MST Solution of UCS 405 (Sep. 2018)

1.a. A - i) $P(\phi) = \{\phi\}$

ii) $P(\{\phi\}) = \{\phi, \{\phi\}\}$

iii) $P(P(\phi)) = \cancel{\{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}} = \{\phi, \{\phi\}\}$

iv) $\{\phi\} \times P(\phi) = \{(\phi, \phi)\}$

v) $\phi \times P(\phi) = \phi$

vi) $P(\phi) \times P(\phi) = \{(\phi, \phi)\}$

1.b. A - i) $1+1=3$ F Yes, it is a proposition.

ii) $(A \cup B) \subseteq C$ T Yes, it is a proposition.

iii) $A \wedge B$ No, it is not a proposition.

iv) $(8+22)^3/10^2$ No, it is not a proposition.

v) $(B \wedge C) \in 9$ No, it is not a proposition.

vi) C is an infinite set F Yes, it is a proposition.

2.a. A -

	Irreflexive	Symmetric	Antisymmetric	Asymmetric
R_1	Y	Y	N	N
R_2	N	N	Y	N

Explanation:-

i) R_1 is irreflexive because for all $a \in N$, $a = a$ thus $(a, a) \notin R_1$

ii) R_1 is symmetric because for all $a, b \in N$, if $a \neq b$ then $b \neq a$ i.e., if $(a, b) \in R_1$ then $(b, a) \in R_1$.

iii) R_1 is not ~~anti~~asymmetric because there exist $a, b \in N$, $a \neq b$ and $b \neq a$.

iv) R_1 is not Antisymmetric because there exist different a and b in N such that $a \neq b$ and $b \neq a$.

(2)

i) R_2 is not irreflexive because it is reflexive.

for all $a \in N$, $\frac{a}{a} = 1 = 2^0$, thus $(a, a) \in R$

ii) R_2 is not symmetric because if $(a, b) \in R_2$, then $\frac{a}{b} = 2^i$, where $i \geq 0$ but $\frac{b}{a} = 2^{-i}$, where $-i \leq 0$. Therefore $(b, a) \notin R_2$.

iii) R_2 is antisymmetric. if $(a, b) \in R_2$ and $(b, a) \in R_2$

We have $\frac{a}{b} = 2^i$ and

$\frac{b}{a} = 2^j$ where $i, j \geq 0$. Then

$$\frac{a}{b} \times \frac{b}{a} = 1 = 2^{i+j}$$

Thus $i + j = 0$. since $i, j \geq 0$, we have $i = j = 0$.

Therefore $\frac{a}{b} = 1$ and hence $a = b$.

iv) R_2 is not asymmetric. Because if we let $a = b$, we can have both (a, b) and (b, a) in R_2.

2. b) $((A \land B \to C) \to (A \lor B \to C))$

A-

A	B	C	$A \land B$	$A \lor B$	$A \land B \to C$	$A \lor B \to C$	$(A \land B \to C) \to (A \lor B \to C)$
F	F	F	F	F	T	T	T
F	F	T	F	F	T	T	T
F	T	F	F	T	T	F	F
F	T	T	F	T	T	T	T
T	F	F	F	T	T	F	F
T	F	T	F	T	T	T	T
T	T	F	T	T	F	F	T
T	T	T	T	T	T	T	T

DNF: $(\neg A \land \neg B \land \neg C) \lor (\neg A \land \neg B \land C) \lor (\neg A \land B \land C)$
$\lor (A \land \neg B \land C) \lor (A \land B \land \neg C) \lor (A \land B \land C)$

CNF: $(A \lor \neg B \lor C) \land (\neg A \lor B \lor C)$

3. a)

A) Prime Factors of $70 = 2, 5$ and 7

Let U be the set of all integers from 1 through 150.

i.e. $U = \{1, 2, ---, 150\}$

Let A be the subset of U consisting of all integers that are divisible by 2, let B be the subset of U consisting of all integers that are divisible by 5 and let C be the suset of U consisting of all integers that are divisible by 7.

$n(U) = 150$

$\therefore \quad n(A) = \left\lfloor \frac{150}{2} \right\rfloor = 75$

$n(B) = \left\lfloor \frac{150}{5} \right\rfloor = 30$

$n(C) = \left\lfloor \frac{150}{7} \right\rfloor = 21$

$A \cap B$ = subset of U consisting of all integers that are divisible by both 2 and 5.

$B \cap C$ = subset of U consisting of all integers that are divisible by both 5 and 7.

$A \cap C$ = subset of U consisting of integers that are divisible by both 2 and 7.

$A \cap B \cap C$ = subset of U consisting of integers that are divisible by 2, 5 and 7.

$\therefore \quad n(A \cap B) = \left\lfloor \frac{150}{2 \times 5} \right\rfloor = 15$, $n(B \cap C) = \left\lfloor \frac{150}{5 \times 7} \right\rfloor = 4$, $n(A \cap C) = \left\lfloor \frac{150}{2 \times 7} \right\rfloor = 10$

$n(A \cap B \cap C) = \left\lfloor \frac{150}{2 \times 5 \times 7} \right\rfloor = 2$

By Inclusion-Exclusion Principle,

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$= 75 + 30 + 21 - 15 - 4 - 10 + 2$

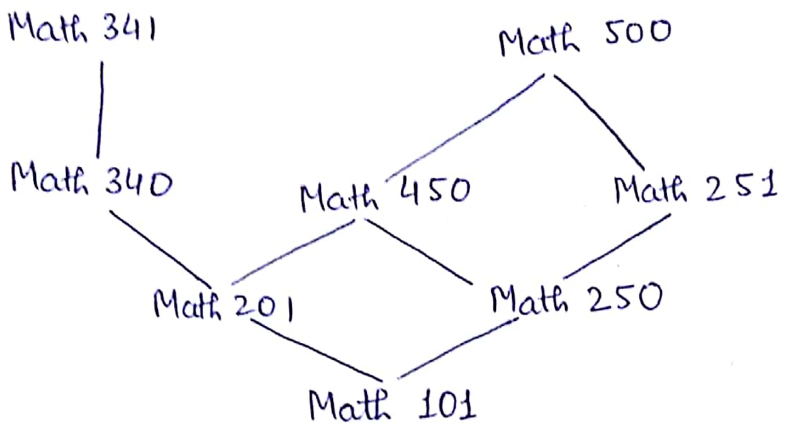
or, $n(A \cup B \cup C) = 99$

Thus, the number of integers n with $1 \le n \le 150$ that are relatively prime to 70

$n(\overline{A \cup B \cup C}) = n(U) - n(A \cup B \cup C) = 150 - 99 = 51$

Ans. 3(b)

(i)



Hasse Diagram for the partial Ordering C of given classes

(ii) Minimal element of C = Math 101

Maximal element of C = Math 341, Math 500

Ans. 4(a)

$A = \{x, y, z\}$

$f: A \times A$, $g: A \times A$

$f = \{(x,y), (y,z), (z,x)\}$

$g = \{(x,y), (y,x), (z,z)\}$

(i) $f \circ g$

$(f \circ g)(x) = f(g(x)) = f(y) = z$

$(f \circ g)(y) = f(g(y)) = f(x) = y$

$(f \circ g)(z) = f(g(z)) = f(z) = x$

$\therefore f \circ g = \{(x,z), (y,y), (z,x)\}$

(ii) $g^{-1}(x) = y$, $g^{-1}(y) = x$, $g^{-1}(z) = z$

$\therefore g^{-1} = \{(y,x), (x,y), (z,z)\}$

⑥

4. b) A - We must find positive integers C and k such that for all $x \geq k$,

$$\frac{3x^4 - 2x}{5x-1} \leq Cx^3$$

To make the fraction $\frac{3x^4 - 2x}{5x-1}$ larger, we can do two things, make the numerator larger and make the denominator smaller:

$$\frac{3x^4 - 2x}{5x-1} \leq \frac{3x^4}{5x-1} \leq \frac{3x^4}{5x-x} = \frac{3x^4}{4x} = \frac{3}{4}x^3$$

In the first step we made the numerator larger and in the second step we made the denominator smaller by subtracting x, not 1. Note that the first inequality requires $x \geq 0$ and second inequality requires $x \geq 1$.

Therefore, if $x \geq 1$

$$\frac{3x^4 - 2x}{5x-1} \leq \frac{3}{4}x^3$$

and hence $\frac{3x^4 - 2x}{5x-1} = O(x^3)$

Ans. 4(c)

Let P: Claghorn has wide support.
Q: Claghorn will be asked to run for the senate.
R: Claghorn yells "Eureka" in Iowa.

Premises:

$P \rightarrow Q$

$R \rightarrow \neg Q$

R

Conclusion: $\neg P$

	Steps	Reasons
1.	$P \rightarrow Q$	Premise
2.	$\neg Q \rightarrow \neg P$	~~Premise~~ Contrapositive
3.	$R \rightarrow \neg Q$	Premise
4.	R	Premise
5.	$\neg Q$	Modus Ponens 3,4
6.	$\neg P$	Modus Ponens 2,5

Therefore, the conclusion follows logically from the premises.