Interference

Some more examples:

Colours of hummingbirds neck and feathers

Peacock feathers.

Soap bubbles.

What if you put two bulbs together
In a room?

Conditions for interference:

(i) The light sources must be coherent.

This means that the plane waves from the sources must maintain a constant phase relation. For example, if two waves are completely out of phase with $\varphi = \pi$, this phase difference must not change with time.

Coherence:

1. **Temporal Coherence:** Correlation between field at a point at two different time : E(x,y,z,t1) and E(x,y,z,t2). If phase diff. Is constant over observation time, then it is temporal coherence. Tell how monochromatic a source is.
2. **Spatial Coherence:** When waves at different points in space preserve a constant phase difference over a time t.

Temporal coherence is characteristic of single beam of light whereas Spatial Coherence is relationship between two separate beams of light.

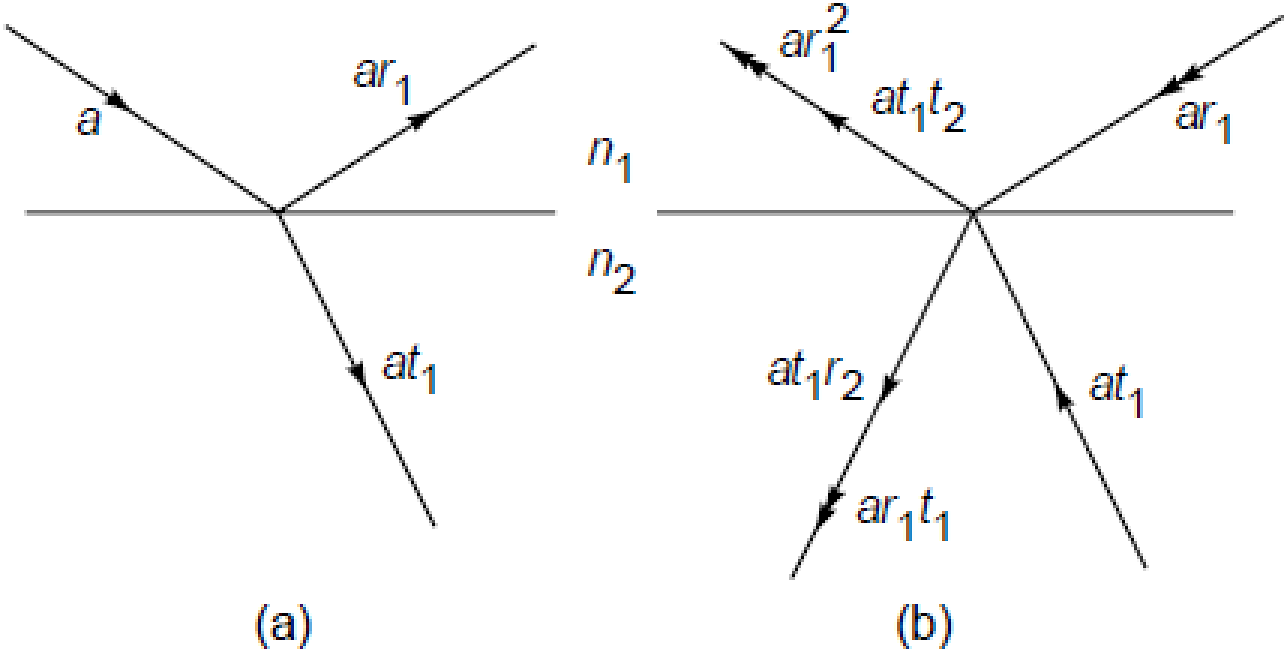
Conditions for interference:

(i) The light sources must be coherent. This means that the plane waves from the sources must maintain a constant phase relation. For example, if two waves are completely out of phase with $\varphi = \pi$, this phase difference must not change with time.

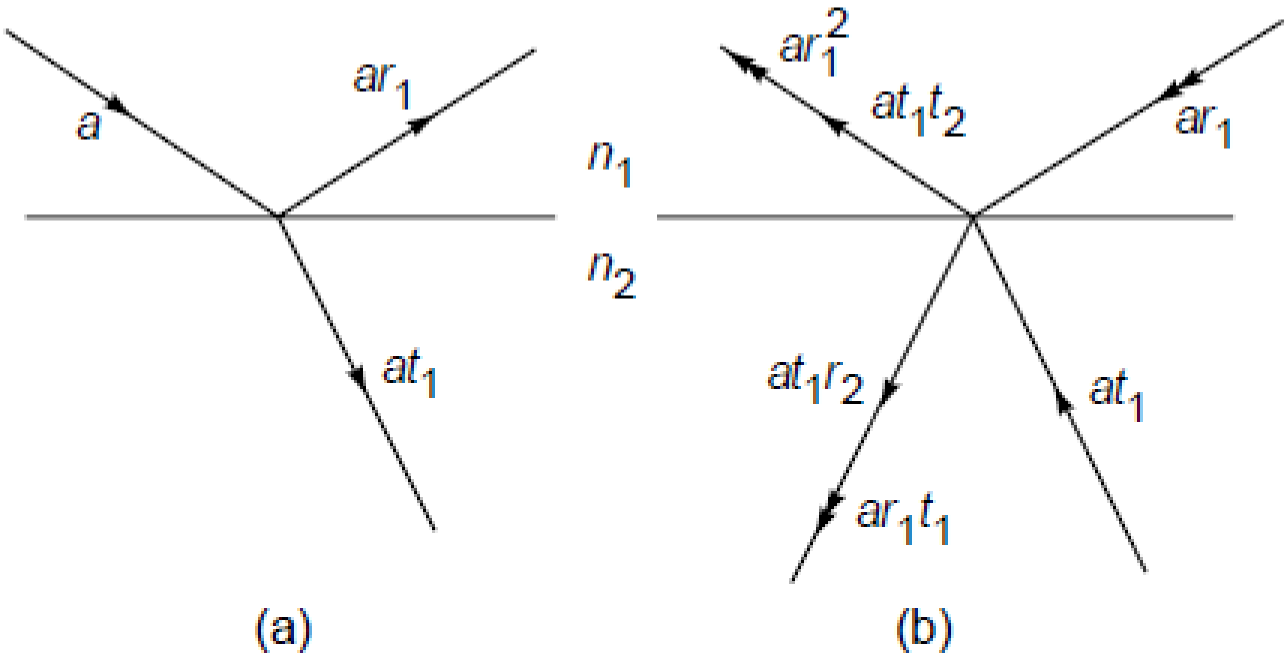
(ii) The light must be monochromatic. This means that the light consists of just one wavelength λ.

(iii) The Principle of Superposition must apply.

Stoke's law: According to this principle, in the absence of any absorption, a light ray that is reflected or refracted will retrace its original path if its direction is reversed.



(a) A ray traveling in a medium of refractive index n_1 incident on a medium of refractive index n_2. (b) Rays of amplitude ar_1 and at_1 incident on a medium of refractive index n_1.



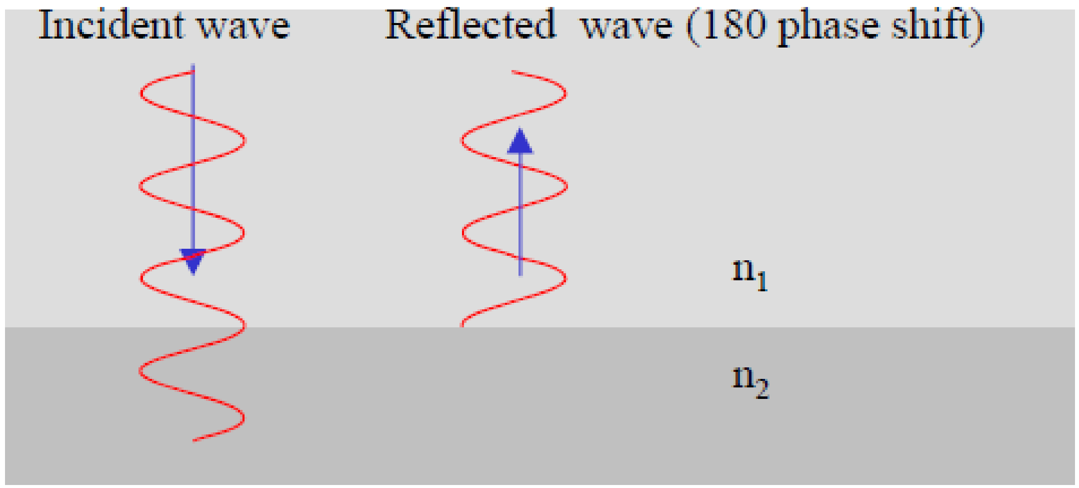
According to the principle of optical reversibility, the two rays of amplitudes $ar_1^{\,2}$ and at_1t_2 must combine to give the incident ray.

$ar_1^{\,2} + at_1t_2 = a$ Or $t_1t_2 = 1-r_1^{\,2}$

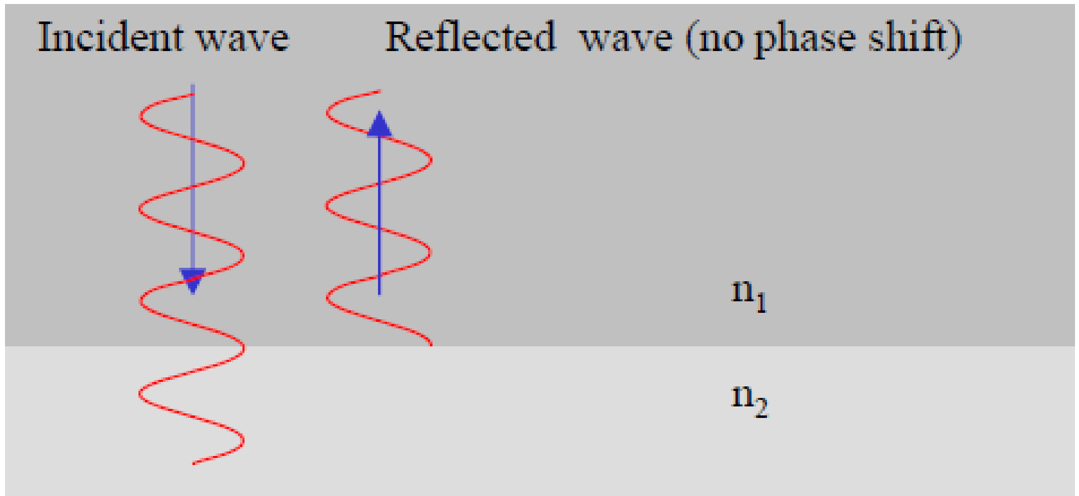
$ar_1t_1 + at_1r_2 = 0$ Or $r_1 = -r_2$

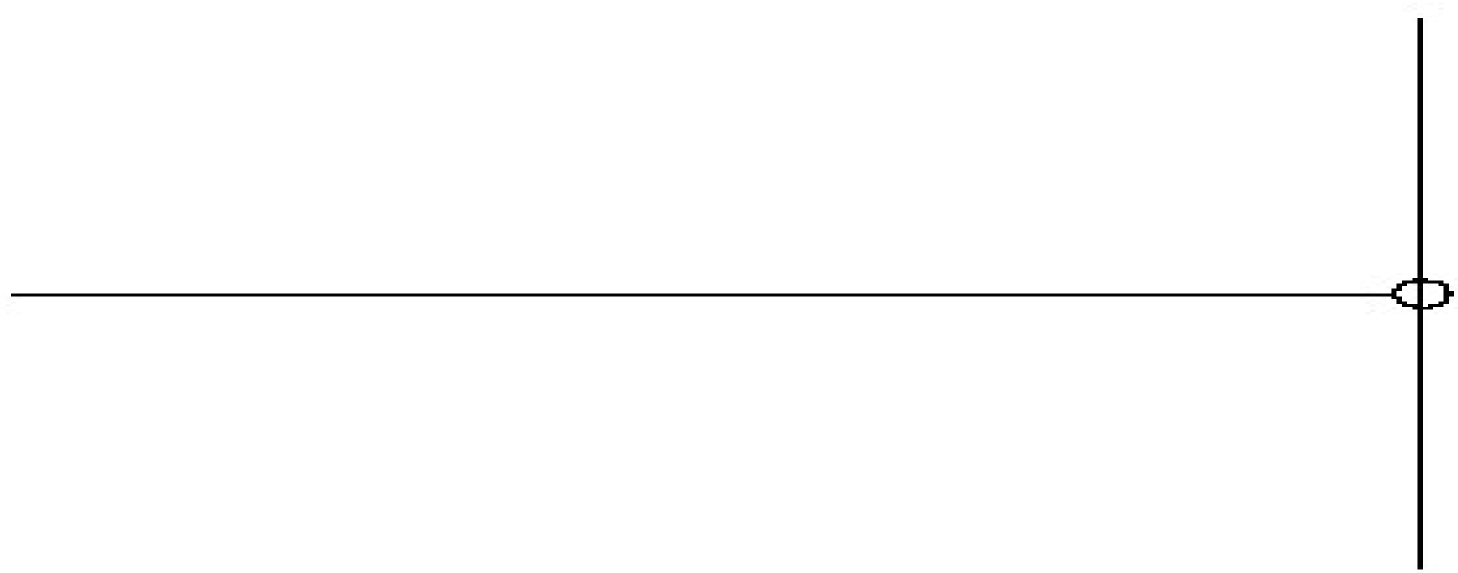
- Interference in thin films:

$n_1<n_2$



$n_1 > n_2$

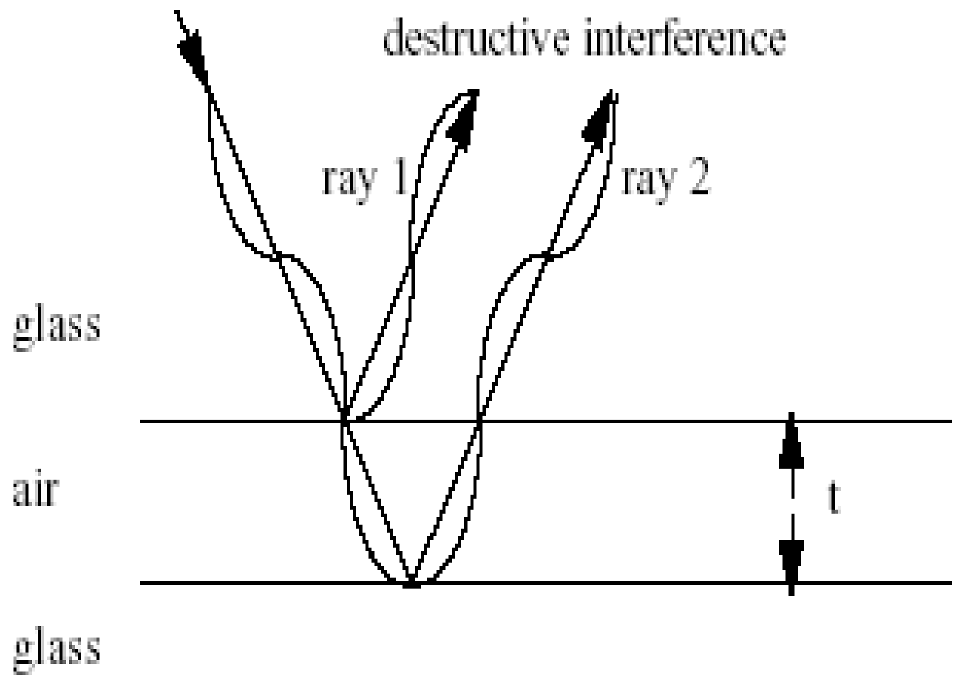




- Ray 2 undergoes a phase change of 180° with respect to the incident ray

- Ray 1, which is reflected from the lower surface, undergoes no phase change with respect to the incident wave

- Refracted waves don't undergo phase transition.



At interface, velocity and wavelength changes but frequency remains same.

Thin Film Interference :

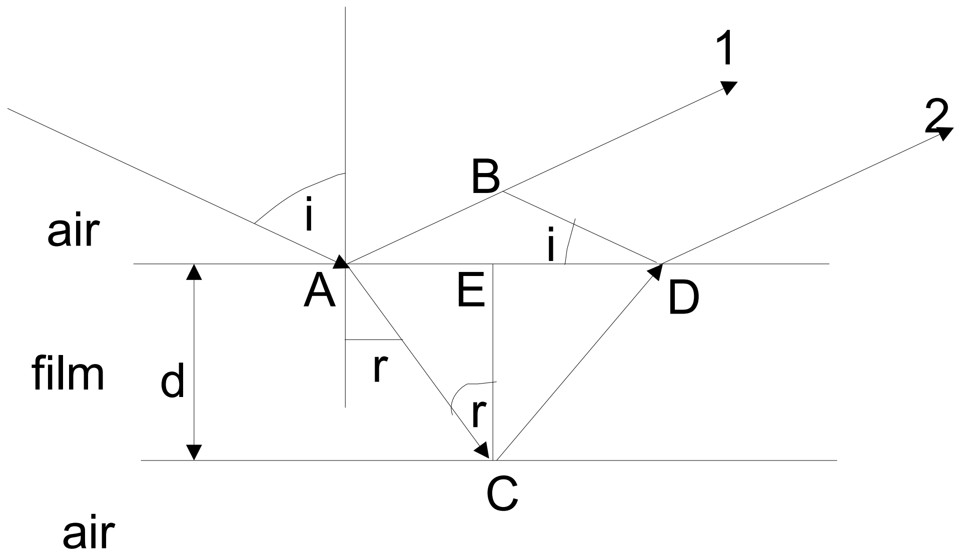
Thin film planes can be parallel to each other or inclined.
Interference in thin films can be studied under two categories, namely,

1. Interference in parallel plate film.
2. Interference in wedge-shaped films.

Optical Path:

- The optical path travelled by a light ray in a medium of refractive index 'μ' is not equal to actual path travel led by the light ray.

Interference in parallel film due to reflected light:



Optical path diff.

$=\mu(AC+CD)-AB$

$$=\frac{2\mu d}{\cos(r)}-AB$$

[AB=AD sin(i)

= 2 AE sin(i)

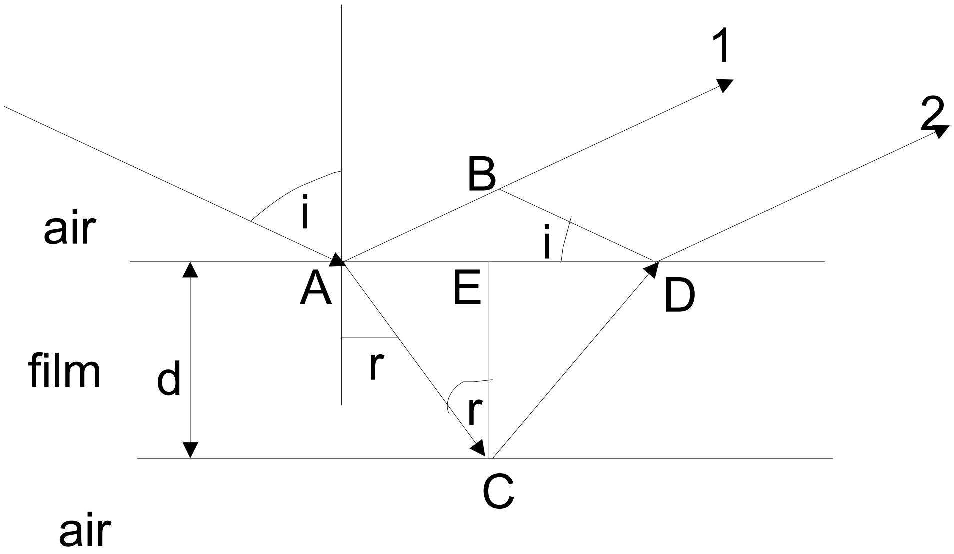
=2d tan(r). sin(i)

=2d tan(r). μ sin(r)]

BD is normal to AB

Hence, Path diff. = 2μd [1/cos(r) – tan(r) sin(r)]

= 2μd cos(r)



There is phase change of 180 or $\lambda/2$ for ray 1 whereas no phase change for ray 2. Therefore,
The condition for maxima will be given by:

$$2\mu d \cos(r) + \lambda/2 = n\lambda$$

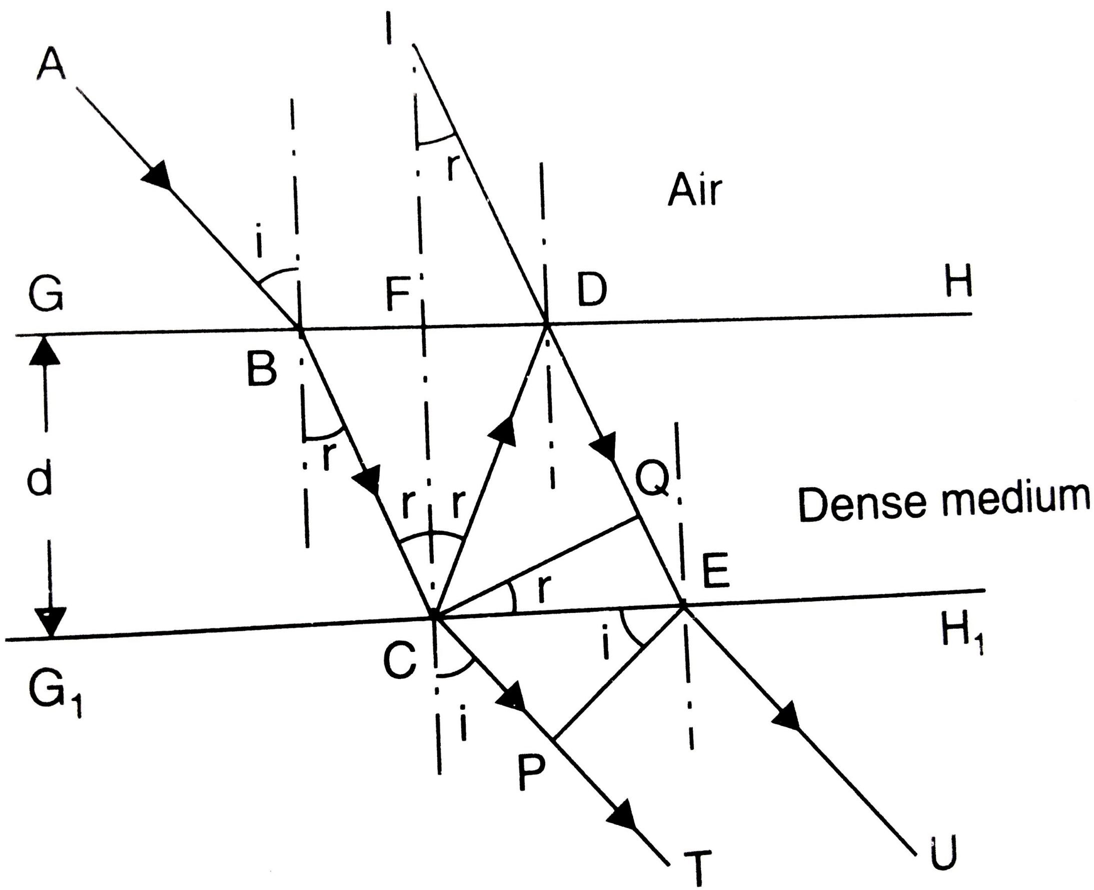
$$2\mu d \cos(r) = (2n-1)\lambda/2 \text{ where } n=1,2,3,\ldots\ldots\ldots$$

Condition for minima will be given by :

$$2\mu d \cos(r) + \lambda/2 = (2n+1)\lambda/2$$

$$2\mu d \cos(r) = n\lambda \text{ where } n=0,1,2,3,\ldots\ldots\ldots\ldots\ldots$$

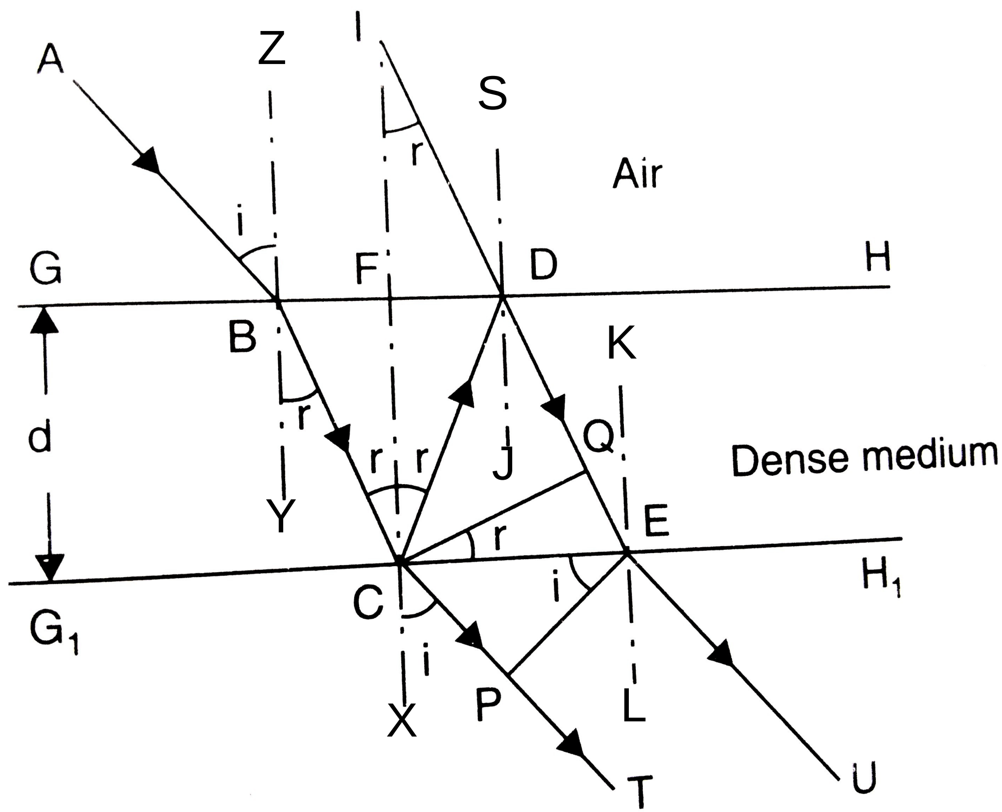
Interference in parallel film due to transmitted light:



Optical path diff.= μ(CD+DE)-CP

CQ is normal to DE and EP is normal to CT. ED is extended backward and it intersects extended CF at I.

Interference in parallel film due to transmitted light:



∡ABZ = ∡XCT
(YZ || IX and AB || CT)

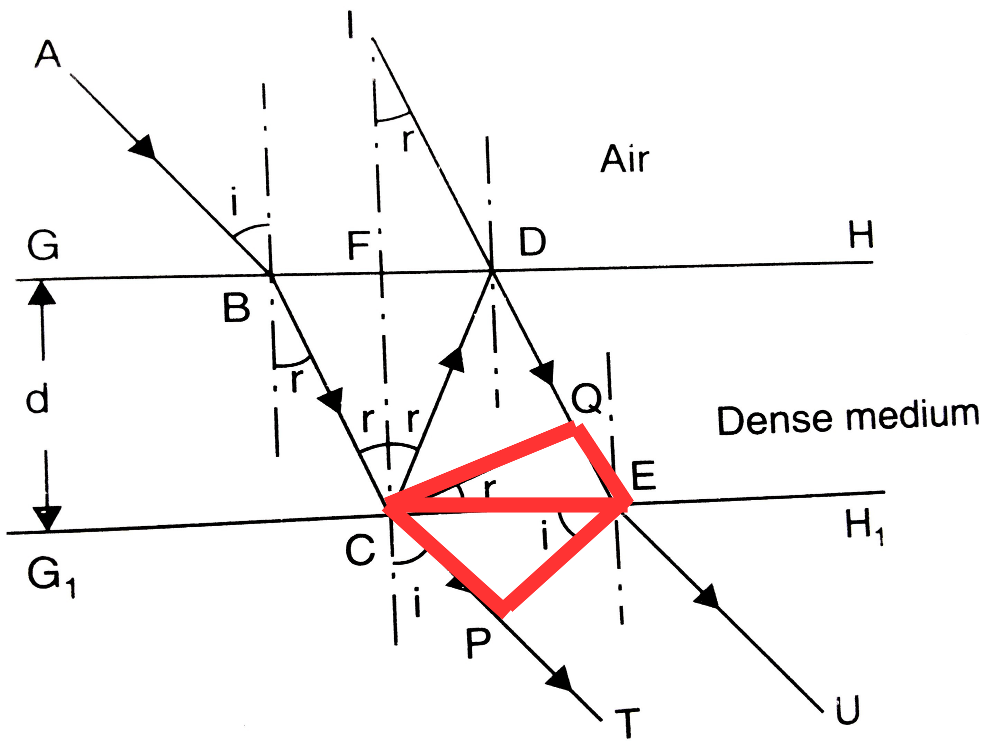
∡PCE = 90-i,
so ∡CEP = i

∡CDE = 2r,
so ∡DCQ = 90-2r,
Hence ∡QCE=r

∡QDJ =∡CID=r

CQ is normal to DE and EP is normal to CT.
DE is extended backward and it intersects extended CF at I.

Interference in parallel film due to transmitted light:



Optical path diff.
=μ(CD+DE)-CP

$$\mu = \frac{\sin(i)}{\sin(r)}$$

$$= \frac{CP/CE}{QE/CE} = \frac{CP}{QE}$$

Hence, CP =μ. QE

So, optical path diff. = μ(CD+DE-QE)

= μ(CD+DQ+QE-QE)

= μ(CD+DQ) =μ(ID+DQ)

= μ.IQ = 2μd.cos(r)

The condition for maxima will be given by:

$$2\mu d \cos(r) = n\lambda \quad \text{where } n=1,2,3,\ldots\ldots\ldots$$

Condition for minima will be given by :

$$2\mu d \cos(r) = (2n+1)\lambda/2 \qquad \text{where } n=0,1,2,3,\ldots\ldots$$

Conditions of maxima and minima in transmitted light are just reverse of conditions for reflected light.

What if monochromatic source is replaced by white light?

What if monochromatic source is replaced by white light?

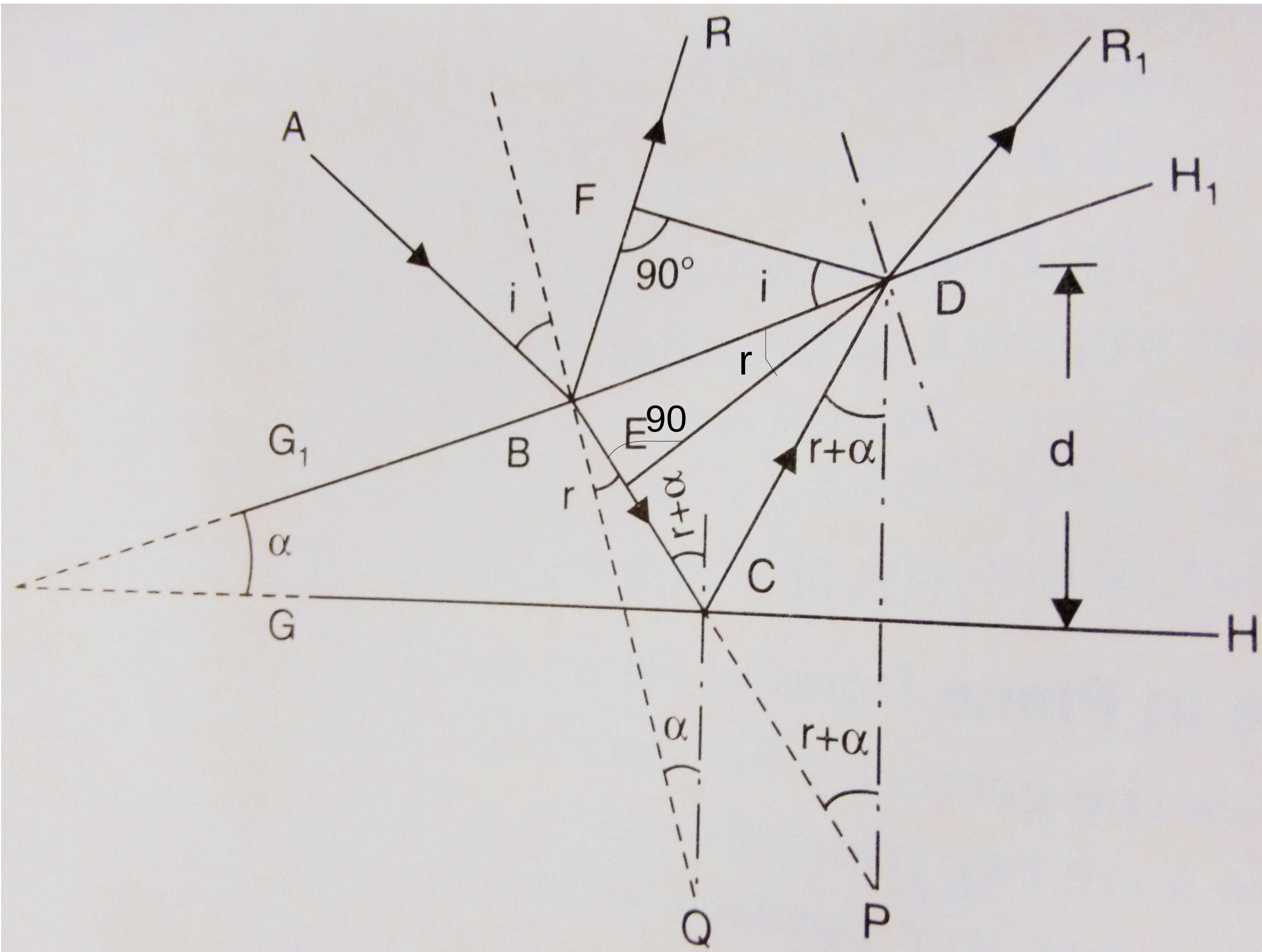
- Path difference is function of μ which in turn depends on wavelength of incident light.

- Depending on region of film and viewing position, condition for maxima is satisfied for some wavelengths giving bright fringes.

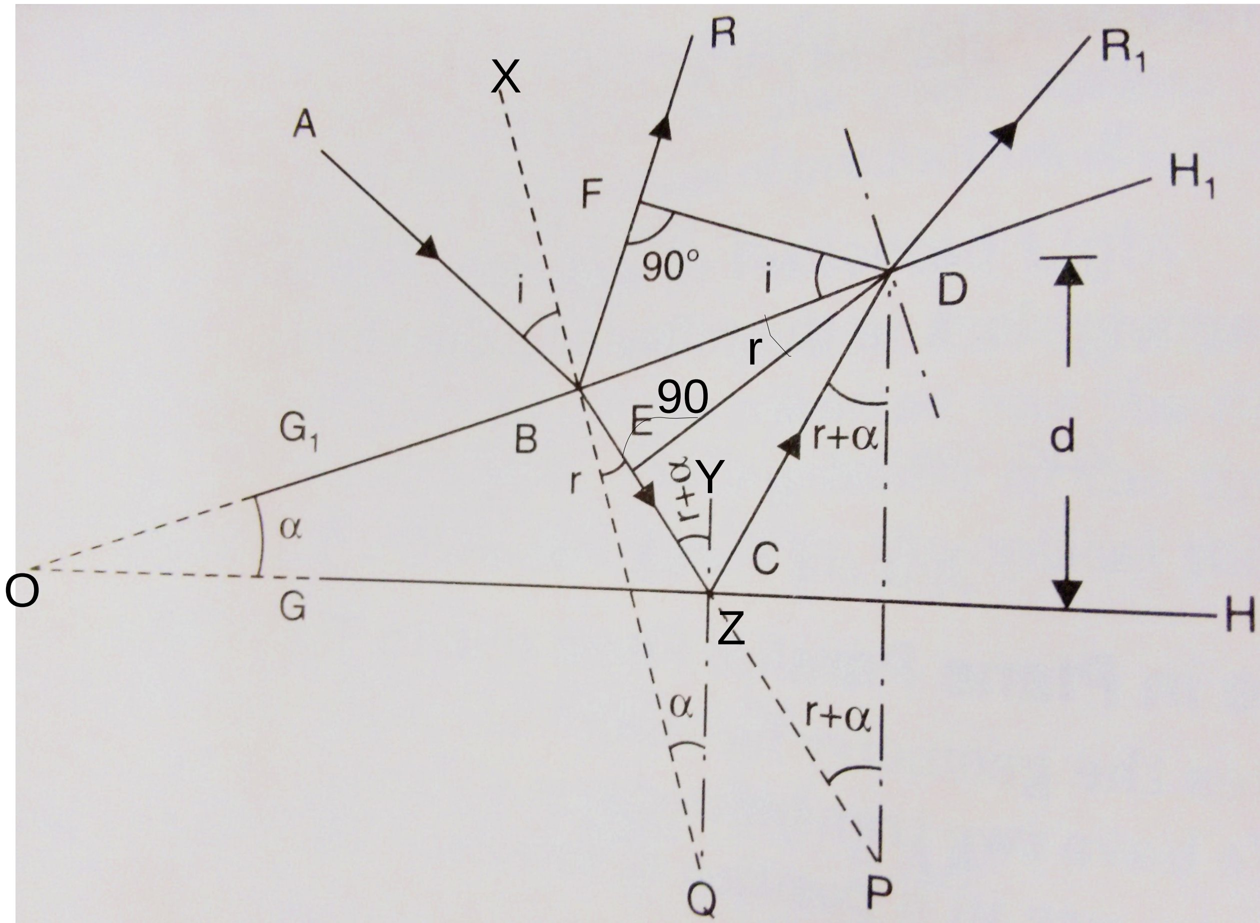
- Wavelength for which minima condition is satisfied would be absent in pattern.

- That is why colours change when we change our viewing angle.

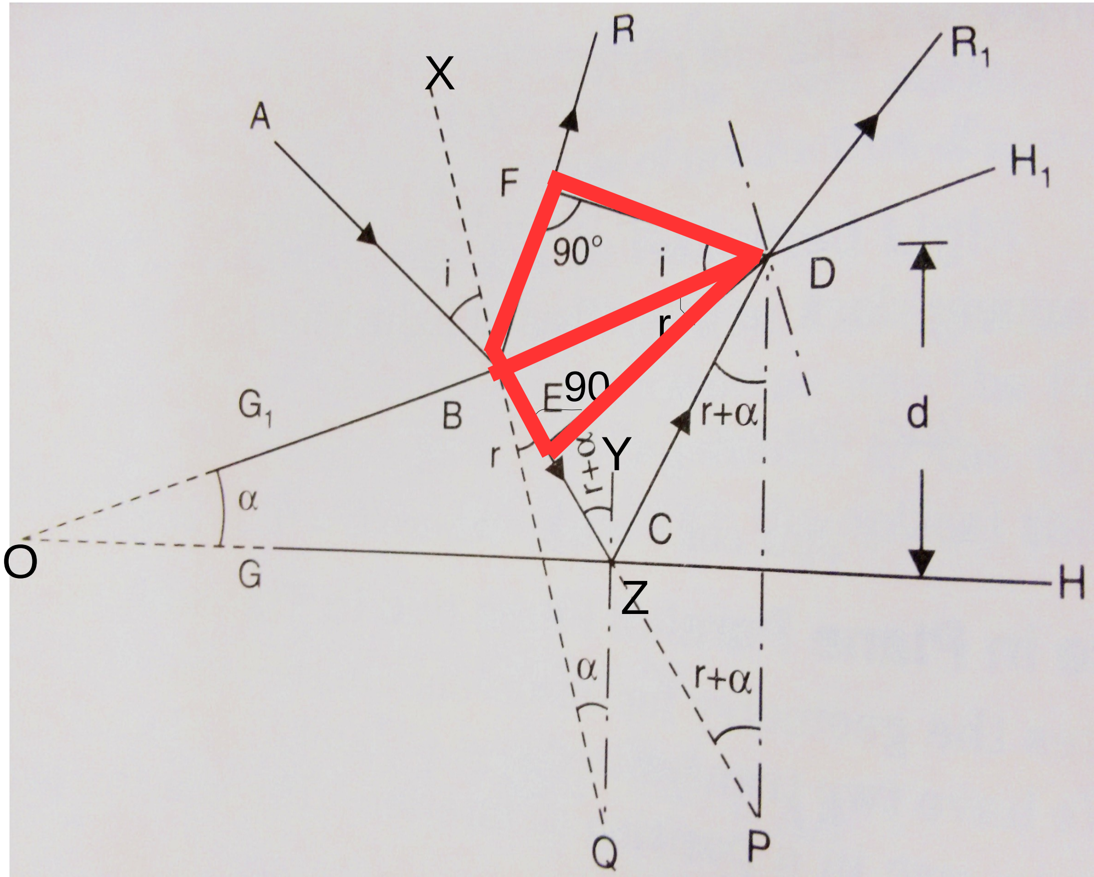
Interference in Wedge shaped film :



Surface GH and G1H1 inclined at an angle α enclose wedge shaped film.



- $QB \perp G_1H_1$ and $CQ \perp GH$, Hence $\measuredangle G_1OG = \measuredangle BQC$
- $\measuredangle BCQ = 180-(r+\alpha)$, so $\measuredangle BZY = (r+\alpha)$.
- $YQ||DP$, so $\measuredangle BZY = \measuredangle ZPD=(r+\alpha)$
- $\measuredangle BZY = \measuredangle YZC = \measuredangle CDP$
- $\measuredangle ABX = \measuredangle XBF$, so $\measuredangle FBD = 90-i$

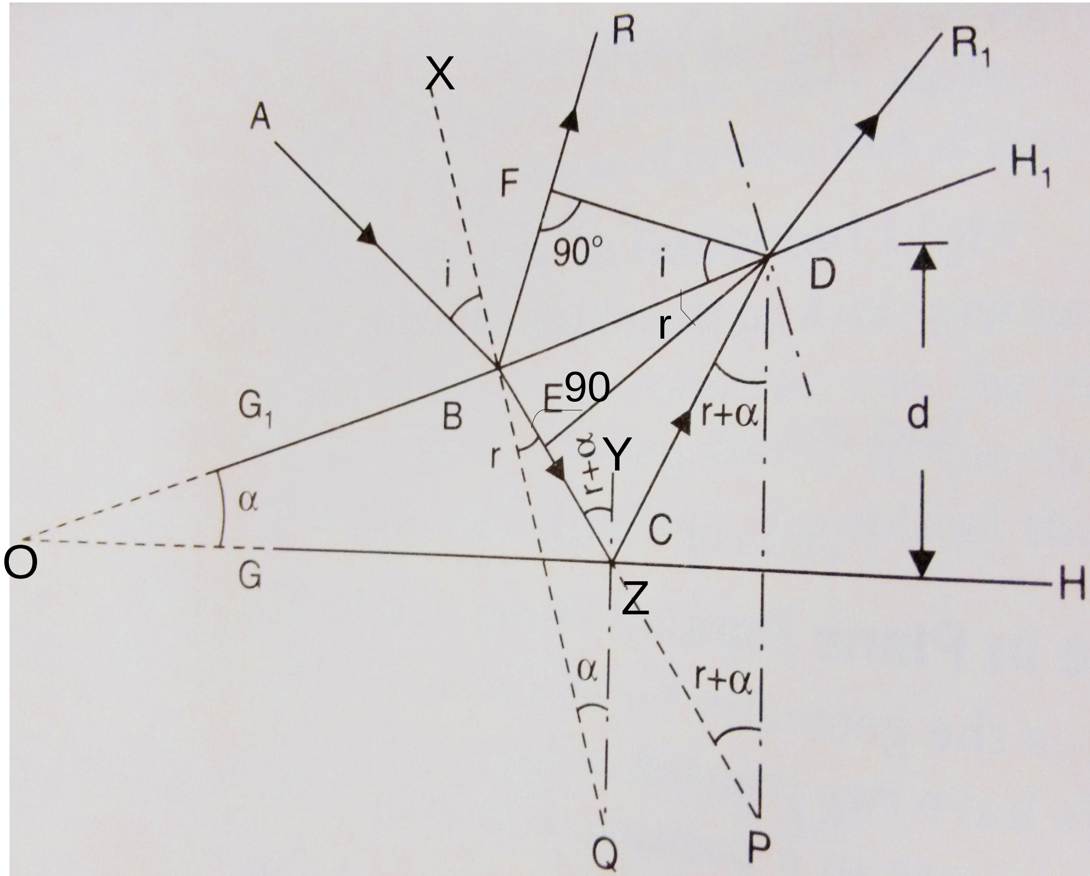


Optical path diff. = μ(BC+CD)-BF =μ(BE+EC+CD)-BF

$$\sin(i)=\frac{BF}{BD};\sin(r)=\frac{BE}{BD}$$

$$\mu=\frac{\sin(i)}{\sin(r)};\mu=\frac{BF}{BE}$$

$$BF=\mu.BE$$



So, optical path diff. = μ(BE+EC+CD)-BF
= μ(BE+EC+CD)-μ.BE =μ(EC+CD)
=μ(EC+CP) =μ.EP
=2μd cos(r+α)

Due to reflection from denser medium a phase difference of 180^o or λ/2 is introduced in ray BR.

For constructive interference or maxima :

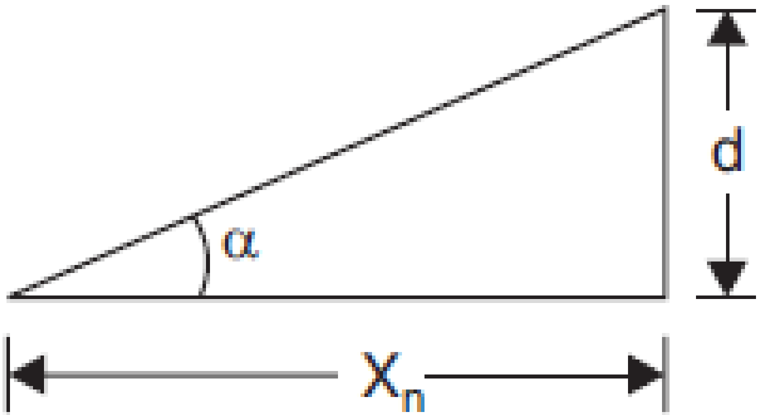
$2\mu d \cos(r+\alpha)+\lambda/2 = n\lambda$ where $n=1,2,3$.......

$2\mu d \cos(r+\alpha) = (2n-1)\lambda/2$

For destructive interference or minima :

$2\mu d \cos(r+\alpha)+\lambda/2 = (2n+1)\lambda/2$ where $n=0,1,2,3$.......

$2\mu d \cos(r+\alpha) = n\lambda$



Nth maximum will be given by :

$2\mu d \cos(r+\alpha) = (2n-1)\lambda/2$

If this maxima is obtained at X_n from edge.

For normal incidence, r=0 and assuming μ=1,

$d = X_n \tan(\alpha)$

So, $2 X_n \tan(\alpha) \cos(\alpha) = (2n-1)\lambda/2$

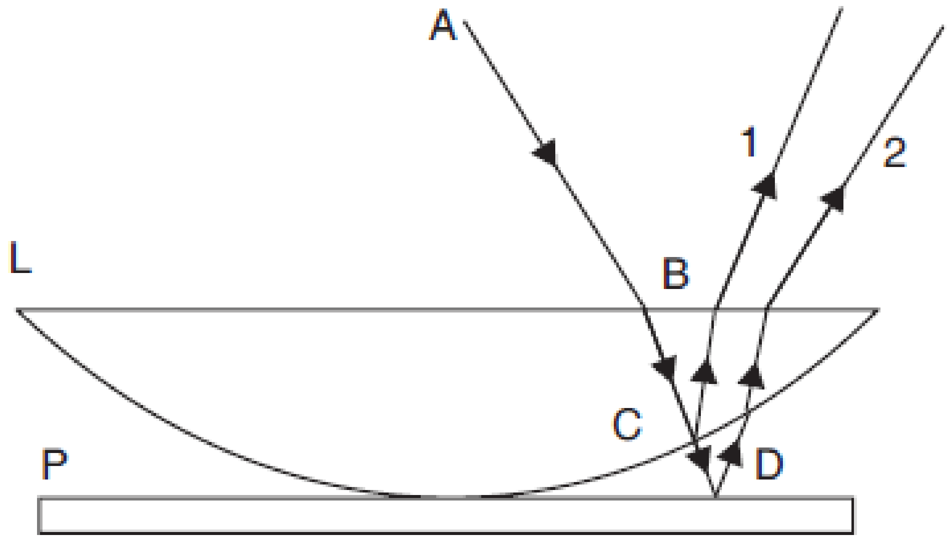
$2 X_n \sin(\alpha) = (2n-1)\lambda/2$

(n+1)th maxima, $2 X_{n+1} \sin(\alpha) = (2n+1)\lambda/2$

$2 (X_{n+1} - X_n)\sin(\alpha) = \lambda$

or fringe spacing $(X_{n+1} - X_n) = \lambda/(2.\sin(\alpha)) = \lambda/(2.\alpha)$

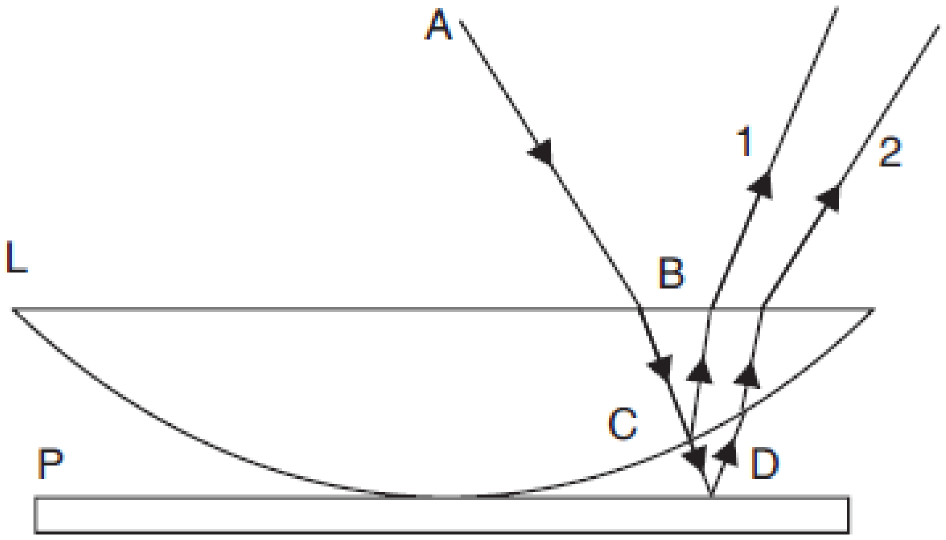
Newton's rings:



A plano-convex lens is placed on plane glass sheet, an air film of increasing thickness is formed.

Intereference occurs between ray 1 and 2 and circular fringes are formed.

Ray 2 undergoes phase change of 180 when reflected from air to glass.



Optical path diff. $=2\mu d \cos(r+\alpha)+\lambda/2$

For air film $\mu=1$, for convex lens of large radius of curvature, α is very small and can be neglected. So,

Optical path diff. $=2d \cos(r)+\lambda/2$

For normal incidence $r = 0$,
Optical path diff. $= 2d +\lambda/2$

For bright fringe or maxima :

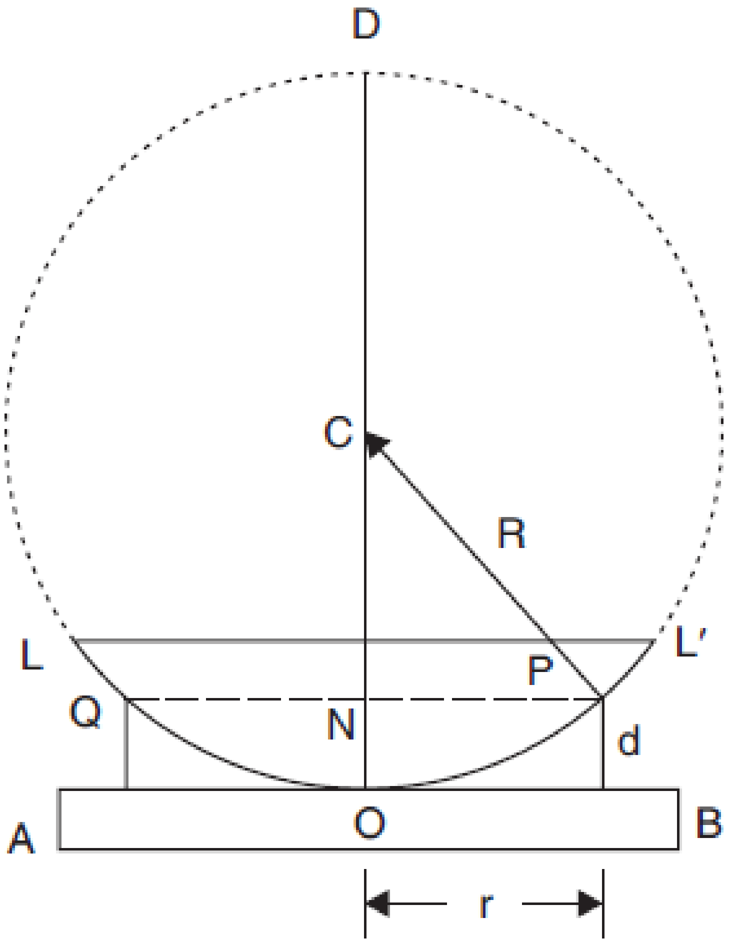
$2d + \lambda/2 = n\lambda$ or $2d = (2n-1)\lambda/2$ where $n=1,2,3\ldots.$

Condition for dark fringe:

$2d + \lambda/2 = (2n+1)\lambda/2$ or $2d = n\lambda$ where $n=0,1,2,3..$

Newton's rings will be observed for transmitted light as well with opposite conditions for minima and maxima.

Diameter of Fringes :



From property of circle :

NP × NQ = NO × ND

i.e. $r \times r = d(2R-d) = 2Rd - d^2$

$d \ll R$, $r^2 \approx 2Rd$

$d = r^2/2R$

For bright fringe:

$2d = (2n-1)\lambda/2$

$$\frac{2r^2}{2R} = \frac{(2n-1)\lambda}{2}; Therefore, r^2 = \frac{(2n-1)\lambda R}{2}$$

If D is diameter of the fringe (r=D/2) then $D_n = \sqrt{2\lambda R}\sqrt{2n-1}$

Similarly for dark fringe: 2d = nλ

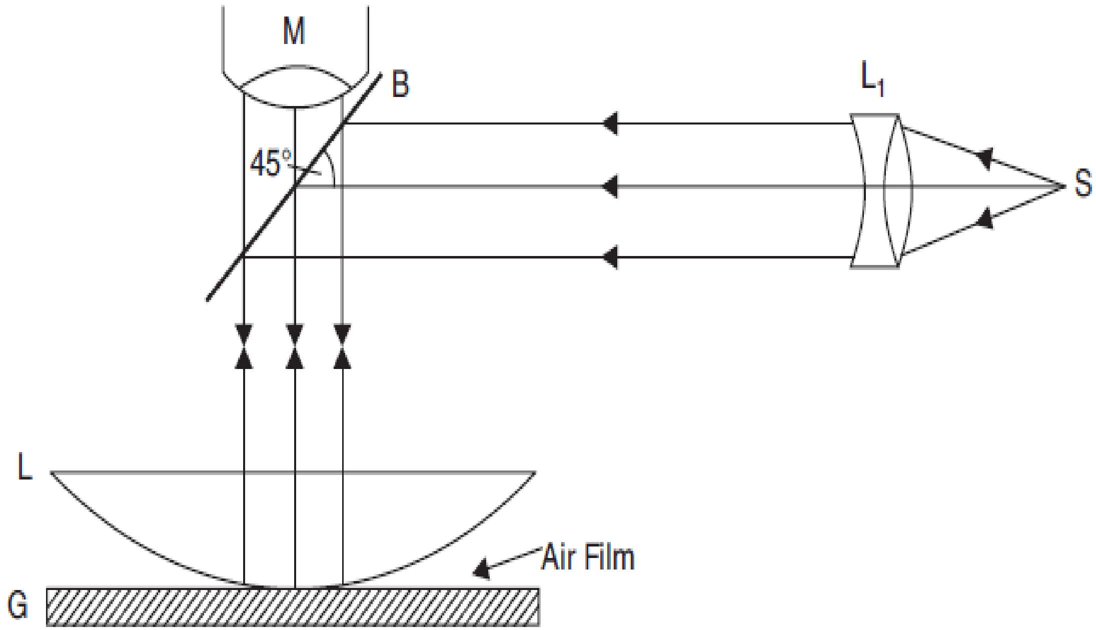
Or

$$\frac{2r^2}{2R} = n\lambda; \Rightarrow r^2 = n\lambda R$$

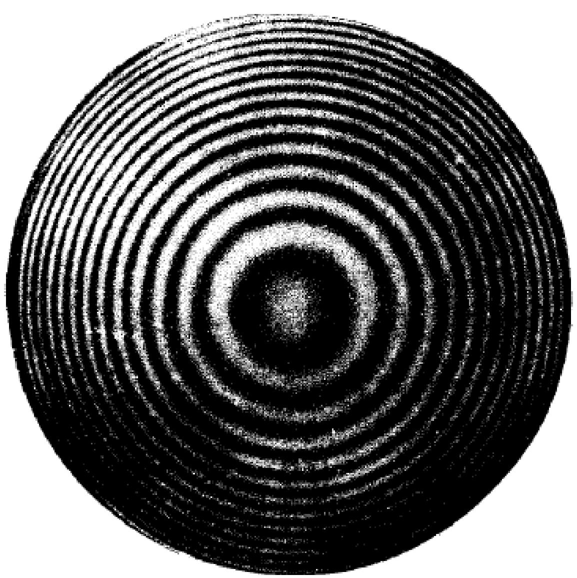
$$\Rightarrow D_n^2 = 4n\lambda R \Rightarrow D_n = 2\sqrt{n\lambda R}$$

Diameters of Newton's rings are proportional to square root of natural numbers.

Experimental Setup:



Newton's rings:



Visibility (V) = (I(max) - I(min))/(I(max) + I(min))

where I(max) represents the measured maximum intensity and I(min) is the corresponding minimum intensity.

V will always lie between 0 and 1.

Measurement of wavelenth:

If D_n and D_{n+p} gives diameter of n^{th} and $(n+p)^{th}$ dark fringe respectively, then

$$D_n^2 = 4n\lambda R$$

$$D_{n+p}^2 = 4(n+p)\lambda R$$

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

Therefore, λ can be calculated using this formula.

Measurement of refractive index:

First D_n and D_{n+p} (diameter of n^{th} and $(n+p)^{th}$ dark fringe respectively) is measured with air film in between :

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

Without disturbing arrangement, liquid is poured in the conatiner. Again, D'_n and D'_{n+p} is measured

$$D_{n+p}^{'2} - D_n^{'2} = \frac{4p\lambda R}{\mu}$$

Diameter of rings reduces in liquid.

Using above equations :

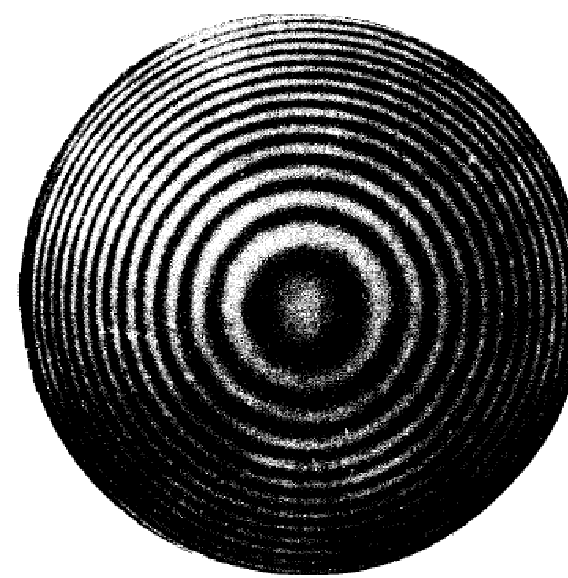
$$\mu = \frac{D_{n+p}^2 - D_n^2}{D_{n+p}^{'2} - D_n^{'2}}$$

Why the center of Newton's rings appears dark in reflected light?

The effective path difference between the reflected rays = $2\mu d\cos(r+\alpha)+\lambda/2$.

At the centre $d=0$ and for a very small angle of wedge $\cos(r+\alpha)=1$.Therefore effective path difference at $\lambda/2$.

This is the condition of minimum intensity. Hence central spot of the ring system appears dark.



Why an excessively thin parallel film appears black?

In Thin parallel film, path diff. for reflected light:
$2\mu d\cos(r) + \lambda/2$

For very thin film $2\mu d\cos(r) << \lambda/2$, so path diff. $= \lambda/2$

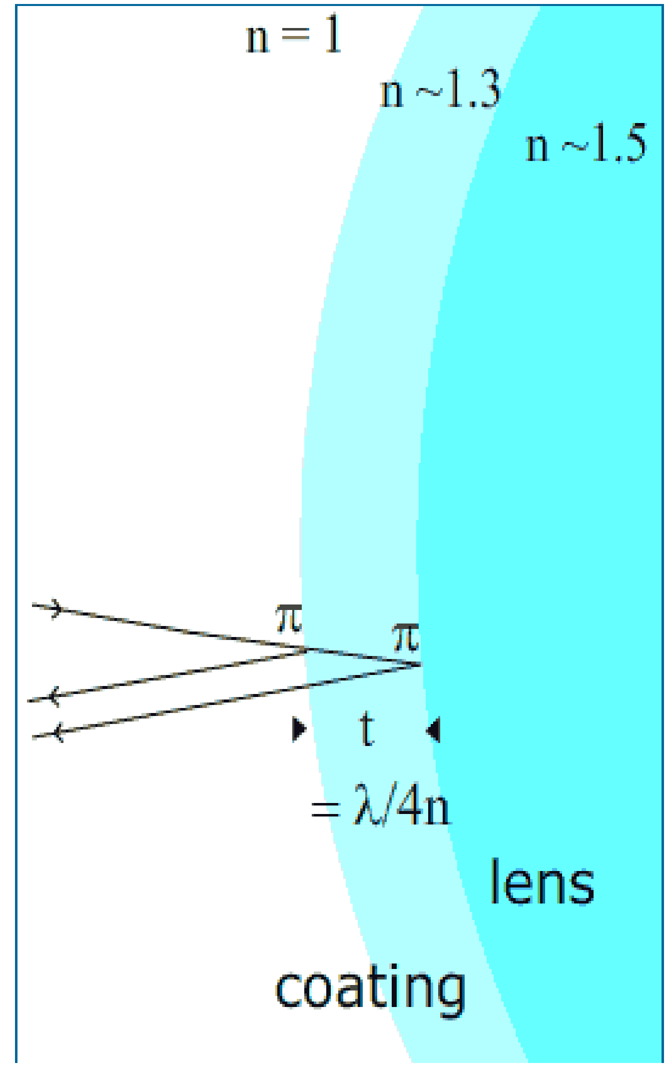
This is the condition of minimum intensity for all wavelengths. Therefore all wavelengths will be absent in reflected system, hence film will appear dark.

Non-reflecting/Anti-reflecting Coatings:

Non-reflective coatings admit more light into cameras and other optical instruments.

Comparing the two reflected rays, there is no component of the phase difference due to reflection: these effects cancel out.

Let's consider a wavelength λ in air and so a wavelength λglass in the glass. Now suppose that thickness of the layer is t = λcoating/4 = λ/4n. So the second reflected ray has travelled λcoating/2 further, so that the phase difference, entirely due to the path difference, is π.



For this wavelength, we have destructive interference: very little power is reflected from the coating, and most is transmitted into the lens: more light is available for the optical instrument, and less is wasted in reflection.

For optical instruments, one would usually choose λ to be in the middle of the visible spectrum (green light at around 550 nm). So, on the axis and with normal incidence, there is maximum destructive interference for green, but still considerable destructive interference for the rest of the visible spectrum.

We've neglected the angle of incidence above. When the angle of refraction in the coating is θ, the pathlength difference is $\lambda/(2n \cos \theta)$: longer by a factor of $1/\cos \theta$.

So the destructive interference is more complete for longer wavelengths – towards the red end of the spectrum – and the destructive interference is less complete for blue and violet.

This explains why the lens in the photo above appears to havea blue-violet tinge – which provides a simple way of recognising such coatings.

To obtain an interference pattern, there should be a definite phase relationship between the waves reflected from the upper surface of the film and from the lower surface of the film.
Thus the path difference should be small compared to the coherence length. For example, if we are using the D1 line of an ordinary sodium lamp (l = 5.890×10^{-5} cm), the coherence length is of the order of 1 cm, and for fringes to be visible, D should be much less than 1 mm. There is no particular value of D for which the fringes disappear; but as the value of D increases, the contrast of the fringes becomes poorer.

A laser beam has a very high coherence length, and fringes can be visible even for path differences much greater than 1 m. On the other hand, if we use a white light source, no fringes will be visible for D >~ 2×10^{-4} cm