Thapar Institute of Engineering and Technology, Patiala
School of Mathematics
Mathematics – II (UMA004): Tutorial Sheet 04

1. Find the Laplace Transform of the following functions:

i) $f(t)=\begin{cases} e^t, & 0<t<1 \\ 0, & t>1 \end{cases}$ ii) $f(t)=\begin{cases} 4, & 0<t<1 \\ 3, & t>1 \end{cases}$ iii) $f(t)=4t^2+\sin(3t)+e^{2t}$ iv) $f(t)=1+2\sqrt{t}+3/\sqrt{t}$

2. Determine the Laplace Transform of the following functions:

i) $e^{-3t}t^4$ ii) $e^{-3t}(2\cos 5t-3\sin 5t)$ iii) $(e^{-at}t^{n-1})/(n-1)!$ iv) $e^{4t}\sin 2t\cos t$

3. Evaluate

i) $L[t(3\sin 2t-2\cos 2t)]$ ii) $L\left[t^2\cos at\right]$ iii) $L\left[t^2e^{3t}\right]$ iv) $L\left[t\,e^{3t}\sin t\right]$

4. Find the Laplace Transform of the following functions:

i) $(1-e^{-t})/t$ ii) $(1-\cos t)/t$ iii) $(\sinh t)/t$ iv) $e^{-t}\sin t/t$

5. Determine the Laplace Transform of following functions:

i) $\int_0^t e^{-2t}t^3dt$ ii) $\int_0^t t\cosh t\,dt$ iii) $e^{-4t}\int_0^t t\sin 3t\,dt$ iv) $L[f'(t)]$, $f(t)=e^{-5t}\sin t$, $f(t)=\dfrac{\sin t}{t}$

6. Show that

i) $\int_0^\infty \dfrac{\cos 6t-\cos 4t}{t}dt=\ln(2/3)$ ii) $\int_0^\infty te^{-3t}\sin t\,dt=3/50$ iii) $\int_0^\infty \dfrac{e^{-3t}-e^{-6t}}{t}dt=\ln(2)$

7. Find the Inverse Laplace Transform of the following functions:

i) $\dfrac{2s-5}{s^2-9}$ ii) $\dfrac{(1+2s)^2}{s^3}$ iii) $\dfrac{1}{s^2+6s+13}$ iv) $\dfrac{3(s^2-2)^2}{2s^5}$

v) $\dfrac{3s-2}{s^{5/2}}-\dfrac{7}{3s+2}$ vi) $\dfrac{s+2}{s^2-4s+13}$ vii) $\dfrac{3s-8}{4s^2+25}$ viii) $\dfrac{3s^2+10s-6}{s^4}$

ix) $\ln\left(\dfrac{s+3}{s+5}\right)$ x) $\dfrac{1}{s}\tan^{-1}\left(\dfrac{1}{s}\right)$ xi) $\dfrac{1}{s^2(s+3)}$ xii) $\dfrac{1}{(2s+3)^{1/2}}$

8. Solve the given differential equations using the Laplace Transformation

i) $y'+y=\cos 2t,\ y(0)=1$ ii) $y''-6y'+9y=t^2e^{3t},\ y(0)=2, y'(0)=6$

iii) $y''+25y=0,\ y(0)=1, y(\pi/2)=-1$ iv) $y''+y=3,\ y(0)=1, y(\pi/2)=1$

9. Check the sufficient condition (piecewise continuity and exponential order) for the existence of the Laplace transform of the following functions:

i) $\sin t/t$ ii) e^t/t iii) $\sin t/t^3$ iv) e^{t^2} v) e^{-t^2}

<u>**P.T.O for Answers**</u>

Answers

1. i) $\dfrac{(e^{1-p}-1)}{1-p}$ ii) $\dfrac{4}{p}-\dfrac{e^{-p}}{p}$ iii) $\dfrac{8}{p^3}+\dfrac{3}{p^2+9}+\dfrac{1}{p-2}$ iv) $\dfrac{1}{p}+\dfrac{\sqrt{\pi}}{p^{3/2}}+\dfrac{3\sqrt{\pi}}{p^{1/2}}$

2. i) $\dfrac{24}{(p+3)^5}$ ii) $\dfrac{2(p+3)}{(p+3)^2+25}-\dfrac{15}{(p+3)^2+25}$ iii) $\dfrac{1}{(p+a)^n}$ iv) $\dfrac{1}{2}\left(\dfrac{3}{(p-4)^2+9}+\dfrac{1}{(p-4)^2+1}\right)$

3. i) $\dfrac{-2p^2+12p+8)}{(p^2+4)^2}$ ii) $\dfrac{2p(p^2-3a^2)}{(p^2+a^2)^3}$ iii) $\dfrac{2}{(p-3)^3}$ iv) $\dfrac{2(p-3)}{\left((p-3)^2+1\right)^2}$

4. i) $\ln\left(\dfrac{p+1}{p}\right)$ ii) $\dfrac{1}{2}\ln\left(1+\dfrac{1}{p^2}\right)$ iii) $\dfrac{1}{2}\ln\left(\dfrac{p+1}{p-1}\right)$ iv) $\dfrac{\pi}{2}-\tan^{-1}\left(p+1\right)$

5. i) $\dfrac{6}{p(p+2)^4}$ ii) $\dfrac{p^2+1}{p(p^2-1)^2}$ iii) $\dfrac{6}{\left(p(p+8)+25\right)^2}$ iv) $\dfrac{p}{p(p+10)+26}$, $p\left(\dfrac{\pi}{2}-\tan^{-1}\left(p\right)\right)-1$

7. i) $\dfrac{1}{6}\left(e^{3t}+11e^{-3t}\right)$ ii) $4+4t+\dfrac{t^2}{2}$ iii) $\dfrac{1}{2}e^{-3t}\sin 2t$ iv) $\dfrac{t^4}{4}-3t^2+\dfrac{3}{2}$

v) $\dfrac{-8t^{3/2}}{3\sqrt{\pi}}+\dfrac{6t^{1/2}}{\sqrt{\pi}}-\dfrac{7e^{-2t/3}}{3}$ vi) $e^{2t}\left(\cos 3t+\dfrac{4}{3}\sin 3t\right)$ vii) $\dfrac{3}{4}\cos\left(\dfrac{5t}{2}\right)-\dfrac{4}{5}\sin\left(\dfrac{5t}{2}\right)$

viii) $-t^3+5t^2+3t$ ix) $\dfrac{1}{t}\left(e^{-5t}-e^{-3t}\right)$ x) $\displaystyle\int_0^t \frac{\sin t}{t}\,dt$ xi) $\dfrac{1}{9}\left(e^{-3t}-1\right)+\dfrac{t}{3}$ xii) $\dfrac{e^{-3t/2}}{\sqrt{2\pi}\sqrt{t}}$

8. i) $\dfrac{1}{5}(4e^{-t}+\cos 2t+2\sin 2t)$ ii) $\dfrac{e^{3t}t^4}{12}+2e^{3t}$ iii) $(\cos 5t-\sin 5t)$ iv) $3-2\cos t-2\sin t$

9. Piecewise continuous: i), iv), v)
Exponential order: v)