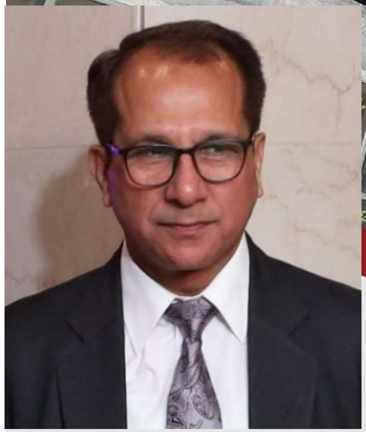
Properties of Plane Surfaces



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MOMENT
OF
INERTIA

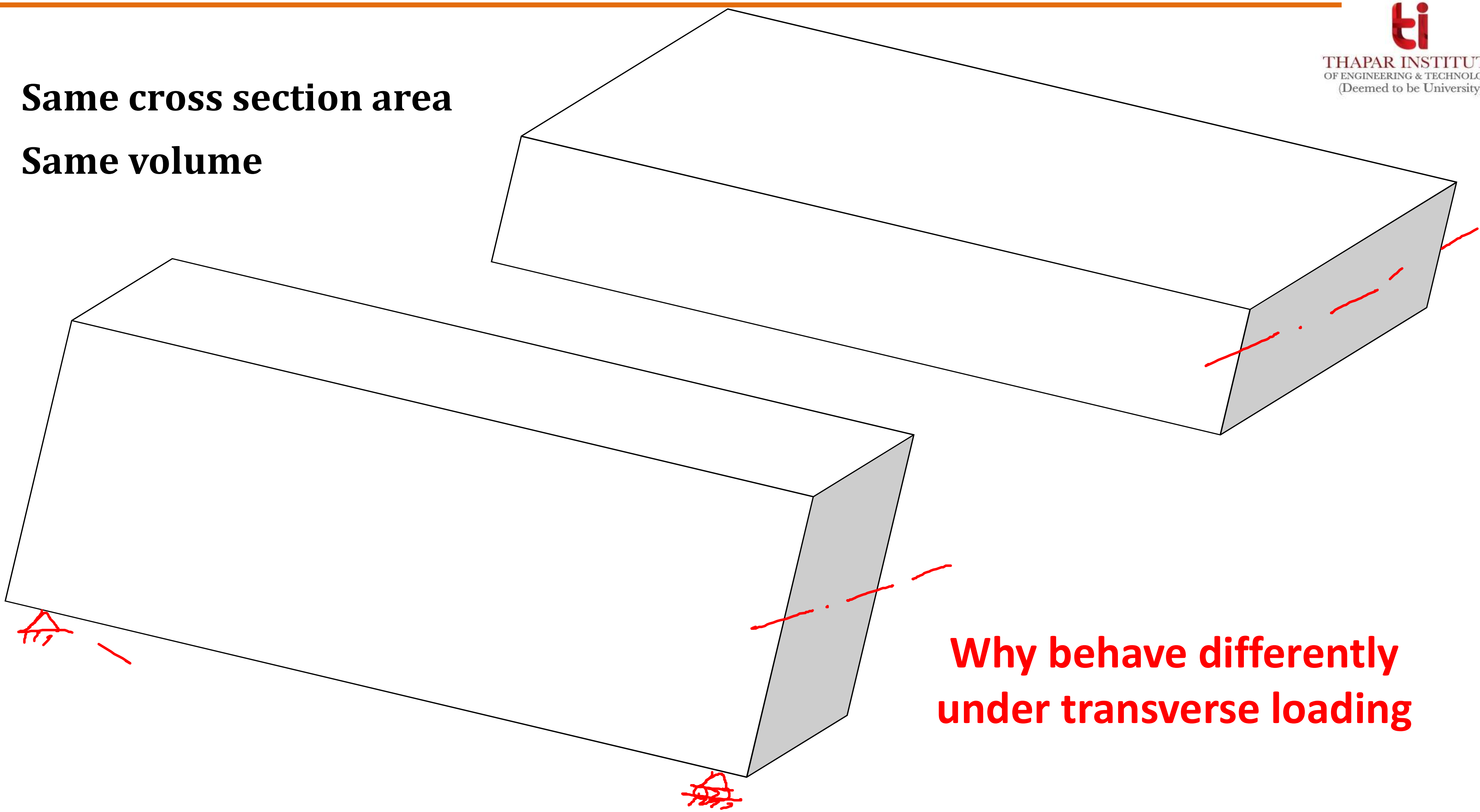
$$\sigma_b = \frac{M \cdot y}{I}$$

What is Moment of inertia?

Capacity of cross section to resist bending

Same cross section area

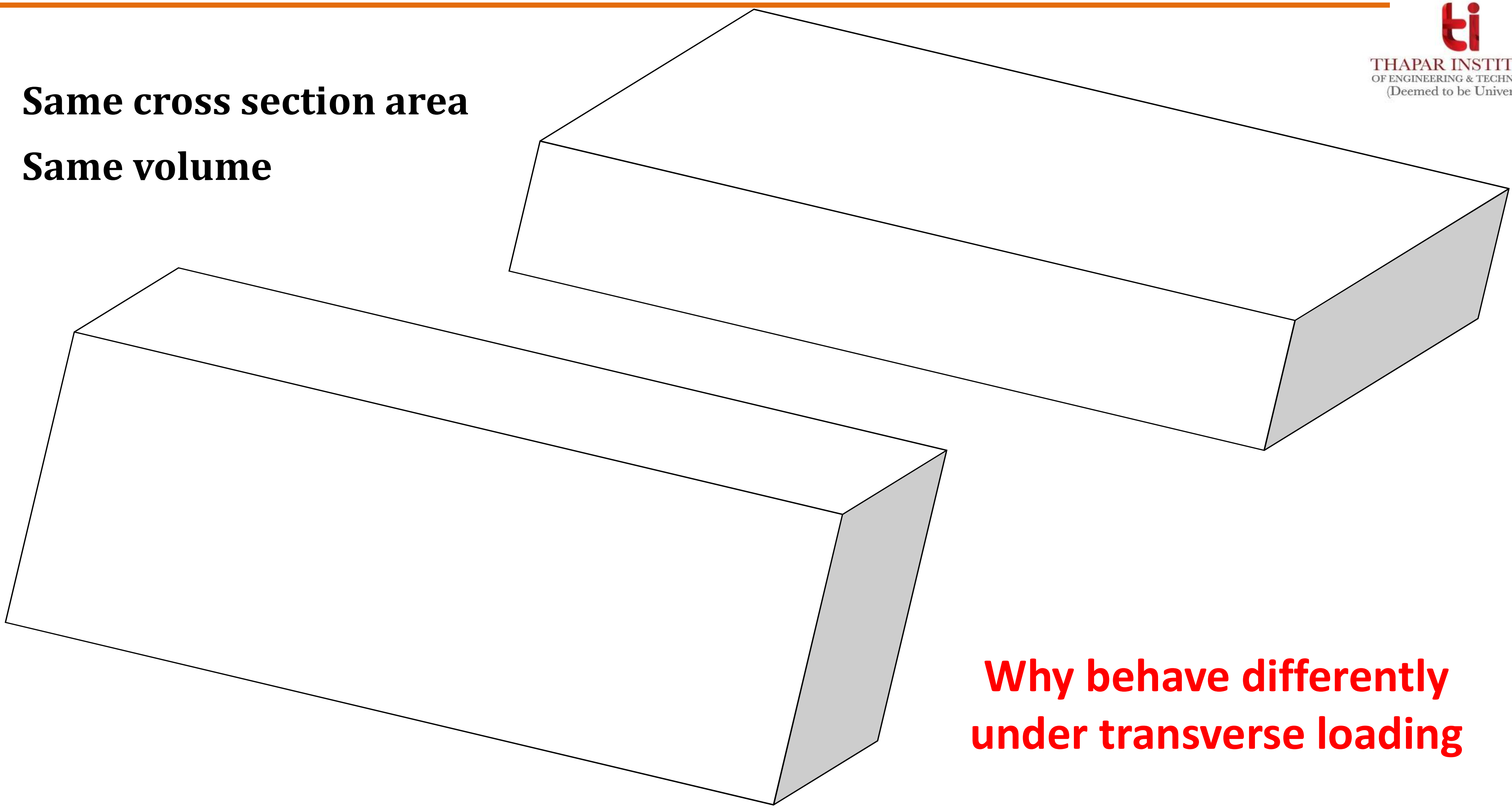
Same volume



Why behave differently under transverse loading

Same cross section area

Same volume



Why behave differently under transverse loading

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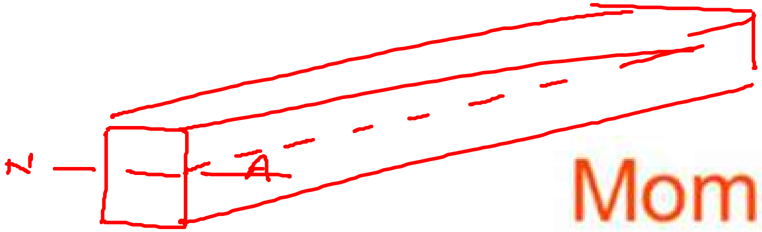
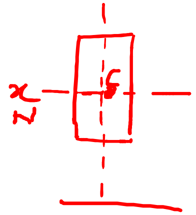
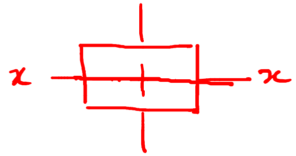


Moment of Inertia (I)





- also known as the **Second Moment of the Area** is a term used to describe the capacity of a cross-section to resist bending.
- It is a mathematical property of a section concerned with a surface area and how that area is distributed about the reference axis. The reference axis is usually a centroidal axis.

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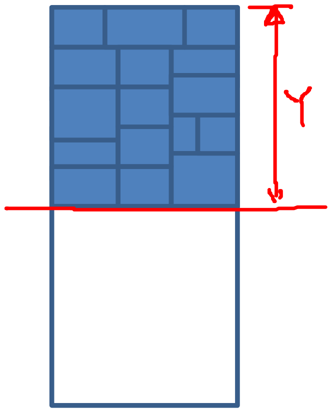

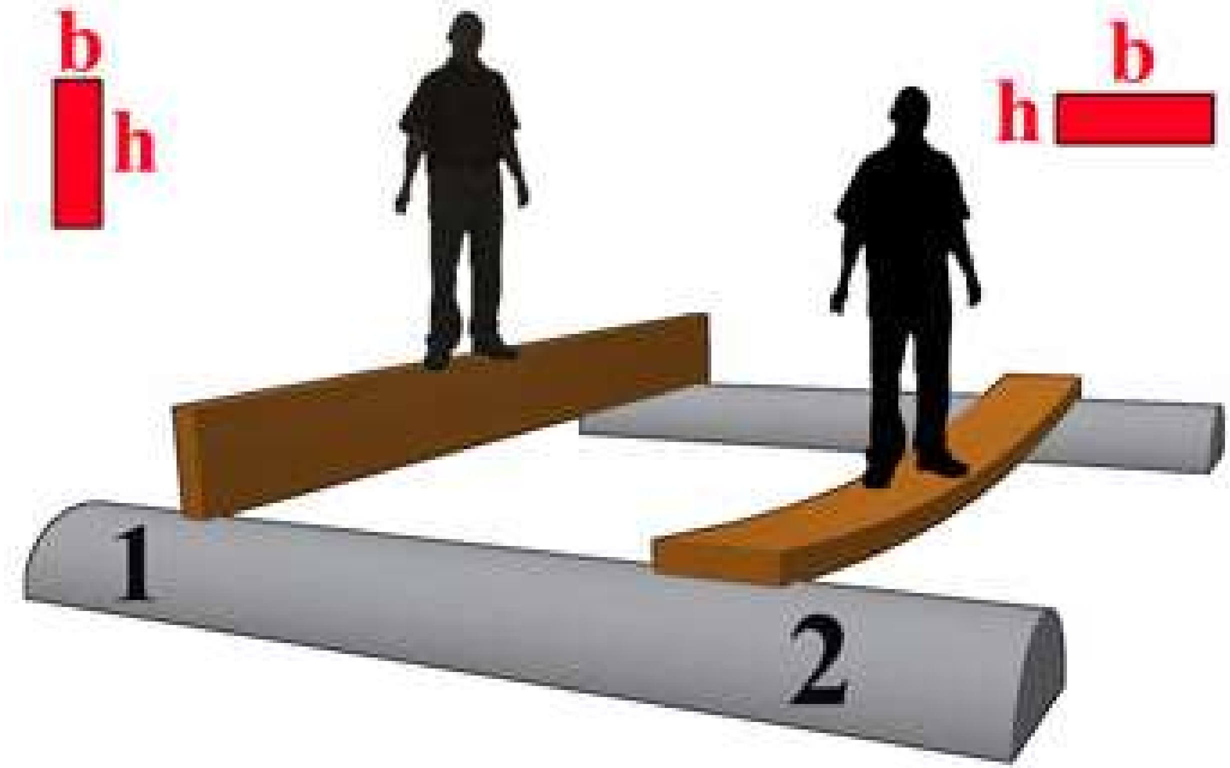

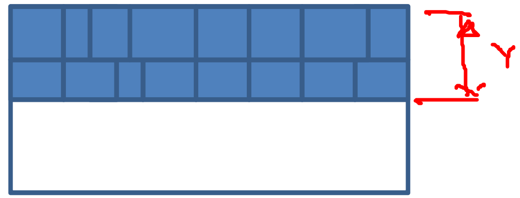

More areas at larger distance from centroidal axis

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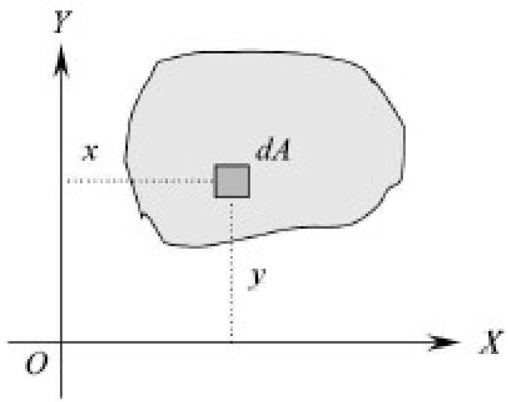

moment of **first moment** of the elemental area dA.

About x axis $y(y\,dA)$

About y axis $x(x\,dA)$

$$I_{xx} = \int y^2\,dA$$

$$I_{yy} = \int x^2\,dA$$

Units-------- mm^4 , cm^4, m^4

Simple rectangle shape

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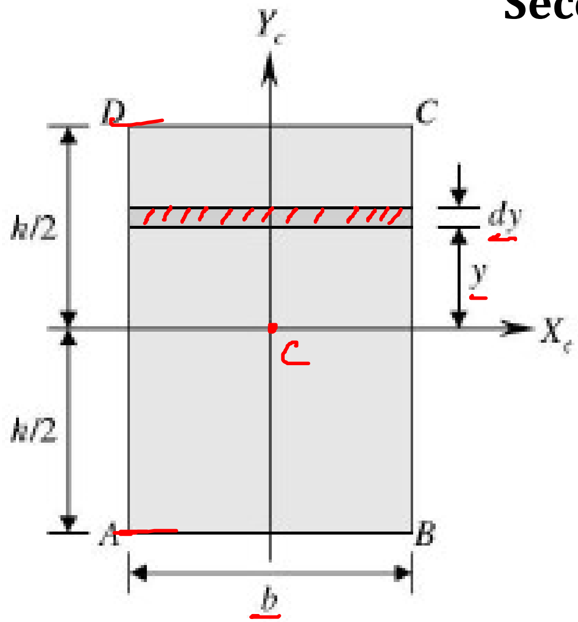
$$dA = bdy$$

Second moment of area of this elemental strip about x-axis



$$dI_{xx} = y^2 dA = y^2 bdy$$

$$I_{xx} = \int_{-h/2}^{h/2} y^2 bdy$$

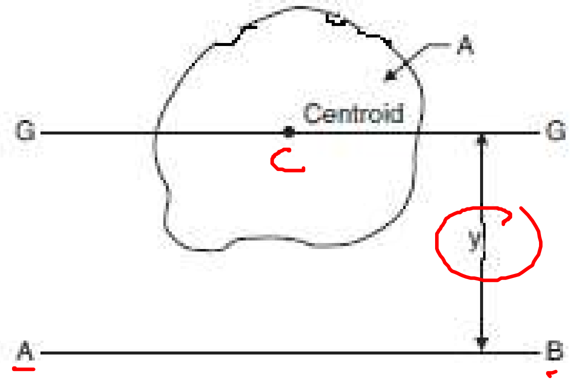
$$= b\left[\frac{y^3}{3}\right]_{-h/2}^{h/2}$$

$$I_{xx} = \frac{bh^3}{12}$$

Parallel Axis Theorem

Moment of inertia about any axis in the plane of an area is equal to the sum of moment of inertia about a parallel centroidal axis and the product of area and square of the distance between the two parallel axis.

$$\mathbf{I_{AB} = I_{GG} + Ay^2}$$



$\mathbf{I_{GG}}$ **= MOI about centroidal axis parallel to AB**

A = the area of the plane figure given and

y = the distance between the axis AB and the parallel centroidal axis GG.

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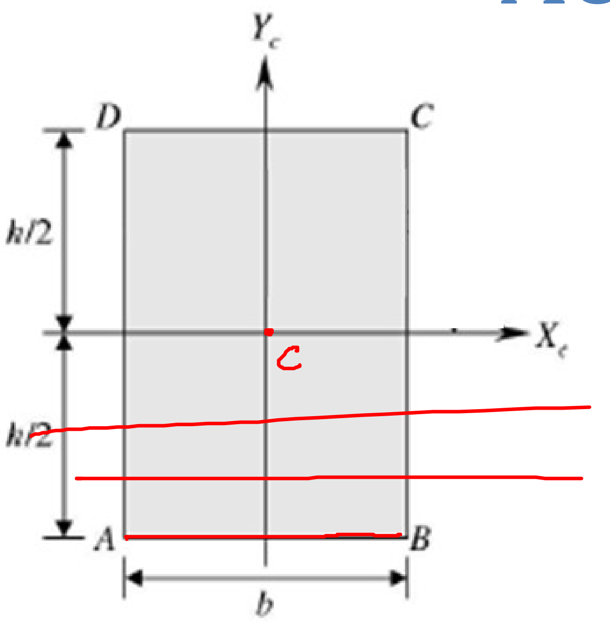
MOI about axis AB



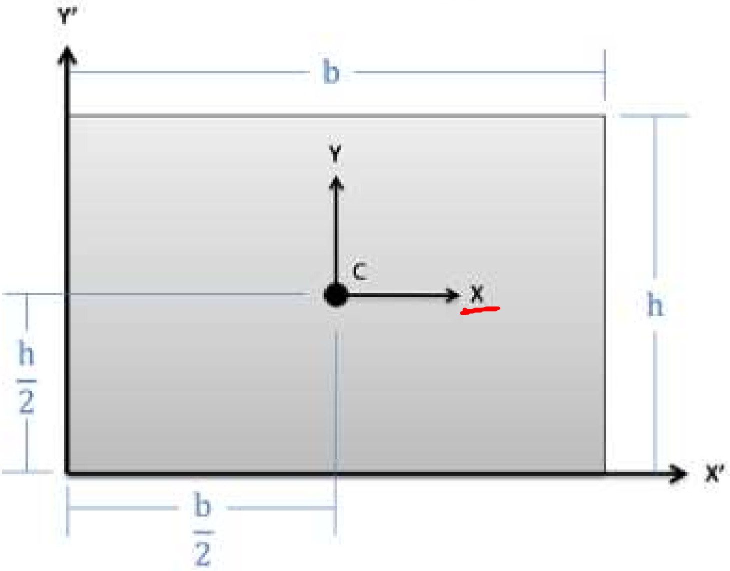
$$I_{AB} = I_{xx} + Ay^2 \quad = \frac{bh^3}{12} + bh\left(\frac{h}{2}\right)^2$$

$$I_{AB} = \frac{bh^3}{12} + bh(h/2)^2$$

$$I_{AB} = \frac{bh^3}{3}$$

Moment of inertia of basic shapes

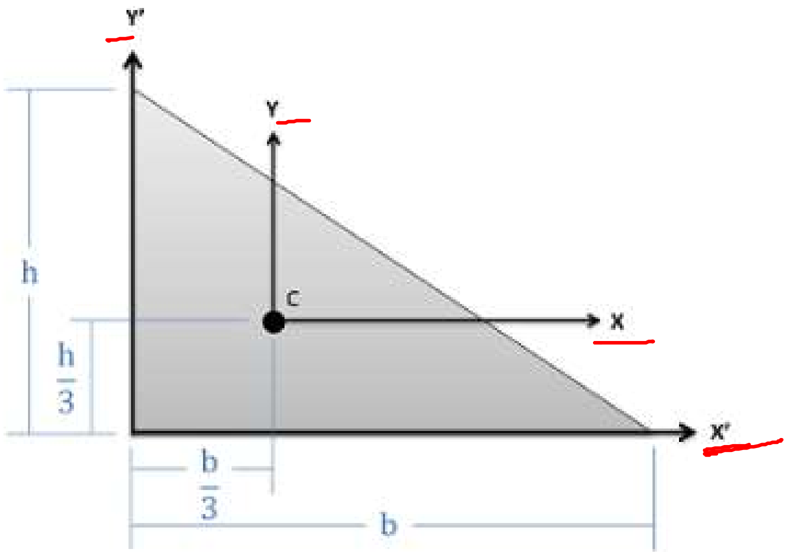
Rectangle



$$I_x = \frac{1}{12} b h^3$$

$$I_y = \frac{1}{12} b^3 h$$

Right Triangle



$$I_x = \frac{1}{36} bh^3$$

$$I_y = \frac{1}{36} b^3 h$$

$$I_{x'} = \frac{1}{12} bh^3$$

$$I_{y'} = \frac{1}{12} b^3 h$$

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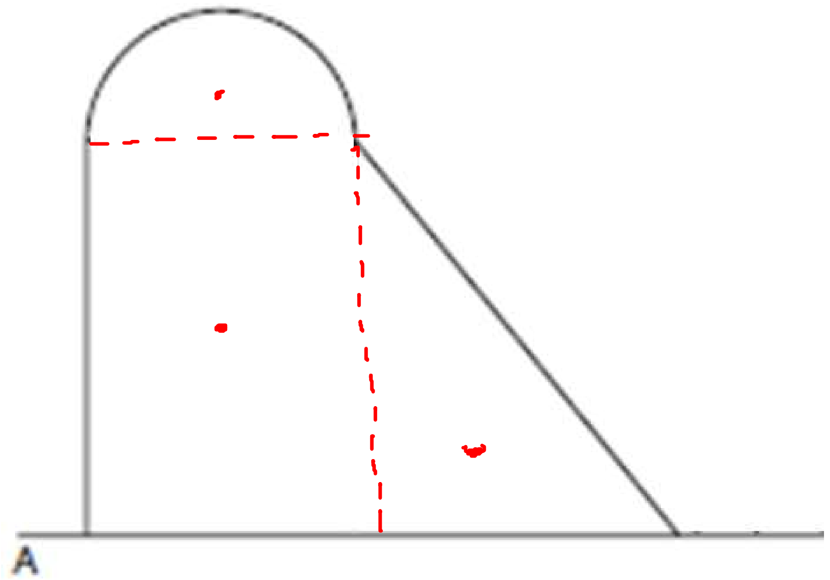
Built-up sections

- It is often advantageous to combine a number of smaller members in order to create a beam or column of greater strength.
- The moment of inertia of such a built-up section is found by adding the moments of inertia of the component parts

Moment of inertia of built-up sections

(1) Divide the given figure into a number of simple figures.

(2) Locate the centroid of each simple figure by inspection or using standard expressions**.**



Moment of inertia of built-up sections

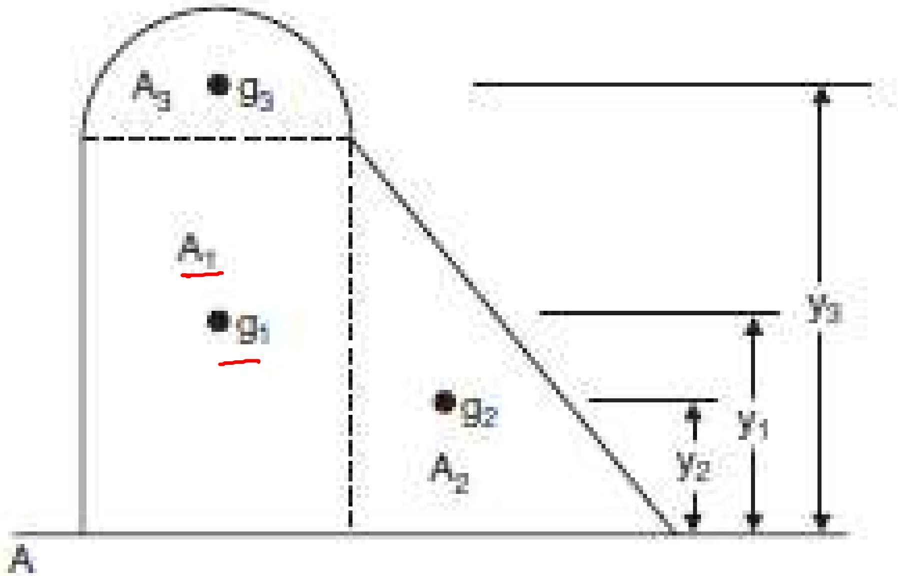
(1) Divide the given figure into a number of simple figures.

(2) Locate the centroid of each simple figure by inspection or using standard expressions.

(3) Find the moment of inertia of each simple figure about its centroidal axis. Add the term Ay^2

where A is the area of the simple figure and y is the distance of the centroid of the simple figure from the reference axis. This gives moment of inertia of the simple figure about the reference axis.

(4) Sum up moments of inertia of all simple figures to get the moment of inertia of the composite section.



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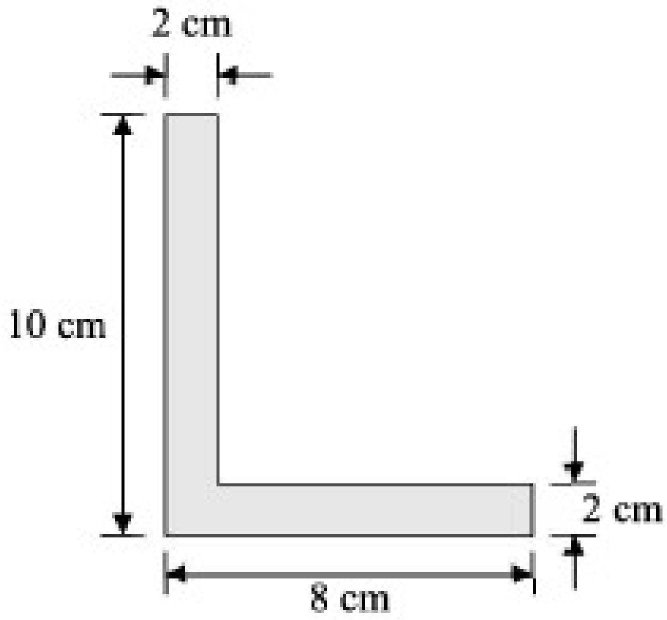
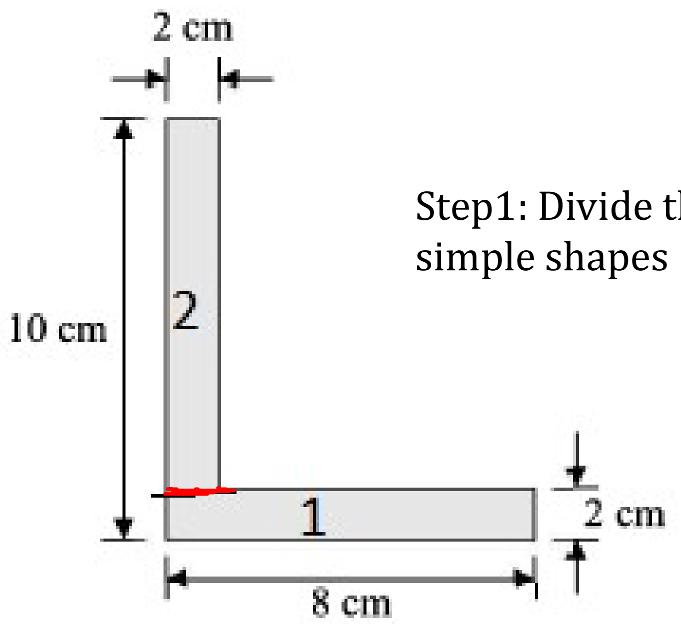
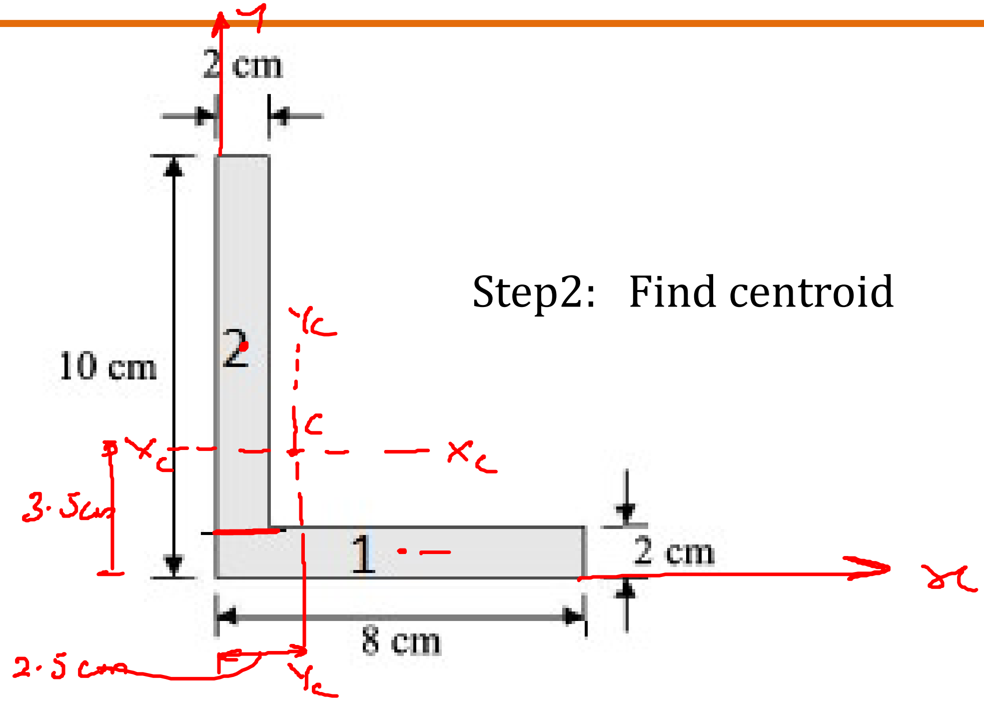
Example: Find moment of inertia about centroidal axis





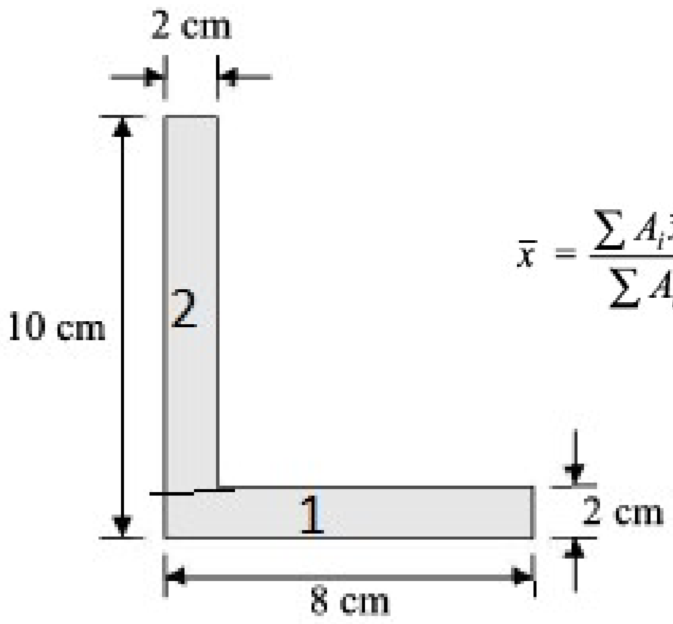
Step1: Divide the section into simple shapes

Step2: Find centroid



S.No	Element	$A_i\ (cm^2)$	$\bar{x}_i\ (cm)$	$\bar{y}_i\ (cm)$	$A_i\bar{x}_i\ (cm^3)$	$A_i\bar{y}_i\ (cm^3)$
1.	Rectangle-(1)	$8 \times 2 = 16$	$8/2 = 4$	$2/2 = 1$	64	16
2.	Rectangle- (2)	$2 \times 8 = 16$	$2/2 = 1$	$2 + (8/2) = 6$	16	96
	$\Sigma =$	32			80	112

$\therefore$ $$\bar{x} = \frac{\sum A_i \bar{x}_i}{\sum A_i} = 2.5 \text{ cm} \qquad \bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = 3.5 \text{ cm}$$



$$\bar{x} = \frac{\sum A_i \bar{x}_i}{\sum A_i} = 2.5 \text{ cm} \qquad \bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = 3.5 \text{ cm}$$

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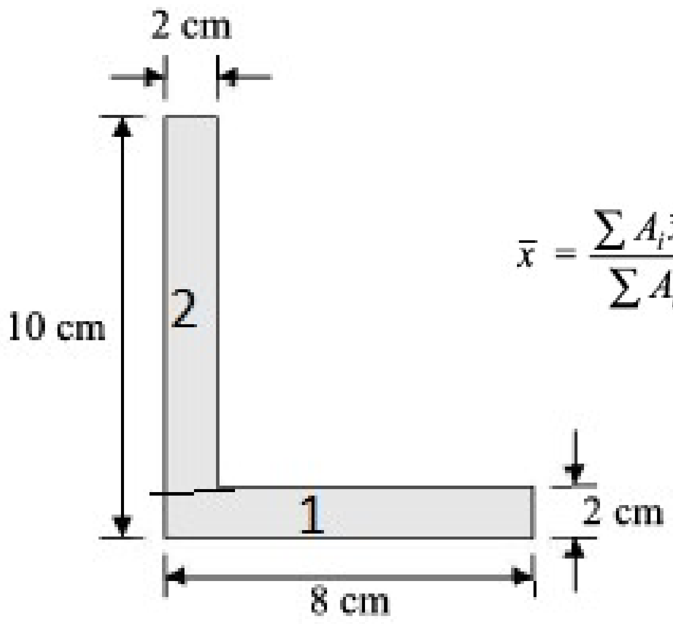


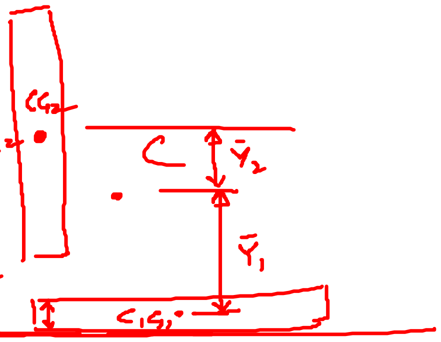
$$\bar{x} = \frac{\sum A_i \bar{x}_i}{\sum A_i} = 2.5 \text{ cm}$$

$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = 3.5 \text{ cm}$$

$I_{x_c x_c} = I_1 + I_2$

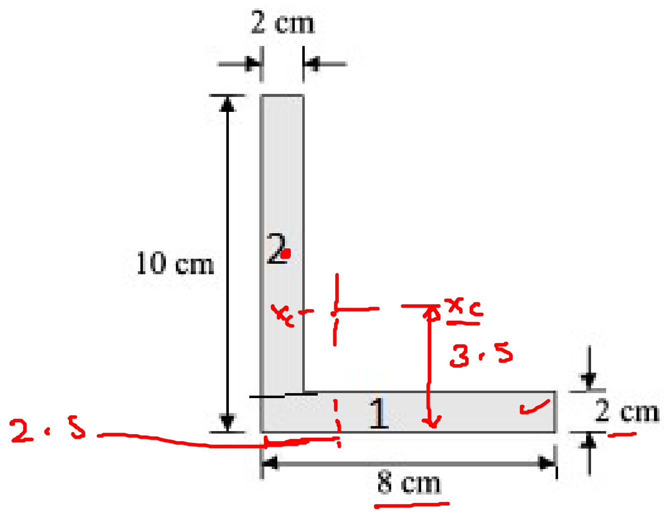
$I_1 = I_{CG_1} + A\bar{y}^2$

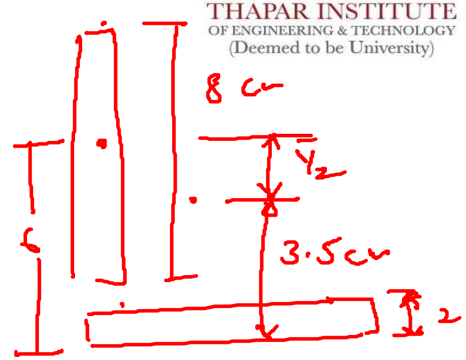
$= \frac{8\times 2^3}{12} + 8\times 2\times(3.5-1)^2$



$I_2 = I_{CG_2} + A\bar{y}_2^2$

$= \frac{2\times 8^3}{12} + 2\times 8\times(6-3.5)^2$





Moments of inertia calculations

S.No	$(\bar{I}_{xx})_i\ cm^4$	$(\bar{I}_{yy})_i\ cm^4$	$A_i(\bar{y}_i-\bar{y})^2 cm^4$	$A_i(\bar{x}_i-\bar{x})^2\ cm^4$
1.	$(1/12)\times 8\times 2^3 = 5.33$	$(1/12)\times 2\times 8^3 = 85.33$	$16(1-3.5)^2 = 100$	$16(4-2.5)^2 = 36$
2.	$(1/12)\times 2\times 8^3 = 85.33$	$(1/12)\times 8\times 2^3 = 5.33$	$16(6-3.5)^2 = 100$	$16(1-2.5)^2 = 36$
Σ =	90.66	90.66	200	72

$I_{x_c x_c}$ = 2nd MOA of the whole area about centroidal x-axis

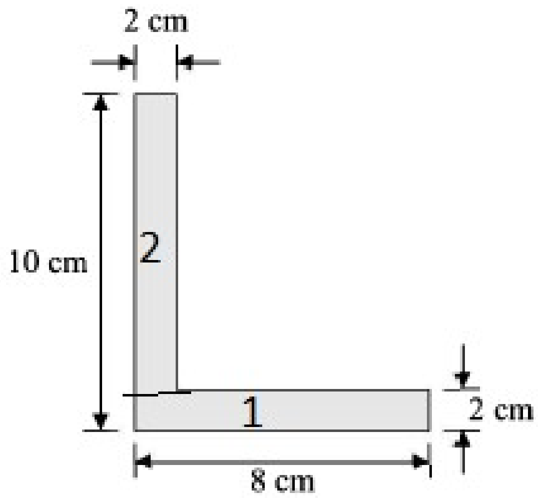
I_1 = " " " 1st area about centroidal axis

I_2 = " " " 2nd " " " "

$$\bar{I}_{xx} = \sum(\bar{I}_{xx})_i + \sum A_i(\bar{y}_i-\bar{y})^2$$
$$= 90.66 + 200 = 290.66\ cm^4$$

$$\bar{I}_{yy} = \sum(\bar{I}_{yy})_i + \sum A_i(\bar{x}_i-\bar{x})^2$$
$$= 90.66 + 72 = 162.66\ cm^4$$

Determine MOI of section about base



Method 1

$$
\begin{aligned}
I_{\text{base}} &= \bar{I}_{xx} + A(d)^2 \\
&= 290.66 + (32)\,(3.5)^2 \qquad [\text{Note that } d = \bar{y} = 3.5 \text{ cm}] \\
&= 682.66 \text{ cm}^4
\end{aligned}
$$

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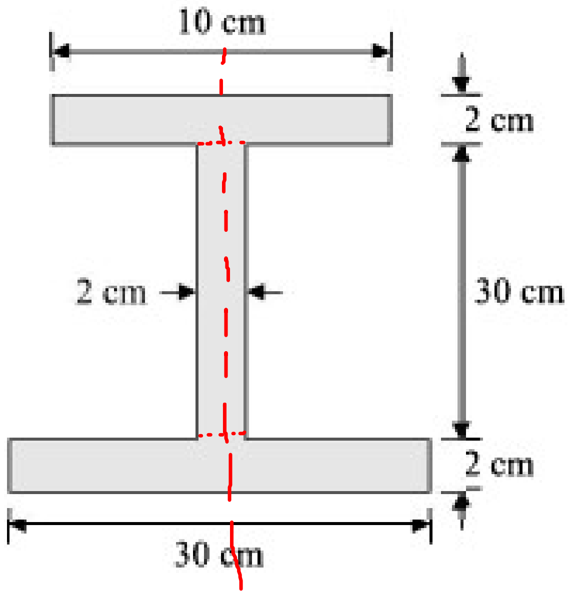
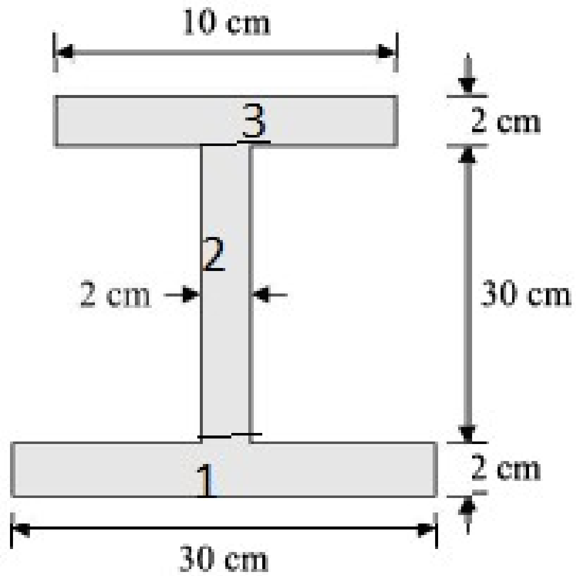
Find moment of inertia about centroidal axis





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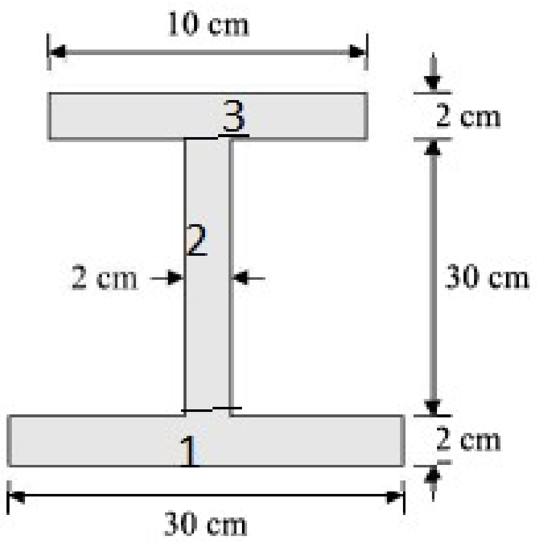
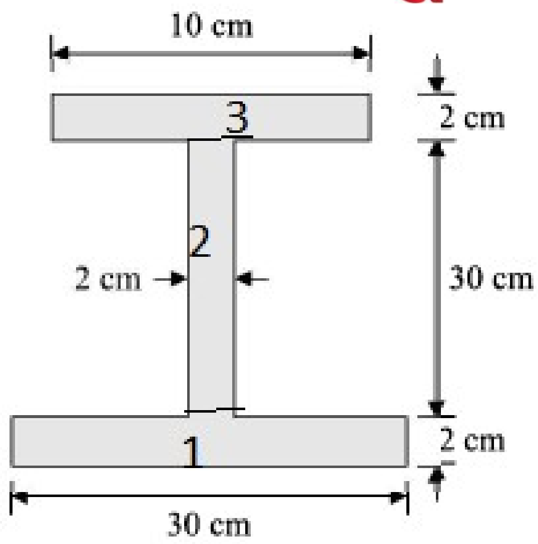
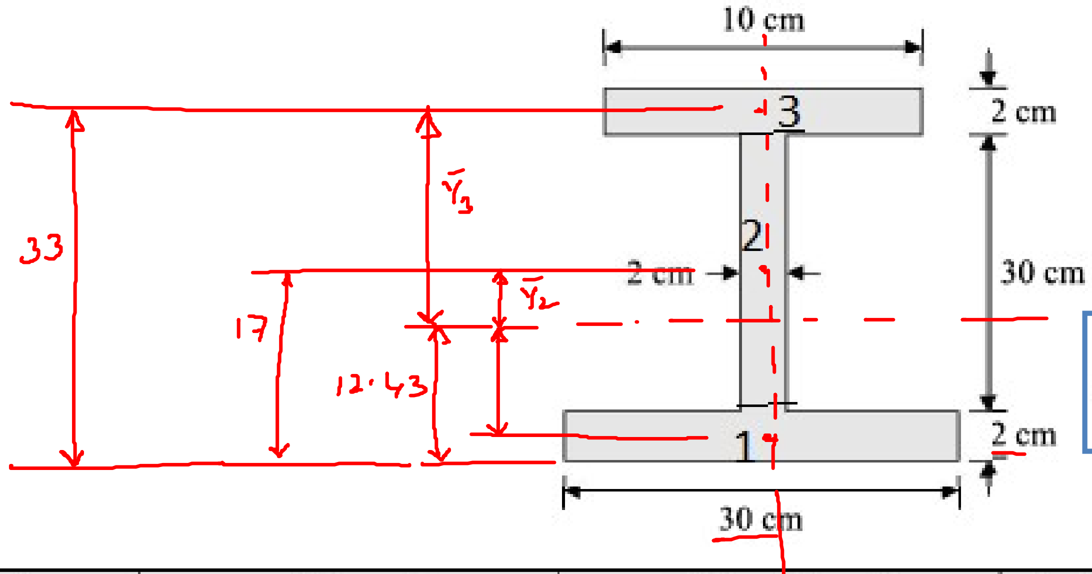


S.No	Element	$A_i\ (cm^2)$	$\bar{y}_i$ (cm)	$A_i\bar{y}_i\ (cm^3)$
1.	Rectangle (1)	$30 \times 2 = 60$	1	60
2.	Rectangle (2)	$30 \times 2 = 60$	17	1020
3.	Rectangle (3)	$10 \times 2 = 20$	33	660
	$\Sigma =$	140		1740

$$\therefore \quad \bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = 12.43 \text{ cm}$$



$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = 12.43 \text{ cm}$$



$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = 12.43 \text{ cm}$$

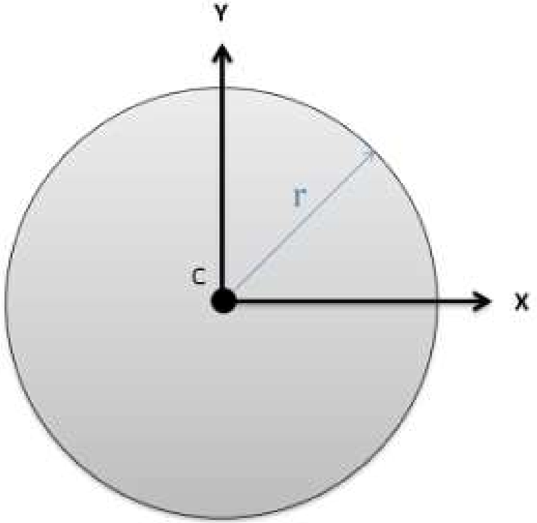
S.No	$(\bar{I}_{xx})_i\ cm^4$	$(\bar{I}_{yy})_i\ cm^4$	$A_i(\bar{y}_i - \bar{y})^2\ cm^4$	$A_i(\bar{x}_i - \bar{x})^2\ cm^4$
1.	$(1/12) \times 30 \times 2^3 = 20$	$(1/12) \times 2 \times 30^3 = 4500$	$60(1 - 12.43)^2 = 7838.69$	0
2.	$(1/12) \times 2 \times 30^3 = 4500$	$(1/12) \times 30 \times 2^3 = 20$	$60(17 - 12.43)^2 = 1253.09$	0
3.	$(1/12) \times 10 \times 2^3 = 6.67$	$(1/12) \times 2 \times 10^3 = 166.67$	$20(33 - 12.43)^2 = 8462.5$	0
$\Sigma =$	4526.67	4686.67	17 554.28	0

$$\bar{I}_{xx} = \sum (\bar{I}_{xx})_i + \sum A_i(\bar{y}_i - \bar{y})^2$$

$$= 4526.67 + 17\,554.28 = 22\,080.95 \text{ cm}^4$$

$$\bar{I}_{yy} = \sum (\bar{I}_{yy})_i + \sum A_i(\bar{x}_i - \bar{x})^2 = 4686.67 \text{ cm}^4$$

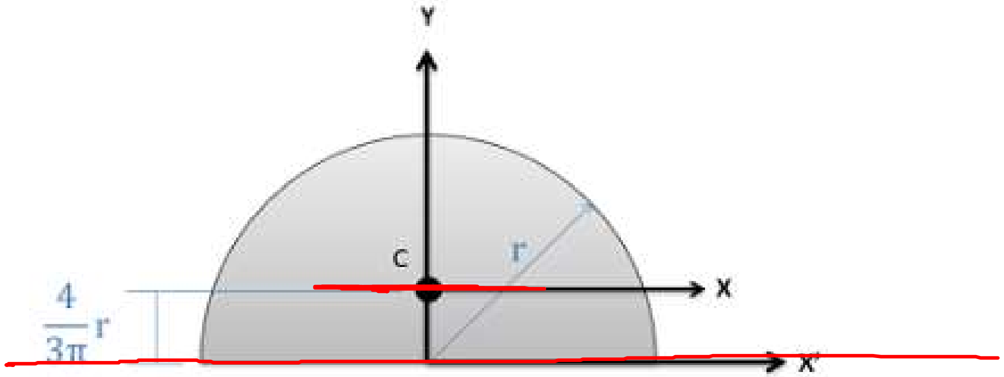
Circle



$$I_x = \frac{\pi}{4} r^4 = \frac{\pi}{64} d^4$$

$$I_y = \frac{\pi}{4} r^4 = \frac{\pi}{64} d^4$$

Semicircle



$$I_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$$

$$I_y = \frac{\pi}{8} r^4$$

$$I_{x'} = \frac{\pi}{8} r^4$$

$$I_{x_c} = I_{x'} - A\bar{y}^2$$

$$\cancel{I_{x'} = I_{x_{cy}} + A\bar{y}^2}$$

$$= \frac{\pi r}{8} - \frac{\pi r^2}{2}\left(\frac{4r}{3\pi}\right)^2$$

$$= 0\ 1059 r^4 = 0.11 r^4$$

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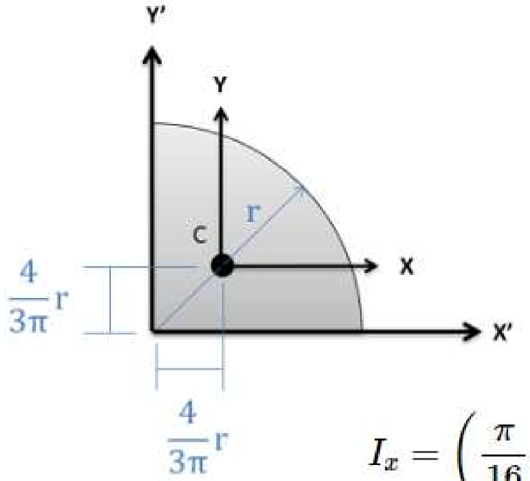
Quarter Circle



$$I_x = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4$$

$$I_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4$$

$= 0.055r^4$

$$I_{x'} = \frac{\pi}{16} r^4$$

$$I_{y'} = \frac{\pi}{16} r^4$$

Find moment of inertia about centroidal axis

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