UCS405 (Discrete Mathematical Structures)

Tutorial Sheet-5 (Relations)

1. Let A = {1, 2, 3, 4}, and R is a relation defined by "a divides b". Write R as a set of ordered pair, draw directed graph.

2. List the ordered pairs in the relation R from A = {0, 1, 2, 3, 4} to B = {0, 1, 2, 3}, where (a, b) ∈ R if and only if
   a) a = b. b) a + b = 4.
   c) a > b. d) b divides a
   e) gcd(a, b) = 1. f) lcm(a, b) = 2.

3. Let A = {1, 2, 3, 4}, give an example of a mapping which is (i) neither symmetric nor antisymmetric, (ii) anti-symmetric and reflexive but not transitive, (iii) transitive and reflexive but not anti-symmetric.

4. For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.
   a) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}
   b) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}
   c) {(2, 4), (4, 2)}
   d) {(1, 2), (2, 3), (3, 4)}
   e) {(1, 1), (2, 2), (3, 3), (4, 4)}
   f) {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}

5. Let R1 = {(1, 2), (2, 3), (3, 4)} and R2 = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)} be relations from {1, 2, 3} to {1, 2, 3, 4}. Find
   a) R1 ∪ R2. b) R1 ∩ R2.
   c) R1 − R2. d) R2 − R1.

6. Let R be the relation {(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)}, and let S be the relation {(2, 1), (3, 1), (3, 2), (4, 2)}. Find S ◦ R.

7. List the 16 different relations on the set {0, 1}.
   I. How many of the 16 different relations on {0, 1} contain the pair (0, 1)?
   II. Which of the 16 relations on {0, 1}, which are
      a) reflexive? b) irreflexive?
      c) symmetric? d) antisymmetric?
      e) asymmetric? f) transitive?

8. Let R be the relation on the set {1, 2, 3, 4, 5} containing the ordered pairs (1, 1), (1, 2), (1, 3), (2, 3), (2, 4), (3, 1), (3, 4), (3, 5), (4, 2), (4, 5), (5, 1), (5, 2), and (5, 4). Find
   a) R^2. b) R^3. c) R^4. d) R^5.