a_1 - a_2a_5) = \frac{a_3}{a_5}(a_6a_3 - a_4a_5)$$

One possible solution for this equation is $a_6 = 1 = a_2, a_1 = 1 = a_5, a_3 = 0 = a_4 = a_0$ We then have the polynomial

$$P(s) = s^6 + s^5 + s^2 + s$$

s^6	1	0	1	0
s^5	1	0	1	0
s^4	5	0	1	0
s^3	ϵ	0.8	0	0
s^2	$\frac{-4}{\epsilon}$	1	0	0
s	$\frac{-3.2-\epsilon^2}{-4}$	0	0	0
1	1	0	0	0

Table 9: R-H Table

These were generated using the following code:

```
clc; clear;
//——————————————
// Question 4
s = poly(0, 's');
// Part a
Ga = s^6 + s^5 + s^4 + s^2 + s + 1;
disp("Part a");
disp(["Routh Table of", "s^6 + s^5 + s^4 + s^2 + s + 1"]);
disp(routh_t(Ga));
disp("==================================================");
//——————————————
// Part b
Gb = Ga * (s^2 + s + 1);
disp("Part b");
disp(["Routh Table of", "s^8 + 2s^7 + 3s^6 + 2s^5 + 2s^4 + 2s^3 + 3s^2 + 2s + 1"]);
disp(routh_t(Gb));
disp("==================================================");
//——————————————
// Part c
Gc = s^6 + s^5 + s^2 + s;
disp("Part c");
disp(["Routh Table of", "s^6 + s^5 + s^2 + s"]);
disp(routh_t(Gc));
disp("==================================================");
```