Analyzed Structures

Van Emde Boas Tree Data Structure

Haania Siddiqui
Khubaib Sattar
Iqra Siddiqui
Shamsa Hafeez

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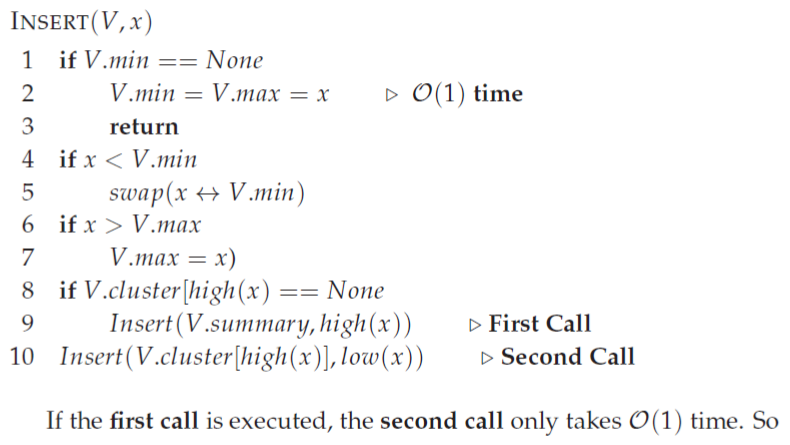
Splitting bit vector into clusters:

Let the bit vector i.e., our universe have u elements. This means that the range of universe would be $0, 1, 2, ..., u-1$. We split it in $\sqrt{u}$ number of clusters. Each cluster would be of size $\sqrt{u}$.
Dividend = Divisor * Quotient + Remainder
$x = i * \sqrt{u} + j$
$low(x) \; = \; x \, mod \sqrt{u} \; = \; j$
$high(x) \; = \left\lfloor \frac{x}{\sqrt{u}} \right\rfloor \; = i$
$V[x] = V.Cluster[i][j]$
$index(i, j) = i\sqrt{u} + j$

Example:
For example x = 12 and u = 16, Then $j = 12 \, mod \sqrt{16} = 0$
$i = \left\lfloor \frac{12}{\sqrt{16}} \right\rfloor = 3$
So, $V[12] = V.Cluster[3][0]$



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Insert($V, x$)
if $V.min == None$
    $V.min = V.max = x$        ▷ $\mathcal{O}(1)$ time
    return
if $x < V.min$
    $swap(x \leftrightarrow V.min)$
if $x > V.max$
    $V.max = x)$
if $V.cluster[high(x) == None$
    $Insert(V.summary, high(x))$        ▷ First Call
$Insert(V.cluster[high(x)], low(x))$        ▷ Second Call
```

If the **first call** is executed, the **second call** only takes $\mathcal{O}(1)$ time. So

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Insertion - Insert(V, x):

First Call: Mark cluster high(x) as non-empty i.e., Insert(V.summary, high[x]). Note that V.summary[i] indicates whether V.cluster[i] is non-empty. Also note that V.summary is of size $\sqrt{u}$.
Second Call: Set V.cluster[high(x)][low(x)] to 1 i.e., Insert(V.cluster[high(x)], low[x]). We know that V.cluster[i] is of size $\sqrt{u}$ ($\forall 0 \leq i \leq \sqrt{u}$)
O(1) time: We store minimum and maximum entry in each structure. This gives an O(1) time overhead for each insert operation.
If the first call is executed, the second call takes only O(1) time. So,
$T(u) = T(\sqrt{u}) + O(1)$
$T(u) = O(log \, log \, u)$

Similar recurrence equation is formed for **Successor** and **Delete** functions. This means that both of these functions are represented by :
$T(u) = T(\sqrt{u}) + O(1)$

Solving the recurrence relation:
Our goal is to solve :
$T(u) = T(\sqrt{u}) + O(1)$....................(i)
According to the master theorem:
$T(n) = aT(\frac{n}{b}) + O(n^d)$................(ii)
We will try to transform our problem from (i) to (ii).
Let u be the size of the universe with range $\{0, 1, 2, ..., u - 1\}$
Consider the following:
$u = 2^k$....................(iii)
Apply square root on both sides of equation (iii).
$\sqrt{u} = \sqrt{2^k}$
$\sqrt{u} = 2^{\frac{2}{k}}$..............(viii)
Turn the recurrence from recurrence in u to a recurrence in $k = log\, u$ ....(iv)

We define:
$S(k) = T(2^n)$...............(v)

Since,
$T(2) \leq O(1)$
$T(u) \leq T(\sqrt{u}) + O(1)$.............(vii)

We have,
$S(1) \leq O(1)$
$S(k) \leq S(\frac{k}{2}) + O(1)$

This means :
$S(k) = O(log\, k)$............(vi)

Put (viii) in (vii)
$T(u) = T(2^{log\, u})$
Using (v):
$T(u) = S(log\, u)$
Using (vi)
$T(u) = O(log\, log\, u)$

Time Complexity of Dijkstra's Algorithm on Van Emde Boas Tree:

Let n be the number of vertices. In worst case
$n = u$
We know that extract minimum is replaced by the successor function:
So the complexity of the Dijkstra's Algorithm under van emde boas tree data structure would be
O(log log u)

Time Complexity of Dijkstra's Algorithm on Min Heap:

Let us assume a graph with M edges and N vertices, then
The decreaseKey() operation in minHeap takes O(LogN) time. So overall time complexity is:
O(M+N)*O(LogN)
which is
O((M+N)*LogN)
= **MLogN**

Conclusions

As it can be noticed that Dijkstra Algorithm presented impressive results when utilized with Van Emde Boas Tree.
It calculated the nearest node in an efficient time complexity of O(log log u) where u is the universe size, while in case of Min Heap, it calculated it in O(MlogN)
Hence, one can conclude that nearest node can be found in an extremely efficient way with Van Emde Boas tree as compared to any other data structure as single source shortest path finding algorithms like Dijsktra Algorithm, reflects amazing results with VEB .

References:

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