Indian Institute of Technology Bombay

EE 324: Control Systems Lab

Problem Sheet 9 Report

April 3, 2021

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Contents

1 Question 1

The open loop transfer function given is

$$G(s) = \frac{10}{s(0.2s+1)(0.05s+1)}$$

The Nyquist diagram of $G(s)$ is given by:

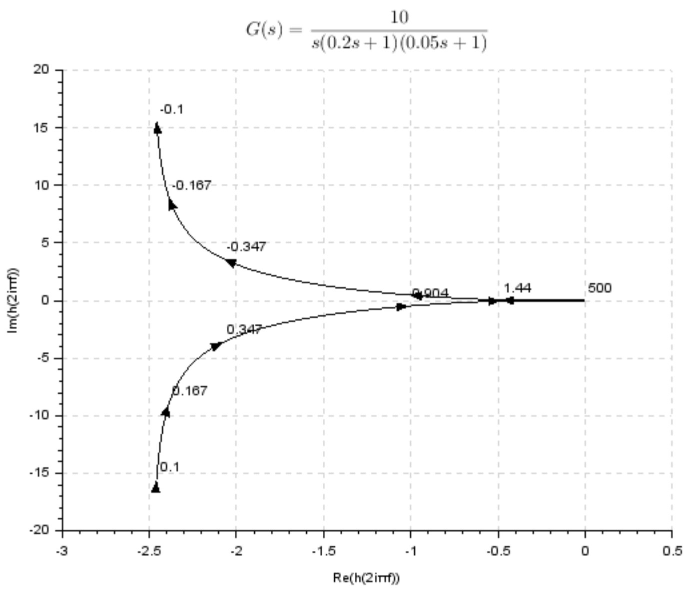

Figure 1: Nyquist diagram of $G(s)$

1.1 Part i

After adding a lag compensator, we have

$$G_1(s) = C(s)G(s) = \frac{10(s+3)}{s(0.2s+1)(0.05s+1)(s+1)}$$

The Nyquist diagram of $G_1(s)$ is given by:

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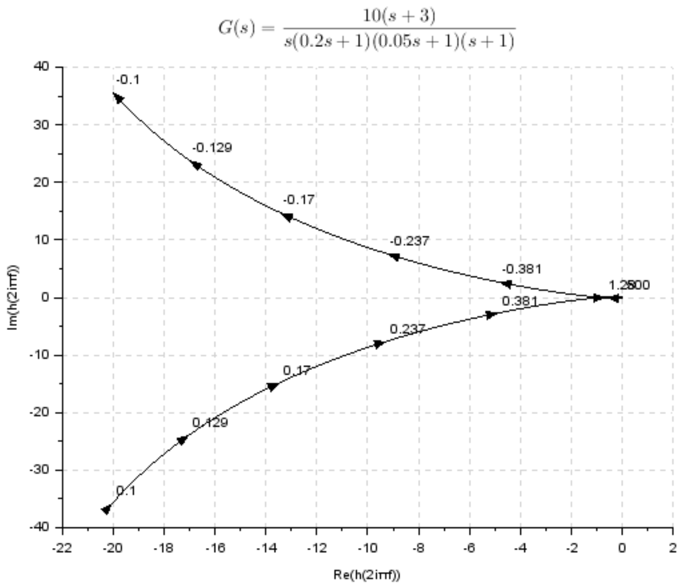

Figure 2: Nyquist diagram of $G(s)$ with lag compensator

1.2 Part ii

After adding a lead compensator, we have

$$G_2(s) = C(s)G(s) = \frac{10(s+1)}{s(0.2s+1)(0.05s+1)(s+3)}$$

The Nyquist diagram of $G_2(s)$ is given by:

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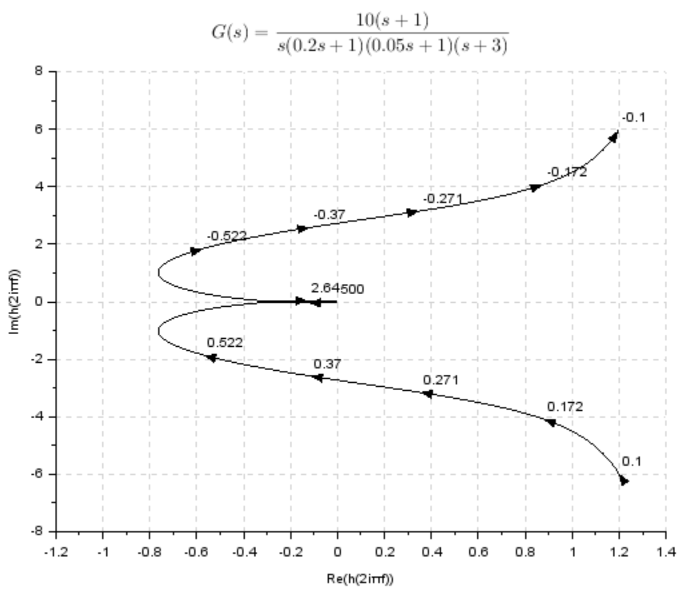

Figure 3: Nyquist diagram of $G(s)$ with lead compensator

Compensator	Gain Margin (in dB)	Phase Margin (in degrees)
Lead	11.75953880	43.17311776
-	7.95880017	22.53594159
Lag	2.07625463	4.02473321

Table 1: Changes in gain and phase margins pn adding compensators

The following code has been used to get the results:

```
clc; clear;
s = poly(0, 's');
g = (10/s) / ((1+s/5) * (1+s/20));
G = syslin('c', g);
scf();
nyquist(G, 0.1, 500);
title("$G(s) = \frac{10}{s(0.2s+1)(0.05s+1)}$", 'fontsize', 3);
disp(sprintf("The gain margin of G(s) is %.8f", g_margin(G)));
disp(sprintf("The phase margin of G(s) is %.8f", p_margin(G)));
lag_comp = (s+3) / (s+1);
g1 = g * lag_comp;
G1 = syslin('c', g1);
scf();
nyquist(G1, 0.1, 500);
title("$G(s) = \frac{10(s+3)}{s(0.2s+1)(0.05s+1)(s+1)}$", 'fontsize', 3);
disp(sprintf("The gain margin of G(s) with lag compensator is %.8f", g_margin(G1)));
disp(sprintf("The phase margin of G(s) with lag compensator is %.8f", p_margin(G1)));
lead_comp = (s+1) / (s+3);
g2 = g * lead_comp;
G2 = syslin('c', g2);
scf();
```

```
nyquist(G2, 0.1, 500);
title("$G(s) = \frac{10(s+1)}{s(0.2s+1)(0.05s+1)(s+3)}$", 'fontsize', 3);
disp(sprintf("The gain margin of G(s) with lead compensator is %.8f", g_margin(G2)));
disp(sprintf("The phase margin of G(s) with lead compensator is %.8f", p_margin(G2)));
```

2 Question 2

The transfer function of a notch filter that rejects a 50 Hz signal is given by

$$G(s) = \frac{s^2 + (100\pi)^2}{s^2 + 2s + (100\pi)^2}$$

The steepness of the magnitude plot can be controlled by varying the real part of the poles of the transfer

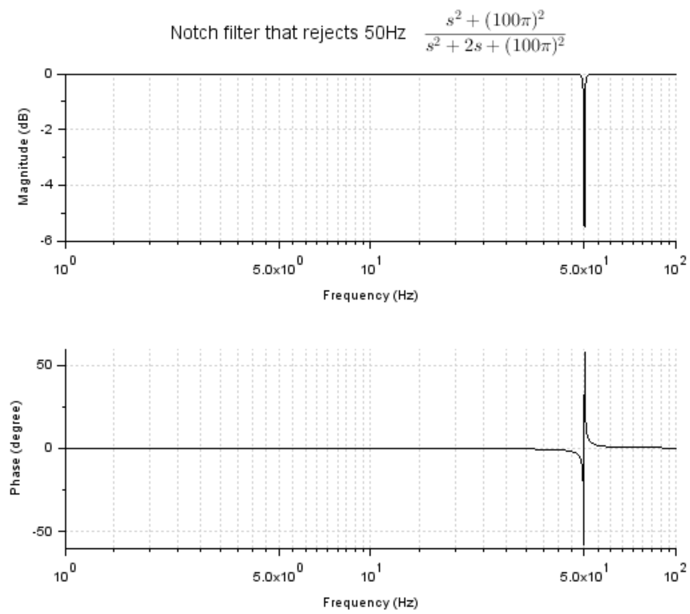

Figure 4: Notch filter that rejects 50Hz signal

function. If the real part of the poles is far away from the origin, the steepness reduces. This can be observed from the figure below.

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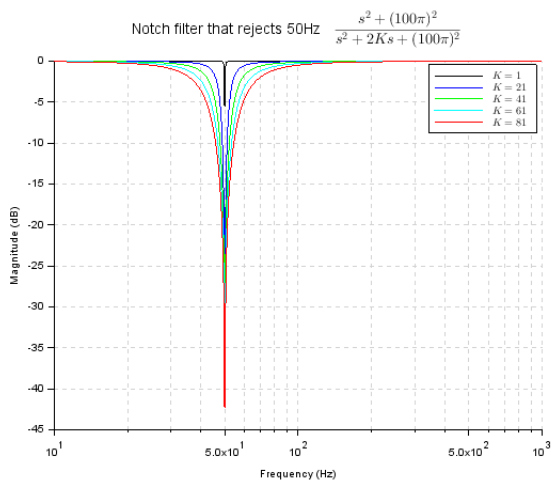

Figure 5: Impulse response transients

The above figures have been generated using the following code:

```
clc; clear;
s = poly(0, 's');
g = ((s^2+100*%pi*%pi*100)/(s^2 + 2*1*s + (100*%pi)^2));
G = syslin('c', g);
scf();
bode(G, 1, 100);
title(["Notch filter that rejects 50Hz", "$\frac{s^2 + (100\pi)^2}{s^2 + 2s + (100\pi)^2}$"],...
'fontsize', 3);
K = 1:20:100;
tfs = [];
labels = [];
for i=1:size(K, 2)
    k = K(i);
    g = ((s^2+100*%pi*%pi*100)/(s^2 + 2*k*s + (100*%pi)^2));
    G = syslin('c', g);
    tfs = [tfs; G];
    str = sprintf("$K = %d$", k);
    labels = [labels; str];
//    gainplot(G, 1, 100, 'r--');
end
scf();
gainplot(tfs, 10, 1000, labels);
title(["Notch filter that rejects 50Hz", "$\frac{s^2 + (100\pi)^2}{s^2 + 2Ks + (100\pi)^2}$"],...
'fontsize', 3);
```

3 Question 3

The transfer function that is given is

$$C(s) = \frac{100}{s+30}$$

The Nyquist diagram of this plant is as follows:

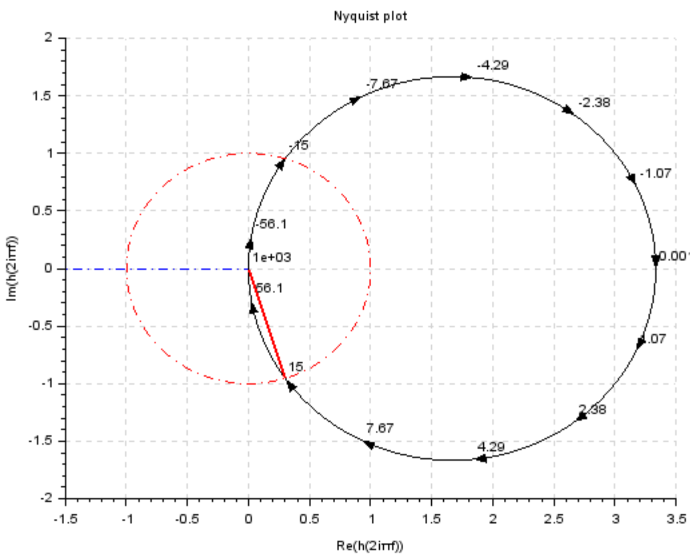

Figure 6: Nyquist diagram of $C(s)$

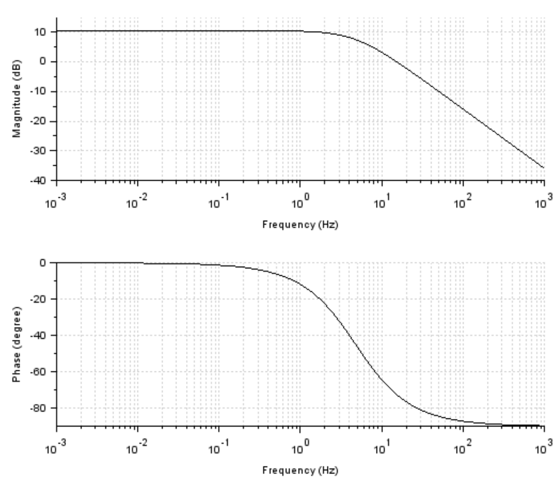

Figure 7: Bode plot of $C(s)$

From the Nyquist diagram, we can conclude that the gain margin of $C(s)$ is 0dB and the phase margin is 107.45760312° with gain crossover frequency of 15.18241393 Hz. Hence, the minimum delay required to destabilize the closed-loop system is

$$T = \frac{PM}{\omega_{gcf}} = 0.0196605s$$

Using Pade's approximation to model e^{-sT} in Scilab as

$$e^{-sT} \approx \frac{1 - 0.5sT + (1/9)s^2T^2 - (1/72)(sT)^3 + (1/1008)(sT)^4 - (1/30240)(sT)^5}{1 + 0.5sT + (1/9)s^2T^2 + (1/72)(sT)^3 + (1/1008)(sT)^4 + (1/30240)(sT)^5}$$

The Nyquist diagram of $C(s)G(s)$ where $G(s)$ is the delay is as follows:

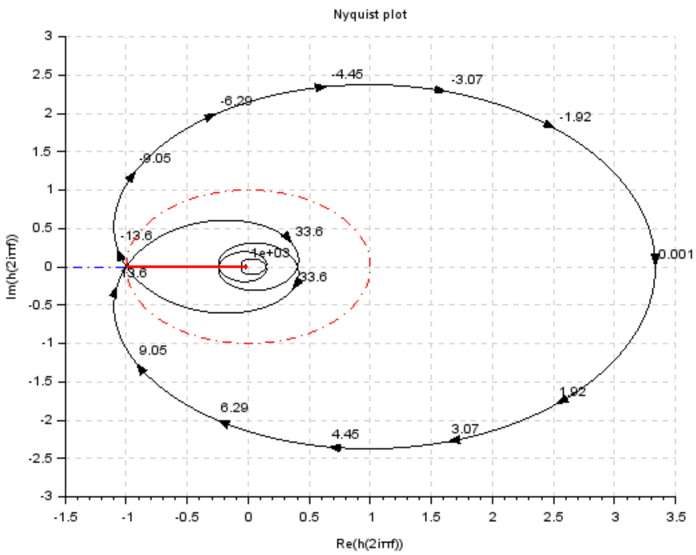

Figure 8: Nyquist diagram of $C(s)G(s)$

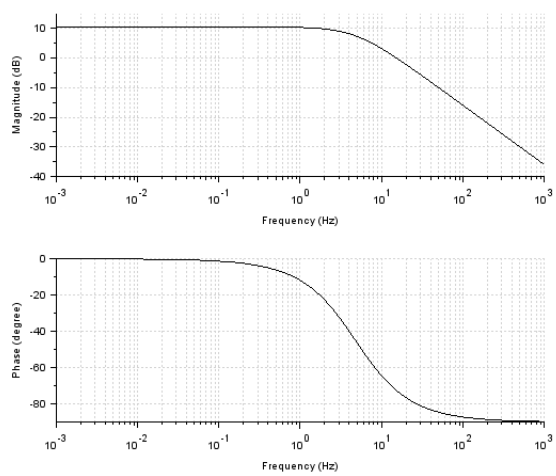

Figure 9: Bode plot of $C(s)G(s)$

The new phase margin is now 0.00000529° and the new gain margin is -0.07814458dB. This discrepency is due to the approximation made. All the above plots were generated using the following code:

```
clc; clear;
clc; clear;
s = poly(0, 's');
c = 100 / (s + 30);
C = syslin('c', c);
[phm, fm] = p_margin(C)
disp(sprintf("The gain margin of C(s) is %.8f", g_margin(C)));
disp(sprintf("The phase margin of C(s) is %.8f at frequenct %.8f Hz", phm, fm));
```

```
scf();
show_margins(C, 'nyquist');
T = phm*(%pi/180)/(2*%pi*fm);
cg = c * (1 - 0.5*s*T + (1/9)*s^2*T**2 - (1/72)*(s*T)^3 + (1/1008)*(s*T)^4 - (1/30240)*(s*T)^5)...
/ (1 + 0.5*s*T + (1/9)*s^2*T**2 + (1/72)*(s*T)^3 + (1/1008)*(s*T)^4 + (1/30240)*(s*T)^5);
CG = syslin('c', cg);
scf();
show_margins(CG, 'nyquist');
disp(sprintf("The gain margin of C(s)G(s) is %.8f", g_margin(CG)));
disp(sprintf("The phase margin of C(s)G(s) is %.8f", p_margin(CG)));
scf();
bode(C);
scf();
bode(CG);
```

4 Question 4

The open-loop system given is

$$G(s) = \frac{1}{s^3 + 3s^2 + 2s}$$

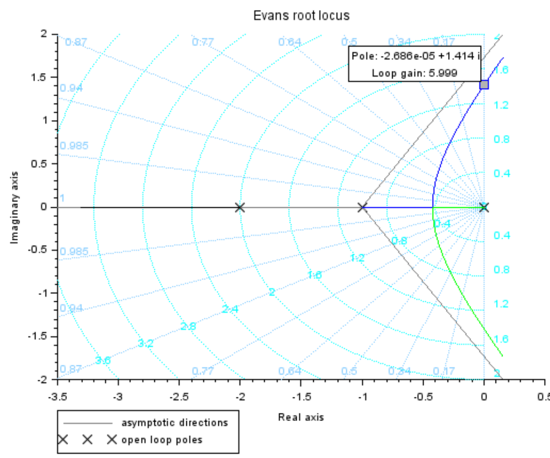

Figure 10: Root Locus

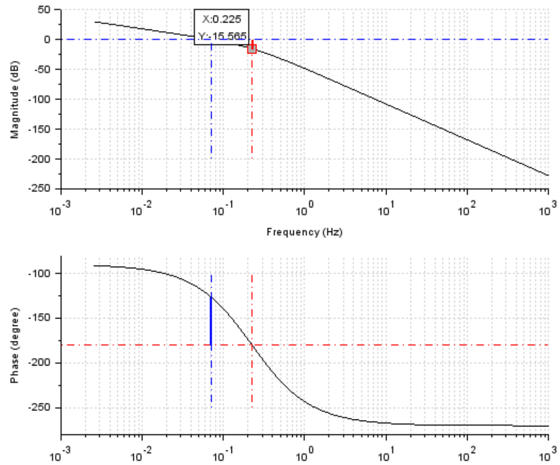

Figure 11: Bode plot

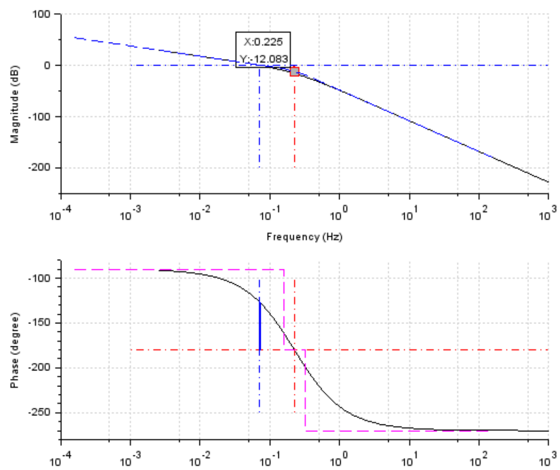

Figure 12: Asymptotic Bode plot

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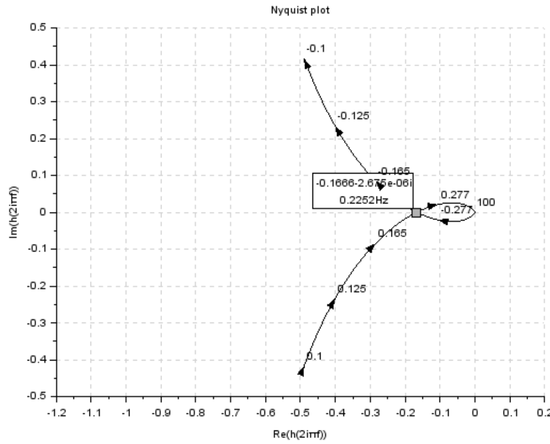

Figure 13: Nyquist diagram

Technique	Gain Margin (in dB)
Root Locus	15.5616
Actual Bode	15.565
Asymptotic Bode	12.803
Nyquist	15.5665

Table 2: Caption

The following code was used to create the plots:

```
clc; clear;
clc; clear;
s = poly(0, 's');
g = 1 / (s^3 + 3*s^2 + 2*s);
G = syslin('c', g);
Kr = kpure(G);
gmr = 20 * log(Kr) / log(10);
disp(sprintf("Gain margin using root locus = %.8f dB", gmr));
scf();
evans(G, 10);
sgrid;
scf();
show_margins(G, 'bode');
bode_asymp(G);
scf();
nyquist(G,  0.1, 100);
```

5 Question 5

The open-loop transfer function is given as follows:

$$G(s) = \frac{10s + 2000}{s^3 + 202s^2 + 490s + 18001}$$

The bode plot of this system is as follows: The gain margin is infinity and the phase margin is not defined.

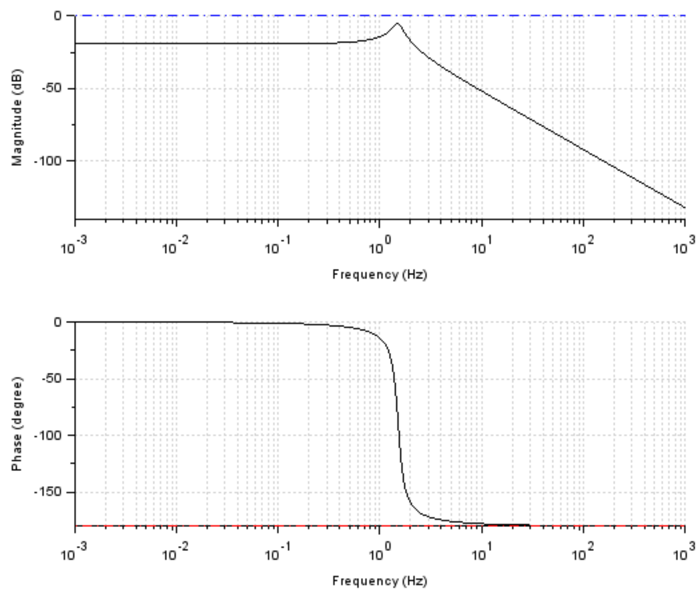

Figure 14: Bode plot

To get the steady state error as 10%, the proportional gain $K = 81.0045$. Hence,

$$G_1(s) = 81.0045 \frac{10s + 2000}{s^3 + 202s^2 + 490s + 18001}$$

The gain margin for G_1 is also infinity and the phase margin is $4.24260124°$ and the gain crossover frequency is 4.76888914 Hz. To improve the phase margin of G_1 such that the new phase margin is greater than or equal to $90°$, without altering the dc gain, we cascade G_1 with $s + 1$.

$$G_2(s) = 81.0045 \frac{(10s + 2000)(s + 1)}{s^3 + 202s^2 + 490s + 18001}$$

As we are adding a unit zero, this does not alter the dc gain. The Bode plot of G_2 is as follows:

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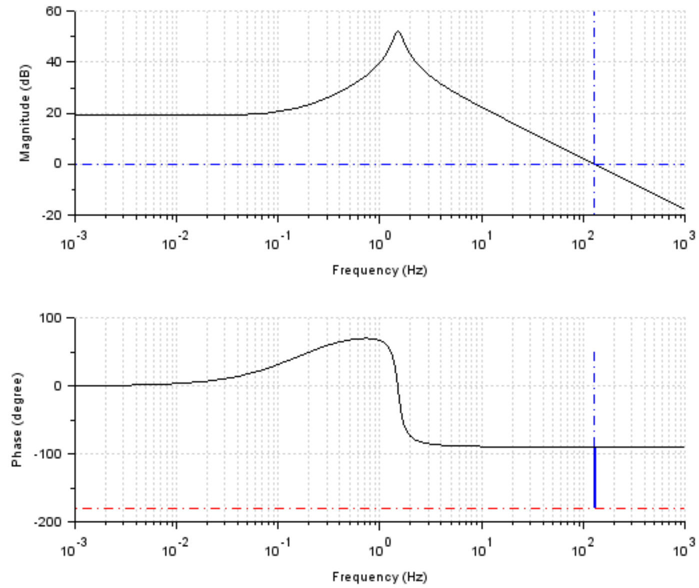

Figure 15: Bode plot

The new gain margin is 0dB, phase margin is 90.07074111° and the gain crossover frequency is 128.94005171 Hz. The closed loop poles of $G_2(s)$ are $-810.93512, -199.99999, -1.1098916$. Hence, it is closed-loop stable.