Supply Chain Management
–Model & Technology-

T. Murata

t-murata@waseda.jp

Room S223

Lecture 2

Lecture plan

1 Introduction (9/29)
2 Inventory theory (1)（10/06)
3 Inventory theory (2) & order operation(10/13)
4 Bullwhip effect & Beer Game(10/20)
5 Playing Beer Game (10/27)
6 No class (11/10) : Reading assignment
7 Network inventory (11/17)
8 Basics of MPS & MRP (11/24)
9 Capacity Planning & MRP-C (12/01)
10 TOC & DBR scheduling (12/08)
11 Just In Time & Lean manufacturing (12/15)
12 Forecasting & Demand Management (12/22)
13 Aggregate Planning and Transportation Planning(1/12)
14 Supplier Selection and B2B e-commerce (1/19)
15 Exam (report) No class (1/26)

Today's : contents

- Inventory model basics
  (under Deterministic Demand)
- Inventory management & EOQ
- Multi-product EOQ & lot aggregation

What is inventory?

- ✓ **A physical resource** that a firm holds in **stock** with the intent of selling it or transforming it into a more valuable state.
- ✓ **A buffering of physical resource against time lag** between demand and production and/or transportation
- ✓ **A buffering of physical resource against uncertain demand**

What is inventory management?

A **set of policies** and **controls** that ;

- ✓ **monitors levels of inventory ,**
- ✓ **determines what levels should be maintained,**
- ✓ when stock should be replenished, and
- ✓ how large orders should be placed.

Functions of Inventory

･To meet anticipated demand
→anticipated sales inventory

･To protect against stock-outs
→ safety stock

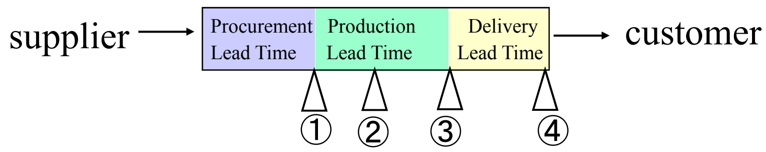
･To take advantage of order cycles
→ **cycle inventory**

･To smooth production requirements
→cushion to apart production from demand

･To hedge against price/demand changes
→ hedge inventory

･To take advantage of quantity discounts
→ hoarding（買いだめ）

Location of Inventories as buffer against time lag



①Raw materials & purchased parts

②Partially completed goods inside production system : *work in progress (WIP)*

③Finished-goods and Spare parts for maintenance inventories at manufacturing firms

④ Finished-goods inventories and Spare parts for maintenance at merchandise (*retail stores*)

Characteristics of Inventory

The inventory is affected by:

◆**Number of items:**

- (ex) Large manufacturer ~ 500,000 items
- (ex) Large Retailer ~ 100,000 items or more

◆**Objective priority**

- **Responsiveness to request**
- **Minimize costs**

◆**Order and demand lot size**

- Unit by unit, in cases, by the dozen, etc.

◆**Lead Time**

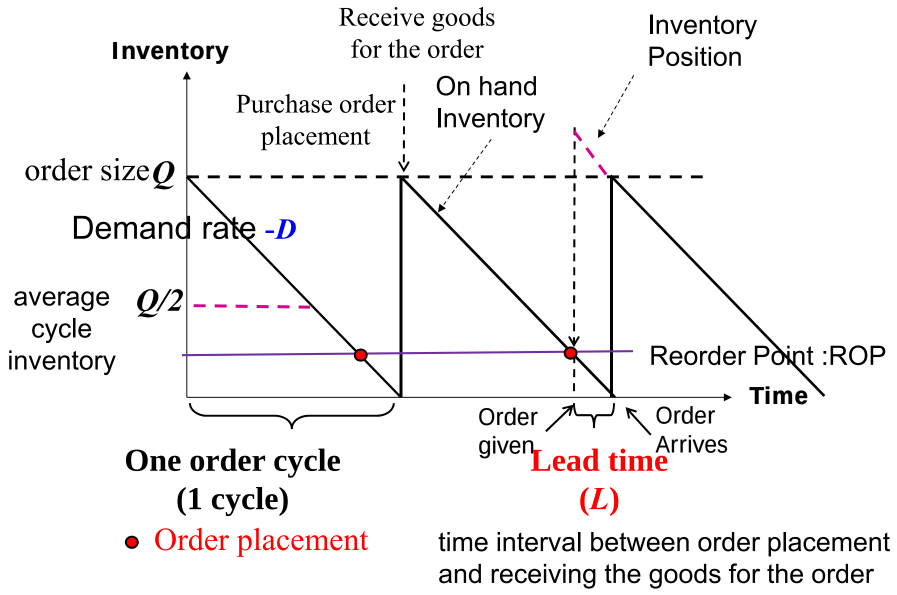
◆**Inventory Check Timing (*Periodic or Continuous)***

◆**Excess demand operation policy** ***(Backorder allowed or not)***

◆**Return policy** ***(Return inventory to supplier allowed or not)***

◆**Changing Inventory** ***(Perishable or Not)***

Cycle Inventory (Deterministic Demand rate)



Basics of Inventory management :
Key Terms of Inventory Costs

Holding (carrying) costs:

cost to carry an item in inventory for a length of time

Insurance, Maintenance and handling, Taxes, Opportunity Costs, Obsolescence (risk of loosing some of its value)

Ordering costs:

costs of ordering and receiving inventory

Shortage costs:

penalty costs for inventory shortage

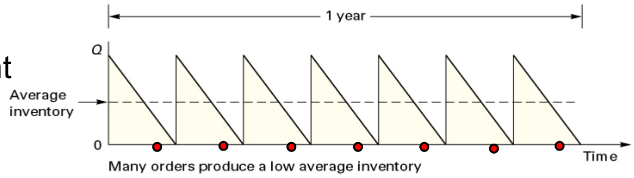
Total Inventory cost is =

Holding cost + Ordering cost + Shortage cost, and depending on order frequency

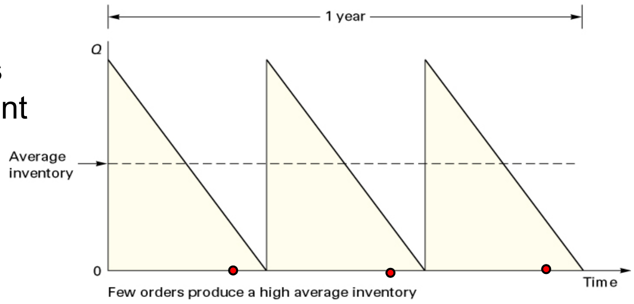
Order frequency & average inventory

Yearly total demand is given deterministically

More frequent



Less frequent



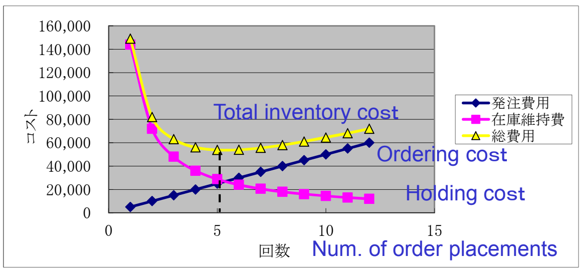
Order frequency affect total inventory cost.

Simulation

Yearly demand : 2,400 units
Ordering cost : 5,000円/order
Holding cost : 120円/unit

発注回数	発注費用	1回発注数	平均在庫	在庫維持費	総費用
1	5,000	2,400	1,200	144,000	149,000
2	10,000	1,200	600	72,000	82,000
3	15,000	800	400	48,000	63,000
4	20,000	600	300	36,000	56,000
5	25,000	480	240	28,800	*53,800
6	30,000	400	200	24,000	54,000
7	35,000	343	171	20,571	55,571
8	40,000	300	150	18,000	58,000
9	45,000	267	133	16,000	61,000
10	50,000	240	120	14,400	64,400
11	55,000	218	109	13,091	68,091
12	60,000	200	100	12,000	72,000

Yearly total Inventory cost is given as a cost function of number of order placements



Basics of Inventory management

Optimal Inventory management is ***;***

total inventory cost *→ MIN*

st. (subject to) satisfying desired service level

What we want to know is optimal inventory management to minimize total inventory cost:

- How often the inventory status (of an item) should be determined?
  (在庫チェックサイクル)
- When a replenishment order should be placed?
  （発注タイミング）
- How large the replenishment order should be?
  （発注量）

⇒Basic theory: EOQ (Economic Order Quantity)

EOQ (Economic Order Quantity)

Problem domains of inventory operation and EOQ

Demand rate / Replenishment decision making	Deterministic (known)	Stochastic (uncertain)
Static*	static EOQ model and extensions	Newsvendor model
Dynamic**	Dynamic EOQ model : Wagner-Whitin	Safety inventory Dynamic replenishment Probabilistic EOQ model

* Static: Past replenishment operation does not influence current replenishment decision

** Dynamic: Past replenishment operation influence current replenishment decision

EOQ (under Deterministic Demand):

Optimal order lot size for minimizing total inventory cost (Developed by Harris in 1913)

Simple limited model but useful to realize essential points

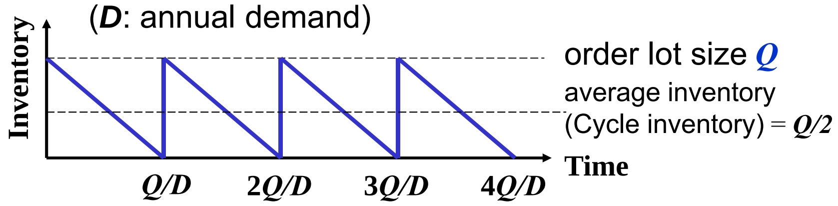
Assumptions;

1. Instantaneous production (no production lead time).
生産のリードタイムは考えない
2. Immediate delivery (no delivery lead time)
品目は発注と同時に入庫される（配送リードタイムゼロ）
3. Constant demand (consumption rate is constant)
品目は一定のスピード（*D*）で消費される(確定的）
4. Known fixed setup costs will be charged per order placement
発注費用は発注ごとに固定で発生する
5. Single product per order and no quantity discount
単一品目の発注である。(発注量による割引なし）

Calculation of **EOQ (1)**

no shortage operation policy case (品切れなし)

(***D***: annual demand)



the number of replenishments per term = D/Q

the ***Replenishment cycle time:*** $T = Q/D$

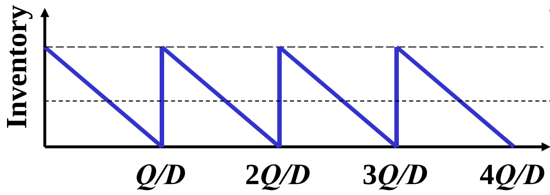
Total inventory cost per term:

$$T_C(Q) = \text{holding cost} + \text{order setup cost} + \text{purchasing cost}$$
$$= hQ/2 + AD/Q + CD$$

where

◆Holding cost incurred per unit per term : h （一個あたりの保管費用/期）
◆Average price per unit purchased: C （品目一個の購入コスト ）
◆Fixed order setup cost incurred per lot : A （発注１回あたり発注コスト ）

Calculation of **EOQ (1)**



Annual demand: D

order lot size Q

average inventory (Cycle inventory) = $Q/2$

Time

Want to decide Q* to minimize total inventory cost per term:

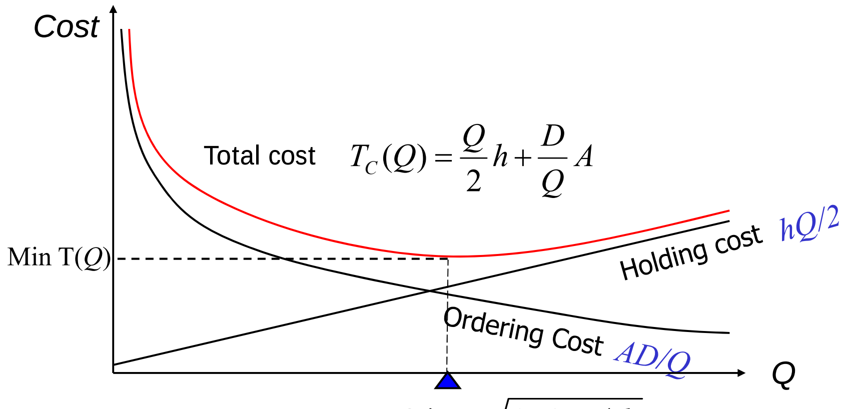
To minimize $T_C(Q)$, take the first derivative with respect to Q and set it to zero then we get Optimal order lot size (EOQ)

$$\frac{d}{dQ} T_C(Q) = \frac{d}{dQ}(hQ/2 + AD/Q + CD) = 0$$

$$Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2(\text{Order Setup Cost})(\text{Annual Demand})}{\text{Holding Cost}}}$$

Calculation of **EOQ (1)**

Total Cost Curve of EOQ is U-Shaped (Convex)



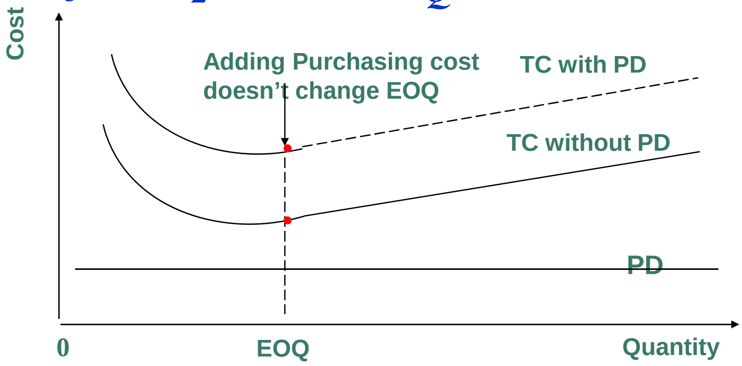
$$Q^* = \sqrt{2AD/h}$$

Calculation of **EOQ (1)**

Total Costs with consider Purchasing Cost

EOQ models isn't influenced by constant purchase price

	Annual carrying cost		Annual ordering cost		Purchasing cost
$T_C =$	$\frac{Q}{2}h$	+	$\frac{D}{Q}A$	+	PD



Calculation of **EOQ (1)**

Optimal order cycle time= $Q^*/D = \sqrt{2A/hD}$

Number of orders per term = $D/Q^* = \sqrt{hD/2A}$

Average Flow time = $Q^*/2D = \sqrt{A/2hD}$

Average holding cost per term = $hQ^*/2 = \sqrt{hAD/2}$

Average ordering cost per term = $AD/Q^* = \sqrt{hAD/2}$

Where $Q^* = \sqrt{2AD/h}$

Findings:

◆If demand increases by a factor of k, the optimal lot size , the number of orders placed per term and inventory holding cost should also increase by a factor of $\sqrt{k}$.

◆ If demand increases by a factor of k, flow time attributed to cycle inventory should decrease by a factor of $\sqrt{k}$.

◆To reduce optimal lot size by a factor of k, the fixed order cost must be reduced by a factor of k^2

Example 2-1 **Calculation of EOQ (1)**

Number 2 pencils at the bookstore are sold at a fairly steady rate of 60 per week. The pencils cost the book store ¥2 each and sell for ¥15 each. It costs the book store ¥1200 to initiate an order, and holding costs are based on an annual interest rate of 25% for pencils cost .

Determine the EOQ (optimum number of pencils for the bookstore to purchase) and the time between placements of orders. What are the yearly holding and order setup costs for the item?

Example 2-1: Solution

- Ordering cost A=¥1200 per order,
- Demand quantity D =60units*52weeks=3120 unit per year
- Inventory holding cost h = 0.25*2=¥ 0. 5 per unit per year

Q* (EOQ) : Optimal Order Quantity that Minimizes Average Total Cost

$= (2\,AD/h)^{1/2} = (2*1200*3120/\ 0.\ 5\ )^{1/2}$

= 3870 units

- Order cycle time = Q^*/D =3870/3120 = 1.24 years
- Number of orders per year = $D/Q^*=$ 0.8
- Average flow time = $Q^*/2D$ = 0.62 years
- Average annual holding cost = $h\,Q^*/2$ =0.5*3870/2=¥967.5
- Average annual ordering cost = AD/Q^* = 1200*3120/3870=¥967.5

Example 2-2

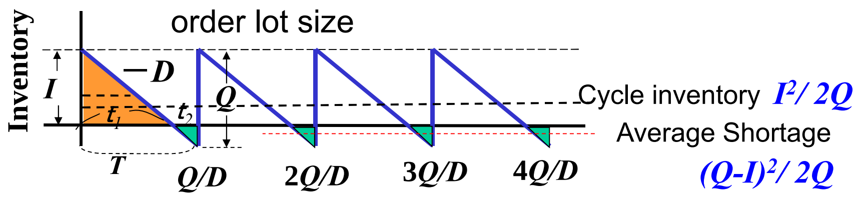
If desired lot size = Q^* = 200 units, what would
Order setup cost A have to be?

･*Demand* D = 12000 units
･Product purchasing cost per unit = ¥500
･*Holding cost* h = 20 % of purchasing cost =500×0.2=100

Solution:
Use EOQ equation and solve for A:
$A = [h\,(Q^*)^2]/2D$ = [100×(200)²] / (2 × 12000) = ¥166.67

Calculation of **EOQ (2)**

Permit shortage operation policy (品切れあり)



･*Order cycle time:* $T = Q/D$

･*Inventory exist time:* $t_1 = T \times (I/Q) = I/D$

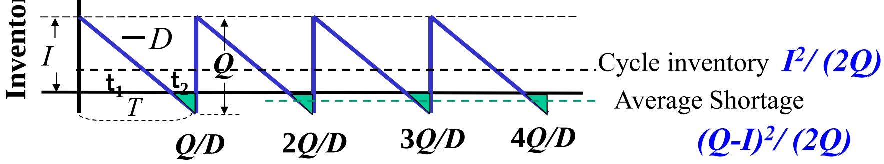
･*Cycle inventory = area* $/T = \{(I \times t1)/2\}/T = I^2/(2Q)$

･*Inventory shortage time:* $t_2 = T \times (Q-I)/Q = (Q-I)/D$

･*Average shortage* = ***area*** $/T = \{((Q-I) \times t2)/2\}\ /T = (Q-I)^2/(2Q)$

Calculation of **EOQ (2)**

Cost model of Permit shortage operation policy



Total inventory cost per term with shortage permission :

$T_C\ (Q, I)$ = *holding cost* + *order setup cost* + *shortage cost* + *purchasing cost*

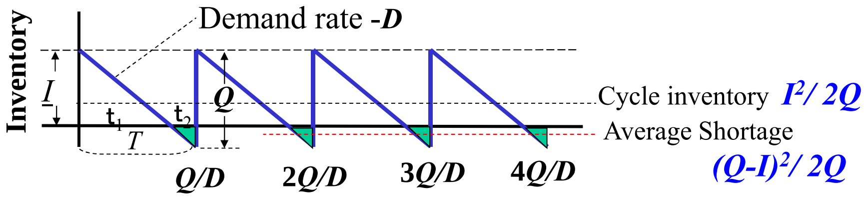
$$= I^2h/(2Q) \ + AD/Q \ + Cs(Q - I)^2/2Q \ + CD$$

where

- ◆Fixed order setup cost incurred per lot : A
- ◆ Holding cost incurred per unit per term : h （一個あたりの保管費用/期）
- ◆ Average price per unit purchased C
- ◆ Initial inventory I
- ◆ shortage cost per unit per term Cs (一個の期当たり品切れコスト)

New trade-off exists : shortage cost *v.s.* holding cost
⇒ Decide optimal Q and I for total cost minimization

Calculation of **EOQ (2)**



To minimize $T_C(Q, I)$, take the first partial derivative with Q and I respectively, and set it to zero for deciding EOQ and I

$$\frac{\partial}{\partial Q} T_C(Q, I) = 0, \quad \frac{\partial}{\partial I} T_C(Q, I) = 0$$

Solve Simultaneous equation

then we get Optimal order lot size (EOQ)

$$\text{EOQ} = Q^* = \sqrt{2AD/h} \times \sqrt{(h + Cs)/Cs}$$

$$I^* = \sqrt{2AD/h} \times \sqrt{Cs/(h + Cs)}$$

Exercise 2-1 : Calculation of EOQ (2)

Company X produces some product and purchases its main part P from company Y. Demand of part P is D = *31,025* unit per year, ordering cost A = *¥4,000* per order, holding cost h = *¥3650* per unit per year, and shortage cost Cs = *¥365,000* per unit per year. Lead time L = *2* days (0.0055 year). Decide *EOQ* and initial inventory I of part P and reorder point r in that case.

X社ではある製品を量産している。その主要部品であるP部品をY社から購入している。P部品の年間あたりの需要 $\boldsymbol{D}$ = 30,000個、発注費用 $\boldsymbol{A}$ = 4,000円/回、一個当たりの在庫保管費用 $\boldsymbol{h}$ = 3,650円/年、一個当たりの品切れ損失$\boldsymbol{Cs}$ = 365,000円/年, リードタイム2日（＝0.0055年）とする。年間のP部品の在庫コストを最小とする最適発注量 *EOQ* と年初の初期在庫量 $\boldsymbol{I}$, 再発注点r をもとめよ。

Extension of EOQ model:

Economic Production Quantity (EPQ)

- Production done in batches or lots
- Capacity to produce a part exceeds the part’s usage or demand rate
- Assumptions of EPQ are similar to EOQ except orders are received incrementally during production

2.4 Extension of EOQ model (1)

Economic Production Quantity

Assumptions

Only one item is involved

Annual demand is known

Usage rate is constant

Usage occurs continually

Production rate is constant

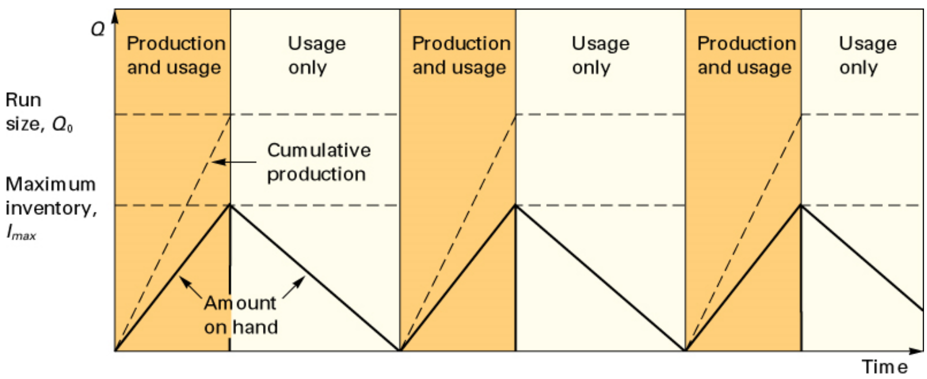
Lead time does not vary

No quantity discounts

Extension of EOQ model (1)

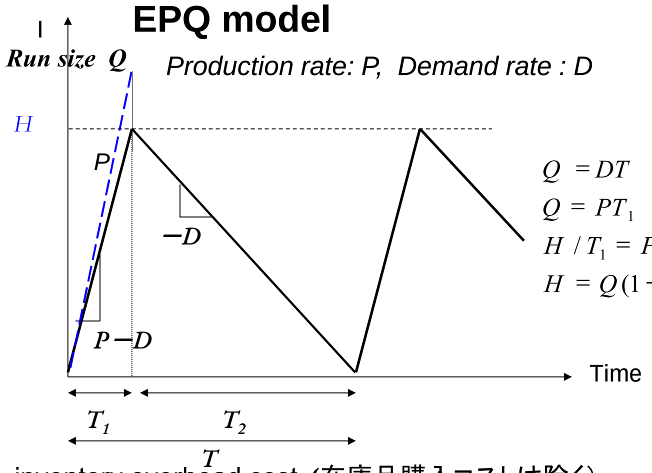
EPQ Model

Run size



EPQ model

Production rate: P, Demand rate : D



$Q = DT$ Demand per cycle

$Q = PT_1$ Supply per cycle

$H / T_1 = P - D$ Gradient

$H = Q(1 - D / P)$

$$Q^* = \sqrt{\frac{2AD}{h'}}$$

$h' = h\,(1 - D/P\,)$

inventory overhead cost (在庫品購入コストは除く)

$$T_C(Q) = \frac{A}{T} + \frac{hH}{2} = \frac{AD}{Q} + \frac{hQ}{2}(1 - \frac{D}{P}) \Rightarrow \frac{d}{dQ}T_C(Q) = 0$$

Fixed order setup cost incurred per lot : A

Holding cost incurred per unit per term : h

Exercise 2.3 EPQ Model

A toy manufacturer uses 48,000 rubber wheels per year for its popular dump truck series (D/year) . The firm makes its own wheels, which it can produce at a rate of 800 per day (P/day). The toy trucks are assembled uniformly over the entire year. Holding cost (h) is \$1 per wheel a year. Setup cost (A) for a production run of wheels is \$45. the firm operates 240 day per year.

Determine the Optimal run size

Extension of EOQ model (2)

EOQ with constraints

制約条件下でのEOQ

In the case that there are upper limit of total available investment on product inventory or upper limit of total storage space.

In these case, EOQ decision for only one kind of product is not suitable for deciding multiple product order lot size.

We need to decide it under constraints of operation on order placement operation.

Extension of EOQ model (2)

Optimal order lot size decision under constraint that there is upper limit of storage space

D : demand for product
A : ordering cost for product
h : holding cost for product
β: necessary storage space for product
Ws: Total available storage space

Formulation problem as

Minimize total inventory overhead cost with Q

$$T_C(Q) = \frac{AD}{Q} + \frac{hQ}{2} \rightarrow Min$$

Subject to $\beta Q \leq Ws$

Extension of EOQ model (2)

Calculate EOQ_j without considering constraints of $\beta Q \le Ws$

If EOQ satisfies the constraints, then EOQ are optimal lot size.

If EOQ do not satisfy the constraints, then we solve the following **NLP optimization problem** using Lagrangian method.

$$T_C(Q) = \frac{AD}{Q} + \frac{hQ}{2} \to Min$$

st. $\beta Q \le Ws$

$$L = \left(\frac{AD}{Q} + \frac{hQ}{2}\right) + \lambda(\beta Q - W_s) \to Min$$

λ: Lagrangean undetermined multiplier

Extension of EOQ model (2)

Optimal solution:

$$L = \left(\frac{AD}{Q} + \frac{hQ}{2}\right) + \lambda(\beta Q - W_s) \to Min$$

partial differentiation with Q and λand set to 0

$$\begin{cases} \partial L / \partial Q = -AD / Q^2 + h/2 + \beta\lambda = 0 \cdots\cdots(1) \\ \partial L / \partial \lambda = \beta Q - W_s = 0 \cdots\cdot(2) \end{cases}$$

Solve the Simultaneous equation

From (1) $Q = \sqrt{2AD/(h+2\beta\lambda)} \cdots\cdots(3)$

How to decide λ and Q :
Substitute (3) to (2) and calculate λ to decide it.
Q can be calculated by formula (3) using value of the decided λ

Example 2-4 **Multi-product EOQ with constraints**

Company A purchases 3 kinds of part A, B, and C. Manager wants to control total payment for order placement of all parts not exceeding ¥4,500,000. No inventory shortage operation and part holding costs are 20% of each price per unit purchased product. Data for each product is shown below. Decide EOQ of each product.

parts	A	B	C
demand (unit)	1000	1000	2000
unit price (¥)	15000	6000	24000
Ordering cost (¥)	15000	15000	15000

Example 2-4 Solution

parts	A	B	C
demand (unit)	1000	1000	2000
unit price (¥)	15000	6000	24000
Order cost (¥)	15000	15000	15000

➢**Calculate *EOQ* Q_j* of part *j* (*j=A, B, C*) : without considering** *total payment for order placement of all parts*

By using EOQ formula $Q_j^* = \sqrt{2A_jD_j/h_j}$ $(j = A, B, C)$,

Q_A*=100, Q_B*=158, Q_C*=112.

These Q_j* do not satisfy the constraints because total investment for parts inventory is 100 × 15000 + 158 × 6000 + 112 × 24000=5,136,000 > 4,500,000

➢**We calculate Q_j by using formula (3) in 3.1 and substitute Q_j to formula (2) to find λ so that Q_j satisfies the constraint of the** *total payment for order placement* .

$$15000Q_A + 6000Q_B + 24000Q_C =$$

$$15000\sqrt{2\times15000\times1000/(3000+30000\lambda)} + 6000\sqrt{2\times15000\times1000/(1200+12000\lambda)}$$

$$+24000\sqrt{2\times15000\times2000/(4800+48000\lambda)} = 4{,}500{,}000$$

Here, we get $\lambda \fallingdotseq -0.0067$, **and**

$Q_A^* \fallingdotseq 103.5 \fallingdotseq 103$, $Q_B^* \fallingdotseq 163.7 \fallingdotseq 164$, $Q_C^* \fallingdotseq 81.8$, $\fallingdotseq 82$

where **Total investment for parts inventory** = ¥ 4,497,000 <4,500,000

Sensitivity of EOQ to Quantity Q (感度解析)

inventory overhead cost (在庫品購入コストを除いた在庫管理費用)

$T_C(Q) = hQ/2 + AD/Q$

Optimal inventory overhead cost per unit per term
(在庫品購入コストを除いた一個あたり維持費用/期)

$$u^* = \frac{T_C(Q^*)}{D} = \frac{hQ^*}{2D} + \frac{A}{Q^*} = \frac{h\sqrt{2AD/h}}{2D} + \frac{A}{\sqrt{2AD/h}}$$

$$= \frac{2A}{\sqrt{2AD/h}}$$

$$Q^* = \sqrt{2AD/h}$$

Optimal inventory overhead cost per term：

$$T_C(Q^*) = u^* \times D = u^* \times D = \sqrt{2ADh}$$

Sensitivity of EOQ to Quantity Q (cont.)

Inventory overhead cost using $Q'(\neq Q^*)$

$$T_C(Q') = \frac{hQ'}{2} + \frac{AD}{Q'}$$

Cost ratio: $$\frac{T_C(Q')}{T_C(Q^*)} = \frac{hQ'/2 + AD/Q'}{\sqrt{2ADh}}$$

$$Q^* = \sqrt{2AD/h} \searrow \; = \frac{hQ'/2 + Q^{*2}h/2Q'}{Q^*h} = \frac{1}{2}\left[\frac{Q'}{Q^*} + \frac{Q^*}{Q'}\right]$$

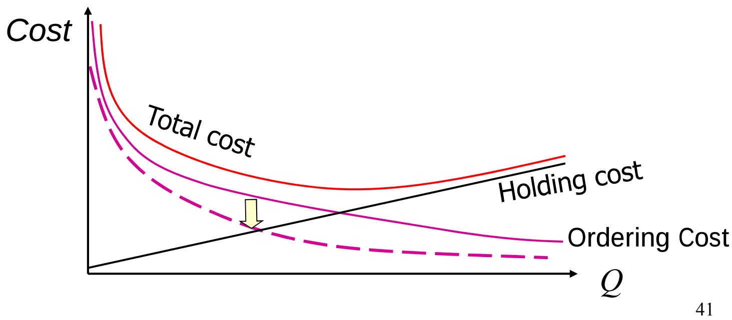
Example: If $Q' = 2Q^*$, then the ratio of the actual to optimal cost is $(1/2)[2 + (1/2)] = 1.25$

This means If we make order lot size **twice as large as EOQ**, total inventory cost without cost of inventory goods purchase cost will only increase **25%**

Advice： We can conclude that $T_C(Q)$ is relatively insensitive to **EOQ**. A firm is often better served by ordering a convenient lot size close to EOQ rather than the precise EOQ

Policy for decreasing total inventory cost

It is not enough to keep only EOQ policy for decreasing total inventory cost. As shown by sensitivity analysis of EOQ, order lot size variation around EOQ does not give so much effect on total inventory cost change, so we should make effort to decrease ordering cost at the same time for decreasing total inventory cost effectively.



Power of two order interval policy (2のべき乗方策)

Optimal order interval with EOQ policy is given by next formula:

$$T^* = Q^*/D = \sqrt{2A/hD}$$

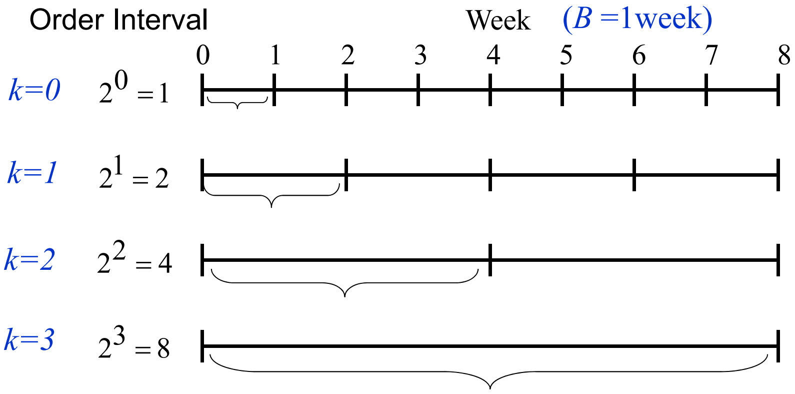
However, value of S^* is given as any real number and it is not feasible to do order operation according to exact interval S^*.

(EOQから計算されるS^*（正のが実数値あるいは丸め値）で発注サイクルを運用することは現実的ではない（受発注オペレーションにおいて実務的不具合を生じやすい。）

In actual order operation, order interval value is usually restricted to a value $B2^k$, where B is a standard unit time (ex. 1day, 1week or 1month) and k is the minimum integer which satisfies the following formula:

$$\sqrt{2}B2^{k-1} \leqq (Q^*/D) \leqq \sqrt{2}B2^{k} \quad \text{where} \quad Q^* = \sqrt{2AD/h}$$

Power of two order interval policy (2のべき乗方策)



Power of two order interval policy is an approximation of EOQ operation.

It has been proved that this policy increases total inventory cost at most 6 % as compared with exact EOQ operation.

Example 2-3

(1) Decide optimal order interval T^* in case of the following by power of two order interval policy

- ***Ordering cost A = ¥500 per order,***
- ***Demand quantity D = 1000 unit per year***
- ***Inventory holding cost h = ¥ 35 per unit per year***

(2) Compare total inventory cost $T_C(Q)$ in case of optimal order interval policy and in case of power of two order interval policy

Example 2-3 Solution

Ordering cost A=¥*500 per order*, **Demand quantity** D=*1000 unit per year*

Inventory holding cost h= ¥ *35 per unit per year*

(1) $Q^* = (2AD/h)^{1/2} = (2*500*1000/35)^{1/2} = 169$ **units**

$S^* = Q^*/D$ =169/1000 years =61.7 days

Round to Nearest Power-of-Two: B=1 (day)

T^* =61.7 is between $2^5\sqrt{2} = 45.25$ and $2^6\sqrt{2} = 90.5$,

therefore $k = 6$ (*i.e.* $2^6 = 64$) and S^* to 64 days

(2) $Q' = T^*D = (64/365)1000 = 175.$

$$Y(Q') = \frac{hQ'}{2} + \frac{AD}{Q'} = \frac{35(175)}{2} + \frac{500(1000)}{175} = \$5,920$$

c.f.

$$Y(Q^*) = \frac{hQ^*}{2} + \frac{AD}{Q^*} = \frac{35(169)}{2} + \frac{500(1000)}{169} = \$5,916$$

Only 0.07% error

Aggregating Multiple Products in a Single Order

- A key to reducing cycle inventory is the reduction of lot size.
- A key to reducing lot size without increasing cost is to reduce the fixed cost associated with each lot.
- This may be achieved by reducing the fixed cost itself or by aggregating lots across multiple products, customers, or suppliers.
- Transportation is a significant contributor to the fixed cost per order. Can possibly combine shipments of different products from the same supplier, can have a single delivery coming from multiple suppliers, or can have a single delivery to multiple retailers
  - **same overall fixed cost**
  - **effective fixed cost is reduced for each product**
  - **lot size for each product can be reduced**

Example 2-5

Suppose there are 3 computer products Litepro, Medpro, and Heavypro. Assume demand per year for these three products D_L= 12000 units for Litepro, D_M=1200 units for Medpro, and D_H=120 units for Heavypro per month. Each model costs $500 and common ordering cost including transportation cost is $4000 per order and an additional fixed cost of $1000 is incurred for each product respectively. Assume holding cost is 20% of unit cost

Demand, D_L = 12000 /year , D_M =1200 /year, D_H =120 /year
Common order cost A = $4000,
Product specific order cost $A_L = A_M = A_H$ = $1000
Unit cost $C_L = C_M = C_H$ = $500
Holding cost ratio r to unit cost = 0.2

Example 2-5 Analysis

Lots are ordered and delivered independently

order setup cost *A' =A+product specific cost($1000) =$5000*

	Litepro	*Medpro*	*Heavypro*
Demand per year	**12,000**	**1,200**	**120**
Fixed cost / order	**$5,000**	**$5,000**	**$5,000**
Optimal order size	**1,095**	**346**	**110**
Order frequency	**11.0 / year**	**3.5 / year**	**1.1 / year**
Annual cost (Including ordering cost)	**$109,544**	**$34,642**	**$10,954**

$EOQ_L = \sqrt{2A'D_L / rC_L}$ (note: r =0.2)

Total Inventory Cost *= $109,544+$34,642+$10,954 =$155,140*

Example 2-6 Analysis cont.

Joint order setup cost $A^* = A + A_L + A_M + A_H = \$7000$

Total Ordering cost = $A^* \times n$ (Number of orders per term)

Order quantity per order = D_L/n , D_M/n, D_H/n

Total Holding cost = $(D_L\, r\, C_L/2n) + (D_M\, r\, C_M/2n) + (D_H\, r\, C_H/2n)$

Optimal number of order $\boldsymbol{n^*}$ *by EOQ (Taking first derivative of Total Holding cost +Total Ordering cost with respect to n and setting it to equal to 0)*

$$= \sqrt{(D_L rC_L + D_M rC_M + D_H rC_H)/2A^*} = \mathbf{9.75/year}$$

	Litepro	*Medpro*	*Heavypro*
Demand per year	**12,000**	**1,200**	**120**
Order frequency	**9.75/year**	**9.75/year**	**9.75/year**
Optimal order size	**1,230**	**123**	**12.3**
Annual holding cost	**$61,512** **(without ordering cost)**	**$6,151**	**$615**

Annual ordering cost = 9.75 × \$7,000 = \$68,250

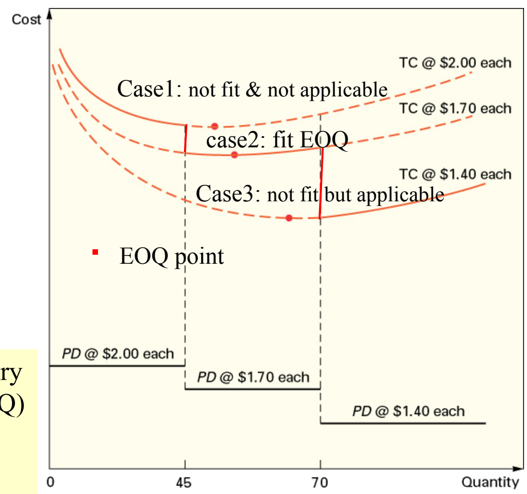
Annual total inventory cost = \$136,528 (12% decreased)

Other Policy for decreasing inventory cost

Purchasing Cost with quantity discount

EOQ models influenced by quantity discount

Order quantity	Unit Price
1 to 44	**$2.00**
45 to 69	**1.70**
70 or more	**1.40**



Need to compare total inventory cost of case 2 (order with EOQ) and of case 3 (order with minimum quantity 70) and select the less cost case.