Solutions
Tutorial Sheet-14 (GroupTheory)

1. 1, 2, 7,20,61

B) $x^2 - 2x - 3 = (x - 3)(x + 1)$; roots $r1 = 3$, $r2 = -1$

$$a_n = c1 3^n + c2(-1)^n$$

C) $a_n = (3/4)3^n + (1/4)(-1)^n$

2. A) 3,6,24,60

B) x2-x-6= (x-3)(x+2) : roots r1=3, r2=-2

$a_n = c1 3^n + c2(-2)^n$

C) $a_n = (9/4)3^n + (3/4)(-2)^n$

3. A) (i) $3 * 4 = 3 + 4 - 3(4) = 3 + 4 - 12 = -5$

(ii) $2 * (-5) = 2 + (-5) + 2(-5) = 2 - 5 + 10 = 7$

(iii) $7 * (1/2) = 7 + (1/2) - 7(1/2) = 4$

B) $(a * b) * c = (a + b - ab) * c = (a + b - ab) + c - (a + b - ab)c$

$= a + b - ab + c - ac - bc + abc = a + b + c - ab - ac - bc + abc$

$a * (b * c) = a * (b + c - bc) = a + (b + c - bc) - a(b + c - bc)$

$= a + b + c - bc - ab - ac + abc$

Hence $*$ is associative and $(Q, *)$ is a semigroup. Also

$a * b = a + b - ab = b + a - ba = b * a$

c) An element e is an identity element if $a * e = a$ for every $a \in Q$. Compute as follows:

$a * e = a$, $a + e - ae = a$, $e - ea = 0$, $e(1 - a) = 0$, $e = 0$

Accordingly, 0 is the identity element.

D) In order for a to have an inverse x, we must have $a * x = 0$ since 0 is the identity element by Part (c).

Compute as follows:

$a * x = 0$, $a + x - ax = 0$, $a = ax - x$, $a = x(a - l)$, $x = a/(a - l)$

Thus if a~= 1, then a has an inverse and it is $a/(a - 1)$.

4. A)

*	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

B) $2^{-1} = 4$, $3^{-1} = 5$ and $6^{-1} = 6$.

C) We have $2^1 = 2$, $2^2 = 4$, but $2^3 = 1$. Hence $|2| = 3$ and $gp(2) = \{1, 2, 4\}$. We have $3^1 = 3$, $3^2 = 2$, $3^3 = 6$,

$3^4 = 4$, $3^5 = 5$, $3^6 = 1$. Hence $|3| = 6$ and $gp(3) = G$.

D) G is cyclic since G = gp(3).

5. we define $U(n)$ to be the set of all positive integers less than n and relatively prime to n

U(3)={1,2}

Order=3

Order of 1=1

Order of 2

$2^1 = 2$, $2^2 = 1$

Hence $|2| = 2$

6. a) Ring-Yes
   ring with unity-Yes
   commutative ring-Yes
   ring with zero divisor-No
   integral domain- Yes
   field- Yes
   b) Ring-Yes
   ring with unity-No
   commutative ring-Yes
   ring with zero divisor-No
   integral domain- Yes
   field- No
   c) Ring-Yes
   ring with unity-No
   commutative ring-Yes
   ring with zero divisor-Yes
   integral domain- No
   field- Yes