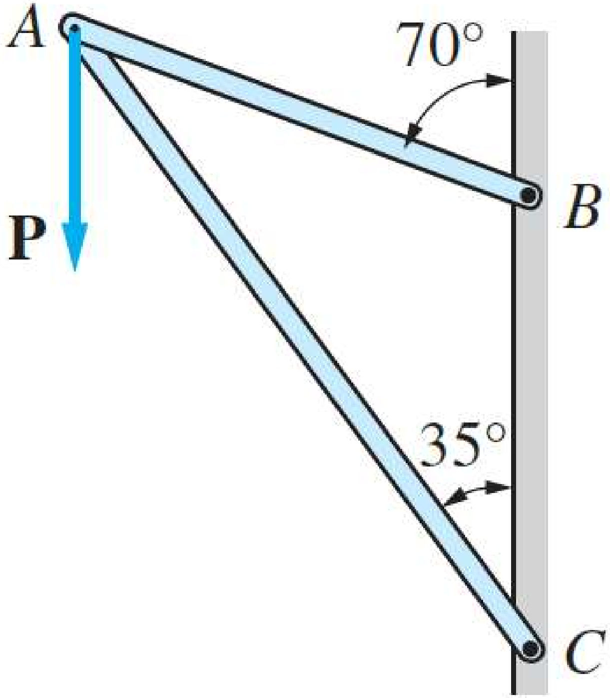
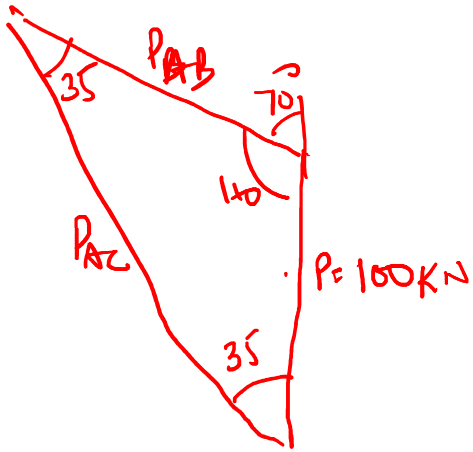
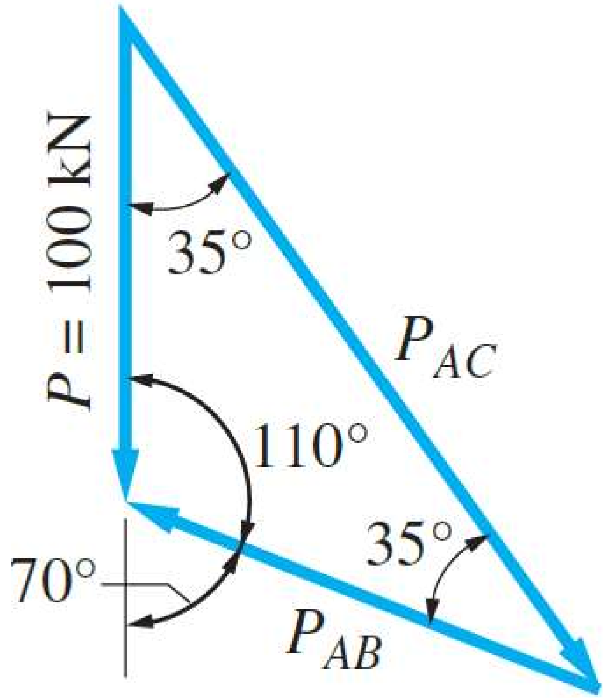
Mechanics: Fundamental Properties of Vectors

Illustration: The vertical force P of magnitude 100 kN is applied to the frame shown in Fig. Resolve P into components that are parallel to the members AB and AC of the frame.







Mechanics: Fundamental Properties of Vectors

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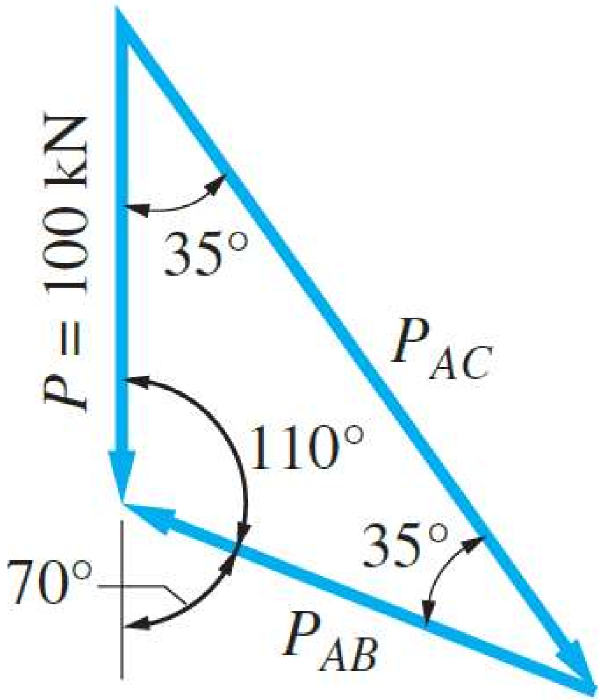


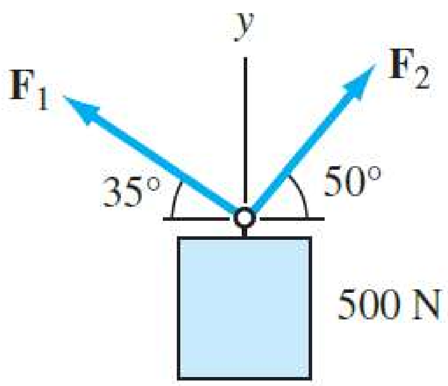
Applying the law of sines to the triangle, we obtain

$$\frac{100}{\sin 35^\circ} = \frac{P_{AB}}{\sin 35^\circ} = \frac{P_{AC}}{\sin 110^\circ}$$

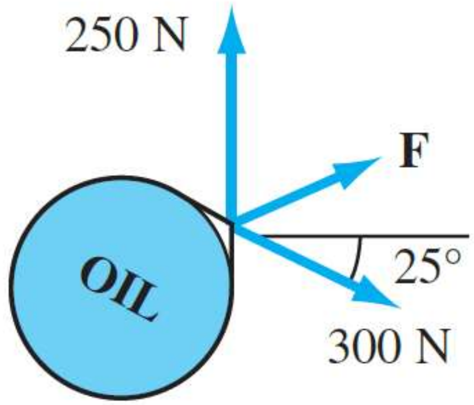
$$\boxed{P_{AB} = 100.0\,\text{kN} \qquad P_{BC} = 163.8\,\text{kN}}$$

Self Exercise

The 500-N weight is supported by two cables, the cable forces being **F**1 and **F**2 as shown in Fig. Knowing that the resultant of **F**1 and **F**2 is a force of magnitude 500N acting in the *y*-direction, determine *F*1 and *F*2.



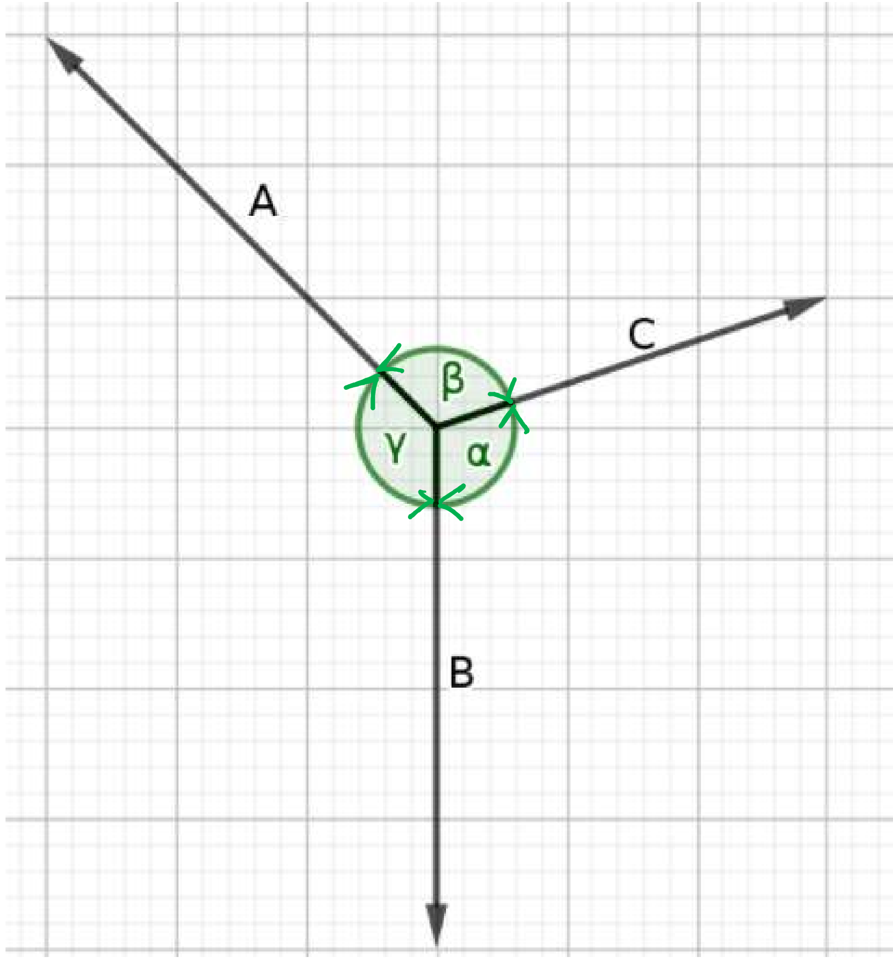
To move the oil drum, the resultant of the three forces shown must have a magnitude of 500 N. Determine the magnitude and direction of the *smallest* force **F** that would cause the drum to move.



Lami's Theorem

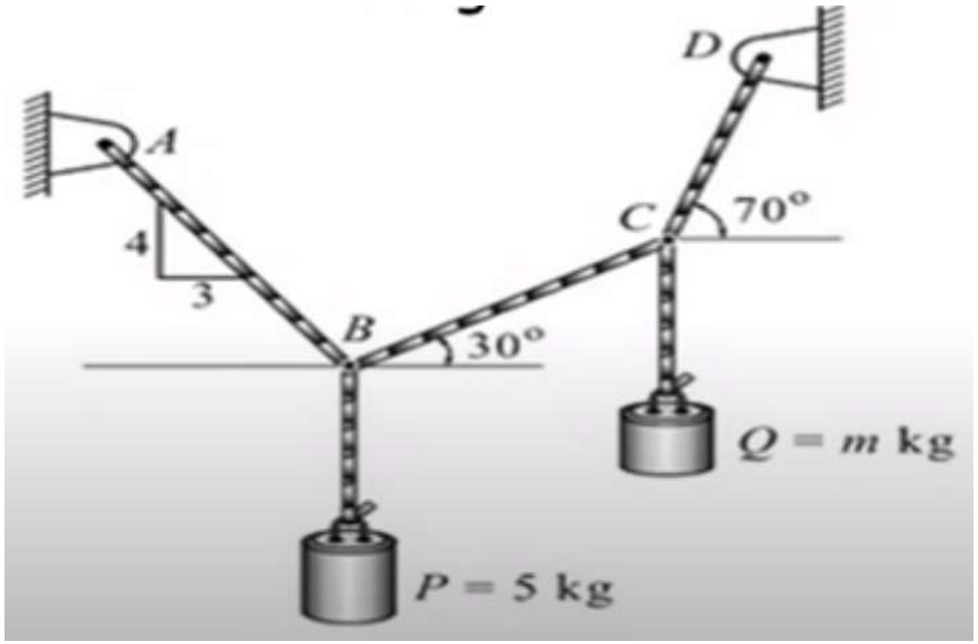
If three coplanar, concurrent forces are in equilibrium then the ratio of each force and the sine of included angle between the other two are constant.

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

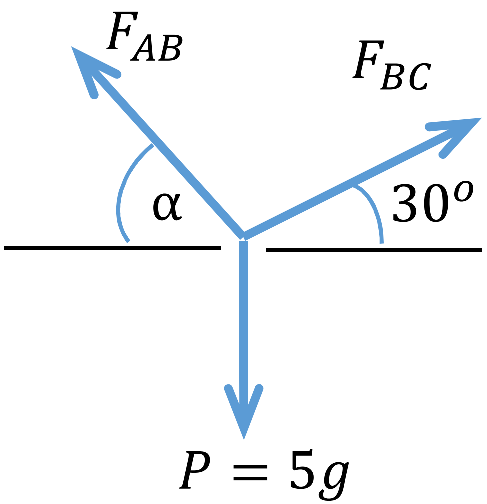


Lami's Theorem

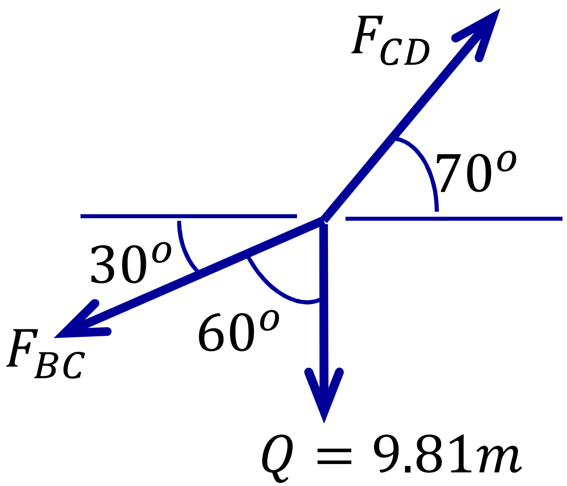
Illustration: The two blocks ***P*** and ***Q*** suspended through a cord ***ABCD*** are in equilibrium. Determine the mass of the block ***Q*** if mass of the block ***P*** is 5 kg.



$$\boxed{tan \propto = \left({}^{4}/_{3}\right) \rightarrow \alpha = 53.13°}$$

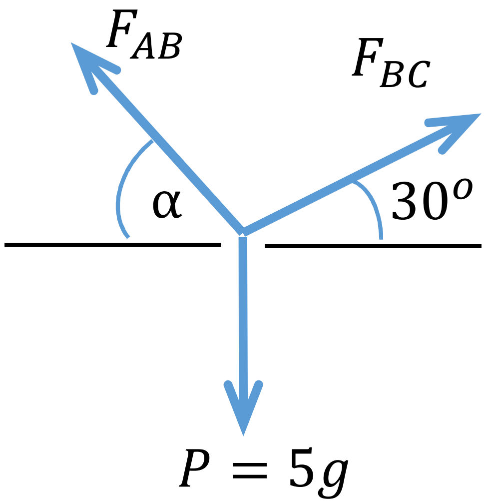


FBD at B



FBD at C

Lami's Theorem

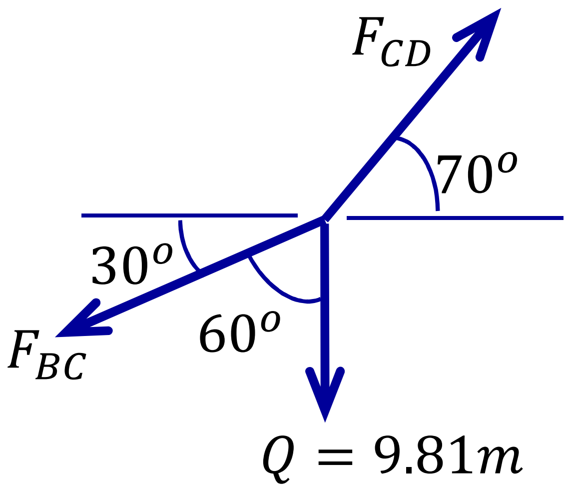


FBD at B

$$\frac{F_{AB}}{\sin 120} = \frac{F_{BC}}{\sin 143.13} = \frac{5 \times 9.81}{\sin 96.87}$$

$$\boxed{F_{BC} = 29.64N}$$

FBD at C



$$\frac{F_{CD}}{\sin 60} = \frac{F_{BC}}{\sin 160} = \frac{9.81m}{\sin 140}$$

$$\boxed{\boldsymbol{m = 5.68kg}}$$

Decomposition of Forces in a Plane

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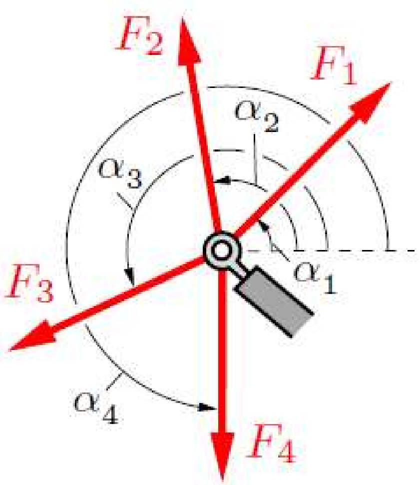
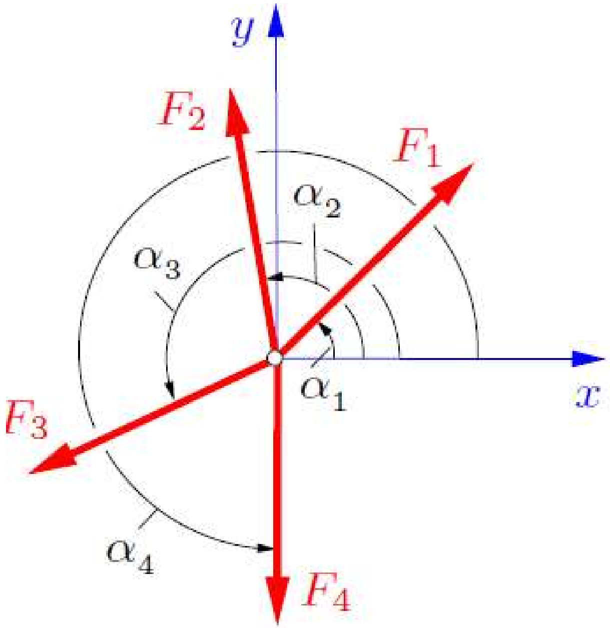
Illustration: An eyebolt is subjected to four forces (F_1 = 12 kN, F_2 = 8 kN, F_3 = 18 kN, F_4 = 4 kN) that act at the given angles (α_1 = 45°, α_2 = 100°, α_3 = 205°, α_4 = 270°) with respect to the horizontal. Determine the magnitude and direction of the resultant.





$$
\begin{aligned}
R_x &= F_{1x} + F_{2x} + F_{3x} + F_{4x} \\
&= F_1 \cos\alpha_1 + F_2 \cos\alpha_2 + F_3 \cos\alpha_3 + F_4 \cos\alpha_4 \\
&= 12\cos 45^\circ + 8\cos 100^\circ + 18\cos 205^\circ + 4\cos 270^\circ \\
&= -9.22\,\mathrm{kN}
\end{aligned}
$$

$$
\begin{aligned}
R_y &= F_{1y} + F_{2y} + F_{3y} + F_{4y} \\
&= F_1 \sin\alpha_1 + F_2 \sin\alpha_2 + F_3 \sin\alpha_3 + F_4 \sin\alpha_4 \\
&= 4.76\,\mathrm{kN}
\end{aligned}
$$

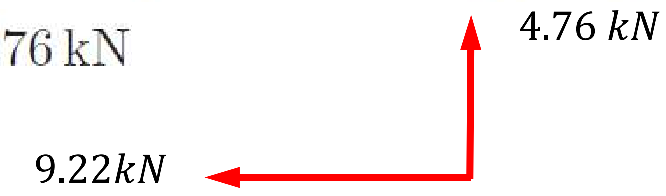
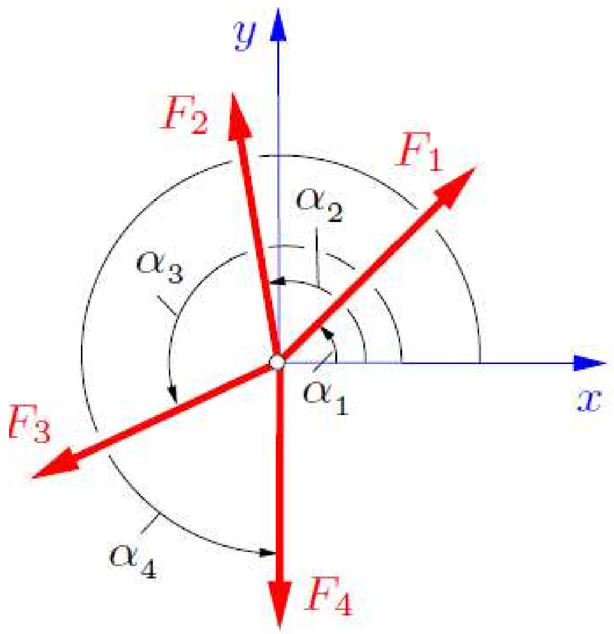
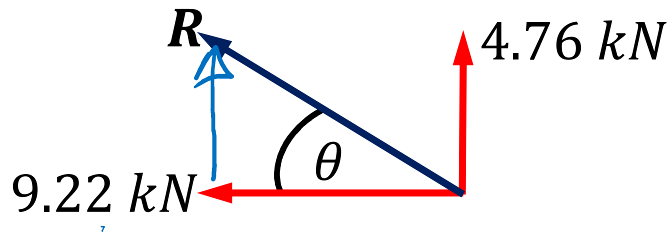


Illustration: An eyebolt is subjected to four forces (F_1 = 12 kN, F_2 = 8 kN, F_3 = 18 kN, F_4 = 4 kN) that act at the given angles (α_1 = 45◦, α_2 = 100◦, α_3 = 205◦, α_4 = 270◦) with respect to the horizontal. Determine the magnitude and direction of the resultant.



$$R = \sqrt{{R_x}^2 + {R_x}^2} = \sqrt{(-9.22)^2 + (4.76)^2}$$

$$R = 10.38\ kN$$



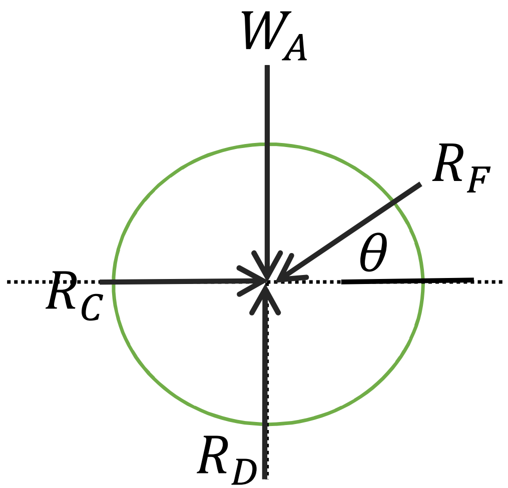
$$\tan\theta = \frac{R_y}{R_x} \quad \rightarrow \quad \theta = tan^{-1}\left(\frac{4.76}{9.22}\right)$$

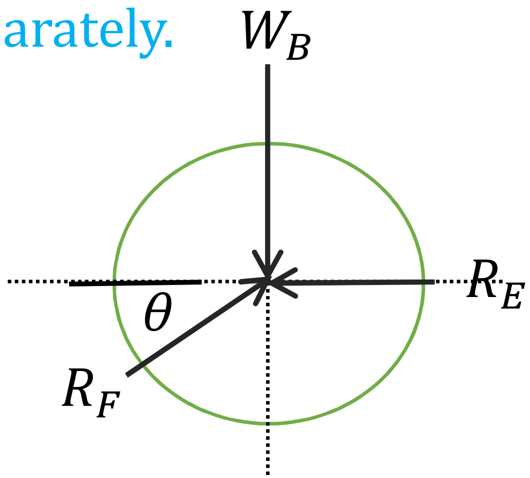
$$\theta = 27.30^{o} \text{ with horizontal}$$

Illustration: Two spherical balls each of radius 20 cm and weight 200 N are kept between two vertical walls 60 cm apart. Determine the reactions at all contact points.

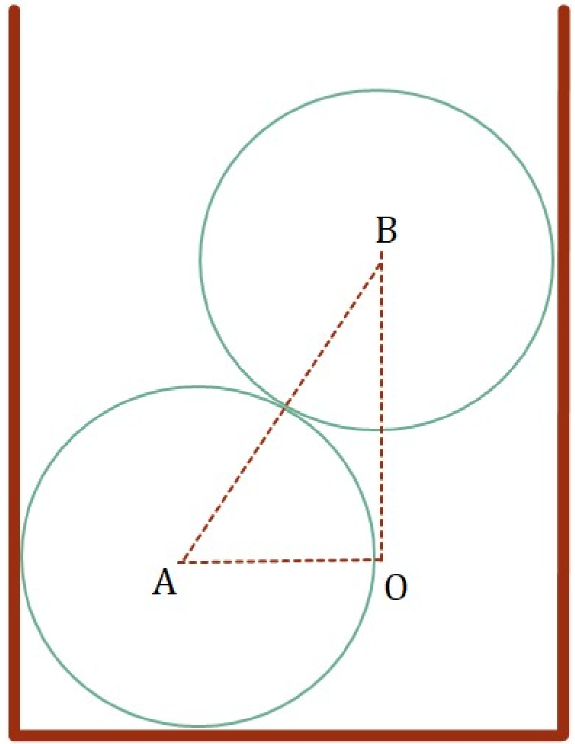
Solution: Draw FBD's of the two balls separately.

FBD of ball **A**





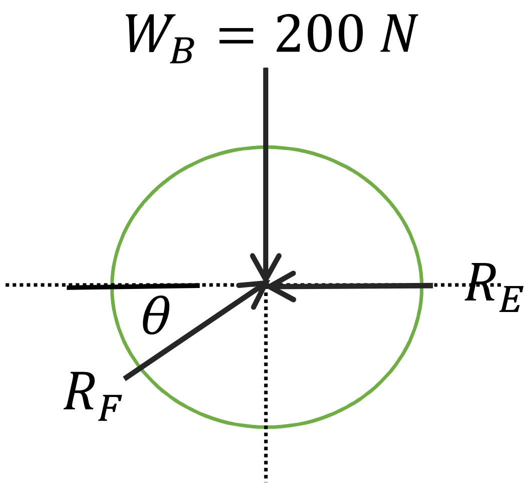
FBD of ball **B**



The balls are paced such that $\boldsymbol{AB}$ is double than $\boldsymbol{AO}$;

i.e. $\boldsymbol{AO}$ = 20 cm and $\boldsymbol{AB}$ = 40 cm;

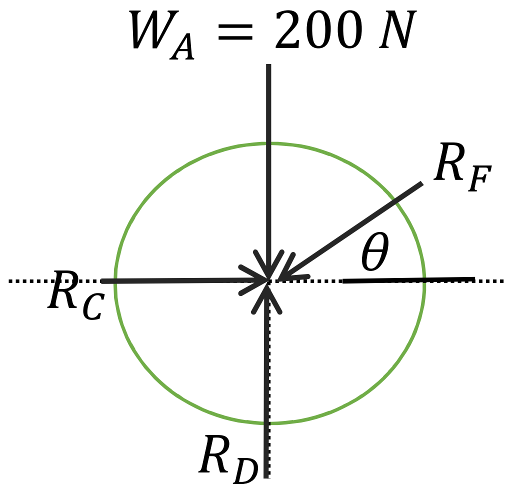
so, $\boldsymbol{\theta} = 60°$



FBD of ball **B**

$\boldsymbol{\theta} = 60°$

FBD of ball **A**



$$\frac{200}{\sin 120} = \frac{R_E}{\sin 150} = \frac{R_F}{\sin 90}$$

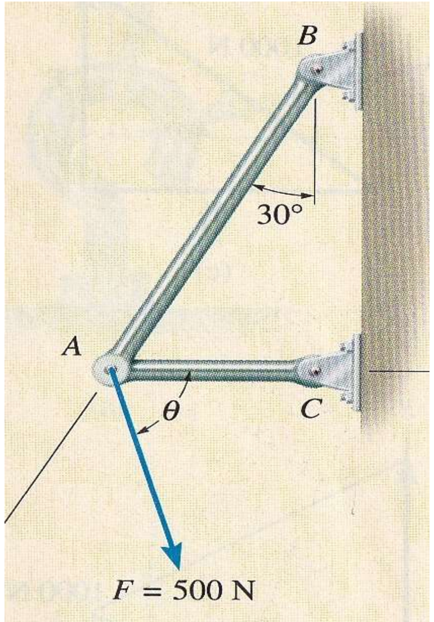
$$\boldsymbol{R_E} = \frac{200 \times \sin 150}{\sin 120} = \boldsymbol{115.47\ N}$$

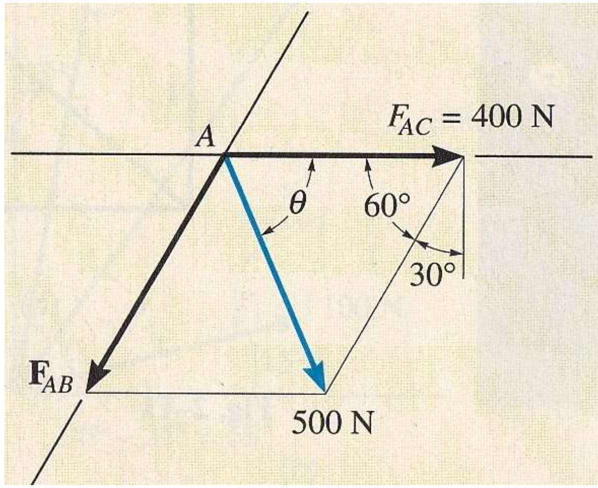
Similarly $\boldsymbol{R_F = 230.94\ N}$

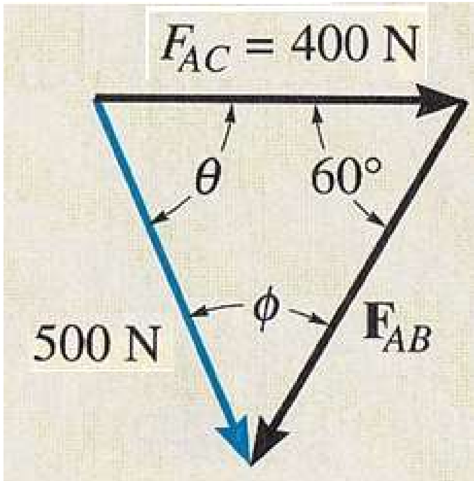
$$\boldsymbol{R_C = 115.47\ N}$$

$$\boldsymbol{R_D = 400\ N}$$

Problem: The force $\boldsymbol{F}$ acting on the frame shown in the figure, has a magnitude of 500 N and is to be resolved into two components acting along members AB and AC. Determine the angle θ, measured below the horizontal, so that the component $\boldsymbol{F}_{\boldsymbol{AC}}$ is directed from A toward C and has a magnitude of 400 N. Also find the magnitude of force in the bar AB.







Using the law of sines

$$\frac{500}{\sin 60} = \frac{F_{AB}}{\sin \theta} = \frac{400}{\sin \Phi}$$

$$\sin \Phi = \frac{400 \times \sin 60}{500} = 43.85°$$

$$\boldsymbol{\theta} = 180 - (60 + 43.85) = \mathbf{76.15°}$$

$$\boldsymbol{F}_{\boldsymbol{AB}} = \frac{500 \times \sin 76.15}{\sin 60} = \mathbf{560.56}\ \boldsymbol{N}$$

THANK YOU