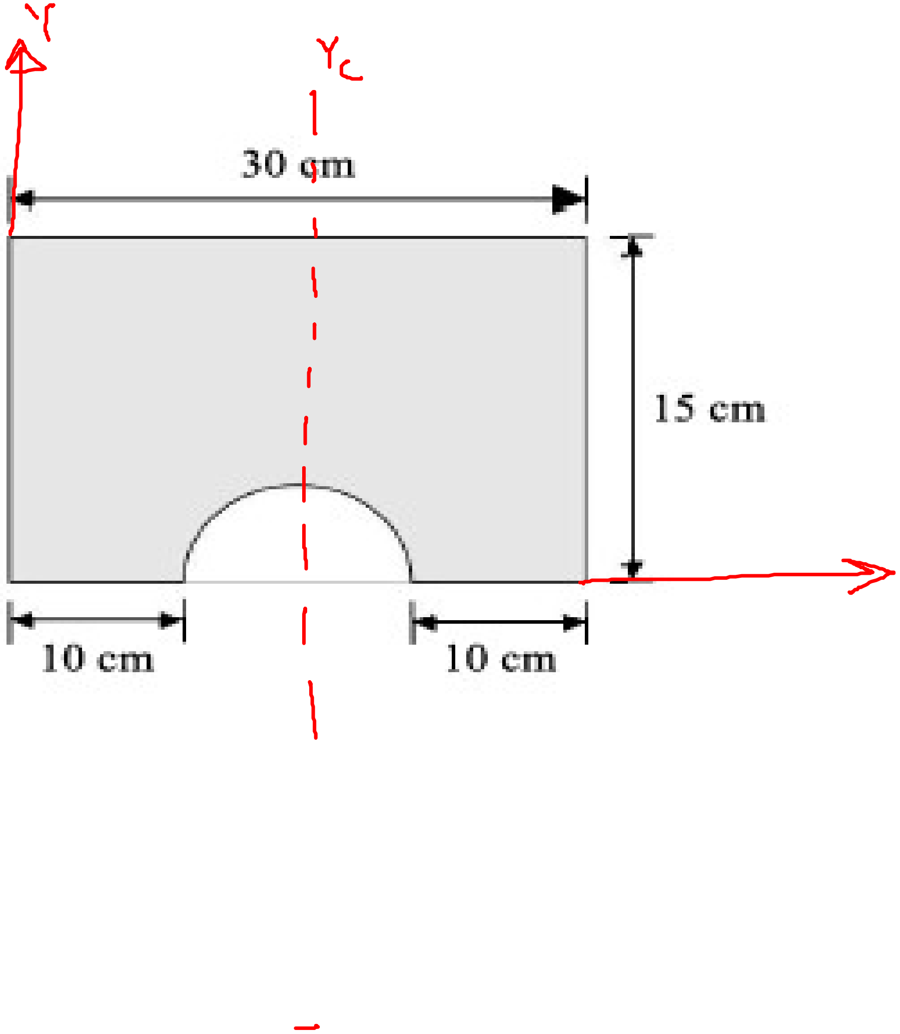
Find moment of inertia about centroidal axis



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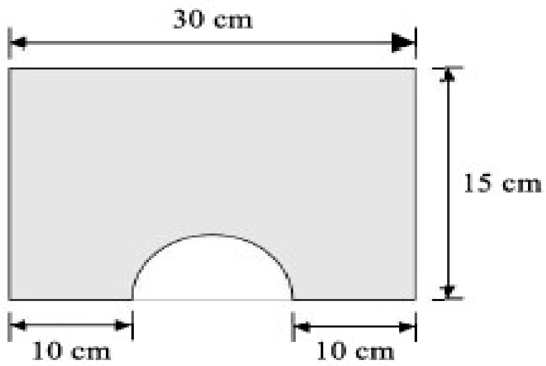
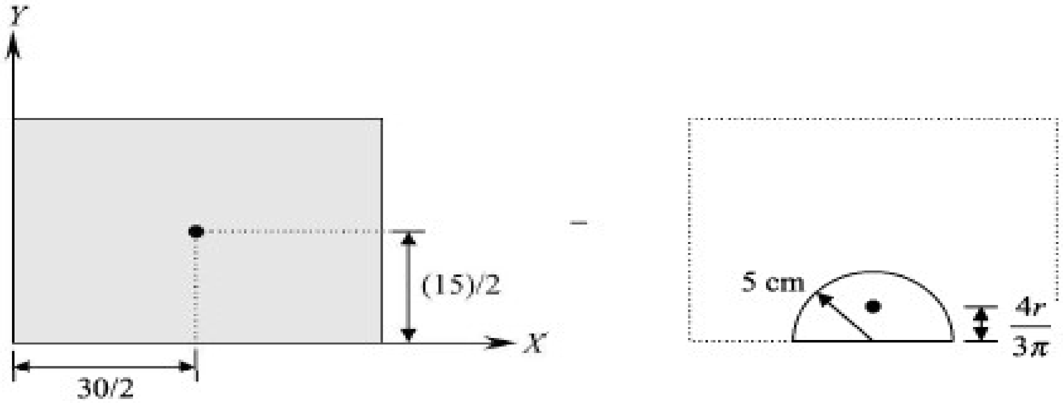
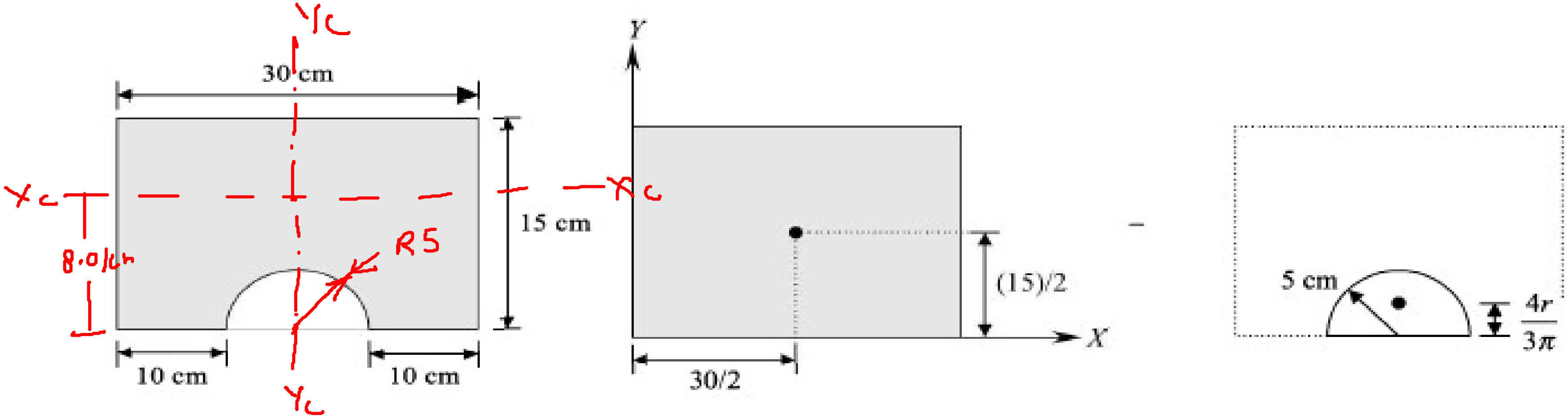


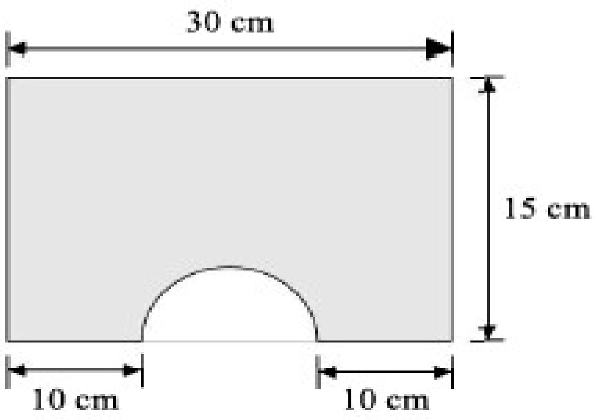




S.No	Element	$A_i (cm^2)$	$\bar{y}_i$ (cm)	$A_i\bar{y}_i (cm^3)$
1.	Rectangle	450	7.5	3375
2.	Semicircle	$-\frac{\pi}{2}(5)^2 = -39.27$	$4(5)/3\pi = 2.12$	–83.25
$\Sigma =$		410.73		3291.75

$$\therefore \qquad \bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = 8.01 \text{ cm}$$

$$I_{x_c x_c} = I_1 - I_2$$



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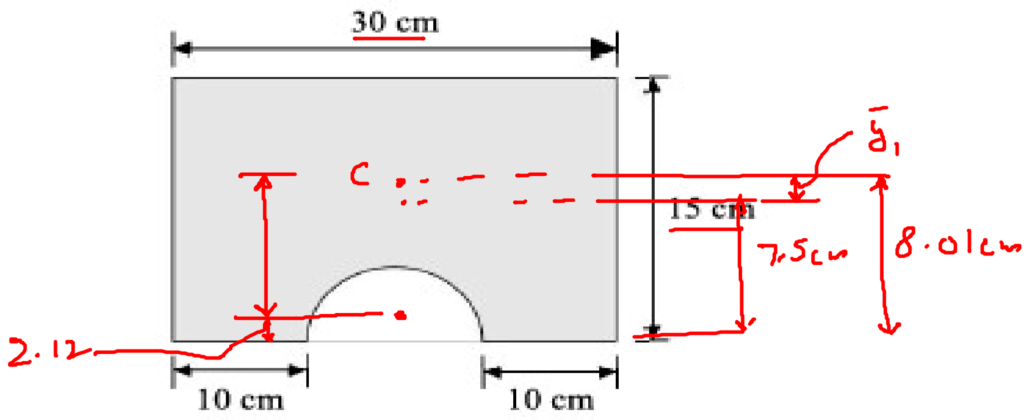


S.No	$(\bar{I}_{xx})_i$ cm^4	$(\bar{I}_{yy})_i$ cm^4	$A_i(\bar{y}_i - \bar{y})^2$ cm^4	$A_i(\bar{x}_i - \bar{x})^2$ cm^4
1.	$(30)(15)^3/12 = 8437.5$	$(15)(30)^3/12 = 33\ 750$	$450(7.5 - 8.01)^2 = 117.05$	0
2.	$-0.11(5)^4 = -68.75$	$-\pi(5)^4/8 = -245.44$	$-39.27(2.12 - 8.01)^2 = -1362.36$	0
$\Sigma =$	8368.75	33 504.56	–1245.31	0

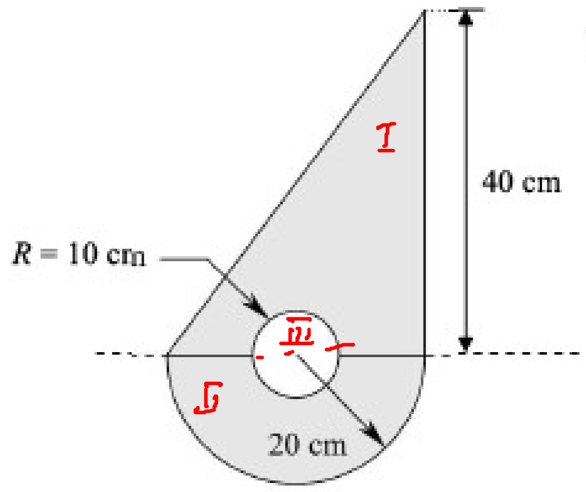
$$\frac{4r}{3\pi} = \frac{4 \times 5}{3 \times \pi} = 2.12$$

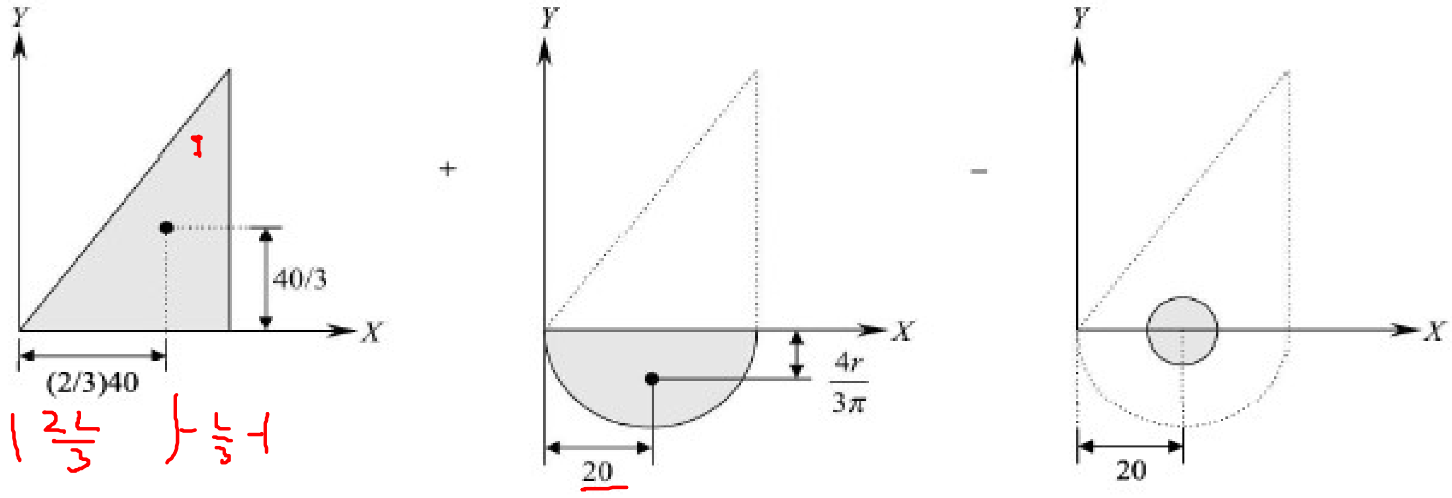
$$= 0.1059\, r^4$$
$$= 0.11\, r^4$$

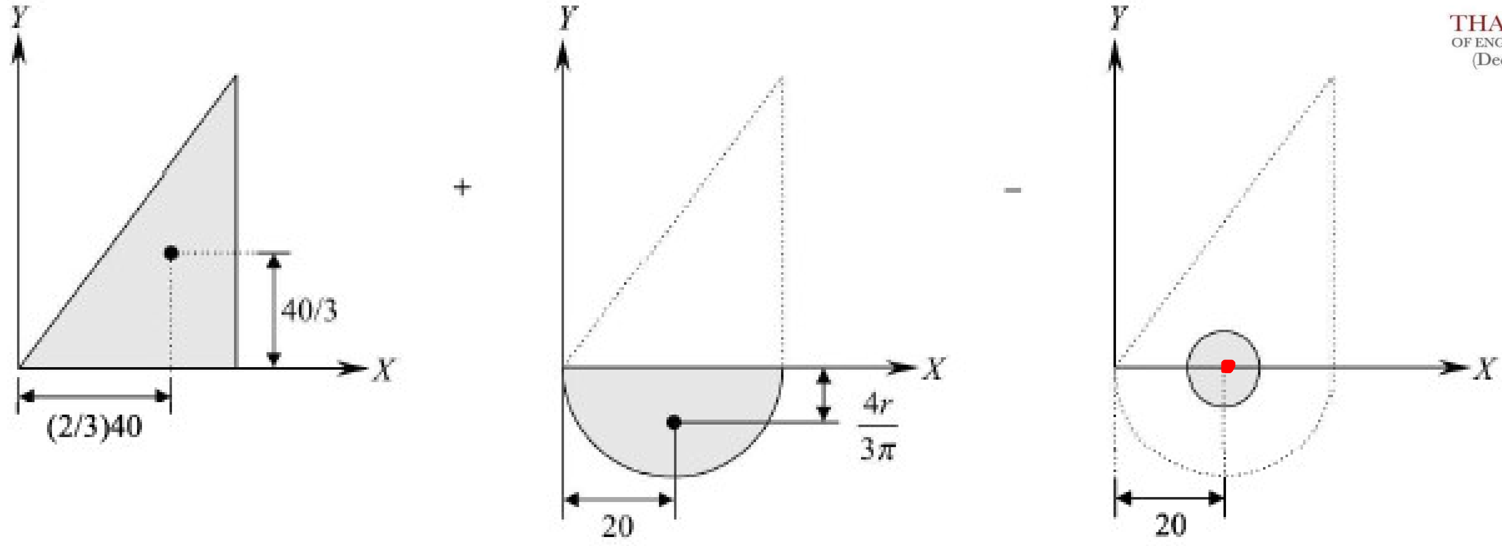
$$\bar{I}_{xx} = \sum (\bar{I}_{xx})_i + \sum A_i(\bar{y}_i - \bar{y})^2$$

$$= 8368.75 - 1245.31 = 7123.44 \text{ cm}^4$$

$$\bar{I}_{yy} = \sum (\bar{I}_{yy})_i + \sum A_i(\bar{x}_i - \bar{x})^2 = 33\ 504.56 \text{ cm}^4$$


I
40 cm
R = 10 cm
III
II
20 cm


Y
I
40/3
X
(2/3)40
+
Y
X
4r
3π
20
-
Y
X
20



S.No	Element	A_i (cm^2)	$\bar{x}_i$ (cm)	$\bar{y}_i$ (cm)	$A_i\bar{x}_i$ (cm^3)	$A_i\bar{y}_i$ (cm^3)
1.	Triangle	$(1/2) \times 40 \times 40 = 800$	$(2/3)40 = 26.67$	$(1/3)40 = 13.33$	21 336	10 664
2.	Semicircle	$\pi(20)^2/2 = 628.32$	20	$-4(20)/3\pi = -8.49$	12 566.4	–5334.44
3.	Circle	$-\pi(10)^2 = -314.16$	20	0	–6283.2	0
	$\Sigma =$	1114.16			27 619.2	5329.56

$$\therefore \quad \bar{x} = \frac{\sum A_i \bar{x}_i}{\sum A_i} = 24.79 \text{ cm} \qquad \bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = 4.78 \text{ cm}$$

from the y-axis

Above the x-axis

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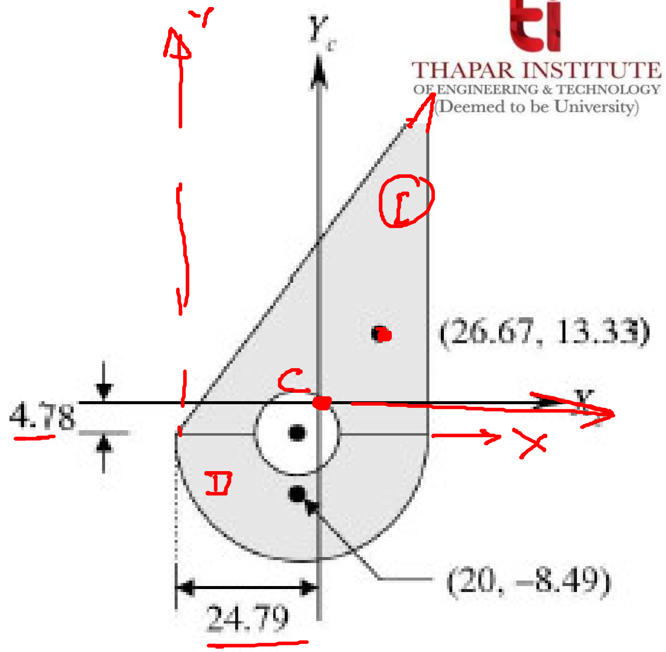
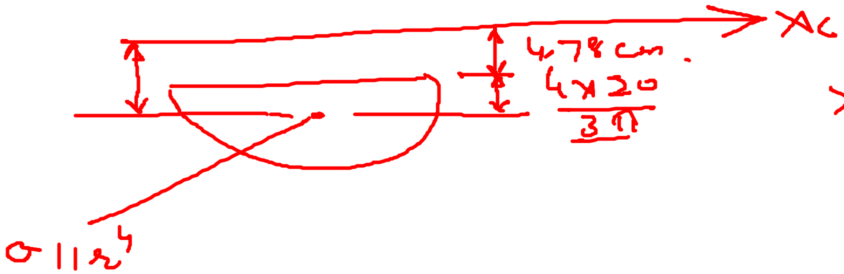
$$I_{x_c x_c} = I_1 + I_2 - I_3$$

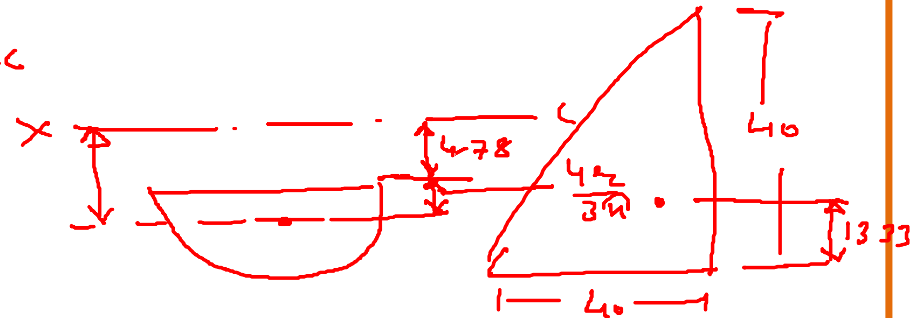
$$I_1 = I_{x_{c_1}} + A\bar{y}_1^2 = \frac{40\times 40^3}{36} + \frac{1}{2}\times 40\times 40\times(4.78-13.33)^2 =$$

$$I_2 = I_{x_{c_2}} + A\bar{y}_2^2 = 0.11\times(20)^4 + \frac{\pi\times(20)^2}{2}\times\left(\frac{4r}{3\pi}+4.78\right)^2 =$$

$$I_3 = I_{x_{c_3}} + A\bar{y}_3^2 = \frac{\pi\times(10)^4}{4} + \pi(10)^2\times(4.78)^2 =$$







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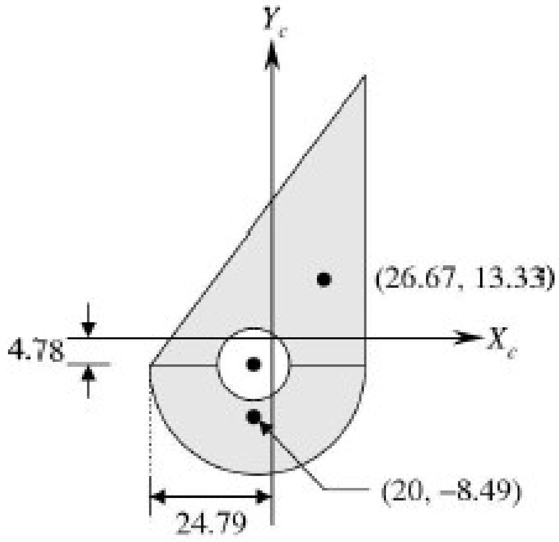

S.No	$(\bar{I}_{xx})_i$ cm^4	$(\bar{I}_{yy})_i$ cm^4	$A_i(\bar{y}_i - \bar{y})^2$ cm^4	$A_i(\bar{x}_i - \bar{x})^2$ cm^4
1.	$40 \times (40)^3/36$ $= 71111.11$	$40 \times (40)^3/36$ $= 71\ 111.11$	$800(13.33 - 4.78)^2$ $= 58\ 482$	$800(26.67 - 24.79)^2$ $= 2827.52$
2.	$0.11(20)^4$ $= 17\ 600$	$\pi(20)^4/8$ $= 62\ 831.85$	$628.32(-8.49 - 4.78)^2$ $= 110\ 642.69$	$628.32(20 - 24.79)^2$ $= 14\ 416.24$
3.	$-\pi(10)^4/4$ $= -7853.98$	$-\pi(10)^4/4$ $= -7853.98$	$-314.16(0 - 4.78)^2$ $= -7178.05$	$-314.16(20 - 24.79)^2$ $= -7208.12$
$\Sigma =$	80 857.13	126 088.98	161 946.64	10 035.64

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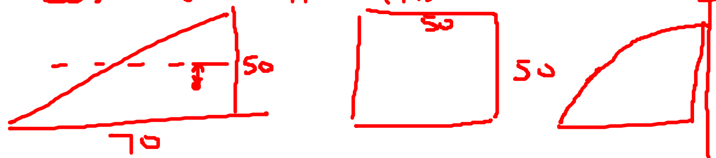
$$\bar{I}_{xx} = \sum(\bar{I}_{xx})_i + \sum A_i(\bar{y}_i - \bar{y})^2$$
$$= 80\,857.13 + 161\,946.64 = 242\,803.77 \text{ cm}^4$$

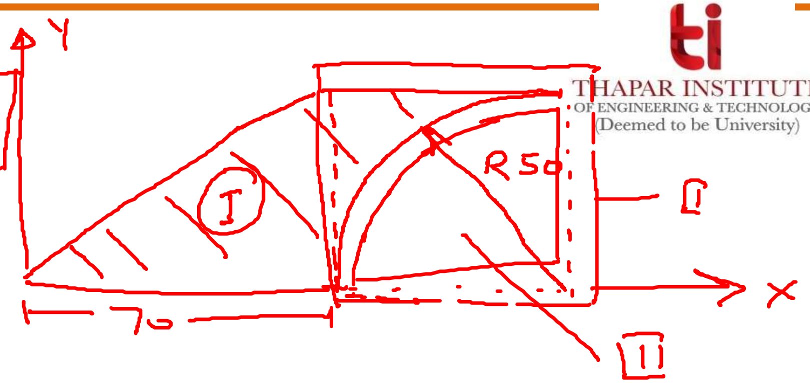
$$\bar{I}_{yy} = \sum(\bar{I}_{yy})_i + \sum A_i(\bar{x}_i - \bar{x})^2$$
$$= 126\,088.98 + 10\,035.64 = 136\,124.62 \text{ cm}^4$$

Compute MOI about the centroidal x-axis.

$X_c = 54.76\ \text{mm}$

$Y_c = 21.87\ \text{mm}$





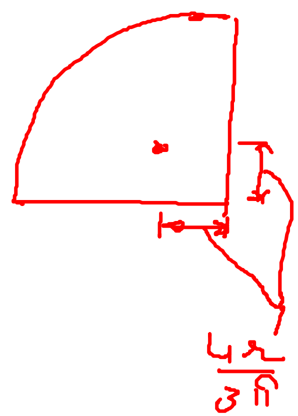
$$I_{x_c} = I_{1c} + I_{2c} - I_{3c}$$

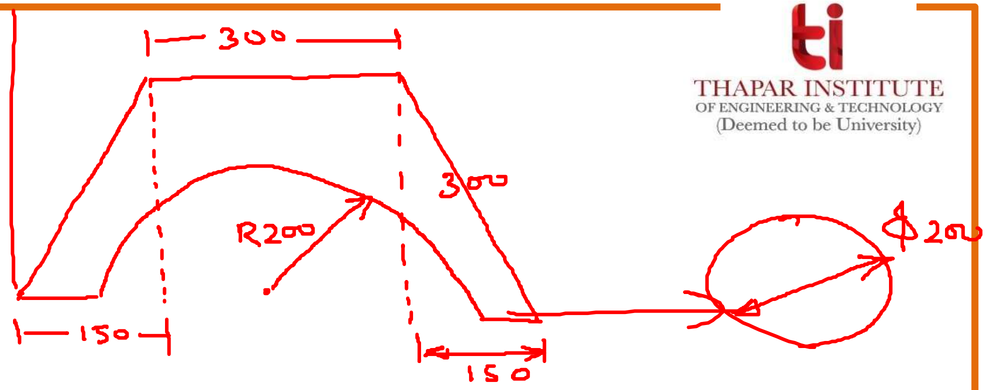
$$I_{1c} = \frac{70\times 50^3}{36} + \frac{1}{2}\times 70\times 50\times\left(\frac{50}{3} - 21.87\right)^2 = 290{,}430.17\ \text{mm}^4$$

$$I_{2c} = \frac{50\times 50^3}{12} + 50\times 50\times(21.87-25)^2 = 545325.58\ \text{mm}^4$$

$$I_{3c} = 0.055(50)^4 + \frac{\pi(50)^2}{4}\times\left(\frac{4\times 50}{3\pi} - 21.87\right)^2 =$$

$$I_{xc} = \underline{290430}.17 + \underline{545325}.58 - \ldots = 491935.67\ \text{mm}^4$$




300
300
R200
150
150
ϕ200

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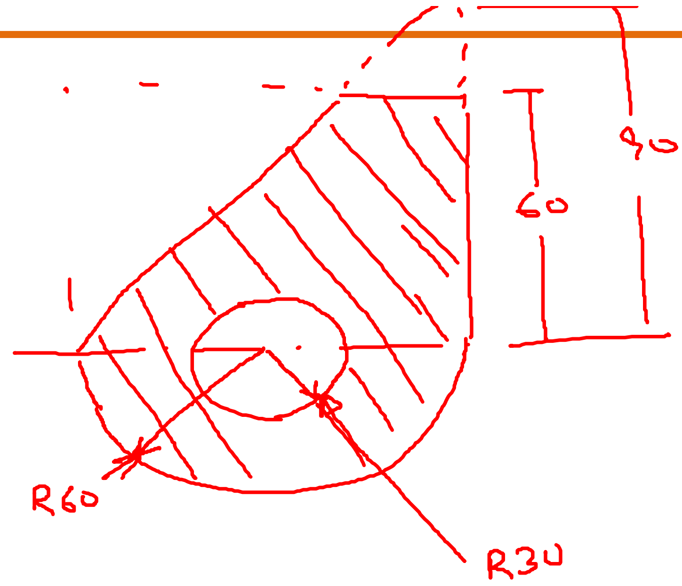

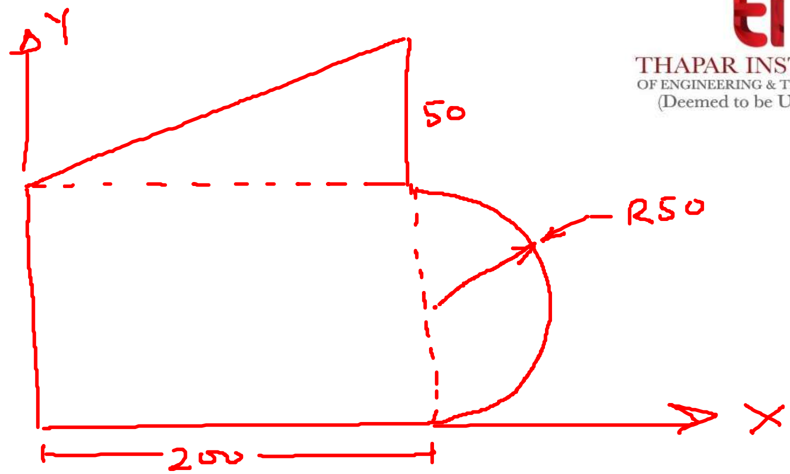

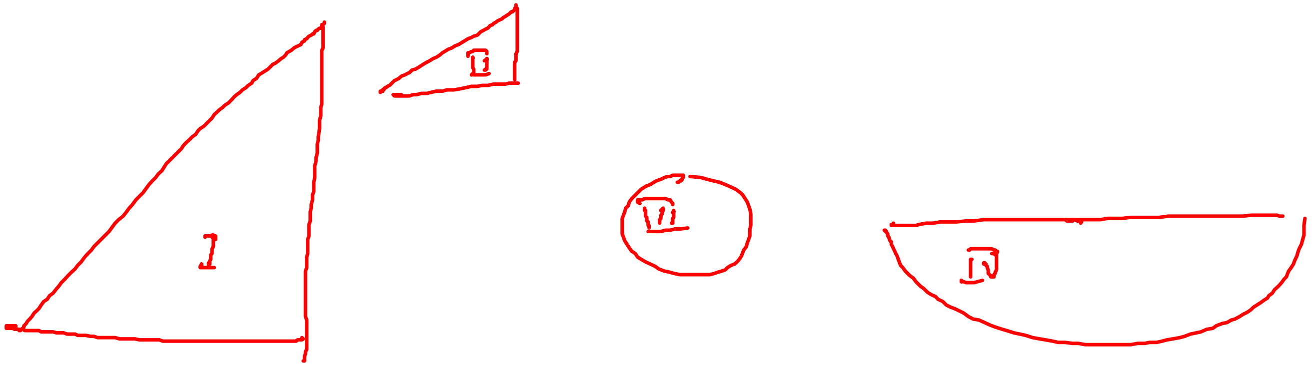

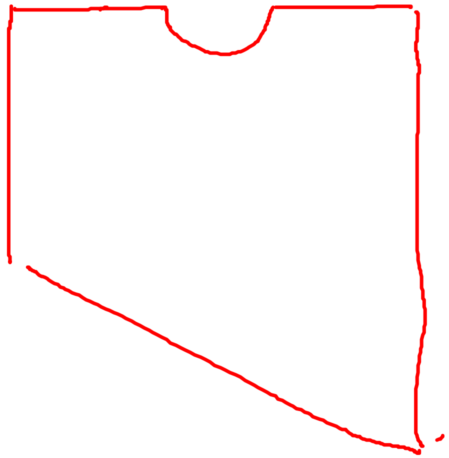

I + IV − II − III

Thank you