School of Mathematics
Thapar Institute of Engineering and Technology, Patiala,
UMA 004: Tutorial Sheet 01

1. Check whether the following differential equations are linear or nonlinear :

(i) $\left(\dfrac{d^2y}{dt^2}\right)^2+3\dfrac{dy}{dt}+x=0$ (ii) $a\dfrac{d^2y}{dt^2}=\left[-6\left(\dfrac{dy}{dt}\right)^3+9y\right]^{4/3}$ (iii) $\left(1+\dfrac{dy}{dt}\right)^2=\dfrac{d^2y}{dt^2}$

(iv) $\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}=x^2+\dfrac{d^2y}{dx^2}$ (v) $\dfrac{d^2y}{dt^2}+\sin(t+y)=\sin t$ (vi) $t^5\dfrac{d^4y}{dt^4}-t^3\dfrac{d^2y}{dt^2}+6y=0$

2. Find the solution of the following differential equations:

(i) $x\dfrac{dy}{dx}=(1-2x^2)\tan y$ (ii) $\dfrac{dy}{dx}=e^{2x-3y}+4x^2e^{-3y}$

(iii) $x(e^{4y}-1)\dfrac{dy}{dx}+(x^2-1)e^{2y}=0, x>0$ (iv) $y-x\dfrac{dy}{dx}=a\left(y^2+\dfrac{dy}{dx}\right)$

3. Solve the following differential equations:

(i) $\dfrac{dy}{dx}=\sin(x+y)+\cos(x+y)$ (ii) $\dfrac{y}{x}\dfrac{dy}{dx}+\dfrac{x^2+y^2-1}{2(x^2+y^2)+1}=0$

(iii) $\dfrac{dy}{dx}=(4x+y+1)^2$ (iv) $\dfrac{dy}{dx}-x\tan(x-y)=1$

4. Verify that the function is a solution of the differential equation on some interval for any choice of the arbitrary constants appearing in the function.

(i) $y=\dfrac{x^2}{3}+\dfrac{c}{x}$; $x\dfrac{dy}{dx}+y=x^2$ (ii) $y=\tan\left(\dfrac{x^3}{3}+c\right)$; $\dfrac{dy}{dx}=x^2(1+y^2)$

(iii) $y=\dfrac{1}{2}+ce^{-x^2}$; $\dfrac{dy}{dx}+2xy=x$ (iv) $y+\sin y=x$; $(y\cos y-\sin y+x)\dfrac{dy}{dx}=y$

5. The initial value problem governing the current I flowing in an L-R circuit, when a step voltage of magnitude E is applied to the circuit is given by

$$IR+L\frac{dI}{dt}=E \quad ; \quad t>0, \quad I(0)=0$$

Find the solution $I(t)$ and the limiting value of I as $t\to\infty$.

6. The temperature of the surface of a steel ball at time t is given by $u(t)=70e^{-kt}+30$ (in ^{o}F) where k is positive constant. Show that u satisfies the first-order equation $\frac{du}{dt}=-k(u-30)$. What is the initial temperature ($t=0$) on the surface of the ball? What happens to the temperature as $t\to\infty$

Answers:

2. (i) $\sin y=cxe^{-x^2}$ (ii) $3e^{2x}-2e^{3y}+8x^3=c$
(iii) $e^{4y}+1=e^{2y}(\log x^2-x^2+c)$ (iv) $(1-ay)(a+x)=cy$

3. (i) $x=\log[1+\tan\frac{1}{2}(x+y)]+c$ (ii) $x^2+2y^2-3\log(x^2+y^2+2)+c=0$
(iii) $4x+y+1=2\tan(2x+c)$ (iv) $\log\sin(x-y)=-\frac{1}{2}x^2+c$

5. (i) $I=\frac{E}{R}(1-e^{-\frac{R}{L}t})$, $I(t\to\infty)=\frac{E}{R}$

6. (i) $u(t=0)=100$, $u(t\to\infty)=30$