Quantum Mechanics

“God does not play dice with the universe.” **A. Einstein**

“Not only does God play dice but... he sometimes throws them where they cannot be seen.” **Stephen Hawking**

“I think I can safely say that nobody understands quantum mechanics.” **Richard Feynman**

Quantum mechanics is the description of the behaviour of matter and light in all its details. In particular, of happenings on an atomic scale.

Atomic behaviour (like proton, neutrons, electrons, photons etc.) is very unlike ordinary behaviour that it is very difficult to understand and get used to it.

Classical mechanics is an approximation of quantum mechanics.

In classical mechanics, future of the particle is completely determined by its initial position, momentum and the forces acting upon it.
In quantum mechanics, the initial state of the particle can not be established with sufficient accuracy, therefore, future of the particle has uncertainty associated with it.

de Broglie hypothesis : A moving body behaves in a certain way as though it has wave nature.

A photon of light of frequency ν has the momentum

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}\,; since\; c = \nu\lambda$$

Which means wavelength of photon is specified by it

momentum according to $\lambda = \frac{h}{p}$ (1)

de Broglie suggested that equation (1) is completely general and applies to material particles as well as photons.

The momentum of a particle of mass ***m*** and velocity ***v*** is given by **p = $\boldsymbol{\gamma}$mv,** and its de Broglie wavelength will be

$$\lambda = \frac{h}{\gamma m v}$$ (de Broglie wavelength)

$\boldsymbol{\gamma}$ is the relativistic factor and is given by $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

For non-relativistic cases, $v \ll c$ and hence $\boldsymbol{\gamma}$= 1.

The greater the particle's momentum, the shorter its wavelength.

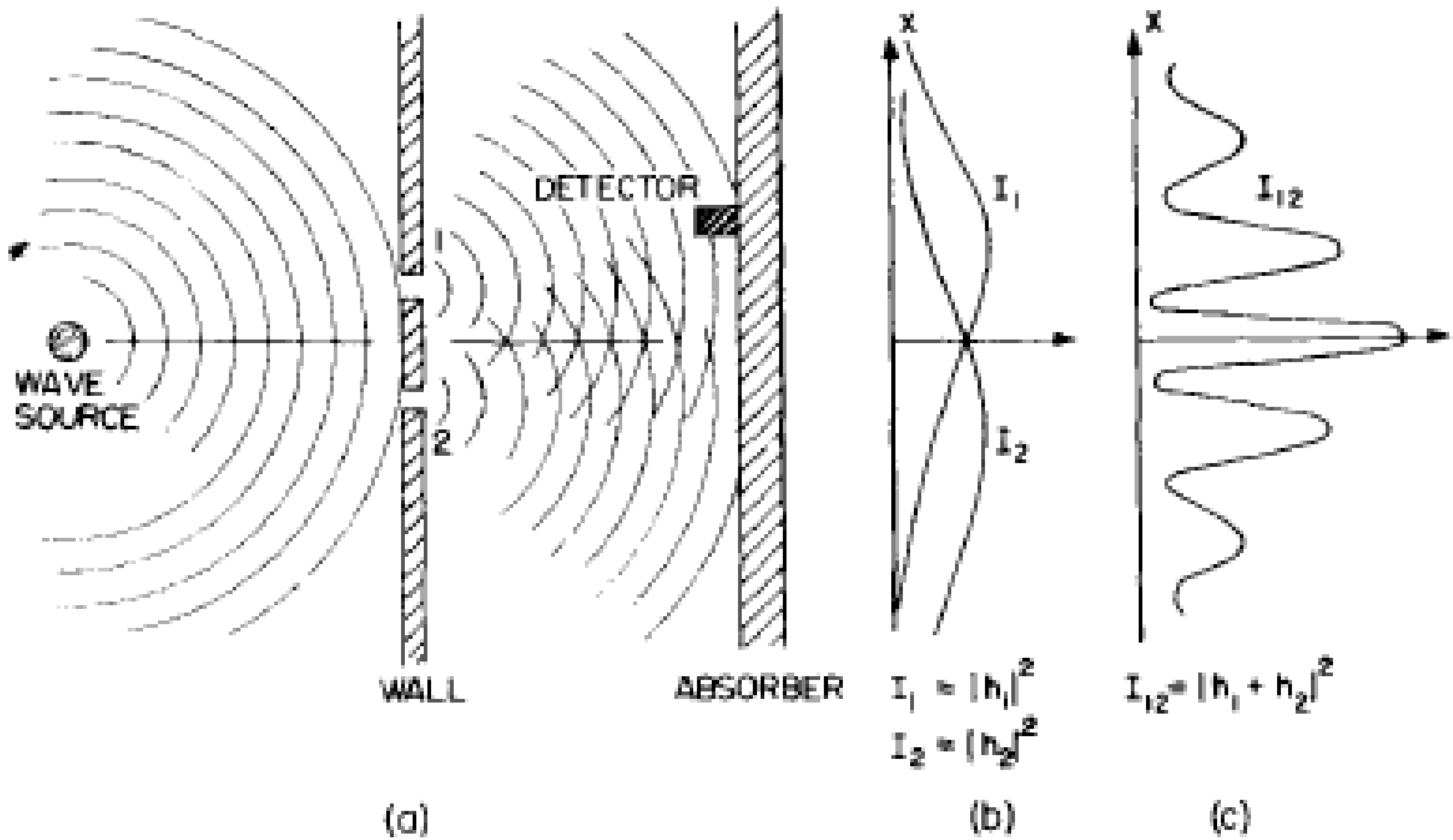
The wave and particle aspects of moving bodies can **never** be observed at the same time.

Under certain situations, a moving body resembles a wave and in others it resembles a particle.

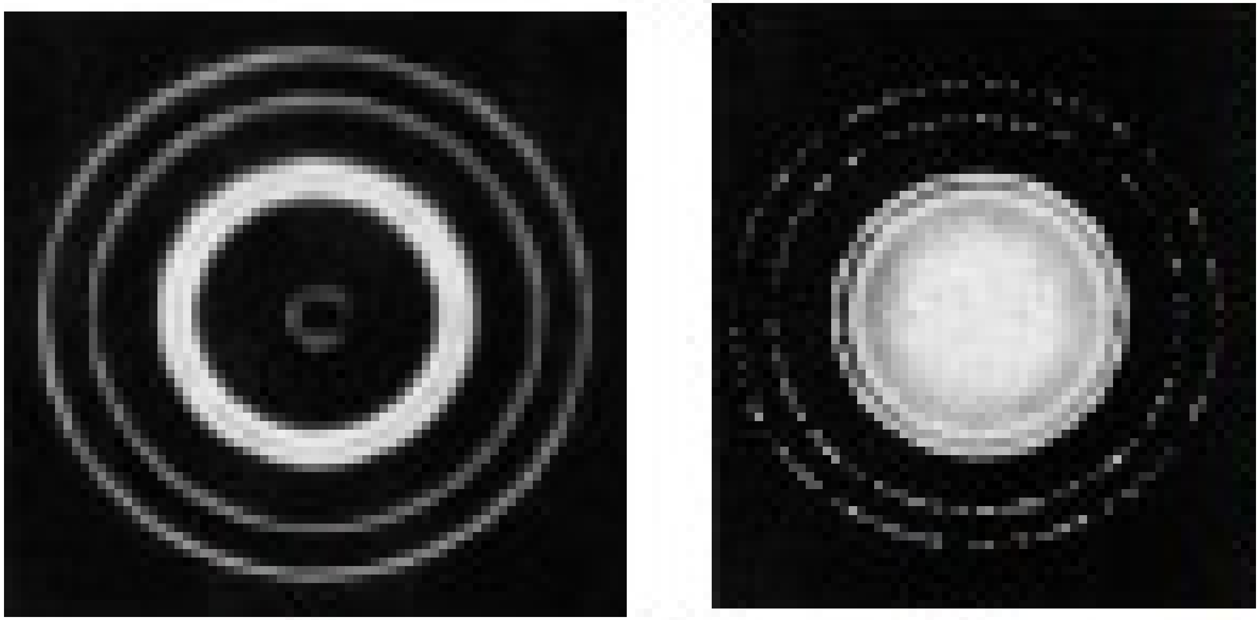
How the de Broglie wavelength compares with its dimensions and the dimension of whatever it interacts with is very important.

de Broglie had no direct experimental evidence to support his conjecture, hence it was called hypothesis.

Proof of de Broglie hypothesis :



What if this wave source is replaced by electron gun and then bullet gun?



(a) (b)

The diffraction pattern of aluminium foil produced (a) by x-rays and (b) by electrons.

"Thomson, the father, was awarded the Nobel Prize for having shown that the electron is a particle, and Thomson, the son, for having shown that the electron is a wave."

Heisenberg uncertainty principle :

"It is impossible to know both the exact position and exact momentum of an object at the same time."

Hence, we can not know the future because we can not know the present.

Mathematically, $$\Delta x \Delta p \geq \frac{h}{4\pi}$$

If you are talking about 3 dimension then the uncertainty principle is still true (you can not measure simultaneously precisely position in x and p_x; y and p_y; z and p_z). But you can measure simultaneously precisely position in x and p_y (or p_z) and so on.

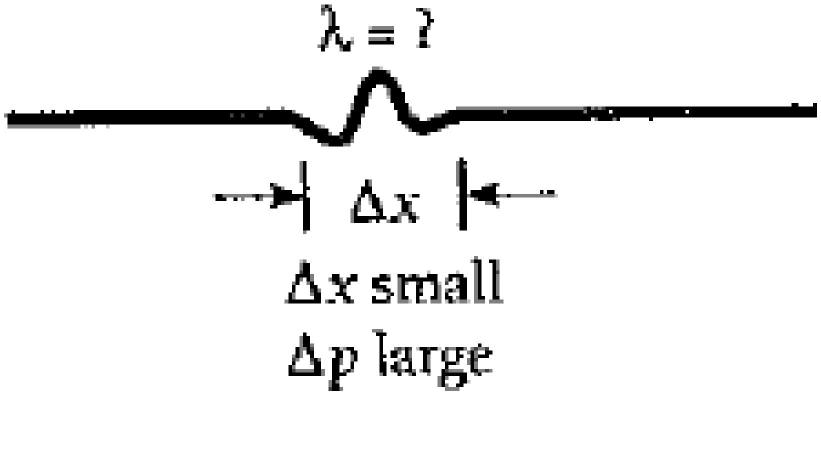
Uncertainty principle in energy and time is given by

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

Planck's constant *h* is so small that limitation imposed by uncertainty principle are significant only in the atomic scale.

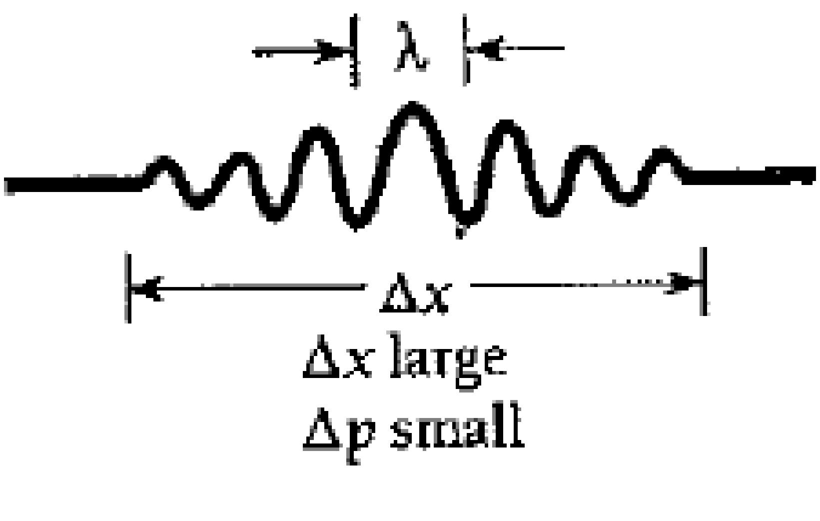
The uncertainties are not due to inadequate apparatus. Doesn't matter how sophisticated an experimental set up is, these uncertainties are fundamentally there!

why position and momentum can not be measured precisely simultaneously



(a)

- Narrower wave group, position can be determined more precisely.
- Wavelength is not well defined; not enough waves to measure λ accurately. $\lambda = h/\gamma mv$, hence momentum is not precise.



(b)

- Wider wave group, position can not be determined precisely.
- Wavelength is well defined; enough waves to measure λ accurately. $\lambda = h/\gamma mv$, hence momentum is well precise.

Significance of wave function :

The wave function is a function of x, y, z and t and is generally a complex quantity having no direct physical significance. The probability of finding a particle at certain point can never be negative as probability can have values between 0 and 1.

The wave function Ψ is not an observable quantity.

The Probability of experimentally finding the body described by the wave function Ψ at point x,y,z, at time t is proportional to the value of $|\Psi|^2$ *there at t.*

A larger value of $|\Psi|^2$ means stronger probability of object's presence. A smaller value of $|\Psi|^2$ means lesser probability of object's presence.

Wave function $\Psi = A + iB$

Where A and B are real functions. The complex conjugate Ψ^* *of* Ψ *is*

Complex conjugate $\Psi = A - iB$

And so $|\Psi|^2 = \Psi^* \Psi = A^2 - i^2B^2 = A^2 + B^2$

Hence, $|\Psi|^2$ *is a positive real quantity as required.*

The linear momentum, angular momentum and energy of the object can be established from wave function Ψ.

Normalization:

The wave function must be normalizable.

Since **|Ψ|²** is the *probability density* of finding object *described by* ***Ψ*,** integral of over all space must be finite—because object has to be somewhere i.e.

$$\int_{-\infty}^{\infty} |\Psi|^2 \, dV = 1 \qquad \text{Normalization}$$

If $\int_{-\infty}^{\infty} |\Psi|^2 \, dV = 0$, the particle doesn't exist.

Obviously this integral can not be infinity and still mean anything.

Non-normalizable wave function can not represent particle. A normalized wave function stays normalized for ever.

Conditions for well behaved wave function :

1. Ψ must be continuous and single valued because probability can have one value at a particular place and time, and continuous.

2. Momentum considerations require that partial derivatives $\partial\Psi/\partial x$, $\partial\Psi/\partial y$ and $\partial\Psi/\partial z$ be finite, continuous, and single valued.

3. Ψ must be normalizable which means that Ψ must go to zero as $x \to \pm\infty$, $y \to \pm\infty$, $z \to \pm\infty$ in order that probability of finding the object over all space is finite constant.

For a normalizable wave function, the probability that the particle will be found between x_1 and x_2 (if motion is restricted in x direction) is given by

$$P_{x_1 x_2} = \int_{x_1}^{x_2} |\Psi|^2 \, dx$$

Expectation value : *how to extract information from a wave function.*

The wave function $\Psi(x,y,z,t)$ contains all information about the particle permitted by uncertainty principle.

Suppose if a particle is confined in x-direction, then **expectation value <x>** of position of the particle described by wave function $\Psi(x,y,z,t)$ can be calculated.

Average position $\bar{x}$ of number of identical particles distributed along x-axis such that N_1 particles are at x_1, N_2 are at x_2, N_3 are at x_3 and so on is given by

$$\bar{x} = \frac{N_1 x_1 + N_2 x_2 + N_3 x_3 + \ldots\ldots}{N_1 + N_2 + N_3 + \ldots\ldots} = \frac{\sum N_i x_i}{\sum N_i} \qquad (2)$$

When dealing with single particle, the number N_i of particles at x_i is replaced by the probability P_i that the particle is found in interval dx at x_i. This probability is :

$$P_i = \left|\Psi_i^2\right| dx$$

Where Ψ_i is the particle wave function at x_i. Making these substitutions in equation (2), the expectation value of the position of a single particle is given by

$$\langle x \rangle = \frac{\int_{-\infty}^{\infty} x \left|\Psi\right|^2 dx}{\int_{-\infty}^{\infty} \left|\Psi\right|^2 dx} \qquad (3)$$

$$\langle x \rangle = \frac{\int_{-\infty}^{\infty} x |\Psi|^2 dx}{\int_{-\infty}^{\infty} |\Psi|^2 dx} \qquad (3)$$

If Ψ is a normalized wave function then denominator in Eq. (3) is 1 (because the particle exists somewhere between x = +∞ and x = -∞). In that case

Expectation value for position

$$\langle x \rangle = \int_{-\infty}^{\infty} x |\Psi|^2 dx$$

Physically <x> is the average of measurements performed on particles all in the state Ψ.
It doesn’t mean that if you measure position of one particle Again and again, <x> will be the average of the results.

Expectation value <G(x)> of any quantity that is function of x for example potential energy can be calculated in this way

Expectation value

$$\langle G(x)\rangle = \int_{-\infty}^{\infty} G(x)\left|\Psi\right|^2 dx$$

The expectation value of momentum **can not** be calculated this way because uncertainty principle doesn't allow that because $p(x) = mv = m\,(dx/dt)$;

If x is specified then $\Delta x = 0$ and then corresponding momentum *p* can not be specified because $\Delta x\, \Delta p \geq h/4\pi$.

The same problem occurs for the expectation value of energy $<E>$ as well. The expectation values $<p>$ and $<E>$ are calculated in other way (operators are used).

Again, this kind of limitation is not observed in classical mechanics.

Operators (another way to calculate expectation value) :

A free particle wave function is given by $\Psi = A\,e^{-(i/\hbar)(Et - px)}$

Differentiating above equation w.r.t. x and t gives

$$\frac{\partial \Psi}{\partial x} = \frac{i}{\hbar}\, p\, A\, e^{-(i/\hbar)(Et-px)} = \frac{i}{\hbar}\, p\, \Psi$$

$$\Rightarrow p\,\Psi = \frac{\hbar}{i}\, \frac{\partial}{\partial x}\, \Psi$$

$$similarly\, \frac{\partial \Psi}{\partial t} = -\frac{i}{\hbar}\, E\, A\, e^{-(i/\hbar)(Et-px)} = -\frac{i}{\hbar}\, E\, \Psi$$

$$\Rightarrow E\,\Psi = i\,\hbar\, \frac{\partial}{\partial t}\, \Psi$$

$$p\Psi=\frac{\hbar}{i}\frac{\partial}{\partial x}\Psi \Rightarrow \hat{p}=-i\hbar\frac{\partial}{\partial x}$$

$$E\Psi=i\hbar\frac{\partial}{\partial t}\Psi \Rightarrow \hat{E}=i\hbar\frac{\partial}{\partial t}$$

An operator tells us what operation to carry on quantity following it. *E* and *p* are operators.
Even if they are derived for free particle, they are entirely general. In that case, E is sum of K.E. and P.E.

$$E=\frac{p^2}{2m}+U$$

multiplying both sides by Ψ

$$\Rightarrow E\Psi=\frac{p^2}{2m}\Psi+U\Psi$$

$$\boldsymbol{i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\Psi}{\partial x^2}\right)+U\Psi}$$

Schrodinger equation in 1-d

Expectation value of *p for normalized wave function* is given by

$$\langle p\rangle=\int_{-\infty}^{\infty}\Psi^{*}\hat{p}\,\Psi\,dx=\int_{-\infty}^{\infty}\Psi^{*}\left(-i\hbar\frac{\partial}{\partial x}\right)\Psi\,dx$$

$$=-i\hbar\int_{-\infty}^{\infty}\Psi^{*}\frac{\partial\Psi}{\partial x}dx$$

Expectation value of *E* is given by

$$\langle E\rangle=\int_{-\infty}^{\infty}\Psi^{*}\hat{E}\,\Psi\,dx=\int_{-\infty}^{\infty}\Psi^{*}\left(i\hbar\frac{\partial}{\partial t}\right)\Psi\,dx$$

$$=i\hbar\int_{-\infty}^{\infty}\Psi^{*}\frac{\partial\Psi}{\partial t}dx$$

Remember operators have to be in between Ψ^* and Ψ.

Expectation value of observable G(x,p) can be calculated as

$$\langle G(x,p)\rangle=\int_{-\infty}^{\infty}\Psi^{*}\hat{G}\,\Psi\,dx$$

Schrodinger Equation :

A basic principle (like Newton's law in classical mechanics) that can not be derived from anything else.

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Psi}{\partial z^2}\right)+U\Psi$$

Where *m* is mass of the particle and *U* is the potential energy of the particle.

If the particle motion is limited to 1-dimension (x,t) the above equation becomes

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\Psi}{\partial x^2}\right)+U\Psi$$

Time-independent Schrodinger equation :

For a particle whose potential energy doesn't depend on time explicitly, forces acting on the particle and hence, potential energy U, vary with position of the particle only. In that case, Schrodinger equation can be simplified by removing reference to *t*.

The wave function can be written as $\Psi(x,t)=\varphi(x)f(t)$

$$\frac{\partial \Psi(x,t)}{\partial x}=\frac{d\varphi(x)}{dx}f(t)$$

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2}=\frac{d^2\varphi(x)}{dx^2}f(t) \qquad (4)$$

$$\frac{\partial \Psi(x,t)}{\partial t}=\varphi(x)\frac{df(t)}{dt} \qquad (5)$$

Using equation (4) and (5) in Schrodinger equation

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\Psi}{\partial x^2}\right)+U\Psi \quad \text{gives}$$

$$i\hbar\varphi(x)\frac{df(t)}{dt}=-\frac{\hbar^2}{2m}f(t)\frac{d^2\varphi(x)}{dx^2}+U\varphi(x)f(t)$$

Dividing both sides by $\varphi(x)f(t)$

$$i\hbar\frac{1}{f(t)}\frac{df(t)}{dt}=-\frac{\hbar^2}{2m}\frac{1}{\varphi(x)}\frac{d^2\varphi(x)}{dx^2}+U$$

L.H.S. is function of *t* only and the R.H.S. is function of *x* only. **This can only be possible if both sides are constant**--Otherwise by varying *t*, we can change L.H.S. without touching R.H.S. and two will no longer be equal.

The constant is energy E. Then

(I am getting rid of writing f as function of t and Φ as function of x).

$$i\hbar \frac{1}{f}\frac{df}{dt}=E$$

$$\Rightarrow \frac{df}{dt}=-\frac{iE}{\hbar}f \qquad (6)$$

$$\text{and}, -\frac{\hbar^2}{2m}\frac{d^2\varphi}{dx^2}+U\varphi=E\varphi \qquad (7)$$

Equation (7) is called **time-independent (steady state) Schrodinger equation**. General solution for equation (6) is given by

$$f(t)=C\,e^{\frac{-iEt}{\hbar}}$$

$$\text{Hence}, \Psi(x,t)=\varphi(x)\,e^{\frac{-iEt}{\hbar}}$$

Constant C is absorbed in Φ(x).

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi}{dx^2}+U\varphi=E\varphi \qquad (7)$$

In three dimesions, equation (7) can be written as

$$-\frac{\hbar^2}{2m}\left(\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz^2}\right)+U\varphi=E\varphi$$

Eigenvalues and Eigenfunctions :

The values of energy E_n for which Schrodinger's steady-state equation can be solved are called **eigenvalues** and the corresponding wave functions Φ_n are called **eigenfunctions.**

The condition that the variable G be quantized is that the following condition should be satisfied:

Eigenvalue equation : $\hat{G}\varphi_n = G_n\varphi_n$

Each G_n is a real number.

If measurements of G are made on a number of identical systems all in states described by the particular eigenfunction Φ_n, each measurement will yield the single value G_n.

Schrodinger equation in 1-d is given by

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi}{dx^2}+U\varphi=E\varphi \qquad (7)$$

$$\Rightarrow \hat{H}=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+U$$

Hamiltonian operator

in such steady-state Schrodinger equation

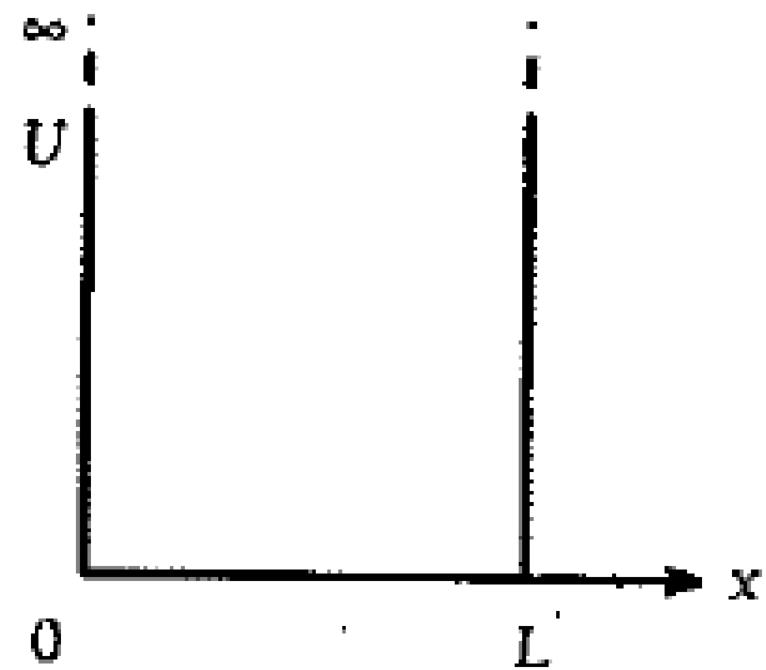
$$\hat{H}\varphi_n=E_n\varphi_n$$

This leads to quantization of energy.

Particle in a box or infinite well : particle in a box with infinitely hard walls (particle doesn't lose energy each time it strikes a wall).

Consider a particle of mass *m* trapped in a box of length L, the particle motion is restricted to x = 0 to x = L. For convenience, the potential energy U is taken to be zero inside the box and infinite for x < 0 and x > L.

Since particle can not have infinite energy, it can not exist outside box, and so the wave function Φ is zero for $x \leq 0$ and $x \geq L$.



The Schrodinger equation is given by

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi}{dx^2}+U\varphi=E\varphi$$

For particle inside the box, above equation becomes

$$\frac{d^2\varphi}{dx^2}+\frac{2mE}{\hbar^2}\varphi=0$$

(Remember U=0 inside box)

The general solution of this equation (S.H.O.) is given by

$$\varphi=A\sin\frac{\sqrt{2mE}}{\hbar}x+B\cos\frac{\sqrt{2mE}}{\hbar}x \qquad (8)$$

The solution given by (8) is subjected to boundary condition: wave function must vanish for x=0 and x=L (i.e. Φ=0 for x=0 and x=L). A and B are constants to be calculated.

$$\varphi = A\sin\frac{\sqrt{2mE}}{\hbar}x + B\cos\frac{\sqrt{2mE}}{\hbar}x \qquad (8)$$

At x=0, cos (0) =1, therefore second term in equation (8) can not describe the particle because it does not vanish at x=0 (remember well behaved wave function must be continuous). Therefore, B=0.

$$\varphi = A\sin\frac{\sqrt{2mE}}{\hbar}x \qquad (9)$$

At x=0, sin(0)=0. Hence Φ=0 at x=0. But Φ must also vanish at x=L which is possible if

$$\frac{\sqrt{2mE}}{\hbar}L = n\pi \qquad n = 1,2,3\ldots\ldots\ldots\ldots \qquad (10)$$

$$\frac{\sqrt{2mE}}{\hbar}L=n\pi \qquad n = 1,2,3\ldots\ldots\ldots\ldots \qquad (10)$$

From equation (10) it is clear that energy of the particle can have certain values, which are ***eigenvalues****. These eigenvalues constitute energy levels of the system and are given by (10) as*

$$\sqrt{(2mE_n)}=\frac{n\pi\hbar}{L}\Rightarrow 2mE_n=\frac{n^2\pi^2\hbar^2}{L^2}$$

$$E_n=\frac{n^2\pi^2\hbar^2}{2mL^2} \qquad n=1,2,3\ \ldots\ldots \qquad (11)$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2 m L^2} \qquad n=1,2,3 \ldots\ldots \qquad (11)$$

Each permitted energy is called an **energy level** and the integer ***n*** specifying an energy level E_n is called its **quantum number**.

Conclusions :

1. Trapped particle can not have an arbitrary energy. Boundary conditions or its confinement restricts its wave function and hence particle is allowed to have only certain specific energies and no others (No counterpart in classical). Exact energies depend on mass of particle and details how it is trapped.
2. Because Planck's constant is so small – quantization of energy clearly noticeable only when *m* and *L* are also small.

3. A trapped particle can not have zero energy. The de Broglie wavelength ($\lambda = h/mv$) is infinite when $v=0$. There is no way a trapped particle can have an infinite wavelength, so particle must have at least some kinetic energy. Exclusion of $E=0$ has no counterpart in classical physics.

4. Energy levels are not equally spaced. $E_n \propto n^2$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \qquad n=1,2,3 \ldots\ldots \qquad (11)$$

Wavefunction :

The wavefunction of particle in a box whose energy is E_n is given by

$$\varphi_n = A \sin \frac{\sqrt{2mE_n}}{\hbar} x$$

Putting

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

We get

$$\varphi_n = A \sin \frac{\sqrt{2m \frac{n^2 \pi^2 \hbar^2}{2mL^2}}}{\hbar} x = A \sin \frac{\frac{n \pi \hbar}{L}}{\hbar} x = A \sin \frac{n \pi x}{L}$$

n=1,2,3 ……

$$\varphi_n = A \sin \frac{n\pi x}{L}$$ n=1,2,3 ……

Φ_n is a well behave wave function because :

1. For each value of n, Φ_n is finite, single valued function of x and Φ_n and $\partial\Phi_n/\partial x$ are continuous.

2. The integral $|\Phi_n|^2$ over all space is finite.

$$\int_{-\infty}^{\infty} |\varphi_n|^2 dx = \int_0^L |\varphi_n|^2 dx = A^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{A^2}{2} \int_0^L \left(1 - \cos\left(\frac{2n\pi x}{L}\right)\right) dx$$

$$= \frac{A^2}{2} \left[\int_0^L dx - \int_0^L \cos\left(\frac{2n\pi x}{L}\right) dx\right]$$

$$= \frac{A^2}{2}[\int_0^L dx - \int_0^L \cos(\frac{2n\pi x}{L})dx]$$

$$= \frac{A^2}{2}[x - \frac{L}{2n\pi}\sin(\frac{2n\pi x}{L})]_0^L$$

$$= \frac{A^2}{2}[L - \frac{L}{2n\pi}\sin(\frac{2n\pi L}{L}) - 0 - \frac{L}{2n\pi}\sin(\frac{2n\pi 0}{L})]$$

$$= A^2(\frac{L}{2})$$

i.e. $$\Rightarrow \int_{-\infty}^{\infty} |\varphi_n|^2 dx = A^2(\frac{L}{2}) \qquad (12)$$

But if Φ is to be normalized that means A should be assigned a value such that equation (12) should be equal to 1.

$$\Rightarrow \int_{-\infty}^{\infty} |\varphi_n|^2 dx = A^2\left(\frac{L}{2}\right) = 1$$

(because Φ_n has to be normalized)

$$\Rightarrow A = \sqrt{\frac{2}{L}}$$

Therefore the normalized wave function Φ_n is given by

$$\varphi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

n = 1,2,3…………..

$$\varphi_1 = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}; \varphi_2 = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}$$

$$\varphi_3 = \sqrt{\frac{2}{L}} \sin \frac{3\pi x}{L}$$

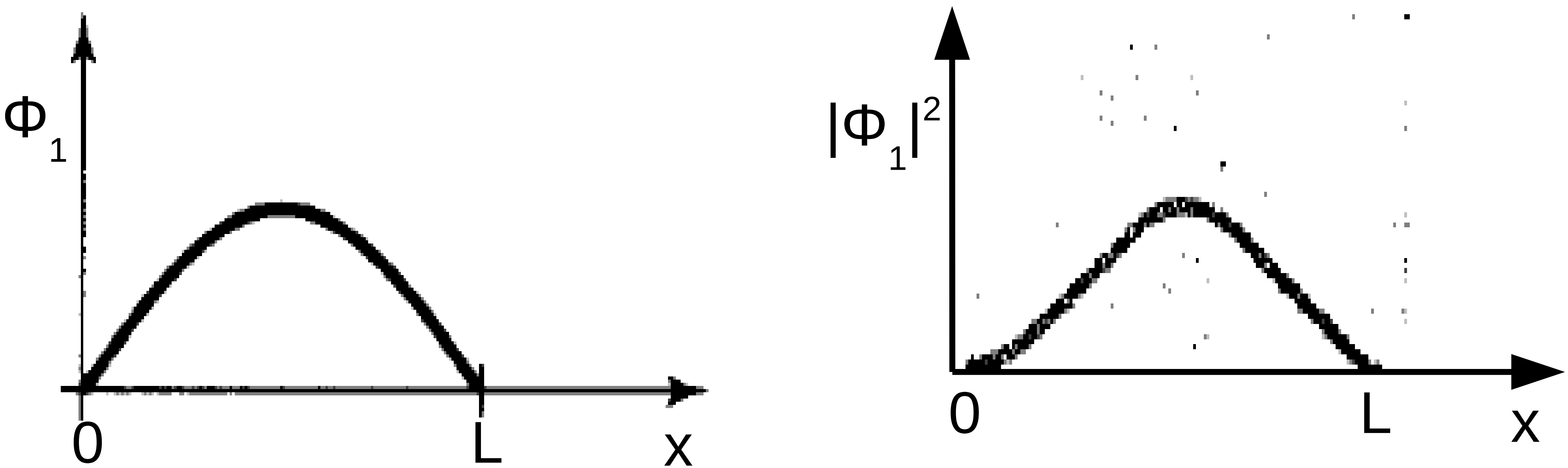
The ground state (lowest energy) of this particle :

$$\varphi_1 = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L};$$

when x = 0, $\Phi_1 = 0$

when x = L/2, Φ_1 = max

when x = L, $\Phi_1 = 0$



Particle has the maximum probability to be in the middle of box in the lowest energy state.

$$\varphi_2 = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}$$

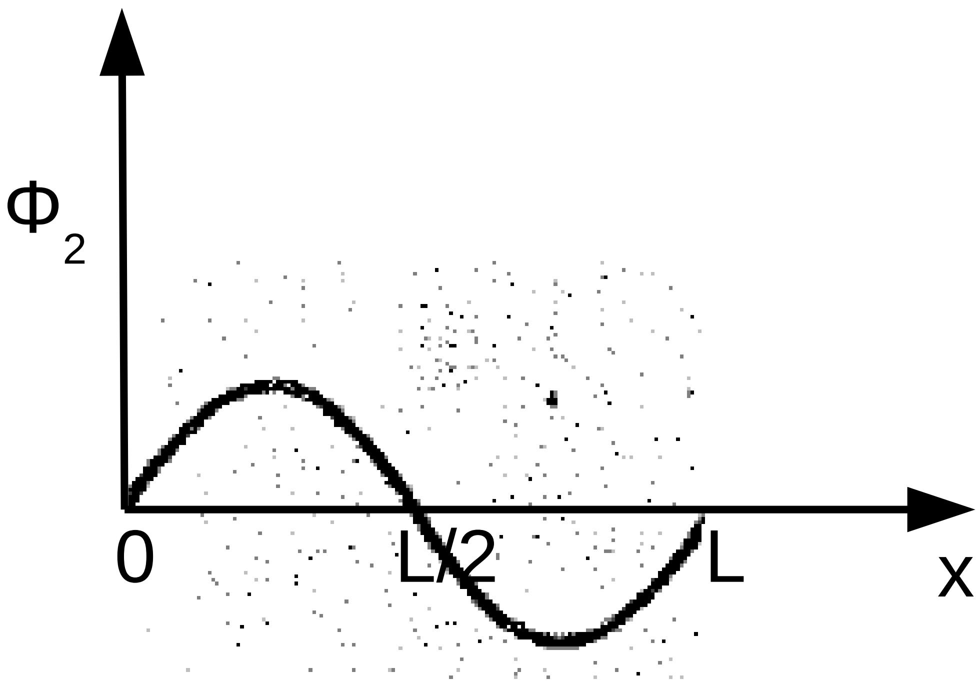
when x = 0, $\Phi_2 = 0$

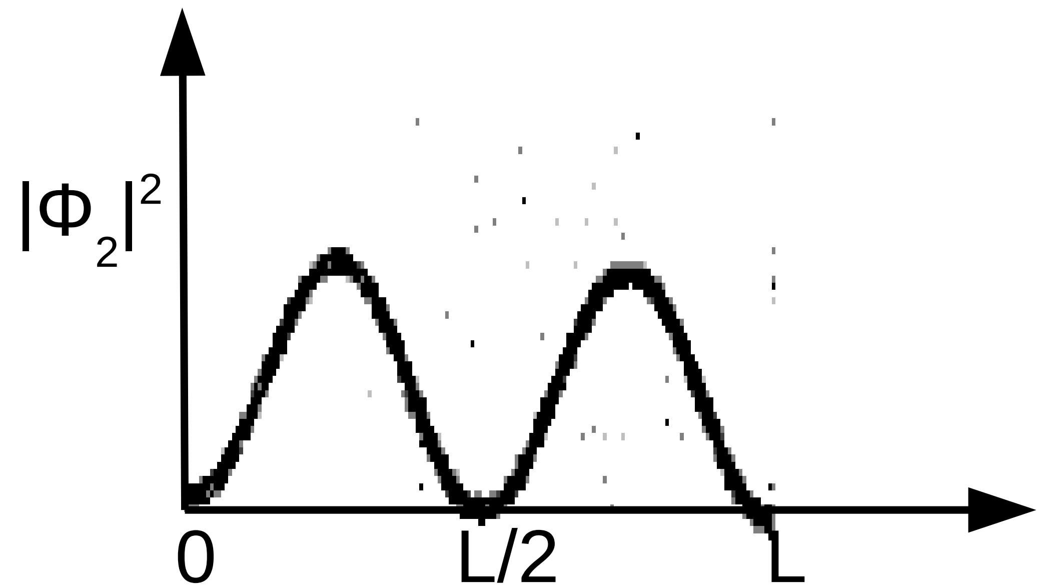
when x = L/4, Φ_2 = max

when x = L/2, $\Phi_2 = 0$

when x = 3L/4, Φ_2 = min (-ve)

when x = L, $\Phi_2 = 0$

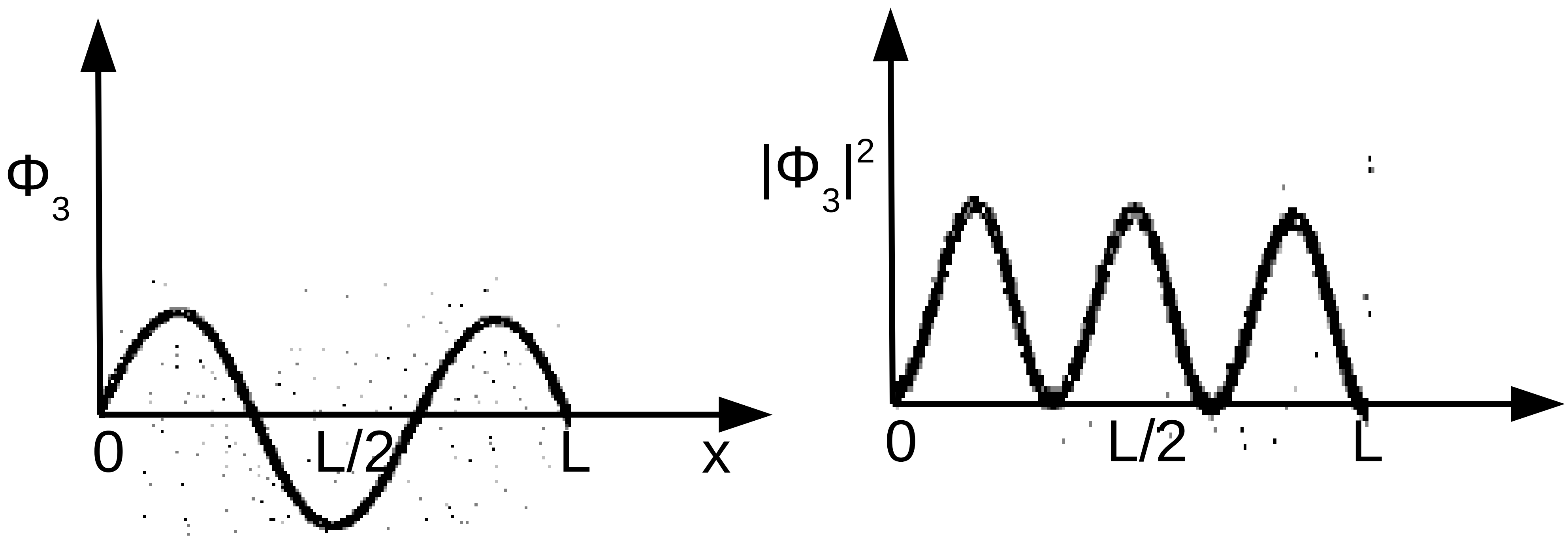




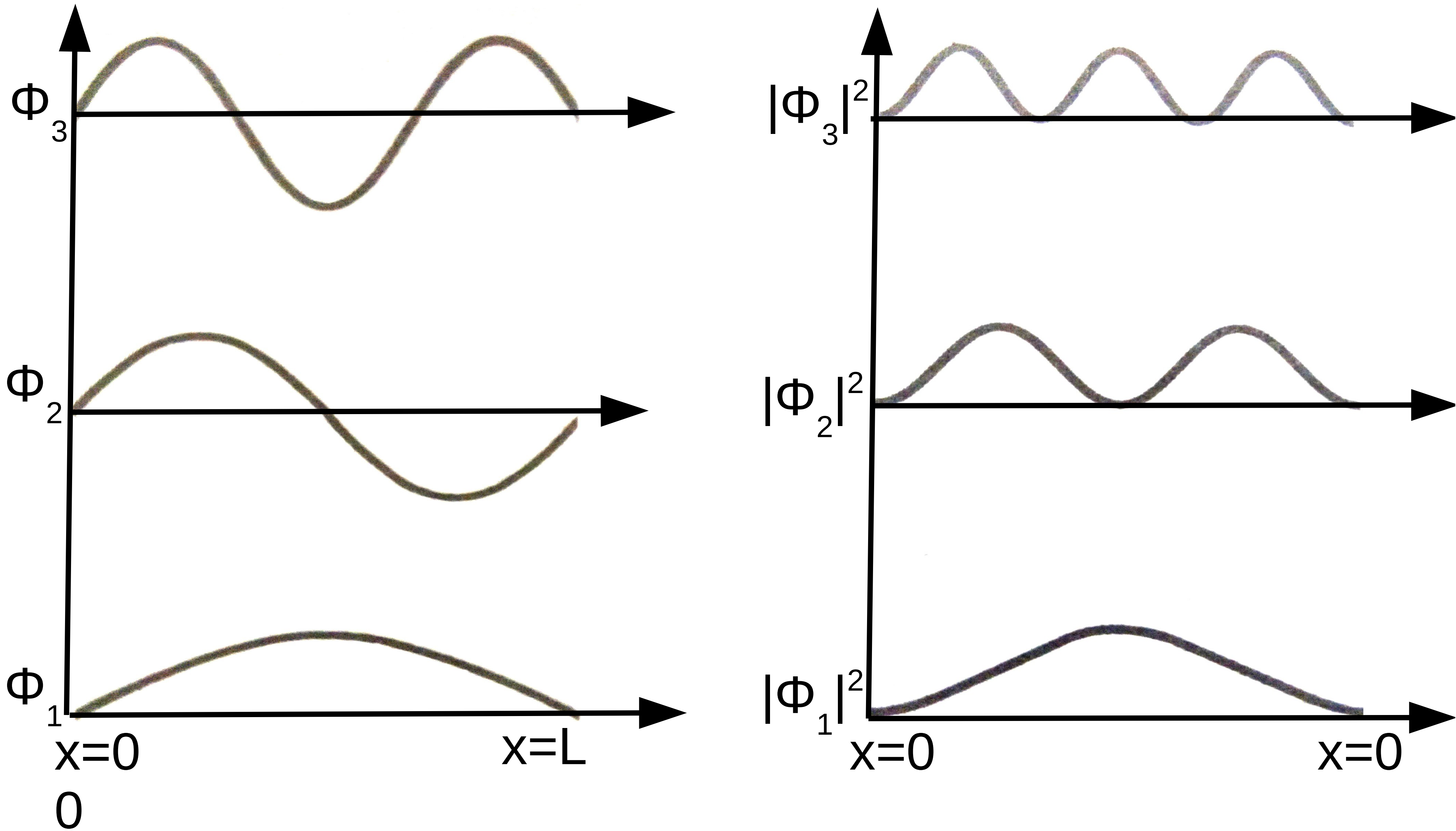
Particle has the minimum probability to be in the middle of box in the first excited energy state.

$$\varphi_3 = \sqrt{\frac{2}{L}} \sin \frac{3\pi x}{L}$$

(work it out yourself)



Comparison of different states :

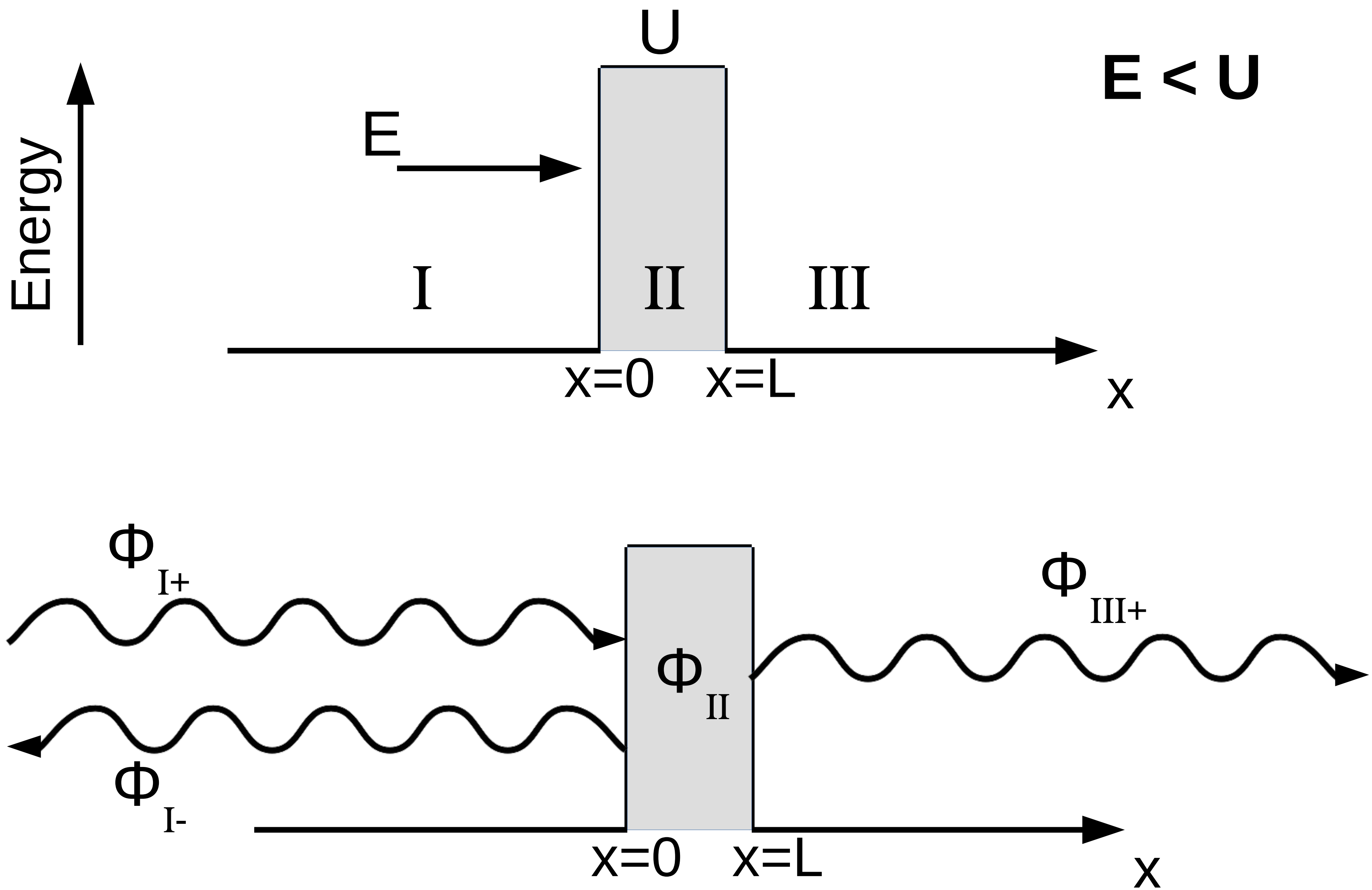


Classical physics, of course, suggests same probability for the particle being anywhere in the box.

Tunneling : Another phenomenon that can be explained only quantum mechanically. No classical counter part.

Potential energies are never infinite in real world and well with infinitely hard walls (infinite potential well) has no physical counterpart.

In real world, we deal with potential barriers of finite height.



On both sides of barrier U=0, no forces act on the particle there. Barrier height is “U” and width is “L”.

Φ_{I+} : Incoming beam of particles moving to the right.

Φ_{I-} : Reflected particles moving to the left.

Φ_{II} : Particles inside the barrier, some of which end up in region III while others return to I.

Φ_{III} : Transmitted particles moving to the right.

The **transmission probability** for a particle to pass through The barrier is equal to fraction of incident beam that gets through the barrier.

Transmission probability is given by $T = e^{-2k_2 L}$ where

$$k_2 = \frac{\sqrt{2m(U-E)}}{\hbar}$$

m is mass of the particle, E is K.E. and U is barrier height, L is width of the barrier.

Classical Mechanically, when energy of the particle is less than barrier height U, particle must be reflected back. In QM, the de Broglie waves corresponding to particle are partly reflected and partly transmitted.

If the barrier is infinitely thick then the transmission probability will be zero. But if it is of finite thickness there is finite probability-however small- for particle to tunnel through region II and emerge in region III.

Particle doesn't go over the top of the barrier, but tunnels through the barrier.

The higher the barrier and wider it is, less will be the chance that the particle can get through the barrier.