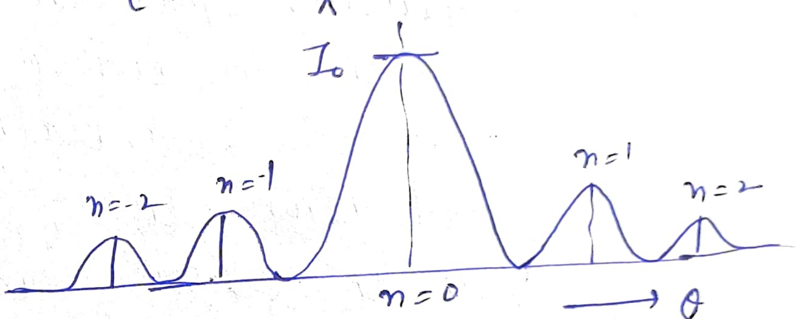
Tut - 6
Diffraction

Ques 1 :- For a single slit diffraction fringe. find the percentage intensities of Ist and 2nd order maxima w.r.t. that of central maximum.

Sol^n :- $I = I_0 \dfrac{\sin^2 \beta}{\beta^2}$ $\left[\beta = \dfrac{\pi b \sin\theta}{\lambda} \right.$



what are those conditions for maxima.

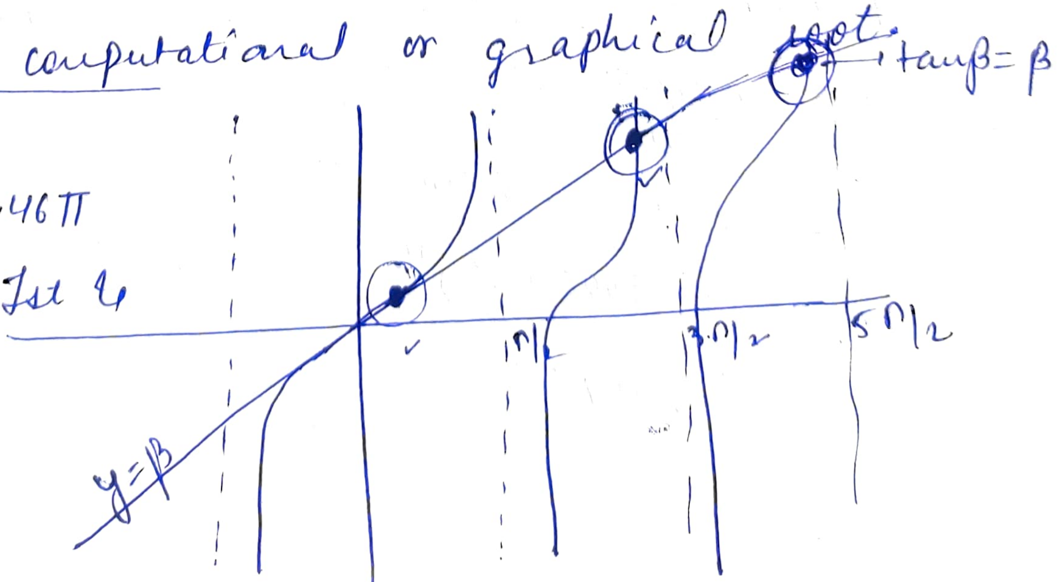
$\therefore I = I_0 \dfrac{\sin^2 \beta}{\beta^2}$

$\dfrac{dI}{d\beta} = 0 \Rightarrow I_0 \dfrac{2 \sin\beta \cos\beta \, \beta^2 - 2\beta \sin^2 \beta}{\beta^2} = 0$

$\boxed{\tan \beta = \beta}$ transcendial eqn

→ use either computational or graphical root.

trivial sol^n
$\beta = 0, 1.43\pi, 2.46\pi$
At these value Ist & 2ND maxima.



for $n=1$, $I = \frac{I_0 \sin^2(1.43\pi)}{(1.43\pi)^2}$

$\frac{I}{I_0} = 0.04719 = 4.719\%$

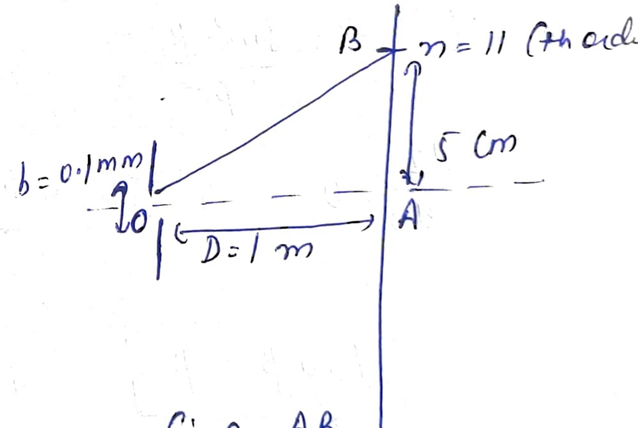
for $n=2$

$\frac{I}{I_0} = \frac{\sin^2(2.46\pi)}{(2.46\pi)^2} = 0.016 = 1.6\%$

<u>Ques 2</u>:- The eleventh order minima of single slit diffraction pattern are found at distance of 5 cm on either side of central max. Find the wavelength of monochromatic radiation used, while the distance b/w slit & screen is 1m & slit width is 0.1 mm.

<u>Solⁿ</u>: $\lambda = ?$

$b = .1\ mm$
$D = 1\ m$



Condition for minima in single slit

$$\boxed{b \sin\theta = n\lambda}$$

$$\lambda = \frac{b \sin\theta}{n}$$

$$= \frac{0.01\ cm \times 5\ cm}{11\,(100^2 + 5^2)^{1/2}}$$

$$\boxed{\lambda = 4540\ A^0}$$

$\sin\theta = \frac{AB}{OB}$

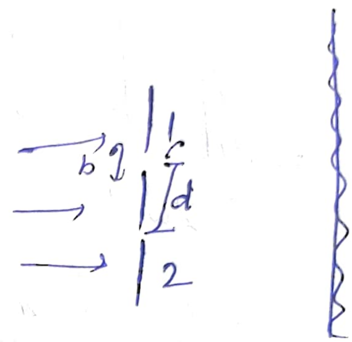
$= \frac{5\ cm}{(100^2 + 5^2)^{1/2}}$

Ques 3 :- A thin needle is placed at centre of aperture having width thrice that of needle. If a laser beam incidents normally on this arrangement, which order spectrum will be absent from the diffraction pattern?

Soln :- Because double slit :-

$$I = \underbrace{\frac{I_0 \sin^2 \beta}{\beta}}_{\text{diffraction}} \underbrace{\cos^2 \gamma}_{\text{interference}}$$

diffraction → like single slit



$\beta = \frac{b \sin \theta}{\lambda}$, $\gamma = \frac{d \sin \theta}{\lambda}$

Because of both slit Diffraction observed on screen. & because two diff. patterns are there then as a result you will a interference of two diffraction.

- Blue is diffraction pattern because of two slits.
- Red → Interference curve
- Black is due to single slit

when combine Black & Red

→ at centre - Red & Black are maximum & Blue is maxima.

- Red decrease oscillatory, Blue also decrease ---

Diffraction minima

Diff. maxima

. Cos is maximum.

→ $\beta = n\pi$

$\frac{b \sin \theta}{\lambda} = n\pi$ $\boxed{b \sin \theta = n\lambda}$ Diffraction minima

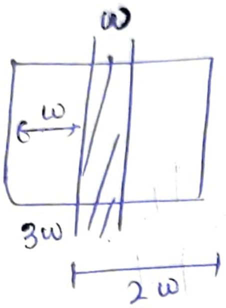
interference maxima

$\gamma = m\pi$

$\boxed{d \sin \theta = m\lambda}$

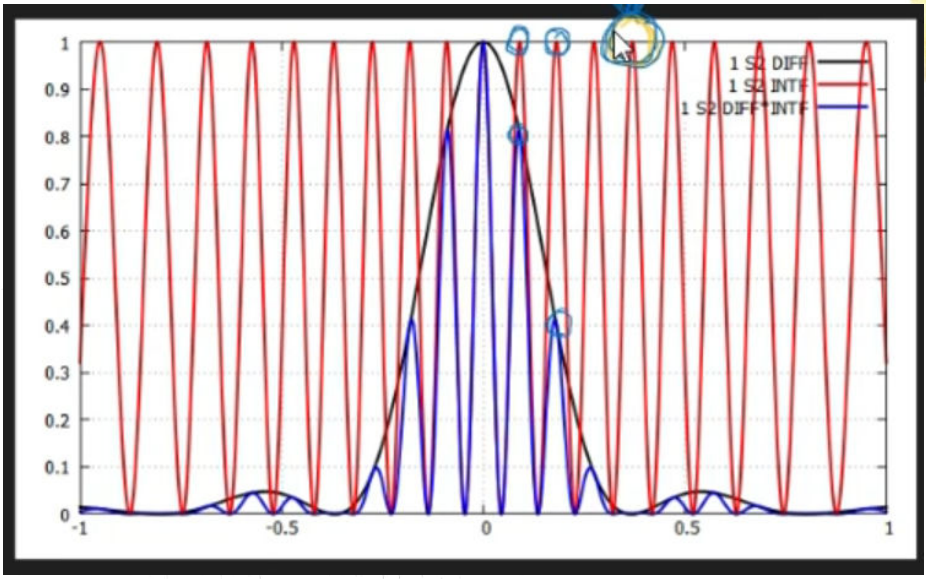
Take Ratio

$$\boxed{\frac{b}{d} = \frac{n}{m}}$$



$d = 2w$

$b = w$



$$\frac{1}{2} = \frac{n}{m} \Rightarrow m = 2n$$

$$\boxed{n \to 1, 2, 3, 4 \ldots}$$

Ques 4 : A double slit each slit having width 0.05 Cm and a seperation of 0.5 Cm b/w them, forms diffraction pattern on a screen placed 1.5 m away from slits. If the diffraction fringe width is 0.15 mm. Find wavelength of monochromatic light used.

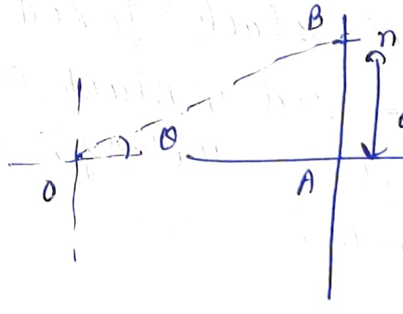
Sol^n :

$b = 0.05$ Cm

$d = 0.5$ Cm

$D = 1.5$ m

$\beta = 0.15$ mm

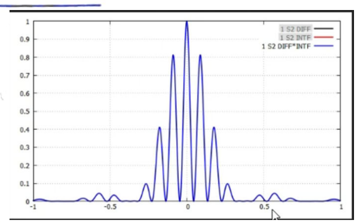


$d \sin\theta = n\lambda$

$\sin\theta = \frac{n\lambda}{d}$

$\tan\theta \simeq \sin\theta = \frac{AB}{OA}$

$= \frac{d_n}{D}$



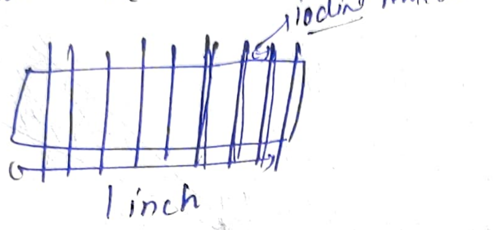
$$\boxed{\frac{d_n}{D} = \frac{n\lambda}{d}} \Rightarrow \boxed{d_n = \frac{n\lambda D}{d}} \quad \Big| \quad d_{n+1} = \frac{(n+1)\lambda D}{d},$$

Fringe width $\beta = d_{n+1} - d_n \Rightarrow \frac{\lambda D}{d}$

$$\lambda = \frac{\beta d}{D} = \frac{0.15\text{ mm} \times 0.5\text{ Cm}}{1.5\text{ m}} = 5000\ \mathring{A}.$$

Ques 5 :- 15000 numbers of long chain iodine molecules (opaque) are arranged parallel on a transparent thin film of length 1 inch. Let film is illuminated by light of wavelength 5600 A°. How many bright spots will be observed on screen? Label this order.

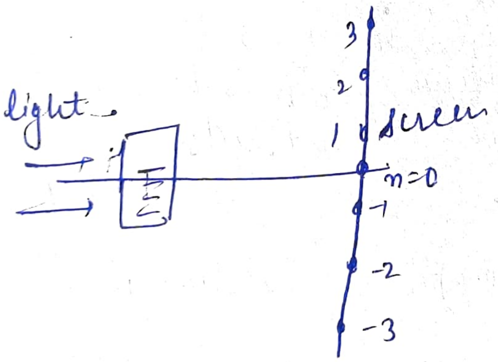
Soln :-



$$d = \frac{1 \text{ inch}}{15000 - 1} = \frac{2.54 \text{ cm}}{14999}$$

$$\boxed{d \sin\theta = n\lambda}$$

order of dark / bright



$\lambda = 5600$ A°

n, λ are fixed so maxima acco to $\sin\theta$

$$n = \frac{d}{\lambda} = \frac{2.54 \text{ cm}}{14999 \times 5600 \text{ A}^\circ}$$

$$= 3.024 \simeq 3$$

no of bright spots will be

$$1 + 2(n) = 7$$

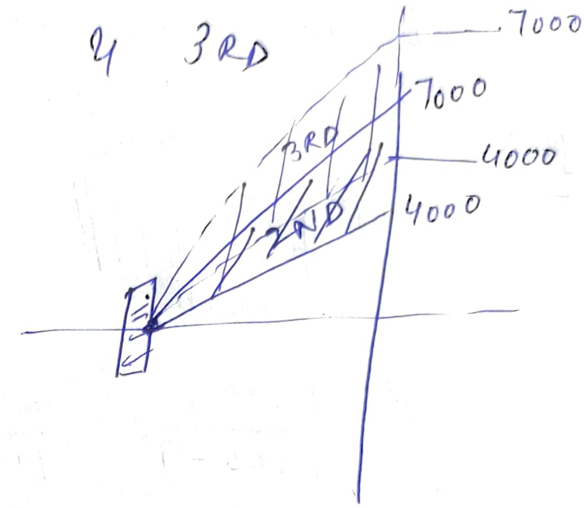
6. Prove that for white light ($\lambda = 4000\,A^\circ$ to $7000\,A^\circ$) the 2nd & 3rd order of spectrum will partially overlap for any Grating.

Sol^n: λ range $\rightarrow$ 4000 - 7000 A°

$\lambda_1 = 4000\,A^\circ$

$\lambda_2 = 7000\,A^\circ$

2nd & 3RD



$d \sin\theta = n\lambda$

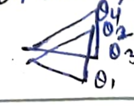
$\theta = \sin^{-1}\left(\frac{n\lambda}{d}\right)$

$d \rightarrow$ ~~distance~~ seperation b/w

The edge colour of the spectrum	$n = 2$ 2nd order spectrum extends from θ_1 to θ_2	$n = 3$ 3RD order spectrum extends from θ_3 to θ_4
$\lambda_1 = 4000\,A^\circ$	$\theta_1 = \sin^{-1}\left(\frac{2 \times 4000}{d}\right)$ $\theta_1 = \sin^{-1}\left(\frac{8000}{d}\right)$	$\theta_3 = \sin^{-1}\left(\frac{3 \times 4000}{d}\right)$ $= \sin^{-1}\left(\frac{12000}{d}\right)$
$\lambda_2 = 7000\,A^\circ$	$\theta_2 = \sin^{-1}\left(\frac{2 \times 7000}{d}\right)$ $= \sin^{-1}\left(\frac{14000}{d}\right)$	$\theta_4 = \sin^{-1}\left(\frac{3 \times 7000}{d}\right)$ $= \sin^{-1}\left(\frac{21000}{d}\right)$

from above table, we can say that

1. $\theta_1 < \theta_3 < \theta_2 < \theta_4 \Rightarrow$ The 2ND & 3RD order spectrum will partially overlap.
2. This is true for any gratting element, the overlapping will be for any gratting.



Ques 7 :- A plane transmission grating has 300 rulings per mm. Determine the dispersive power of violet ($\lambda = 4000$ A°) & Red ($\lambda = 6328$ A°) light for second order diffraction pattern.

Sol^n :- The dispersive power of a plane transmission gratting is given by :-

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

for second order $\Rightarrow n = 2$

Gratting element $= a+b = \frac{1mm}{300-1} = \frac{10^{-3}}{299} m$

$$= 3.344 \times 10^{-6} m$$

As we know

$$(a+b)\sin\theta = n\lambda$$

$$\sin\theta = n\lambda/(a+b)$$

$$\cos\theta = \sqrt{1 - \left(\frac{n\lambda}{(a+b)}\right)^2}$$

for Violet, wavelength $\lambda_{violet} = 4000$ A°

$$\cos\theta = \sqrt{1 - \left(\frac{2 \times 4000 \times 10^{-10}}{3.34 \times 10^{-6}}\right)^2} = 0.971$$

Dispersive power for violet $= \frac{n}{(a+b)\cos\theta} = \frac{2}{3.34 \times 10^{-6} \times 0.971}$

$= 0.6159 \times 10^{6}$ rad/m

for Red, $\lambda_{red} = 6328$ A°

$$\cos\theta = \sqrt{1 - \left(\frac{2 \times 6328 \times 10^{-10}}{3.34 \times 10^{-6}}\right)^2} = 0.926$$

Dispersive power for Red $= \frac{n}{(a+b)\cos\theta} = \frac{2}{3.34 \times 10^{-6} \times 0.926}$

$= 0.646 \times 10^{6}$ rad/m

Ques 8 : A plane transmission grating can just resolve two spectral line of $\lambda = 5499.5\ A^\circ$ and $5500.5\ A^\circ$ in the first order diffraction pattern. Determine the min. order of the same grating can resolve, while using another pair of wavelength $6500\ A^\circ$ and $6500.5\ A^\circ$.

Sol^n : Resolving power of plane transmission grating is given by

$$\frac{\lambda}{d\lambda} = nN$$

- For 2 spectral lines of λ $5499.5 A^\circ$ & $5500.5\ A^\circ$

$$\lambda = \frac{5499.5 + 5500.5}{2} = 5500\ A^\circ$$

$$\& \ d\lambda = |5499.5 - 5500.5| = 1\ A^\circ$$

Total no. of rulings = $N = \frac{\lambda}{n d\lambda}$

$\boxed{n = 1}$

$$N = \frac{5500\ A^\circ}{1 \times 1\ A^\circ} = 5500$$

For second case

$$\lambda = \frac{6500 + 6500.5}{2} = 6500.25\ A^\circ$$

$$\& \ d\lambda = |6500 - 6500.5| = 0.5\ A^\circ$$

Now, to determine the min. order spectrum

$$n = \frac{\lambda}{d\lambda N} = \frac{6500.25}{0.5\ A^\circ \times 5500} = 2.36$$

The same grating can resolve $n = 3$ i.e. 3RD order spectrum.

The grating can resolve 3RD order spectrum.