Discrete Mathematical Structures (UCS-405)
Tutorial Sheet-14

1. Consider the following homogeneous recurrence relation:

$$a_n=2a_{n-1}+3a_{n-2,} \quad a_0=1,\ a_1=2$$

   (a) Find the next three terms of the sequence.
   (b) Find the general solution.
   (c) Find the unique solution with the given initial conditions.

2. Consider the following homogeneous recurrence relation:

$$a_n=a_{n-1}+6a_{n-2,} \quad a_0=3,\ a_1=6$$

   (a) Find the next three terms of the sequence.
   (b) Find the general solution.
   (c) Find the unique solution with the given initial conditions.

3. Consider the set Q of rational numbers, and let $*$ be the operation on Q defined by

$$a * b = a + b - ab$$

   (a) Find: (i) $3 * 4$; (ii) $2 * (-5)$; (iii) $7 * (1/2)$.
   (b) Is $(Q, *)$ a semigroup? Is it commutative?
   (c) Find the identity element for $*$.
   (d) Do any of the elements in Q have an inverse? What is it?

4. Consider the group G = {1, 2, 3, 4, 5, 6} under multiplication modulo 7.
   (a) Find the multiplication table of G.
   (b) Find 2^{-1}, 3^{-1}, 6^{-1}.
   (c) Find the orders and subgroups generated by 2 and 3.
   (d) (d) Is G cyclic?

5. Compute the orders of the following groups and also calculate order of element of groups
   (a) U(3) Under multiplication modulo 3
   (b) U(5) Under multiplication modulo 5
   (c) U(20) Under multiplication modulo 20
   (d) U(15) Under multiplication modulo 15

6. Check Ring, ring with unity, commutative ring, ring with zero divisor, integral domain and field for following algebraic structure:
   a) (Z,+,*)
   b) (E,+,*) // E=set of even number
   c) R={0,1,2,3,4,5}, $(R,+_6,*_6)$