UCS405 (Discrete Mathematical Structures)

Solutions

Tutorial Sheet-2 (Set Theory)

1. A- Intersection of two fuzzy sets

$\mu A \cap B(x) = \mu A(x) \wedge \mu B(x) = \min(\mu A(x), \mu B(x))$

$\mu A(x) = \{0.2, 0.5, 0.6, 0.1, 0.9\}$

$\mu B(x) = \{0.1, 0.5, 0.2, 0.7, 0.8\}$

$\mu A \cap B = \{0.1, 0.5, 0.2, 0.1, 0.8\}$

2. A- Union of two fuzzy sets

$\mu A \cup B(x) = \mu A(x) \vee \mu B(x) = \max(\mu A(x), \mu B(x))$

$\mu A(x) = \{0.6, 0.5, 0.1, 0.7, 0.8\}$

$\mu B(x) = \{0.9, 0.2, 0.6, 0.8, 0.5\}$

$\mu A \cup B(x) = \{0.9, 0.5, 0.6, 0.8, 0.8\}$

Complement of $\mu A \cup B(x) = \{0.1, 0.5, 0.4, 0.2, 0.2\}$

3. A- $P \cup Q = \{a, a, a, b, c, c, d, d\}$
$P \cap Q = \{a, a, c\}$
$P - Q = \{a, d, d\}$

4. A- Then (a) is not a partition of S since 7 in S does not belong to any of the subsets. Furthermore, (b) is not a partition of S since {1, 3, 5} and {5, 7, 9} are not disjoint. On the other hand, (c) is a partition of S.

5. A- Note first that each partition of S contains either 1, 2, 3, or 4 distinct cells. The partitions are as follows:
(1) [{a, b, c, d}]
(2) [{a}, {b, c, d}], [{b}, {a, c, d}], [{c}, {a, b, d}], [{d}, {a, b, c}], [{a, b}, {c, d}], [{a, c}, {b, d}], [{a, d}, {b, c}]
(3) [{a}, {b}, {c, d}], [{a}, {c}, {b, d}], [{a}, {d}, {b, c}], [{b}, {c}, {a, d}], [{b}, {d}, {a, c}], [{c}, {d}, {a, b}]
(4) [{a}, {b}, {c}, {d}]
There are 15 different partitions of S.

6. A-

$A \cap (B \cup C) = \{x \mid x \in A, x \in (B \cup C)\}$
$= \{x \mid x \in A, x \in B \text{ or } x \in A, x \in C\} = (A \cap B) \cup (A \cap C)$

Here we use the analogous logical law $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ where $\wedge$ denotes "and" and $\vee$ denotes "or."

7. A-

We construct a table which shows the membership relations for the sets in the left-hand side and the right-hand side of the identity. The number 1 indicates that an element is in a set, and 0 means that an element is not in a set. The table contains each combination of sets A and B that an element can belong to.

A	B	A^c	$A^c \cap B$	$A \cup (A^c \cap B)$	$A \cup B$
1	1	0	0	1	1
1	0	0	0	1	1
0	1	1	1	1	1
0	0	1	0	0	0

Since the answers in the last two columns are the same, the identity is proved.

8. A-

For any $x \in$ LHS, $x \in (B\text{-}A)$ or $x \in (C\text{-}A)$ [or both].

when $x \in B - A$ $\Longrightarrow (x \in B) \wedge (x \notin A)$

$\Longrightarrow (x \in B \cup C) \wedge (x \notin A)$

$\Longrightarrow x \in (B \cup C) - A$

when $x \in C - A$ $\Longrightarrow (x \in C) \wedge (x \notin A)$

$\Rightarrow (x \in B \cup C) \wedge (x \notin A)$

$\Longrightarrow x \in (B \cup C) - A$

Therefore, LHS $\subseteq$ RHS

For any $x \in$ RHS, $x \in (B \cup C)$ and $x \notin$ A.

when $x \in B$ and $x \notin A$

$(x \in B) \wedge (x \notin A) \Longrightarrow x \in B - A$

$\Longrightarrow x \in (B - A) \cup (C - A)$

when $x \in C$ and $x \notin A$,

$(x \in C) \wedge (x \notin A) \Longrightarrow x \in C - A$

$\Longrightarrow x \in (B - A) \cup (C - A)$

Therefore, RHS $\subseteq$ LHS

With LHS $\subseteq$ RHS and RHS $\subseteq$ LHS, we can conclude that **LHS = RHS**