Solutions
Tutorial Sheet-11 (Basic Logic)

Ans 1: a) Sharks have not been spotted near the shore.
b) Swimming at the New Jersey shore is allowed, and sharks have been spotted near the shore.
c) Swimming at the New Jersey shore is not allowed, or sharks have been spotted near the shore.
d) If swimming at the New Jersey shore is allowed, then sharks have not been spotted near the shore.
e) If sharks have not been spotted near the shore, then swimming at the New Jersey shore is allowed.
f) If swimming at the New Jersey shore is not allowed, then sharks have not been spotted near the shore.
g) Swimming at the New Jersey shore is allowed if and only if sharks have not been spotted near the shore.
h) Swimming at the New Jersey shore is not allowed, and either swimming at the New Jersey shore is allowed or sharks have not been spotted near the shore.

Ans 2: a) $r \land \neg p$ **b)** $\neg p \land q \land r$ **c)** $r \rightarrow (q \leftrightarrow \neg p)$

d) $\neg q \land \neg p \land r$ **e)** $(q \rightarrow (\neg r \land \neg p)) \land \neg((\neg r \land \neg p) \rightarrow q)$ **f)** $(p \land r) \rightarrow \neg q$

Ans 3: a) Converse: "I will ski tomorrow only if it snows today."
Contrapositive: "If I do not ski tomorrow, then it will not have snowed today."
Inverse: "If it does not snow today, then I will not ski tomorrow."

b) Converse: "If I come to class, then there will be a quiz."
Contrapositive: "If I do not come to class, then there will not be a quiz."
Inverse: "If there is not going to be a quiz, then I don't come to class."

c) Converse: "A positive integer is a prime if it has no divisors other than 1 and itself."
Contrapositive: "If a positive integer has a divisor other than 1 and itself, then it is not prime."
Inverse: "If a positive integer is not prime, then it has a divisor other than 1 and itself."

Ans 4: a) Jan is not rich, or Jan is not happy.
b) Carlos will not bicycle tomorrow, and Carlos will not run tomorrow.
c) Mei does not walk to class, and Mei does not take the bus to class.
d) Ibrahim is not smart, or Ibrahim is not hard working.

Ans 5: a)

p	q	$p \land q$	$(p \land q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

b)

p	q	$p \lor q$	$p \rightarrow (p \lor q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

c)

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

d)

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

Ans 6: In each case we will show that if the hypothesis is true, then the conclusion is also.
a) If the hypothesis $p \wedge q$ is true, then by the definition of conjunction, the conclusion p must also be true.
b) If the hypothesis p is true, by the definition of disjunction, the conclusion $p \vee q$ is also true.
c) If the hypothesis $\neg p$ is true, that is, if p is false, then the conclusion $p \to q$ is true.
d) If the hypothesis $p \wedge q$ is true, then both p and q are true, so the conclusion $p \to q$ is also true.

Ans 7: For $(p \to r) \vee (q \to r)$ to be false, both of the two conditional statements must be false, which happens exactly when r is false and both p and q are true. But this is precisely the case in which $p \wedge q$ is true and r is false, which is precisely when $(p \wedge q) \to r$ is false. Because the two propositions are false in exactly the same situations, they are logically equivalent.

Ans 8: **a)** $p \vee \neg q \vee \neg r$ **b)** $(p \vee q \vee r) \wedge s$ **c)** $(p \wedge \mathbf{T}) \vee (q \wedge \mathbf{F})$

Ans 9: **i)**

- Construct the Truth Table for the proposition
- Pick each row that evaluates to T
  - If a variable r in this row is T then write it as it; otherwise, write the negation of it, i.e., $\neg r$
  - OR these written literals (literal = variable or its complement)

Example: Truth Table for $p \vee q \to \neg r$

p	q	r	$\neg r$	$p \vee q$	$p \vee q \to \neg r$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	T

$p \vee q \to \neg r \equiv$
$(p \wedge q \wedge \neg r) \vee$
$(p \wedge \neg q \wedge \neg r) \vee$
$(\neg p \wedge q \wedge \neg r) \vee$
$(\neg p \wedge \neg q \wedge r) \vee$
$(\neg p \wedge \neg q \wedge \neg r)$

ii)

1. Eliminate implication signs

$$\neg(\neg p \vee q) \vee (\neg r \vee p)$$

2. Move negation inwards; eliminate double negation

$$(p \wedge \neg q) \vee (\neg r \vee p)$$

3. Convert to CNF using associative/distributive laws

$$(p \vee \neg r \vee p) \wedge (\neg q \vee \neg r \vee p)$$