THAPAR INSTITUTE OF ENGINEERING AND TECHNOLOGY, PATIALA
DEPARTMENT OF CIVIL & MECHANICAL ENGINEERING
MECHANICS-UES009

BE FIRST YEAR SESSION: 2020-21

TUTORIAL SHEET-01 (REPRESENTATION OF FORCES AND THEIR RESULTANTS)

1. A force of 100 units acts along the line OP, terminating at P. If the coordinates of point O and P are (-3,1,2) and (10,5,8) respectively, specify the force in terms of unit vectors.
2. Compute the magnitude of the force F, whose components along the x, y, and z direction are 15kN, 26kN and -33KN, respectively. Also compute the inclination with all axes.
3. Forces 30kN, 40kN, 50kN and 60kN, are concurrent at O(1, 2, 3) and are directed through M(6, 3, -2), N(-4, -2, 5), P(-3, 2, 4) and Q(4, -3, 6), respectively. Determine the resultant of the system.
4. Determine the magnitude, direction and sense of the resultant of forces acting as shown in Fig. 1.
5. Two forces of magnitude 10N and 6N act on the ring. If the largest magnitude of the resultant force the ring can support is 14N, determine the angle between the forces, Fig 2.
6. The force F acting on the frame as shown in Fig. 3 has a magnitude of 500N and is to be resolved into two components acting along struts AB and AC. Determine the angle ϕ, measured below the horizontal, so that the component FAC is directed from A towards C and has a magnitude of 400 N.
7. In Fig. 4, if the tensions in the pulley cable are equal i.e. 400N, express the force R exerted on the pulley by the two tensions. Determine the magnitude of R.
8. Find out the magnitude, orientation and sense of the resultant force of the force system shown in Fig. 5. Forces are acting on the sides of the hexagon.

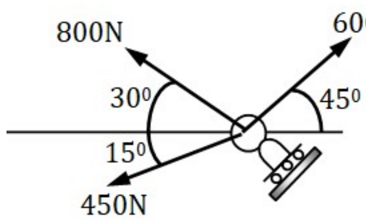


Fig. 1

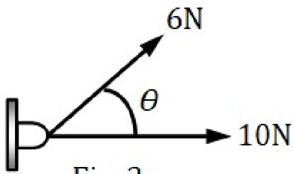


Fig. 2

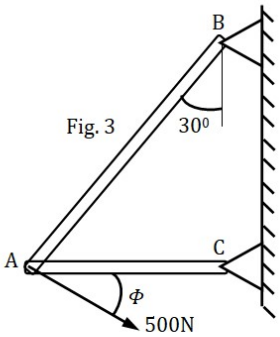


Fig. 3

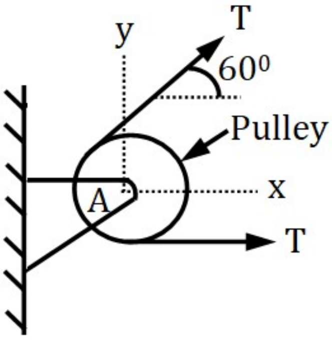


Fig. 4

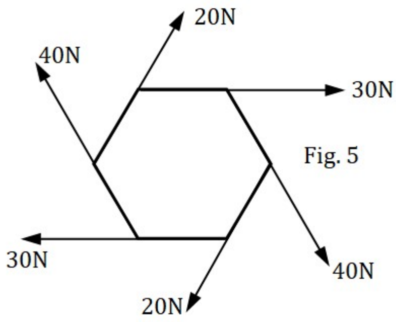


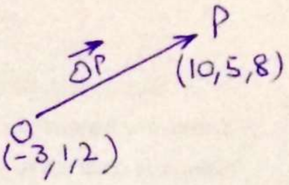
Fig. 5

Tutorial Sheet No 1

1.

Solution 1:

$|F| = 100$ units acting along $\vec{OP}$



$$\hat{OP} = \frac{(10-(-3))\hat{i} + (5-1)\hat{j} + (8-2)\hat{k}}{\sqrt{13^2+4^2+6^2}}$$

$$= \frac{13\hat{i} + 4\hat{j} + 6\hat{k}}{\sqrt{221}} = 0.874\hat{i} + 0.269\hat{j} + 0.404\hat{k}$$

Force vector $\bar{F} = 100 \times$ (unit vector)

$$\boxed{\vec{F} = 87.4\hat{i} + 26.9\hat{j} + 40.4\hat{k}}$$

Solution 2. To find the magnitude of force

$$\bar{F} = 15\hat{i} + 26\hat{j} - 33\hat{k}$$

$$|F| = \sqrt{15^2 + 26^2 + (-33)^2} = \sqrt{1990} = \boxed{44.61 \text{ KN}}$$

Inclination along all the axes

$$\alpha = \cos^{-1}\left(\frac{15}{44.61}\right) \Rightarrow 70.61^\circ$$

$$\beta = \cos^{-1}\left(\frac{26}{44.61}\right) \Rightarrow 54.35^\circ$$

$$\gamma = \cos^{-1}\left(\frac{-33}{44.61}\right) \Rightarrow 137.71^\circ$$

Solution 3. $F_1 = 30$ KN, $F_2 = 40$ KN, $F_3 = 50$ KN, $F_4 = 60$ KN

All the forces are concurrent at 'O'

Coordinates of all the points

O(1,2,3), M(6,3,-2), N(-4,-2,5), P(-3,2,4) and Q(4,-3,6)

Unit vector along OM

$$\hat{OM} = \frac{(6-1)\hat{i} + (3-2)\hat{j} + (-2-3)\hat{k}}{\sqrt{5^2+1^2+(-5)^2}} = \frac{5\hat{i} + \hat{j} - 5\hat{k}}{7.14}$$

$$\hat{ON} = \frac{(-4-1)\hat{i} + (-2-2)\hat{j} + (5-3)\hat{k}}{\sqrt{(-5)^2+(-4)^2+2^2}} = \frac{-5\hat{i} - 4\hat{j} + 2\hat{k}}{6.71}$$

$$\hat{OP} = \frac{(-3-1)\hat{i} + (2-2)\hat{j} + (4-3)\hat{k}}{\sqrt{(-4)^2+0^2+1^2}} = \frac{-4\hat{i} + \hat{k}}{4.123}$$

$$\hat{OQ} = \frac{(4-1)\hat{i} + (-3-2)\hat{j} + (6-3)\hat{k}}{\sqrt{3^2+5^2+3^2}} = \frac{3\hat{i}-5\hat{j}+3\hat{k}}{6.56}$$

$\bar{R} = \vec{F_1} + \bar{F_2} + \bar{F_3} + \bar{F_4}$

$F_1 = 30(\hat{OM})$
$F_2 = 40(\hat{ON})$
$F_3 = 50(\hat{OP})$
$F_4 = 60(\hat{OQ})$

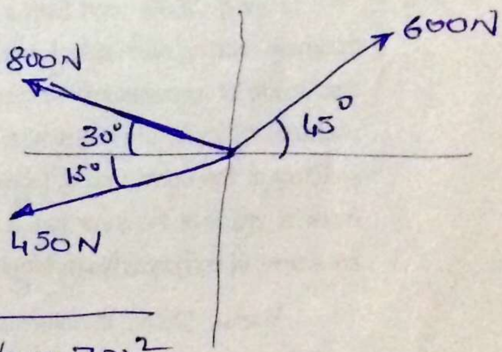
$$\bar{R} = 30\left(\frac{5\hat{i}+\hat{j}-5\hat{k}}{7.14}\right) + 40\left(\frac{-5\hat{i}-4\hat{j}+2\hat{k}}{6.71}\right) + 50\left(\frac{-4\hat{i}+\hat{k}}{4.123}\right) + 60\left(\frac{3\hat{i}-5\hat{j}+3\hat{k}}{6.56}\right)$$

$$= 4.201(5\hat{i}+\hat{j}-5\hat{k}) + 5.96(-5\hat{i}-4\hat{j}+2\hat{k}) + 12.127(-4\hat{i}+\hat{k}) + 9.146(3\hat{i}-5\hat{j}+3\hat{k})$$

$$= (21\hat{i}+4.20\hat{j}-21\hat{k}) + (-29.8\hat{i}-23.84\hat{j}+11.92\hat{k}) + (-48.51\hat{i}+12.127\hat{k}) + (27.44\hat{i}-45.73\hat{j}+27.44\hat{k})$$

$$\boxed{\bar{R} = -29.87\hat{i} - 65.37\hat{j} + 30.49\hat{k}}$$

Sol. 4



$\Sigma F_x = 0$

$= 600\cos 45° - 800\cos 30° - 450\cos 15°$

$= -703.22\ N$

$\Sigma F_y = 0$

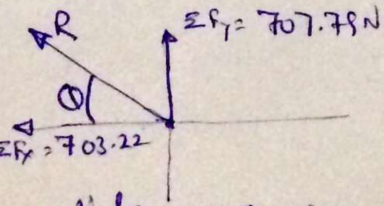
$= 600\sin 45° + 800\sin 30° - 450\cos 15$

$= 707.79\ N$

$\vec{R} = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{(-703.22)^2 + (707.79)^2}$

$\boxed{|R|} = \boxed{= 997.74\ N}$

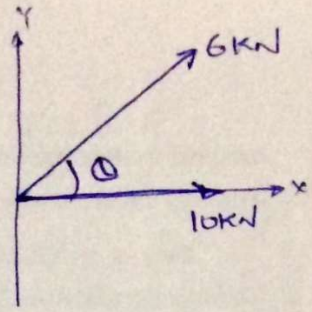
$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{707.79}{-703.22}\right)$ $\boxed{= -45.18° \text{ with horizontal}}$



Sense is anticlockwise.

Sol. 5

$R = 14\,kN, \ \theta = ?$



$\Sigma F_x = 10 + 6\cos\theta$

$\Sigma F_y = 6\sin\theta$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} =$

$14 = \sqrt{(10 + 6\cos\theta)^2 + (6\sin\theta)^2}$ Square both sides

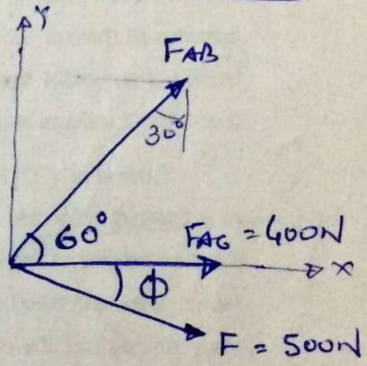
$196 = 100 + 36\cos^2\theta + 120\cos\theta + 36\sin^2\theta$

$= 100 + 36(\cos^2\theta + \sin^2\theta) + 120\cos\theta$

$120\cos\theta = 196 - 136 = 60$

$\cos\theta = 0.5, \quad \boxed{\theta = 60^\circ}$

Sol. 6



By sine law

$\dfrac{400}{\sin\theta} = \dfrac{500}{\sin 60} = \dfrac{F_{AB}}{\sin\phi}$

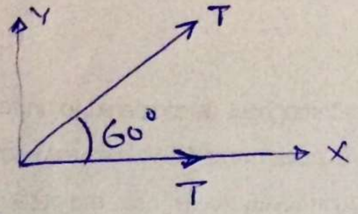
$\sin\theta = \dfrac{400}{500}\sin 60^\circ, \Rightarrow \theta = 43.85^\circ$

$\phi = 180 - (60 + 43.85) = \boxed{76.14^\circ}$

$F_{AB} = \dfrac{500 \times \sin 76.14}{\sin 60} = 560.54\,N$

Sol 7.

FBD at 'A'



$T = 400\,N$.

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$\Sigma F_x = T + T\cos 60 = 400 + 200 = 600\,N$.

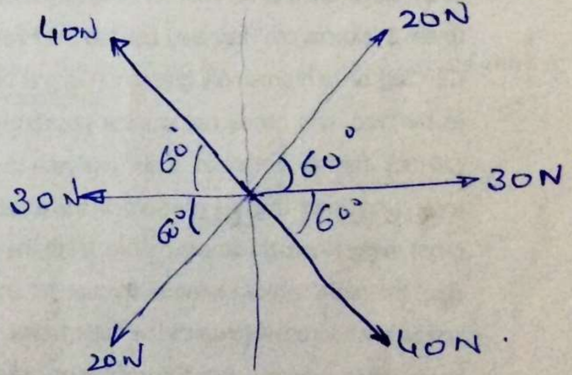
$\Sigma F_y = T\sin 60 = 400 \times 0.866 = 346.41\,N$.

$$R = \sqrt{(600)^2 + (346.41)^2} = \boxed{692.82\,N}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{346.41}{600}\right)$$

$$\boxed{= 29.99^\circ}$$

Sol. 8.



$\Sigma F_x = 0$

$= 30 + 20\cos 60 - 40\cos 60 - 30$
$\quad - 20\cos 60 + 40\cos 60°$
$= 0$

Similarly $\Sigma F_y = 0$

$\boxed{R = 0}$ $\boxed{\theta = 0°}$

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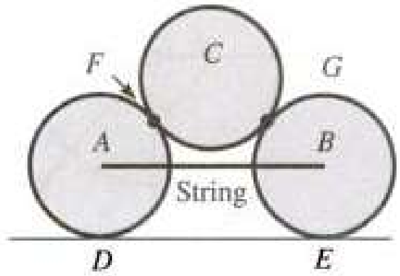
BE FIRST YEAR SESSION: 2020-21

TUTORIAL SHEET-02 (EQUILIBRIUM OF FORCES-PART 1)

1. Two smooth circular cylinders each of weight 1000 N and radius 15 cm, are connected at their centres by a string *AB* of length of 40 cm and rest upon a horizontal plane, supporting above them a third cylinder of weight 2000 N and radius 15 cm, as shown in Fig. 1. Find force in the string *AB* and the pressure produced on the floor at the points of contact *D* and *E*.
2. A rod *AB* as shown in Fig. 2, is held by a ball and socket point at *A* and supports a mass *C* weighing 1000 N at end *B*. The rod is in *x*-*y* plane and is inclined to *y*-axis at an angle of 18°. The rod is 12 m long and has negligible weight. Find the forces in the cable *DB* and *EB*.
3. Two weights are suspended from *B* and *C* points of a rope as shown in Fig. 3. If the distance *AD* is 6 m, how much will be the magnitude of ***W*** to maintain its equilibrium.
4. Two spheres weighing, 60N and 100N, are connected by a flexible string *AB*, and rest on two mutually perpendicular planes *PQ* and *QR* (Fig. 4). Find the tension in the string which passes freely through slots in smooth inclined planes *PQ* and *QR*.
5. A tower of 20 m height is supported by three ropes as shown in Fig. 5. If force in member *AB* is 50 N, find the force in *AC* and *AD* so that the resultant of these forces should be vertical.
6. Blocks *A* and *B* have masses 400 kg and 200 kg, respectively and rest on 27° incline as shown in Fig. 6. Blocks are attached to a post by cords and the post is held fixed by action of force *F*. Assuming all contact surfaces smooth and cords parallel to incline, determine the value of *F*. Also determine the ground reaction.

Fig. 1



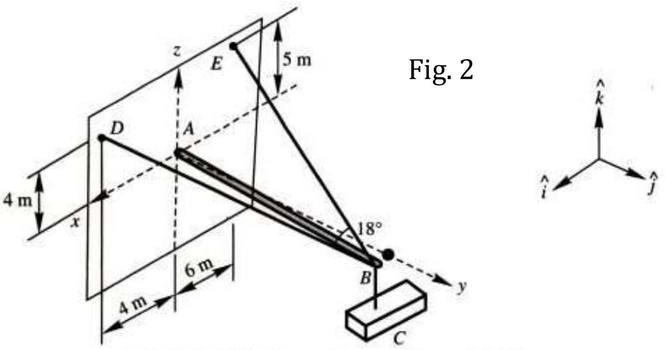
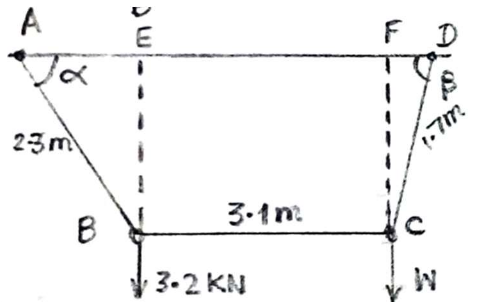


Fig. 2

Fig. 3



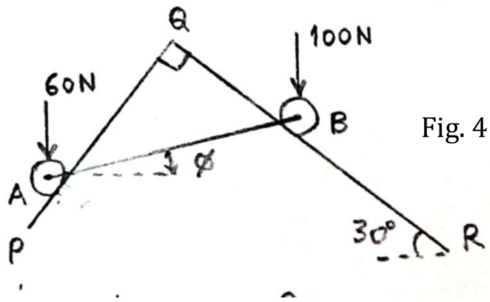


Fig. 4

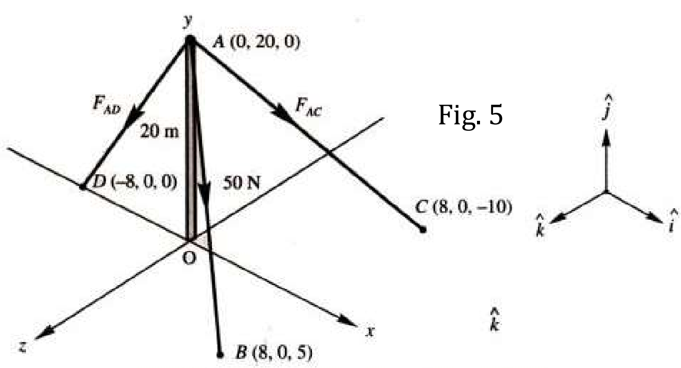
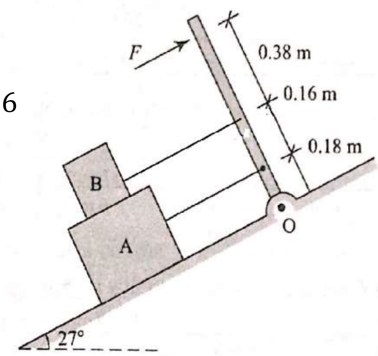


Fig. 5

Fig. 6



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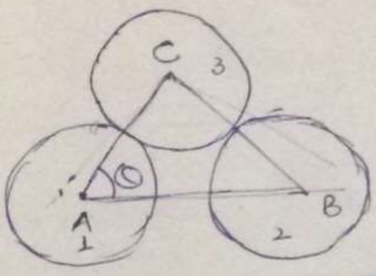
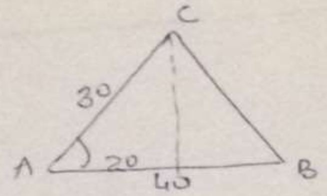
1.

Solutions: Tutorial Sheet No 2

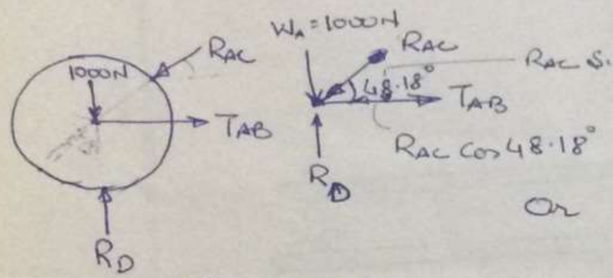




$\cos\theta = \frac{20}{30} = 0.6666$

$\theta = 48.18^\circ$

FBD of cylinder A



$\Sigma F_x = 0$

$T_{AB} = R_{AC}\cos 48.18^\circ$ — I

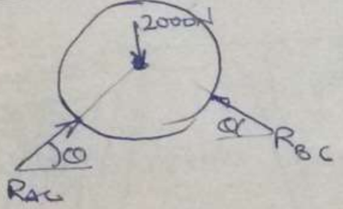
$\Sigma F_Y = 0$

$1000 + R_{AC}\sin 48.18 = R_D$ — II

Or From eq. I
$T_{AB} - 0.66 R_{AC} = 0$ — I

$R_D - 0.745 R_{AC} = 1000$ — II

FBD of C



$\Sigma F_x = 0$

$R_{AC}\cos 48.18 = R_{BC}\cos 48.18^\circ$

$\boxed{R_{AC} = R_{BC}}$

$\Sigma F_Y = 0$

$2 R_{AC}\sin 48.18 = 2000$

$\boxed{R_{AC} = 1342\ N}$, Put in eqn I

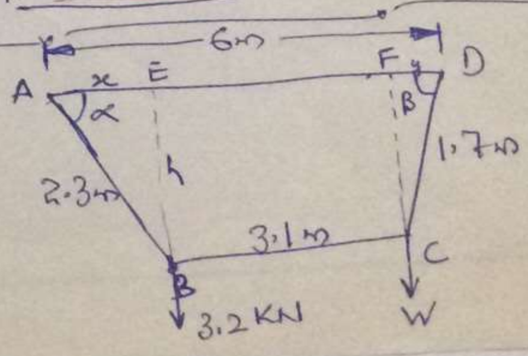
$T_{AB} - 0.66 \times 1342 = 0$, $\boxed{T_{AB} = 895\ N}$

From eqn II

$R_D - 0.745 \times 1342 = 1000$

$\boxed{R_D = 2000\ N} = \boxed{R_E = 2000\ N}$

Solution 3: Let $AE = x$, $FD = y$



$3.1 + x + y = 6$

$x + y = 6 - 3.1 = 2.9\ m$

$y = 2.9 - x$

From ΔABE

$(2.3)^2 = x^2 + h^2$ — (1) $BE = h$

In Δ CFD

$1.7^2 = h^2 + y^2 = h^2 + (2.9-x)^2$ — II

$(2.3)^2 = h^2 + x^2$

$(1.7)^2 = h^2 + x^2 + (2.9)^2 - 2\times 2.9\,x$

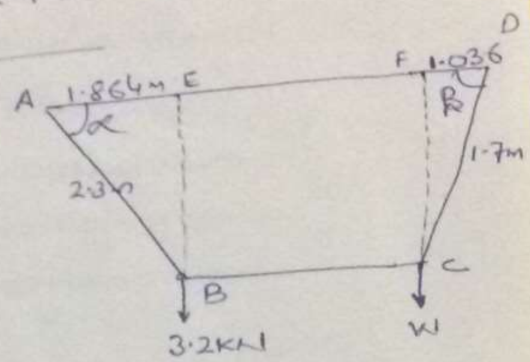
$2.4 = -8.41 + 5.8x$

$x = 1.864\text{ m}$

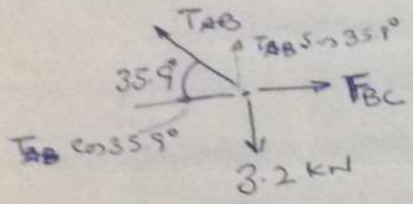
$y = \cancel{1.064\text{ m}}\ 1.036\text{ m}$.

$\alpha = 35.86^\circ$

$\beta = 52.45^\circ$



FBD of joint 'B'



$\Sigma F_y = 0$

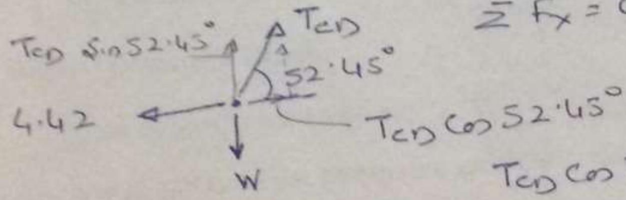
$T_{AB}\sin 35.9 = 3.2$

$T_{AB} = 5.46\text{ KN}$

$\Sigma F_x = 0 \qquad T_{AB}\cos 35.9^\circ = F_{BC}$

$F_{BC} = 4.42\text{ KN}$.

FBD of joint 'C'



$\Sigma F_x = 0$

$T_{CD}\cos 52.45 = 4.42$

$T_{CD} = 7.25\text{ KN}$.

$\Sigma F_y = 0$

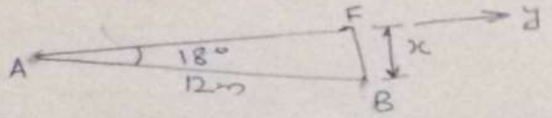
$W = T_{CD}\sin 52.42$

$\boxed{W = 5.75\text{ KN}}$

Solution 2:

Co-ordinates of point A(0,0,0)

To find coordinates of point 'B'



$\sin 18^\circ = \frac{x}{12}$ $\quad x = 3.708\,m.$

$\cos 18 = \frac{AF}{12}$, $AF = 11.412\,m.$

co-ordinates of point 'B' $= (3.708, 11.412, 0)$

" " " $D = (4, 0, 4)$

" " " $E = (-6, 0, 5)$

To find F_{BD} and F_{BE}

Unit vector along $\hat{DB} = \frac{(4-3.708)i + (0-11.412)j + (4-0)k}{\sqrt{(0.292)^2 + (11.412)^2 + (4)^2}}$

$\hat{DB} = \frac{0.292i - 11.412j + 4k}{12.096} = 0.024i - 0.943j + 0.33k$

$\hat{EB} = \frac{(-6-3.708)i + (0-11.412)j + (5-0)k}{\sqrt{(9.708)^2 + (11.412)^2 + (5)^2}}$

$= -0.6146i - 0.7225j + 0.3165k$

$\overline{W} = -1000k$

$\overline{F_{BD}} + \overline{F_{BE}} + \overline{W} = 0$

$\overline{F_{BD}} = |F_{BD}| \times \hat{DB}$, $\overline{F_{BE}} = |F_{BE}| \times \hat{EB}$

$F_{BD}[0.024i - 0.943j + 0.33k] + F_{BE}[-0.6146i - 0.7225j + 0.3165k]$
$-1000k = 0$

$0.33F_{BD} + 0.3165F_{BE} - 1000 = 0$ — I

$0.024F_{BD} - 0.6146F_{BE} = 0$ —— II

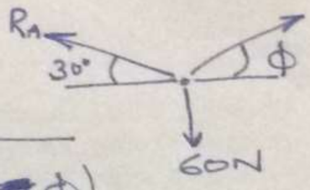
Solving equations I and II

$\boxed{\begin{array}{l} F_{BD} = 2920.92N \\ F_{BE} = 114.05N \end{array}}$ Ans

Solution 4:

Let T_{AB} be the tension in the string.

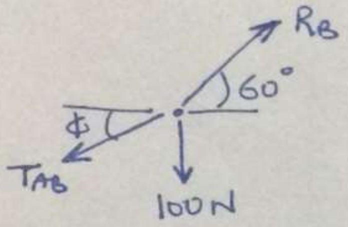
FBD of sphere 'A'



$$\frac{T_{AB}}{\sin 120} = \frac{R_A}{\sin(90+\phi)} = \frac{60}{\sin(180-30-\phi)}$$

$$T_{AB} = \frac{60 \sin 120}{\sin(90+60-\phi)} = \frac{51.96}{\cos(60-\phi)} \quad \text{— I}$$

FBD of sphere 'B'



$$\frac{T_{AB}}{\sin 150} = \frac{R_B}{\sin(90-\phi)} = \frac{100}{\sin(90+30+\phi)}$$

$$T_{AB} = \frac{100 \times \sin 150}{\cos(30+\phi)} \quad \text{— II}$$

$$= \frac{50}{\cos(30+\phi)}$$

From eqn (I) & (II)

$$\frac{51.96}{\cos(60-\phi)} = \frac{50}{\cos(30+\phi)} \Rightarrow \frac{\sin(30+\phi)}{\cos(30+\phi)} = 1.04$$

$$\boxed{\phi = 16.1^\circ}$$

From eq. I or II

$$T_{AB} = \frac{51.96}{\cos(60-16.1)} = 72.11 \text{ N}$$

$$\boxed{T_{AB} = 72.11 \text{ N}}$$

Solution 5. $F_{AB} = 50N$. Unit vector

$$\hat{AB} = \frac{(8-0)\hat{i} + (0-20)\hat{j} + (5-0)\hat{k}}{\sqrt{8^2+20^2+5^2}} = \frac{8\hat{i}-20\hat{j}+5\hat{k}}{\sqrt{489}} = \frac{8\hat{i}-20\hat{j}+5\hat{k}}{22.113}$$

$$\hat{AC} = \frac{(8-0)\hat{i} + (0-20)\hat{j} + (-10-0)\hat{k}}{\sqrt{8^2+20^2+10^2}} = \frac{8\hat{i}-20\hat{j}-10\hat{k}}{\sqrt{564}} = \frac{8\hat{i}-20\hat{j}-10\hat{k}}{23.748}$$

$$\hat{AD} = \frac{(-8-0)\hat{i} + (0-20)\hat{j} + (0-0)\hat{k}}{\sqrt{8^2+20^2}} = \frac{-8\hat{i}-20\hat{j}}{\sqrt{464}} = \frac{-8\hat{i}-20\hat{j}}{21.54}$$

$$\hat{AB} = 0.3617\hat{i} - 0.9044\hat{j} + 0.2261\hat{k}$$
$$\hat{AC} = 0.3368\hat{i} - 0.8422\hat{j} - 0.4211\hat{k}$$
$$\hat{AD} = -0.3714\hat{i} - 0.9285\hat{j}$$

For the resultant need to be vertical, $\hat{i}$, $\hat{k}$ component will be zero.

$$\bar{R} = 0\hat{i} + x\hat{j} + 0\hat{k}$$
$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$$

$$0\hat{i} + x\hat{j} + 0\hat{k} = \frac{50(8\hat{i}-20\hat{j}+5\hat{k})}{22.113} + F_{AC}\frac{(8\hat{i}-20\hat{j}-10\hat{k})}{23.748} + \frac{F_{AD}(-8\hat{i}-20\hat{j})}{21.54}$$

considering $\hat{i}$ and $\hat{k}$ component only

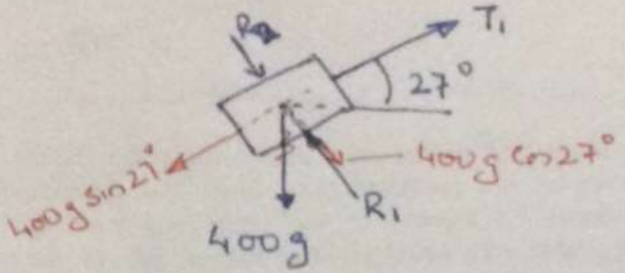
$$18.088 + 0.3368F_{AC} - 0.3714F_{AD} = 0 \quad \text{— I}$$
$$11.3055 - 0.4211F_{AC} - 0F_{AD} = 0 \quad \text{— II}$$

From eqn - II, $F_{AC} = \frac{11.3055}{0.4211} \Rightarrow \boxed{F_{AC} = 26.85\,N}$

$$18.088 + 0.3368 \times 26.85 = 0.3714F_{AD}$$

$$\boxed{F_{AD} = 73.05\,N}$$

Solution 6: FBD of block 'A'

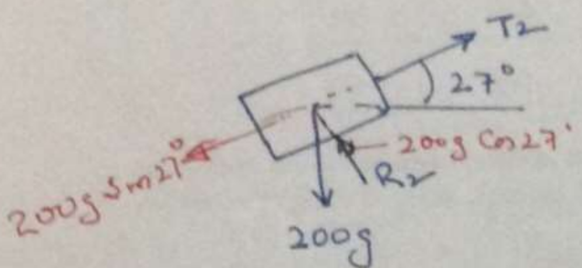


Considering equilibrium along the plane:

$T_1 = 400 \times 9.81 \sin 27^\circ$

$= 1781.46\ N$

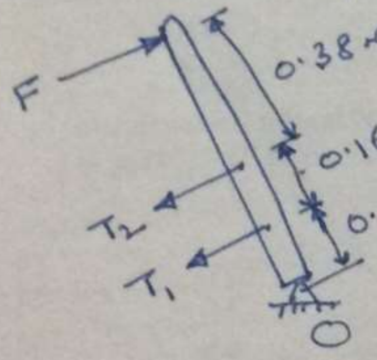
FBD of block 'B'



$T_2 = 200 \times 9.81 \sin 27^\circ$

$= 890.73\ N$

FBD of the rod



$\Sigma M_O = 0$

$0.72F = 0.18\,T_1 + 0.34\,T_2$

$= 0.18 \times 1781.46 + 0.34 \times 890.73$

$= 623.51\ N$

$\Rightarrow \boxed{F = 866\ N}$

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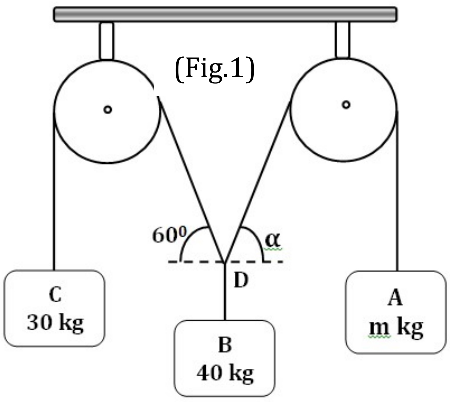
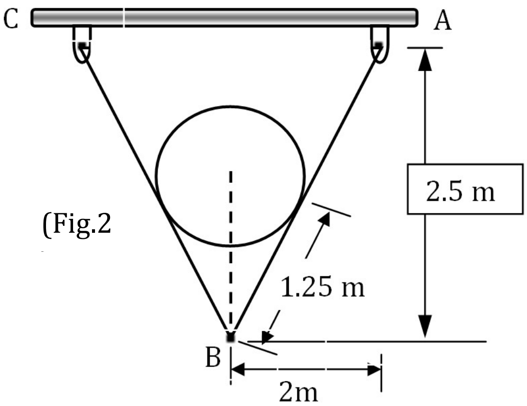
TUTORIAL SHEET-03 (EQUILIBRIUM OF FORCES-PART 2)

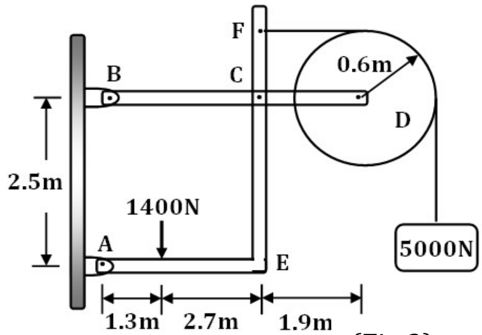
1. Determine the mass that must be supported at A and the angle α of the connecting rod in order to hold the system as shown in Fig. 1 in equilibrium.
2. A 2m diameter cylindrical tank is to be supported at each end of a hanger arranged as shown in Fig. 2. The total weight supported by the two hangers is 15 kN. Determine the force in the pins A and B due to the weight of the tanks. Hangers are weightless and contacts are frictionless.
3. Two weightless bars BCD and AECF are hinged together at C. The pulley at D has a diameter of 1.2m and a mass of 200 kg. Making use of free body diagram, determine forces transmitted from one bar to another at C (Fig. 3).
4. Beam MN supports beam PKQ with the help of three bars PN, PJ, and KN as shown in Fig. 4. Determine the reaction at supports M and N of the lower beam.
5. A bracket is subjected to a coplanar force system as shown in Fig. 5. Determine the magnitude and the line of action of the single resultant of the system. If the resultant is to pass through the point *B*, what should be the magnitude and direction of the couple?
6. Replace the forces acting on the pipe as shown in Fig. 6 by an equivalent single force and couple system acting at point *A*.
7. A ladder of length 5m has a weight of 200N. The foot of the ladder rests in the floor and the top of it leans against the vertical wall. Both the wall and the floor are smooth. The ladder is inclined at 60° with the floor; a weight of 300N is suspended at the top of the ladder. Find the value of the horizontal force to be applied at the foot of the ladder to keep it in equilibrium.



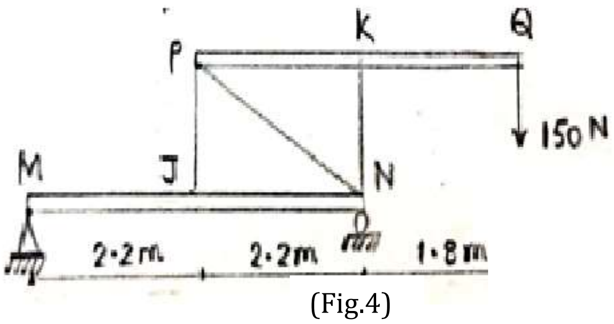
(Fig.1)



(Fig.2

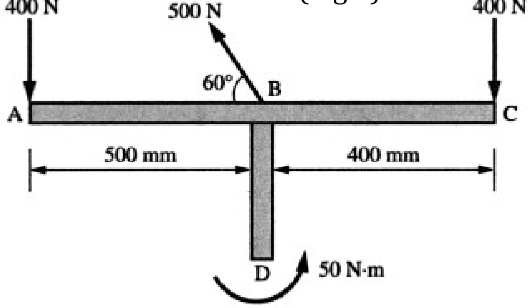


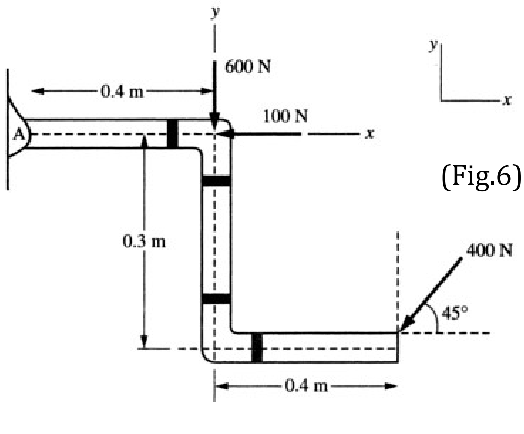
(Fig.3)



(Fig.4)

(Fig.5)



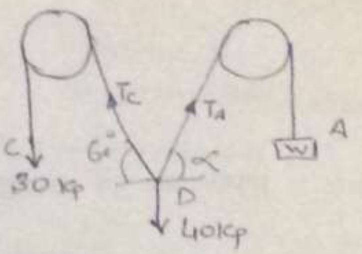


(Fig.6)

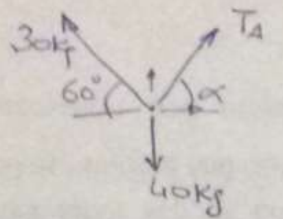
1.

Solution 1 Tutorial sheet No 3.

FBD at 'D'



$T_c = 30\,kg$



$\Sigma F_x = 0$ $T_A \cos\alpha = 30 \times 9.81 \cos 60 = 147.15\,N$ —— I

$\Sigma F_y = 0$ $T_A \sin\alpha + 30 \times 9.81 \sin 60 = 40 \times 9.81$

$T_A \sin\alpha = 40 \times 9.81 - 30 \times 9.81 \sin 60 = 137.53\,N$ — II

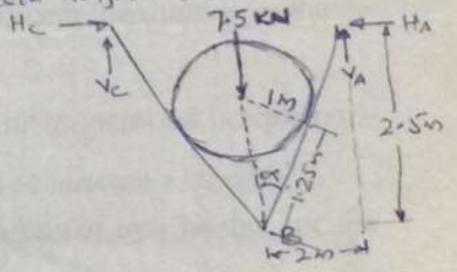
$\tan\alpha = \dfrac{137.53}{147.15} = 0.9346$

$\boxed{\alpha = 43.06^\circ}$

$T_A = \dfrac{147.15}{\cos 43.06} = 201.41\,N$

$\boxed{T_A = 201.41\,N = W}$

Solution 2 Weight supported by each hanger = 7.5 kN.



$H_A = H_C$

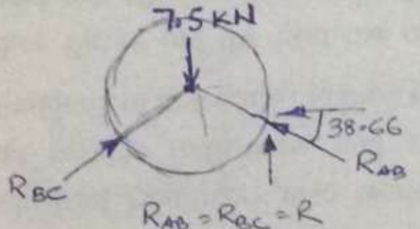
$V_A + V_C = 7.5$

$V_A = V_C = 3.75\,kN$

$\tan\alpha = \dfrac{1}{1.25}$

$\alpha = 38.66^\circ$

FBD of Cylinder



$R_{AB} = R_{BC} = R$

$\Sigma F_y = 0$

$2R \sin 38.66 = 7.5$

$R = 6\,kN$

$\Sigma M_B = 0$

$6 \times 1.25 - 3.75 \times 2 - 2.5 H_A = 0$

$H_A = 0 = H_C$

$\Sigma F_x = 0$

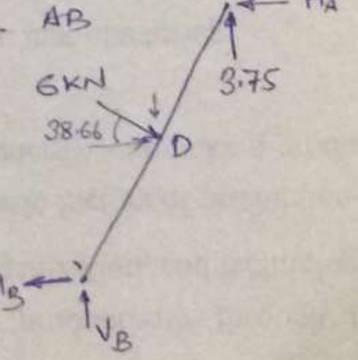
$6 \cos 38.66 = \boxed{H_B = 4.69\,kN}$

$\Sigma F_y = 0$

$3.75 + V_B = 6 \sin 38.66$

$\boxed{V_B = 0}$

FBD of link AB

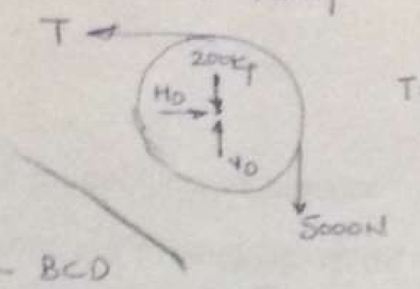


Solution 3:

3.

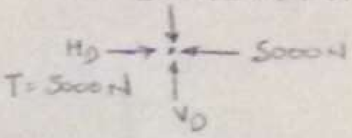
Tension in the wire, T = 5000 N

FBD of Pulley –



FBD at D

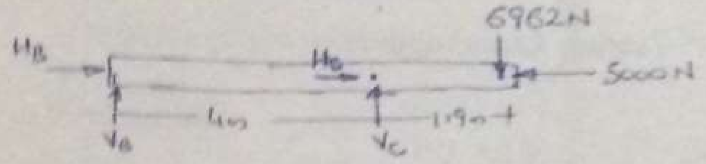
$5000 + 200 \times 9.81 = 6962$ N



$H_D = 5000$ N

$V_D = 6962$ N

FBD of link BCD

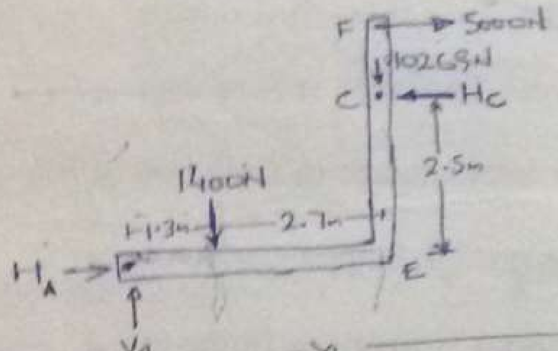


$\Sigma M_B = 0$

$6962 \times 5.9 = 4 V_C$

$\boxed{V_C = 10269 \text{ N}}$

FBD of link AECF



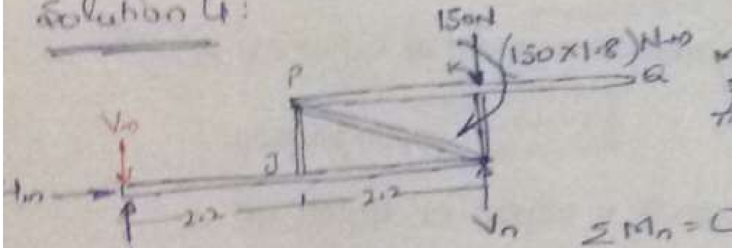
$\Sigma M_A = 0$

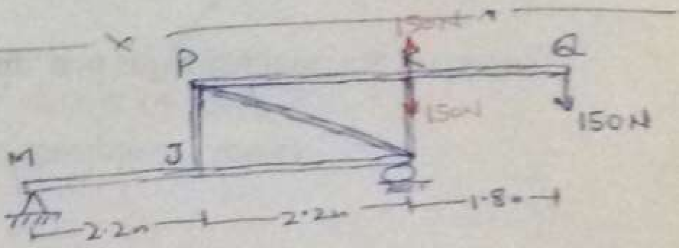
$1400 \times 1.3 - 2.5 H_C + 10269 \times 4 + 5000 \times 3.1 = 0$

$1820 - 2.5 H_C + 41076 + 15500 = 0$

$\boxed{H_C = 23358 \text{ N}}$

Solution 4:





$V_m + V_n = 150$ N

$H_m = 0$

$\Sigma M_n = 0$

$4.4 V_m + 150 \times 1.8 = 0$

$\boxed{V_m = -61.36 \text{ N}}$

$V_n = 150 - V_m$

$= 150 - (-61.36) =$

$\boxed{V_n = 211.36 \text{ N}}$

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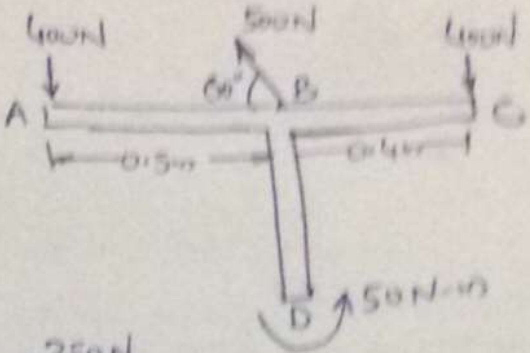
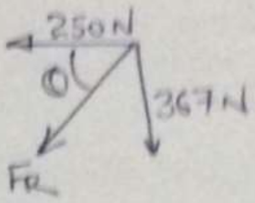
Solution 5



$\Sigma F_x = 500\cos 60 = 250\,N \leftarrow$

$\Sigma F_y = 400 - 500\sin 60 + 400$
$= 367\,N \downarrow$

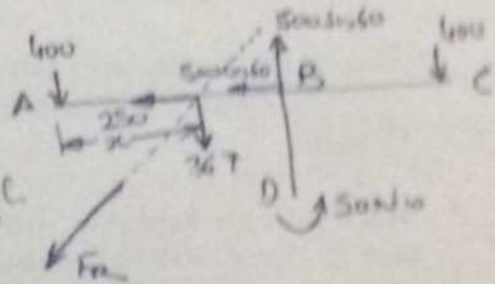


$F_R = \sqrt{(250)^2 + (367)^2}$
$= \boxed{444.06\,N}$

$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{367}{250}\right) \Rightarrow 55.74°$

$\Sigma M_A =$
$-500\sin 60 \times 0.5 + 400 \times 0.9 - 50 = 93.5\,N\text{-}m$

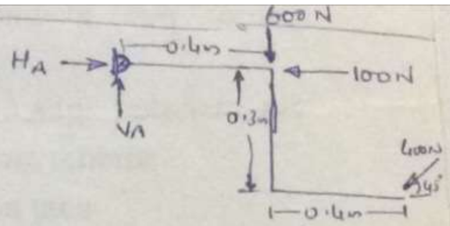
$367 \cdot x = 93.5$
$x = 0.255\,m$



If the resultant is to pass through 'B'

$\Sigma M_B = -400 \times 0.5 + 400 \times 0.4 - 50 + M = 0$
$-200 + 160 - 50 + M = 0$
$\boxed{M = 90\,N\text{-}m}$

Solution 6



$\Sigma F_x = 0$

$H_A - 100 - 400\cos 45 = 0$

$H_A = 382.84\ N$

$\Sigma F_y = 0 \quad V_A - 600 - 400\sin 45° = 0$

$V_A = 882.84\ N$

$\Sigma M_A = 0 = 600 \times 0.4 + 400\sin 45 \times 0.8 + 400\cos 45 \times 0.3$

$= 240 + 226.27 + 84.85 =$

$= 551.12\ Nm$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{(382.84)^2 + (882.84)^2}$

$= 962.27\ N.$

$\theta = \tan^{-1}\left(\frac{\Sigma F_Y}{\Sigma F_x}\right) = 66.56°$

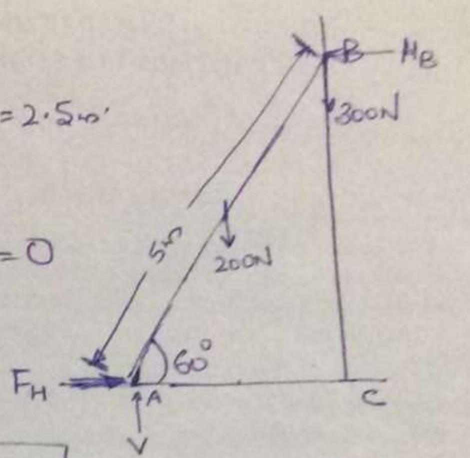
$d = \frac{\Sigma M_A}{R} = \frac{551.12}{962.27} = 0.5727\ m.$

$\bar{x} = \frac{\Sigma M_A}{\Sigma F_Y} = \frac{551.12}{882.84} = 0.624\ m.$

$\bar{y} = \frac{\Sigma M_A}{\Sigma F_x} = \frac{551.12}{382.84} = 1.439\ m.$

Magnitude and couple system ⇒ One has to find Resultant, direction and distance. ie Resultant is the opposite force which is acting on the member so, that the body is at equilibrium.

Solution 7



$\cos 60 = \frac{AC}{5}$

$AC = 5\cos 60 = 2.5\ m$

$BC = 4.33\ m.$

$\Sigma M_A = 0$

$300 \times 2.5 - 4.33\ H_B + 200 \times 1.25 = 0$

$750 + 250 = 4.33\ H_B$

$H_B = 230.94\ N$

$\Sigma F_x = 0$

$\boxed{F_H = H_B = 230.94\ N}$

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TUTORIAL SHEET-04 (ANALYSIS OF TRUSSES- METHOD OF JOINTS)

1. If the roller at B can sustain a maximum load of 3 kN, determine the largest magnitude of each of the three forces F that can be supported by the truss. (Fig.1)
2. A signboard, 3 m x 4 m area, is supported by two frames as shown in Fig.2, Calculate the forces in each member due to horizontal wind load of 1 kN/m^2 on the signboard. Assume all truss members are pinned and that two-sixth of the total load is concentrated at B and one-sixth at C.
3. Determine the member forces in the truss shown in Fig.3.
4. Determine the forces in all the members of the truss shown in Fig.4.
5. Determine the forces in all the members of the truss shown in Fig.5. Indicate tension or compression. Also, indicate all zero force members.

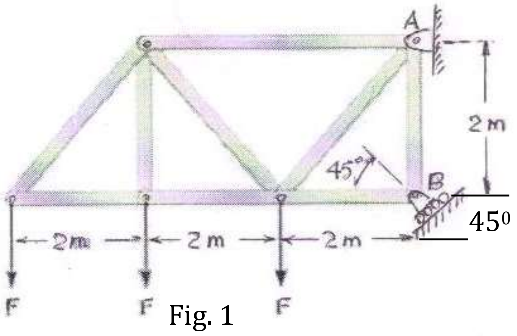


Fig. 1

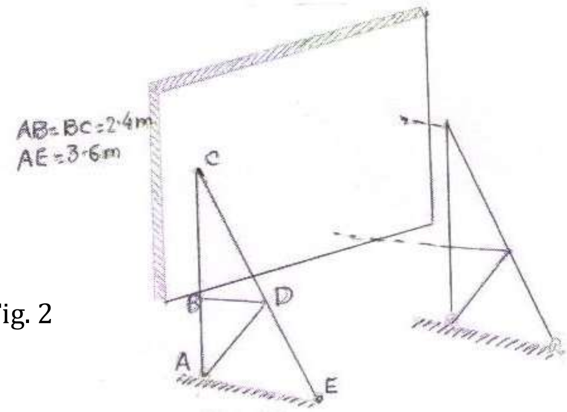
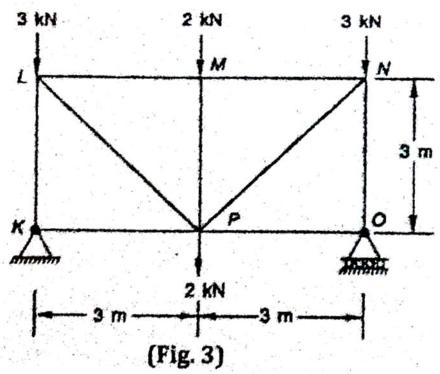


Fig. 2



(Fig. 3)

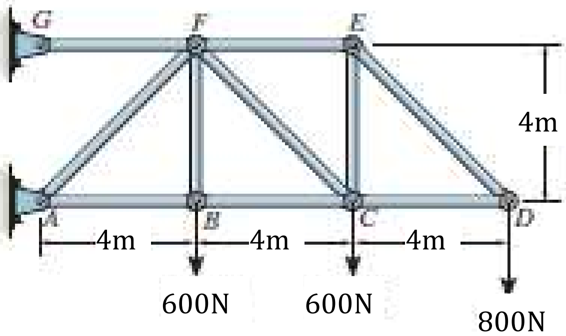


Fig. 4

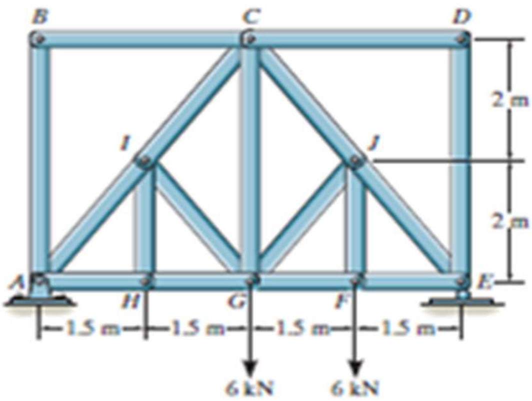


Fig. 5

Tutorial Sheet NO 4.

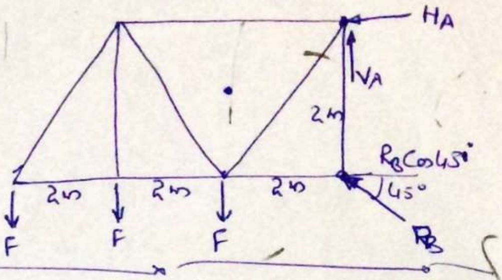
Solution 1

$\Sigma M_A = 0$

$2F + 4F + 6F = 2R_B \cos 45^\circ$

$12F = 2 \times 3 \times 0.707$

$\boxed{F = 0.353 \text{ KN}}$



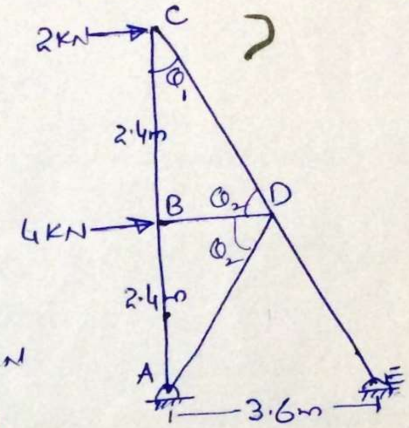
Solution 2

Area $= 3 \times 4 = 12 m^2$

Total load $= 12 \times 1 = 12$ KN

Load at B $= \frac{2}{6} \times 12 = 4$ KN

" " $C = \frac{1}{6} \times 12 = 2$ KN.

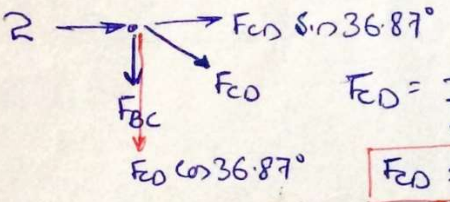


Equilibrium of joint 'C'

$\tan \theta_1 = \frac{3.6}{4.8}$

$\theta_1 = 36.87^\circ$

$\theta_2 = 53.13^\circ$

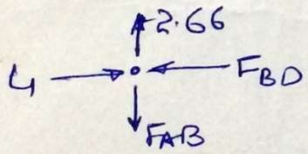


$F_{CD} = \frac{-2}{\sin 36.87} = -3.33$ KN

$\boxed{F_{CD} = 3.33 \text{ KN (comp)}}$

$F_{BC} + F_{CD} \cos 36.87 = 0$, $F_{BC} = -(-3.33) \cos 36.87^\circ$. $\boxed{F_{BC} = 2.66 \text{ KN (T)}}$

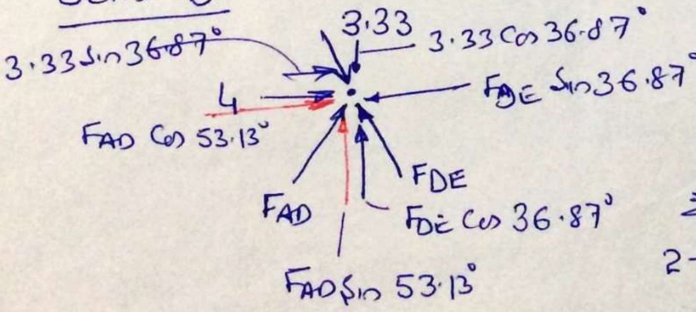
Joint 'B'



$\boxed{F_{BD} = 4 \text{ KN (comp)}}$

$\boxed{F_{AB} = 2.66 \text{ KN (T)}}$

Joint 'D'



$\Sigma F_V = 0$

$-2.66 + 0.8 F_{DE} + 0.8 F_{AD} = 0$

$0.8(F_{AD} + F_{DE}) = +2.66$

$F_{AD} + F_{DE} = +3.325$ — I.

$\Sigma F_H = 0$

$2 + 4 + 0.6 F_{AD} - F_{DE} \times 0.6 = 0$

$0.6(F_{AD} - F_{DE}) = -6$

$F_{AD} - F_{DE} = -10$ — II

$F_{AD} + F_{DE} = 3.325$ — I

$2F_{AD} = -6.675$

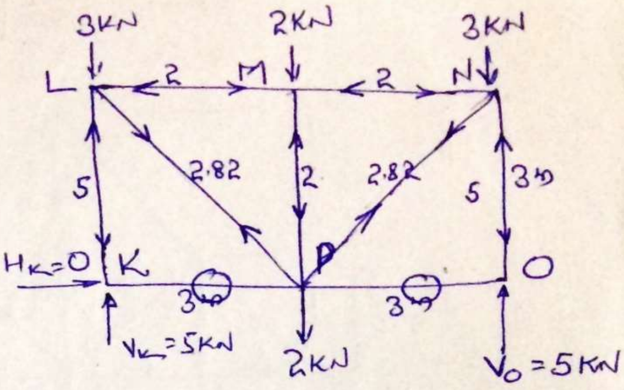
$\boxed{F_{AD} = 3.37 \text{ KN (T)}}$

$\boxed{F_{DE} = 6.66 \text{ KN (comp)}}$

...tion 3:

Joint 'K'

$\boxed{F_{KL} = 5\,KN\ (\text{comp})}$



Joint 'L'

$\Sigma F_V = 0 \quad F_{PL}\sin 45^\circ = 5-3 = 2, \quad \boxed{F_{PL} = 2.82\,KN\ (T)}$

$F_{LM} + F_{PL}\cos 45^\circ = 0, \quad F_{LM} = -2KN, \quad \boxed{F_{LM} = 2KN\ (\text{comp})}$

Joint 'M'

$\boxed{F_{PM} = 2KN\ (C)}$ $\boxed{F_{MN} = 2KN\ (\text{comp})}$

Joint 'N'

$F_{PN}\sin 45^\circ = 2$

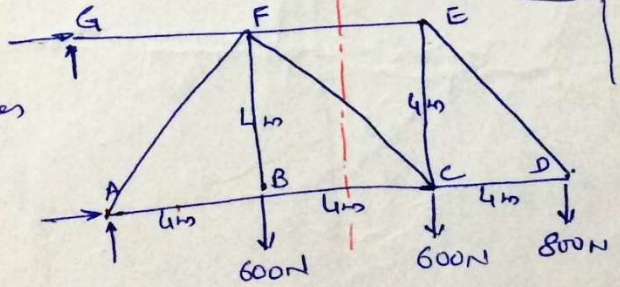
$\boxed{F_{PN} = 2.82\,KN\ (T)}$

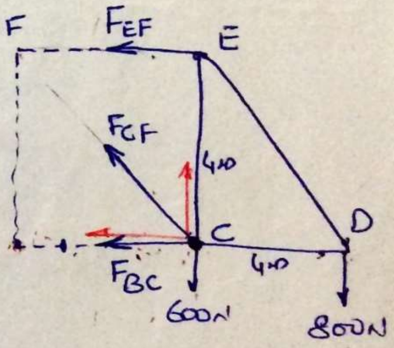
$F_{NO} - F_{PN}\cos 45^\circ = 3$

$\boxed{F_{NO} = 3+2 = 5\,KN\ (\text{comp})}$

Solution 4:

Determine forces in members BC, EF & CF.





$\Sigma M_C = 0$

$-4F_{EF} + 800\times 4 = 0, \quad \boxed{F_{EF} = 800N\ (T)}$

$\Sigma M_F = 0$

$4F_{BC} + 600\times 4 + 800\times 8 = 0, \quad F_{BC} = \frac{-8800}{4} = -2200\,N$

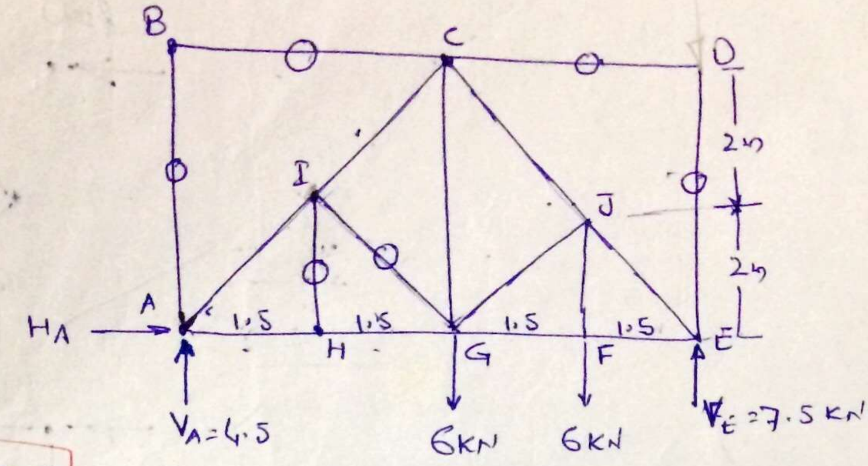
$\boxed{F_{BC} = 2200\,N\ (\text{comp})}$

$\Sigma F_Y = 0$

$F_{CF}\sin 45^\circ = 600 + 800 = 1400$

$F_{CF} = \frac{1400}{\sin 45}, \quad \boxed{F_{CF} = 1980\,N\ (T)}$

Solution 5.



$\boxed{H_A = 0}$

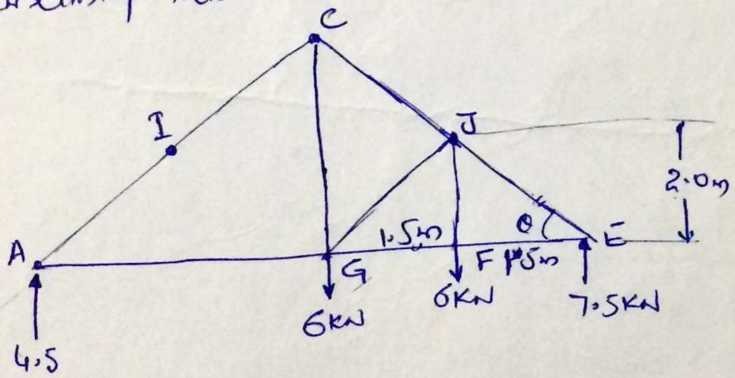
$\Sigma M_A = 0 \qquad 6V_E = 6\times 3 + 6\times 4.5 \qquad V_E = 7.5\text{ KN}.$

$V_E + V_A = 6 + 6 = 12, \qquad V_A = 4.5\text{ KN}$

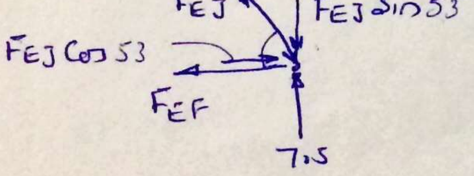
Zero force member

$F_{AB} = F_{BC} = F_{CD} = F_{DE} = F_{HI} = F_{GI} = 0$

The remaining truss will be



Equilibrium of joint E



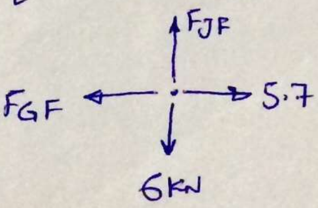
$F_{EJ}\sin 53° = 7.5$

$F_{EJ} = \dfrac{7.5}{\sin 53} =$

$\boxed{F_{EJ} = 9.4\text{KN (Comp)}}$

$F_{EJ}\cos 53 = F_{EF} = 5.7\text{KN (T)}$

Equilibrium of joint F



$F_{FJ} = 6\text{ KN (T)}$

$\boxed{F_{GF} = 5.7\text{ KN (T)}}$

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TUTORIAL SHEET-05 (ANALYSIS OF TRUSSES- METHOD OF SECTION)

1. Determine the forces in the members *EF* and *GI* of the truss shown in Fig. 1. Forces acting at joints *C*, *G* and *K* are in kN and all distances in m.
2. Determine the forces in the members *BD, DE* and *EG* of the roof truss shown in Fig. 2.
3. Determine the forces in the members *FH, GH* and *GI* of the roof truss shown in Fig. 3.
4. Find the forces in the members BC, CE and DE for the truss shown in Fig. 4.
5. Determine the forces in members GH, CD and CG for the truss loaded and supported as shown in Fig. 5.
6. Find the forces in the members GH, GD and CD for the truss shown in Fig. 6.

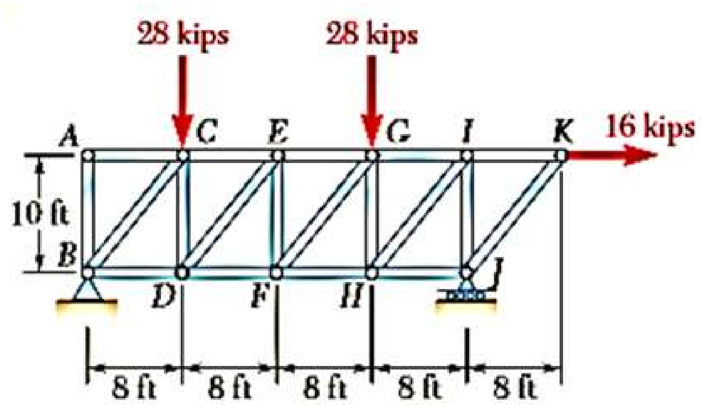


Fig. 1

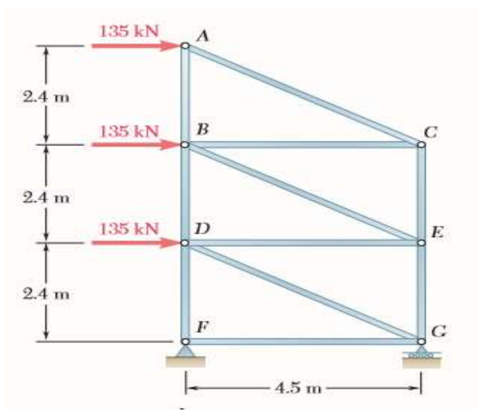


Fig. 2

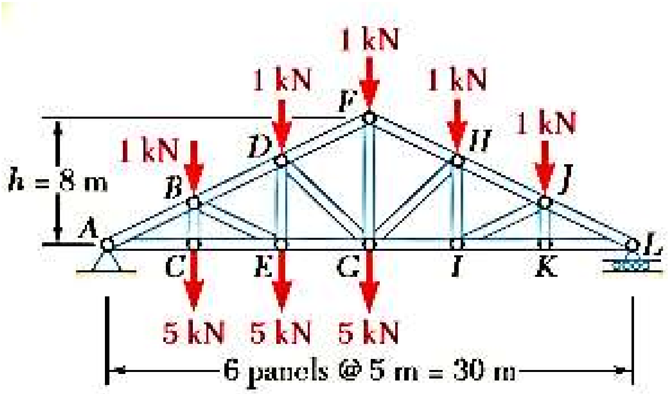


Fig. 3

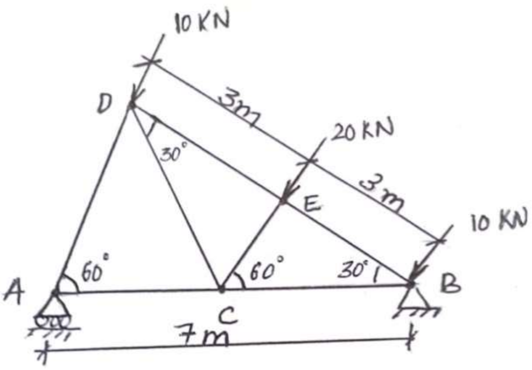


Fig. 4

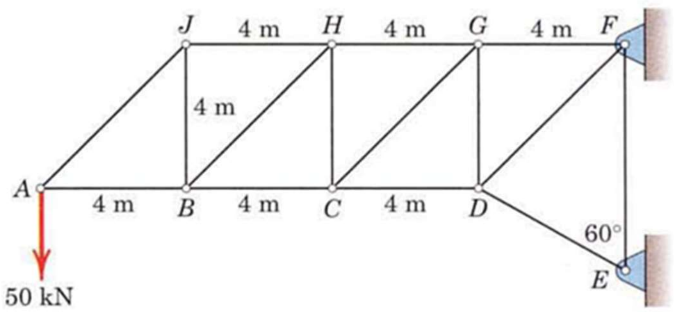


Fig. 5

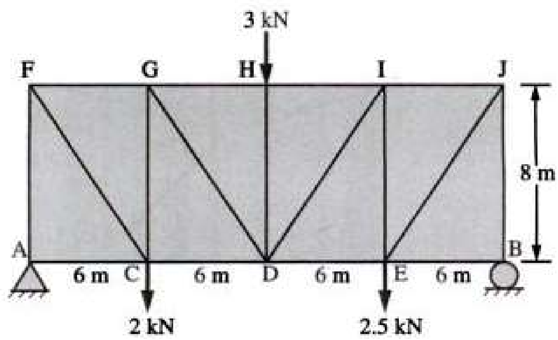
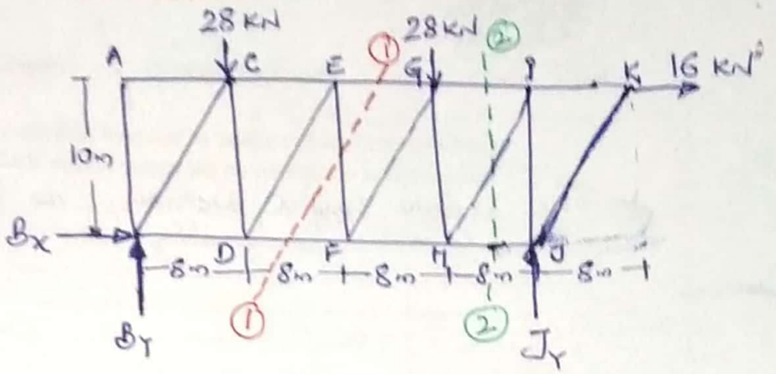


Fig. 6

Solutions

Tutorial Sheet No 5

Solution 1.



$\Sigma M_B = 0$

$28\times8 + 28\times24 + 16\times10 - J_Y \times 32 = 0$

$J_Y = 33\,kN$

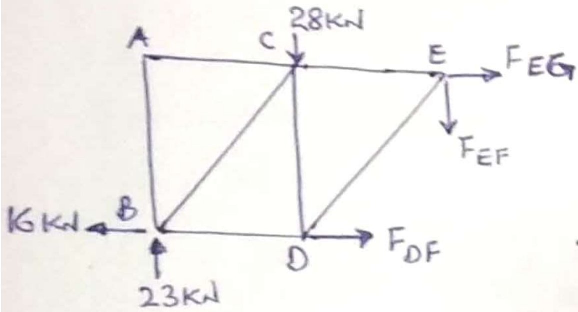
$\Sigma F_x = 0$

$B_x + 16 = 0, \quad B_x = -16\,kN$

$\Sigma F_Y = 0$

$B_Y + J_Y = 28 + 28 = 56\,kN$

$B_Y = 56 - 33 = 23\,kN.$



To find force in member EF:

Take section 1-1 and draw LHS portion of the truss as a free body

$\Sigma F_Y = 0$

$23 - 28 - F_{EF} = 0$

$F_{EF} = -5\,kN$

The nature of force in bar EF was assumed as tension, but the -ve sign indicates that it is compression.

$\boxed{F_{EF} = 5\,kN\,(comp)}$

Similarly forces in member EF and DF can also be determined

To find force in member GI, take another section 2-2 that intersects the member GI and ~~and~~ only two more members and draw FBD of right hand portion of the truss.

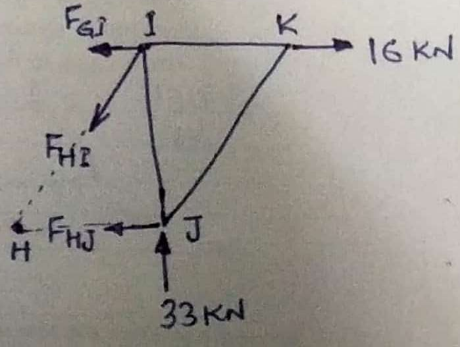
Extend the lines of forces HI and HJ to meet at H.

$\Sigma M_H = 0$

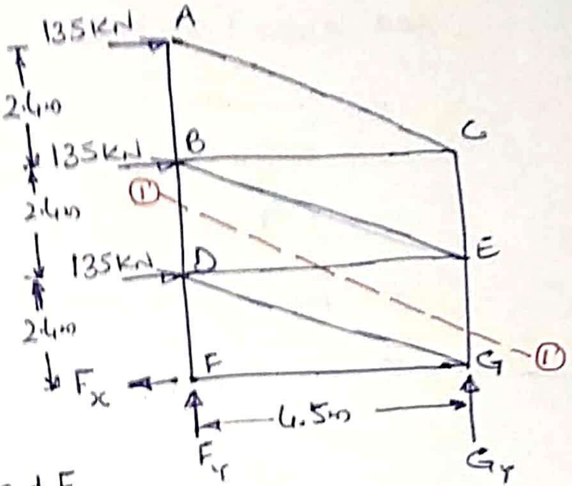
$16\times10 - F_{GI}\times10 - 33\times8 = 0$

$F_{GI} = -10.4\,kN$

$\boxed{F_{GI} = 10.4\,kN\,(comp)}$



Solution 2



$\Sigma F_x = 0$

$F_x = 135 + 135 + 135 = 405$ kN

$\Sigma F_Y = 0$ $\quad F_Y + G_Y = 0;\ F_Y = -G_Y$

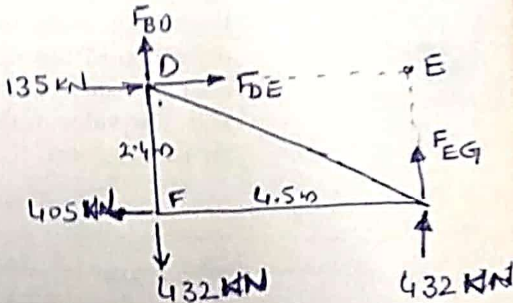
$\Sigma M_F = 0$

$135 \times 2.4 + 135 \times 4.8 + 135 \times 7.2 - G_Y \times 4.5 = 0$

$\boxed{G_Y = 432 \text{ kN}}$

$\boxed{F_Y = -432 \text{ kN}}$

Take section 1-1 to find F_{BD}, F_{DE} and F_{EG}



$\Sigma F_x = 0$

$135 + F_{DE} - 405 = 0$

$\boxed{F_{DE} = 270 \text{ kN (T)}}$

$\Sigma M_E = 0$

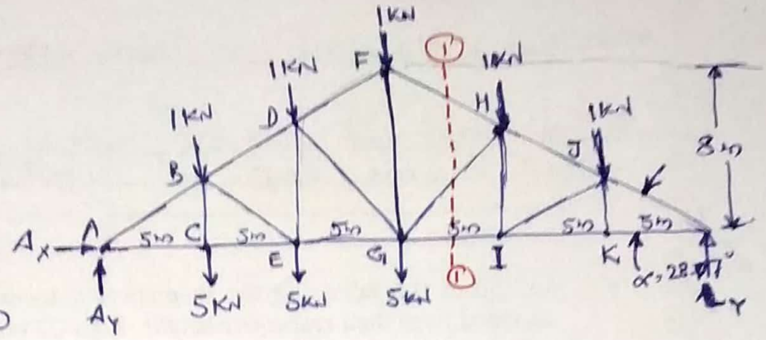
$F_{BD} \times 4.5 + 405 \times 2.4 - 432 \times 4.5 = 0$

$\boxed{F_{BD} = 216 \text{ kN (T)}}$

$\Sigma F_Y = 0$ $\quad F_{BD} + F_{EG} - 432 + 432 = 0$

$F_{EG} = -216$ kN or $\boxed{F_{EG} = 216 \text{ kN(comp)}}$

Solution 3:



$\Sigma F_x = 0$, $A_x = 0$

$\Sigma F_Y = 0$

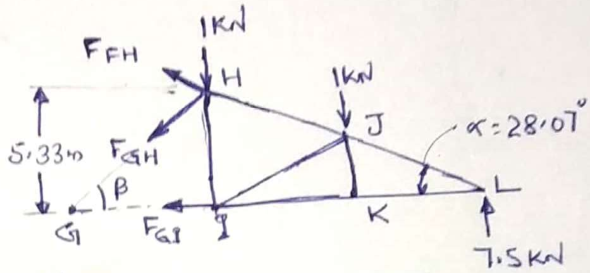
$A_Y + L_Y = 20\,KN$

$\Sigma M_A = 0$

$(5+1)\times 5 + 6\times 10 + 6\times 15 + 1\times 20 + 1\times 25 - L_Y \times 30 = 0$

$L_Y = 7.5\,KN$, $A_Y = 12.5\,KN$

Take a section 1–1 to intersect with FH, GH and GI and draw FBD of the RHS portion.



$\tan 28.07 = \frac{HI}{IL}$

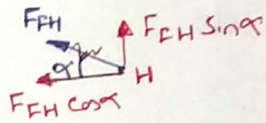
$HI = 0.533 \times 10 = 5.33\,m$.

$\Sigma M_H = 0$

$F_{GI} \times 5.33 + 1\times 5 - 7.5\times 10 = 0$

$\boxed{F_{GI} = 13.13\,KN\,(T)}$

To find F_{FH}, $\Sigma M_G = 0$



$1\times 5 + 1\times 10 - 7.5\times 15 - F_{FH}\sin\alpha \times 5 - F_{FH}\cos\alpha \times 5.33 = 0$

$F_{FH} = -13.83\,KN$, $\boxed{F_{FH} = 13.83\,KN\,(\text{Comp})}$

$\tan\beta = \frac{5.33}{5.00}$, $\beta = 46.83°$ To find F_{GH}

$\Sigma F_x = 0$

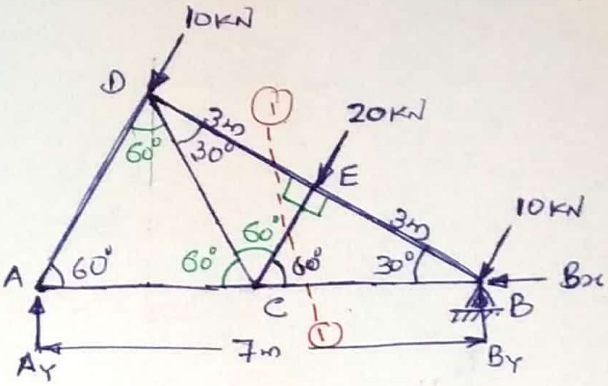
$F_{FH}\cos\alpha + F_{GH}\cos\beta + F_{GI} = 0$

$(-13.83)\cos 28.07 + F_{GH}\cos 46.83 + 13.13 = 0$

$F_{GH} = -1.35\,KN$

$\boxed{F_{GH} = 1.35\,KN\,(\text{Comp})}$

Solution 4:



$\Sigma M_B = 0$

$-20\times 3 - 10\times 6 + 7\times A_Y = 0$

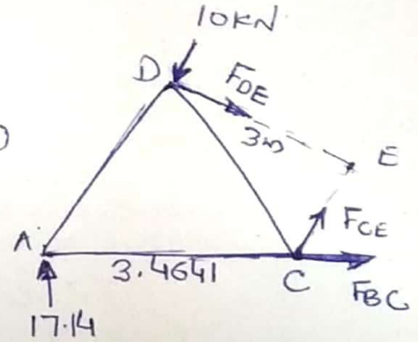
$\boxed{A_Y = 17.143\,kN}$

In ΔCDE

$\tan 30° = \dfrac{CE}{DE} \Rightarrow CE = 1.732\,m.$

In ΔABD

$\tan 30 = \dfrac{AD}{BD} \Rightarrow AD = 3.4641\,m. = AC$



$\Sigma M_C = 0$

$F_{DE}\times 1.732 - 10\times 3 + 17.143\times 3.4641 = 0$

$F_{DE} = -16.9659\ kN$

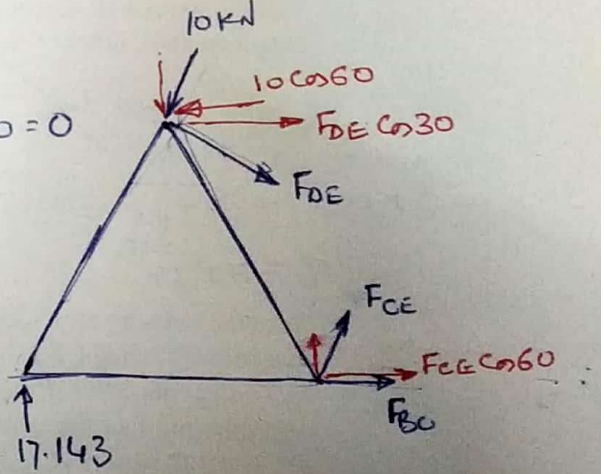
$\boxed{F_{DE} = 16.97\ kN\ (Comp)}$

$\Sigma M_A = 0$

$F_{DE}\times 3.4641 - F_{CE}\times 3 = 0$

$-16.97\times 3.4641 = 3F_{CE} \Rightarrow F_{CE} = -19.60\,kN$

$\boxed{F_{CE} = 19.60\,kN\ (comp)}$



$\Sigma F_x = 0$

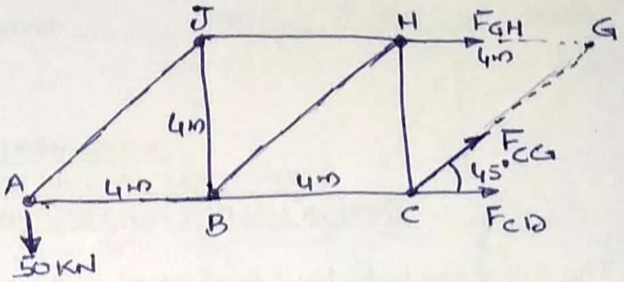
$F_{BC} + F_{CE}\cos 60 + F_{DE}\cos 30 - 10\cos 60 = 0$

$F_{BC} + 9.8 + 14.70 - 5 = 0$

$F_{BC} = -19.50\ kN$

$\boxed{F_{BC} = 19.50\ kN\ (comp)}$

Solution 5:



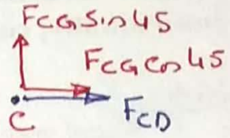
$\Sigma M_C = 0$

$F_{GH} \times 4 - 50 \times 8 = 0$

$\boxed{F_{GH} = 100\ KN (T)}$

$\Sigma F_Y = 0$

$F_{CG} \sin 45 = 50$



$\boxed{F_{CG} = \frac{50}{\sin 45} = 70.7\ KN (T)}$

$\Sigma M_G = 0$

$- F_{CD} \times 4 - 50 \times 12 = 0$

$F_{CD} = -150\ KN.$

$\boxed{F_{CD} = 150\ KN (Comp)}$

Solution 6:

$A_x = 0$

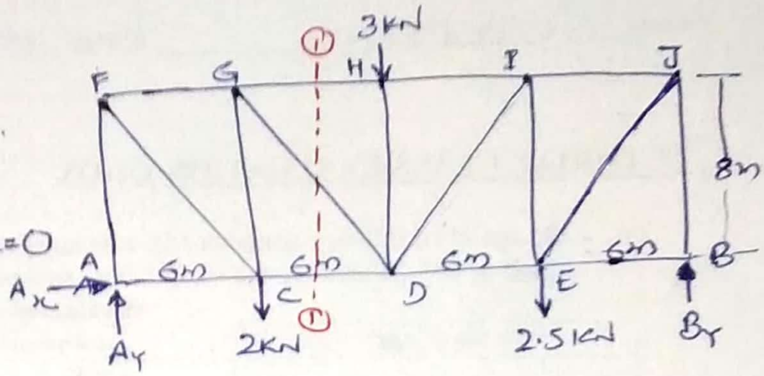
$A_Y + B_Y = 3 + 2 + 2.5 = 7.5\,kN$

$\Sigma M_A = 0$

$2\times6 + 2.5\times18 - 24B_Y + 3\times12 = 0$

$B_Y = 3.875\,kN$

$A_Y = 3.625\,kN$

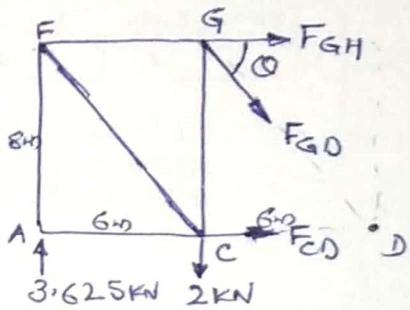


$\Sigma M_D = 0$

$F_{GH}\times 8 + 3.625\times12 - 2\times6 = 0$

$F_{GH} = -3.937\,kN$

$\boxed{F_{GH} = 3.937\,kN\,(comp)}$

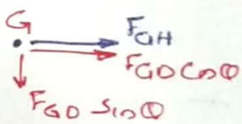


$\Sigma M_G = 0$

$3.625\times6 - F_{CD}\times8 = 0$

$\boxed{F_{CD} = 2.72\,kN\,(T)}$

$\tan\theta = \frac{8}{6} \Rightarrow \theta = 53.13^\circ$



$\Sigma F_Y = 0$

$3.625 - 2 - F_{GD}\sin\theta = 0$

$\boxed{F_{GD} = 2.03\,kN\,(T)}$

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TUTORIAL SHEET-06 (Centroid and Moment of inertia)

1. For the plane area shown in Fig.1, determine (a) the first moment with respect to the x and y axis (b) the location of the centroid
2. Determine the centroid of the area shown in Fig.2 with respect to x and y axis

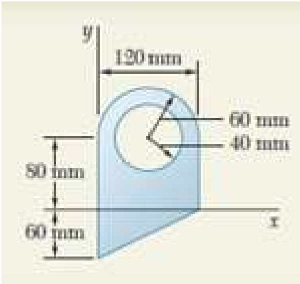


Fig.1

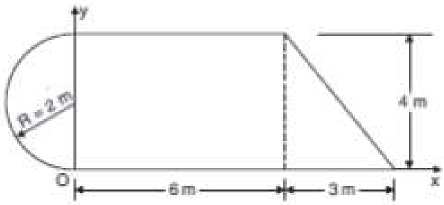


Fig.2

3. Determine the coordinates of the centroid of the plane area with reference to the axis shown in Fig.3.Take x=40mm
4. Find moment of inertia of shaded area about edge AB. Radius of semi-circle =10cm

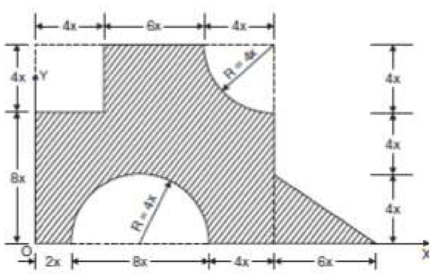


Fig.3

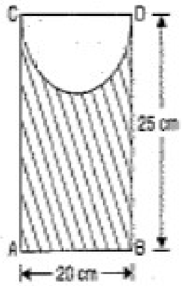
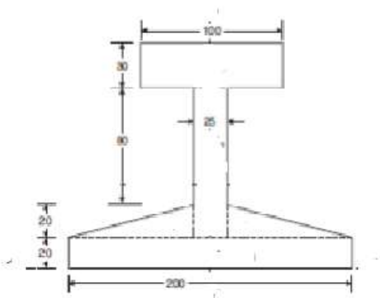
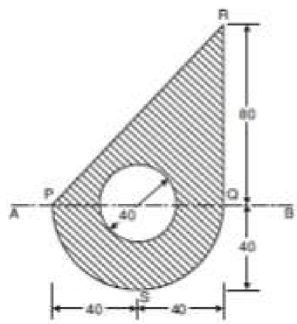


Fig.4

5. Determine the moment of inertia of the built-up section about centroidal x and y axis (Fig.5)
6. Find moment of inertia of shaded area about axis AB(Fig.6)





7. Find the second moment of shaded portion about its centroidal axis (Fig.7)
8. Compute the moment of inertia of given cross section about horizontal AA axis ((Fig.8)

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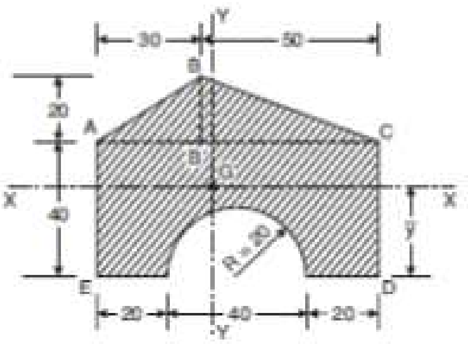


Fig.7

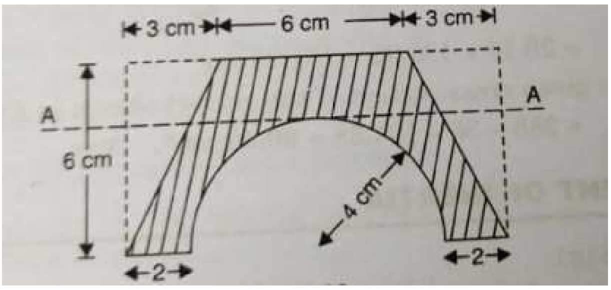
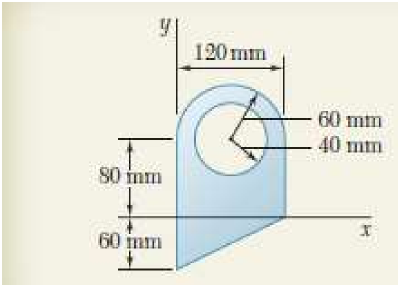


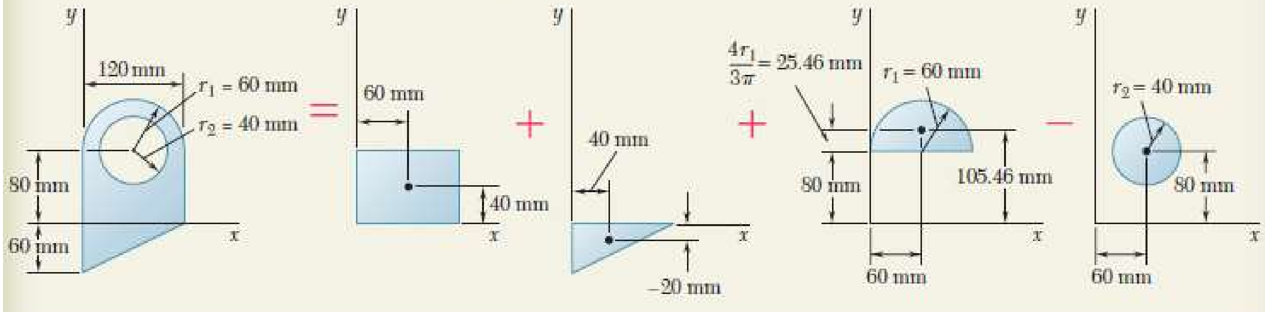
Fig.8



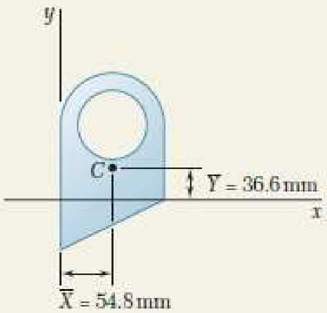
Q.1

SOLUTION

Components of Area. The area is obtained by adding a rectangle, a triangle, and a semicircle and by then subtracting a circle. Using the coordinate axes shown, the area and the coordinates of the centroid of each of the component areas are determined and entered in the table below. The area of the circle is indicated as negative, since it is to be subtracted from the other areas. We note that the coordinate $\overline{y}$ of the centroid of the triangle is negative for the axes shown. The first moments of the component areas with respect to the coordinate axes are computed and entered in the table.



Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	-20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma \bar{x}A = +757.7 \times 10^3$	$\Sigma \bar{y}A = +506.2 \times 10^3$



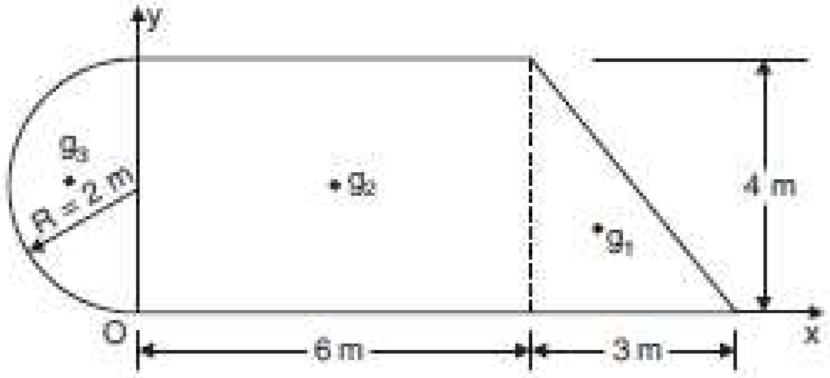
a. First Moments of the Area. Using Eqs. (5.8), we write

$$Q_x = \Sigma \bar{y}A = 506.2 \times 10^3 \text{ mm}^3 \qquad Q_x = 506 \times 10^3 \text{ mm}^3$$
$$Q_y = \Sigma \bar{x}A = 757.7 \times 10^3 \text{ mm}^3 \qquad Q_y = 758 \times 10^3 \text{ mm}^3$$

b. Location of Centroid. Substituting the values given in the table into the equations defining the centroid of a composite area, we obtain

$$\bar{X}\Sigma A = \Sigma \bar{x}A: \qquad \bar{X}(13.828 \times 10^3 \text{ mm}^2) = 757.7 \times 10^3 \text{ mm}^3$$
$$\bar{X} = 54.8 \text{ mm}$$
$$\bar{Y}\Sigma A = \Sigma \bar{y}A: \qquad \bar{Y}(13.828 \times 10^3 \text{ mm}^2) = 506.2 \times 10^3 \text{ mm}^3$$
$$\bar{Y} = 36.6 \text{ mm}$$

Q.2



Solution: The composite section is divided into three simple figures, a triangle, a rectangle and a semicircle

Now, area of triangle $A_1 = \frac{1}{2} \times 3 \times 4 = 6 \text{ m}^2$

Area of rectangle $A_2 = 6 \times 4 = 24 \text{ m}^2$

Area of semicircle $A_3 = \frac{1}{2} \times \pi \times 2^2 = 6.2832 \text{ m}^2$

$\therefore$ Total area $A = 36.2832 \text{ m}^2$

The coordinates of centroids of these three simple figures are:

$$x_1 = 6 + \frac{1}{3} \times 3 = 7 \text{ m}$$

$$y_1 = \frac{4}{3} \text{ m}$$

$$x_2 = 3 \text{ m}$$

$$y_2 = 2 \text{ m}$$

$$x_3 = \frac{-4R}{3\pi} = -\frac{4 \times 2}{3\pi} = -0.8488 \text{ m}$$

$$y_3 = 2 \text{ m} \qquad \text{(Note carefully the sign of } x_3\text{).}$$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A}$$

$$= \frac{6 \times 7 + 24 \times 3 + 6.2832 \times (-0.8488)}{36.2832}$$

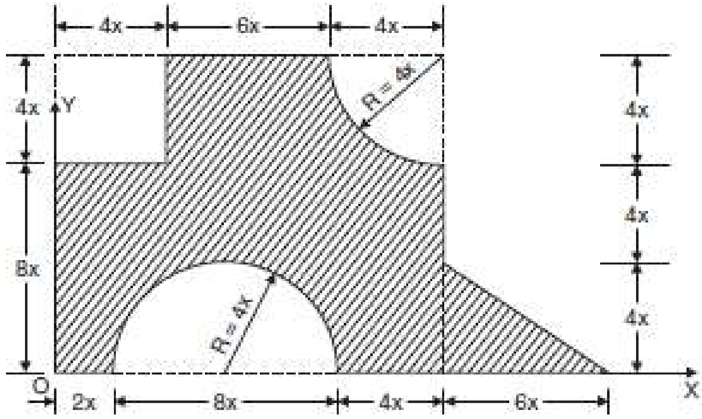
i.e., $\bar{x} = \mathbf{2.995\ m}$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A}$$

$$= \frac{\frac{6 \times 4}{3} + 24 \times 2 + 6.2832 \times 2}{36.2832}$$

i.e., $\bar{y} = \mathbf{1.890\ m}$

Q.3



Solution: The composite figure is divided into the following simple figures:

(1) A rectangle $A_1 = (14x) \times (12x) = 168x^2$

$$x_1 = 7x;\ y_1 = 6x$$

(2) A triangle $$A_2 = \frac{1}{2}\ (6x) \times (4x) = 12x^2$$

$$x_2 = 14x + 2x = 16x$$

$$y_2 = \frac{4x}{3}$$

(3) A rectangle to be subtracted

$$A_3 = (-4x) \times (4x) = -16x^2$$

$$x_3 = 2x;\ y_3 = 8x + 2x = 10x$$

(4) A semicircle to be subtracted

$$A_4 = -\frac{1}{2}\pi\ (4x)^2 = -8\pi x^2$$

$$x_4 = 6x$$

$$y_4 = \frac{4R}{3\pi} = 4 \times \frac{4(x)}{3\pi} = \frac{16x}{3\pi}$$

(5) A quarter of a circle to be subtracted

$$A_5 = -\frac{1}{4} \times \pi\ (4x)^2 = -4\pi x^2$$

$$x_5 = 14x - \frac{4R}{3\pi} = 14x - (4)\left(\frac{4x}{3\pi}\right) = 12.3023x$$

$$y_5 = 12x - 4 \times \left(\frac{4x}{3\pi}\right) = 10.3023x$$

Total area $A = 168x^2 + 12x^2 - 16x^2 - 8\pi x^2 - 4\pi x^2$

$= 126.3009x^2$

$$\bar{x} = \frac{\Sigma A_i x_i}{A}$$

$$\Sigma A_i x_i = 168x^2 \times 7x + 12x^2 \times 16x - 16x^2 \times 2x - 8\pi x^2 \times 6x - 4\pi x^2 \times 12.3023x$$

$$= 1030.6083x^3$$

$$\therefore \quad \bar{x} = \frac{1030.6083x^3}{126.3009x^2}$$

$$= 8.1599x = 8.1599 \times 40 \qquad \text{(since } x = 40 \text{ mm)}$$

$$= \mathbf{326.40\ mm}$$

$$\bar{y} = \frac{\Sigma A_i y_i}{A}$$

$$\Sigma A_i y_i = 168x^2 \times 6x + 12x^2 \times \frac{4x}{3} - 16x^2 \times 10x - 8\pi x^2 \times \frac{16x}{3\pi} - 4\pi x^2 \times 10.3023x$$

$$= 691.8708x^3$$

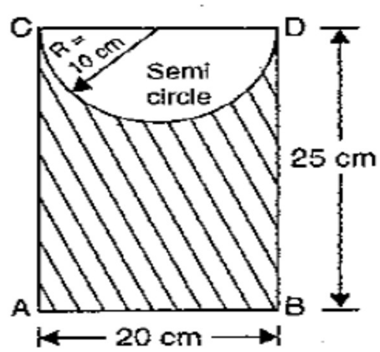
$$\therefore \quad \bar{y} = \frac{691.8708x^3}{126.3009x^2}$$

$$= 5.4780x$$

$$= \mathbf{219.12\ mm} \qquad \text{(since } x = 40 \text{ mm)}$$

Centroid is at (326.40, 219.12).

Q.4



Sol. Given :

Radius of semi-circle, $R = 10$ cm

Width of rectangle, $b = 20$ cm

Depth of rectangle, $d = 25$ cm

Moment of inertia of the shaded portion about AB

= M.O.I. of rectangle $ABCD$ about AB – M.O.I. of semi-circle on DC about AB

M.O.I. of rectangle $ABCD$ about AB

$$= \frac{bd^3}{3} \qquad \text{[see equation (5.11)]}$$

$$= \frac{20 \times 25^3}{12} = 104.167 \text{ cm}^4$$

M.O.I. of semi-circle about DC

$$= \frac{1}{2} \times \text{[M.O.I. of a circle of radius 10 cm about a diameter]}$$

$$= \frac{1}{2} \times \left[\frac{\pi}{64} d^4\right] = \frac{1}{2} \times \frac{\pi}{64} \times 20^4 = 3.925 \text{ cm}^4$$

Distance of C.G. of semi-circle from DC

$$= \frac{4r}{3\pi} = \frac{4 \times 10}{3\pi} = 4.24 \text{ cm}$$

Area of semi-circle, $A = \dfrac{\pi r^2}{2} = \dfrac{\pi \times 10^2}{2} = 157.1 \text{ cm}^2$

M.O.I. of semi-circle about a line through its C.G. parallel to CD

= M.O.I. of semi-circle about CD – Area × [Distance of C.G. of semi-circle from DC]^2

$$= 3925 - 157.1 \times 4.24^2$$

$$= 3925 - 2824.28 = 1100.72 \text{ cm}^4$$

Distance of C.G. of semi-circle from AB

$$= 25 - 4.24 = 20.76 \text{ cm}$$

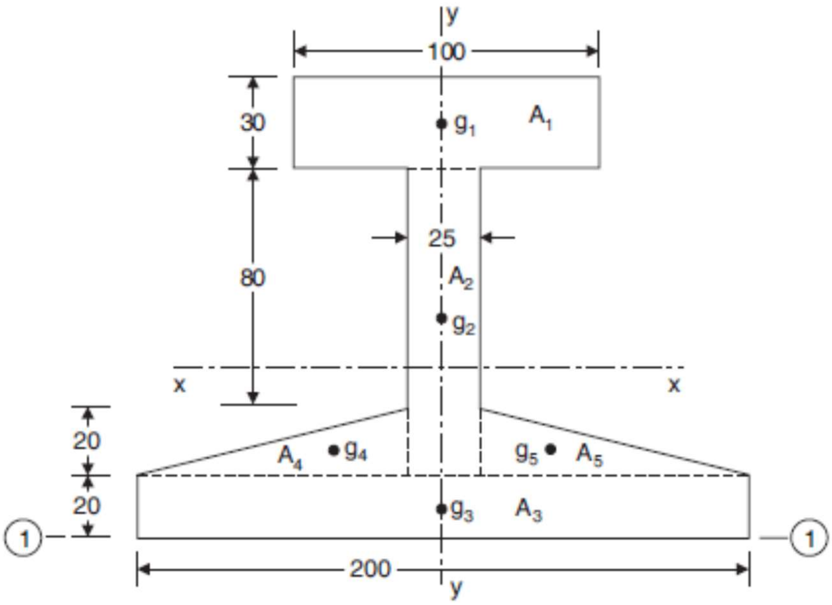
M.O.I. of semi-circle about $AB = 1100.72 + 157.1 \times 20.76^2$

$$= 1100.72 + 67706.58 = 68807.30 \text{ cm}^4$$

∴ M.O.I. of shaded portion about AB

$$= 104.167 - 68807.30 = \mathbf{35359.7 \text{ cm}^4}. \quad \textbf{Ans.}$$

Q.5



Solution: The given composite section may be divided into simple rectangles and triangles as shown in the Fig. 4.58

Area $A_1 = 100 \times 30 = 3000 \text{ mm}^2$

Area $A_2 = 100 \times 25 = 2500 \text{ mm}^2$

Area $A_3 = 200 \times 20 = 4000 \text{ mm}^2$

Area $A_4 = \frac{1}{2} \times 87.5 \times 20 = 875 \text{ mm}^2$

Area $A_5 = \frac{1}{2} \times 87.5 \times 20 = 875 \text{ mm}^2$

Total area $A = 11250 \text{ mm}^2$

Due to symmetry, centroid lies on the axis *y-y*.

A reference axis (1)–(1) is choosen as shown in the figure.

The distance of the centroidal axis from (1)–(1)

$$\bar{y} = \frac{\text{sum of moment of areas about (1)–(1)}}{\text{Total area}}$$

$$= \frac{3000 \times 135 + 2500 \times 70 + 4000 \times 10 + 875\left(\frac{1}{3} \times 20 + 20\right) \times 2}{11250}$$

$= 59.26$ mm

With reference to the centroidal axis x-x and y-y, the centroid of the rectangle A_1 is g_1 (0.0, 75.74), that of A_2 is g_2(0.0, 10.74), that of A_3 is g_3 (0.0, 49.26), the centroid of triangle A_4 is g_4 (41.66, 32.59) and that of A_5 is g_5 (41.66, 32.59).

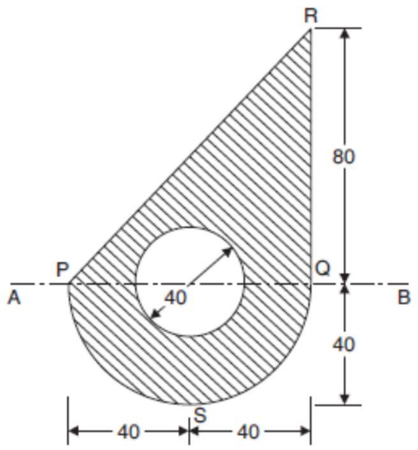
$$I_{xx} = \frac{100\times 30^3}{12} + 3000 \times 75.74^2 + \frac{25\times 100^3}{12} + 2500 \times 10.74^2 + \frac{200\times 20^3}{12} + 4000 \times 49.26^2 + \frac{87.5\times 20^3}{36} + 875 \times 32.59^2 + \frac{87.5\times 20^3}{36} + 875 \times 32.59^2$$

$$\mathbf{I_{xx} = 3{,}15{,}43{,}447\ mm^4}$$

$$I_{yy} = \frac{30\times 100^3}{12} + \frac{100\times 25^3}{12} + \frac{20\times 200^3}{12} + \frac{20\times 87.5^3}{36} + 875 \times 41.66^2 + \frac{20\times 87.5^3}{36} + 875 \times 41.66^2$$

$$\mathbf{I_{yy} = 1{,}97{,}45{,}122\ mm^4.}$$

Q.6



Solution: The section is divided into a triangle PQR, a semicircle PSQ having base on axis AB and a circle having its centre on axis AB.

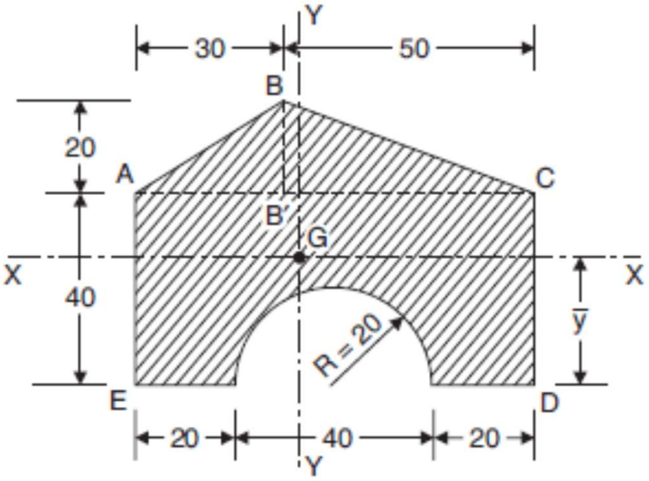
Now,

Moment of inertia of the section about axis AB = Moment of inertia of triangle PQR about AB + Moment of inertia of semicircle PSQ about AB – moment of inertia of circle about AB

$$= \frac{80\times 80^3}{12} + \frac{\pi}{128} \times 80^4 - \frac{\pi}{64} \times 40^4$$

$$\mathbf{I_{AB} = 42{,}92{,}979\ mm^4.}$$

Q.7



Solution: The section is divided into three simple figures viz., a triangle *ABC*, a rectangle *ACDE* and a semicircle.

Total Area = Area of triangle *ABC* + Area of rectangle *ACDE* – Area of semicircle

$$A = \frac{1}{2} \times 80 \times 20 + 40 \times 80 - \frac{1}{2} \times \pi \times 20^2$$

$$= 3371.68$$

$$A\,\bar{y} = \frac{1}{2} \times 80 \times 20 \left(\frac{1}{3} \times 20 + 40\right) + 40 \times 80 \times 20 - \frac{1}{2} \times \pi \times 20^2 \times \frac{4 \times 20}{3\pi}$$

$$= 95991.77$$

$$\therefore \quad \bar{y} = \frac{95991.77}{3371.6} = 28.47 \text{ mm}$$

$$A\,\bar{x} = \frac{1}{2} \times 30 \times 20 \times \frac{2}{3} \times 30 + \frac{1}{2} \times 50 \times 20 \times \left(\frac{1}{3} \times 50 \times 30\right)$$

$$+ 40 \times 80 \times 40 - \frac{1}{2} \times \pi \times 20^2 \times 40$$

$$= 132203.6$$

$$\therefore \quad \bar{x} = \frac{A\bar{x}}{A} = \frac{132203.6}{3371.68} = 37.21 \text{ mm}$$

$$\left.\begin{array}{l}\text{Moment of inertia about} \\ \text{centroidal } x\text{-}x \text{ axis}\end{array}\right\} = \left\{\begin{array}{l}\text{Moment of inertia of triangle } ABC \text{ about} \\ x\text{-}x \text{ axis} + \text{Moment of inertia of rectangle} \\ \text{about } x\text{-}x \text{ axis} - \text{moment of semicircle} \\ \text{about } x\text{-}x \text{ axis}\end{array}\right.$$

$$\therefore \qquad I_{xx} = \frac{80\times 20^3}{36} + \frac{1}{2} \times 80 \times 20 \left(60 - \frac{2}{3} \times 20 - 28.47\right)^2$$

$$+ \frac{80\times 40^3}{12} + 80 \times 40 \times (28.47 - 20)^2$$

$$- \left[0.0068598 \times 20^4 + \frac{1}{2}\pi \times 20^2 \left(28.47 - \frac{4\times 20}{3\pi}\right)^2\right]$$

$$\boldsymbol{I_{xx} = 6,86,944\ \text{mm}^4.}$$

Similarly,

$$I_{yy} = \frac{20\times 30^3}{36} + \frac{1}{2} \times 20 \times 30 \left(39.21 - \frac{2}{3} \times 30\right)^2 + \frac{20\times 50^3}{36}$$

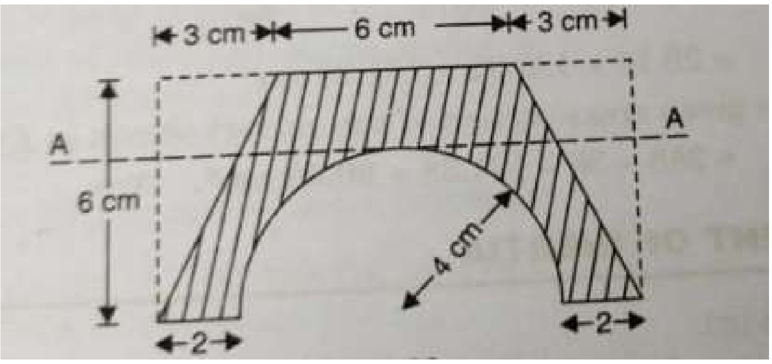
$$+ \frac{1}{2} \times 20 \times 50 \times \left[39.21 - \left(30 + \frac{1}{3} \times 50\right)\right]^2 + \frac{40\times 80^3}{12}$$

$$+ 40 \times 80(39.21 - 40)^2 - \frac{1}{2} \times \frac{\pi}{64} \times 40^4 - \frac{1}{2} \times \frac{\pi}{4}$$

$$\times 40^2 (40 - 39.21)^2$$

$$\boldsymbol{= 1868392\ \text{mm}^4.}$$

Q.8



Sol. Enclose the given cross-section of the culvert shown in Fig. 7.28 into a rectangle of dimensions 6 × 12. Then the moment of inertia of the given cross-section about horizontal axis A-A will be equal to the M.O.I. of rectangle of size 6 × 12 about axis A-A minus the M.O.I. of two triangles about A-A minus the M.O.I. of the semi-circle about axis A-A. Now M.O.I. of rectangle about axis A-A.

= M.O.I. of rectangle about an axis parallel to axis A-A and passing through its C.G. plus area of rectangle multiplied by the square of the distance between two axes.

$$= \frac{bd^3}{12} + A \times x^2, \text{ where } b = 12,\ d = 6,\ A = 12 \times 6 = 72 \text{ and } x = 4 - 3 = 1$$

$$= \frac{12 \times 6^3}{12} + 72 \times 1^3 = 216 + 72 = 288 \text{ cm}^4$$

M.O.I. of two triangles about axis A-A. (Both triangles are similar and equal)

= M.O.I. of two triangle about an axis parallel to axis A-A and passing through the C.G. of the triangles plus areas of two triangles multiplied by the square of the distance between two axes.

$$= 2\left[\frac{b_1 h_1^{\,3}}{36} + A_1 x_1^{\,2}\right], \text{ where } b_1 = 3,\ h_1 = 6,\ A_1 = \frac{3 \times 6}{2} = 9 \text{ cm}^2,\ x_1 = 0$$

($\because$ The axis A-A passes through the C.G. of the triangle. Hence $x_1 = 0$)

$$= 2\left[\frac{3 \times 6^3}{36} + 9 \times 0\right] = 36 \text{ cm}^4$$

M.O.I. of semi-circle about axis A-A.

= M.O.I. of the semi-circle about an axis parallel to A-A and passing through its C.G. + area of semi-circle × square of distance between two axis

$$= 0.11r^4 + \frac{\pi r^2}{2} \times \left[r - \frac{4r}{3\pi}\right]^2$$

$$\left[\because \text{ M.O.I. of semi-circle about an axis passing through its C.G.} = 0.11r^4 \text{ and distance of the C.G.} = \frac{4r}{3\pi}\right]$$

$$= 0.11 \times 4^4 + \frac{\pi \times 4^2}{2} \times \left[4 - \frac{4 \times 4}{3\pi}\right]^2$$

$$= 28.16 + 133.22 = 161.38 \text{ cm}^4$$

$\therefore$ M.O.I. of the given cross-section of the culvert shown in Fig. 7.28

$= 288 - 36 - 161.38 =$ **90.62 cm^4. Ans.**

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MECHANICS-UES009

BE FIRST YEAR SESSION: 2020-21

Tutorial No. 7 (Friction)

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1. A uniform 10 Kg ladder in Fig. 1 rests against the smooth wall at B, and the end A rests on the rough horizontal plane for which the coefficient of static friction is μ=0.3. Determine the angle of inclination θ of the ladder and the normal reaction at B if the ladder is on the verge of slipping.

If the ladder is placed at an angle 80 degree, up to what height a man of weight 40 Kg can climb on the ladder.

2. Blocks A and B have a mass of 3 Kg and 9 Kg, respectively, and are connected to the weightless links shown in Fig 2. Determine the largest vertical force P that can be applied at the pin C without causing any movement. The coefficient of static friction between the blocks and the contacting surface is μ_s=0.3.

3. Determine the least weight of the top block necessary to prevent downward motion of the 1100 N block. Assume the coefficient of friction between the contact surface is 0.3. Solve the problem for (1) drum is frictionless. (2) Drum has a 0.25 fiction coefficient.

4. A block overlaying a 10° wedge on a horizontal floor (Fig. 4), leaning against a vertical wall, and weighing 2000 N is to be raised by applying a horizontal force to the wedge. Assuming coefficient of frictionfor all the contact surfaces 0.25, determine the minimum horizontal force to be applied to raise the block.

5. Referring to Fig 5, compute the force P applied through 21° wedge A necessary to impend the motion of the block B weighing 2 kN. Assume the angle of limiting friction for all the contiguous surfaces is 21°.

6. A rotating drum (Fig. 6) is braked by a flat belt attached to the lever ABC hinged at B. Determine the breaking moment exerted by a vertical weight of 150 N while the coefficient of friction between the belt and drum is 0.43.

7. A cord is wrapped twice around one capstan A and three times around capstan B (Fig 7). Finally, the cord goes around a half barrel section as shown an supports a mass of 500 kg. What is the force T required to maintain the equilibrium. Take μ_s=0.3 for all the contact surfaces.

8. The turnbuckle shown in Fig. 8 has a square thread with a mean radius of 5 mm and a lead of 2 mm. If the coefficient of static friction between the screw and turnbuckle is μ_s=0.25, determine the moment M that must be applied to draw the end screws closer together.

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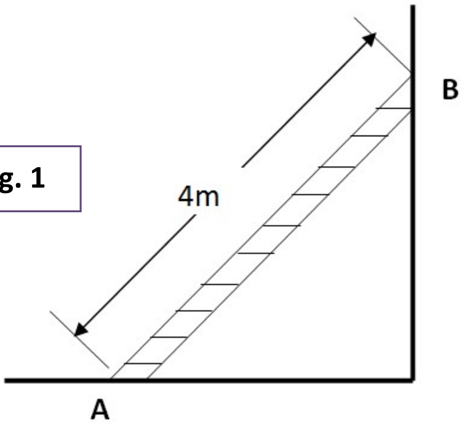

Fig. 1

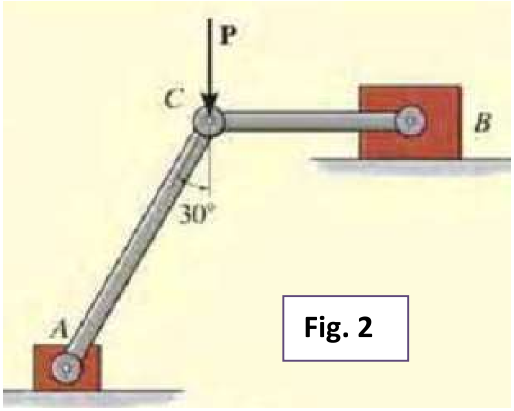

Fig. 2

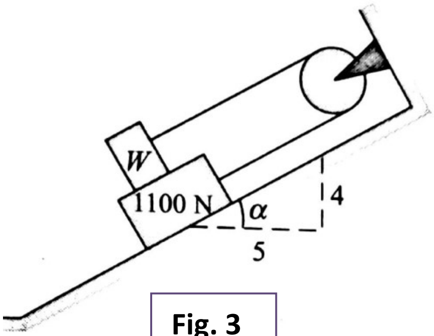

Fig. 3

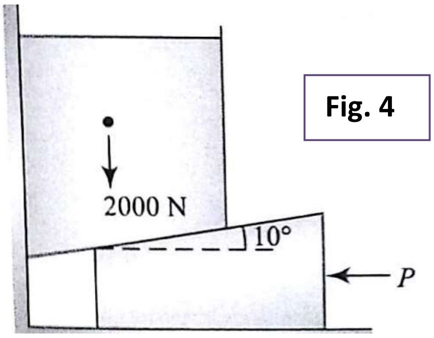

Fig. 4

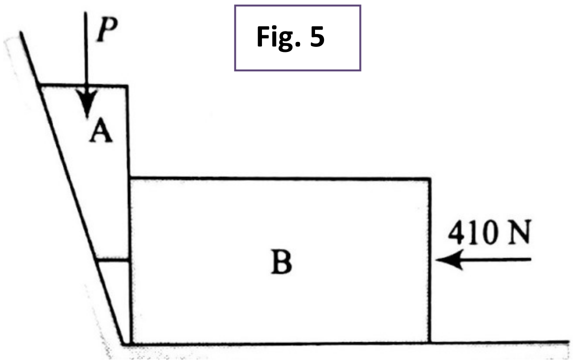

Fig. 5

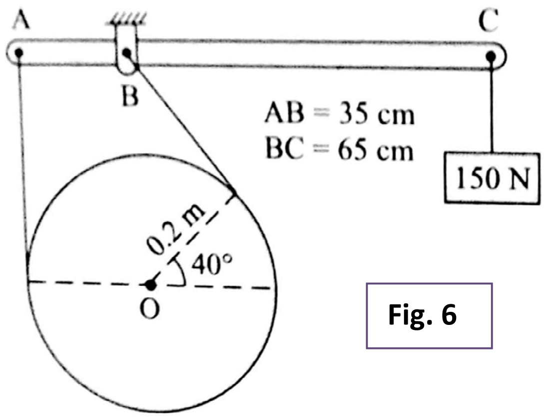

Fig. 6

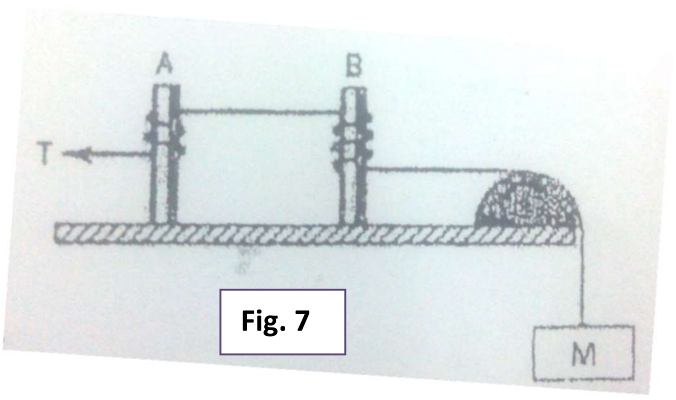

Fig. 7

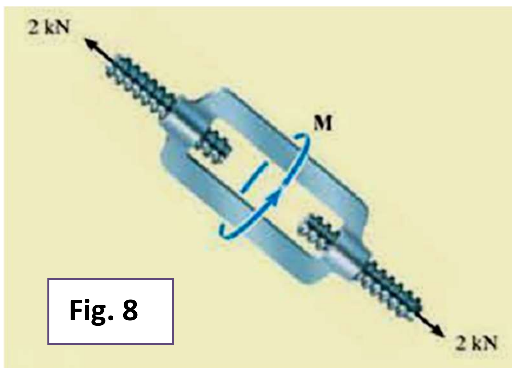

Fig. 8

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Q1.

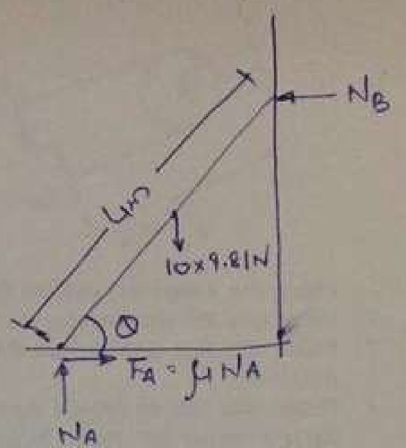
$F_A = \mu N_A = 0.3 N_A$

Case I

$\Sigma F_x = 0, \quad 0.3 N_A = N_B$

$\Sigma F_y = 0 \quad N_A = 98.1\,N$

$N_B = 0.3 \times 98.1 = 29.43\,N$



$\Sigma M_A = 0$

$N_B \times 4 \sin\theta = 98.1 \times 2 \cos\theta$

~~29.43×~~

$117.72 \sin\theta = 196.2 \times \cos\theta$

$\tan\theta = \frac{196.2}{117.72}$

$\boxed{\theta = 59^\circ}$

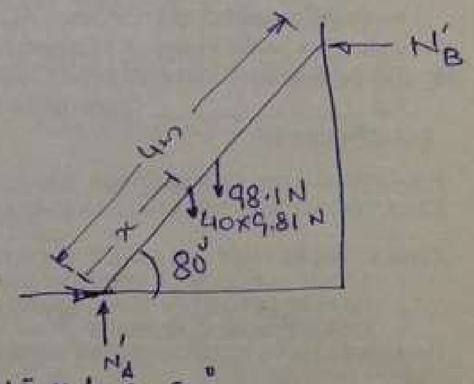
Case II

$\Sigma F_y = 0$

$N_A' = 98.1 + 40 \times 9.81 = 490.5\,N$

$F_A' = 0.3 \times 490.5 = 147.15\,N$

$N_B' = F_A' = 147.15\,N$



$\Sigma M_A = 0$

~~392.4 x + 98.1 × 2 cos 80~~

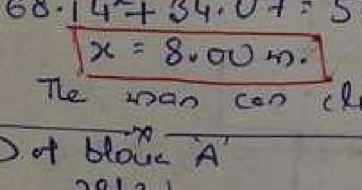
$392.4 \times x \cos 80 + 98.1 \times 2 \cos 80^\circ = 147.15 \times 4 \sin 80^\circ$

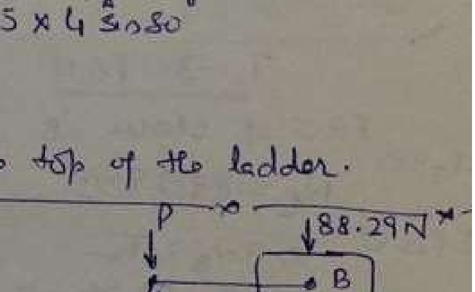
$68.14x + 34.07 = 579.66$

$\boxed{x = 8.00\,m.}$

The man can climb up to the top of the ladder.

Q2. FBD of block 'A'





$\Sigma F_y = 0$

$N_A = 29.43 + T_1 \cos 30^\circ$

$\Sigma F_x = 0 \quad F_A = 0.3 N_A = T_1 \sin 30^\circ$

or $0.3(29.43 + T_1 \cos 30) = T_1 \sin 30^\circ$

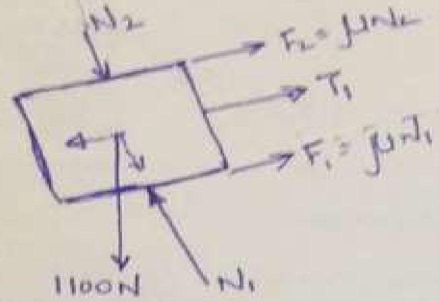
$8.829 + 0.2598\,T_1 = 0.5\,T_1$

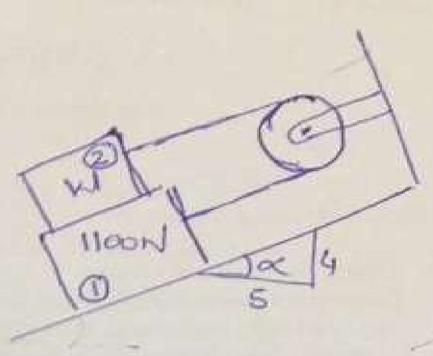
$0.2402\,T_1 = 8.829$

$T_1 = 36.76\,N$

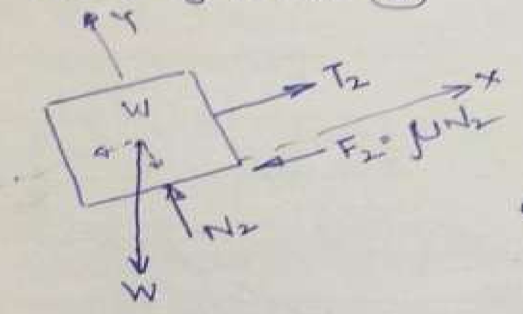
3. $\tan\alpha = 4/5$, $\alpha = 38.7^\circ$

FBD of block (1)





FBD of block (2)



$\Sigma F_Y = 0$

$N_2 = W\cos 38.7^\circ$ — I

$\Sigma F_x = 0$

$T_2 = 0.3N_2 + W\sin 38.7$

or $T_2 = 0.3(W\cos 38.7) + W\sin 38.7$

$= 0.234W + 0.625W$

$T_2 = 0.86W$ — II

From FBD of block (1)

$\Sigma F_x = 0$

$T_1 + 0.3N_1 + 0.3N_2 = 1100\sin 38.7^\circ = 687.77N.$

$T_1 + 0.3N_1 + 0.3\times W\cos 38.7 = 687.77$ ($\because N_2 = W\cos 38.7$ eq. I)

$T_1 + 0.3N_1 + 0.234W = 687.77$ — III

$\Sigma F_Y = 0$

$N_1 = N_2 + 1100\cos 38.7^\circ$

$= W\cos 38.7 + 1100\cos 38.7$

$N_1 = 0.78W + 858.47$

Put N_1 in eq. - III

$T_1 + 0.3(0.78W + 858.47) + 0.234W = 687.77$

$T_1 + 0.234W + 257.54 + 0.234W = 687.77$

$T_1 + 0.468W = 430.23$ — IV

Also $T_1 = T_2$, put in eq. IV, from eq. II

$0.86W + 0.468W = 430.23$

$\Rightarrow \boxed{W = 323.97N.}$

(2) $\mu = 0.25$, $\beta = 180^\circ = \pi$ rad. (Angle of wrap)

$\frac{T_1}{T_2} = e^{\mu\beta} = 2.193$

$T_1 = 2.193T_2$

From eq. II, $T_2 = 0.86W$

$T_1 = 2.193T_2 = 1.886W$

Put T_1 in eq. IV

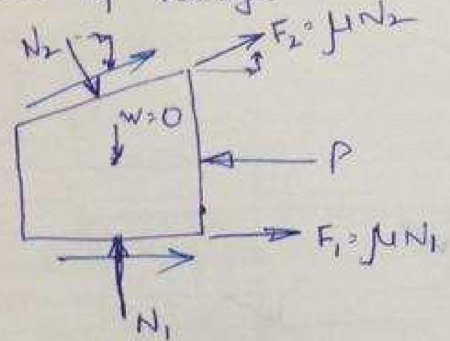
$1.886W + 0.468W = 430.23$

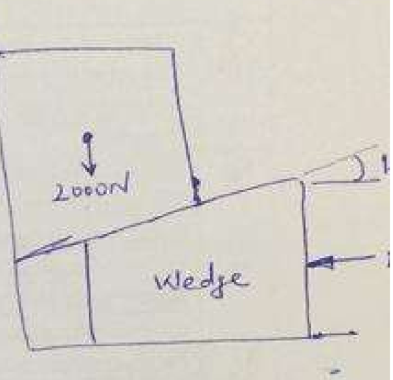
$2.354W = 430.23$

$\boxed{W = 182.77\ N}$

Q.4 Let W for the wedge = 0

FBD of wedge





$\Sigma F_x = 0$ $\quad 0.25N_1 + 0.25N_2\cos 10^\circ + N_2\sin 10^\circ = P$

$0.25N_1 + 0.246N_2 + 0.174N_2 = P$

$0.25N_1 + 0.42N_2 = P$ —— I

$\Sigma F_y = 0$

$N_1 + 0.25N_2\sin 10^\circ = N_2\cos 10^\circ$

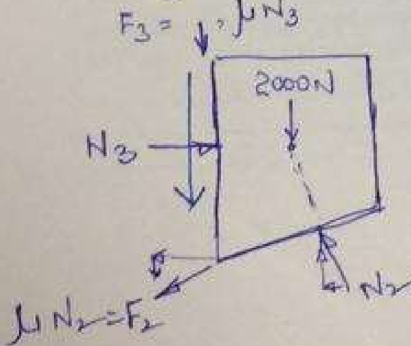
$\Rightarrow N_1 = 0.941N_2$ —— II

Put N_1 in eq I

$0.25(0.941N_2) + 0.42N_2 = P$

$P = 0.655N_2$ —— III

FBD of block



$\Sigma F_x = 0$

$N_3 = 0.25N_2\cos 10^\circ + N_2\sin 10^\circ$

$N_3 = 0.42N_2$ — IV

$\Sigma F_y = 0$

$N_2\cos 10^\circ = 2000 + 0.25N_3 + 0.25N_2\sin 10^\circ$

$0.9848N_2 - 0.25(0.42N_2) - 0.0434N_2 = 2000$

$0.8364N_2 = 2000$

$N_2 = 2391\ N$, $\boxed{P = 1566\ N}$

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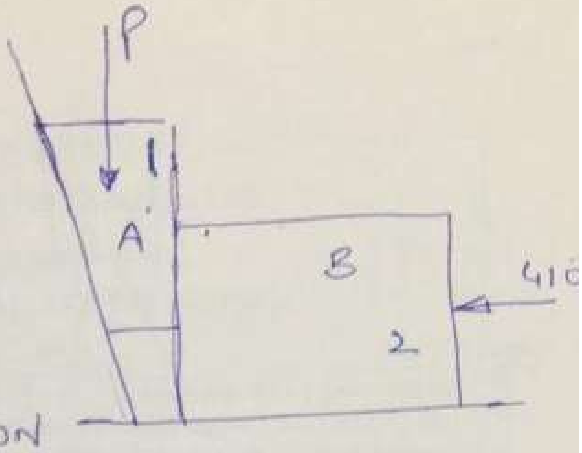
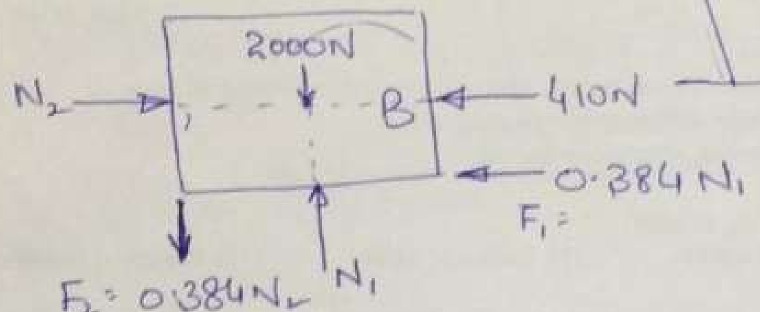
5.

$\mu = \tan\phi = \tan 21^\circ$

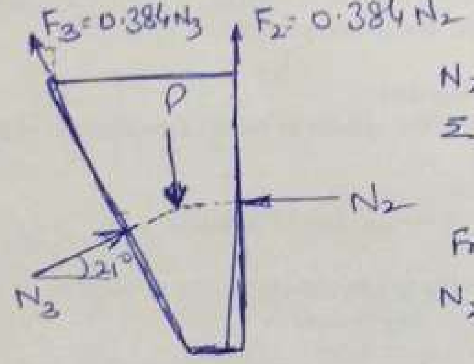
$= 0.384$



FBD of block 'B'



FBD of block 'A'



$\Sigma F_x = 0$

$N_2 = 0.384 N_1 + 410$ — I

$\Sigma F_y = 0$

$N_1 = 2000 + 0.384 N_2$ — II

From eq I

$N_2 = 0.384(2000 + 0.384 N_2) + 410$

$= 768 + 0.147 N_2 + 410$

$0.8526 N_2 = 1178$

$N_2 = 1381.66 = 1382 \text{ N}.$

$\Sigma F_x = 0$

$N_3 \cos 21 = N_2 + 0.384 N_3 \sin 21^\circ$

$0.9336 N_3 = 1382 + 0.1376 N_3$

$N_3 = 1736.18 = 1736 \text{ N}.$

$\Sigma F_y = 0$

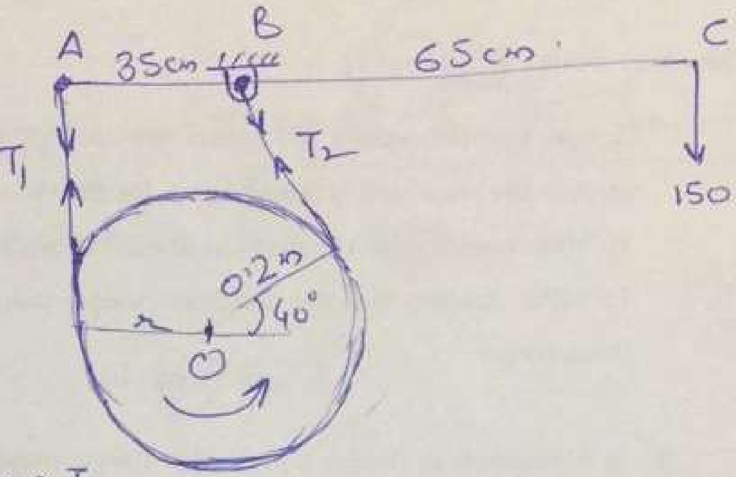
$N_3 \sin 21 + 0.384 N_3 \cos 21 + 0.384 N_2 = P$

$1736 \times 0.3584 + 0.3585 \cancel{N_3} \times 1736 + 0.384 \times 1382 = P$

$622.18 + 622.36 + 530.69 = P$

$\boxed{P = 1775 \text{ N}}$

Q6



$\mu = 0.43$

$\theta = 220° \; (180+40)$

$= 3.84$ rad

$e^{\mu\theta} = 5.212$

$\frac{T_1}{T_2} = 5.212$ or $T_1 = 5.212 T_2$

$\Sigma M_B = 0$

$35 \times T_1 = 65 \times 150$

$T_1 = 278.57$ N

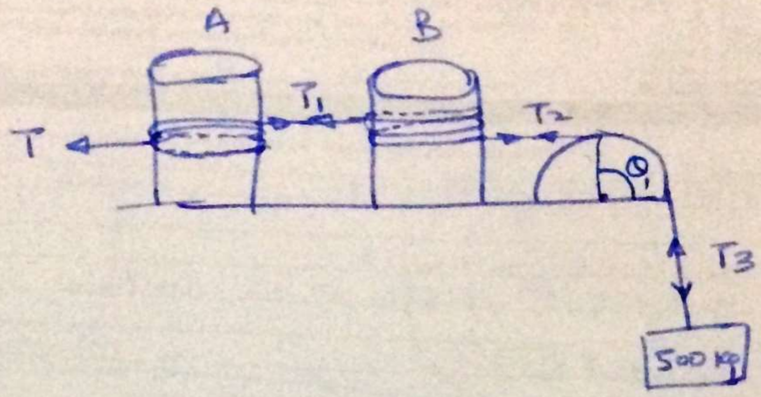
$T_2 = 53.5$ N.

Torque $= (T_1 - T_2) \times R$

$= (278.57 - 53.5) \times 0.2$

$\boxed{\text{Torque} = 45 \text{ N-m}}$

Q7



$\mu_s = 0.3$

$T = ?$

$\frac{T_3}{T_2} = e^{\mu \theta_1}$

$\theta_1 = \pi/2$ rad

$\frac{T_3}{T_2} = e^{\left(0.3 \times \frac{\pi}{2}\right)} = 1.60$

$T_3 = 500 \times 9.81 = 4905$

$\frac{4905}{T_2} = 1.60$ $\boxed{T_2 = 3065.62\ N}$

$\frac{T_2}{T_1} = e^{\mu \theta_B}$ $\theta_B = 2\pi \times 3 = 6\pi$ rad

$\frac{T_2}{T_1} = e^{0.3 \times 6\pi} = 285.68$

$3065.62 = 285.68\, T_1 \Rightarrow \boxed{T_1 = 10.73\ N}$

$\frac{T_1}{T} = e^{\mu \theta_A}$ $\theta_A = 4\pi$

$\frac{T_1}{T} = e^{0.3 \times 4\pi} = 43.38$

$10.73 = 43.38\, T$

$\boxed{T = 0.25\ N}$

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TUTORIAL SHEET-08 (PRINCIPLE OF VIRTUAL WORK)

1. For the mechanism shown in Figure 1, calculate the value of moment M required to maintain the equilibrium. Use principle of virtual work.
2. Determine the ratio P/Q of the forces required for the equilibrium of the system shown in Fig.2.
3. In Figure 3, the pressure 'p' driving the piston is 2 MPa and diameter of the piston is 100 mm. For what value weight W, the configuration shown shall be in equilibrium, if the lengths AB = BC = 300 mm. Neglect friction.
4. A load W of magnitude 600 N is applied to the linkage at B (Fig. 4). The constant of the spring, k = 2.5 kN/m, and the spring is unstretched when AB and BC are horizontal. Neglecting the weight of the linkage and knowing that l = 300 mm, determine the value of θ corresponding to equilibrium.
5. A 4.4 m long ladder weighing 310 N, is supported on a 2.9 m high wall (Figure 3). A man of weight 720 N stands on a particular rung of the ladder as shown in the figure. Use principle of virtual work to calculate the force P required to maintain the equilibrium of the ladder. Assume all contact surfaces smooth.
6. If the force P = 100 N is applied at the end A of the toggle press (Figure 6), calculate the force delivered at block E using the virtual work method. Neglect the weight of the block.

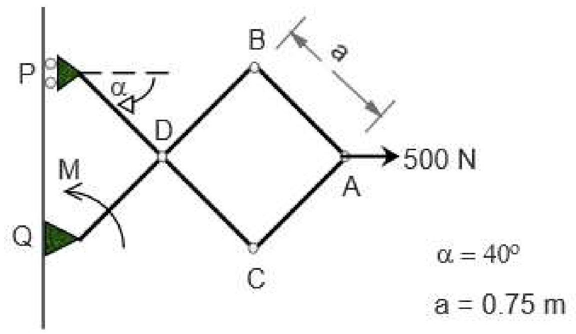

Fig. 1

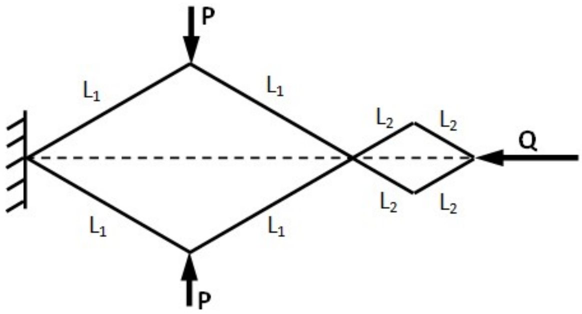

Fig. 2

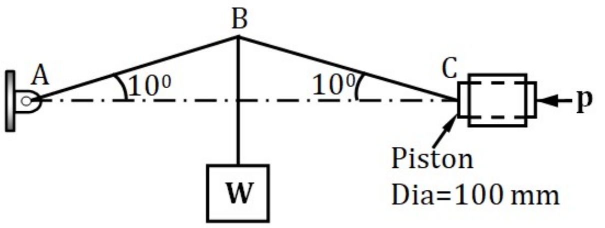

Fig. 3

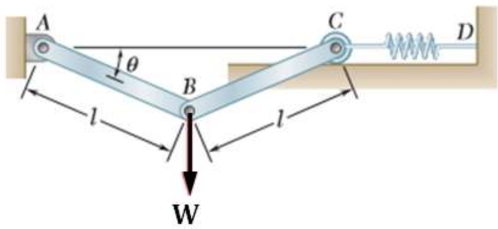

Fig. 4

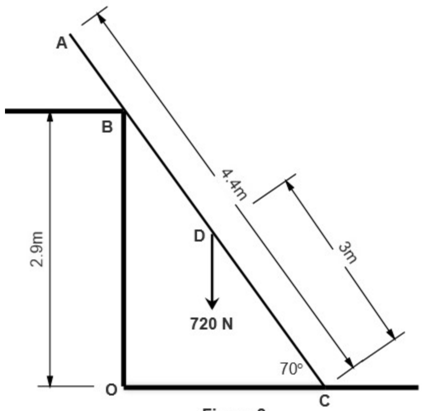

Fig. 5

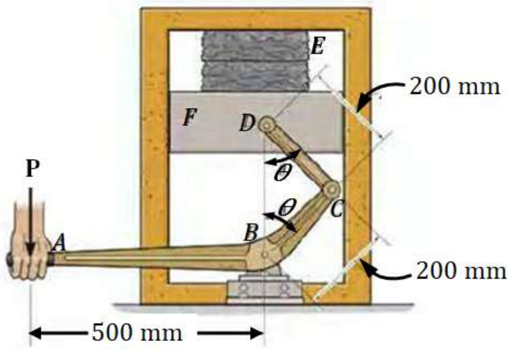

Fig. 6

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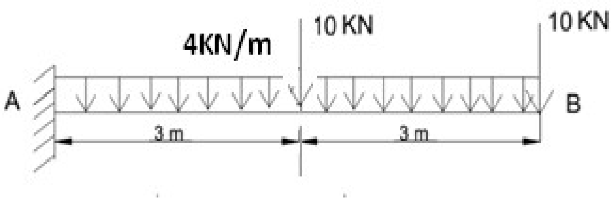
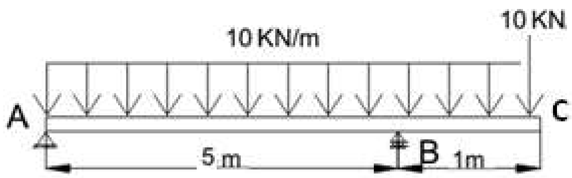
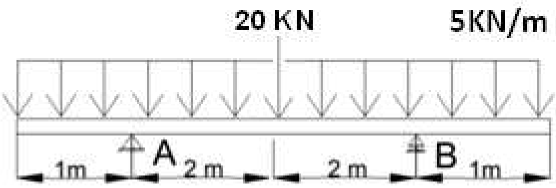
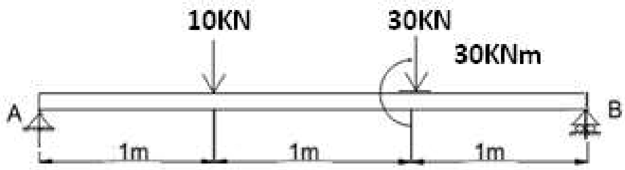
TUTORIAL SHEET-09 (SFD and BMD)

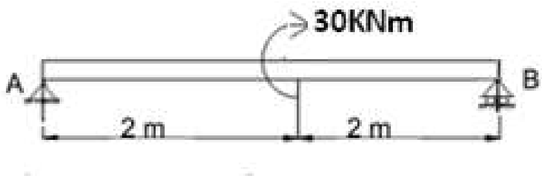
Obtain the expressions for the shear force and bending moment for each segment of the beams shown below. Draw the SFD and BMD indicating the values of the salient points. Also find the location of the point of contra-flexure, wherever applicable.



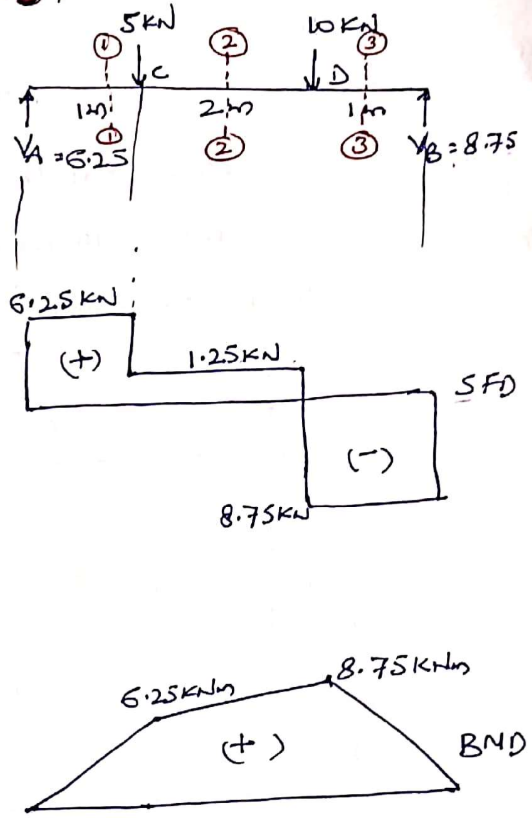








Tutorial Sheet No 9



1. $V_A + V_B = 5 + 10 = 15\,kN$.

$\Sigma M_A = 0$

$5 \times 1 + 10 \times 3 - 4V_B = 0$

$V_B = 8.75\,kN$ $V_A = 6.25\,kN$

$V_{11} = 6.25\,kN$. $M_{11} = 6.25x$

$V_A = V_C = 6.25\,kN$

$M_A = 0$, $M_C = 6.25\,kNm$.

$V_{22} = 1.25\,kN$.

$\Sigma M_{22} = 0$

$-M_{22} + 6.25x - 5(x-1) = 0$

$M_{22} = 6.25x - 5(x-1)$

$M_C = 6.25\,kNm$. $M_D\big|_{x=3} = 8.75\,kNm$.

$V_{33} = -8.75\,kN$

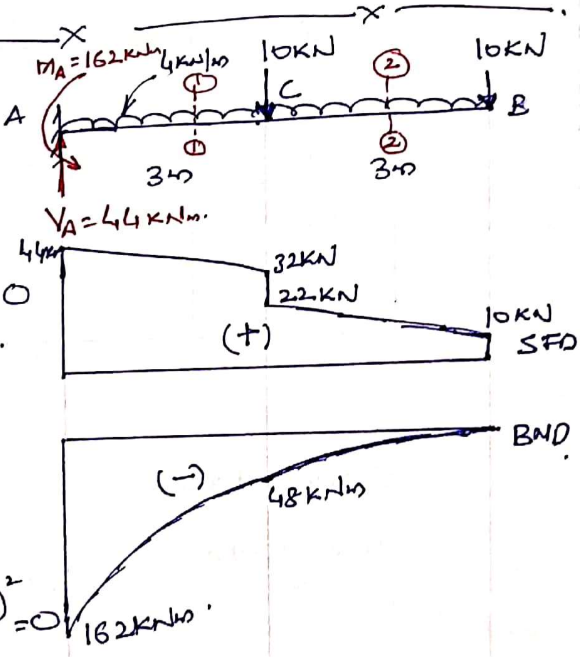
$V_D = V_B = -8.75\,kN$

$\Sigma M_{33} = 0$

$M_{33} = 8.75(4-x)$

$M_D = 8.75\,kNm$, $M_B = 0$

Sol 2.



$V_A = 10 + 10 + 24 = 44\,kN$.

$\Sigma M_A = 0;\ -M_A + 30 + 60 + 72 = 0$

$M_A = 162\,kNm$.

$\Sigma M_{11} = 0$

$-162 + 44x - 4\frac{x^2}{2} - M_{11} = 0$

$M_{11} = 44x - 2x^2 - 162$.

$44 - 4x - V_{11} = 0$

$V_{11} = 44 - 4x$, $V_A = 44\,kN$, $V_C = 44 - 12 = 32\,kN$.

$M_A = -162\,kNm$. $M_C\big|_{x=3} = 132 - 18 - 162 = -48\,kNm$.

$V_{22} = 4(6-x) + 10$

$V_C = 22\,kN$. $V_B = 10\,kN$

$\Sigma M_{22} = 0$, $M_{22} + 10(6-x) + \frac{4(6-x)^2}{2} = 0$

$M_{22} = -10(6-x) - 2(6-x)^2$

$M_C = -30 - 18 = -48\,kNm$.

$M_B = 0$

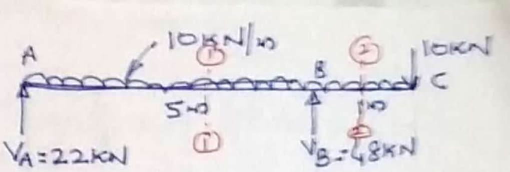
S:

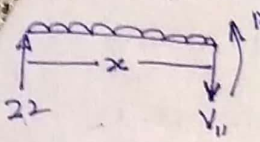
$V_A + V_B = 70\,kN$

$\Sigma M_A = 0$

$\times 6 \times 3 + 10 \times 6 - 5V_B = 0$

$V_B = 48\,kN \quad V_A = 22\,kN.$





$22 - 10x - V_{11} = 0$

$V_{11} = 22 - 10x.$

$\underline{V_A = 22\,kN}.\quad V_B = 22 - 50$

$V_B = \underline{-28\,kN}$

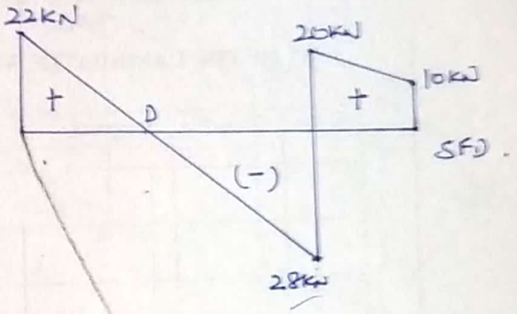
$\Sigma M_{11} = 0$

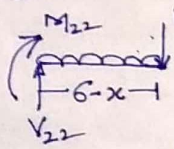
$22x - 10\frac{x^2}{2} - M_{11} = 0, \quad M_{11} = 22x - 5x^2$

$\underline{M_A = 0}, \quad M_B = 110 - 125 = \underline{-15\,kNm}.$

$V_{11} = 0, \quad 22 - 10x = 0, \quad 10x = 22, \quad x = 2.2\,m.$

$M_{max}\Big|_{x=2.2} = 48.4 - 24.2 = \underline{24.2\,kNm}.$

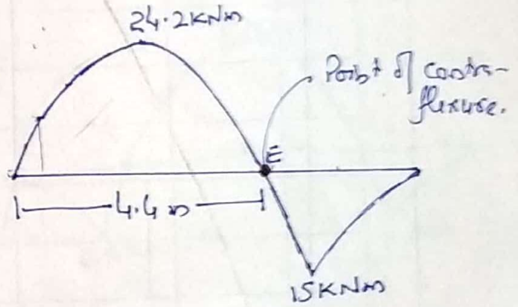




$V_{22} - 10(6-x) - 10 = 0$

$V_{22} = 10(6-x) + 10$

$\underline{V_B = 20\,kN}. \quad \underline{V_C = 10\,kN}.$



$\Sigma M_{22} = 0$

$M_{22} + 10(6-x) + 10\frac{(6-x)^2}{2} = 0$

$M_{22} = -10(6-x) - 5(6-x)^2$

$M_B\Big|_{x=5} = -10 - 5 = \underline{-15\,kNm}.$

$\underline{M_C\Big|_{x=6} = 0}$

$M_{11} = 0, \quad 22x - 5x^2 = 0$

$5x^2 = 22x$

$5x = 22$

$\underline{x = 4.4\,m}.$

$\underline{M_{11} = 22x - 5x^2}$

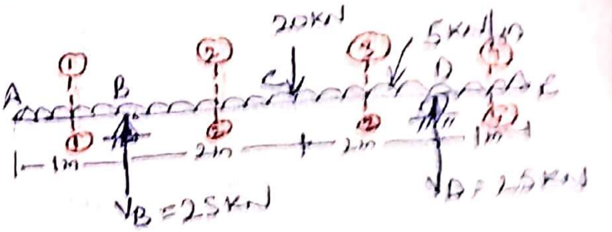
$\frac{dM}{dx} = 22 - 10x$

$\frac{d^2M}{dx^2} = -10$

At $x = 1$, $M = 17\,kNm$

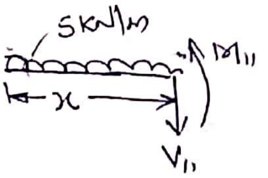
$x = 2$, $M = 24\,kN$

...on 4



$V_B + V_D = 5\times6 + 20 = 50$ kN.

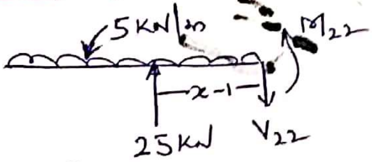
$V_B = V_D = \frac{50}{2} = 25$ kN.



$5x + V_{11} = 0$, $V_{11} = -5x$ $V_A = 0$, $V_B = -5$ kN

$-\frac{5x^2}{2} - M_{11} = 0$, $M_{11} = -\frac{5x^2}{2}$

$M_A = 0$, $M_B = -2.5$ kNm



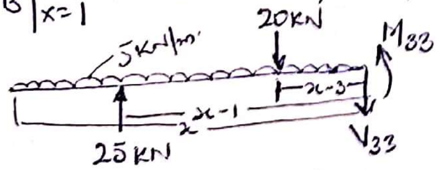
$25 - 5x - V_{22} = 0$, $V_{22} = 25 - 5x$

$V_B = 20$ kN, $V_C = 10$ kN

$\Sigma M_{22} = 0$

$-\frac{5x^2}{2} + 25(x-1) = M_{22}$

$M_B|_{x=1} = -2.5$ kNm $M_C|_{x=3} = -22.5 + 50 = +27.5$ kNm



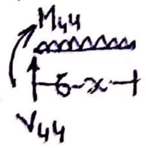
$25 - 5x - 20 = V_{33}$, $V_C|_{x=3} = -10$ kN

$V_D|_{x=5} = 25 - 25 - 20 = -20$ kN.

$\Sigma M_{33} = 0$ $25(x-1) - 5\frac{x^2}{2} - 20(x-3) = M_{33}$

$M_C|_{x=3} = 50 - 22.5 = 27.5$ kNm

$M_D|_{x=5} = 100 - 62.5 - 40 = -2.5$ kNm



$V_{44} - 5(6-x) = 0$

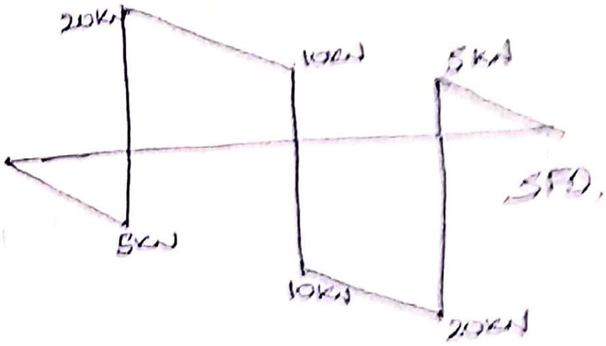
$V_{44} = 5(6-x)$ $V_D|_{x=5} = 5$ kN

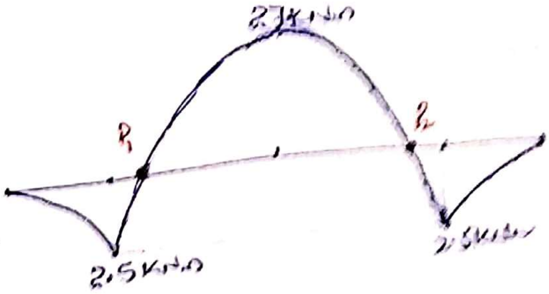
$V_E|_{x=6} = 0$

$\Sigma M_{44} = 0$

$M_{44} + \frac{5(6-x)^2}{2} = 0$ $M_{44} = -\frac{5}{2}(6-x)^2$

$M_D|_{x=5} = -2.5$ kNm. $M_E|_{x=6} = 0$





P_1, P_2 are points of contraflexure

P_1 $-M_{22} = 0$

$-\frac{5x^2}{2} + 25(x-1) = 0$

$25x - 25 - 02.5x^2 = 0$

$02.5x^2 - 25x + 25 = 0$

$x = \frac{25 \pm \sqrt{625 - 250}}{2\times2.5}$

$= \frac{25 \pm 19.36}{5}$

$x = 1.128$ m

$P_1 = 1.128$ m $P_2 = 6 - 1.128 = 4.872$ m

$\boxed{P_1 = 1.13\text{ m}}$ $\boxed{P_2 = 4.87\text{ m}}$