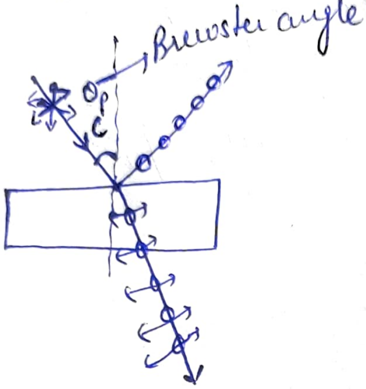
Tut - 7

Polarization

Ques 1: A glass plate is to be used as polarizer. Find the angle of polarization for it. Also find the angle of refraction. Given μ for glass as $= 1.54$.

Sol^n:



light is unpolarized so its direction of electric & magnetic fields of all photons present in this beam of light are randomly oriented so on polarized light so there exists a particular angle of incident for which the reflected ray will be polarized so here the plane of polarization is this plane which contain incident, reflected & normal to the surface
Refracted Ray not polarized.

$$\boxed{\mu = \tan \theta_p}$$

$$\theta_p = \tan^{-1}\mu = \tan^{-1}(1.54)$$
$$= 57^\circ$$

Reflection Angle

$$\mu = \frac{\sin(i)}{(\sin r)}$$

$$\frac{1}{\mu} = \frac{\sin r}{\sin \theta_p}$$

$$\sin^{-1}\left(\frac{(\sin \theta_p)}{\mu}\right) = r$$

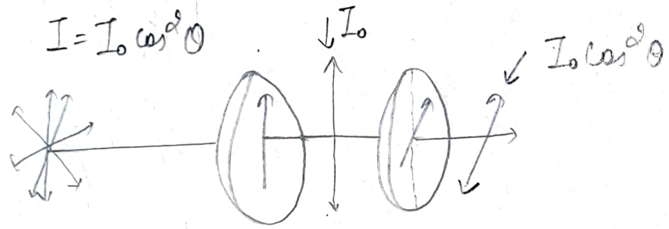
$$r = \sin^{-1}\left(\frac{\sin \theta_p}{\mu}\right)$$

$$= \sin^{-1}\left(\frac{\sin 57^\circ}{1.54}\right) = 33^\circ$$

$$\theta_p + r = 90^\circ$$
$$r = 90^\circ - 57^\circ = 33^\circ$$

Ques 2: Two polarizing sheets have ~~their~~ their polarizing directions parallel so that the intensity of transmitted light is a max. Through what angle either sheet is turned so that intensity becomes one half of initial value.

Sol^n:- According to malus, when completely plane polarized light is incident on the analyzer, the intensity I of the light transmitted by analyzer is directly proportional to the square of cosine of angle b/w the transmission axis of the analyzer & the polarizer.



We have

$$I = I_0 \cos^2\theta$$

$$I = I_0/2$$

$$\frac{\cancel{I_0}}{2} = \cancel{I_0} \cos^2\theta$$

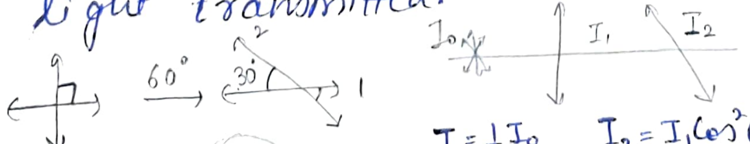
$$\cos^2\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)^{1/2}$$

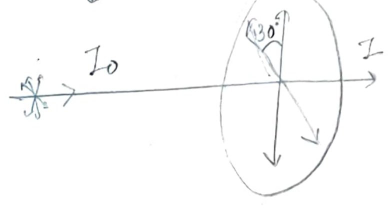
$$\boxed{\theta = \pm 45, \pm 135 \ldots}$$

Ques 3: Two nicols are first crossed & then one of them is rotated through 60°. Calculate the % of incident light transmitted.

Sol^n:-



$I_1 = \frac{1}{2} I_0$ $I_2 = I_1 \cos^2\theta$



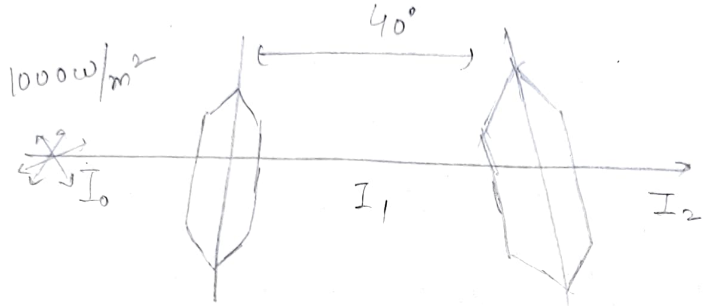
$$I_2 = \frac{I_0}{2} \cos^2(30°)$$

$$= \frac{3}{8} I_0$$

$$= 0.375\, I_0$$

Ques 4 :- Light of irradiance 1000 W/m^2 is shone through two polarizers, with their transmission axes placed at a relative angle of 40°. What is the intensity of transmitted light? If third polarizer is placed at an angle of 20° b/w other two. what is irradiance?

Sol^n :-

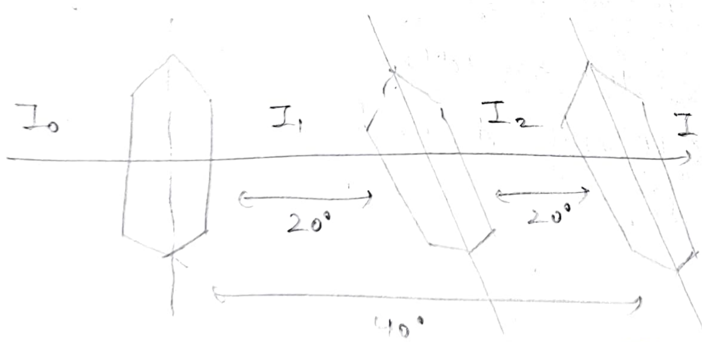


$$I_2 = \frac{I_0}{2} \cos^2\theta$$

$\theta = 40°$

$I_0 = 1000$

$$I_2 = 1000 \cos^2(40°)$$
$$= 293.412 \; W/m^2$$



$$I_3 = I_2 \cos^2(20°)$$

$$I_2 = I_1 \cos^2(20°)$$

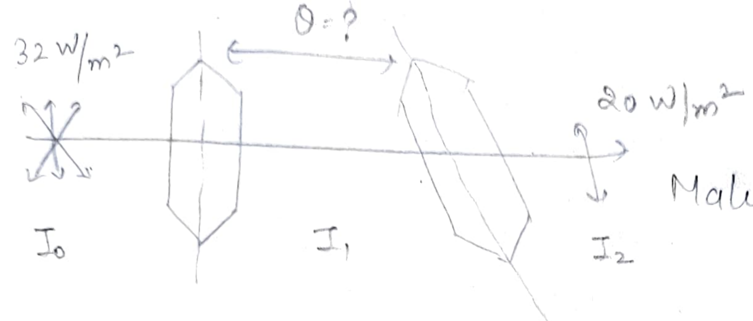
$$\Rightarrow I_3 = [I_1 \cos^2(20°)] \cos^2(20°)$$

$$= \frac{I_0}{2} \cos^4(20°) = \frac{1000}{2} \cos^4(20°)$$

$$= 389.86 \; W/m^2$$

Irradiance :- It may be defined as the power incident on unit area & usually expressed as W/m^2.

Ques 5 :- Unpolarized light of intensity that the 32 W/m² passes through two polaroids such that the intensity of emerging light is 20 W/m². What is the angle between the axes of polaroids?



$$I_2 = I_1 \cos^2\theta$$

$$I_2 = \frac{I_0}{2}\cos^2\theta \quad \text{or} \quad \cos^2\theta = \frac{2 \times I_2}{I_0}$$

$$\Rightarrow \frac{20 \times 2}{32} = \cos^2\theta$$

$$\frac{40}{32} = \cos^2\theta$$

which is > 1

$\cos^2\theta > 1$, so such a situation can never help simply

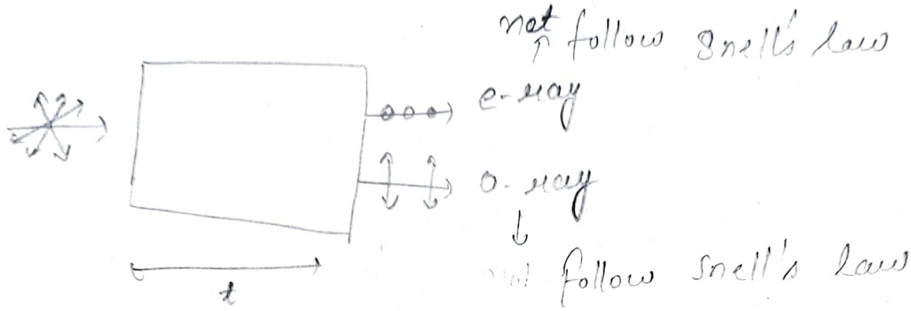
If incident is 32 then in middle it should be half is 16 W/m², after crossing second polaroids, intensity cannot ↑ from 16 to 20.

So $\cos^2\theta$ ↑ θ is impossible

Therefore the above configuration is not possible for any value of angle b/w axis of polaroids.

Ques 6 Calculate the thickness of double refracting crystal to introduce a path difference of $\lambda/2$ b/w E and O rays, when $\lambda = 6000 A^\circ$, $\mu = 1.55$ & $\mu_E = 1.54$.

Solⁿ :



$\mu_o = 1.55$

$\mu_e = 1.54$

$\lambda = 6000 A^\circ$

$\Delta = \lambda/2 \ , \ \phi = \pi$

optical path for e-ray $\rightarrow \mu_e t$

o-ray $\rightarrow \mu_o t$

path diff. $= (\mu_e - \mu_o)\, t = \lambda/2$

$$t = \frac{\lambda}{2}\left(\frac{1}{\mu_e - \mu_o}\right)$$

$$= \frac{6000\ A^\circ}{2}\left(\frac{1}{1.55 - 1.54}\right)$$

$$= 3 \times 10^{-3}\ cm$$

Natural light

E-ray (Extraordinary) → Not follow Snell's law

O-ray (ordinary) → follow Snell's law.

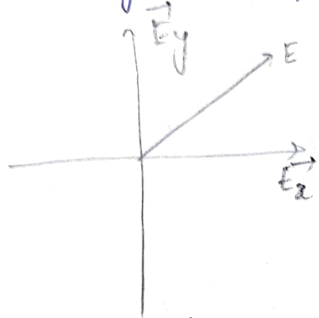
Because if angle of incident is zero, angle of reflection should also be zero. However we have seen e-ray bends even if angle of incident is zero mean not follow snell's law.

Ques 7 :- Discuss the state of polarization where x & y component of electric fields are given by following equations

(a) $E_x = E_0 \cos(\omega t + kz)$ and $E_y = E_0 \cos(\omega t + kz + \pi)$

(b) $E_x = E_0 \cos(\omega t + kz)$ and $E_y = E_0 \cos(\omega t + kz + \pi/2)$.

Sol^n :- Lissajous figures



$$\vec{E} = \vec{E_x} + \vec{E_y}$$

$$\vec{E_x} = E_{x0} \cos(\omega t + kz + \phi_1)\,\hat{i} \quad -(1)$$

$$\vec{E_y} = E_{y0} \cos(\omega t + kz + \phi_2)\,\hat{j} \quad -(2)$$

$$\vec{E} = \vec{E_x} + \vec{E_y}$$

$$\boxed{\left(\frac{E_x}{E_{x0}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\,\frac{E_x}{E_{x0}}\frac{E_y}{E_{y0}}\cos(\phi_2 - \phi_1) = \sin^2(\phi_2 - \phi_1)} \quad -(3)$$

(a) $E_x = E_0 \cos(\omega t + kz)$

$E_y = E_0 \cos(\omega t + kz + \pi)$

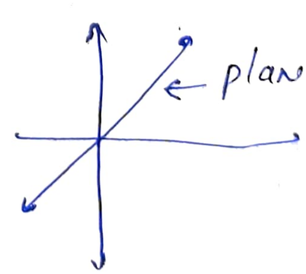
$\phi_1 = 0, \quad \phi_2 = \pi, \quad \phi_2 - \phi_1 = \pi$

from eq (3)

$$\left(\frac{E_x}{E_0}\right)^2 + \left(\frac{E_y}{E_0}\right)^2 - 2\,\frac{E_x E_y}{E_0^2}(-1) = 0$$

$$(E_x + E_y)^2 = 0$$

$\boxed{E_y = -E_x}$ Eq^n of straight line.



(b) $\phi_1 = 0, \quad \phi_2 = \pi/2, \quad \phi_2 - \phi_1 = \pi/2$

$$\left(\frac{E_x}{E_0}\right)^2 + \left(\frac{E_y}{E_0}\right)^2 - 2\,\frac{E_x E_y}{E_0^2}(0) = 1$$

$\boxed{E_x^2 + E_y^2 = E_0^2}$ → Eq^n of circle.

(circular polarization.