

School of Mathematics
Thapar University, Patiala,
UMA 004 (Mathematics-II): Tutorial Sheet 03

1. The indicated function $y_1(x)$ is a solution of the given differential equation. Find the second solution $y_2(x)$.

(i) $y'' - 4y' + 4y = 0$; $y_1 = e^{2x}$ (ii) $x^2y'' - 3xy' + 4y = 0$; $y_1 = x^2 \ln x$
 (iii) $x^2y'' - 7xy' + 16y = 0$; $y_1 = x^4$ (iv) $x^2y'' - xy' + 2y = 0$; $y_1 = x \sin(\ln x)$
 (v) $(1 - x^2)y'' + 2xy' = 0$; $y_1 = 1$
 (vi) $(1 - 2x - x^2)y'' + 2(1 + x)y' - 2y = 0$; $y_1 = x + 1$

2. Find the general solution of the following second-order differential equation.

(i) $4y'' + y' = 0$ (ii) $y'' - y' - 6y = 0$ (iii) $y'' + 8y' + 16y = 0$
 (iv) $12y'' - 5y' - 2y = 0$ (v) $y'' + 9y = 0$ (vi) $y'' - 4y' + 5y = 0$

3. Solve each differential equation (by Cauchy-Euler equation):

(i) $x^2y'' - 2y = 0$ (ii) $xy'' + y' = 0$ (iii) $x^2y'' + xy' + 4y = 0$
 (iv) $x^2y'' - 3xy' - 2y = 0$ (v) $25x^2y'' + 25xy' + y = 0$ (vi) $x^2y'' + 5xy' + y = 0$
 (vii) $3x^2y'' + 6xy' + y = 0$

4. Find the general solution of the following differential equations

(i) $(x + 2)^2y'' + (x + 2)y' - y = 24x$ (ii) $(3x + 1)^2y'' + (3x + 1)y' + y = 6x$

5. Find the general solution of the following differential equations by the method of undetermined coefficients.

(i) $y'' - 3y' - 10y = 1 + x^2$ (ii) $4y'' - y = e^x + e^{3x}$ (iii) $3y'' + 2y' - y = e^{-2x} + x$
 (iv) $y'' + 3y' + 2y = \cos x + \sin x$ (v) $y'' + 16y = 16 \sin 4x$ (vi) $y'' + 2y' + 10y = e^{-x} \sin 3x$
 (vii) $y'' + 4y' + 4y = 6e^{-2x} \cos^2 x$ (viii) $y'' + y' = 10x^4 + 2$

6. Solve each differential equation by variation of parameters method.

(i) $y'' + y = \sec x$ (ii) $y'' + y = \sin x$ (iii) $y'' + y = \cos^2 x$
 (iv) $xy'' - 4xy = e^{2x}$ (v) $y'' + 3y' + 2y = \frac{1}{1+e^x}$ (vi) $y'' + 3y' + 2y = \sin e^x$
 (vii) $y'' + 2y' + y = e^{-x} \ln x$ (viii) $3y'' - 6y' + 6y = e^x \sec x$

Answers:

1. (i) $y_2 = xe^{2x}$ (ii) $y_2 = x^2$ (iii) $y_2 = x^4 \ln |x|$
 (iv) $y_2 = x \cos(\ln x)$ (v) $y_2 = x - x^3/3$ (vi) $y_2 = x^2 + x + 2$
 2. (i) $y = c_1 + c_2e^{-x/4}$ (ii) $y = c_1e^{3x} + c_2e^{-2x}$ (iii) $y = c_1e^{-4x} + c_2xe^{-4x}$
 (iv) $y = c_1e^{2x/3} + c_2e^{-x/4}$ (v) $y = c_1 \cos 3x + c_2 \sin 3x$ (vi) $y = e^{2x}(c_1 \cos x + c_2 \sin x)$
 3. (i) $y = c_1x^{-1} + c_2x^2$ (ii) $y = c_1 + c_2 \ln x$ (iii) $y = c_1 \cos(2 \ln x) + c_2 \sin(2 \ln x)$
 (iv) $y = c_1x^{2-\sqrt{6}} + c_2x^{2+\sqrt{6}}$ (v) $y = c_1 \cos(\frac{1}{5} \ln x) + c_2 \sin(\frac{1}{5} \ln x)$
 (vi) $y = c_1x^{-2} + c_2x^{-2} \ln x$ (vii) $y = x^{-1/2} \left[c_1 \cos\left(\frac{1}{6}\sqrt{3} \ln x\right) + c_2 \sin\left(\frac{1}{6}\sqrt{3} \ln x\right) \right]$
 4. (i) Set $x + 2 = z$, $y = c_1(x + 2) + \frac{c_2}{x+2} + 24(x + 2)^2 + 12[\ln(x + 2) - 6](x + 2) + 24$
 (ii) Set $3x + 1 = z$, $y = [A + B \ln(3x + 1)](3x + 1)^{1/3} + 3(x - 1)/2$
 5. (i) $y = Ae^{-2x} + Be^{5x} - (50x^2 - 30x + 69)/500$ (ii) $y = Ae^{x/2} + Be^{-x/2} + (35e^x + 3e^{3x})/105$ (iii) $y = Ae^{-x} + Be^{x/3} + (e^{-2x} - 7x - 14)/7$ (iv) $y = Ae^{-x} + Be^{-2x} + (2 \sin x - \cos x)/5$ (v) $y = A \cos 4x + B \sin 4x - 2x \cos x$ (vi) $y = e^{-x}(A \cos 3x + B \sin 3x) - (xe^{-x} \cos 3x)/6$ (vii) $y = (Ax + B)e^{-2x} + 3e^{-2x}(2x^2 - \cos 2x)/4$ (viii) $y = A + Be^{-x} + 2x^5 - 10x^4 + 40x^3 - 120x^2 + 242x$
 6. (i) $y = c_1 \cos x + c_2 \sin x + x \sin x + \cos x \ln |\cos x|$ (ii) $y = c_1 \cos x + c_2 \sin x - \frac{1}{2}x \cos x$
 (iii) $y = c_1 \cos x + c_2 \sin x + \frac{1}{2} - \frac{1}{6} \cos 2x$ (iv) $y = c_1e^{2x} + c_2e^{-2x} + \frac{1}{4} \left(e^{2x} \ln |x| - e^{-2x} \int_{x_0}^x \frac{e^{4t}}{t} dt \right)$, $x_0 > 0$
 (v) $y = c_1e^{-x} + c_2e^{-2x} + (e^{-x} + e^{-2x}) \ln(1 + e^x)$ (vi) $y = c_1e^{-2x} + c_2e^{-x} - e^{-2x} \sin e^x$
 (vii) $y = c_1e^{-x} + c_2xe^{-x} + \frac{1}{2}x^2e^{-x} \ln x - \frac{3}{4}x^2e^{-x}$ (viii) $y = c_1e^x \sin x + c_2e^x \cos x + \frac{1}{3}xe^x \sin x + \frac{1}{3}e^x \cos x \ln |\cos x|$