**UCS405 (Discrete Mathematical Structures)**

**Solutions**

**Tutorial Sheet-4 (Functions)**

1. A-

| | Steps 1 through 3 | Step 4 | Hash Value |
|---|---|---|---|
| Smith | $\rightarrow 19 \cdot 2 + 13 \cdot 2^2 + 9 \cdot 2^3 + 20 \cdot 2^4 + 8 \cdot 2^5 = 738$ | $= 11 \cdot 63 + 45 \rightarrow$ | 45 |
| Jones | $\rightarrow 10 \cdot 2 + 15 \cdot 2^2 + 14 \cdot 2^3 + 5 \cdot 2^4 + 19 \cdot 2^5 = 880$ | $= 13 \cdot 63 + 61 \rightarrow$ | 61 |
| Brown | $\rightarrow 2 \cdot 2 + 18 \cdot 2^2 + 15 \cdot 2^3 + 23 \cdot 2^4 + 14 \cdot 2^5 = 1012$ | $= 16 \cdot 63 + 4 \rightarrow$ | 4 |
| Zento | $\rightarrow 26 \cdot 2 + 5 \cdot 2^2 + 14 \cdot 2^3 + 20 \cdot 2^4 + 15 \cdot 2^5 = 984$ | $= 15 \cdot 63 + 39 \rightarrow$ | 39 |
| Ruster | $\rightarrow 18 \cdot 2 + 21 \cdot 2^2 + 19 \cdot 2^3 + 20 \cdot 2^4 + 5 \cdot 2^5 + 18 \cdot 2^6 = 1904$ | $= 30 \cdot 63 + 14 \rightarrow$ | 14 |

2. A-

Let $c = 20$ and $n_0 = 1$.
Must show that $0 \leq f(n)$ and $f(n) \leq cg(n)$.
$0 \leq 15n^3 + n^2 + 4$ for all $n \geq n_0 = 1$.
$f(n) = 15n^3 + n^2 + 4 \leq 15n^4 + n^4 + 4n^4$
$15n^4 + n^4 + 4n^4 = 20n^4 = 20g(n) = cg(n)$

$T(n) = 15n^3 + n^2 + 4$

$T(n) = O(n^3)$.

$T(n) = O(n^4)$.

$O(n)$ is an upper bound

"=" is not really equality. It's used as "set inclusion" $\in$ here.

Don't use $O(n) = T(n)$

3. A-

Let $c = 15$ and $n_0 = 1$.
Must show that $0 \leq cg(n)$ and $cg(n) \leq f(n)$.
$0 \leq 15n^3$ for all $n \geq n_0 = 1$.
$cg(n) = 15n^2 \leq 15n^3 \leq 15n^3 + n^2 + 4 = f(n)$

$T(n) = 15n^3 + n^2 + 4$
$T(n) = \Omega(n^3)$.
$T(n) = \Omega(n^2)$.
$\Omega(n)$ is a lower bound

4. A-

When $n \geq 1$,

$$n^2 + 5n + 7 \leq n^2 + 5n^2 + 7n^2 \leq 13n^2$$

When $n \geq 0$,

$$n^2 \leq n^2 + 5n + 7$$

Thus, when $n \geq 1$

$$1n^2 \leq n^2 + 5n + 7 \leq 13n^2$$

Thus, we have shown that $n^2 + 5n + 7 = \Theta(n^2)$ (by definition of Big-$\Theta$, with $n_0 = 1$, $c_1 = 1$, and $c_2 = 13$.)

5. A-

**a)**h(317) = 317 mod 31 = 7 (because 317 div 31 = 10, and 317 mod 31 = 317 - 31 * 10 = 7)
h(918) = 918 mod 31 = 19 (because 918 div 31 = 29, and 918 mod 31 = 918 - 31 * 29 = 19)
h(007) = 007 mod 31 = 7 - collision with car 317 (spot is already taken)
h(110) = 110 mod 31 = 17 (because 110 div 31 = 3, and 110 mod 31 = 110 - 31 * 3 = 17)
h(111) = 111 mod 31 = 18 (using the result above)
h(310) = 310 mod 31 = 0

**b)**one of the solutions is to try to occupy the next (consecutive available spot)

6. $n^3 + \log(n^2 + 1) = O(n^3)n! + 2^n = O(n!)$
Therefore, $(n! + 2^n)(n^3 + \log(n^2 + 1)) = O(n^3.n!)$