

School of Mathematics
Thapar University, Patiala,
UMA 004: Tutorial Sheet 02

1. Solve the following differential equation:

$$\begin{array}{ll} \text{(i)} \quad 2x(x+y) \frac{dy}{dx} = 3y^2 + 4xy; \quad y(1) = 1 & \text{(ii)} \quad (x^2 + y^2) \frac{dy}{dx} = xy; \quad y(1) = 2 \\ \text{(iii)} \quad x^2 \frac{dy}{dx} = y^2 + 2xy & \text{(iv)} \quad (x^3 + y^3) dx - xy^2 dy = 0 \end{array}$$

2. Solve the differential equation:

$$\text{(i)} \quad \frac{dy}{dx} = \frac{x+y+4}{x+y-6} \qquad \text{(ii)} \quad (2x+3y-1)dx - 4(x+1)dy = 0$$

3. Determine which of the following equations are exact or not and if so the find its solution.

$$\text{(i)} \quad \cos x \cos^2 y \, dx + 2 \sin x \sin y \cos y \, dy = 0 \qquad \text{(ii)} \quad (2xy^4 + \sin y) \, dx + (4x^2y^3 + x \cos y) \, dy = 0$$

4. Find the value of n for which the equation $(xy^2 + nx^2y)dx + (x^3 + x^2y)dy = 0$ is exact and solve for that value of n .

5. Solve the following equations by finding an integrating factor

$$\begin{array}{ll} \text{(i)} \quad y \, dx - x \, dy = xy^3 \, dy & \text{(ii)} \quad x \, dy - y \, dx = (2x^2 - 3)dx \\ \text{(iii)} \quad x \, dy = (x^5 + x^3y^2 + y) \, dx & \text{(iv)} \quad (2xy^2 - y) \, dx + x \, dy = 0 \\ \text{(v)} \quad (3x^2 - y^2) \, dy - 2xy \, dx = 0 & \text{(vi)} \quad (x+2) \sin y \, dx + x \cos y \, dy = 0 \\ \text{(vii)} \quad (3x^2y^3e^y + y^3 + y^2) \, dx + (x^3y^3e^y - xy) \, dy = 0 & \text{(viii)} \quad (5x^3 + 12x^2 + 6y^2) \, dx + 6xy \, dy = 0 \end{array}$$

6. Solve the following linear equations

$$\text{(i)} \quad y' + y = \frac{1}{1 + e^{2x}} \qquad \text{(ii)} \quad y' + y \cot x = 2x \csc x$$

7. The equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$ which is known as Bernoulli's equation, is linear when $n = 0$ or 1 . Show that it can be reduced to a linear equation for any other value of n by the change of variable $z = y^{1-n}$ and apply this method to solve the following equations:

$$\text{(i)} \quad xy' + y = x^4y^3 \qquad \text{(ii)} \quad x \, dy + y \, dx = xy^2 \, dx$$

Answers:

$$\begin{array}{llll} 1. \quad \text{(i)} \quad y^2 + 2xy = 3x^3 & \text{(ii)} \quad 2y^2 \ln(cy) = x^2, & \text{(iii)} \quad y = \frac{cx^2}{1-cx} & \text{(iv)} \quad y^3 = x^3 \log(cx^3) \\ 2. \quad \text{(i)} \quad y - x = 5 \log(x + y - 1) + c & & \text{(ii)} \quad (2x - y + 3)^4 = c(x + 1)^3 & \\ 3. \quad \text{(i)} \quad \text{Not Exact} & & \text{(ii)} \quad x^2y^4 + x \sin y = c & \\ 4. \quad n = 3; \quad x^2y^2 + 2x^3y = c & & & \\ 5. \quad \text{(i)} \quad \log \frac{x}{y} = \frac{y^3}{3} + c & \text{(ii)} \quad y = 2x^2 + 3 + cx & \text{(iii)} \quad \tan^{-1} \frac{x}{y} = -\frac{x^4}{4} + c & \text{(iv)} \quad y = \frac{x}{x^2+c} \\ & \text{(v)} \quad x^2 - y^2 = cy^3 & \text{(vi)} \quad x^2 e^x \sin y = c & \text{(vii)} \quad x^3 e^y + x + \frac{x}{y} = c \quad \text{(viii)} \quad x^5 + 3x^4 + 3x^2y^2 = c \\ 6. \quad \text{(i)} \quad y = e^{-x} \tan^{-1} e^{-x} + ce^{-x} & & \text{(ii)} \quad y = x^2 \csc x + c \csc x & \\ 7. \quad \text{(i)} \quad \frac{1}{y^2} = -x^4 + cx^2 & & \text{(ii)} \quad 1 + xy \log x = cxy & \end{array}$$