

Q 1).

Given a random sample (x_1, x_2, \dots, x_n)

$\mu = \theta_1$ (mean) & $\sigma^2 = \theta_2$ (variance)

$$\text{Likelihood function } L(\theta_1, \theta_2) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\theta_2}} \exp\left(-\frac{(x_i - \theta_1)^2}{2\theta_2}\right)$$

To maximize take log on both sides

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^n \left[-\frac{1}{2} \ln(2\pi\theta_2) - \frac{(x_i - \theta_1)^2}{2\theta_2} \right]$$

(i) differentiate w.r.t θ_1 [for θ_1]

$$\frac{d}{d\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^n \frac{x_i - \theta_1}{\theta_2} = 0$$

$$x_i - n\theta_1 = 0$$

$$\theta_1 = \frac{\sum_{i=1}^n x_i}{n} \quad [\text{mean}]$$

(ii) differentiate w.r.t θ_2 [for θ_2]

$$\frac{d}{d\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^n \left[-\frac{1}{2\theta_2} + \frac{(x_i - \theta_1)^2}{2\theta_2^2} \right] = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^n (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^n (x_i - \theta_1)^2 \quad [\text{Variance}]$$

Q2

Binomial distribution $B(n, \theta)$ $p = \theta$ $q = 1 - \theta$

PMF (Prob mass function)

$$f(x; n, \theta) = {}^n C_x \theta^x (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^n {}^n C_{x_i} \theta^{x_i} (1-\theta)^{n-x_i}$$

$$\ln L(\theta) = \sum_{i=1}^n \left[\ln {}^n C_{x_i} + x_i \ln \theta + (n-x_i) \ln (1-\theta) \right]$$

differentiate w.r.t θ

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^n \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right] = 0$$

Find θ

$$\sum_{i=1}^n \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right] = 0$$

$$\sum_{i=1}^n \left[\frac{x_i}{\theta} - \frac{(n-x_i)\theta}{(1-\theta)} \right] = 0$$

$$\sum_{i=1}^n \left[(1-\theta)x_i - (n-x_i)\theta \right] = 0$$

$$\theta \sum_{i=1}^n x_i = \sum_{i=1}^n x_i n$$

$$\theta = \frac{\sum_{i=1}^n x_i}{n \cdot n}$$

MLE