

Roll Number: _____

Thapar Institute of Engineering & Technology, Patiala

Department of Computer Science and Engineering

MID SEMESTER EXAMINATION

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| B. E. (Second Year): Semester-III (2018/19) (COE) | Course Code: UCS405 Course Name: Discrete Mathematical Structures |
| September 25 th , 2018 | Time: 10:30 A.M. - 12:30 P.M. |
| Time: 2 Hours, M. Marks: 25 | Name of Faculty: Dr. Manju, Dr. Smita Agrawal, Mr. Ashish Girdhar, Mr. Shatrughan Modi, Ms. Urvashi |

Note: Attempt all questions in a proper sequence with justification.
Assume missing data, if any, suitably.

Q1(a) List the elements of the following sets: (3)

- $P(\emptyset)$
- $P(\{\emptyset\})$
- $P(P(\emptyset))$
- $\{\emptyset\} \times P(\emptyset)$
- $\emptyset \times P(\emptyset)$
- $P(\emptyset) \times P(\emptyset)$

Q1(b) Let A, B and C are three sets such that (3)

$$A = \{1, \dots, 10\}, B = \{3, 7, 11, 12\}, C = \{0, 1, \dots, 20\}$$

Which of the following are propositions?

- $1 + 1 = 3$
- $(A \cup B) \subseteq C$
- $A \cap B$
- $(8 + 22)^3 / 10^2$
- $(B \cap C) \in 9$
- C is an infinite set

Q2(a) Let N be the set of all positive integers, and R_1, R_2 be two relations on N defined as follows: (4)

- Definition of R_1 : For all $a, b \in N$,
 $(a, b) \in R_1$ iff $a \neq b$.
- Definition of R_2 : For all $a, b \in N$,
 $(a, b) \in R_2$ iff $\frac{a}{b} = 2^i$ for some integer $i \geq 0$

Mark the following table properly (Y/N) to indicate the properties of the relations R_1 and R_2 have along with the explanation:

| | Irreflexive | Symmetric | Antisymmetric | Asymmetric |
|-------|-------------|-----------|---------------|------------|
| R_1 | | | | |
| R_2 | | | | |

Q2(b) Using truth table find CNF and DNF for $((A \wedge B \rightarrow C) \rightarrow (A \vee B \rightarrow C))$. (3)

Q3(a) Using Inclusion-Exclusion principle, calculate the number of integers n with $1 \leq n \leq 150$ which are relatively prime to 70. (2)

Q3(b) Prerequisites in college is a familiar partial ordering of available classes. We write $A < B$ if course A is a prerequisite for course B . Let C be the ordered set consisting of the mathematics courses and their prerequisites appearing in table given below: (2+1)

- Draw the Hasse diagram for the partial ordering C of these classes.
- Find all minimal and maximal elements of C .

| Class | Prerequisites |
|----------|--------------------|
| Math 101 | None |
| Math 201 | Math 101 |
| Math 250 | Math 101 |
| Math 251 | Math 250 |
| Math 340 | Math 201 |
| Math 341 | Math 340 |
| Math 450 | Math 201, Math 250 |
| Math 500 | Math 450, Math 251 |

Q4(a) With $A = \{x, y, z\}$, let $f, g: A \times A$ be given by $f = \{(x, y), (y, z), (z, x)\}$ and $g = \{(x, y), (y, x), (z, z)\}$. (2)

Determine each of the following functions. Write your answers as a collection of ordered pairs.

- $f \circ g$
- g^{-1}

Q4(b) Use the definition of big-O to prove that $\frac{3x^4 - 2x}{5x - 1} = O(x^3)$. (2)

Q4(c) Consider (3)

Premises: If Claghorn has wide support, then he'll be asked to run for the senate. If Claghorn yells "Eureka" in Iowa, he will not be asked to run for the senate. Claghorn yells "Eureka" in Iowa.

Conclusion: Claghorn does not have wide support.

Determine whether the conclusion follows logically from the premises. Explain by representing the statements symbolically and using rules of inference.