Discrete Mathematical Structures (UCS-405)

Tutorial Sheet-13

1. Find the minimum number of students in a class to be sure that three of them are born in the same month.

2. Suppose five points are chosen from the interior of a square S where each side has length two inches. Show that the distance between two of the points must be less than $\sqrt{2}$ inches.

3. Find the minimum number of elements that one needs to take from the set $S = \{1, 2, 3, \ldots, 9\}$ to be sure that two of the numbers add up to 10.

4. Suppose a department contains 13 professors, then find the minimum number of professors who were born in the same month.

5. Prove using mathematical induction that the sum of the first n odd numbers is n^2; that is,

$$P(n): 1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

6. Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive 0s.

7. Prove the following proposition (for $n \geq 0$):

$P(n)$: $1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1$

8. Disprove by counterexample that for any a, b $\in$ Z, if $a^2 = b^2$, then a=b.

9. Prove that every integer that is a perfect cube is a multiple of 9, or is 1 more than a multiple of 9, or is 1 less than a multiple of 9.

10. Prove: $1 + 4 + 7 + \cdots + 3n - 2 = n(3n-1)/2$