UCS405 (Discrete Mathematical Structures)

Solutions

Tutorial Sheet-1 (Set Theory)

1. To show that $23 \in A$, we must find a natural number $\mathbf{k_0}$ such that
$23 = 3k_0 + 5$
Since, every element of A has the form 3k + 5 for some $k \in N$. To find out if there is such a *k,* we simply solve the equation for $\mathbf{k_0}$ and see if the solution is an integer.

$$3k_0 + 5 = 23$$
$$3k_0 = 23 - 5$$
$$k_0 = 18/3 = 6$$

Since **6** is a natural number, we know $3 - 6 + 5 = 23 \in A$.
We use the template for element membership in a set to develop a template for proving that one set is a subset of another set.

2(i). $2^x - 1$ is always an odd number for all positive integral values of x. In particular, $2^x - 1$ is an odd number for $x = 1, 2, \ldots, 9$. Thus, A = {1, 2, 3, 4, 5, 6, 7, 8, 9}.

(ii) $x^2 + 7x - 8 = 0$
or, $(x + 8)(x - 1) = 0$
giving $x = - 8$ *or* $x = 1$

Thus, $B = \{- 8, 1\}$

3. (i) True
Since, 37 has exactly two positive factors, 1 and 37, 37 belongs to the set.

(ii) False
7747 is not a multiple of 37.

(iii) True
Since, the sum of positive factors of 28
= 1 + 2 + 4 + 7 + 14 + 28
= 56 = 2(28)

4. (i) $A = \{x \mid x \in D \; and \; x \; is \; odd\} = \{1, 3, 5, 7, \ldots, 99\}$

(ii) $B = \{y \mid y = x + 2, x \in D\}$
So, for $1 \in D, y = 1 + 2 = 3$
$2 \in D, y = 2 + 2 = 4,$
And, so on. Therefore, $B = \{3, 4, 5, 6, \ldots, 100\}$

5. Given $E = \{2, 4, 6, 8, 10\}$
(i) Let $A = \{x \mid x = n + 1, n \in E\}$
Thus, for $2 \in E, x = 3$
$4 \in E, x = 5,$
And, so on. Therefore, $A = \{3, 5, 7, 9, 11\}$.

(ii) Let $B = \{x \mid x = n^2, n \in E\}$

So, for $2 \in E, x = (2)^2 = 4, 4 \in E, x = (4)^2 = 16, 6 \in E, x = (6)^2 = 36,$ and so on. Hence, $B = \{4, 16, 36, 64, 100\}$

6. For $X = \{1, 2, 3, 4, 5, 6\}$, it is the given that $n \in X$, but $2n \notin X$.

Let, $A = \{x \mid x \in X \text{ and } 2x \notin X\}$

Now, $1 \notin A$ as $2.1 = 2 \in X$

$2 \notin A$ as $2.2 = 4 \in X$

$3 \notin A$ as $2.3 = 6 \in X$

But $4 \in A$ as $2.4 = 8 \notin X$

$5 \in A$ as $2.5 = 10 \notin X$

$6 \in A$ as $2.6 = 12 \notin X$

So, $A = \{4, 5, 6\}$

(ii) Let $B = \{x \mid x \in X \text{ and } x + 5 = 8\}$

Here, $B = \{3\}$

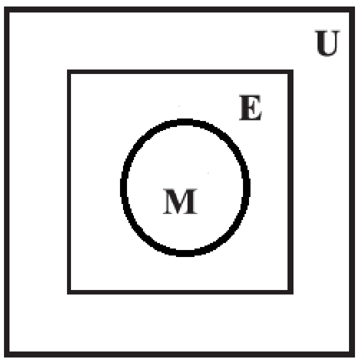
As, $x = 3 \in X$ and $3 + 5 = 8$ and there is no other element belonging to such that $x + 5 = 8$.

(iii) Let $C = \{x \mid x \in X, x > 4\}$ Therefore, $C = \{5, 6\}$

7.(i). Since all of the students who study mathematics study English, but some students who study English do not study Mathematics.

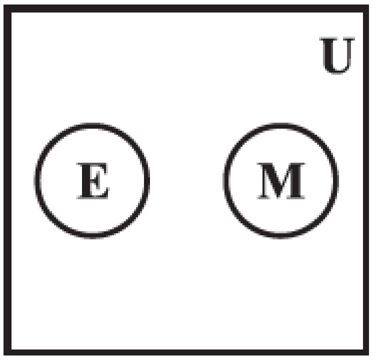
Therefore, $M \subset E \subset U$

Thus the Venn diagram is

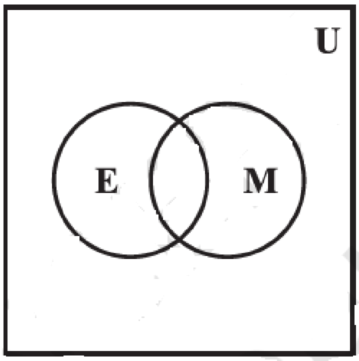


(ii). Since there is no student who study both English and Mathematics

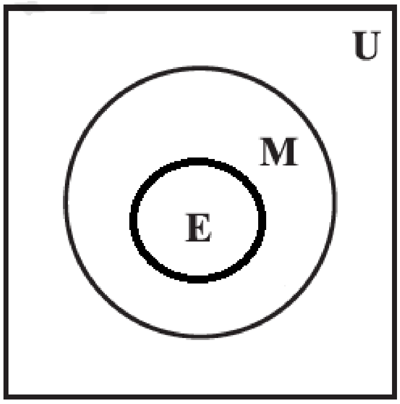
Hence, $E \cap M = \phi$.



(iii). Since there are some students who study both English and Mathematics, some English only and some Mathematics only.
Thus, the Venn diagram is



(iv). Since every student studying English studies Mathematics.
Hence, $E \subset M \subset U$



8. B U C = {1, 2, 3, 4, 5, 6, 7, 8}. *B ∩ C = {3, 4, 5} complement of* C = {1, 2, 9}. *B-C={1*, 2}. Therefore,

UNION (B, C) = 111111110
INTER (B, C) = 001110000
COMP (C) = 110000001
DIFF (B, C) = 110000000

9. (i) We start with the intersection: $H \cap F$ = {dog, rabbit}. Now we union that result with W: $(H \cap F) \cup W$ = {dog, duck, rabbit, deer, frog, mouse}

(ii) We start with the union: $F \cup W$ = {dog, cow, rabbit, duck, pig, deer, frog, mouse}. Now we intersect that result with H: $H \cap (F \cup W)$ = {dog, rabbit, mouse}.

(iii)We start with the intersection: $H \cap F$ = {dog, rabbit}. Now we want to find the elements of W that are *not* in $H \cap F$. $(H \cap F)^c \cap W$ = {duck, deer, frog, mouse}.

10.

$\overline{A \cup B} = \overline{\{1, 2, 3, 4, 5, 6, 8\}} = \{7\}$. $\overline{A} = \{5, 6, 7, 8\}$. $\overline{B} = \{1, 2, 4, 7\}$. $\overline{A} \cap \overline{B} = \{7\}$. It now follows that $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

11.

Let A, B, C denote the set of students in algebra, biology, and chemistry class, Respectively. Then A U BU C is the set of students in one of the three classes, A∩B is the set of students in both algebra and biology, and so forth. To count the number of Students in all three classes, i.e. count | A U BU C |, we can first add all the number of students in all three classes:

$$|A| + |B|+|C|$$

However, now we've counted the students in two classes too many times. So we subtract out the students who are in each pair of classes:

$$-|A \cap B|-|A \cap C|-|B \cap C|$$

For students who are in two classes, we've counted them twice, then subtracted them once, so they're counted once. But for students in all three classes, we counted them 3 times, then subtracted them 3 times. Thus we need to add them again: $|A \cap B \cap C|$

Thus

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$55 = 28 + 30 + 24 - 8 - 16 - 5 + |A \cap B \cap C|$$

Thus $|A \cap B \cap C| = 2$, i.e. there are 2 students in all three classes.

12. We need to find out the number of students who took at least one of the three subjects and subtract that number from the overall 120 to get the number of students who did not opt for any of the three subjects.

Number of students who took at least one of the three subjects can be found by finding out A U B U C, where A is the set of those who took Physics, B the set of those who took Chemistry and C the set of those who opted for Math.

Now, $A \cup B \cup C = A + B + C - (A \cap B + B \cap C + C \cap A) + (A \cap B \cap C)$
A is the set of those who opted for Physics = 120/2 = 60 students
B is the set of those who opted for Chemistry = 120/5 = 24
C is the set of those who opted for Math = 120/7 = 17.

The 10th, 20th, 30th..... Numbered students would have opted for both Physics and Chemistry.
Therefore, $A \cap B = 120/10 = 12$

The 14th, 28th, 42nd..... Numbered students would have opted for Physics and Math. Therefore, C ∩ A = 120/14 = 8

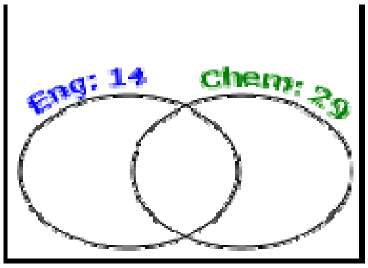
The 35th, 70th.... numbered students would have opted for Chemistry and Math. Therefore, B ∩ C = 120/35 = 3

And the 70th numbered student would have opted for all three subjects.

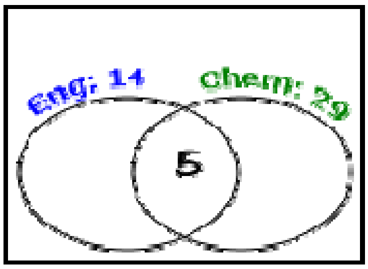
Therefore, AUBUC = 60 + 24 + 17 - (12 + 8 + 3) + 1 = 79.

Number of students who opted for none of the three subjects = 120 - 79 = 41.

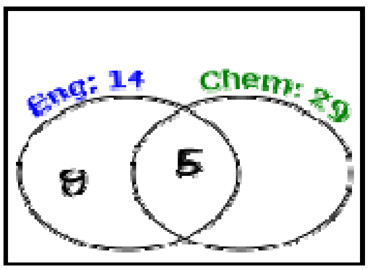
13. There are two classifications in this universe: English students and Chemistry students. First I'll draw my universe for the forty students, with two overlapping circles labeled with the total in each:



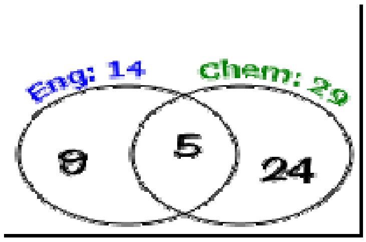
Five students are taking both classes, so I'll put "5" in the overlap:



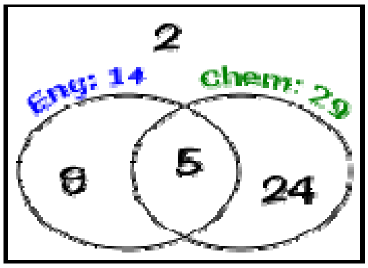
I've now accounted for five of the 14 English students, leaving nine students taking English but not Chemistry, so I'll put "9" in the "English only" part of the "English" circle:



I've also accounted for five of the 29 Chemistry students, leaving 24 students taking Chemistry but not English, so I'll put "24" in the "Chemistry only" part of the "Chemistry" circle:



This tells me that a total of 9 + 5 + 24 = 38 students are in either English or Chemistry (or both). This gives me the answer to part (b) of this exercise. This also leaves two students unaccounted for, so they must be the ones taking neither class, which is the answer to part (a) of this exercise. I'll put "2" inside the box, but outside the two circles:



The last part of this exercise asks me for the probability that a given student is taking Chemistry but not English. Out of the forty students, 24 are taking Chemistry but not English, which gives me a probability of: 24/40 = 0.6 = 60%

14.a) $\{2,4,6,8,10\}$

b) $\{A, U, S, T, R, L, I\}$

c) $\{6,9,12,15\}$

d) $\{10,15,20,25,30\}$

e) $\{3\}$

f) $\{-7,7\}$