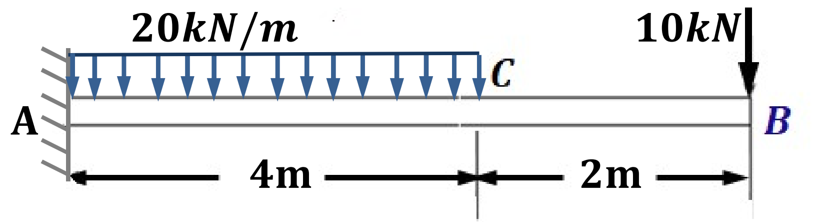
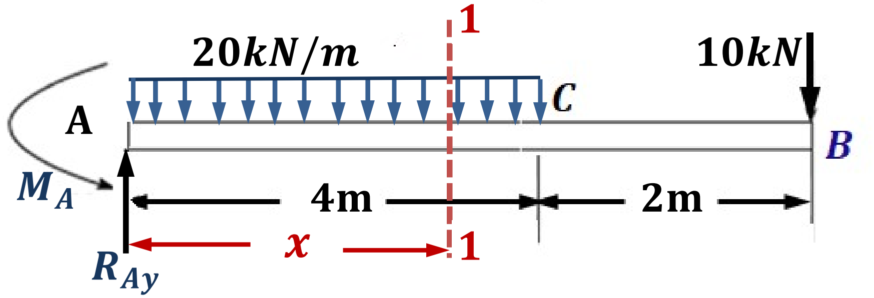
THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY
(Deemed to be University)

Ex. Draw SF and BM diagrams for the given cantilever beam loaded as shown.



Solution:

1.Draw FBD of the beam, and find reactions.

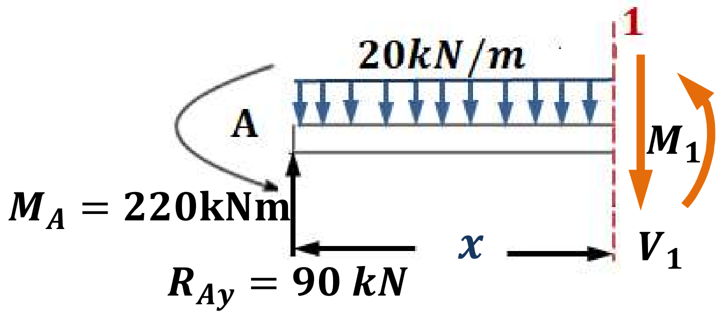


$R_{Ay} = 10 + 20 \times 4 = 90\ kN;$

$\Sigma M_A = 0;$

$M_A = (20 \times 4 \times 2) + (10 \times 6) = 220\ kNm;$

2. Take a section 1-1 at a distance x somewhere between **A** and **C** and draw FBD of LHS or RHS of the section.



4. $\Sigma F_y = 0, \quad V_1 - 90 + 20x = 0;$

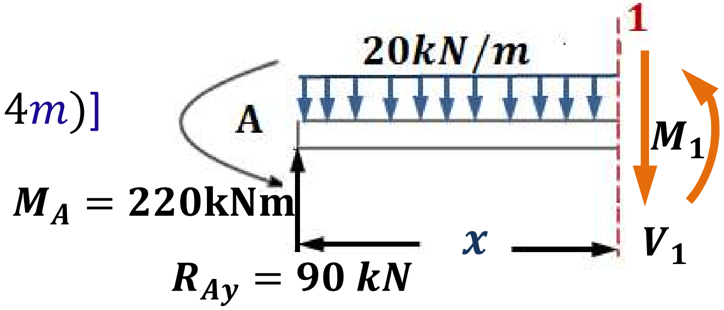
$\boldsymbol{V_1 = 90 - 20x}$ ....(1)

[Eq. for S.F. for the first segment of the Cantilever beam where ($0 \leq x \leq 4m$)]

5. $\Sigma M_{1-1} = 0, \quad 90.x - M_A - 20.x.\frac{x}{2} - M_1 = 0;$

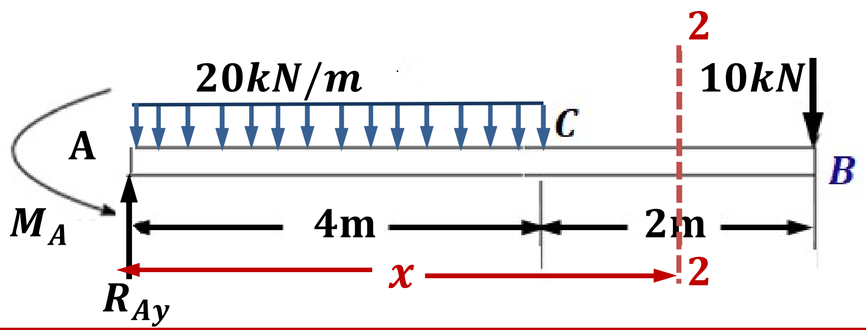
$\boldsymbol{M_1 = 90.x - 220 - 10.x^2}$ ....(2)

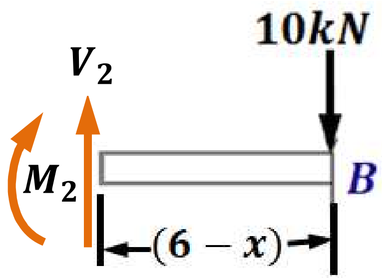
[Eq. for B.M. for the first segment of the Cantilever beam where ($0 \leq x \leq 4m$)]

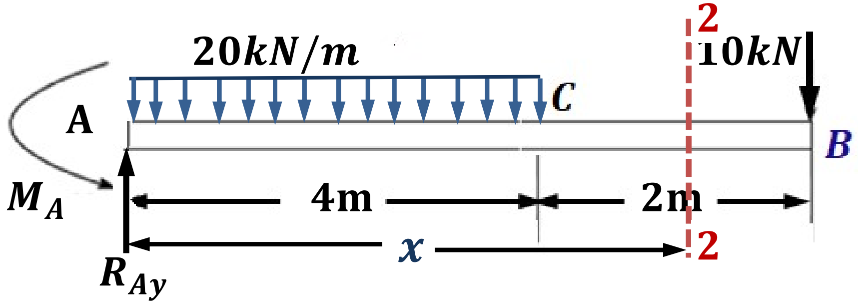


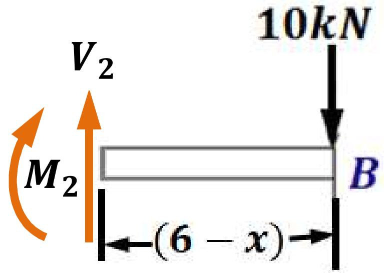
6. Take another section 2-2 between ***C*** and ***B***, at a distance ***x*** from point ***A***

and draw FBD of the RHS of the section.









7. $\Sigma F_y = 0,\quad V_2 - 10 = 0;\ \boldsymbol{V_2 = 10}$ ....(3)

[Eq. for S.F. for the IInd segment of the Cantilever beam where $(4m \le x \le 6m)$]

8. $\Sigma M_{2-2} = 0,\quad M_2 + 10.(6 - x) = 0;\ \boldsymbol{M_2 = -10(6 - x)}$ ....(4)

[Eq. for B.M. for the first segment of the Cantilever beam where $(4m \le x \le 6m)$]

To find S.F. and B.M.

After obtaining equations for S.F. and B.M., put values of $\boldsymbol{x}$ in these equations to obtain the magnitude of shear force and bending moment at all the salient points , i.e. at ***A***, $\boldsymbol{x = 0}$, at ***C***, $\boldsymbol{x = 4m}$ and at ***B***, $\boldsymbol{x = 6m}$

$$V_1 = 90 - 20x \text{.........}(1) \qquad (0 \leq x \leq 4m)$$

$$V_2 = 10 \text{ ....}(3) \qquad (4m \leq x \leq 6m)$$

$$M_1 = 90.x - 220 - 10.x^2 \dots\dots\dots\dots(2) \quad (0 \leq x \leq 4m)$$

$$M_2 = -10(6 - x) \text{ ....}(4) \qquad (4m \leq x \leq 6m)$$

To find S.F. and B.M.

at ***A***, $x = 0$, at ***C***, $x = 4m$ and at ***B***, $x = 6m$

From eq. (1), S.F. at ***A***, $V_{A(x=0)} = 90 - 20 \times 0 = 90\ kN$

and ***C***, $V_{C(x=4)} = 90 - 20 \times 4 = 10\ kN$ **variation of S.F. will be linear.**

From Eq. 3, $V_C = V_B = 10\ kN$ **S.F. will be constant from C to B.**

Eq. (2), B.M. at ***A***, $M_A(x = 0) = 90 \times 0 - 220 - 10 \times 0 = -220\ kNm$;

B.M. at **C**, $M_C(x = 4m) = 90 \times 4 - 220 - 10 \times 4^2 = -20\ kNm$, **variation of B. M. will be parabolic.**

and B.M. at **C**, $M_C(x = 4m) = -10(6 - 4) = -20\ kNm$ **(from eq. 4)**

and B.M. at **B**, $M_B(x = 6m) = -10(6 - 6) = 0$ **variation will be linear.**

To find S.F. and B.M.

$V_A = 90\ kN,\ V_C = 10\ kN,$

variation of S.F. will be linear from A to C.

$V_C = V_B = 10\ kN$ S.F. will be constant from C to B.

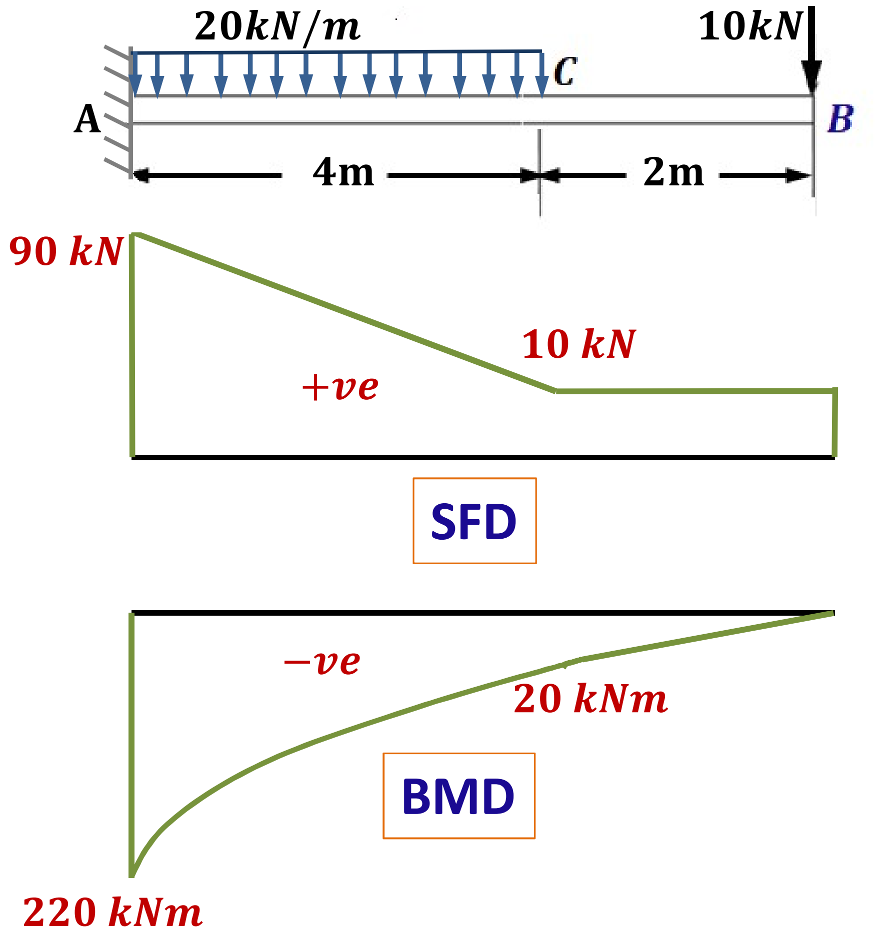
$M_A(x = 0) = -220\ kNm;$

$M_C(x = 4m) = -20kNm,$

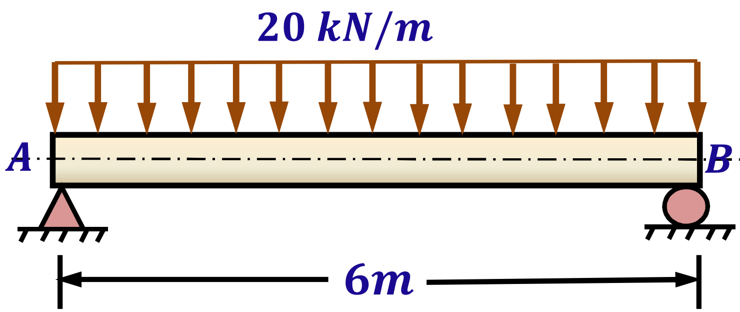
variation will be parabolic from A to C.

$M_C(x = 4m) = -20\ kNm$

$M_B(x = 6m) = 0$ variation will be linear from C to B.



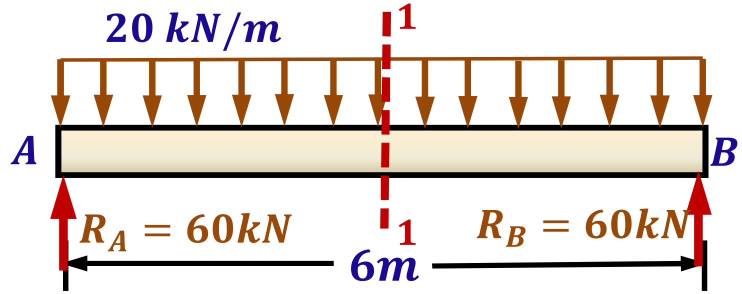
Ex: Draw the shear force and bending-moment diagrams for the beam and loading shown.



Solution: Draw FBD and find reactions.

$R_A = R_B = 60\ kN;$

Take a section **1-1** at a distance $\boldsymbol{x}$ somewhere between **A** and **B** and draw FBD of LHS.

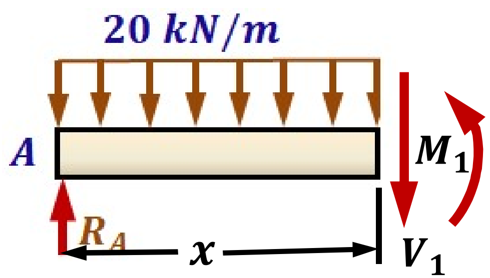


$\Sigma F_y = 0; 60 - 20.x - V_1 = 0;$

$\boldsymbol{V_1 = 60 - 20.x}$......(1)

[Eq. for S.F. in the beam where (0$\leq x \leq 6m$)]

$\Sigma M_{1-1} = 0;\ \ 60.x - 20.x.\frac{x}{2} - M_1 = 0;$



$\boldsymbol{M_1} = 60.x - 10.x^2$ ....(2)

[Eq. for B.M. in the beam where (0$\leq x \leq 6m$)]

$V_1 = 60 - 20.x$

S.F. at A, $x = 0$, $\boldsymbol{V_A = 60\ kN}$,

at B, $x = 6m$, $\boldsymbol{V_B = -60\ kN}$,

Shear force varies linearly between points A and B, and changes from +ve to –ve.

Somewhere it will be zero. So, put

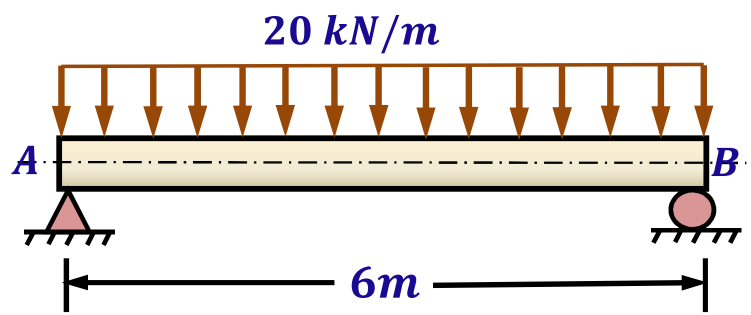
$V_1 = 0;\ 60 - 20.x = 0;$

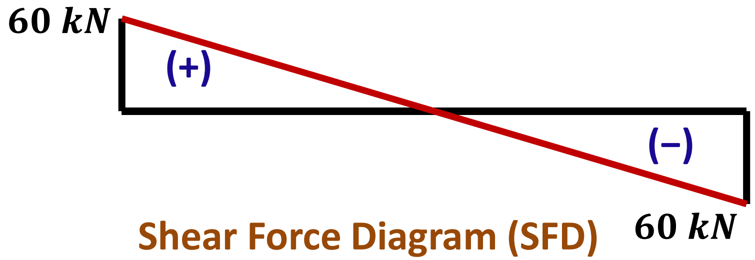
$$\boxed{\boldsymbol{x = 3m}}$$

$M_1 = 60.x - 10.x^2$

B,M. at A, $x = 0$, $\boldsymbol{M_A = 0}$,

at B, $x = 6m$, $\boldsymbol{M_B = 0}$,





Shear Force Diagram (SFD)

To find maxima of B.M., differentiate M_1 w.r.t. x

and put it equal to zero. $\frac{dM}{dx} = 0$;

$= 60 - 20x$;

which is equal to shear force, so,

$$\frac{dM}{dx} = V;$$

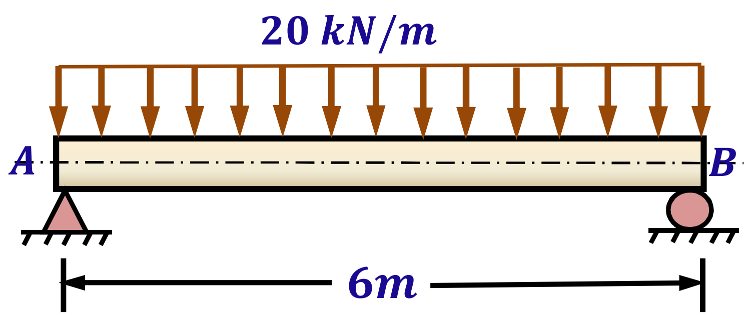
$$\frac{dM}{dx} = 0; \quad 60 - 20.x = 0; \quad \boxed{x = 3m}$$

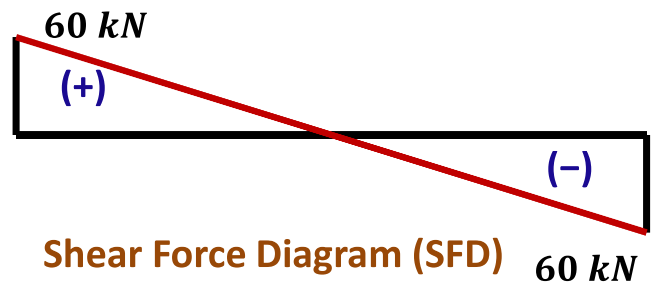
$$M_{max}(at\ x = 3m) = 60 \times 3 - 10 \times 3^2$$

$$M_{max} = 90kNm$$

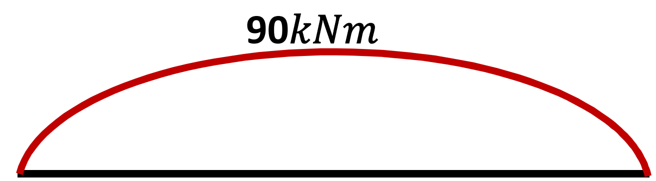
$$\frac{d^2M}{dx^2} = -20;$$

Concavity will be downward.





Shear Force Diagram (SFD)



Bending Moment Diagram (BMD)

THANK YOU