



Linear and Planar density

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Why some materials show anisotropic properties?

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Is this due to different number of atoms along different directions?

<i>Metal</i>	<i>Modulus of Elasticity (GPa)</i>		
	<i>[100]</i>	<i>[110]</i>	<i>[111]</i>
Aluminum	63.7	72.6	76.1
Copper	66.7	130.3	191.1
Iron	125.0	210.5	272.7
Tungsten	384.6	384.6	384.6

YES

The density of atoms along a direction called as linear density

The density of atoms in a plane called as planar density

- Only atoms whose center of mass lies on the plane has to be count.
- In the BCC crystal, the (111) plane partially intersects the atom at the body center ($\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$). This atom has to be excluded from the calculation.

LD = No. of atoms along a particular direction

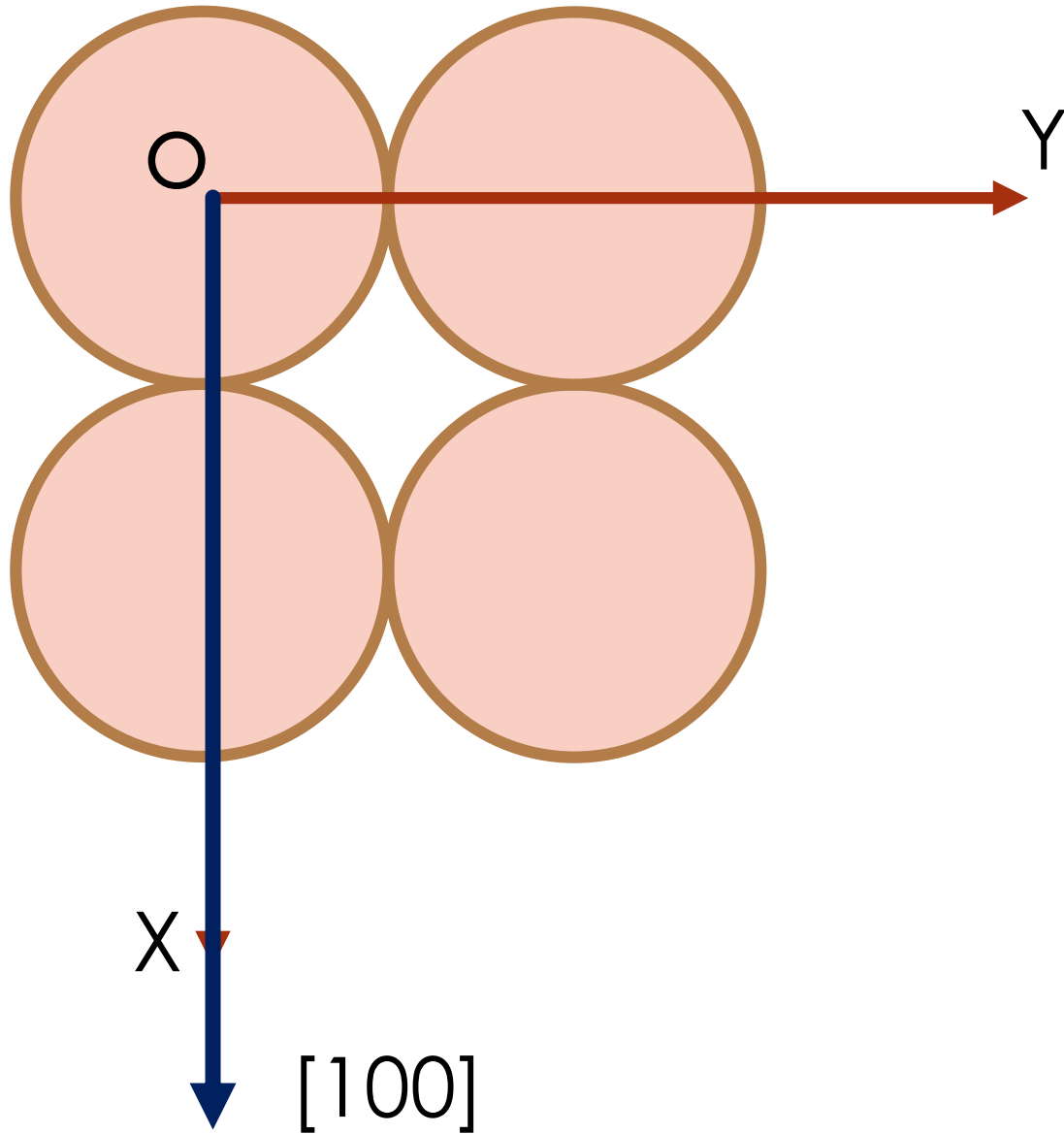
The density of atoms along a direction

$$LD = \frac{\text{No. of atoms along a direction}}{\text{Unit Length}}$$

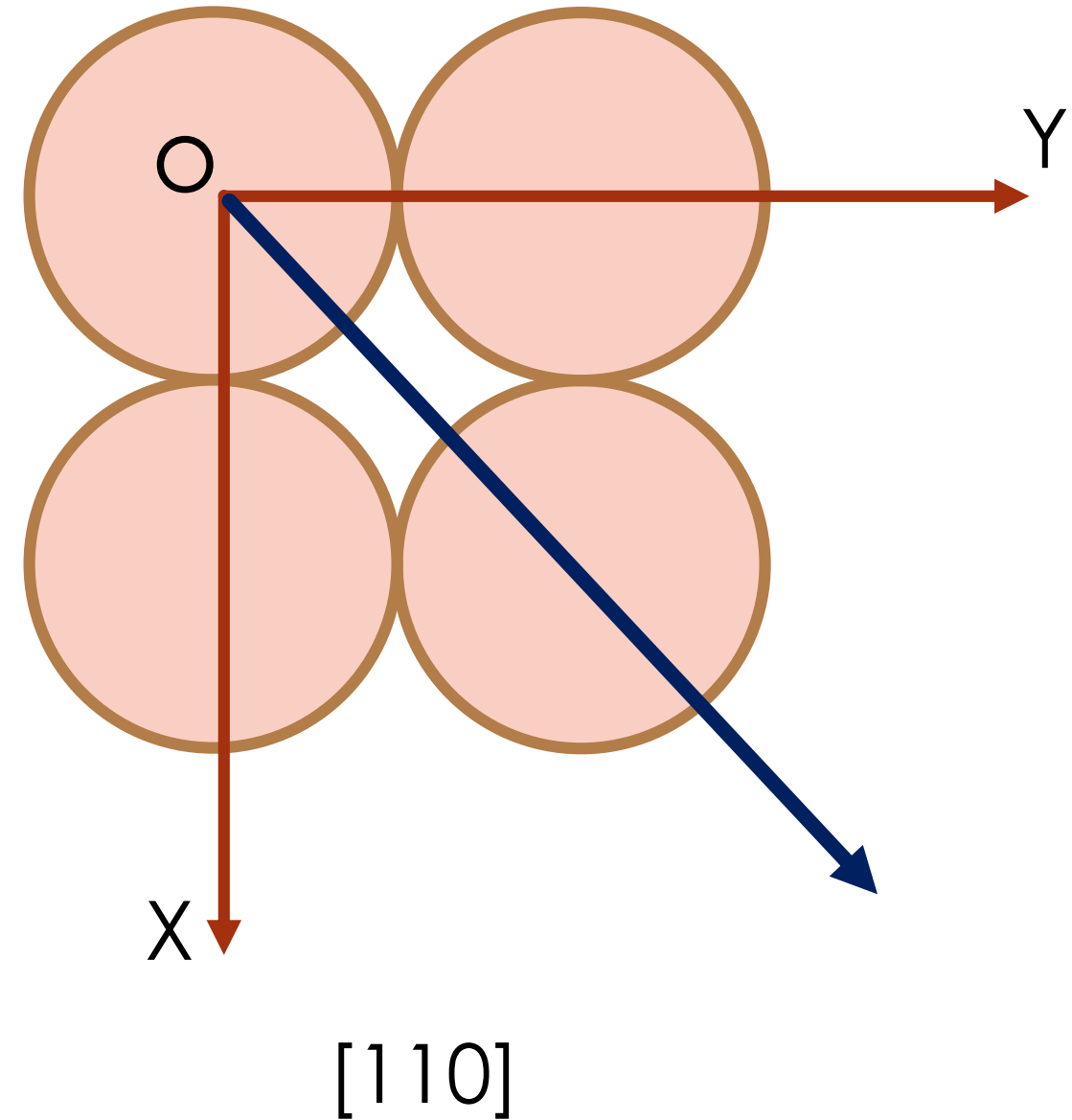
Linear density (LD)

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Simple Cubic (top view)



$$LD = 1/a$$



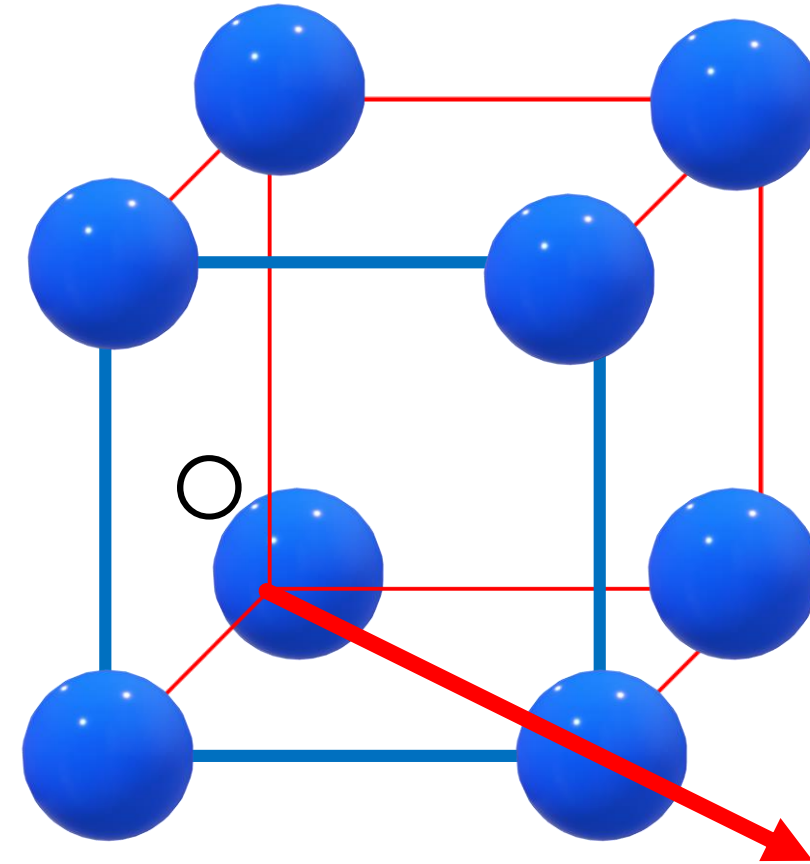
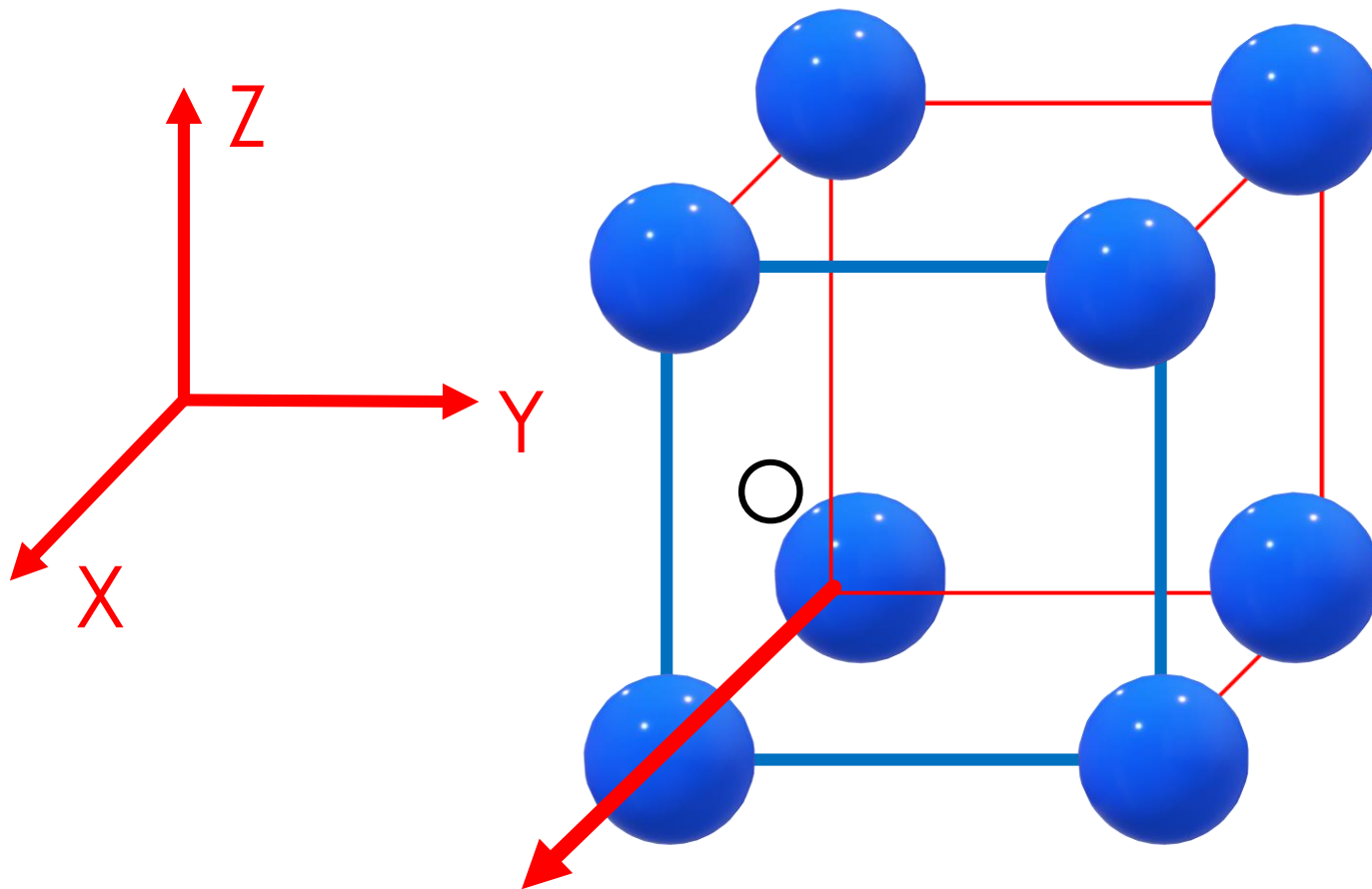
$$LD = 1/a\sqrt{2}$$

Linear density (LD)

Simple Cubic

[100]

[110]



Find out LD for the following directions.

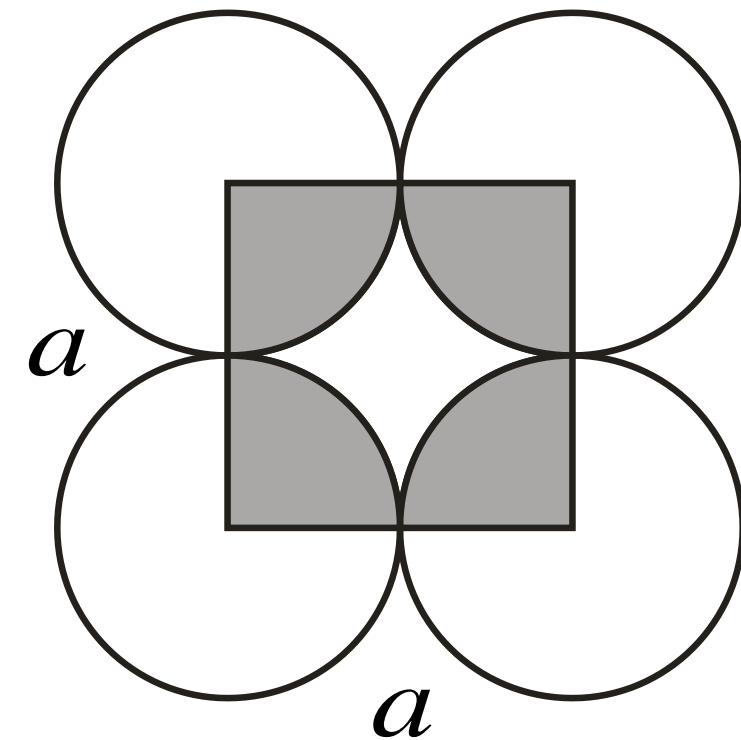
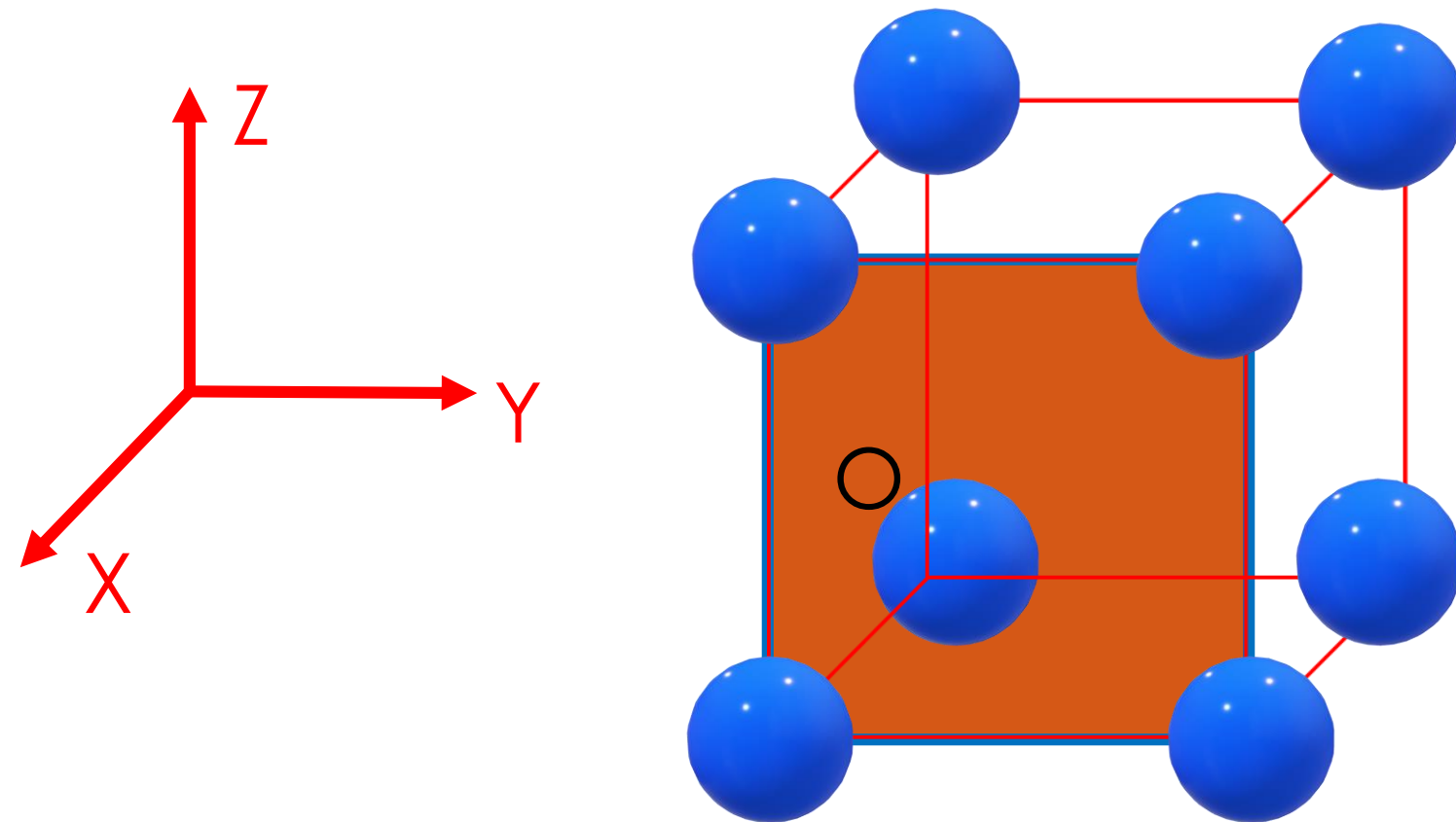
Structure	LD [100]	LD [110]	LD [111]
SC	$1/a$	$1/a\sqrt{2}$	$1/a\sqrt{3}$
BCC			
FCC			

The density of atoms in a plane

$$PD = \frac{\text{No. of atoms lying in a plane}}{\text{Area of the plane}}$$

Planar Density (PD)

Simple Cubic (100)



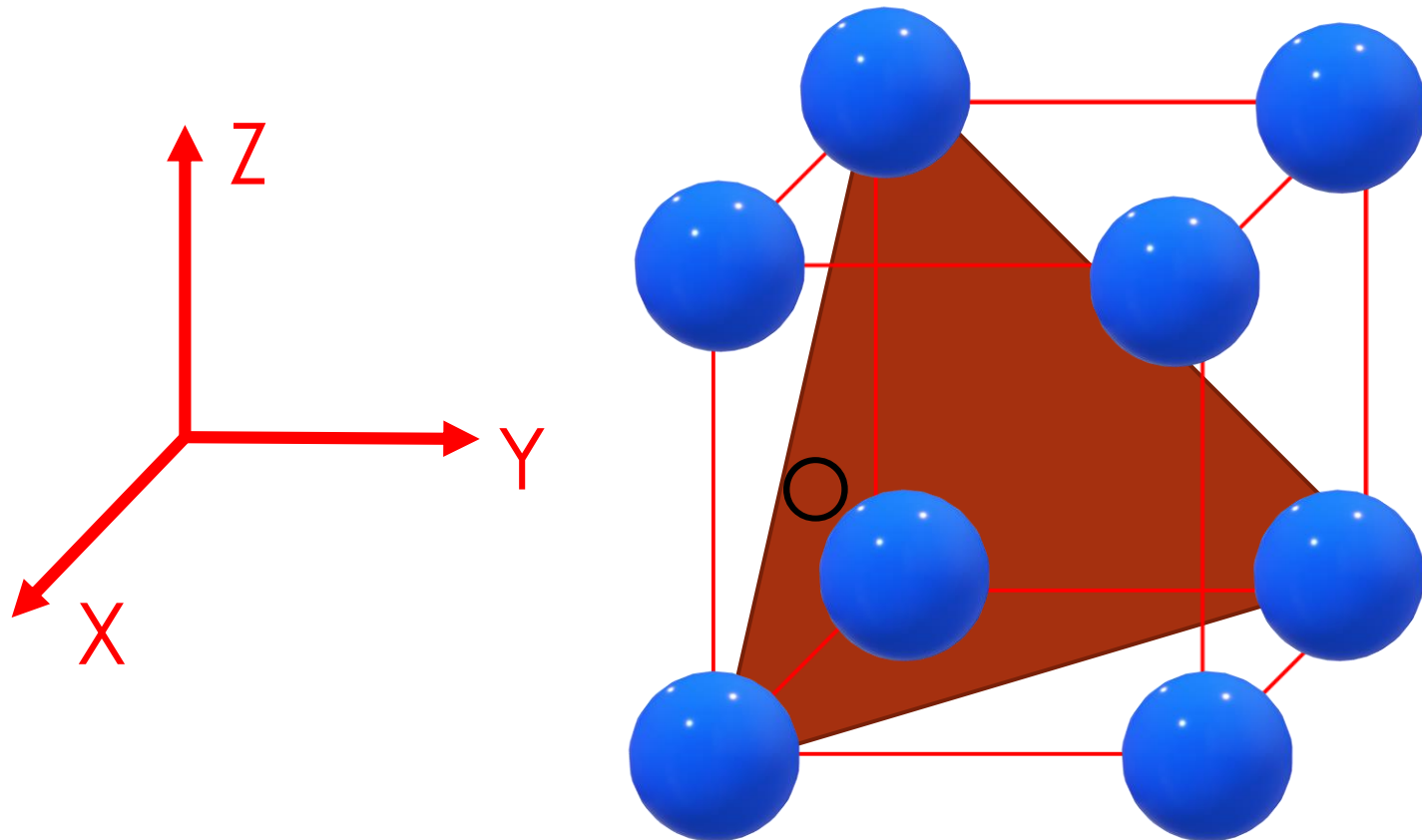
$$PD = 1/a^2$$

Planar Density (PD)

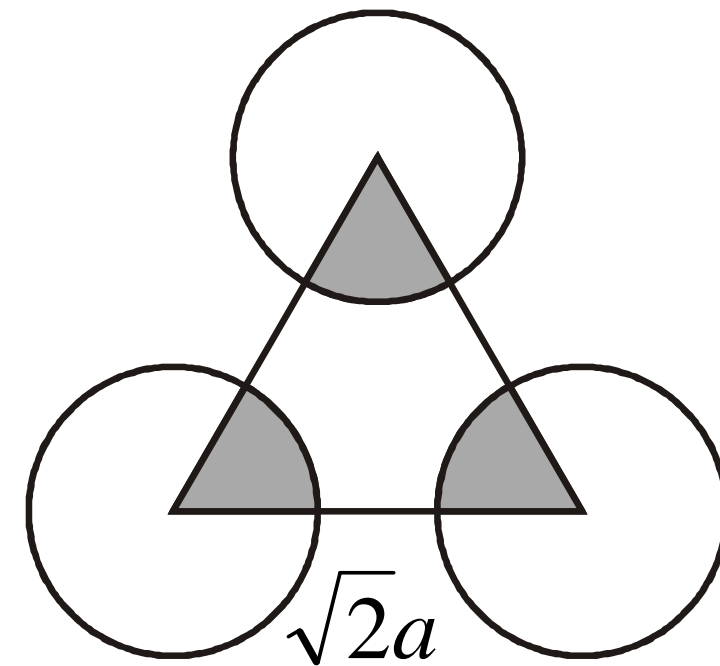
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Simple Cubic

(111)



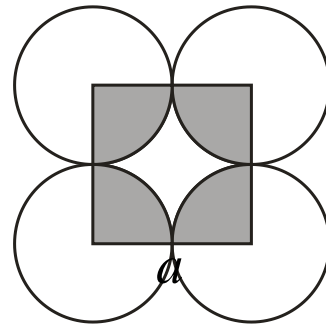
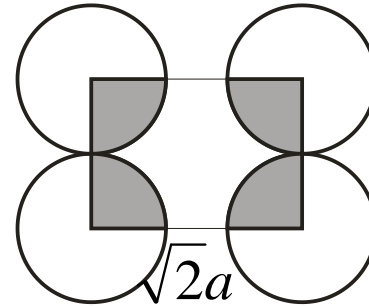
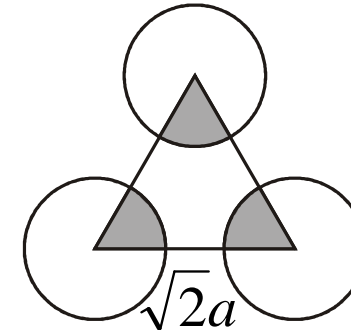
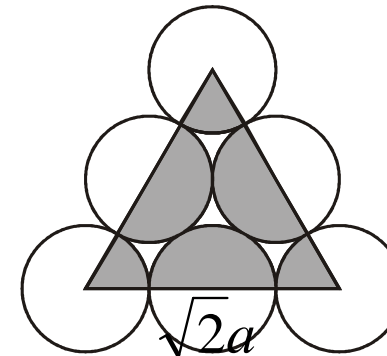
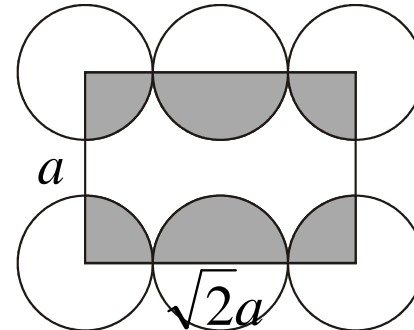
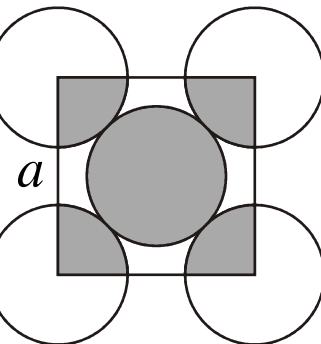
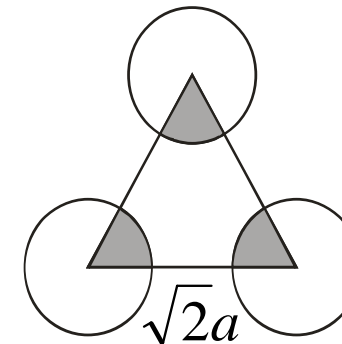
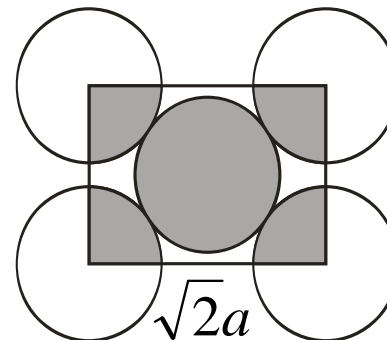
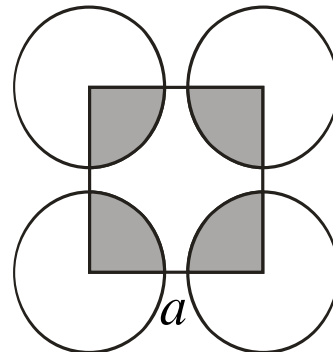
Area of equilateral
triangle = $\frac{1}{2} \times b \times h$



$$PD = \frac{\frac{1}{6} \times 3}{\frac{1}{2} \times a\sqrt{2} \times a\sqrt{\frac{3}{2}}}$$

$$PD = \frac{1}{\sqrt{3}} a^2$$

Planes showing atomic fractions

SC (100)  (110)  (111) **FCC****BCC**

Find out the planar density for following planes.

Structure	PD (100)	PD (110)	PD (111)
SC	$1/a^2$	$1/a^2\sqrt{2}$	$1/a^2\sqrt{3}$
BCC			
FCC			

1. Linear density and planar density are the analogous to atomic packing fraction in one dimension and two dimension respectively.
2. Linear density varies with the direction. This is the origin of anisotropic properties.

1. Compute and compare the linear densities for $[100]$, $[110]$ and $[111]$ for copper. Given $a = 0.3615$ nm.
2. Calculate the linear atomic density in the $[110]$ direction in the copper crystal lattice in atoms per square millimeter. The lattice constant of copper is 0.361 nm.
3. A metal crystallizes in the FCC structure. Calculate the linear atomic density along $[110]$ and $[111]$ direction. Assume lattice constant $a = 0.3923$ nm.
4. Compute the planar density for the BCC and FCC (100), (111) and (110) planes in terms of atomic radius r .
5. Calculate the planar density for (110) plane of BCC iron lattice in atoms per square millimeter. The lattice constant of iron is 0.287 nm.