Anelastic and Plastic behavior

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Anelastic Deformation

Recoverable deformation that occur as a function of time.

Elastic deformation continues after the application of load due to some relaxation process within the material.

This is a time-dependent process.

$$X = 1 - e^{-t/t_r}$$

X = fraction of relaxation process that is completed till time 't'

t_r = relaxation time. 10^{-13} (atomic) to 10^{6} (grain boundaries sliding) seconds

What will happen if we apply load for a time duration

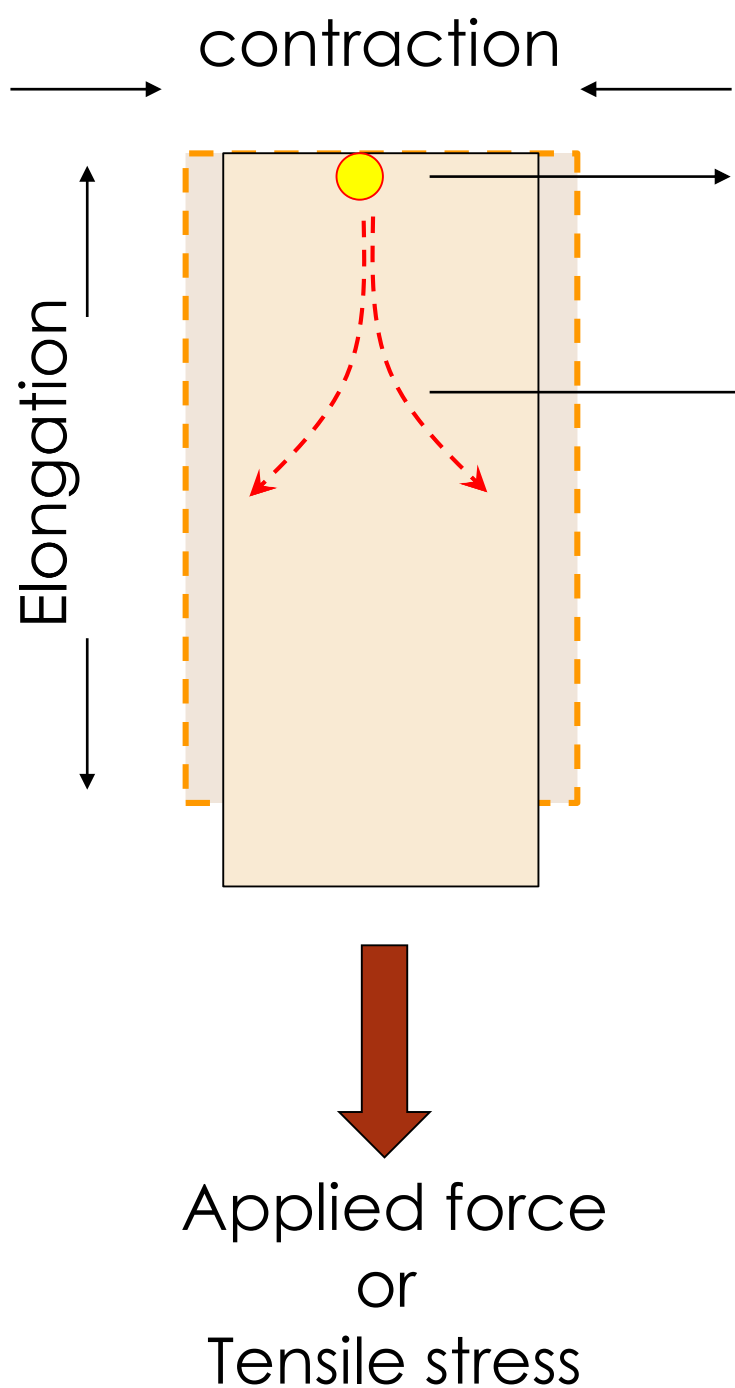
t << tr

t >>tr

or t ~ tr

Define relaxation time t_r.

Anelastic deformation



Carbon atom will experience stress

It will jump to places where stress is lower

- "Jumping" of carbon atom from one place to other place is via diffusion.
- Diffusion is a time dependent process, usually ~100 s at room temperature.

What will happen if we apply load for

(i) 2 s

(ii) 10,000 s

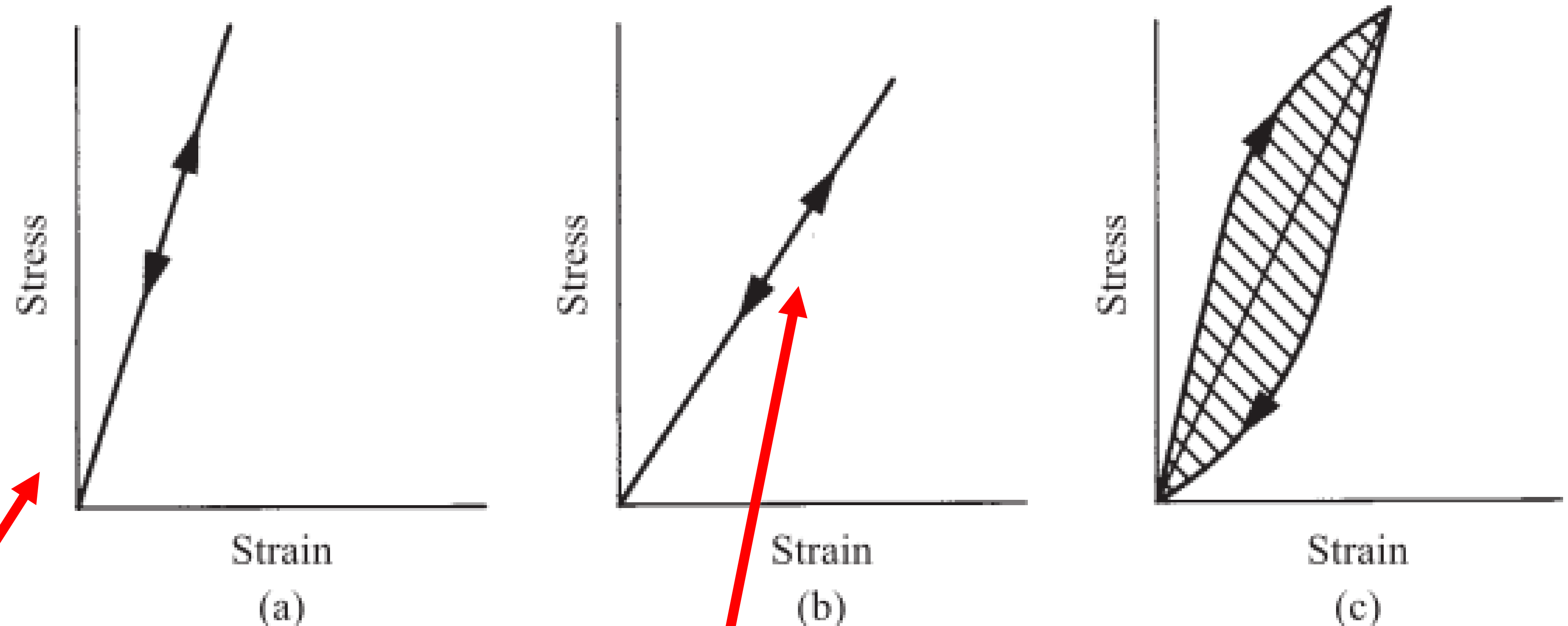
(iii) 120 s

Anelastic deformation

$t \ll t_r$ — 2 sec

$t \gg t_r$ — 10000 sec

$t \sim t_r$ — 120 sec



Strain due to bond stretching only

Strain due to bond stretching + carbon atom redistribution

Higher strain
Strain due to applied force + strain due to jump of carbon atoms

Relaxation process

1. Strongly temperature dependent.
2. Temperature variation varies the relaxation time t_r.
3. Decreases with increase in temperature.

Damping Capacity

Fraction of energy dissipated

1. Materials with good damping capacity are required to make base pads of machines to damp out the vibrations.

Materials with good damping capacity : grey cast iron, polymer, rubber.

2. Materials with low damping capacity are used in temple bells and piezoelectric device.

Energy loss during a cycle is frequency dependent.

In common tyres the energy loss is high at speed ~ 1000 kmph.

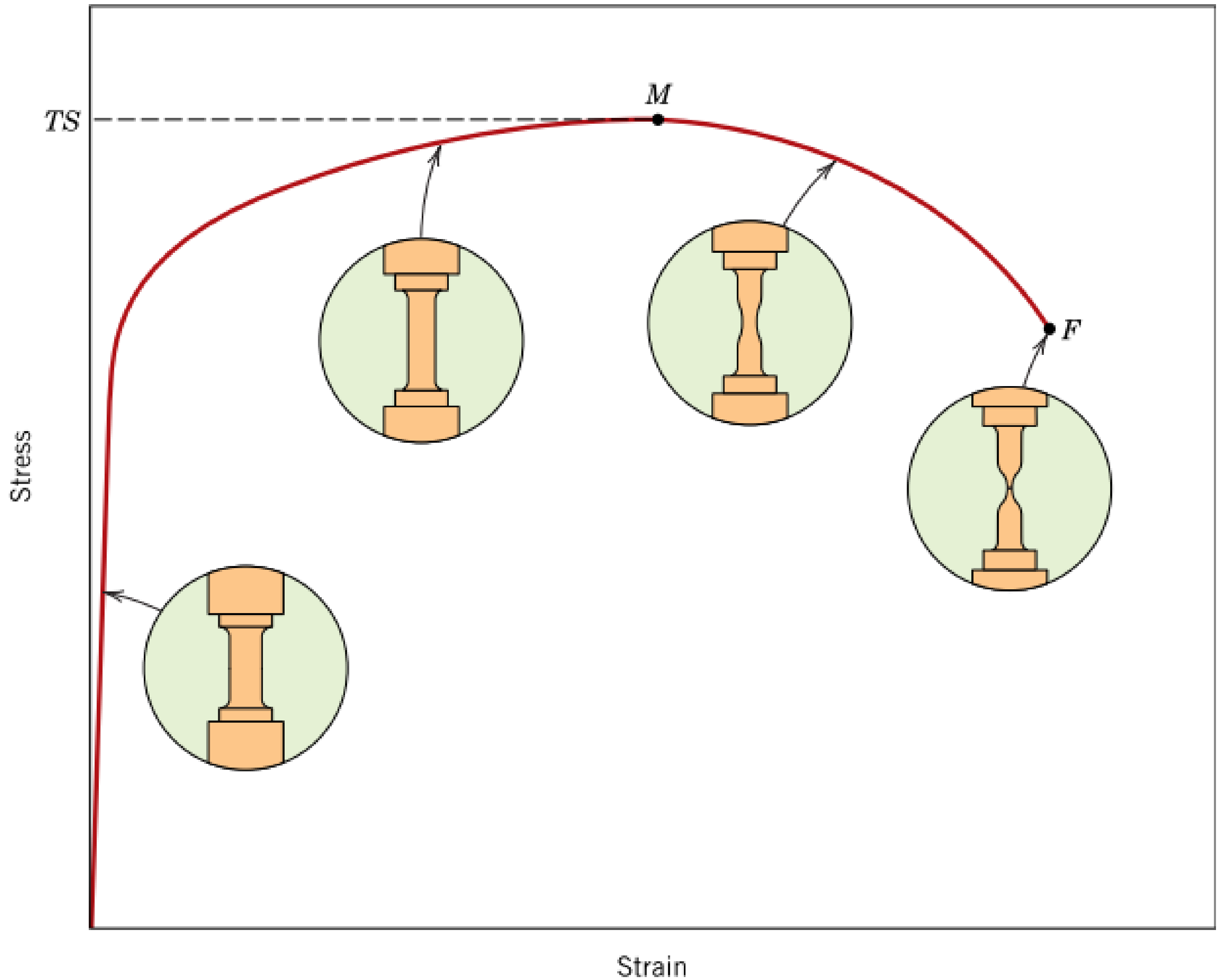
Plastic deformation

This is a permanent deformation or change in shape of a solid body without fracture under the action of a sustained force.

In Plastic deformation

1. Stress is not proportional to strain
2. Deformation is not reversible
3. Deformation occurs by breaking a re-arrangement of atomic bonds

Typical stress-strain curve



After point M, stress decreases.

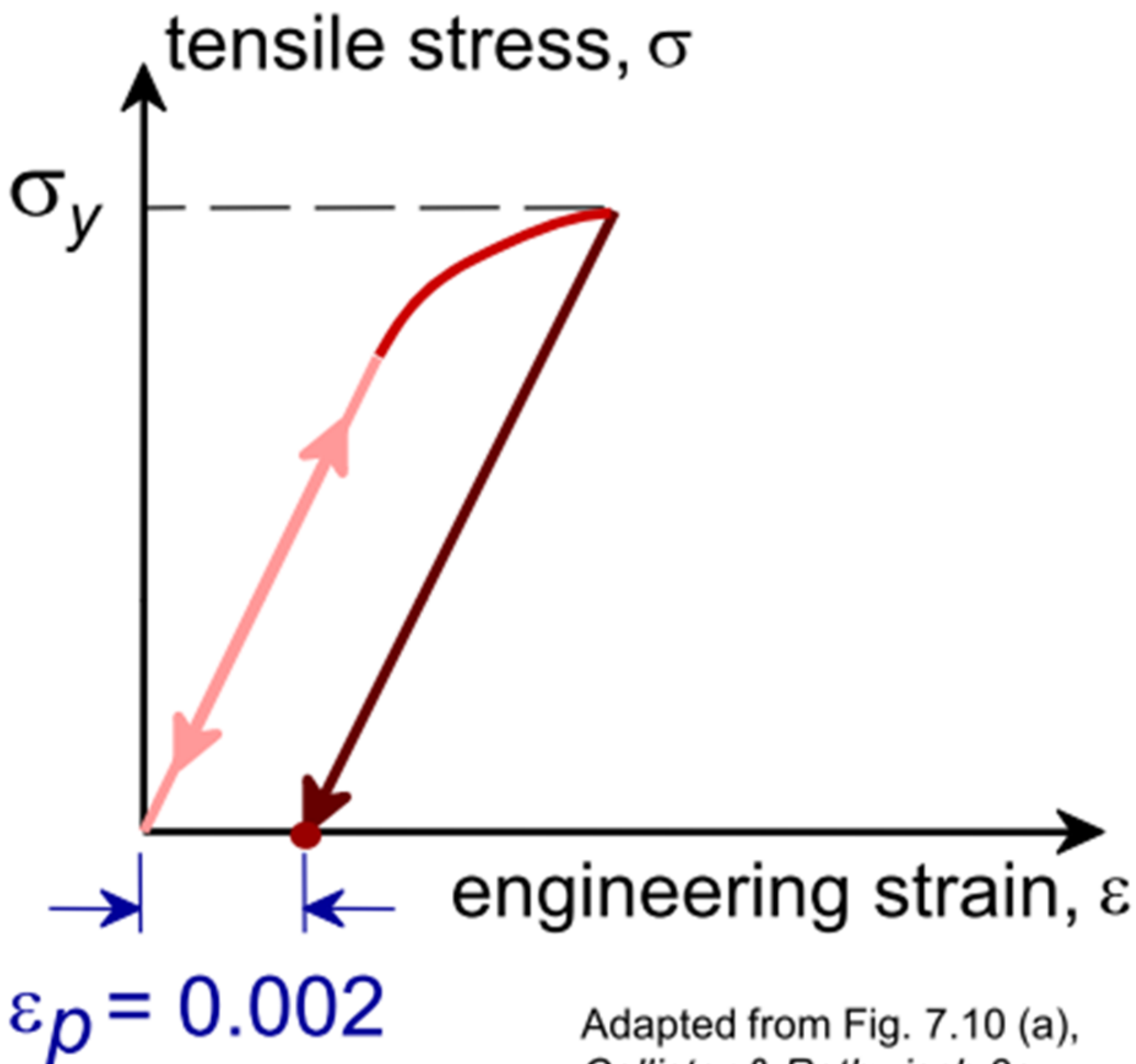
Does this mean that the material is becoming weaker?

Necking leads to smaller cross sectional area!

Yield Strength

Stress at which noticeable plastic deformation occurred.

when $\varepsilon_p = 0.002$



Adapted from Fig. 7.10 (a), *Callister & Rethwisch 3e.*

σ_y = yield strength

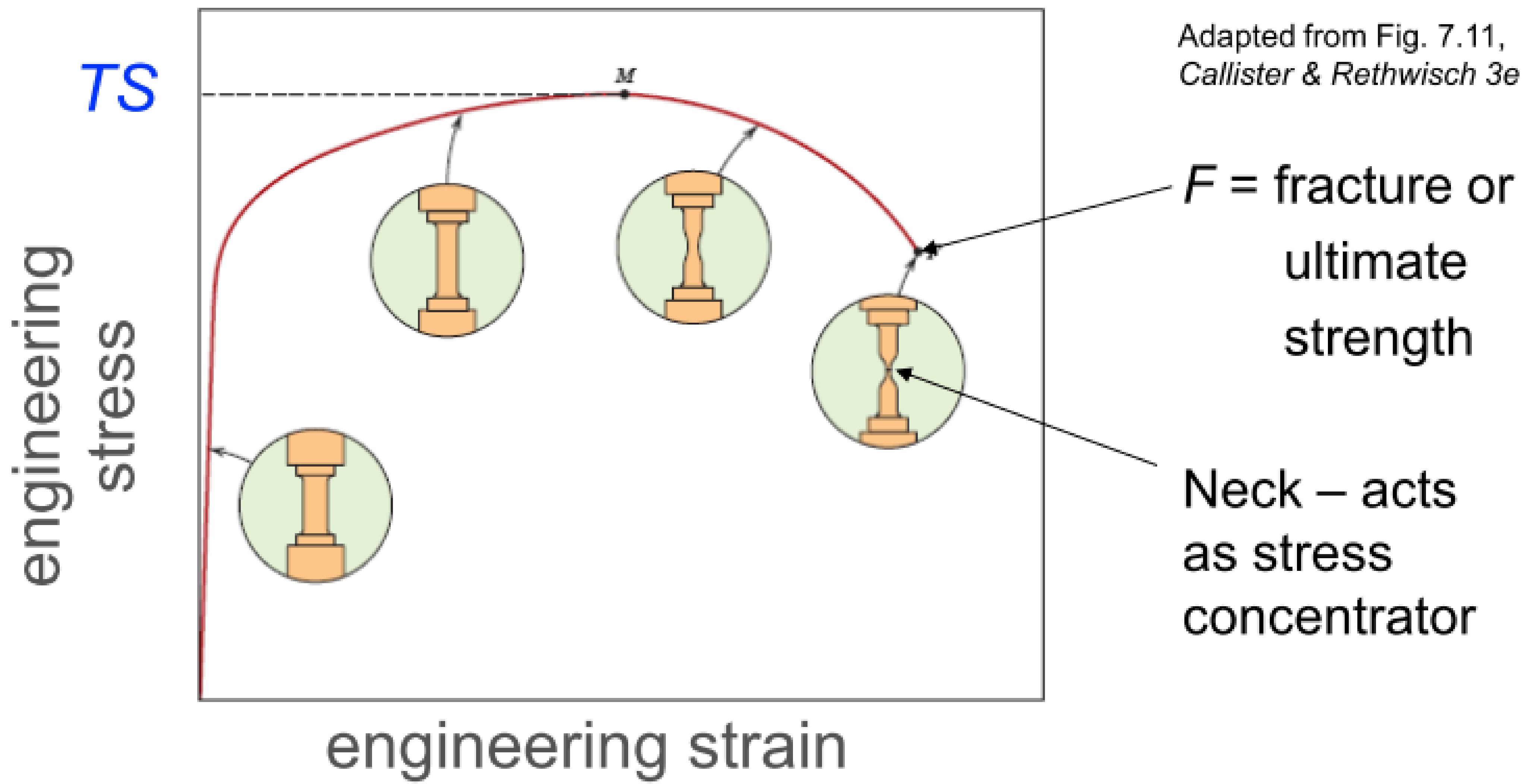
Note: for 2 inch sample

$\varepsilon = 0.002 = \Delta z/z$

$\therefore \Delta z = 0.004$ in

Ultimate Tensile Strength

Maximum stress on engineering stress-strain curve



Adapted from Fig. 7.11, *Callister & Rethwisch 3e*

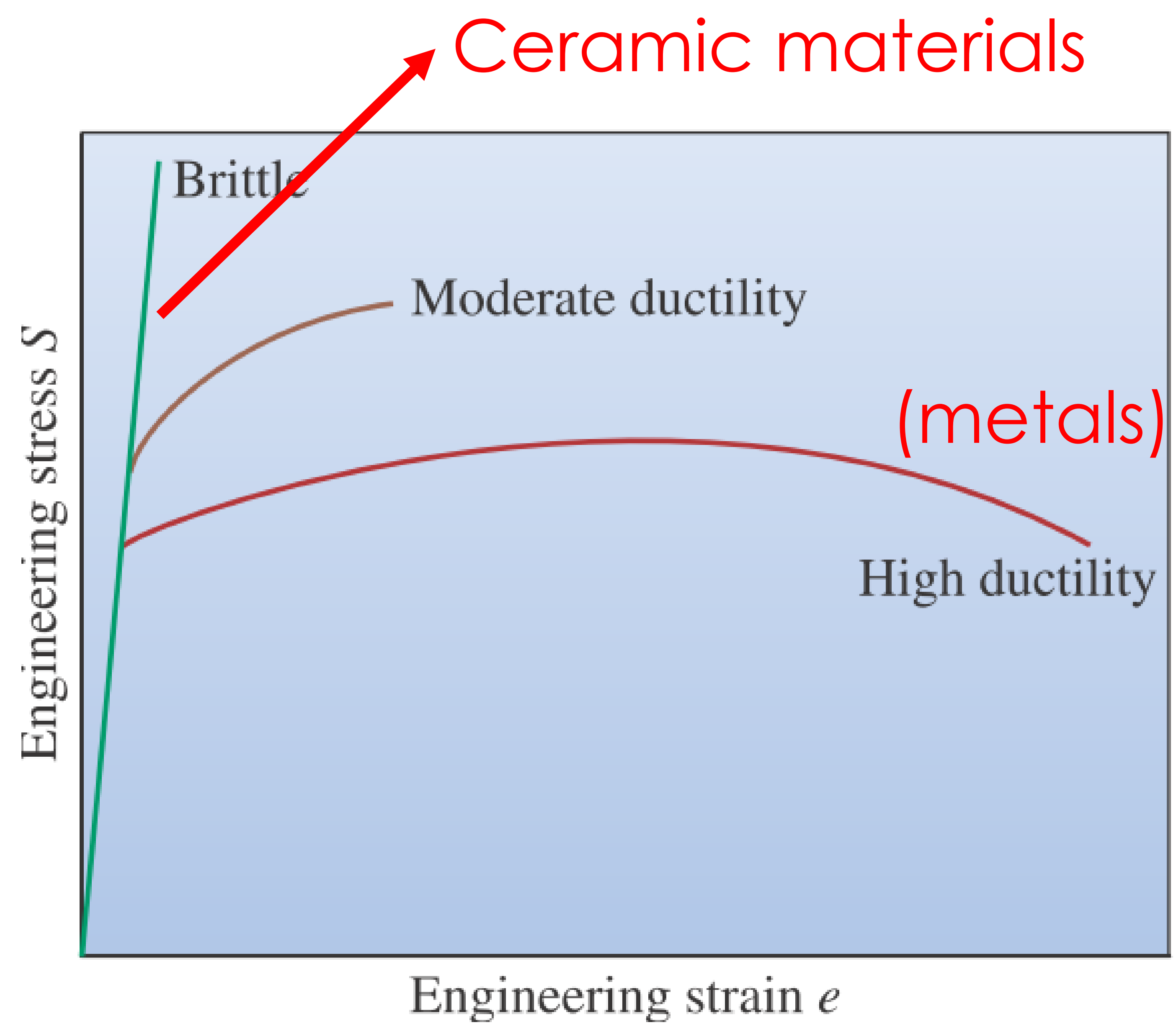
- Metals: occurs when noticeable necking starts.
- Polymers: occurs when polymer backbone chains are aligned and about to break.

Ductility

Measure of degree of plastic deformation that has been sustained at fracture.

Ductile materials can undergo significant plastic deformation before fracture.

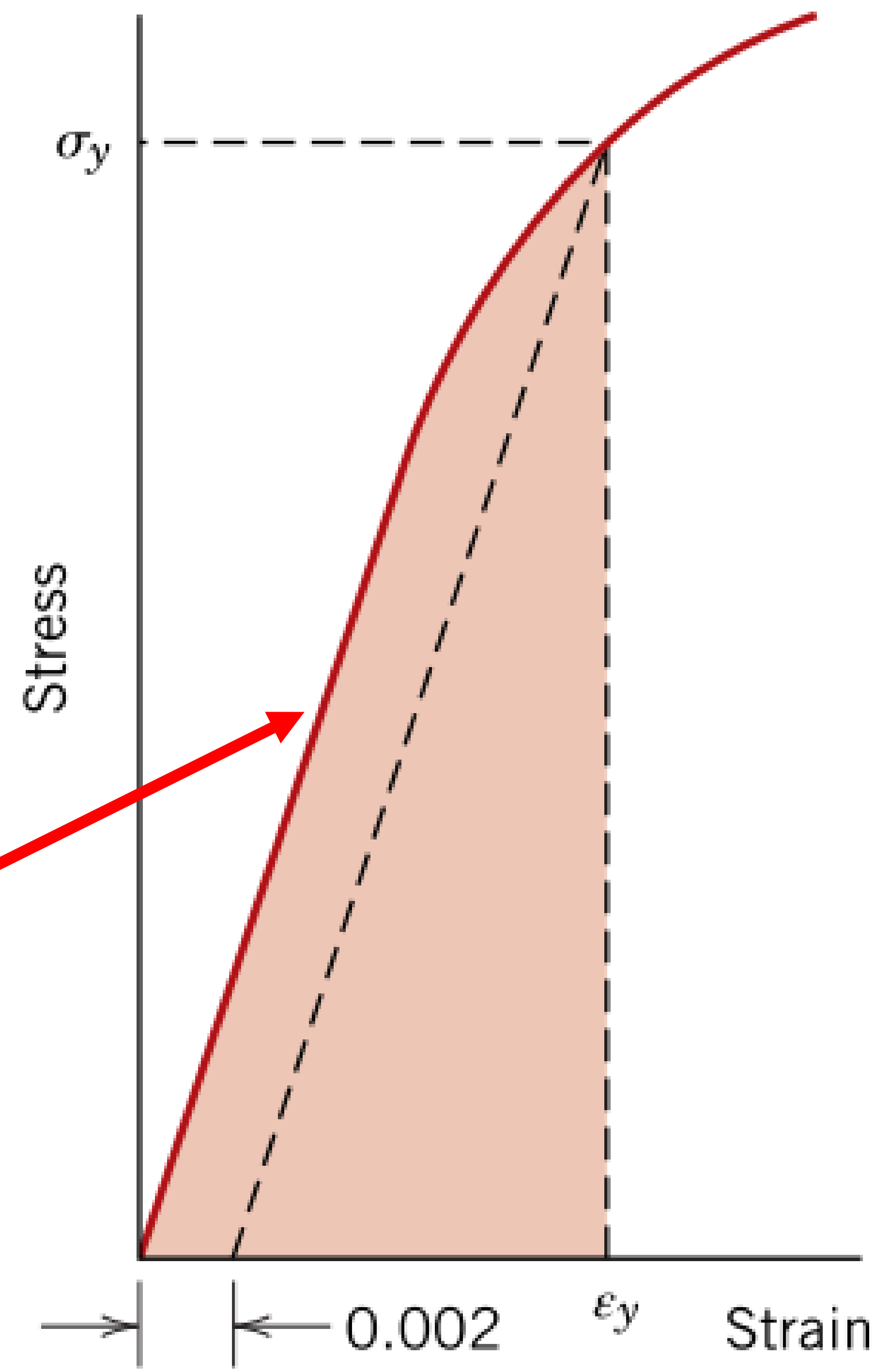
Brittle materials can tolerate only very small plastic deformation.



Stiffness

Stiffness is the extent to which an object resists deformation in response to an applied force. The complementary concept is flexibility or pliability.

The more flexible an object is, the less stiff it is.

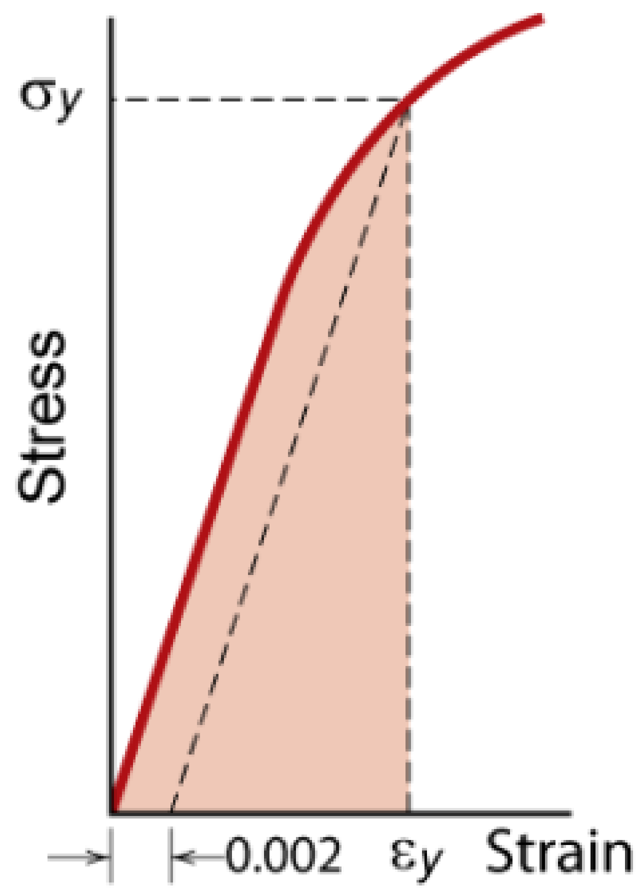


Slope = Stiffness

Resilience

Ability of a material to store energy

Energy stored best in elastic region



$$U_r = \int_0^{\varepsilon_y} \sigma d\varepsilon$$

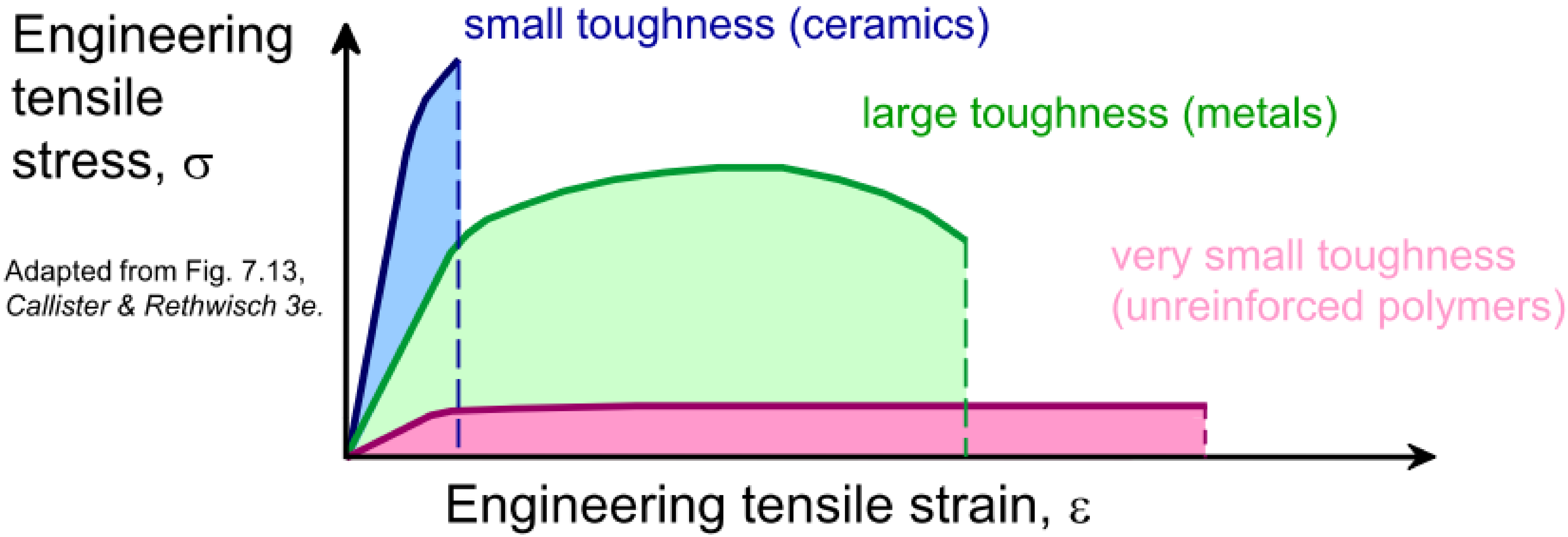
If we assume a linear stress-strain curve this simplifies to

$$U_r \cong \frac{1}{2}\sigma_y \varepsilon_y$$

Toughness

Energy to break a unit volume of a material

Approximate by the area under the stress-strain curve.



Adapted from Fig. 7.13, *Callister & Rethwisch 3e.*

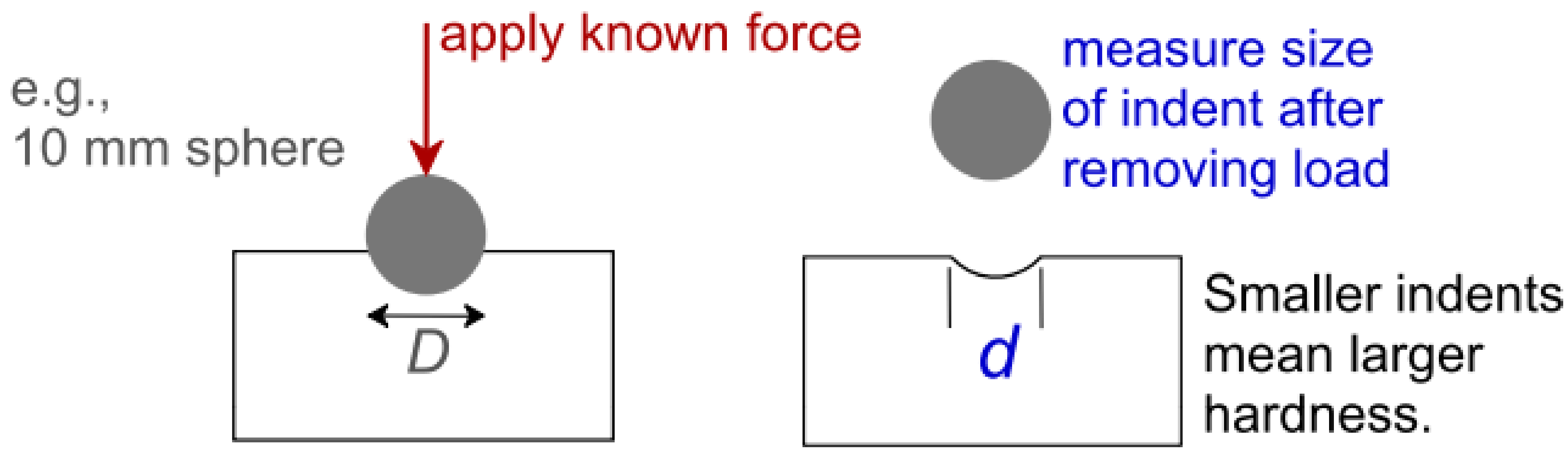
Brittle fracture: elastic energy
Ductile fracture: elastic + plastic energy

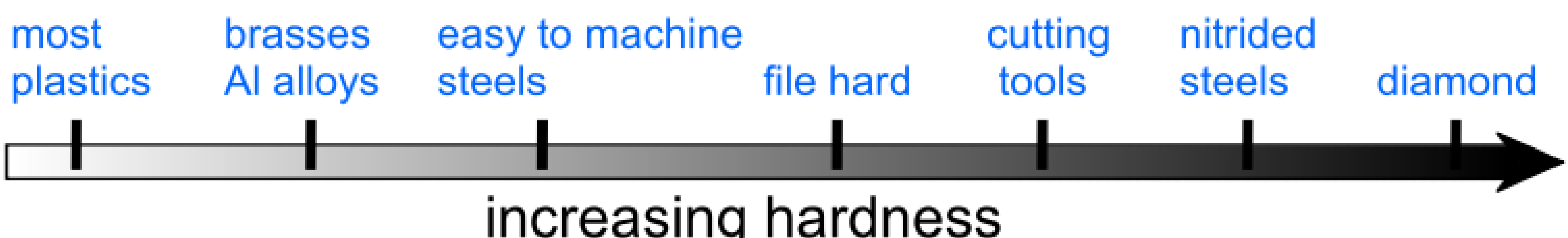
Hardness

Resistance to permanent deformation on the surface.

Large hardness means

- Resistance to plastic deformation or cracking in the compression.
- Better wear properties.





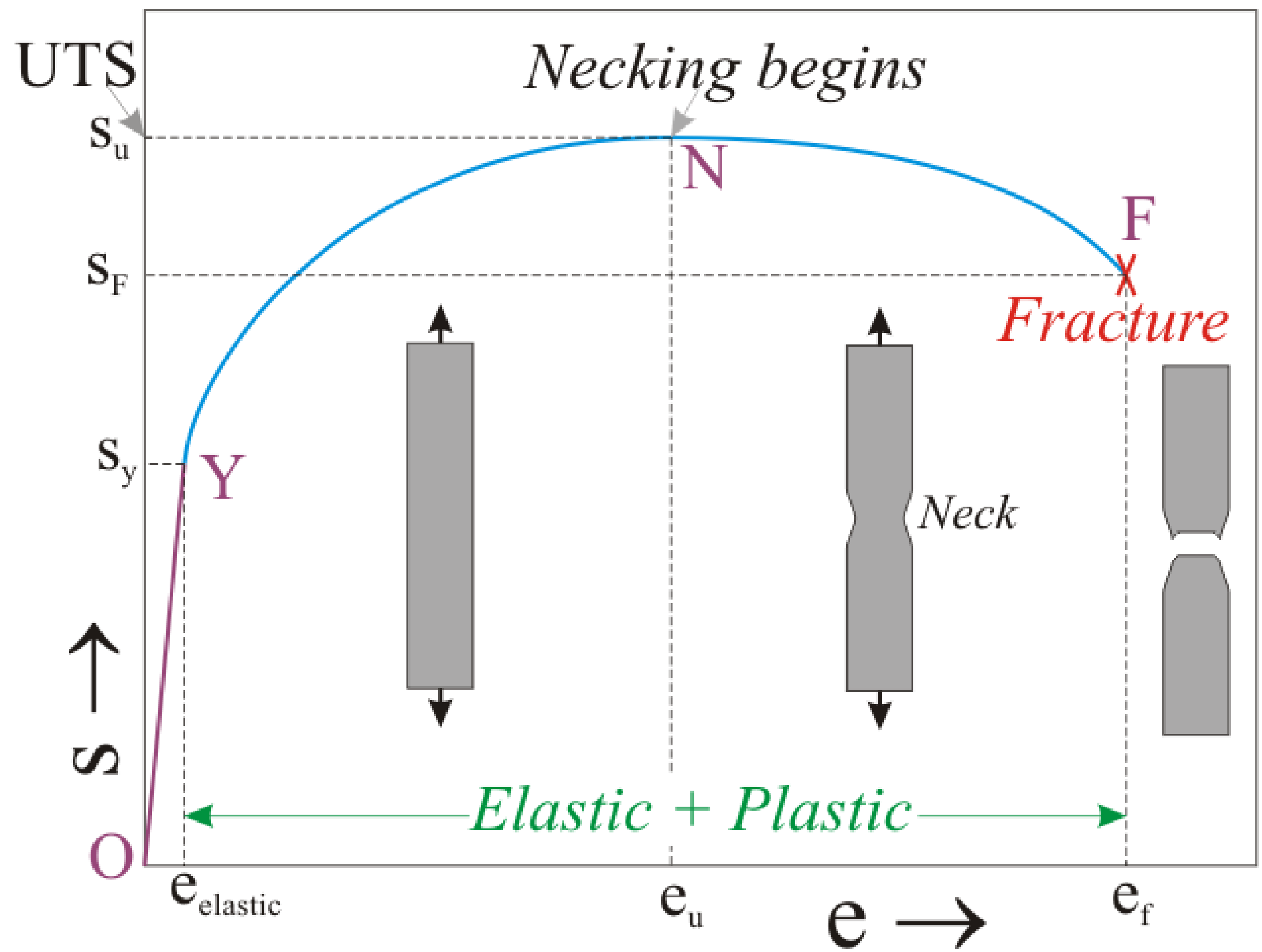
Engineering Stress – Engineering Strain

Engineering stress

$$\sigma(S) = \frac{F}{A_0} = \frac{\text{Applied stress}}{\text{Initial cross} - \text{sectional area}}$$

Engineering strain

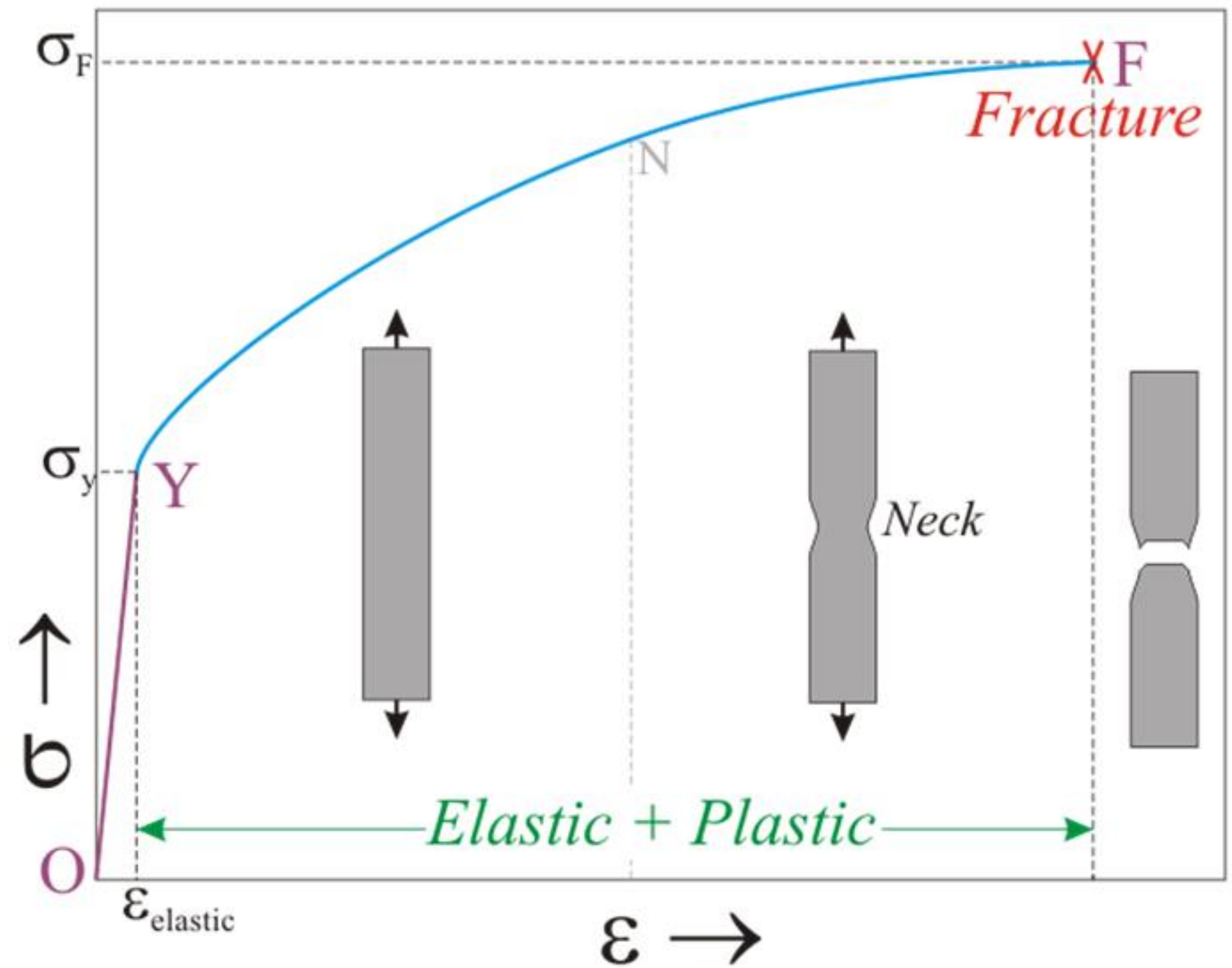
$$\epsilon(e) = \frac{\Delta l}{l_0} = \frac{\text{Change in length}}{\text{Original length}}$$



True Stress – True Strain

True stress $$\sigma_T = \frac{F}{A_i} = \frac{\text{Applied stress}}{\text{Actual (instantaneous)area}}$$

True strain $$\epsilon_T = \int_{l_0}^{l} \frac{dl}{l}$$



Relationship in engineering and true stress-strain

True stress $\sigma_T = \frac{F}{A_i} = \frac{\text{Applied stress}}{\text{Actual (instantaneous)area}}$

True strain $\epsilon_T = \int_{l_0}^{l} \frac{dl}{l}$

For an applied force F,

cross-sectional area = A_i

change in length = $l_i - l_o$

$$\therefore \boldsymbol{\epsilon_T = \int_{l_0}^{l_i} \frac{dl}{l} = \ln\frac{l_i}{l_0} = \ln(1+\epsilon)}$$

$$\boldsymbol{\sigma_T = \frac{F}{A_i} = \frac{F}{\frac{A_0 l_0}{l_i}} = \frac{F}{A_0}\frac{l_i}{l_0} = \sigma(1+\epsilon)}$$

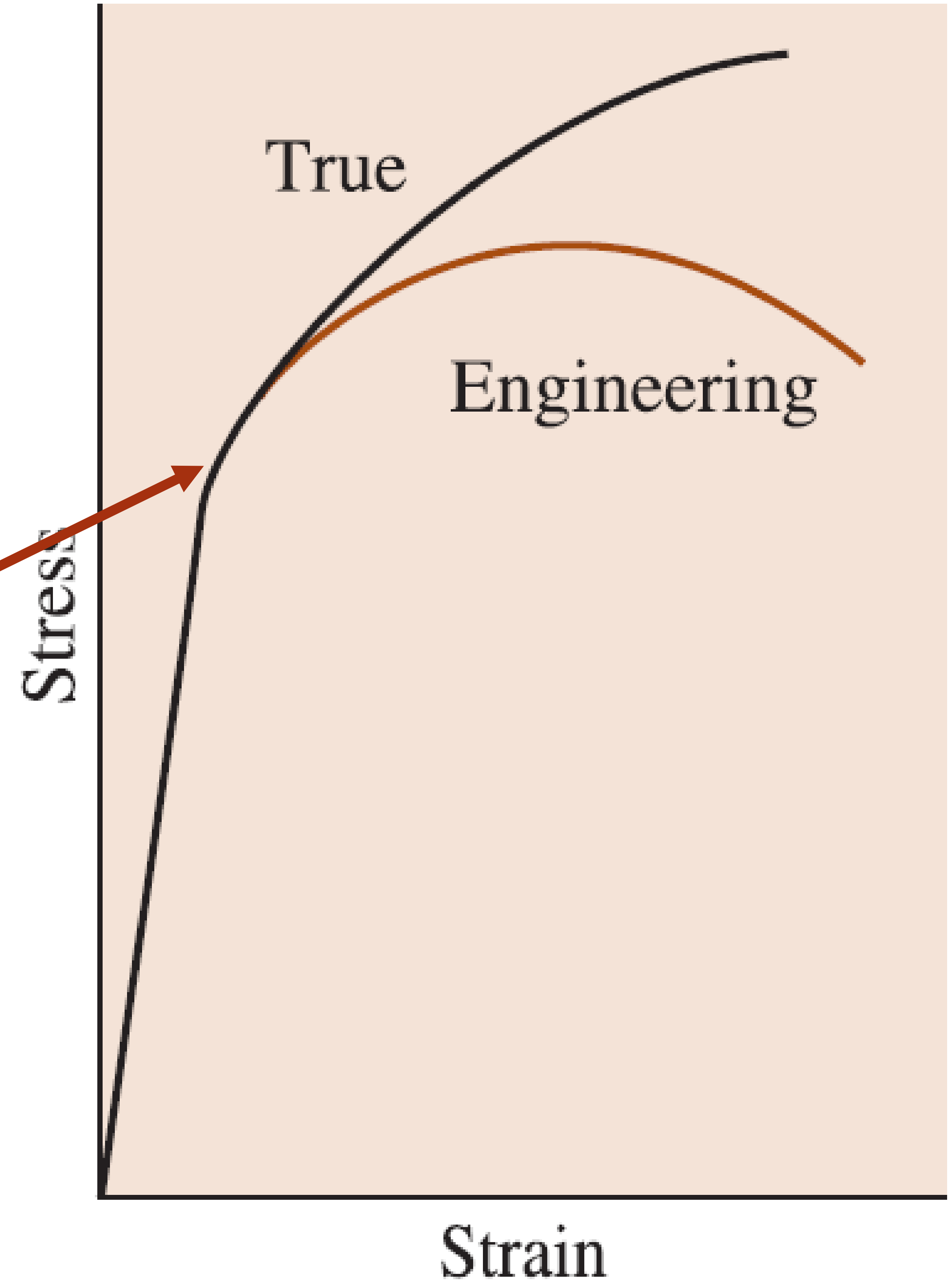
Assuming that volume remains same, i.e., $A_0 l_0 = A_i l_i$

Engineering strain

$$\epsilon = \frac{l_i - l_0}{l_0} = \frac{l_i}{l_0} - 1$$

$$\Rightarrow 1 + \epsilon = \frac{l_i}{l_0}$$

Engineering and true stress strain curve

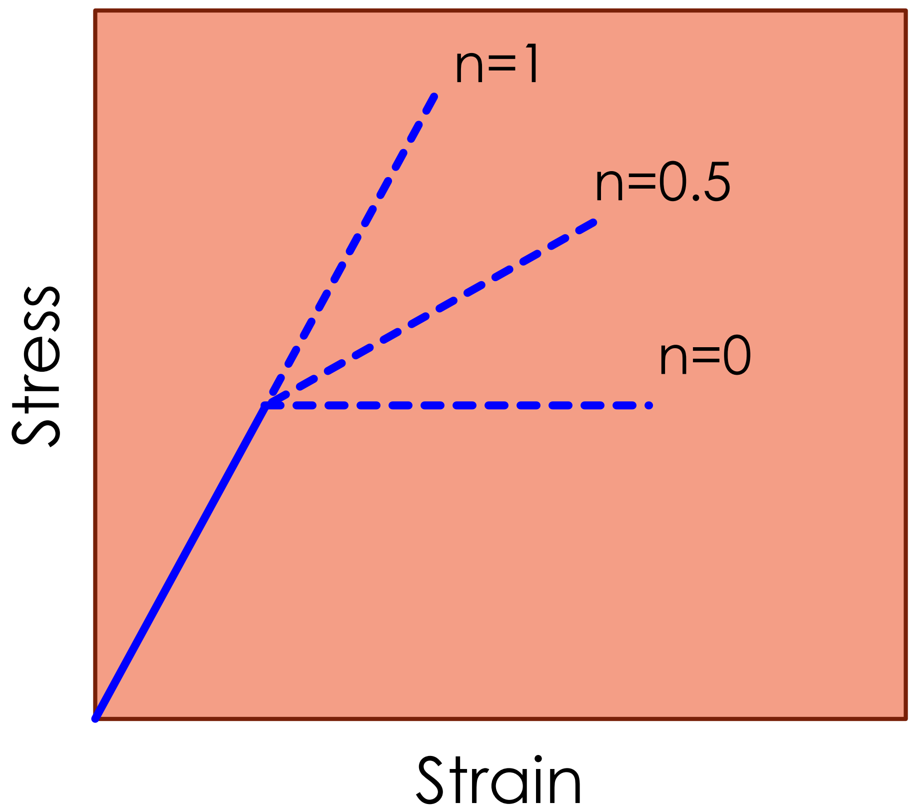


Similar till yield point

Work hardening: time independent

$$\sigma_T = K\epsilon_T^n$$

K = strength coefficient
n = work hardening exponent
n = 0.5 → Large plastic strain (copper and brass)
n = 0.15 → less strain (heat treated steel)



Material	n	K MPa	K psi
Low-carbon steel (annealed)	0.21	600	87,000
4340 steel alloy (tempered at 315°C)	0.12	2650	385,000
304 stainless steel (annealed)	0.44	1400	205,000
Copper (annealed)	0.44	530	76,500
Naval brass (annealed)	0.21	585	85,000
2024 aluminum alloy (heat-treated—T3)	0.17	780	113,000
AZ-31B magnesium alloy (annealed)	0.16	450	66,000

Work hardening: time dependent

$$\sigma_T = A\left(\frac{d\epsilon}{dt}\right)^m$$

A = Constant
m = strain rate sensitivity

m = 0 → stress is strain rate independent
m = 0.2 for common metals
m = 0.4 – 0.9 → super-plastic behavior (fine grained steel)
- material can deform without necking
m=1 → material behave like viscous liquid

Summary

1. Anelastic process is a time dependent elastic deformation process.
2. For may practical applications the relaxation time should be larger than the load unload cycle.
3. Relaxation is temperature dependent process.
4. True stress-strain curve is the best to find out the mechanical properties of materials.