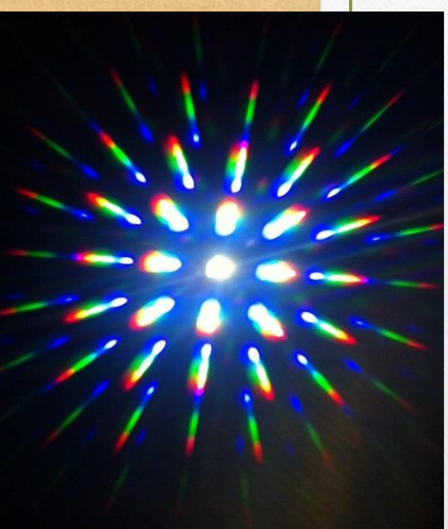
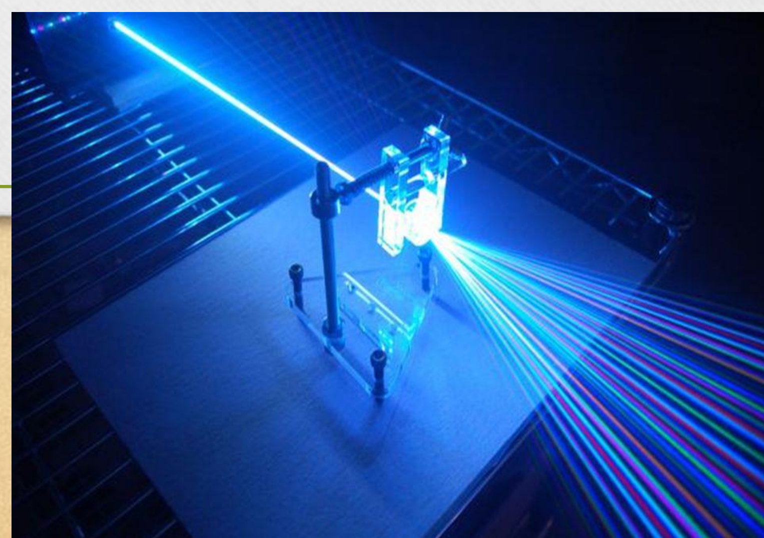
Diffraction of Light waves

Optics: ***Interference:*** Parallel and wedge-shape thin films, Newton rings, Applications as Non-reflecting coatings, Measurement of wavelength and refractive index. ***Diffraction:*** Single and Double slit diffraction, and Diffraction grating, Applications - Dispersive and Resolving Powers. ***Polarization:*** Production, detection, Applications – Anti-glare automobile headlights, Adjustable tint windows. ***Lasers:*** Basic concepts, Laser properties, Ruby, HeNe, and Semiconductor lasers, Applications – Optical communication and Optical alignment.



Topics to be covered for this tutorial

1. **Single Slit diffraction**
2. **Double slit , multiple slit**
3. **Diffraction Grating**
4. **Dispersive power of grating**
5. **Resolving power of grating**



Diffraction definition:

Bending/spreading out of waves as they pass by some objects or through a finite-width aperture.

No one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage, and there is no specific, important physical difference between them. The best we can do is, roughly speaking, is to say that when there are only a few sources, say two, interfering, then the result is usually called interference, but if there is a large number of them, it seems that the word diffraction is more often used.

Huygens's principle:

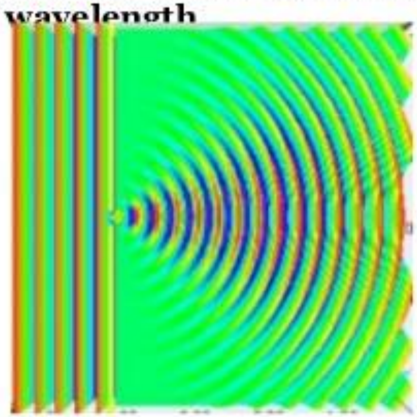
Every unobstructed point on a wavefront will act a source of secondary spherical waves which spread out in the forward direction. The new wavefront is the surface tangent to all the secondary spherical waves.

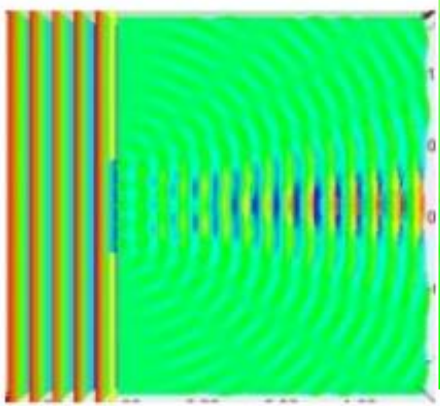
How Light Bends Around an Object

- Light bends around an object by diffraction, obviously; but what does that mean?
- When an object interferes with the passage of light, the waves will pass through a slit in a objects. Then they will spread out in a certain way on the other side of the object.

Examples

- **Slit size equal to wavelength** **Slit size 5x the wavelength**

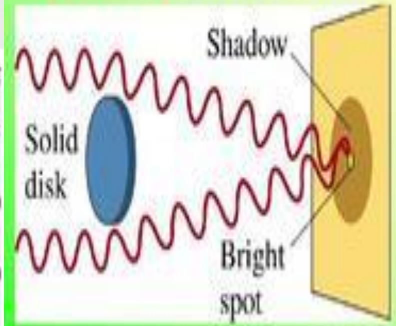




31-1. Diffraction of Light (P703)

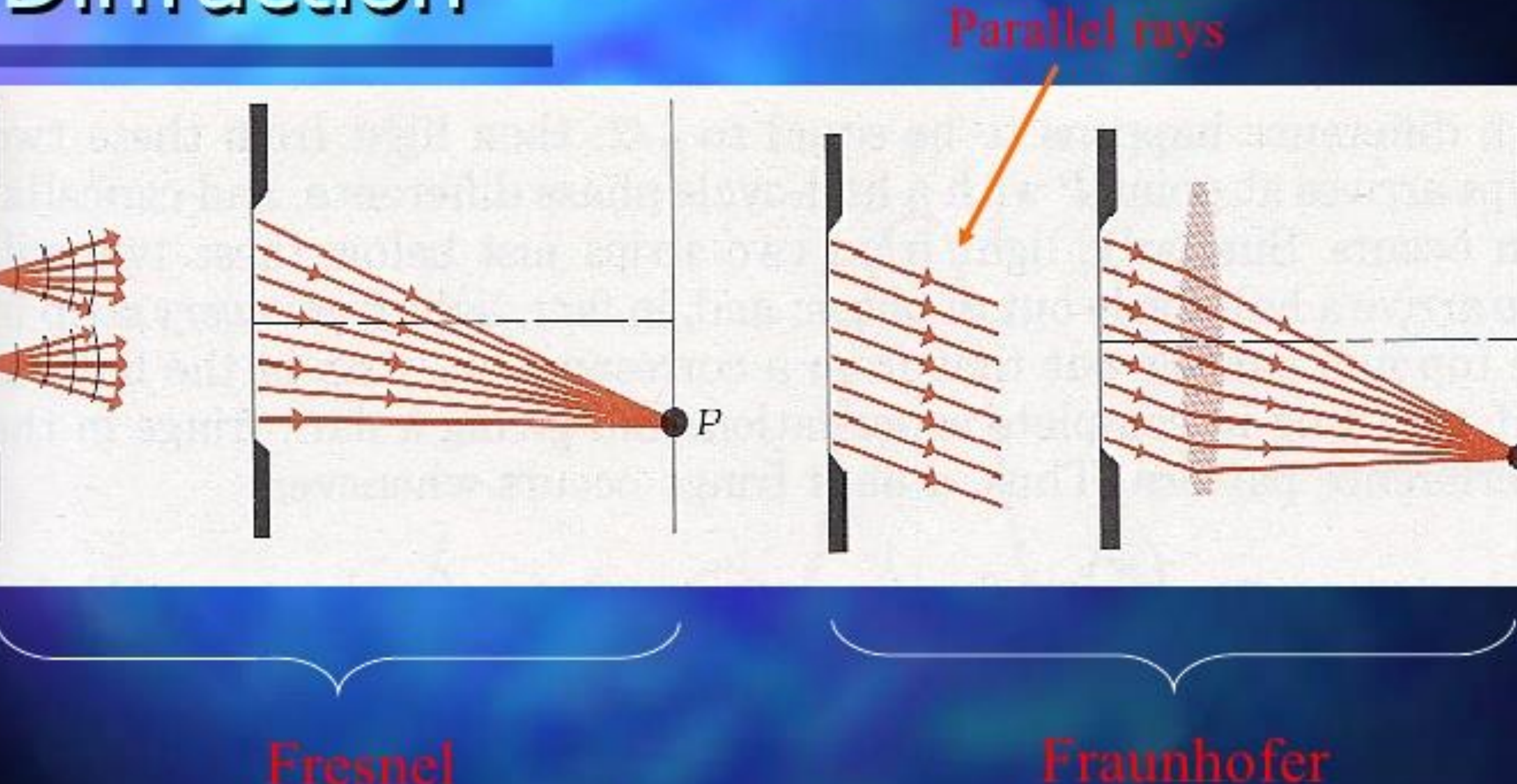
1. Diffraction :

When light encounters an obstacle, it spreads out and bend into the geometric shadow. The diffraction pattern is formed on the screen.



Diffraction, like interference, characterizes the wave nature of light.

Fresnel and Fraunhofer Diffraction



- Relation of Fresnel diffraction to Fraunhofer diffraction by a single slit

I. FRESNAL DIFFRACTIO

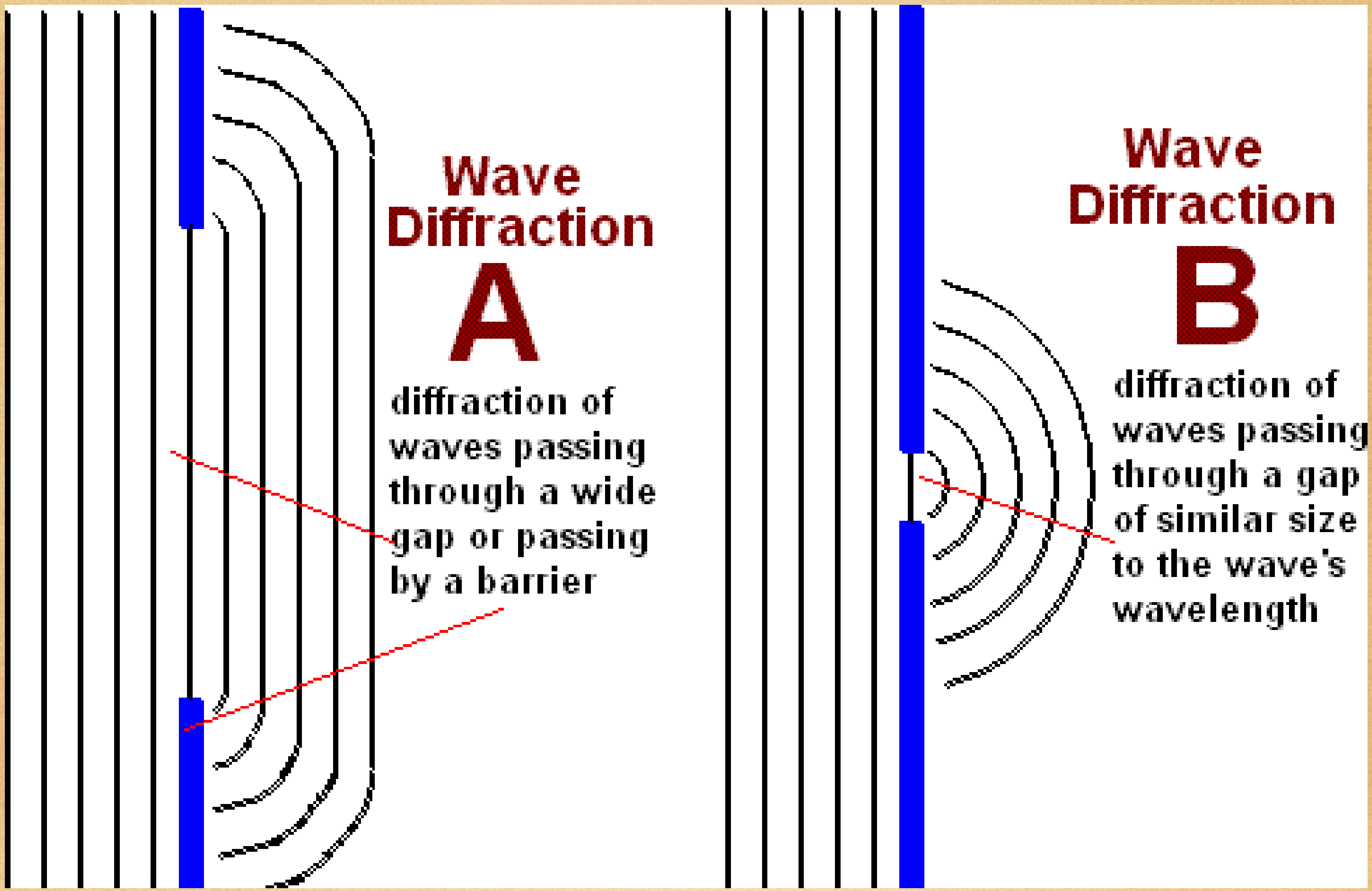
When the distance between the slit ab an source of light s as well as between slit a and the screen is finite, the diffraction is called Fresnal diffraction.

In Fresnal diffraction the waves are eithe spherical or cylindrical.

I. FRAUNHOFER DIFFRACTION

If light incident on slit ab is coming from inf distance, the distance between obstacle a and screen c is infinite, the diffraction is called Fraunhofer diffraction.

In Fraunhofer diffraction the incident waves should have plane wavefronts.


Wave Diffraction
A
diffraction of waves passing through a wide gap or passing by a barrier
Wave Diffraction
B
diffraction of waves passing through a gap of similar size to the wave's wavelength

Difference between interference and diffraction:

- In Interference, minima are usually perfectly dark while this is not the case for diffraction.

- In interference, all maxima are of same intensity but they have varying intensity in diffraction.

- Fringe width could be equal in some cases in interference while they are never equal in diffraction.

- In interference, interaction takes place between two separate wavefronts originating from two coherent sources while in diffraction, interaction takes place between secondary wavelets originating from same wavefront.

Difference between interference and diffraction:

1. If you have two infinitely-narrow double slits, there will be just interference, but for finite-width slits there can be both interference and diffraction effects.

2. Finite width slit: the width of slit is comparable with the wavelength λ.

3. One continuous wide slit is equivalent to the $N \rightarrow \infty$ limit of the N-slit result ofinterference.

4. Diffraction is simply the $N \rightarrow \infty$ limit of interference, there is technically no need to introduce a new term for it. But on the other hand, a specific kind of pattern arises, so it makes sense to give it its own name.

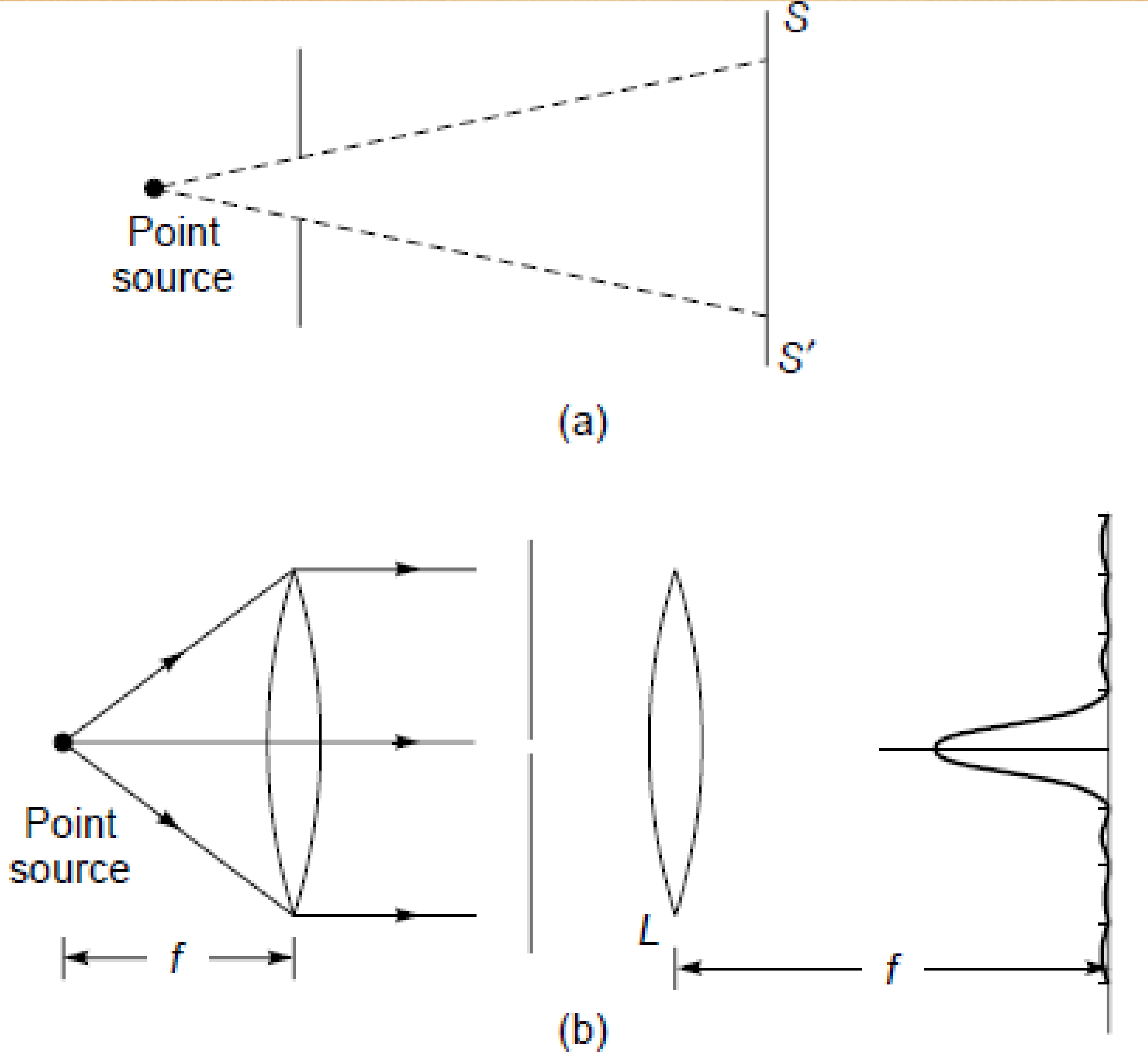
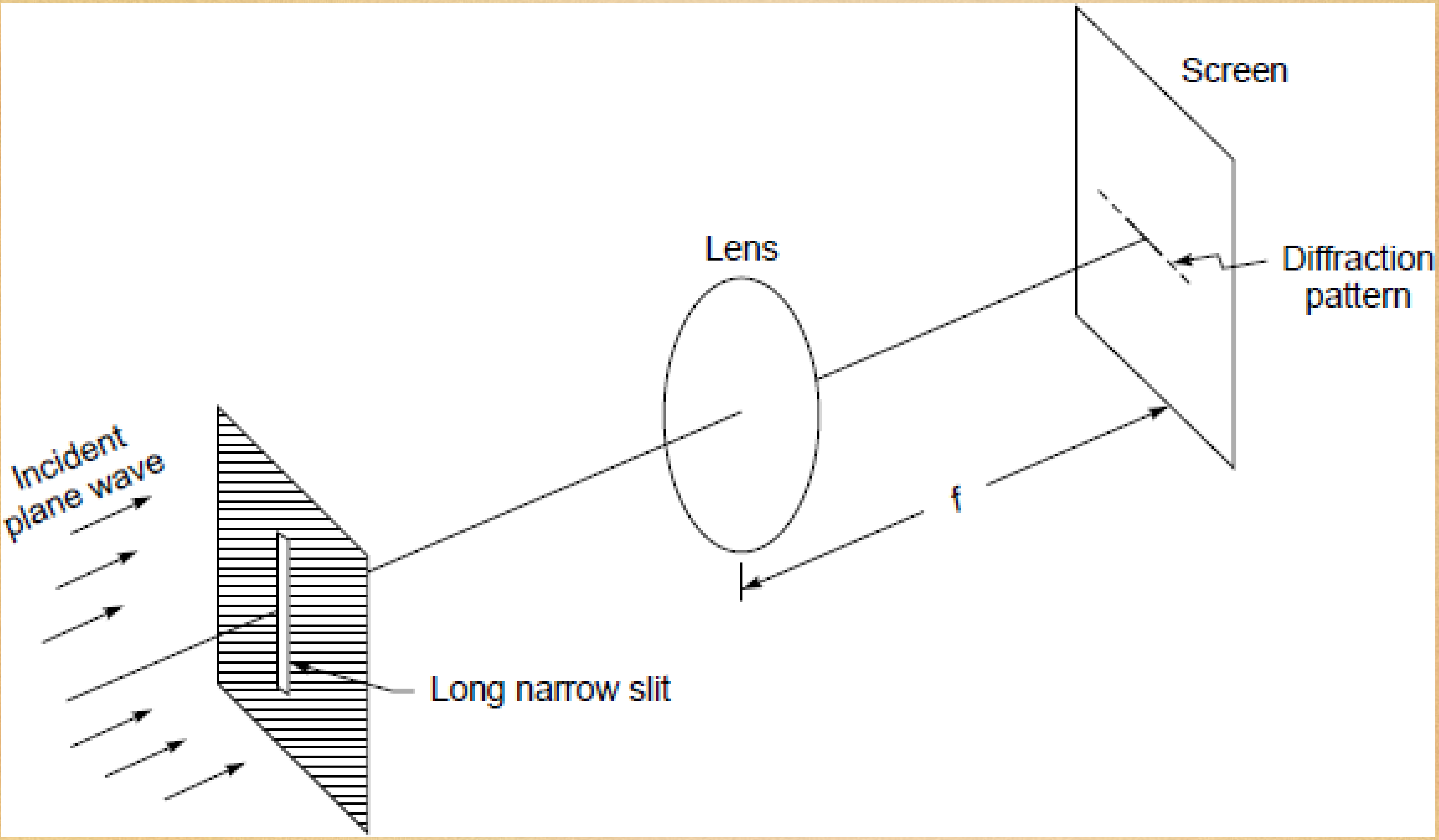
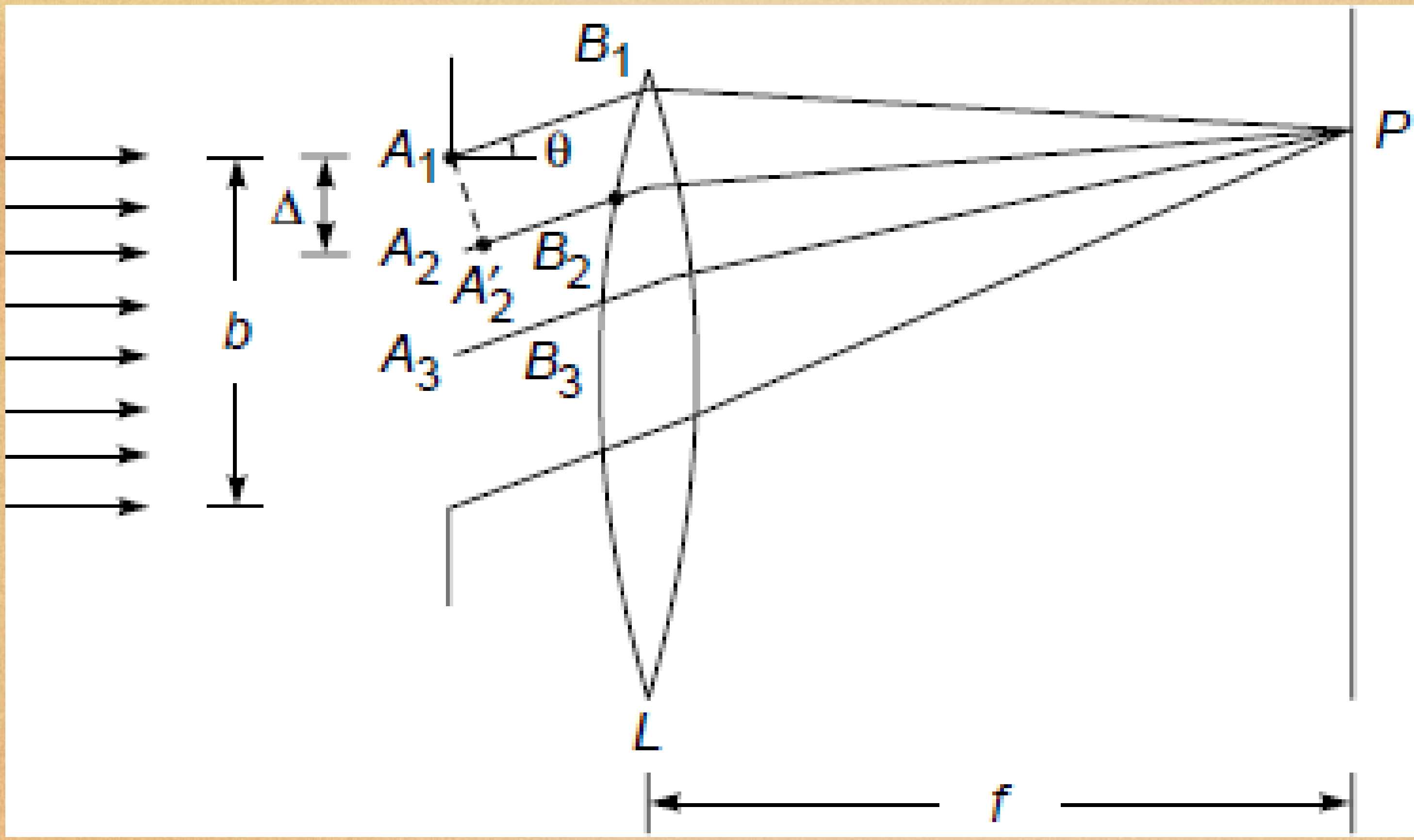


Fig. 18.2 (a) When either the source or the screen (or both) is at a finite distance from the aperture, the diffraction pattern corresponds to the Fresnel class. (b) In the Fraunhofer class both the source and the screen are at infinity.

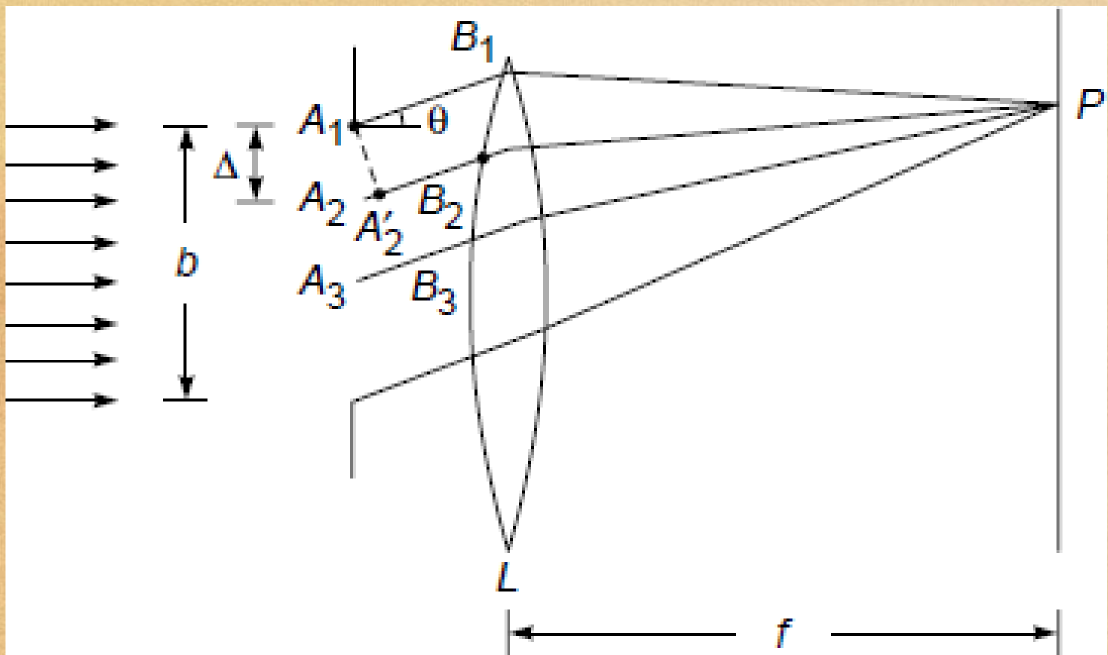
Diffraction at single slit:

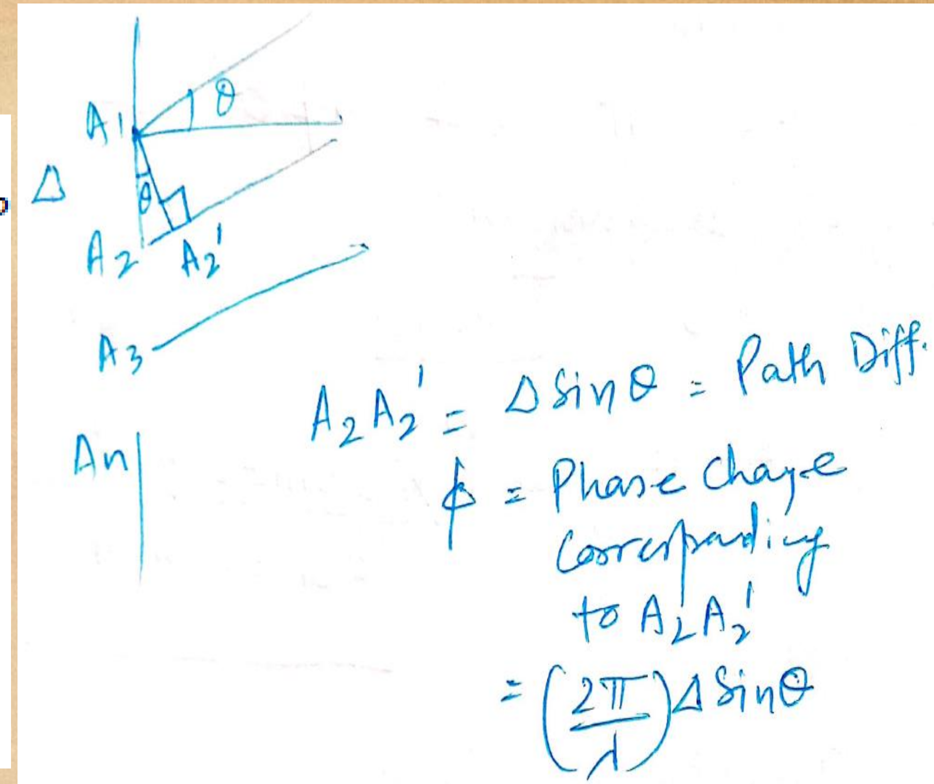


Diffraction at single slit:



Here the distance between two consecutive points be Δ. Thus, if the number of point sources is n, then $b = (n - 1)\Delta$.





The additional path traversed by the disturbance emanating from point A2 will be A2 A'2 , where

A2 A'2 =Δ.sin(θ).

Corresponding phase difference, φ =(2π/λ).Δ.sin(θ)

if the field at point P due to the disturbance emanating from point A1 is “a cos(ωt)”, then the field due to the disturbance emanating from A2 is “a cos (ωt – φ)"

$$E = a[\cos \omega t + \cos (\omega t - \phi) + \cdots + \cos [(\omega t - (n-1)\phi)]$$

where

$$\phi = \frac{2\pi}{\lambda}\Delta \sin \theta$$

$$\cos \omega t + \cos (\omega t - \phi) + \cdots + \cos [\omega t - (n-1)\phi]$$

$$= \frac{\sin (n\phi/2)}{\sin (\phi/2)} \cos\left[\omega t - \frac{1}{2}(n-1)\phi\right] \qquad \textbf{(4)}$$

Thus

$$E = E_0 \cos\left[\omega t - \tfrac{1}{2}(n-1)\phi\right] \qquad \textbf{(5)}$$

where the amplitude E_θ of the resultant field is given by[1]

$$E_\theta = a \frac{\sin (n\phi/2)}{\sin (\phi/2)} \qquad \textbf{(6)}$$

In the limit of $n \to \infty$ and $\Delta \to 0$ in such a way that $n\Delta \to b$, we have

$$\frac{n\phi}{2} = \frac{\pi}{\lambda} n\Delta \sin \theta \to \frac{\pi}{\lambda} b \sin \theta$$

Further

$$\phi = \frac{2\pi}{\lambda}\Delta \sin\theta = \frac{2\pi}{\lambda}\frac{b\sin\theta}{n}$$

will tend to zero, and, so we may write

$$E_\theta \approx \frac{a\sin(n\phi/2)}{\phi/2}$$

$$= na\frac{\sin(\pi b\sin\theta/\lambda)}{(\pi b\sin\theta/\lambda)}$$

$$= A\frac{\sin\beta}{\beta} \qquad A = na \tag{7}$$

$$\beta = \frac{\pi b\sin\theta}{\lambda} \tag{8}$$

Thus

$$E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta) \tag{9}$$

The corresponding intensity distribution is given by

$$I = I_0\frac{\sin^2\beta}{\beta^2} \tag{10}$$

where I_0 represents the intensity at $\theta = 0$.

Positions of Maxima and Minima:

$$I = I_0 \frac{\sin^2\beta}{\beta^2}$$

The intensity is zero when $\beta = m\pi$ $\quad m \neq 0$

When $\beta = 0$, $(\sin\beta)/\beta = 1$ and $I = I_0$, which corresponds to the maximum of the intensity.

$$but\ \beta = \frac{\pi b \sin(\theta)}{\lambda}$$

$$\frac{\pi b \sin(\theta)}{\lambda} = m\pi$$

$$Hence, b\sin\theta = m\lambda; where\ m = \pm 1, \pm 2, \pm 3, \ldots\ldots (minima)$$

First minimum occurs at $\theta = \pm \sin^{-1}(\lambda/b)$, second minimum occurs at $\theta = \pm \sin^{-1}(2\lambda/b)$ and so on. Can be used for fringe width calculations.

Upper limit of $\sin(\theta)$ is 1 so max. value of m is integer closest to b/λ.

Condition for maxima

When $\beta = 0$, $(\sin\beta)/\beta = 1$ and $I = I_0$, which corresponds to the maximum of the intensity. Substituting the value of β, one obtains

$$b\sin\theta = m\lambda$$

Where m=0 for central maxima

Intensity distribution is given by

$$I = I_0 \frac{\sin^2\beta}{\beta^2}$$

To find the secondary maximas, we have to differentiate I with respect to β

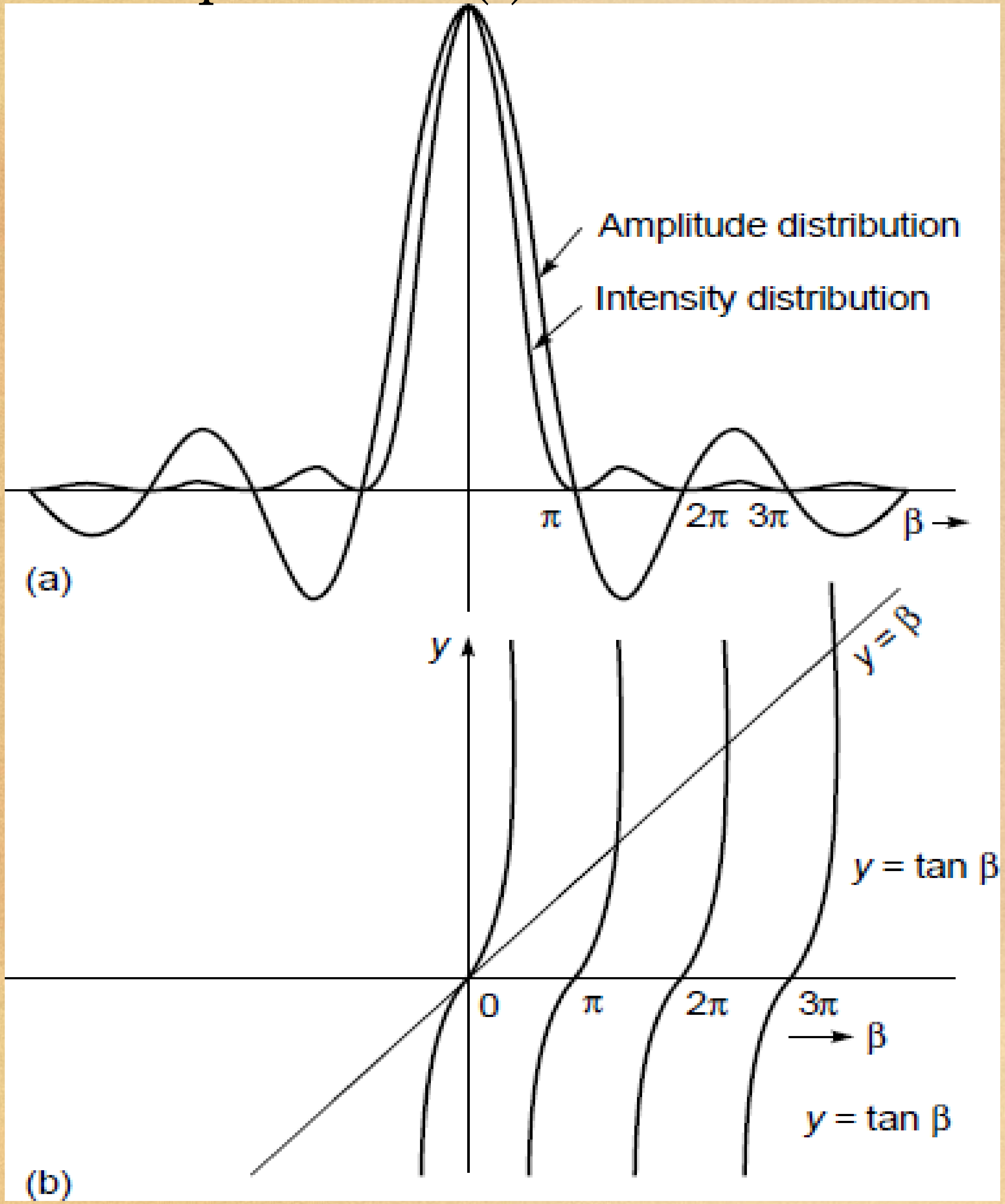
$$\frac{dI}{d\beta} = I_0\left(\frac{2\sin\beta\cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3}\right) = 0$$

$$\Rightarrow \sin\beta(\beta - \tan\beta) = 0$$

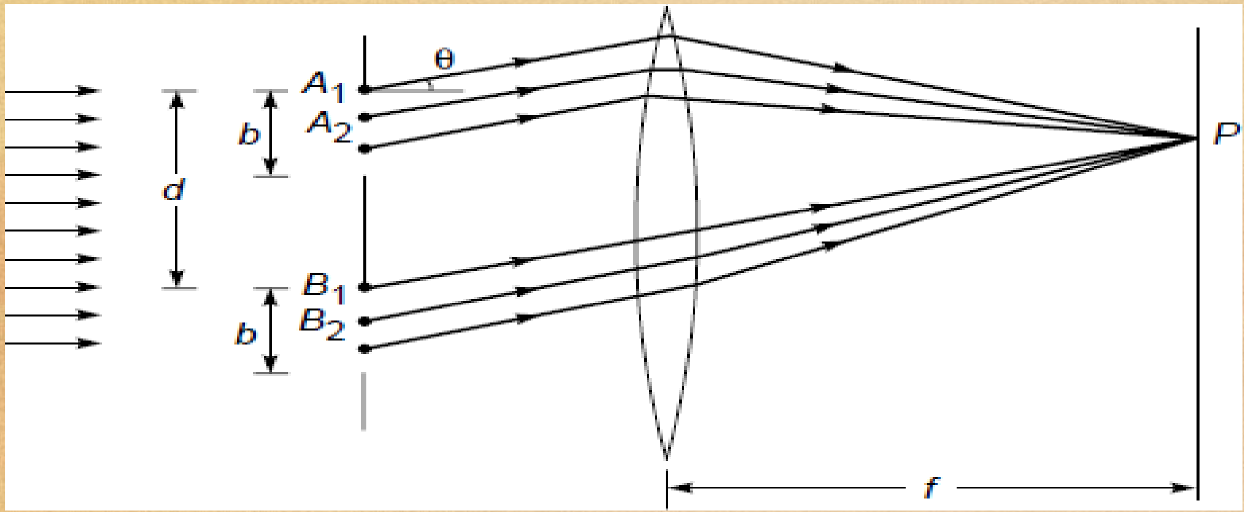
Condition sin(β) = 0 or β = mπ (m≠0) gives minima.

Condition tan(β) = β gives maxima. β=0 gives central Maxima. Rest of roots are found by intersection of curves y = β and y = tan(β)

(a) Intensity distribution with ß
(b) roots of equation tan(ß) =ß. Roots are: ß=1.43π, 2.46π and so on.



Two slit diffraction pattern :



$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

Similarly, the second slit will produce a field

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \Phi_1)$$

at point P, where

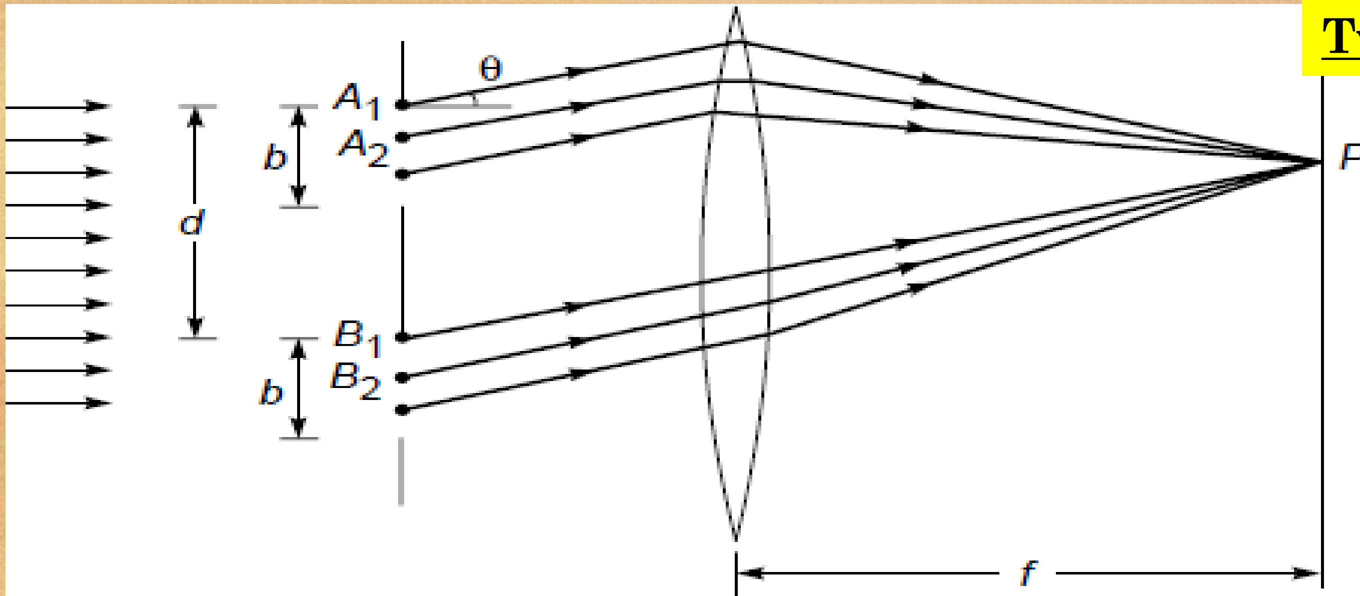
$$\Phi_1 = \frac{2\pi}{\lambda} d\sin\theta$$

represents the phase difference between the disturbances (reaching point P) from two corresponding points on the slits; by corresponding points we imply pairs of points such as (A_1, B_1), (A_2, B_2), . . . which are separated by a distance d.

$$E = E_1 + E_2$$

$$= A\frac{\sin\beta}{\beta}[\cos(\omega t - \beta) + \cos(\omega t - \beta - \Phi_1)]$$

Two slit diffraction pattern :



which represents the interference of two waves, each of amplitude $A \sin\beta/\beta$ and differing in phase by Φ_1. The above equation can be rewritten in the form

$$E = 2A \frac{\sin \beta}{\beta} \cos \gamma \cos \left(\omega t - \beta - \frac{1}{2} \Phi_1 \right)$$

where

$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d \sin \theta \qquad \textbf{(44)}$$

The intensity distribution will be of the form

$$I = 4I_0 \frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma \qquad \textbf{(45)}$$

18.6.1 Positions of Maxima and Minima

Equation (45) tells us that the intensity is zero wherever

$$\beta = \pi, 2\pi, 3\pi, \ldots \qquad \beta = \frac{\pi b \sin\theta}{\lambda}$$

or when

$$\gamma = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \qquad \gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d \sin\theta$$

$$\left.\begin{array}{ll} b \sin\theta = m\lambda & m = 1, 2, 3, \ldots \\ d \sin\theta = \left(n + \frac{1}{2}\right)\lambda & n = 1, 2, 3, \ldots \end{array}\right\}$$

The interference maxima occur when

$$\gamma = 0, \pi, 2\pi, \ldots$$

or when

$$d \sin\theta = 0, \lambda, 2\lambda, 3\lambda, \ldots$$

$$I = 4I_0 \frac{\sin^2\beta}{\beta^2} \cos^2\gamma$$

Interference term due to two slit

Diffraction term

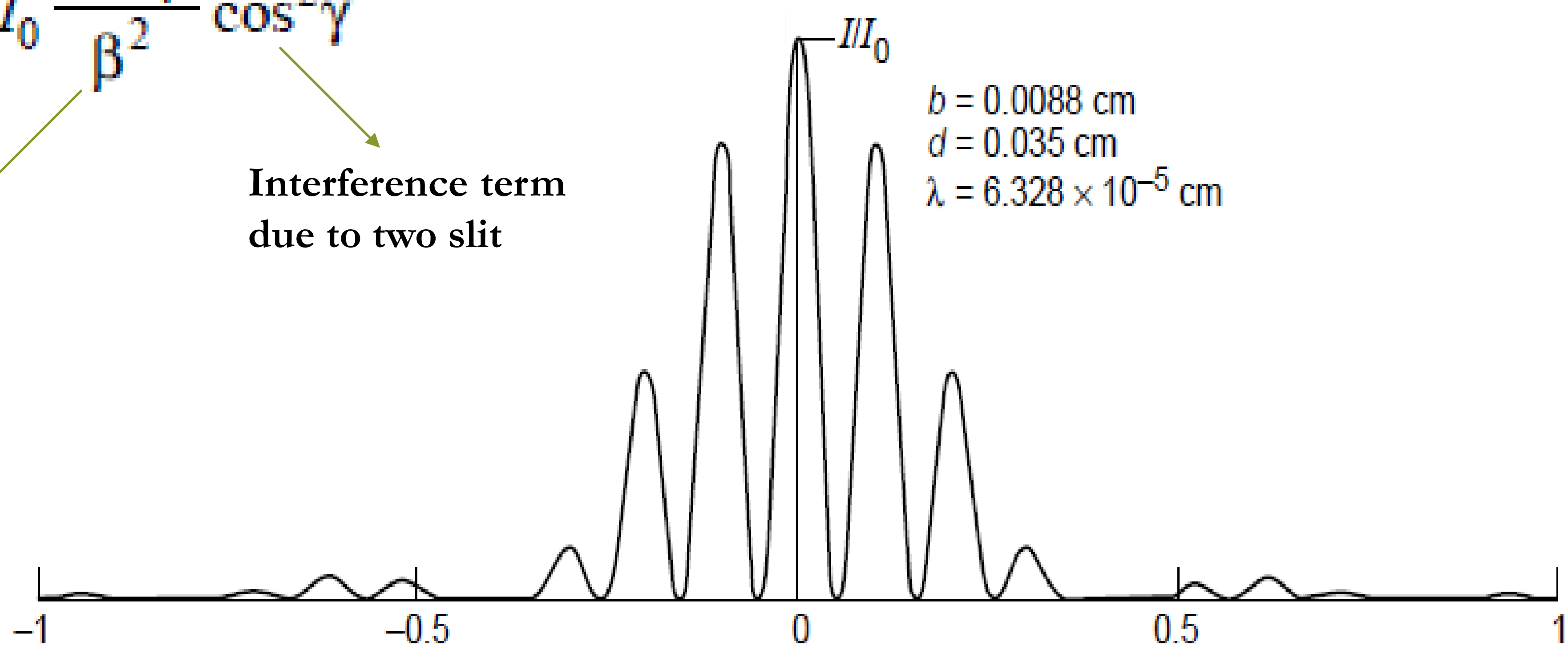


Fig. 18.31 The double-slit intensity distribution as predicted by Eq. (45) corresponding to b = 0.0088 cm, $\lambda = 6.328 \times 10^{-5}$ cm, and d = 0.035 cm.

The actual positions of the maxima will approximately occur at the above angles provided the variation of the diffraction term is not too rapid. Further, a maximum may not occur at all if θ corresponds to a diffraction minimum, i.e., if $b \sin\theta = \lambda, 2\lambda, 3\lambda, \ldots$. These are usually referred to as missing orders. For example, in Fig. 18.31 we can see that for $b = 0.0088$ cm, the interference maxima are extremely weak around $\theta \doteq 0.41°$; this is so because at

$$\theta = \sin^{-1}\left(\frac{\lambda}{b}\right)$$

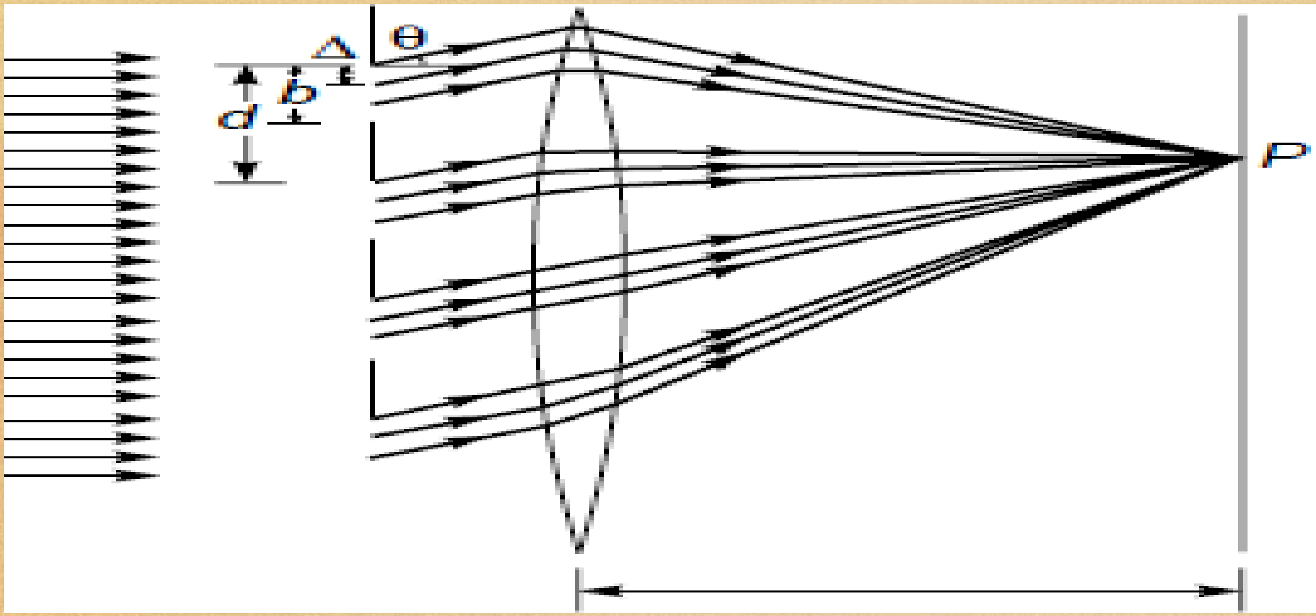
$$= \sin^{-1}\left(\frac{6.328\times 10^{-5}}{8.8\times 10^{-3}}\right) = \sin^{-1}(7.19\times 10^{-3})$$

$$\doteq 0.00719 \text{ rad}$$

$$\doteq 0.412°$$

the first minimum of the diffraction term occurs.

N-slit (Multiple slit) diffraction pattern:



As before, we assume that each slit consists of n equally spaced point sources with spacing Δ (see Fig. 18.33). Thus the field at an arbitrary point P will essentially be a sum of N terms:

$$E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta) + A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \Phi_1)$$

$$+ \cdots + A\frac{\sin\beta}{\beta}\cos[\omega t - \beta - (N-1)\Phi_1] \qquad \textbf{(48)}$$

$$E = \frac{A \sin \beta}{\beta} \{\cos(\omega t - \beta) + \cos(\omega t - \beta + \Phi_1)$$

$$+ \cdots + \cos[\omega t - \beta - (N-1)\Phi_1]\}$$

$$= \frac{A \sin \beta}{\beta} \frac{\sin N\gamma}{\sin \gamma} \cos\left[\omega t - \beta - \frac{1}{2}(N-1)\Phi_1\right] \qquad \textbf{(49)}$$

where

$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d \sin \theta$$

The corresponding intensity distribution will be

$$I = I_0 \frac{\sin^2 \beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2 \gamma} \qquad \textbf{(50)}$$

where $I_0\,(\sin^2\beta)/\beta^2$ represents the intensity distribution produced by a single slit. As can be seen, the intensity distribution is a product of two terms; the first term $(\sin^2\beta)/\beta^2$ represents the diffraction pattern produced by a single slit, and the second term $(\sin^2 N\gamma)/\sin^2\gamma$ represents the interference pattern produced by N equally spaced point sources. For $N = 1$, Eq. (50) reduces to the single-slit diffraction pattern [see Eq. (10)] and for $N = 2$, to the double-slit diffraction pattern [see Eq. (45)]. In Fig. 18.34 we have given a plot of the function

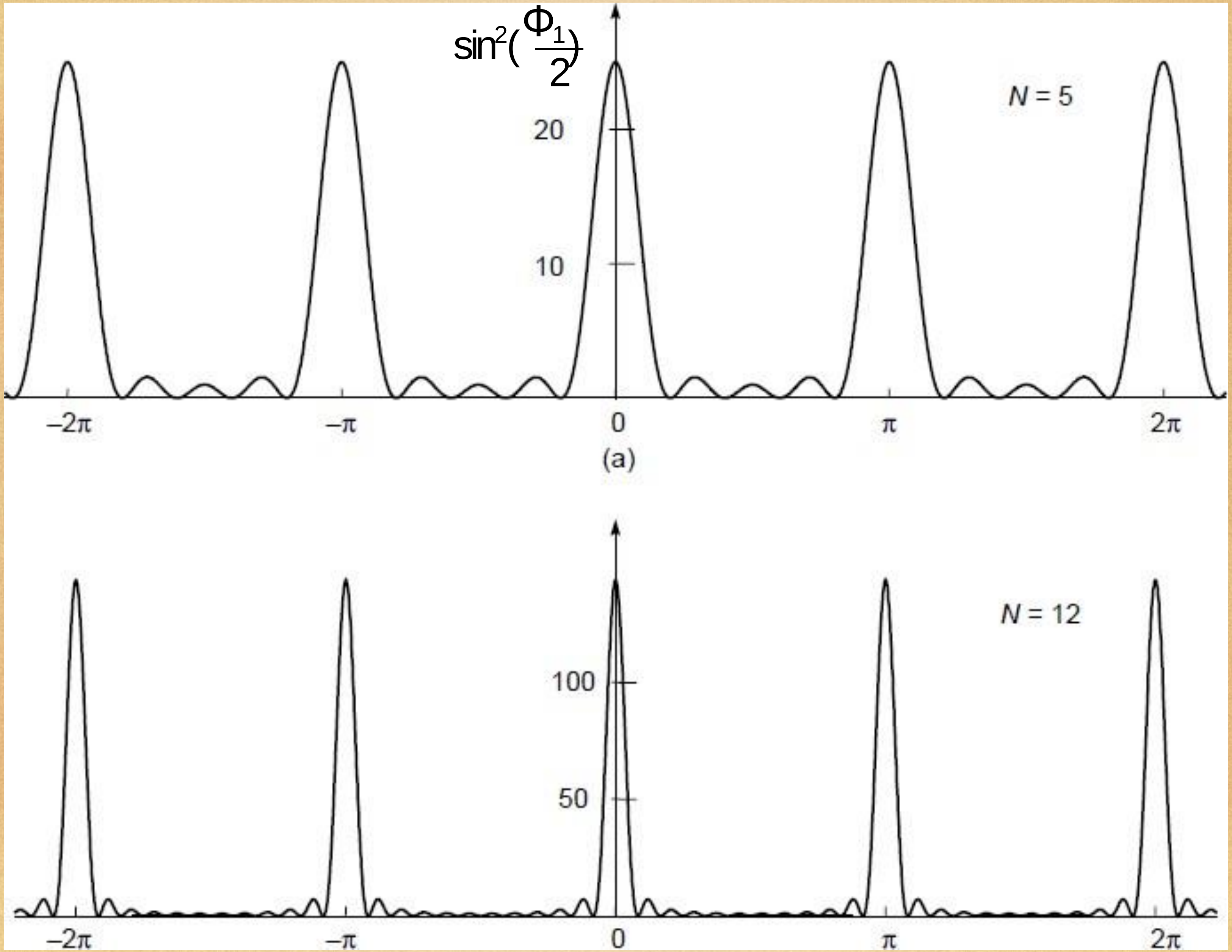
$$\frac{\sin^2 N\gamma}{\sin^2\gamma}$$

as a function of γ for $N = 5$ and $N = 11$. One can immediately see that as the value of N becomes very large, the above function becomes very sharply peaked at $\gamma = 0, \pi, 2\pi, \ldots$. Between the two peaks, the function vanishes when

$$\gamma = \frac{p\pi}{N} \qquad p = \pm 1, \pm 2, \ldots \qquad \text{but} \qquad p \neq 0, \pm N, \pm 2N$$

which are referred to as secondary minima.

$$\frac{\sin^2\left(\frac{N\,\Phi_1}{2}\right)}{\sin^2\left(\frac{\Phi_1}{2}\right)}$$



$$\rightarrow \frac{\Phi_1}{2}$$

Position of principal maxima

$$I = I_0 \frac{\sin^2\beta}{\beta^2}\frac{\sin^2 N\gamma}{\sin^2\gamma}$$

18.7.1 Positions of Maxima and Minima

When the value of N is very large, one obtains intense maxima at $\gamma \simeq m\pi$, i.e., when

$$d \sin\theta = m\lambda \qquad m = 0, 1, 2, \ldots \tag{51}$$

This can be easily seen by noting that

$$\lim_{\gamma\to m\pi} \frac{\sin N\gamma}{\sin\gamma} = \lim_{\gamma\to m\pi} \frac{N\cos N\gamma}{\cos\gamma} = \pm N$$

thus, the resultant amplitude and the corresponding intensity distributions are given by

$$E = N\frac{A\sin\beta}{\beta} \tag{52}$$

and

$$I = N^2 I_0 \frac{\sin^2\beta}{\beta^2} \tag{53}$$

where

$$\beta = \frac{\pi b\sin\theta}{\lambda} = \frac{\pi b}{\lambda}\frac{m\lambda}{d} = \frac{\pi b m}{d} \tag{54}$$

From Eq. (50) it can be easily seen that the intensity is zero when either

$$b \sin\theta = n\lambda \qquad n = 1, 2, 3, \ldots \tag{55}$$

or

$$N\gamma = p\pi \qquad p \neq N, 2N, \ldots \tag{56}$$

Equation (55) gives us the minima corresponding to the single-slit diffraction pattern. The angles of diffraction corresponding to Eq. (56) are

$$d \sin\theta = \frac{\lambda}{N}, \frac{2\lambda}{N}, \ldots, \frac{(N-1)\lambda}{N}, \frac{(N+1)\lambda}{N}, \frac{(N+2)\lambda}{N},$$

$$\ldots, \frac{(2N-1)\lambda}{N}, \frac{(2N+1)\lambda}{N}, \frac{(2N+2)\lambda}{N}, \ldots \tag{57}$$

Thus, between two principal maxima we have $N - 1$ minima. Between two such consecutive minima the intensity has to have a maximum; these maxima are known as secondary maxima. Typical diffraction patterns for $N = 1, 2, 3$, and 4 are shown in Fig. 18.35, and the intensity distribution as predicted by Eq. (50) for $N = 4$ is shown in Fig. 18.36. When N is very large, the principal maxima will be much more intense in comparison to the secondary maxima. We mention here two points:

Fi

$$N\gamma = p\pi \qquad p \neq N, 2N, 3N \cdots \quad (1)$$

If $p = N, 2N, 3N$

then $\cancel{N}\gamma = \cancel{N}\pi, 2\cancel{N}\pi, 3\cancel{N}\pi$

$$\gamma = \pi, 2\pi, 3\pi$$ (which is the case of maximas)

$$\gamma = m\pi$$

using (1)

$$N\left(\frac{\cancel{\pi} d\sin\theta}{\lambda}\right) = p\cancel{\pi} \qquad \left(\because \gamma = \frac{\pi d\sin\theta}{\lambda}\right)$$

$$d\sin\theta = \frac{p\lambda}{N}; \; p \neq N, 2N \cdots$$

So taking $p = 1, 2 \cdots$ $\quad d\sin\theta = \lambda/N, \frac{2\lambda}{N} \cdots$

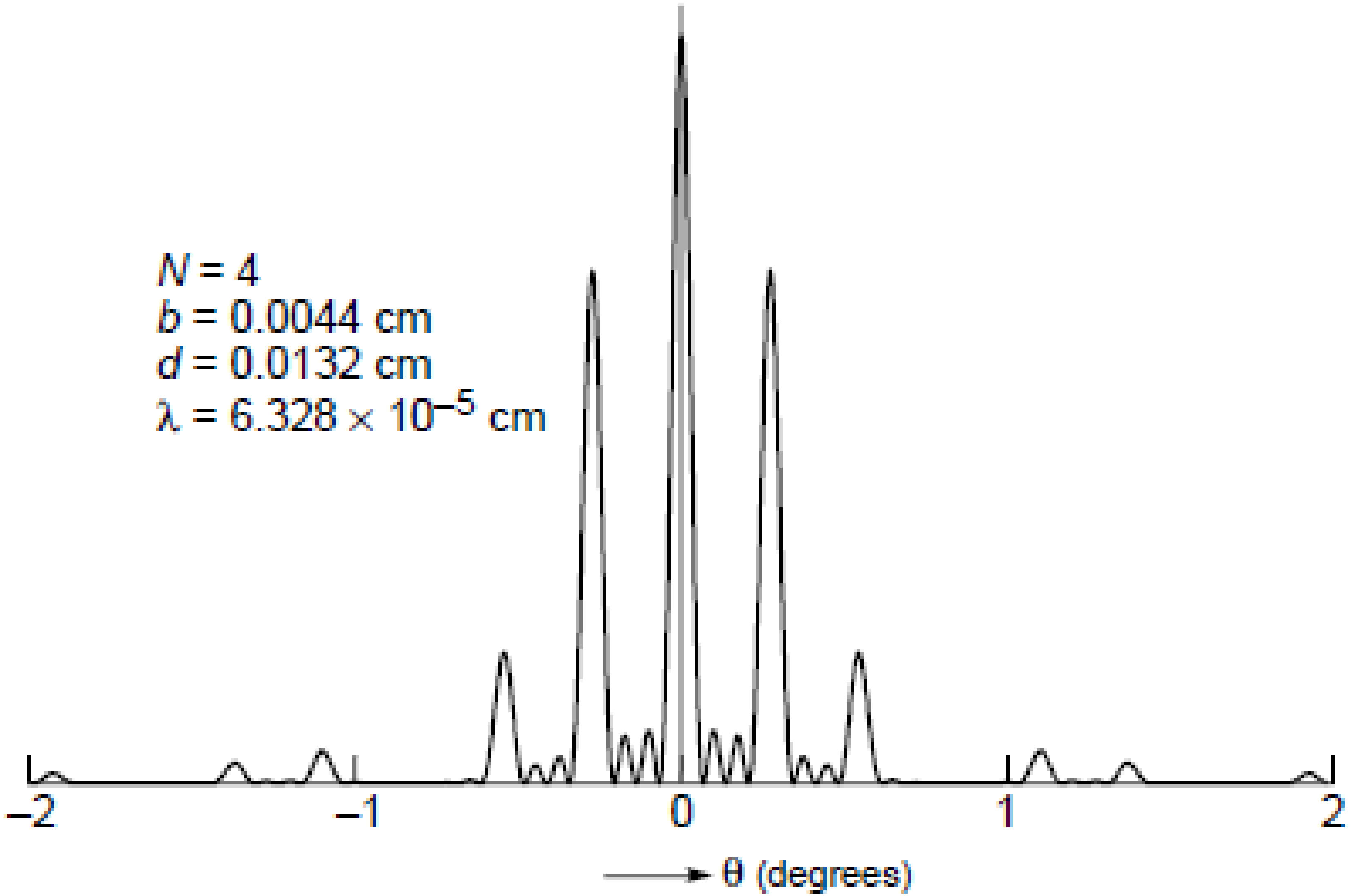


Fig. 18.36 The intensity distribution corresponding to the four-slit Fraunhofer diffraction pattern as pre-

Missing order

What if angle of principal maxima is same as diffraction minima?

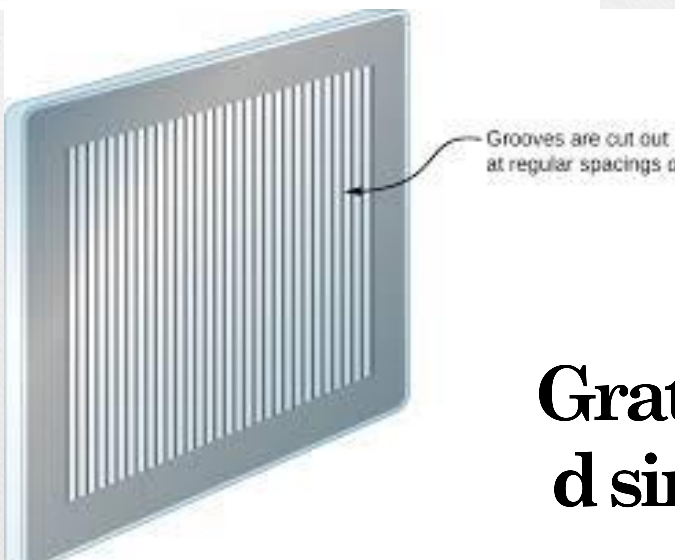
This will happen when these conditions are satisfied simultaneously

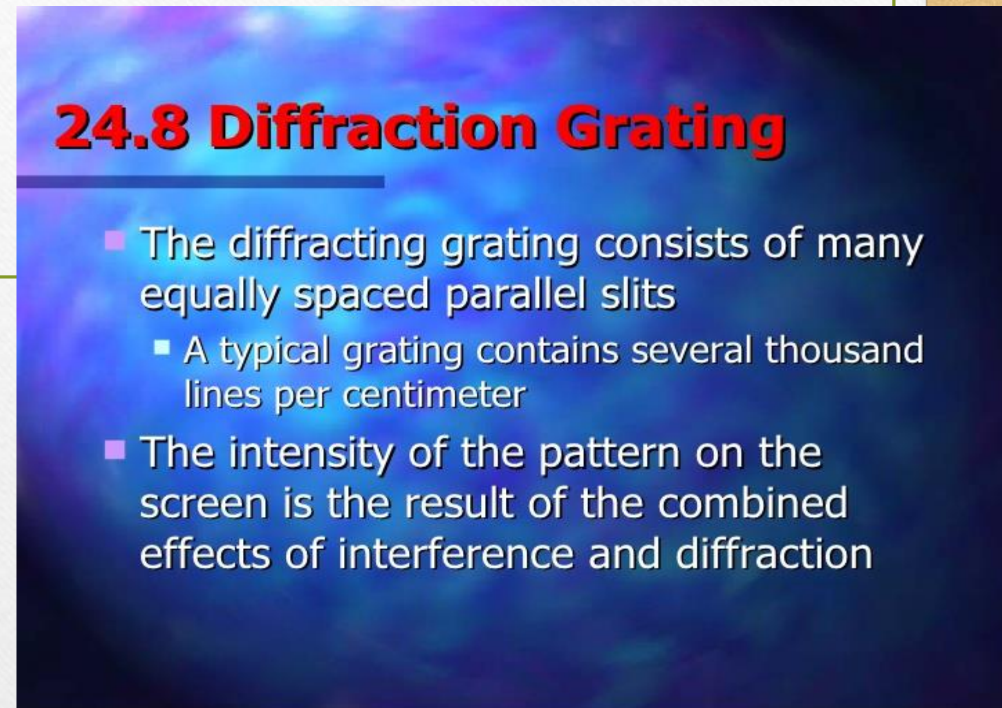
$d \sin(\theta) = m\lambda$ $m=0,1,2,\ldots\ldots$ (Principal maxima) And

$b \sin(\theta) = \lambda, 2\lambda, 3\lambda\ldots\ldots$ (diffraction minima)

These are referred as **missing order.**

Diffraction grating:





Grating Equation: Equation for maxima

$d \sin(\theta) = m\lambda \qquad m=0,1,2,\ldots\ldots$

Diffraction grating:

A very large number of equidistant slits is called diffraction grating.

Corresponding diffraction pattern is called as the grating spectrum. Principal maxima:

$$d \sin(\theta) = m\lambda \qquad m=0,1,2,\ldots\ldots..$$

As it depends on wavelength, so principal maxima ($m \neq 0$) for different λ will give different θ. Can be used for measurement of λ.

More is number of slits, narrower will be principal maxima. Usually 15,000 per inch slits are there.

Lines should be as equally spaced as possible.

Grating Spectrum :

Principal maxima: $\mathbf{d \sin(\theta) = m\lambda}$

m= 0,1,2,…….. This equation is also called the **grating equation**.

The zeroth order principal maxima occurs at $\theta=0$ irrespective of wavelength. Thus for white light, central maximum will be white.

For $m\neq0$, θ are different for different λ, various spectral components appear at different locations.

Dispersive power of grating :

Principal maxima: **$d \sin(\theta) = m\lambda$** m=0,1,2,……...

Differentiating this equation:

$$d\cos(\theta)\Delta\theta = m\,\Delta\lambda$$

$$\Rightarrow \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos(\theta)}$$

$\Delta\theta/\Delta\lambda$ is called **dispersive power.**

Conclusions:

1. Dispersive power is proportional to "*m*" (order of principal maximum). Higher is m, well separated will be maxima corresponding to 2 close wavelengths like sodium doublet. Zeroth order principal maxima will overlap.

Conclusions contd:

2.Dispersive power is inversely proportional to "*d*" (the grating element). Smaller is "*d*", larger will be angular dispersion.

2. Dispersive power is inversely proportional to cos(θ). if θ is very small then cos(θ)≃1,

$$\frac{d\theta}{d\lambda} = \frac{m}{d\cos\theta} \Rightarrow \frac{d\theta}{d\lambda} = \frac{m}{d}$$

Such spectrum is known as **normal spectrum**. For this dθ is directly proportional to dλ.

Resolving power of grating :

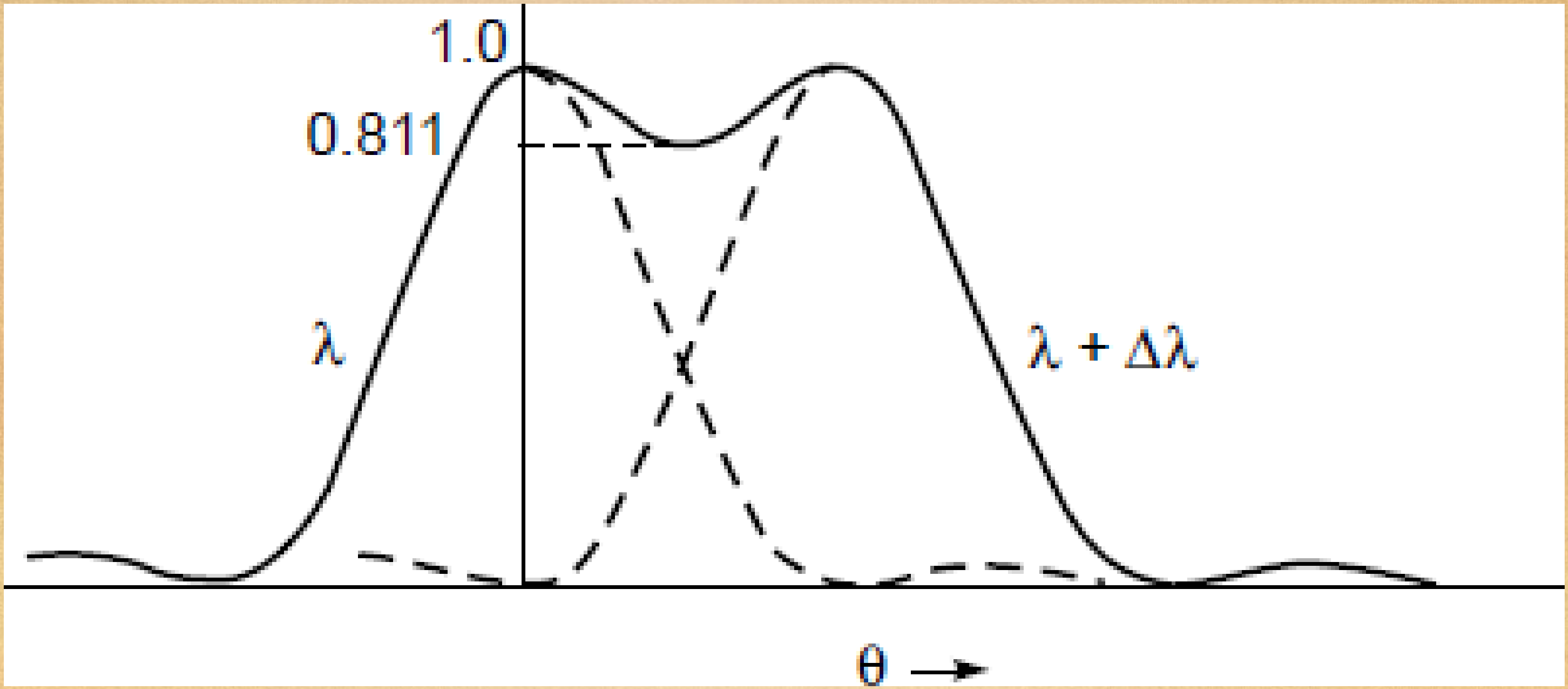
Minimum separation at which two objects look separate is Called "limit of resolution". Smaller is separation between 2 objects an instrument can Resolve, higher is its resolving power and better is the Instrument.

In case of diffraction grating, resolving power is power or ability of the grating to distinguish two nearby spectral lines.

Resolving power of grating :

Rayleigh's Criterion :

If the principal maximum corresponding to wavelength $\lambda + \Delta\lambda$ falls on first minimum (on either side) of the wavelength λ, then the two wavelengths λ and $\lambda + \Delta\lambda$ are said to be just resolved.



Resolving power of grating :

- If angle θ is the angle corresponding to m^{th} order spectrum then these conditions are satisfied simultaneously :

- principal maximum for wavelength $\lambda+\Delta\lambda$:
- $d \sin(\theta) = m(\lambda+\Delta\lambda)$

- minimum for wavelength λ:
- $d \sin(\theta) = m\lambda+\lambda/N$

Or $m\Delta\lambda = \lambda/N$

Or $\lambda/\Delta\lambda = m N$

- Equating both sides: $m(\lambda+\Delta\lambda) = m\lambda+\lambda/N$

$\lambda/\Delta\lambda$ is called the **resolving power of a grating**.

Resolving power of grating :

Resolving power; $\lambda/\Delta\lambda = m N$

- Resolving power depends on total number of lines in grating exposed to incident light (N).
- Resolving power is proportional to "order of spectrum".

Thank you