UCS405 (Discrete Mathematical Structures)

Solutions

Tutorial Sheet-3 (Functions)

1. A- domain (F) = {1, 2, 3, 4, 5};
Co-domain (F) = {a, b, c, d, e};
Range (F) = {a, b, c, d};
$F^{-1}(a)$ = does n't exist;
F^{-1} ({a, b, c)) = could n't able to find out

2. A- F(3) = 2F(2) + 3F(1) + F(0) = 10 + 9 + 2 = 21
F(4) = 2F(3) + 3F(2) + F(l) = 42 + 15 + 3 = 60
F(5) = 2F(4) + 3F(3) + F(2) = 120+ 63 + 5 = 188

3. A- f and g is a function. But h is not a function.

4. A- i. f is not surjective.
ii. g is not surjective. There is no x ∈ {1, 2, 3} (the domain) for which g(x) = b, so b, which is in the co-domain, is not in the range. Notice that there is an element from the co-domain "missing" from the bottom row of the matrix.
iii. h is surjective. Every element of the co-domain is also in the range. Nothing in the co-domain is missed.

5. A-

$A = R - \{3\}$

$B = R - \{1\}$

$f : A \to B$

$$f(x) = \frac{x-2}{x-3}$$

$$f(x_1) = f(x_2)$$

$$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$(x_2-3)(x_1-2) = (x_2-2)(x_1-3)$$

$$x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$$

$$-3x_1 - 2x_2 = -3x_2 - 2x_1$$

$$-x_1 = -x_2$$

$$x_1 = x_2$$

So, $f(x)$ is one-one

$$f(x) = \frac{x-2}{x-3}$$

$$y = \frac{x-2}{x-3}$$

$$y\left(x-3\right)=x-2$$

$$yx-3y=x-2$$

$$yx-x=3y-2$$

$$x(y-1)=3y-2$$

$$x=\frac{3y-2}{(y-1)}$$

$$f\left(x\right)=\frac{x-2}{x-3}$$

$$=\frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3}$$

$$=\frac{\frac{3y-2-2(y-1)}{y-1}}{\frac{3y-2-3(y-1)}{y-1}}$$

$$=\frac{3y-2-2y+2}{3y-2-3y+3}$$

$$=\frac{3y-2y}{-2+3}$$

$$=y$$

$$f\left(x\right)=y$$

$f\left(x\right)$ is onto.

So $f\left(x\right)$ is bijective and invertible

$$f\left(x\right)=\frac{x-2}{x-3}$$

$$y=\frac{x-2}{x-3}$$

$$x=\frac{y-2}{y-3}$$

$$x(y-3)=y-2$$

$$xy-3x=y-2$$

$$xy-y=3x-2$$

$$y(x-1)=3x-2$$

$$y=\frac{3x-2}{x-1}$$

$$f^{-1}(x)=\frac{3x-2}{x-1}$$

6. A- $(f \circ g)(x) = f(g(x)) = f(2x + 1) = 2x + 1 + 2 = 2x + 3$
$(g \circ f)(x) = g(f(x)) = g(x + 2) = 2(x+2) + 1 = 2x + 5$
Hence, $(f \circ g)(x) \neq (g \circ f)(x)$

7. A- We have
$h \circ (g \circ f)(x) = h(g \circ f(x)) = h(g(f(x))) = h(g(2x))$
$= h(3(2x) + 4) = h(6x + 4) = \sin(6x + 4)$ for x in N.
Also,
$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = (h \circ g)(2x) = h(g(2x))$
$= h(3(2x) + 4) = h(6x + 4) = \sin(6x + 4)$, for x in N.
This shows that $h \circ (g \circ f) = (h \circ g) \circ f$.