Thapar Institute of Engineering and Technology, Patiala
School of Mathematics
Mathematics – II (UMA004): Tutorial Sheet 05

1. Use Convolution theorem to find the Inverse Laplace Transform of the following functions:

i) $\dfrac{6}{s(s+3)}$ ii) $\dfrac{1}{(s+1)^2(s+2)}$ iii) $\dfrac{s}{(s^2+4)(s^2+9)}$ iv) $\dfrac{s}{(s^2+\pi^2)^2}$

2. Obtain the Laplace Transform of following functions:

i) $(t-1)^2\,u(t-1)$ ii) $e^{-2t}\,u(t-3)$ iii) $4u(t-\pi)\cos t$ iv) $e^{-t}\sin t\; u(t-\pi)$

3. Express the following functions in terms of Unit Step Function and hence find its Laplace Transform

i) $f(t)=\begin{cases}1-e^{-t}, & 0<t<2\\ 0, & \text{otherwise}\end{cases}$ ii) $f(t)=\begin{cases}\sin\omega t, & 0<t<\pi/\omega\\ 0, & \text{otherwise}\end{cases}$

iii) $f(t)=\begin{cases}t/\alpha, & 0<t<\alpha\\ 1, & t>\alpha\end{cases}$ iv) $f(t)=\begin{cases}t^2, & 0<t<2\\ t-1, & 2<t<3\\ 7, & t>3\end{cases}$

4. Using shifting theorems to determine the Inverse Laplace Transform of the following functions:

i) $L^{-1}\left[\dfrac{e^{-as}}{s(s-2)}\right]$ ii) $L^{-1}\left[\dfrac{e^{-2s}}{(s-2)^2}\right]$ iii) $L^{-1}\left[\dfrac{e^{4-3s}}{(s+4)^{5/2}}\right]$ iv) $L^{-1}\left[\dfrac{3(1+e^{-s\pi})}{s^2+9}\right]$

5. Prove that

i) $L^{-1}\left[\dfrac{2s^2-1}{(s^2+1)(s^2+4)}\right]=-\sin(t)+\dfrac{3\sin(2t)}{2}$

ii) $L^{-1}\left[\dfrac{e^{-s\pi}}{s^2+1}\right]=-\sin(t)\;u_{\pi}(t)$

6. Solve the given differential equations using the Laplace Transform

i) $y''+16y=\cos 4t,\; y(0)=0, y'(0)=0$

ii) $y''+9y=\sin(3t),\; y(0)=1, y(\pi/2)=1$

iii) $y''-3y'+2y=u_1(t),\quad y(0)=1, y'(0)=1$

iv) $y''+9y=f(t)$, where $f(t)=\begin{cases}8\sin t, & 0<t<\pi\\ 0, & t>\pi\end{cases}$ and $y(0)=0, y'(0)=4$

v) $y''+y=\delta(t-\pi)-\delta(t-2\pi),\quad y(0)=0, y'(0)=1$

P.T.O for Answers

Answers

1. i) $2\left(1-e^{-3t}\right)$ ii) $(t-1)e^{-t}+e^{-2t}$ iii) $\dfrac{(\cos 2t-\cos 3t)}{5}$ iv) $\dfrac{t\sin \pi t}{2\pi}$

2. i) $\dfrac{2e^{-p}}{p^3}$ ii) $\dfrac{e^{-3(p+2)}}{p+2}$ iii) $\dfrac{-4pe^{-\pi p}}{p^2+1}$ iv) $\dfrac{-e^{-\pi(p+1)}}{p(p+2)+2}$

3. i) $\dfrac{1-e^{-2p}}{p}+\dfrac{e^{-2(p+1)}-1}{p+1}$ ii) $\dfrac{\omega}{p^2+\omega^2}\left(e^{-\pi p/\omega}+1\right)$ iii) $\dfrac{1}{p^2\alpha}\left(1-e^{-p\alpha}\right)$

iv) $\dfrac{2}{p^3}-e^{-2p}\,\dfrac{3p^2+3p+2}{p^3}+e^{-3p}\,\dfrac{5p-1}{p^2}$

4. i) $\dfrac{(e^{2(t-a)}-1)}{2}$ ii) $e^{2(t-2)}(t-2)u(t-2)$ iii) $e^4\left(\dfrac{4e^{-4(t-3)}(t-3)^{3/2}}{3\sqrt{\pi}}u(t-3)\right)$

iv) $\sin 3t\left(1-u(t-\pi)\right)$

6. i) $\dfrac{t\sin 4t}{8}$

ii) $(\cos 3t-\sin 3t)-\dfrac{t\cos 3t}{6}$

iii) $e^t+\left([1-2e^{t-1}+e^{2(t-1)}]u(t-1)\right)/2$

iv) $(\sin t+\sin 3t)-\left(\sin t-\dfrac{\sin 3t}{3}\right)u(t-\pi)$

v) $\sin t\left(1-u(t-\pi)-u(t-2\pi)\right)$