Example: Compute the horizontal and vertical components of all the forces acting on each of the members (neglect self weight).

Solution: Draw FBD of the whole frame

$\Sigma F_y = 0$, $A_y - (400 \times 9.81 \times 10^{-3}) = 0$
$\Rightarrow A_y =$

$$\Sigma M_A = 0;$$

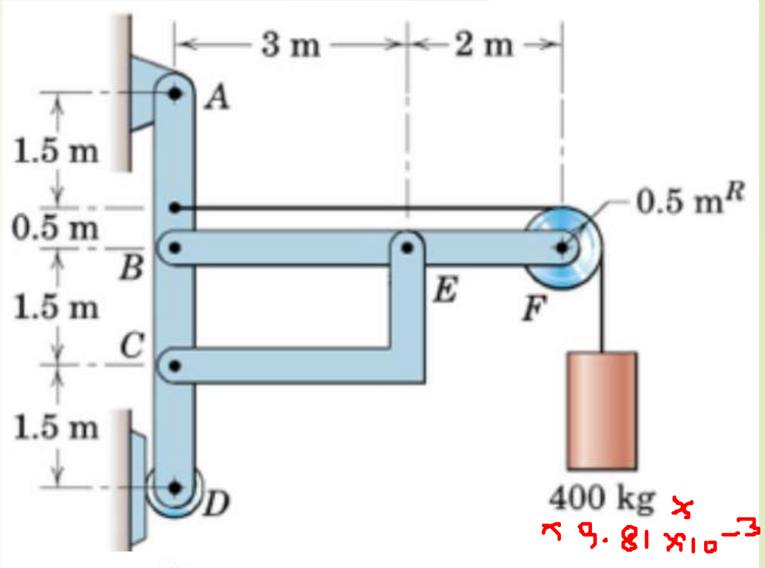
$$(400 \times 9.81 \times 10^{-3}) \times 5.5 - 5R_D = 0; \quad \rightarrow R_D = 4.32\ kN$$

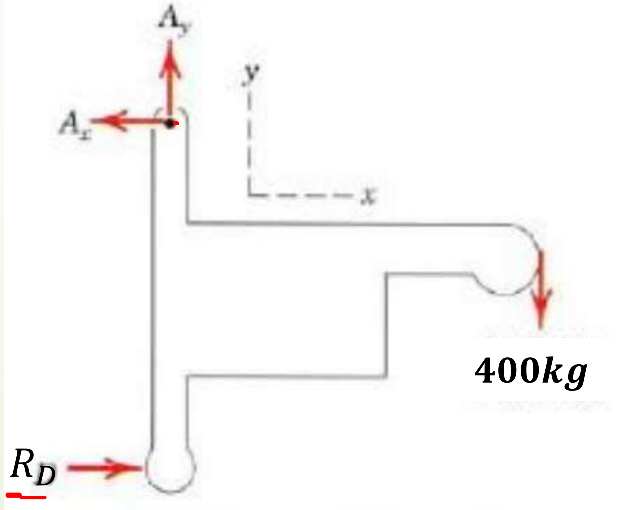
$$\Sigma F_x = 0;$$

$$A_x - 4.32 = 0; \quad \rightarrow \quad A_x = 4.32\ kN$$

$$\Sigma F_y = 0;$$

$$A_y - 3.92 = 0; \quad \rightarrow \quad A_y = 3.92\ kN$$

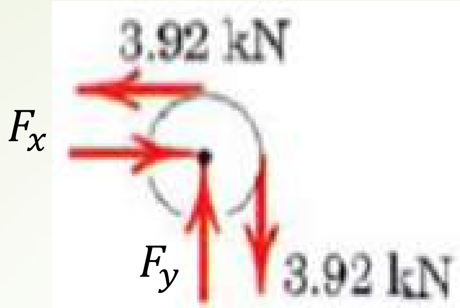


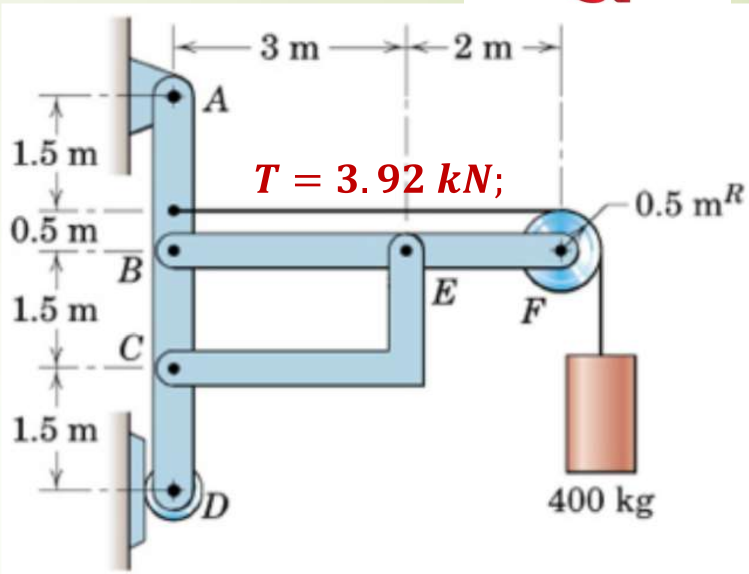


Draw FBD of the pulley

$F_x = 3.92\ kN;$

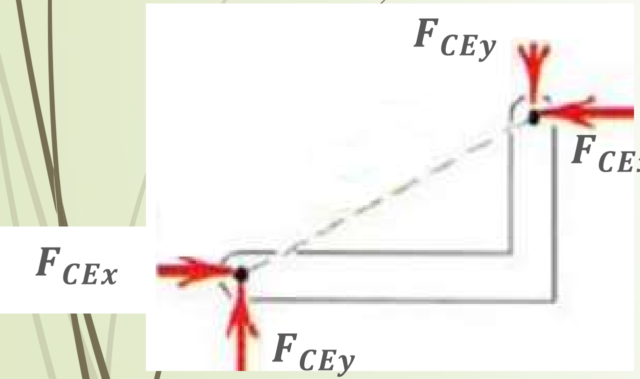
$F_y = 3.92\ kN;$





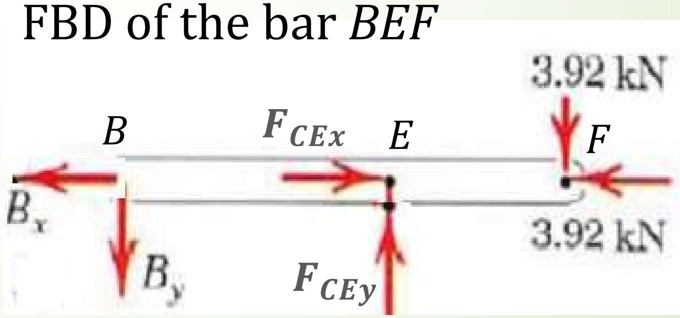
FBD of the link *CE*

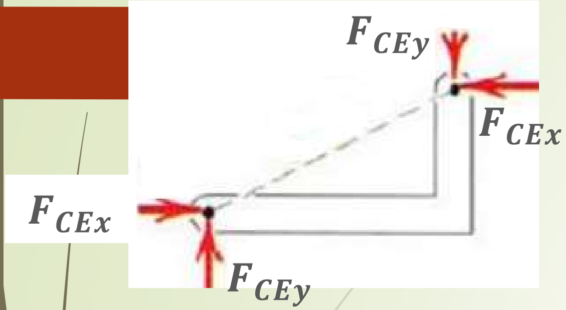
(Two force member)



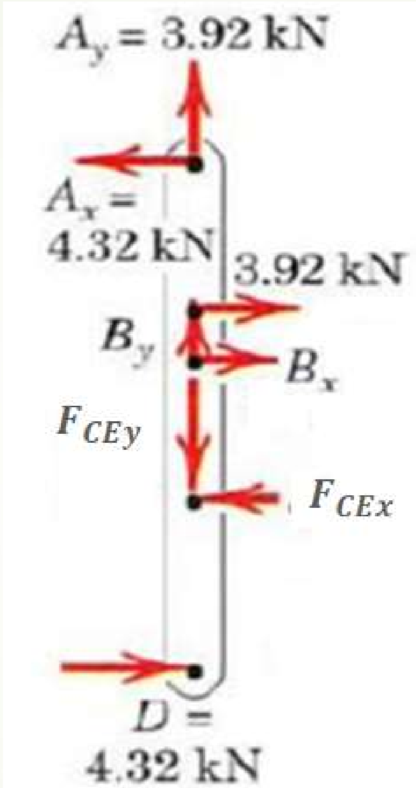
$\theta = tan^{-1}\left(\frac{1.5}{3}\right) = 26.56°;$

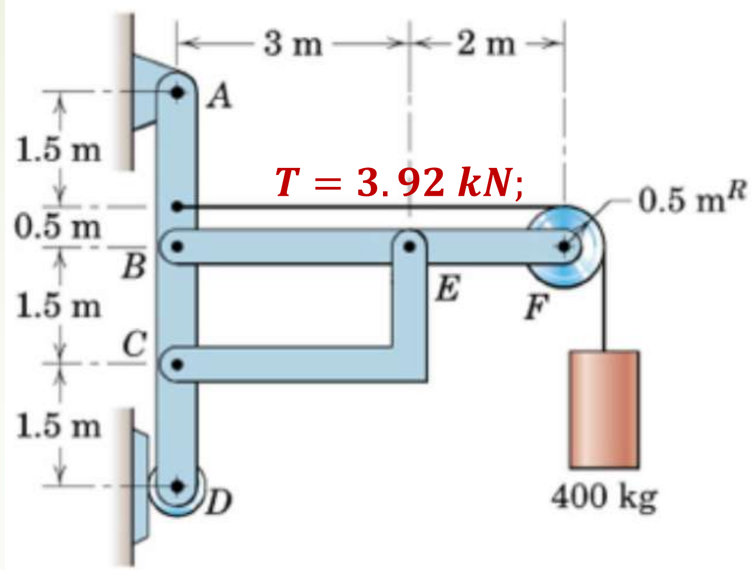
FBD of the bar *BEF*



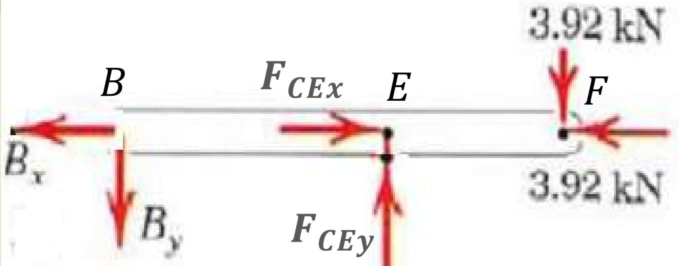


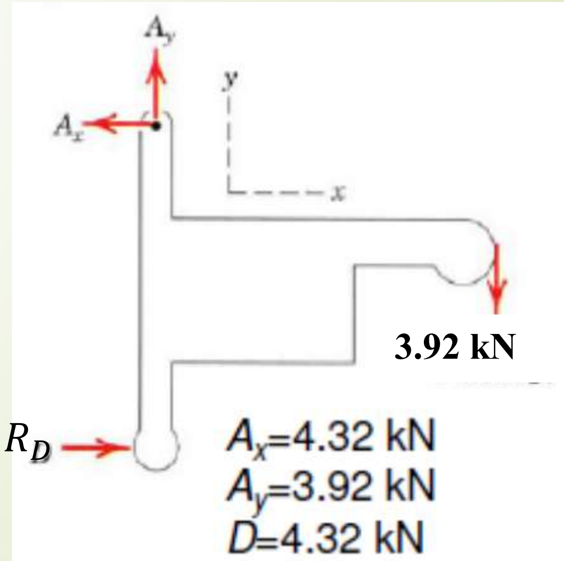
FBD of the bar *ABCD*

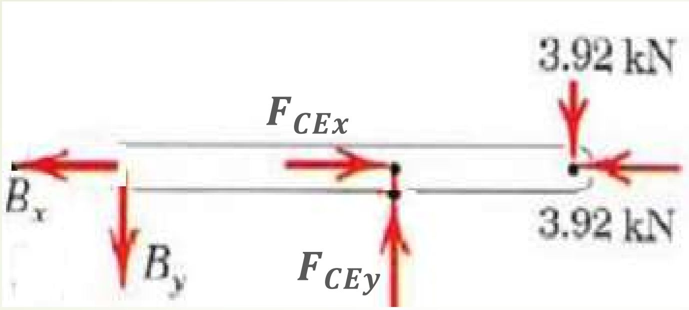


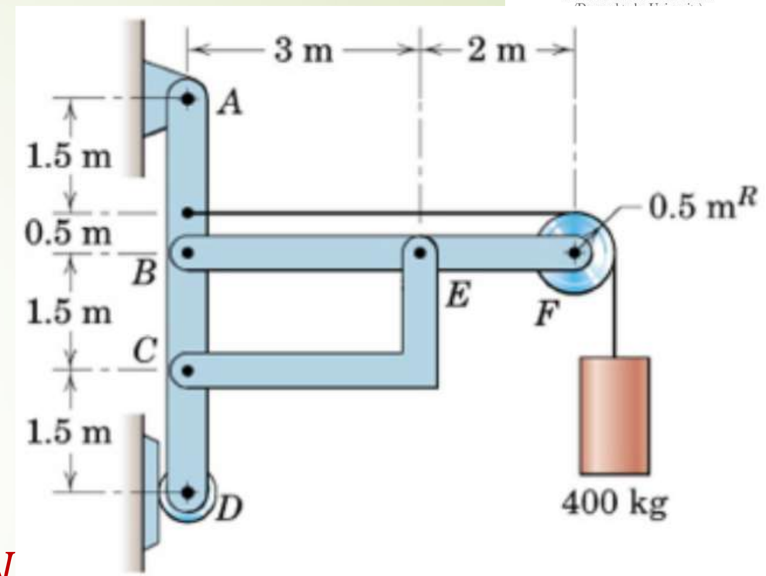


FBD of the bar *BEF*









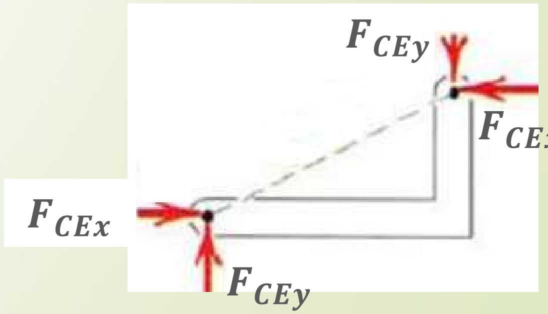
$$\Sigma \boldsymbol{M_B} = \mathbf{0};$$

$$(F_{CEy} \times 3) - 3.92 \times 5 = 0; \qquad \rightarrow F_{CEy} = 6.53\ kN$$

$$F_{CE} \sin 26.56 \times 3 = 3.92 \times 5 = 0 \qquad \rightarrow F_{CE} = 14.61\ kN$$

$$F_{CEx} = F_{CE} cos\theta = 14.61 \cos 26.56 = 13.07\ \ kN$$

$$\theta = tan^{-1}\left(\frac{1.5}{3}\right) = 26.56°;$$



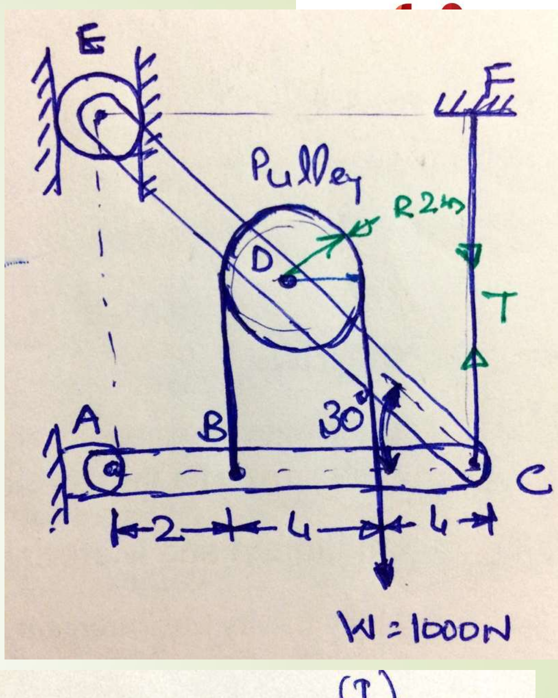
$$\Sigma \boldsymbol{F_x} = \mathbf{0};$$

$$B_x - F_{CEx} + 3.92 = 0 \qquad B_x = 9.15\ kN$$

$$\Sigma \boldsymbol{F_y} = \mathbf{0};$$

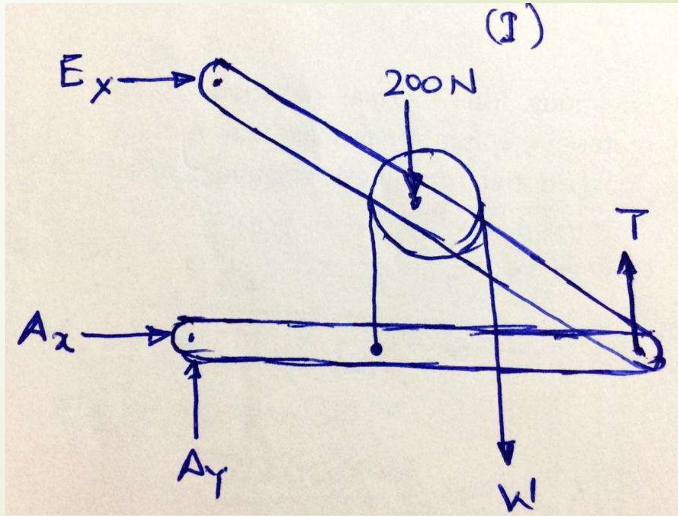
$$B_y - F_{CEy} + 3.92 = 0 \qquad B_y = 2.61\ kN$$

Example: A pulley of radius 2m, weighing 200N is connected at point D on the bar EC. Joint A and C are pin joints and E is a roller joint. Find the tension in the wire FC, if both the bars are weightless and pulley is frictionless.



Solution: (I) Draw FBD of the complete system by removing external supports

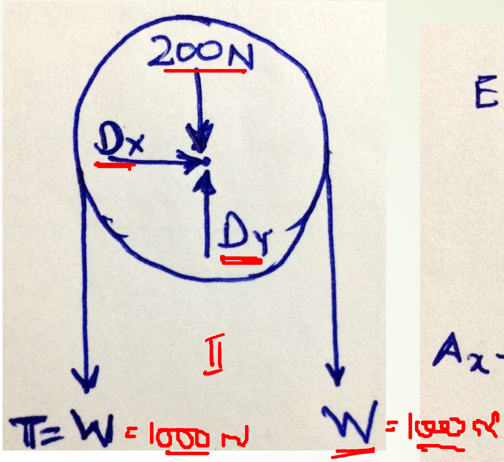
Although the pulley at D is not removed but still its weight is to act because it has effect on both the supports as well as the wire having tension $\boldsymbol{T}$.

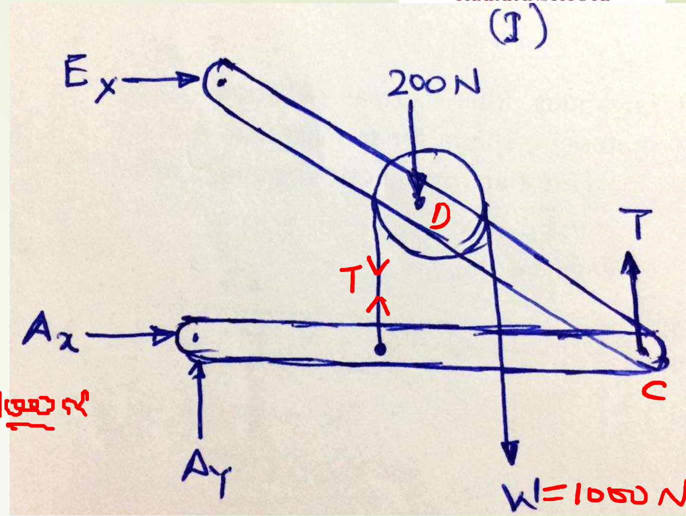


(II) Draw FBD of the pulley

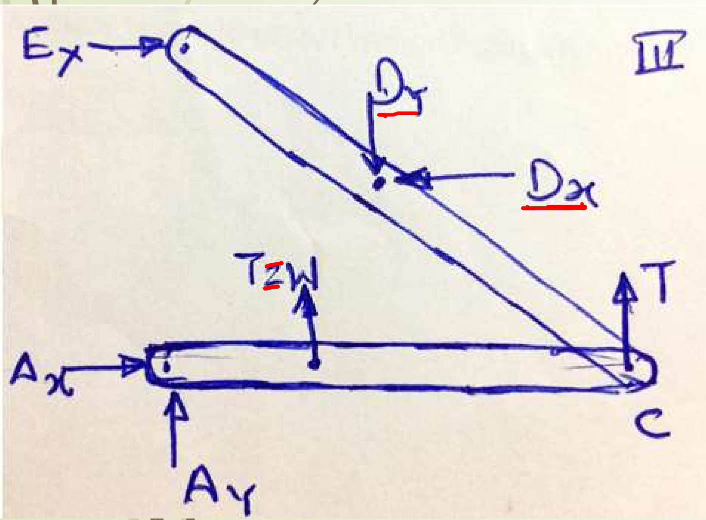
$\Sigma F_x = 0 \Rightarrow D_x = 0$

$\Sigma F_Y = 0 \Rightarrow D_Y = 200 + 1000 + 1000 = 2200\,N$



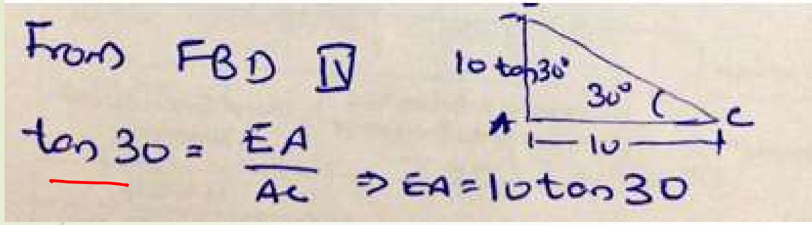


(III) Draw FBD of the complete system by removing the pulley

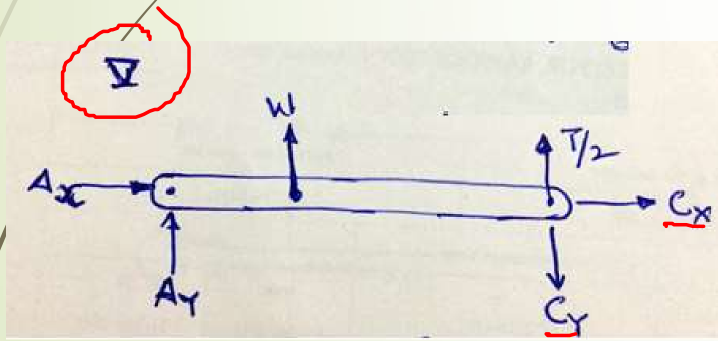


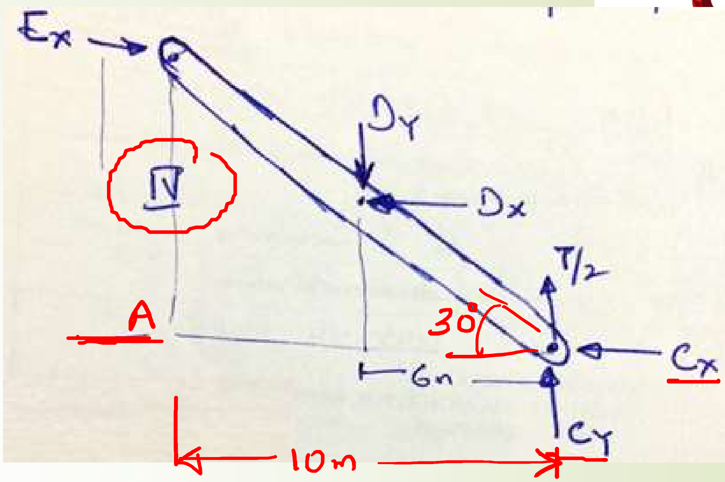
In this FBD weight of the pulley is not to be shown because it has been taken care by the reaction D_Y.
D_X and D_Y are the reactions on the bar, so their directions will be reversed .

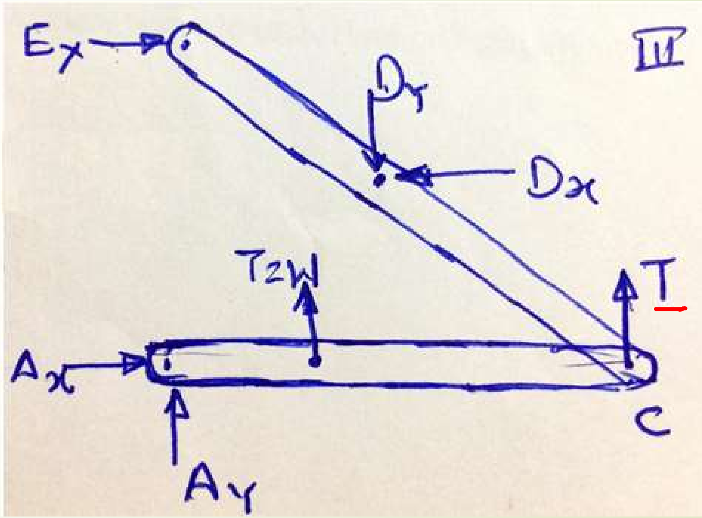
(IV) Draw FBD of the inclined bar *EDC*



(V) Draw FBD of the horizontal bar *ABC*





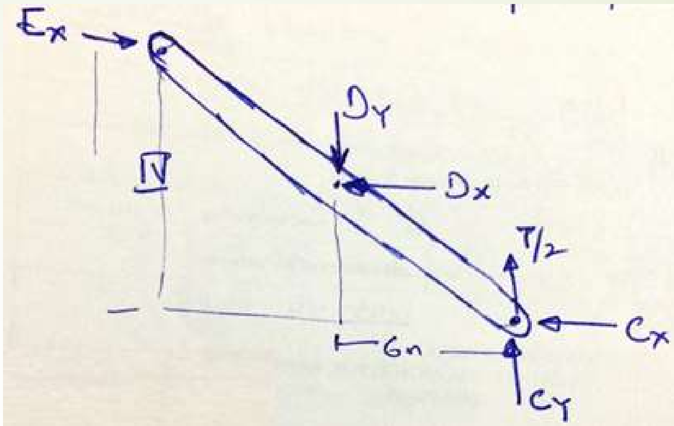


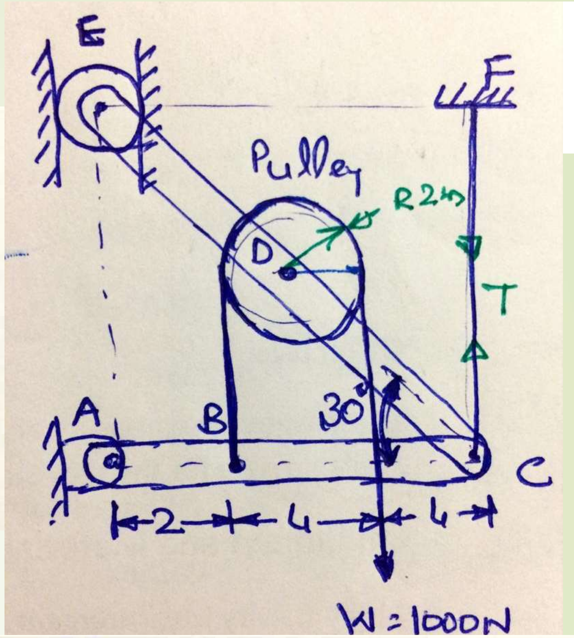
In FBD IV, take moments about C,

$$\Sigma \boldsymbol{M_c} = \mathbf{0};$$

$$E_x \times 10\tan 30 - D_y \times 6 = 0$$

$$E_x = \frac{2200 \times 6}{10tan30} = 2286.31\ N$$



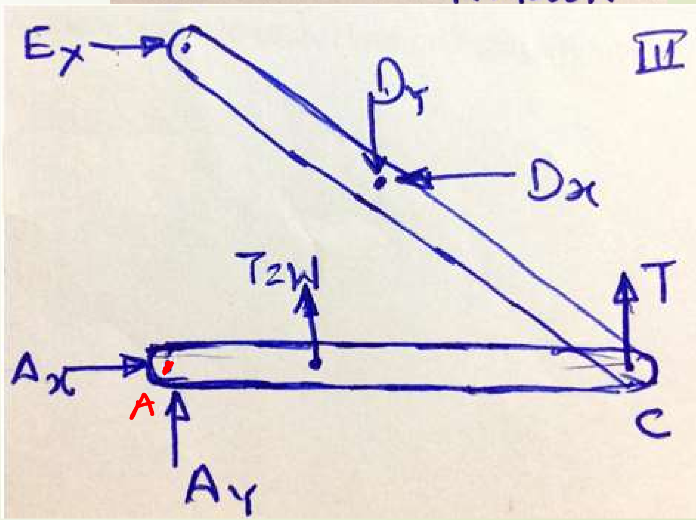


Considering FBD -III, take moments about A, $\Sigma M_A = 0;$

$$E_x \times 10\tan 30 + D_y \times 4 - W \times 2 - T \times 10 = 0$$

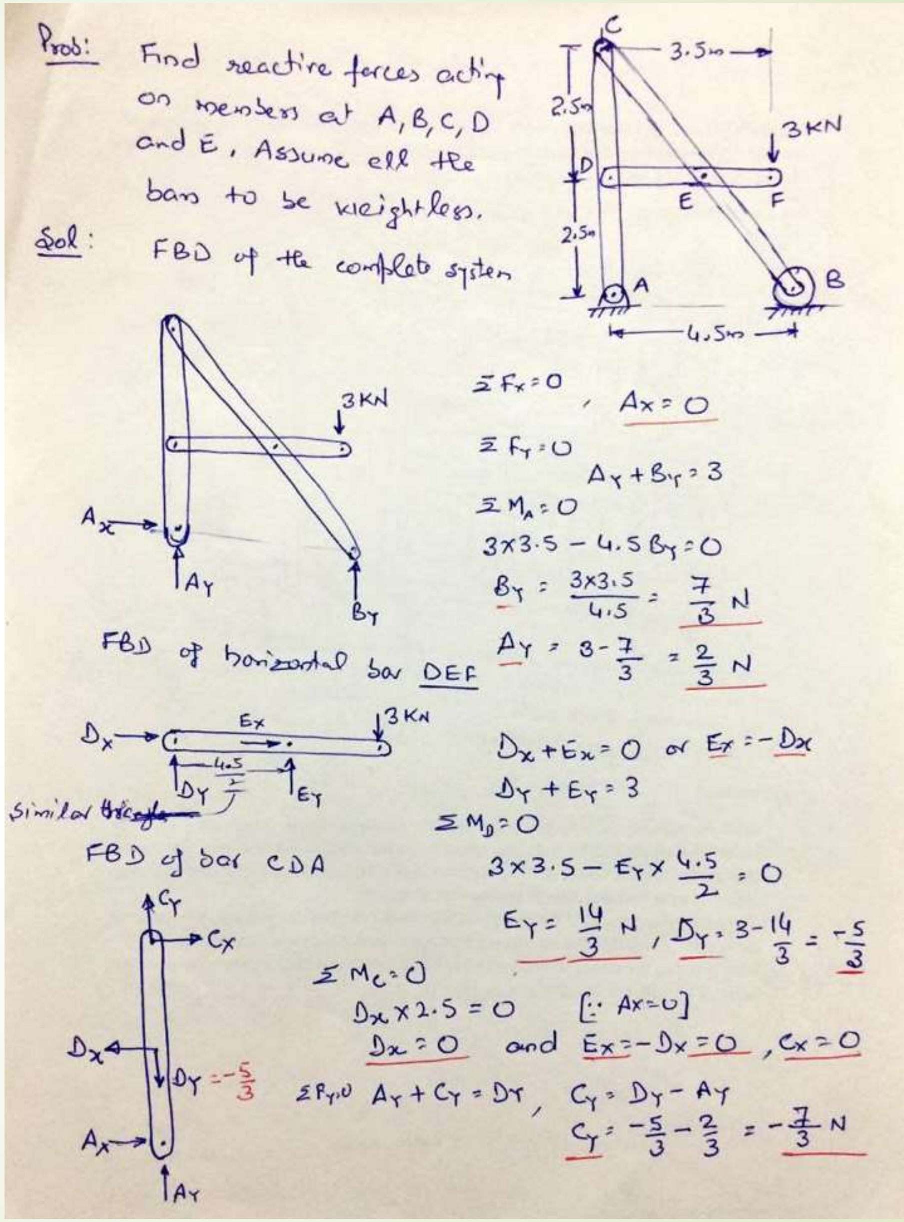
$$2286.31 \times 10\tan 30 + 2200 \times 4 - 1000 \times 2 = 10T$$

$$\boldsymbol{T = 1000N}$$



Example: Find reactive forces acting on members at A, B, C, D and E. Assume all the bars to be weightless.

Solution: Draw FBD of the complete system, FBD's of the bars separately and solve equations of equilibrium to find out reactions at all the points.



Prob: Find reactive forces acting on members at A, B, C, D and E. Assume all the bars to be weightless.

Sol: FBD of the complete system

$\Sigma F_x = 0$, $A_x = 0$

$\Sigma F_y = 0$

$A_Y + B_Y = 3$

$\Sigma M_A = 0$

$3 \times 3.5 - 4.5 B_Y = 0$

$B_Y = \frac{3 \times 3.5}{4.5} = \frac{7}{3}$ N

$A_Y = 3 - \frac{7}{3} = \frac{2}{3}$ N

FBD of horizontal bar DEF

$D_x + E_x = 0$ or $E_x = -D_x$

$D_Y + E_Y = 3$

$\Sigma M_D = 0$

$3 \times 3.5 - E_Y \times \frac{4.5}{2} = 0$

$E_Y = \frac{14}{3}$ N, $D_Y = 3 - \frac{14}{3} = \frac{-5}{3}$

Similar ~~triangle~~

FBD of bar CDA

$\Sigma M_C = 0$

$D_x \times 2.5 = 0$ $[\because A_x = 0]$

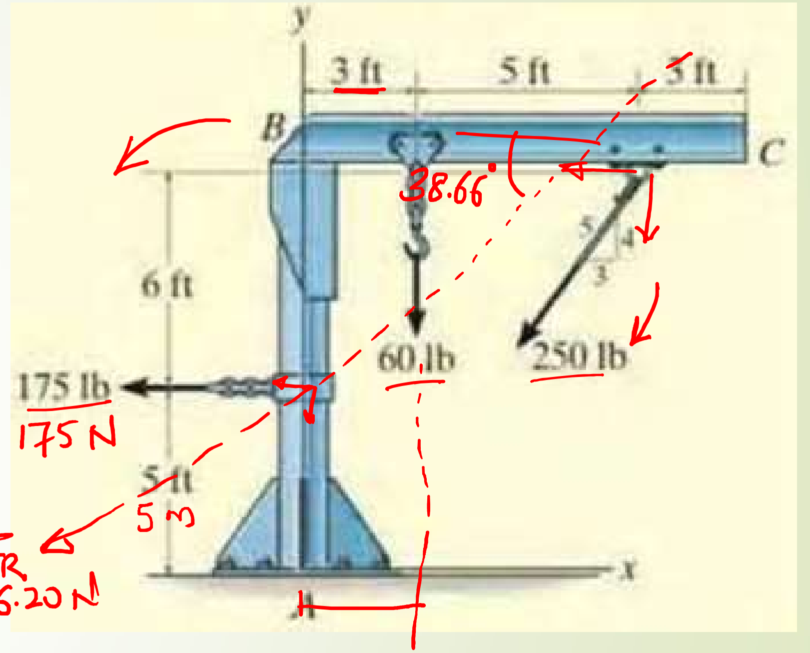
$D_x = 0$ and $E_x = -D_x = 0$, $C_x = 0$

$\Sigma F_y = 0$ $A_Y + C_Y = D_Y$, $C_Y = D_Y - A_Y$

$C_Y = -\frac{5}{3} - \frac{2}{3} = -\frac{7}{3}$ N

Equivalent Force

Example: The jib crane shown in the figure is subjected to three coplanar forces. Replace the loading by an equivalent resultant force and specify where the resultant's line of action intersects with column *AB* and boom *BC*.



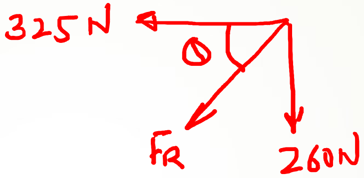
Solution: $F_{Rx} = \Sigma F_x$

$F_{Rx} = 175 + 250 \times 3/5 = 325\,N \leftarrow$

$\Sigma F_y = F_{Ry} = 60 + 250 \times 4/5 = 260\,N \downarrow$

$F_R = \sqrt{(325)^2 + (260)^2} = \boxed{416.20\,N}$

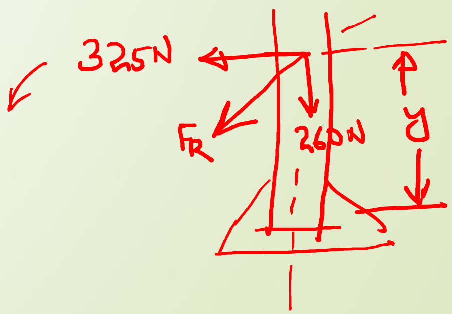
$\tan\theta = \frac{260}{325} \Rightarrow \theta = 38.66^\circ$



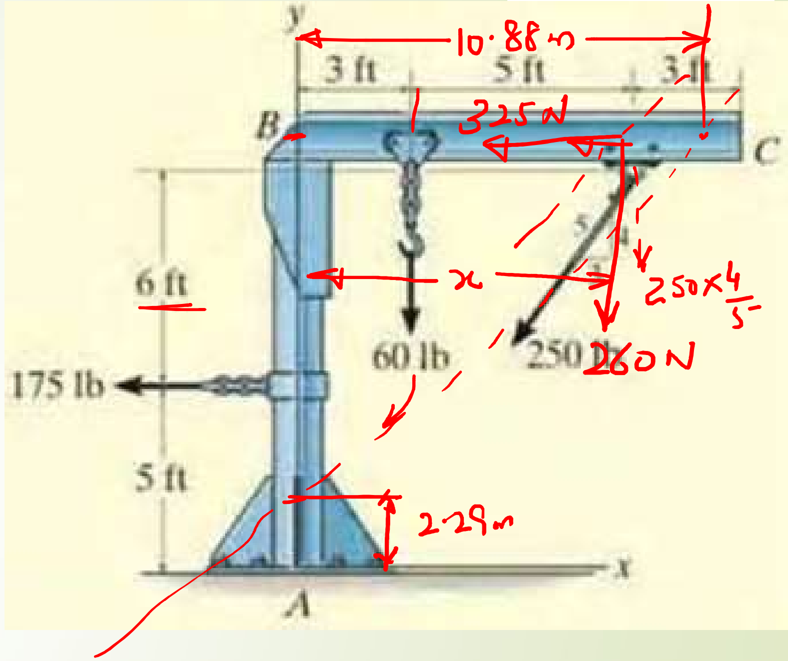
Moment summation; $M_{RA} = \Sigma M_A$

$-175 \times 5 + 60 \times 3 - 250 \times 3/5 \times 11 + 250 \times 4/5 \times 8 = -325 \times y$

$+325\,y = +745 \Rightarrow \boxed{y = 2.29\,m}$



Example: The jib crane shown in the figure is subjected to three coplanar forces. Replace the loading by an equivalent resultant force and specify where the resultant's line of action intersects with column *AB* and boom *BC*.



Solution:

$\Sigma M_B = M_{RB}$

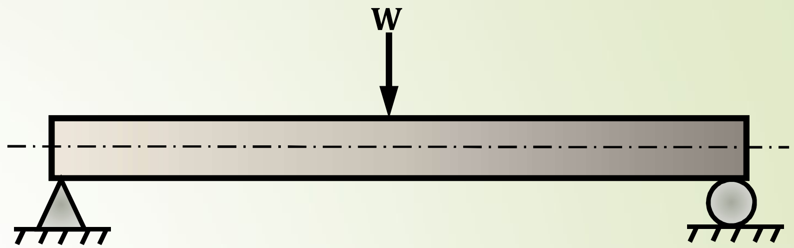
$175 \times 6 + 60 \times 3 + \underbrace{250 \times \frac{4}{5}}\times 8 = 260\,(x)$

$x = \dfrac{1050 + 180 + 1600}{260} = \underline{10.88\ \text{ft}}$

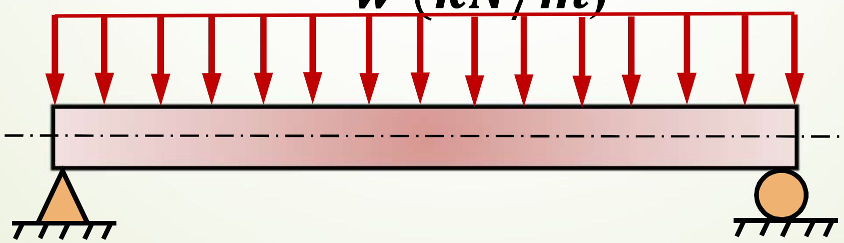
Equilibrium of Rigid Bodies – Part II

Different Types of Loads

Point load: A point load or concentrated load is one which is considered to be act at a point.



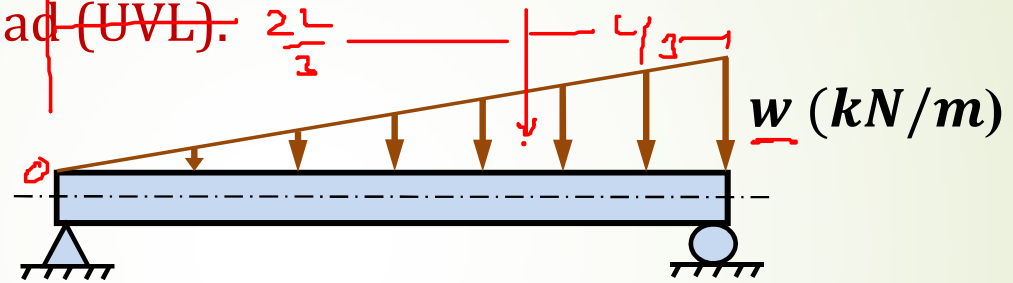
Distributed load: A distributed load is one which is distributed or spread in some manner over the length of the beam. If the spread is uniform (i.e. at the uniform rate w kN/m), it is said to be uniformly distributed load (UDL).



Uniformly Distributed load (UDL)

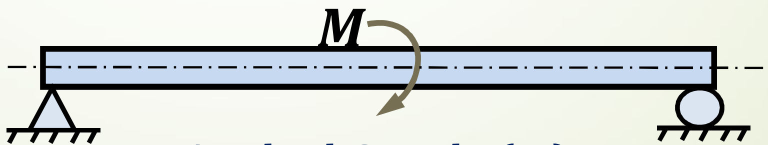
Different Types of Loads

Distributed load: A distributed load is one which is distributed or spread in some manner over the length of the beam. If the spread is varying at uniform rate it is said to be uniformly varying load ~~(UVL)~~.



Uniformly Varying Load(UVL)

Applied Couple: Some times beams are also subjected to couples (clockwise or counter-clockwise)



Applied Couple (M)