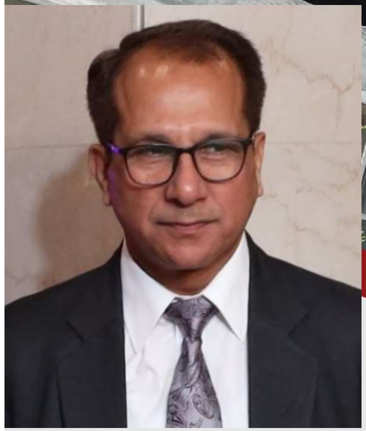
Properties of Plane Surfaces



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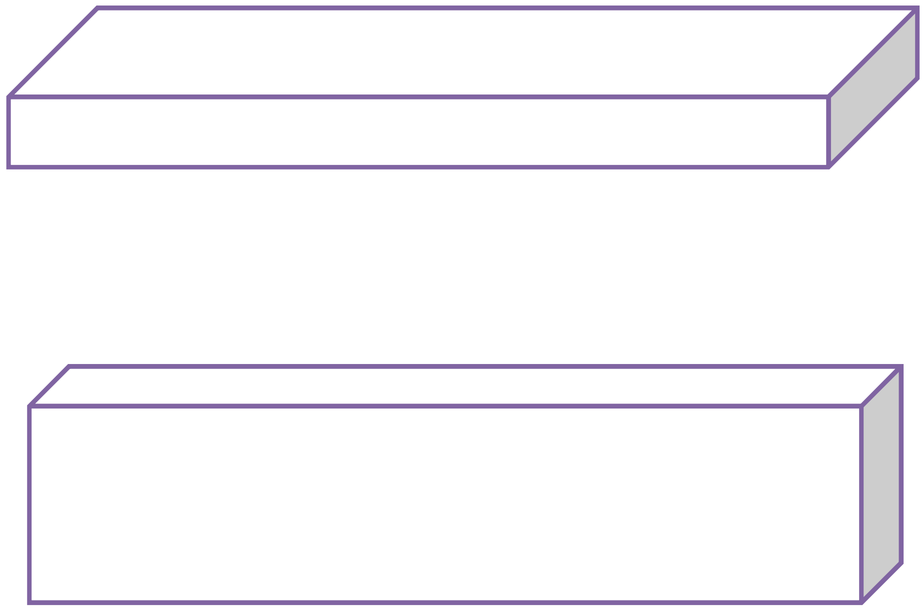
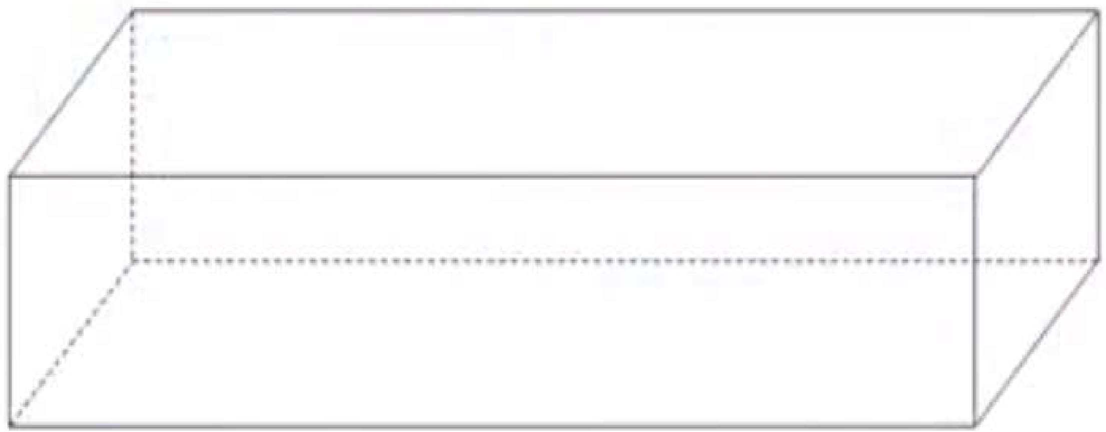
PROPERTIES OF PLANE SURFACES

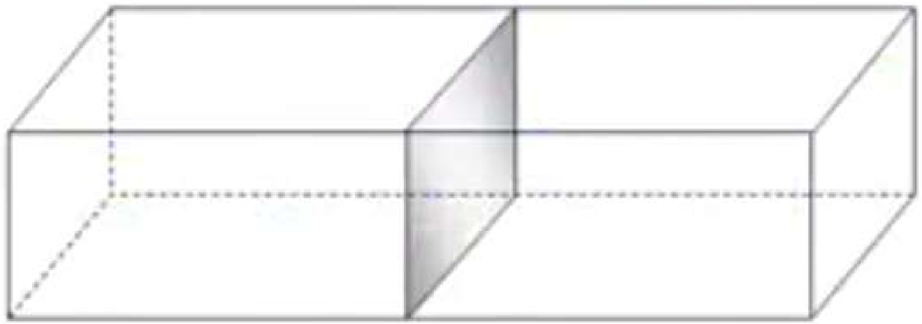
Properties of plane surfaces

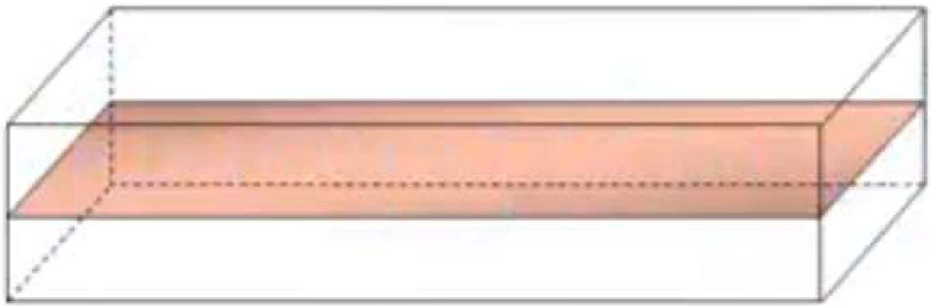
- Area
- First moment of area
- Second moment of area

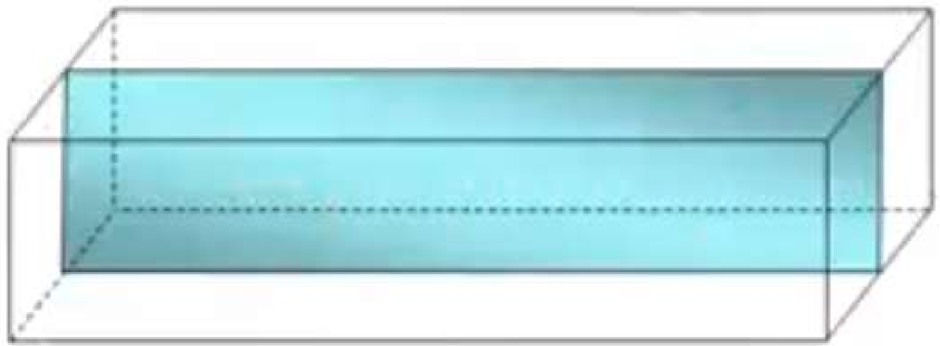




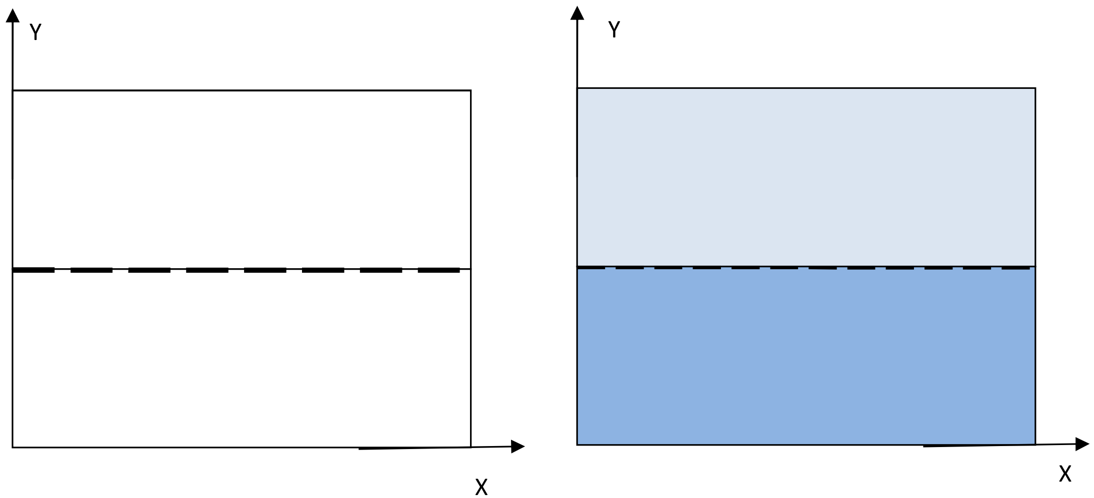
Planes of symmetry





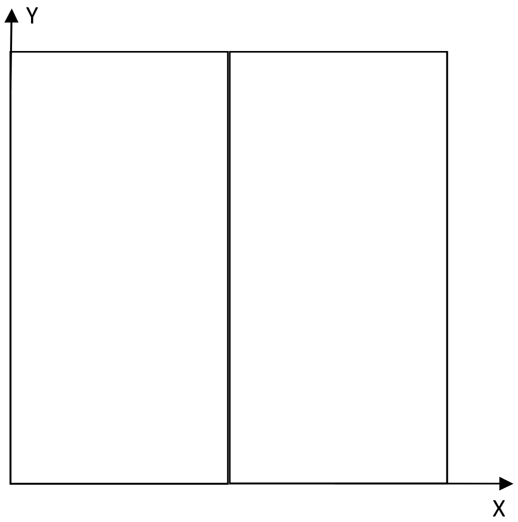


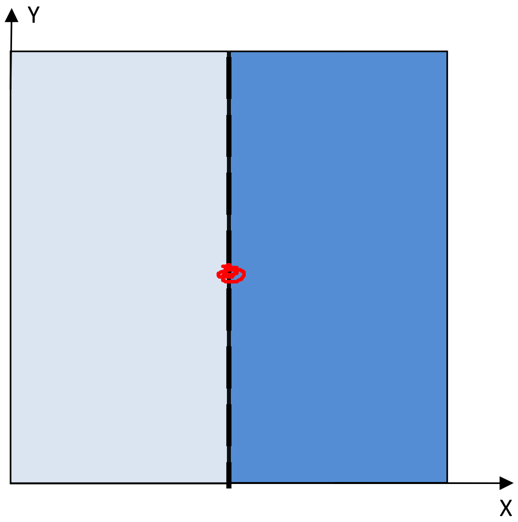
Centroidal X axis



A line parallel to x axis which divides it into two equal areas

Centroidal   Y axis





A line parallel to y axis which divides it into two equal areas

Centroid

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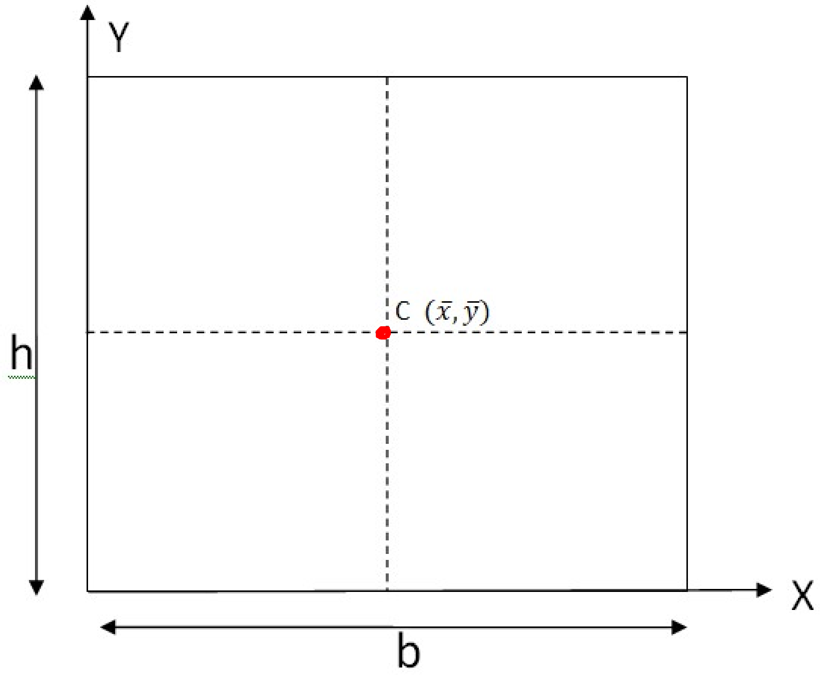
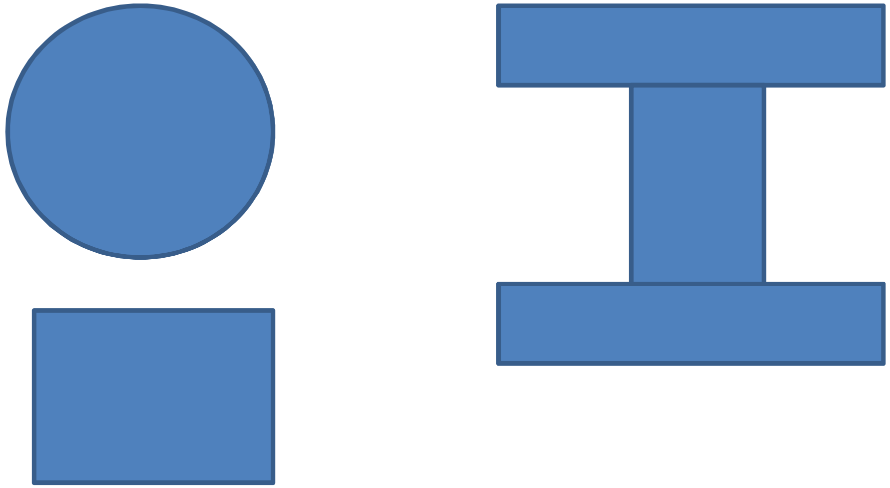


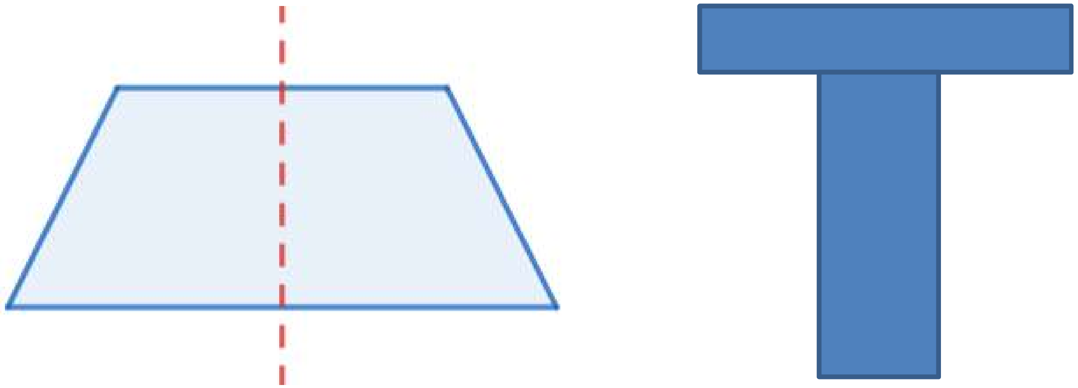
Centroid is point of intersection of centroidal x axis and y axis

Centroid is geometrical centre of area

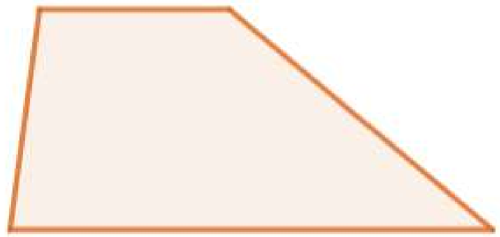
Double symmetrical figure

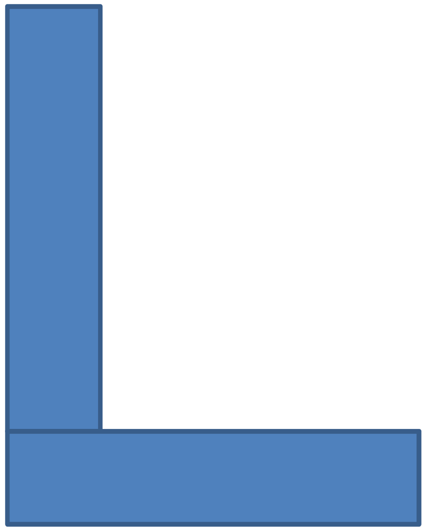


Single symmetrical figure



Unsymmetrical Figures





To find centroid we use first moment of area

How to find centroid of unsymmetrical figure

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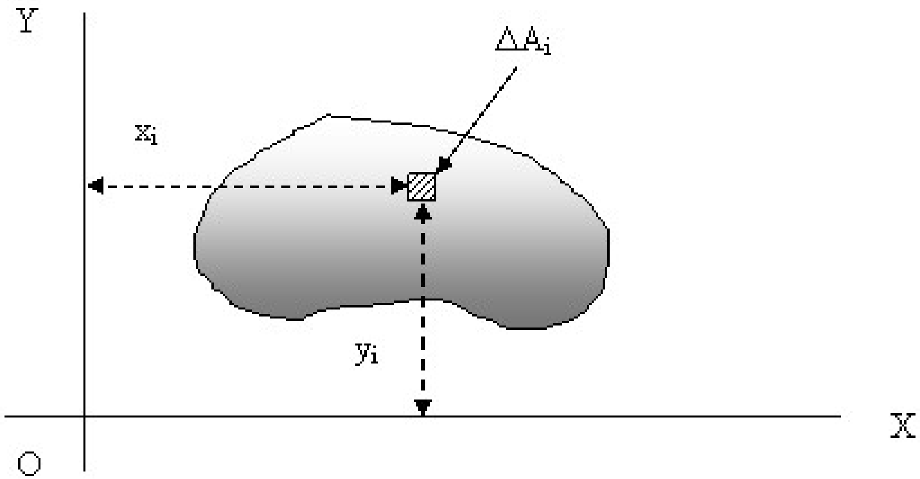
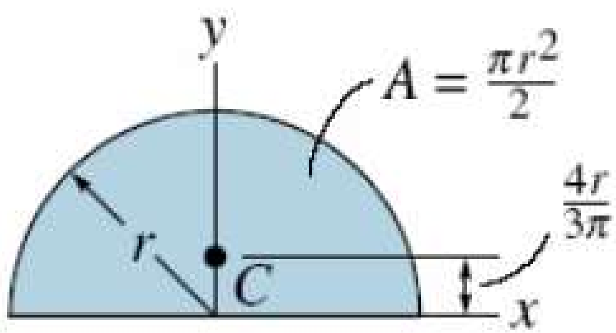
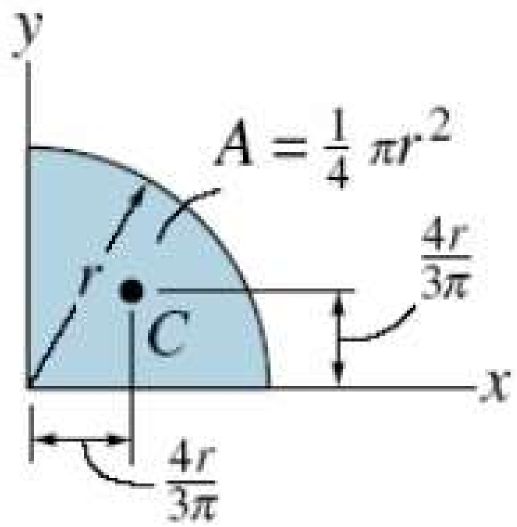
$$\bar{y} = \frac{\int y\,dA}{A}$$

$$\bar{x} = \frac{\int x\,dA}{A}$$



Centroid of semi circle and quarter circle





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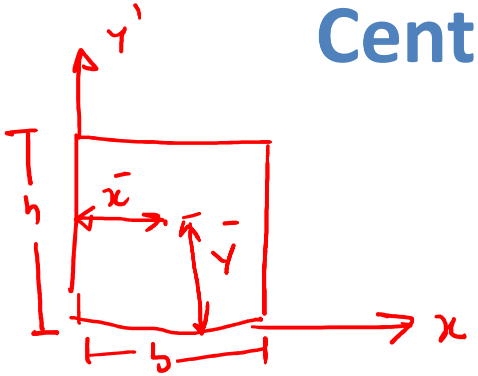
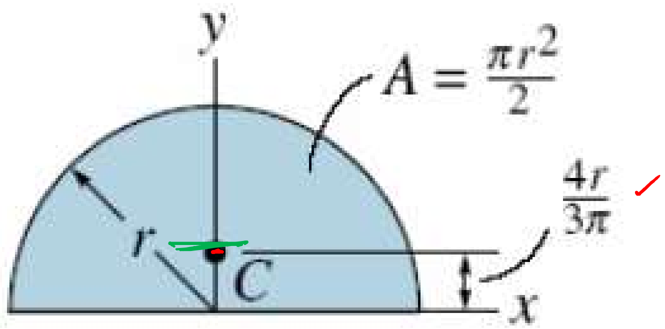
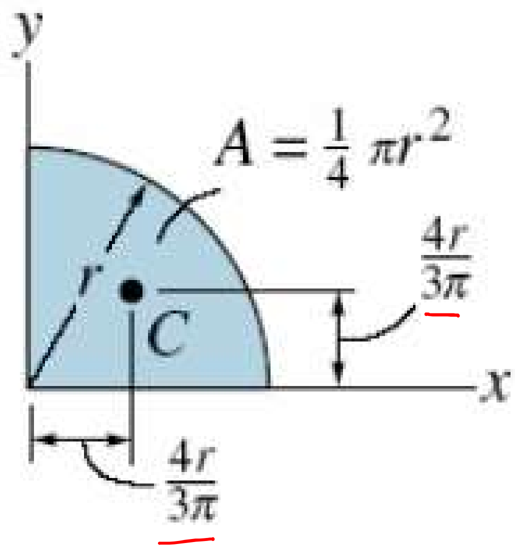
Centroid of semi circle and quarter circle

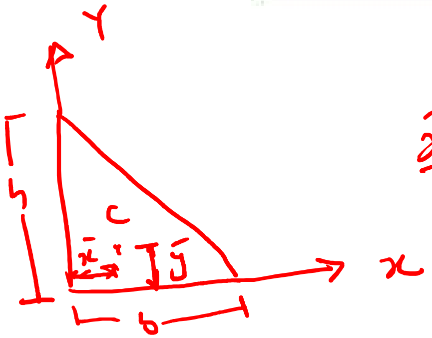


$\bar{x} = \frac{b}{2}$

$\bar{y} = \frac{h}{2}$

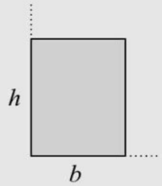
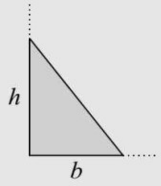
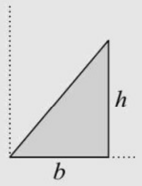
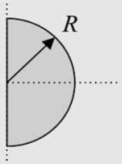
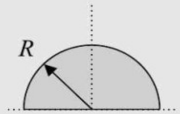
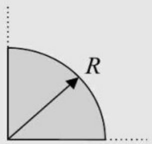






$\bar{x} = \frac{b}{3}$

$\bar{y} = \frac{h}{3}$

S.No	*Shape*	*Figure*	*Area*	$\bar{x}$	$\bar{y}$
1.	Rectangle		bh	$\frac{b}{2}$	$\frac{h}{2}$
2.	Right-angled triangle		$\frac{1}{2}bh$	$\frac{b}{3}$	$\frac{h}{3}$
3.	Right-angled triangle		$\frac{1}{2}bh$	$\frac{2}{3}b$	$\frac{h}{3}$
4.	Semicircle		$\frac{\pi R^2}{2}$	$\frac{4R}{3\pi}$	0
5.	Semicircle		$\frac{\pi R^2}{2}$	0	$\frac{4R}{3\pi}$
6.	Quadrant		$\frac{\pi R^2}{4}$	$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$

Steps to find centroid of composite figure

- ❑Break up the figure into parts whose centroid is known like rectangle, circle, triangle etc.
- ❑Find area of each part A_1, A_2, A_3----A_n
- ❑Find location of centroid of each part separately (x_1,y_1), (x_2,y_2), (x_3,y_3),--- (x_n,y_n)
- ❑Find total area A

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3 - - - A_nx_n}{A}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 - - - A_ny_n}{A}$$

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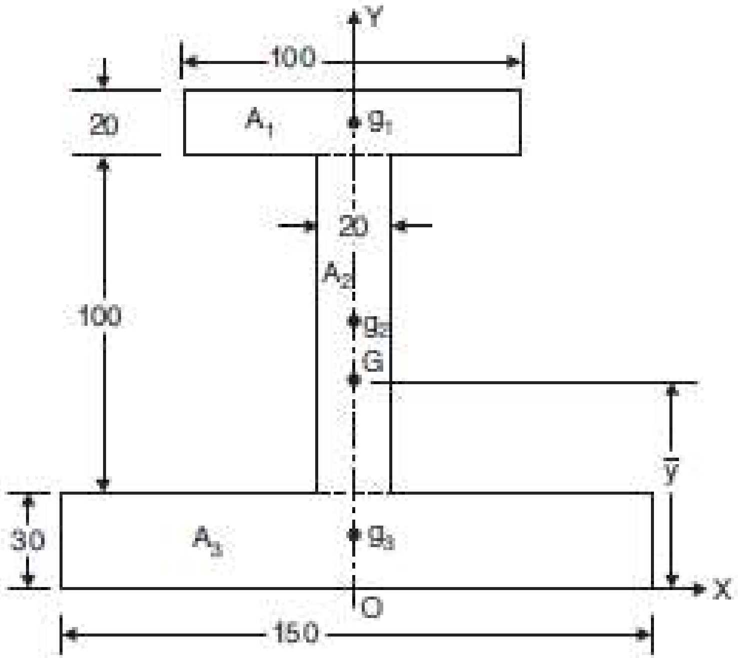


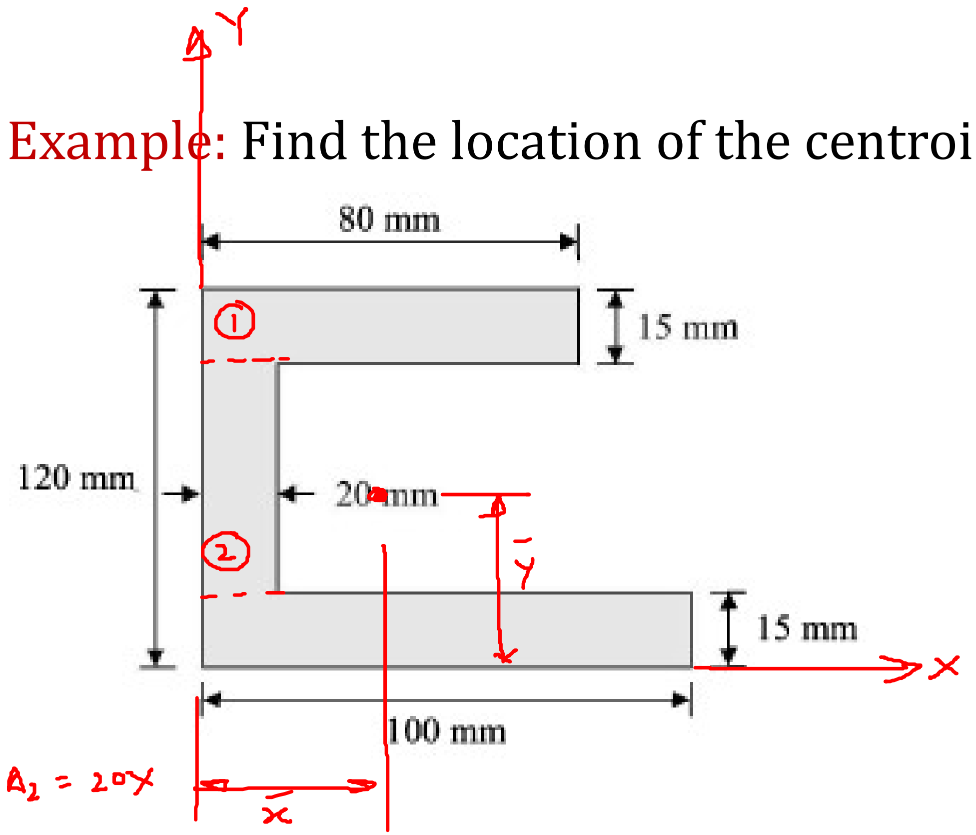
$As\ section\ is\ symmetrical\ about\ y\ axis$

$$\bar{x} = 0$$

	Area	y	Ay
1	20x100	140	20x100x140
2	20x100	80	20x100x80
3	150x30	15	150x30x15

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = 59.71\ mm$$

Example: Find the location of the centroid of the shaded area



$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 - - - A_n x_n}{A}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 - - - A_n y_n}{A}$$

Solution: Divide the composite area into simple areas

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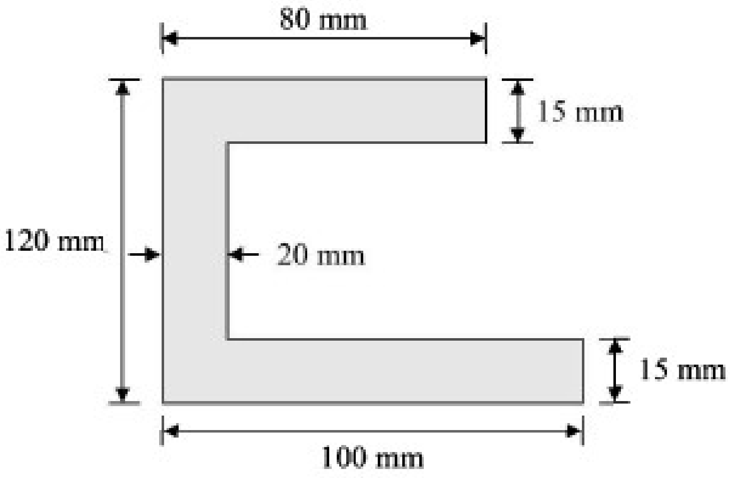
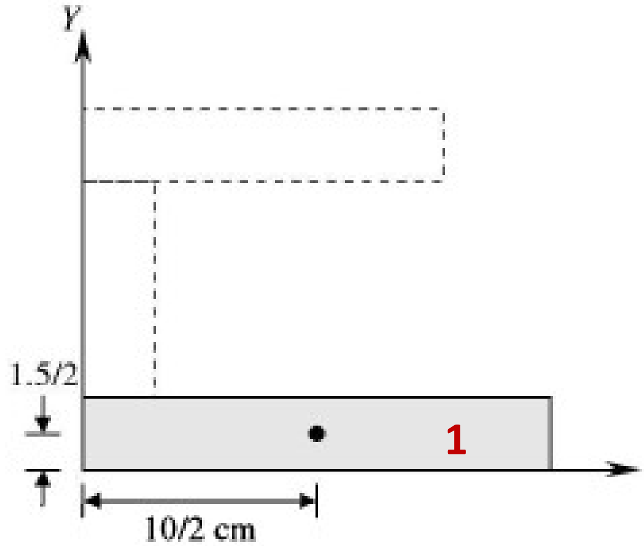
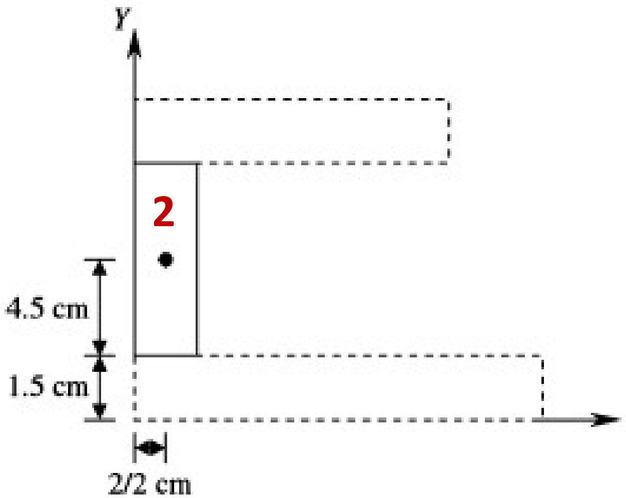
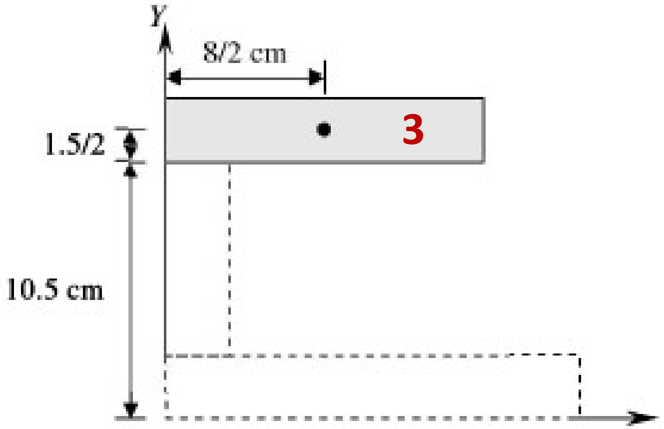
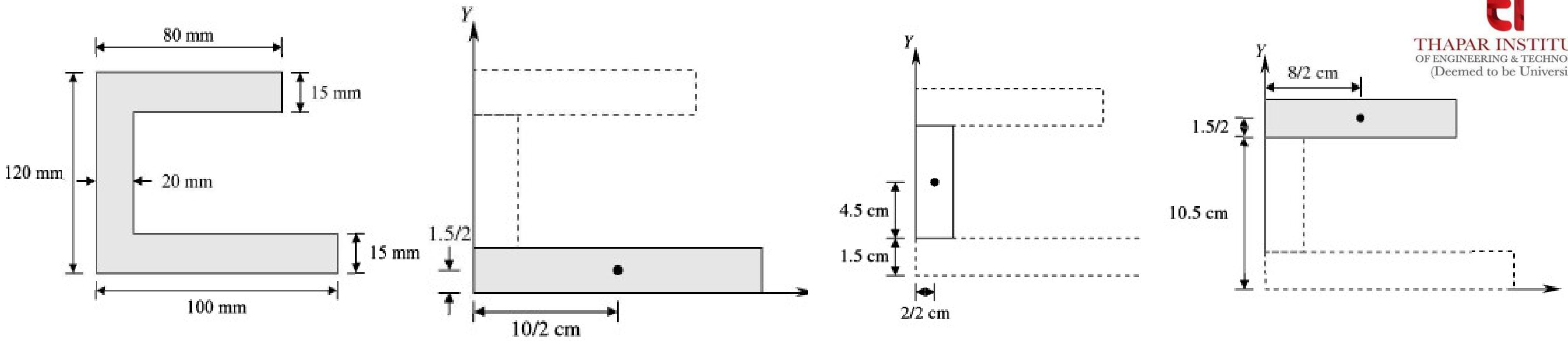








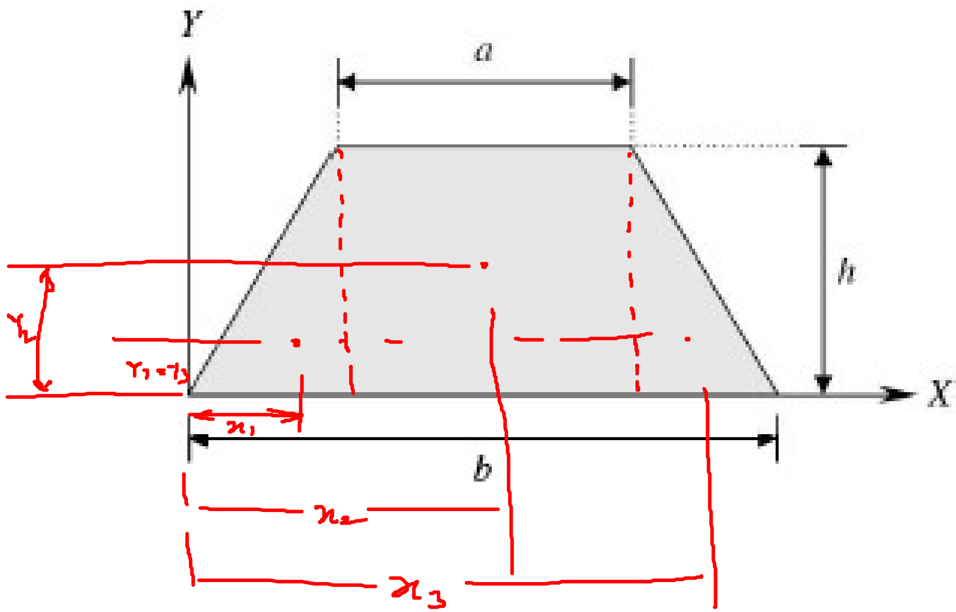


S.No	Element	$A_i\ (cm^2)$	$\bar{x}_i\ (cm)$	$\bar{y}_i\ (cm)$	$A_i\bar{x}_i\ (cm^3)$	$A_i\bar{y}_i\ (cm^3)$
1.	Rectangle-(1)	$10 \times 1.5 = 15$	$10/2 = 5$	$1.5/2 = 0.75$	75	11.25
2.	Rectangle-(2)	$[12 - 2(1.5)] \times 2 = 18$	$2/2 = 1$	$12/2 = 6$	18	108
3.	Rectangle-(3)	$8 \times 1.5 = 12$	4	$12 - 1.5/2 = 11.25$	48	135
	$\Sigma =$	45			141	254.25

$$\bar{x} = \frac{\sum A_i \bar{x}_i}{\sum A_i} = \frac{141}{45} = 3.13 \text{ cm (or) } 31.3 \text{ mm}$$

$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = \frac{254.25}{45} = 5.65 \text{ cm (or) } 56.5 \text{ mm}$$

Example: Find the location of the centroid of the shaded area



Solution: Divide the composite area into simple areas

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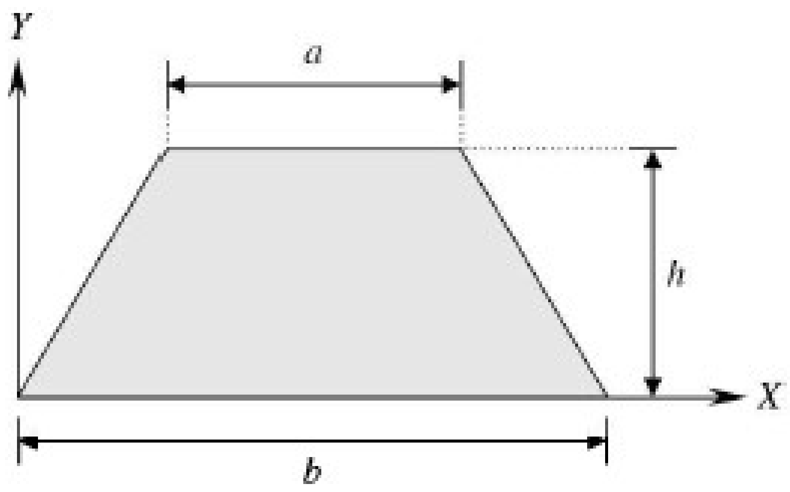

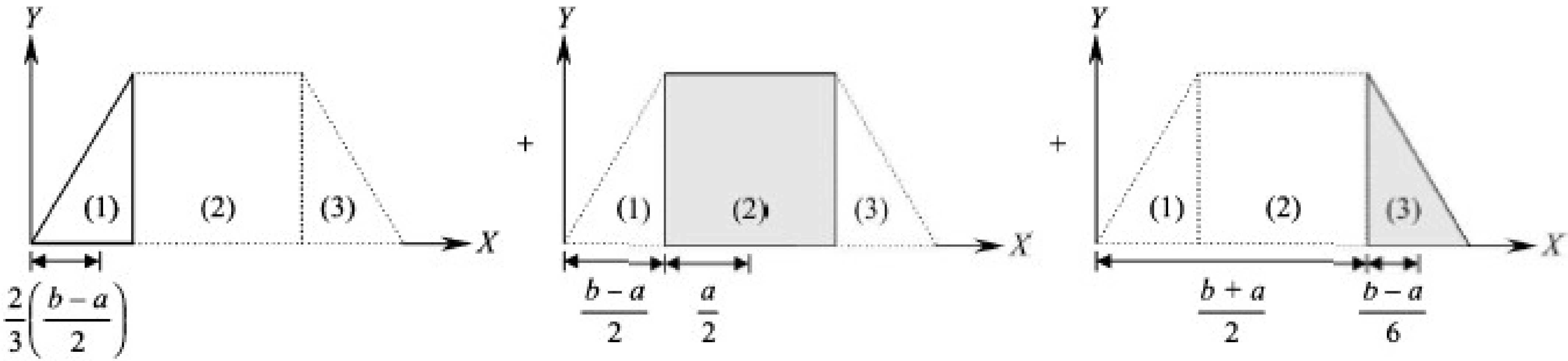

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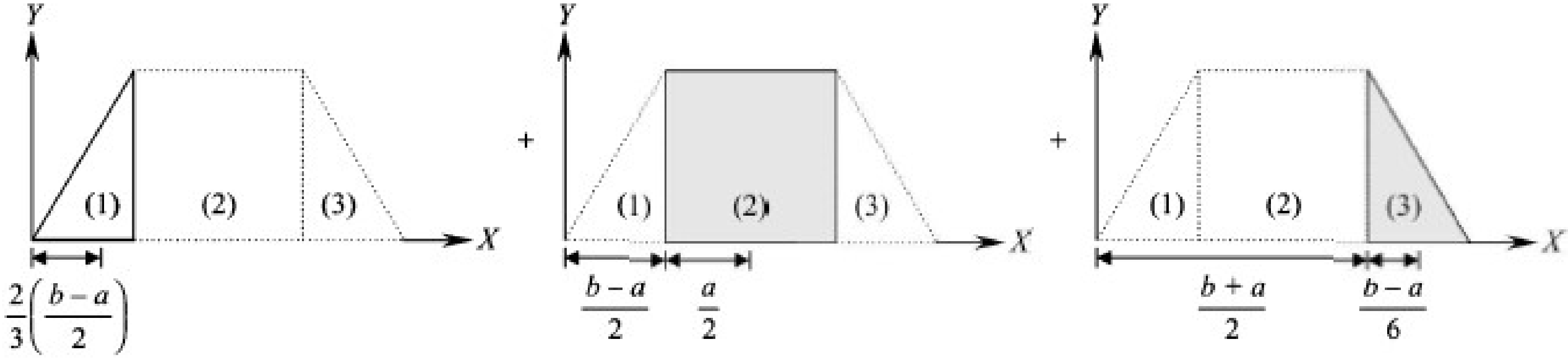
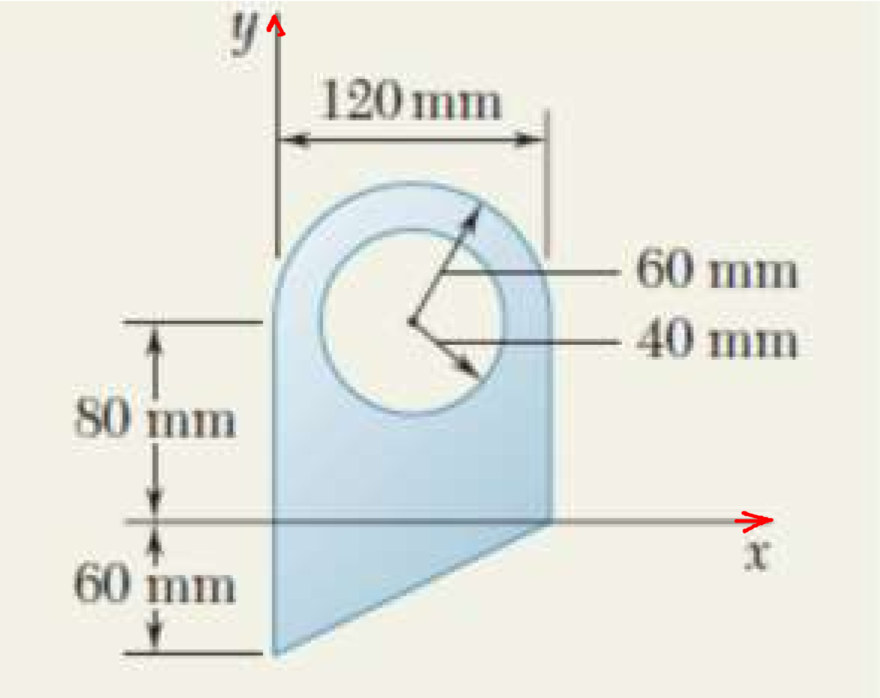
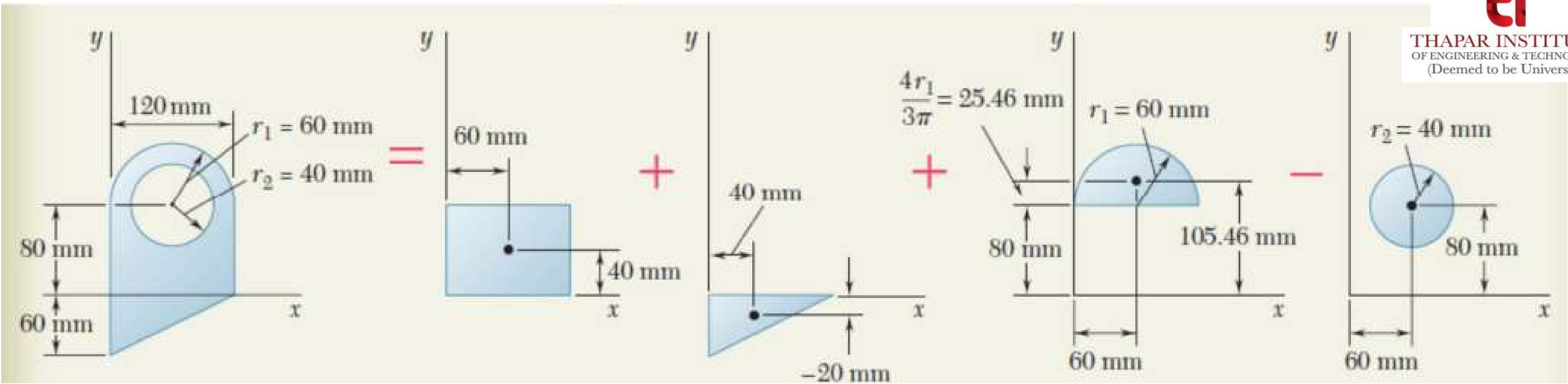


S.No	Element	A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
1.	Triangle-(1)	$\frac{1}{2}\left(\frac{b-a}{2}\right)h$	$\frac{2}{3}\left(\frac{b-a}{2}\right)$	$\frac{h}{3}$	$\frac{(b-a)^2}{12}h$	$\frac{(b-a)h^2}{12}$
2.	Rectangle-(2)	ah	$\frac{b}{2}$	$\frac{h}{2}$	$\frac{abh}{2}$	$\frac{ah^2}{2}$
3.	Triangle-(3)	$\frac{1}{2}\left(\frac{b-a}{2}\right)h$	$\frac{2b+a}{3}$	$\frac{h}{3}$	$\frac{(2b+a)(b-a)h}{12}$	$\frac{(b-a)h^2}{12}$
	$\Sigma =$	$\frac{(b+a)}{2}h$			$\frac{bh}{4}(b+a)$	$\frac{h^2}{6}(b+2a)$

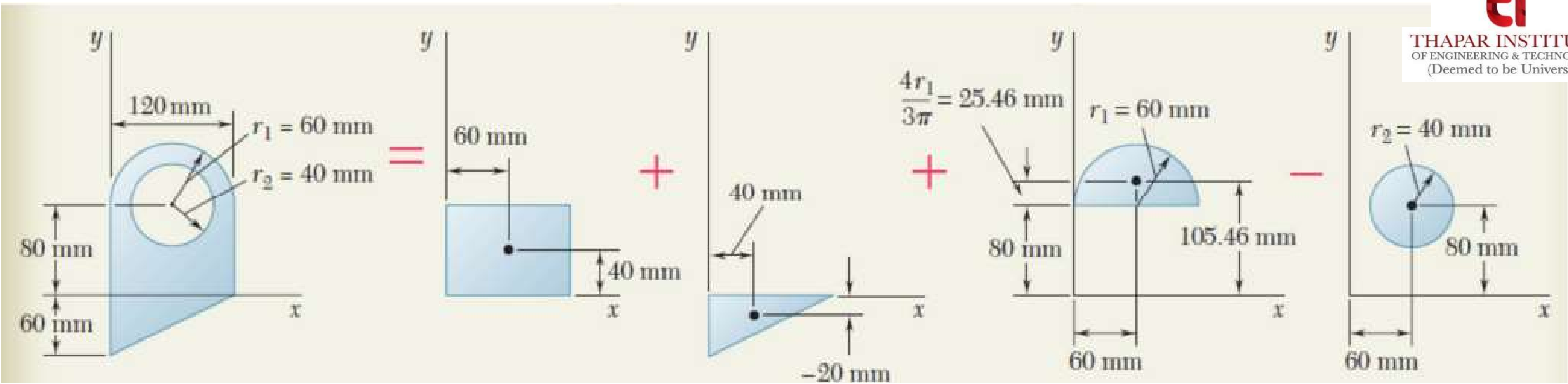
$$\bar{x} = \frac{\sum A_i \bar{x}_i}{\sum A_i} = \frac{b}{2} \qquad \bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = \frac{h}{3}\left[\frac{b+2a}{b+a}\right]$$

Find the location of the centroid of the shaded area





120 mm
$r_1 = 60$ mm
$r_2 = 40$ mm
80 mm
60 mm
60 mm
40 mm
40 mm
−20 mm
$\frac{4r_1}{3\pi} = 25.46$ mm
$r_1 = 60$ mm
80 mm
105.46 mm
60 mm
$r_2 = 40$ mm
80 mm
60 mm



120 mm
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80 mm
105.46 mm
60 mm
$r_2 = 40$ mm
80 mm
60 mm

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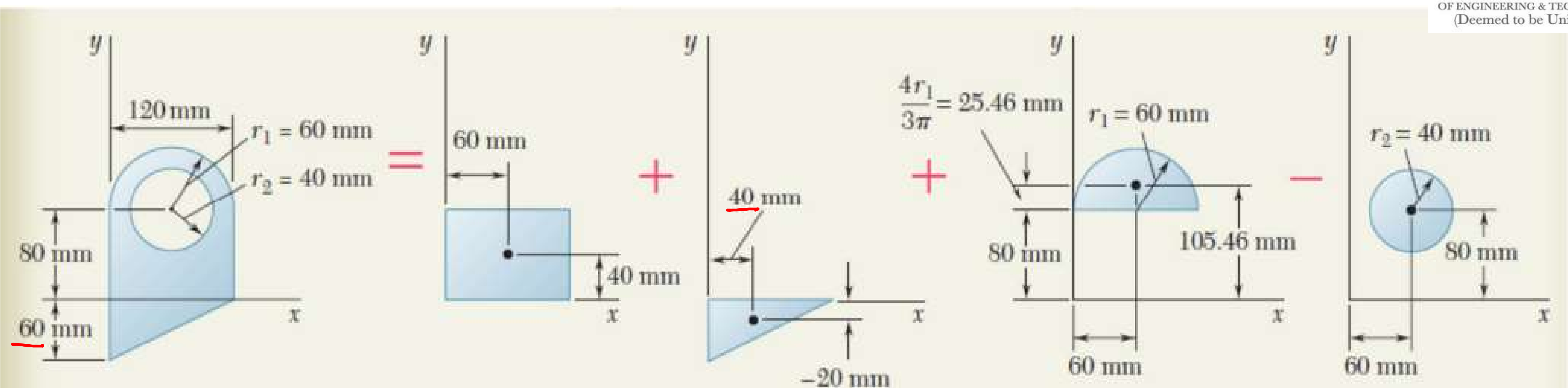

Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$ ✓	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$ ✓	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$ ✓	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

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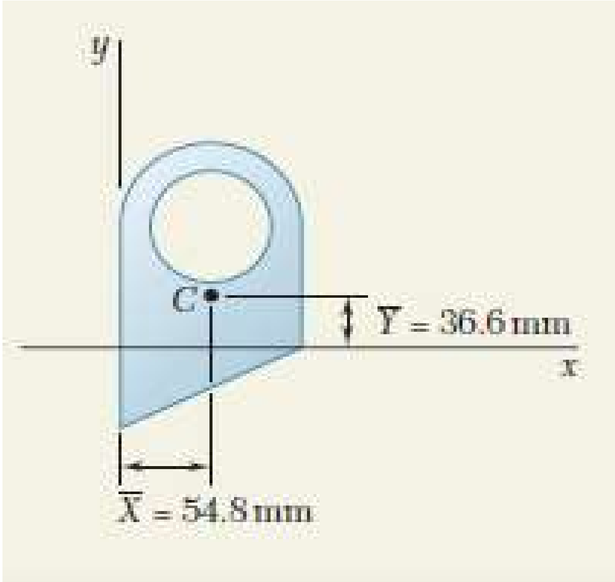

$$\overline{X} = \frac{\sum A\overline{X}}{\sum A} = \frac{757.7 \times 10^3}{13.828 \times 10^3} = 54.79\ mm$$

$$\overline{Y} = \frac{\sum A\overline{Y}}{\sum A} = \frac{506.2 \times 10^3}{13.828 \times 10^3} = 36.61\ mm$$

THANK YOU