$$i_{C_1} = I_S \exp\left(\frac{v_{BE1}}{V_T}\right)$$

$$i_{C_2} = I_S \exp\left(\frac{v_{BE2}}{V_T}\right)$$
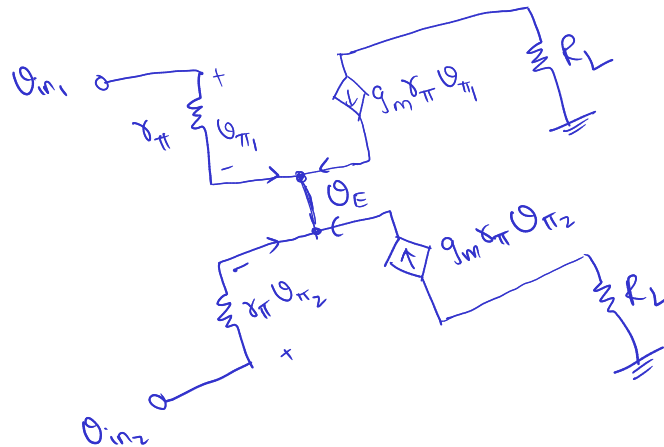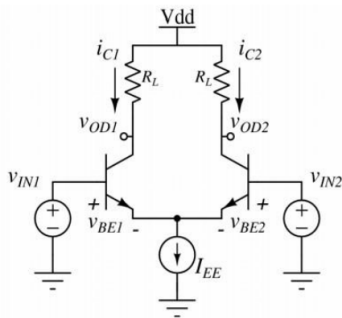
$$i_{e_1} + i_{e_2} = I_{EE}$$

$$\Rightarrow \quad i_{C_1} + i_{C_2} = \alpha I_{EE}$$

$$\Rightarrow \quad I_S = \frac{\alpha I_{EE}}{\exp\left(\frac{v_{BE1}}{V_T}\right) + \exp\left(\frac{v_{BE2}}{V_T}\right)}$$
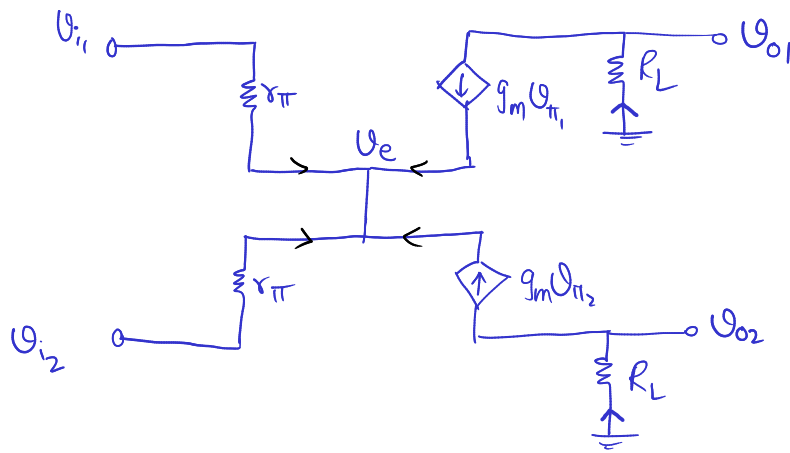
$$\Rightarrow \quad i_{C_1} = \frac{\alpha I_{EE}}{1 + \exp\left(-\frac{v_{ID}}{V_T}\right)}$$

$$i_{C_2} = \frac{\alpha I_{EE}}{1 + \exp\left(\frac{v_{ID}}{V_T}\right)}$$

$$v_E = g_m r_\pi v_{\pi_1} R_L = g_m r_\pi v_{\pi_2} R_L$$

$$\Rightarrow v_{\pi_1} = v_{\pi_2}$$

$$v_{o1} = -R_L\, g_m\, (v_{i_1} - v_e)$$

$$v_{o2} = -R_L\, g_m\, (v_{i_2} - v_e)$$

$$v_{o1} - v_{o2} = -R_L\, g_m\, (v_{i_1} - v_{i_2})$$

$\Rightarrow$

$$\Rightarrow \quad \text{differential gain} = -g_m R_L = -\frac{\alpha\, I_{EE}\, R_L}{2 V_T} \qquad \left(g_m = \frac{\alpha\, I_{EE}}{2 V_T} \right)$$

for small signals