Software Project Management

Week 4

1

Today's Lecture

- Developing Work-breakdown structure
- Estimating size of a software
- Developing Budgets and Schedules

2

Scheduling

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PMI Terminology

- Project Time Management
  - includes the processes required to accomplish timely completion of the project.
- Time Management Processes
  - Activity Definition – identifying the specific schedule activities that need to be performed to produce the various project deliverables.
  - Activity Sequencing – identifying and documenting dependencies among schedule activities.
  - Activity Resource Estimating – estimating the type and quantities of resources required to perform each schedule activity.
  - Activity Duration Estimating – estimating the number of work periods that will be needed to complete individual schedule activities.
  - Schedule Development – analyzing activity sequences, durations, resource requirements, and schedule constraints to create the project schedule.
  - Schedule Control – controlling changes to the project schedule

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Project Time Management Processes

- **6.1 Activity Definition –** identifying the specific schedule activities that need to be performed to produce the various project deliverables.
- **6.2 Activity Sequencing –** identifying and documenting dependencies among schedule activities.
- **6.3 Activity Resource Estimating –** estimating the type and quantities of resources required to perform each schedule activity.
- **6.4 Activity Duration Estimating –** estimating the number of work periods that will be needed to complete individual schedule activities.
- **6.5 Schedule Development –** analyzing activity sequences, durations, resource requirements, and schedule constraints to create the project schedule.
- **6.6 Schedule Control –** controlling changes to the project schedule.

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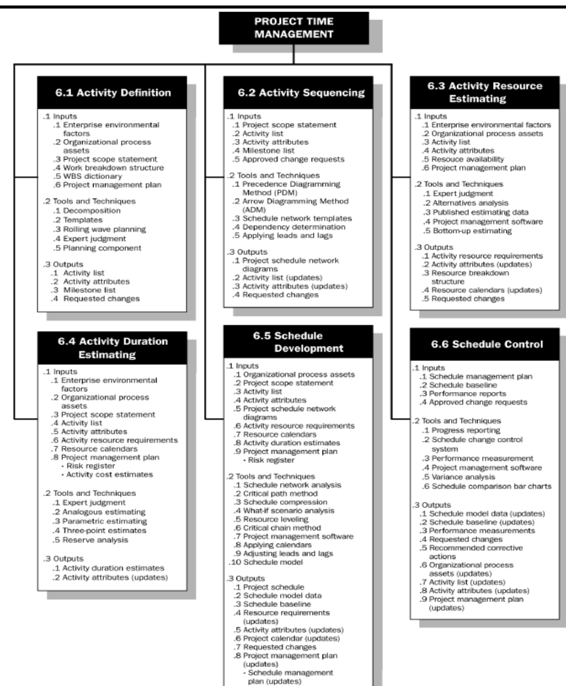

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Figure 6-1. Project Time Management Overview

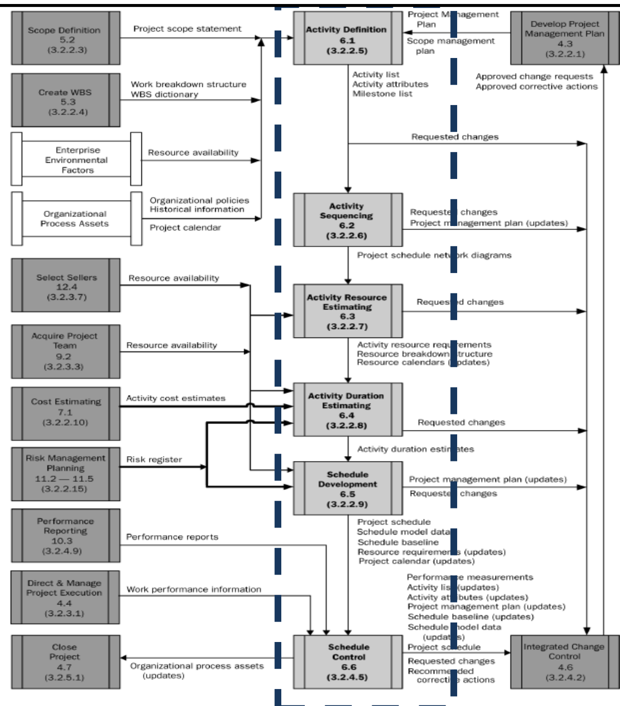

Figure 6-2. Project Time Management Process Flow Diagram

Note: Not all process interactions and data flow among the processes are shown.

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6.1. Activity Definition

Inputs

.1 Enterprise environmental factors
.2 Organizational process assets
.3 Project scope statement
.4 Work breakdown structure
.5 WBS dictionary
.6 Project management plan

Tools & Techniques

.1 Decomposition
.2 Templates
.3 Rolling wave planning
.4 Expert judgment
.5 Planning component

Outputs

.1 Activity list
.2 Activity attributes
.3 Milestone list
.4 Requested changes

Figure 6-3. Activity Definition: Inputs, Tools & Techniques, and Outputs

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6.2. Activity Sequencing

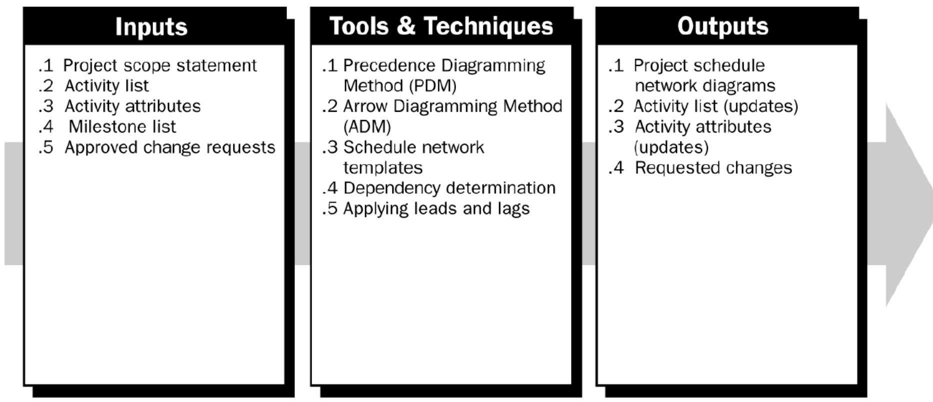

Figure 6-4. Activity Sequencing: Inputs, Tools & Techniques, and Outputs

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Networking Fundamentals

- Interdependencies of activities
- Project completion time
- Impact of late start
- Impact of early start
- Trade-offs between resources and time
- “What if” exercises
- Cost of a crash program
- Slippages in planning/performance
- Evaluation of performance

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Arrow Diagram Method (ADM)

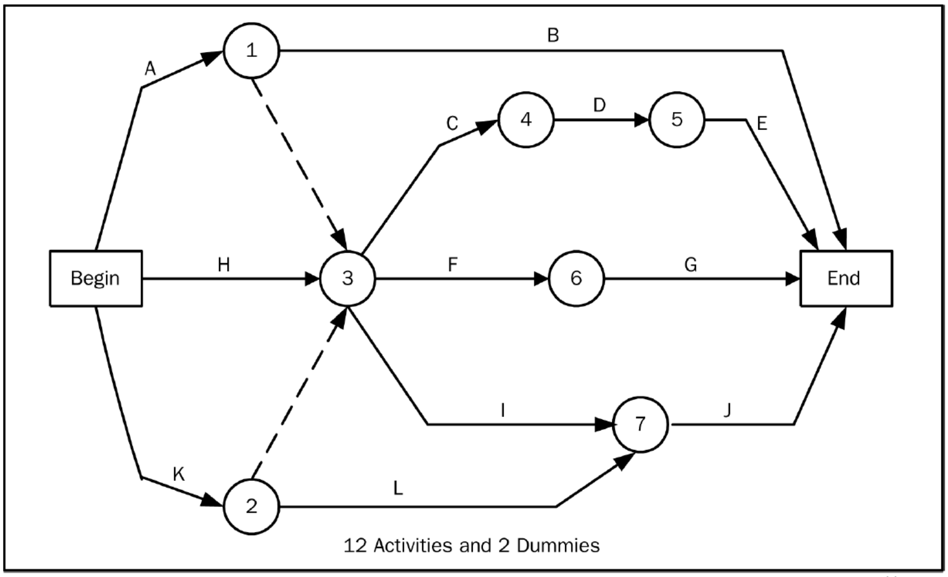

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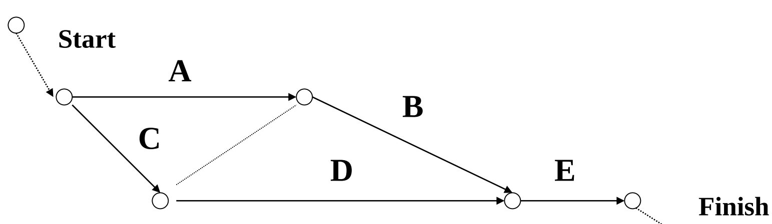
Figure 6-6. Arrow Diagram Method

ADM

- **Arrow diagramming method (ADM)** – Uses *arrows* to represent activities and *connecting nodes* to show dependencies



- **ADM** uses ***finish-to-start*** dependencies only and uses ***dummy activities*** to show logical relationships

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Some more arrow diagramming methods: CPM/PERT

- Critical Path Method (CPM)
  - E I Du Pont de Nemours & Co. (1957) for construction of new chemical plant and maintenance shut-down
  - Deterministic task times
  - Activity-on-node network construction
  - Repetitive nature of jobs
- Project Evaluation and Review Technique (PERT)
  - U S Navy (1958) for the POLARIS missile program
  - Multiple task time estimates (probabilistic nature)
  - Activity-on-arrow network construction
  - Non-repetitive jobs (R & D work)

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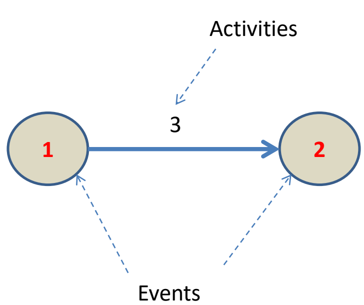
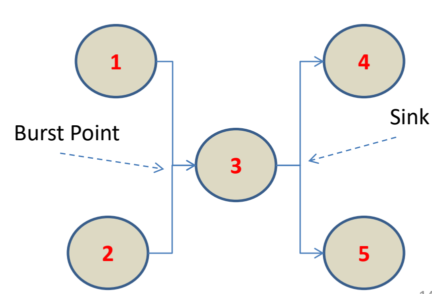
Nomenclature

- Networks are composed of events and activities





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Project Network

- Network analysis is the general name given to certain specific techniques which can be used for the planning, management and control of projects

- Use of nodes and arrows

  Arrows
  - ➔ An arrow leads from tail to head directionally
  - Indicate ACTIVITY, a time consuming effort that is required to perform a part of the work.

  Nodes
  - ● A node is represented by a circle
  - - Indicate EVENT, a point in time where one or more activities start and/or finish.
- Activity
  - A task or a certain amount of work required in the project
  - Requires time to complete
  - Represented by an arrow
- Dummy Activity
  - Indicates only precedence relationships
  - Does not require any time of effort

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Project Network

- Event
  - Signals the beginning or ending of an activity
  - Designates a point in time
  - Represented by a circle (node)
- Network
  - Shows the sequential relationships among activities using nodes and arrows

◆Activity-on-node (AON)

nodes represent activities, and arrows show precedence relationships

◆Activity-on-arrow (AOA)

arrows represent activities and nodes are events for points in time

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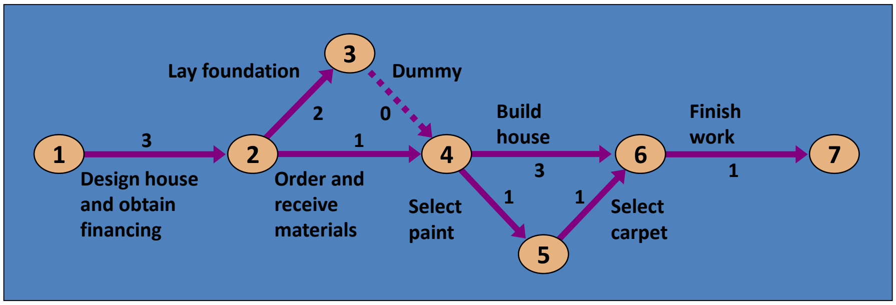
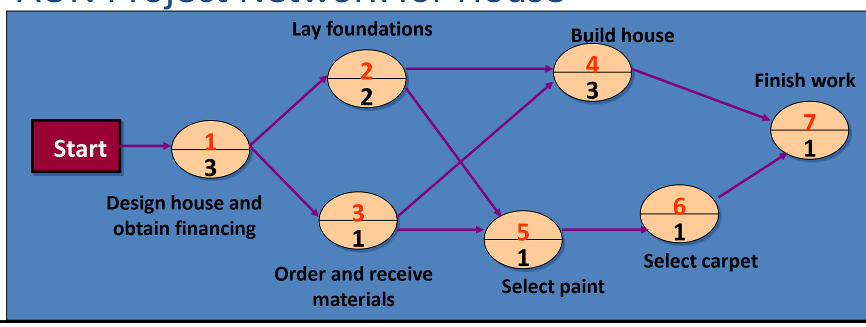
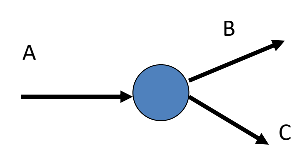
AOA Project Network for House



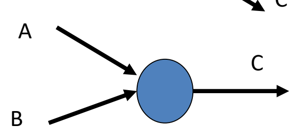
AON Project Network for House



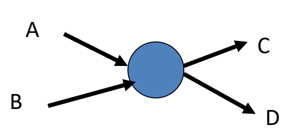
Situations in network diagram



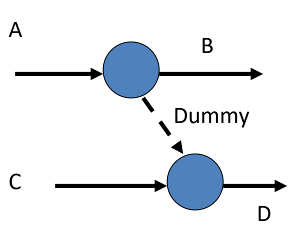
A must finish before either B or C can start



both A and B must finish before C can start



both A and C must finish before either of B or D can start



A must finish before B can start

both A and C must finish before D can start

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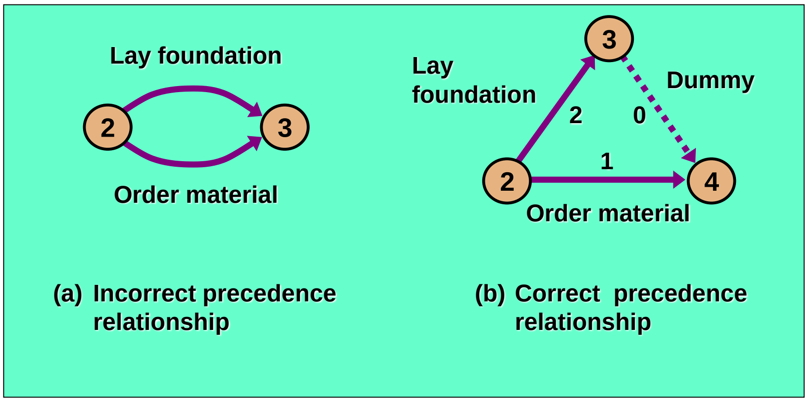
Concurrent Activities



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Network example

Illustration of network analysis of a minor redesign of a product and its associated packaging.

The key question is: ***How long will it take to complete this project ?***

Activity number		Completion time (weeks)
1	Redesign product	6
2	Redesign packaging	2
3	Order and receive components for redesigned product	3
4	Order and receive material for redesigned packaging	2
5	Assemble products	4
6	Make up packaging	1
7	Package redesigned product	1
8	Test market redesigned product	6
9	Revise redesigned product	3
10	Revise redesigned packaging	1
11	Present results to the Board	1

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For clarity, this list is kept to a minimum by specifying only immediate relationships, that is relationships involving activities that "occur near to each other in time".

Activity number		Activity number	
1	must be finished before	3	can start
2		4	
3		5	
4		6	
5,6		7	
7		8	
8		9	
8		10	
9,10		11	

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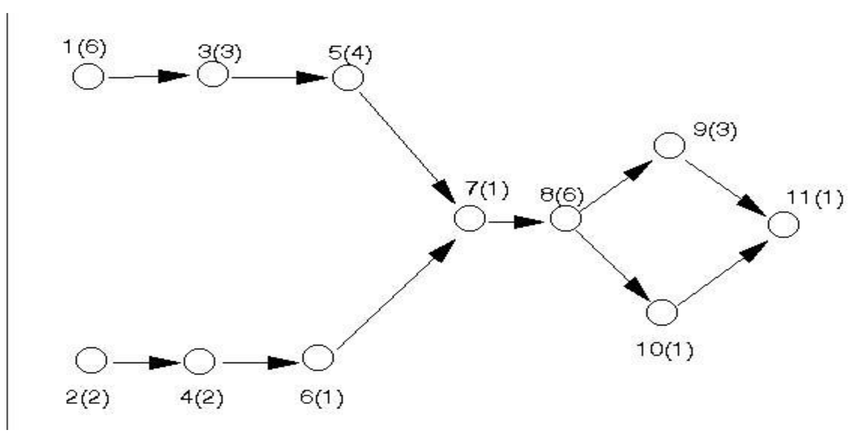
Questions to prepare activity network

- Is this a Start Activity?
- Is this a Finish Activity?
- What Activity Precedes this?
- What Activity Follows this?
- What Activity is Concurrent with this?



CPM calculation

- Path
  - A connected sequence of activities leading from the starting event to the ending event
- Critical Path
  - The longest path (time); determines the project duration
- Critical Activities
  - All of the activities that make up the critical path

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Constructing CPM

- A project network may have only one start node
- A project network may have only one end node
- A link has duration
- Nodes have no duration
- Time moves from left to right
- Nodes are numbered sequentially
- A network may not contain loops
- A network may not contain dangles
- Precedents are the immediate preceding activities

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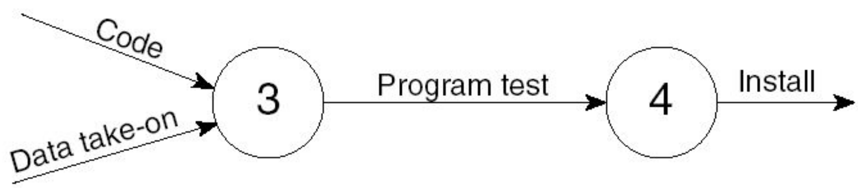
Fragment of a CPM network



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Using Dummy Activities

- Two Paths with a common node
- Two paths linked by a dummy activity
- Two activities between same two nodes
- Representing lagged activities

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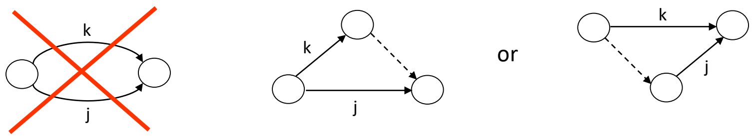
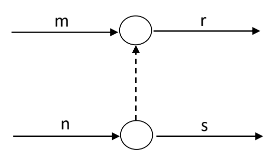
Dummy Activities



Dashed lines are called *dummy activities*

Activity	Predecessor
m	–
n	–
r	m, n
s	n



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CPM Network of a sample Project

- The forward pass
- The backward pass
- Identifying the critical path
- Activity Float
  - Free Float
  - Interfering Float
- Shortening the project duration
- Identifying critical activities

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Forward Pass

- Earliest Start Time (ES)
  - earliest time an activity can start
  - ES = maximum EF of immediate predecessors
- Earliest finish time (EF)
  - earliest time an activity can finish
  - earliest start time plus activity time

$EF= ES + t$

Backward Pass

- Latest Start Time (LS)

  Latest time an activity can start without delaying critical path time

  $LS= LF - t$
- Latest finish time (LF)

  latest time an activity can be completed without delaying critical path time

  LS = minimum LS of immediate predecessors

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CPM analysis

- Draw the CPM network
- Analyze the paths through the network
- Determine the float for each activity
  - Compute the activity's float

    float = LS - ES = LF - EF
  - Float is the maximum amount of time that this activity can be delay in its completion before it becomes a critical activity, i.e., delays completion of the project
- Find the critical path is that the sequence of activities and events where there is no "slack" i.e.. Zero slack
  - Longest path through a network
- Find the project duration is minimum project completion time

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Example – 1 (AOA)

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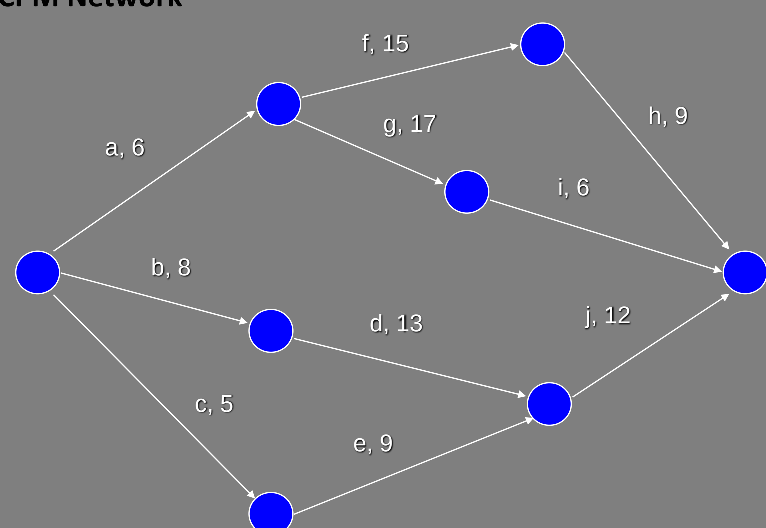
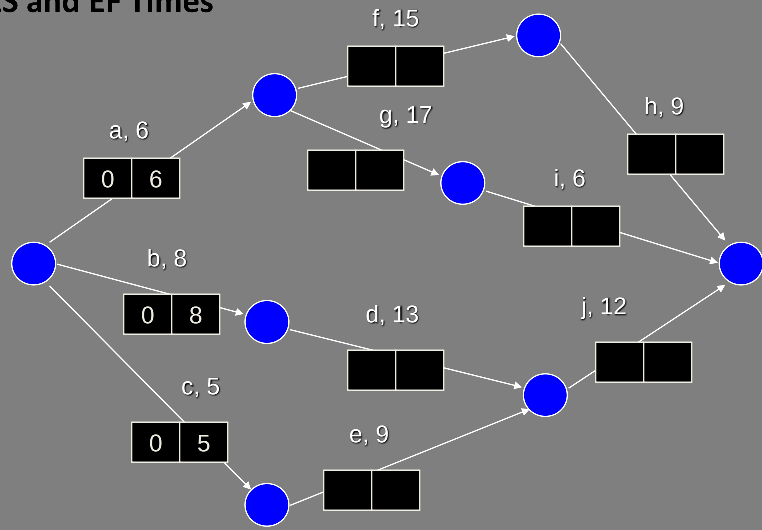
CPM Example:

- **CPM Network**



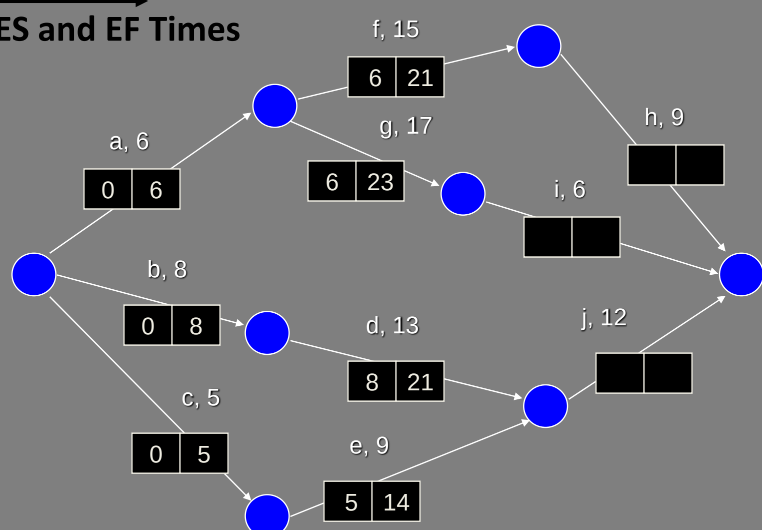
CPM Example

- **ES and EF Times**



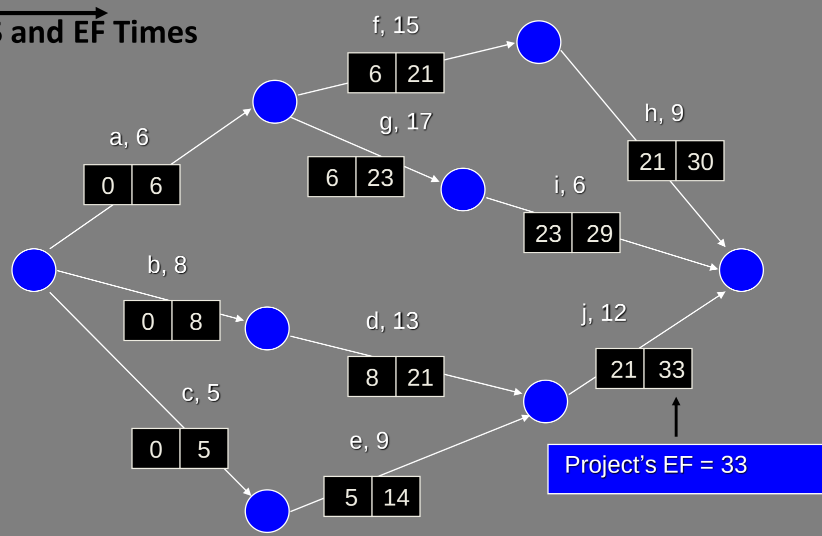
CPM Example

- **ES and EF Times**



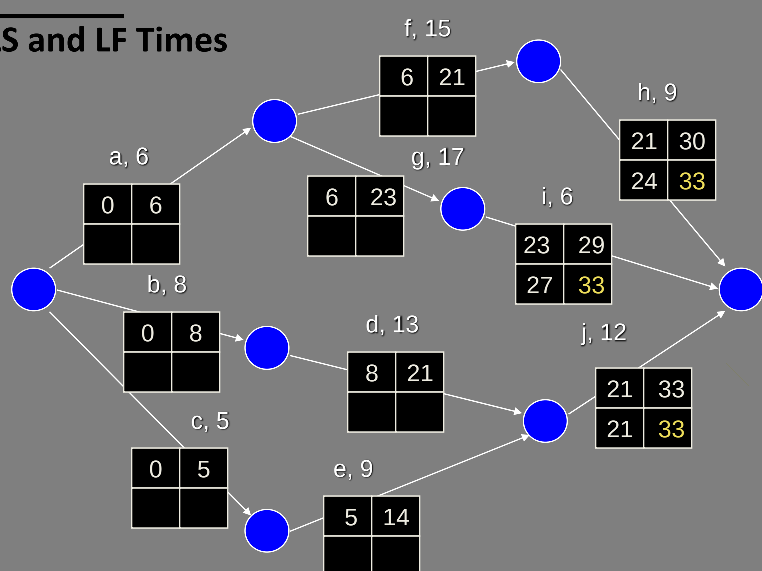
CPM Example

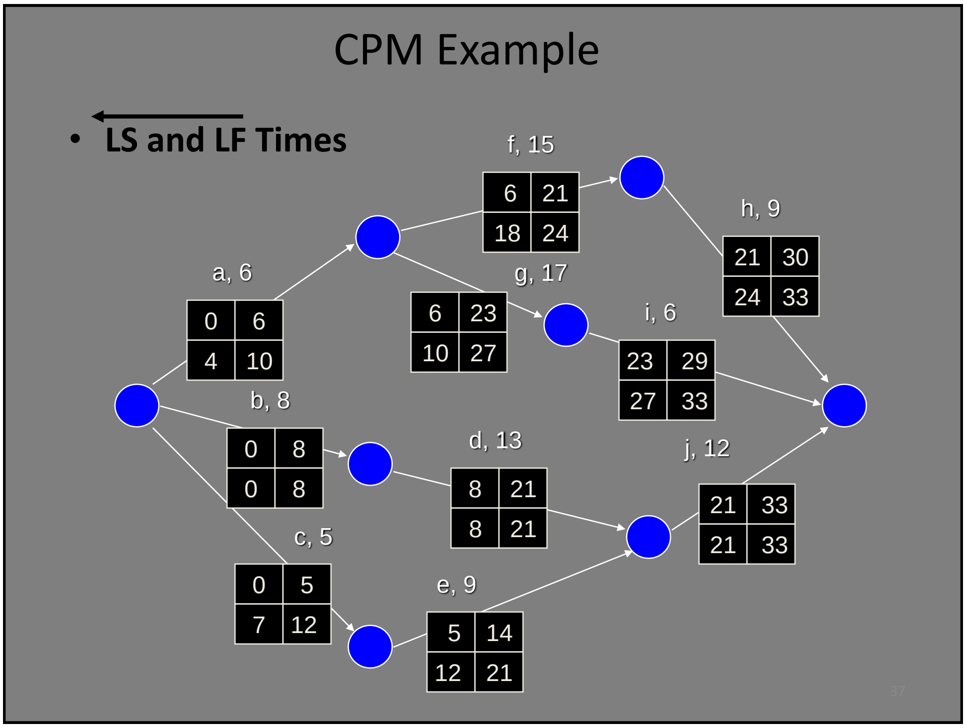
- ES and EF Times

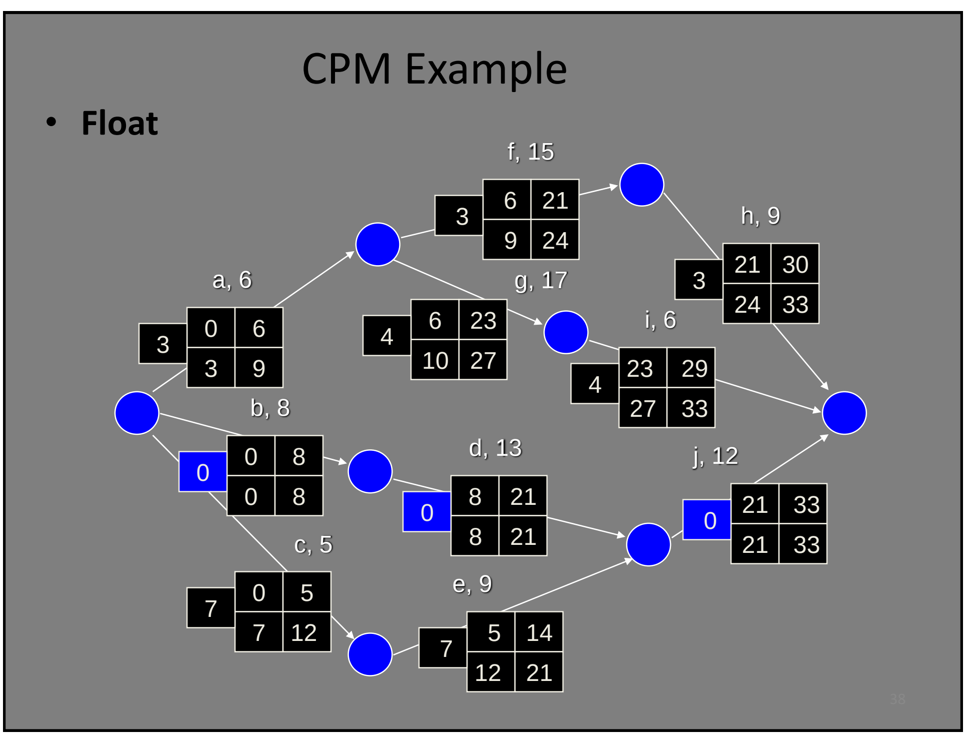


CPM Example

- LS and LF Times




CPM Example
• LS and LF Times
f, 15
6 21
18 24
h, 9
21 30
24 33
a, 6
0 6
4 10
g, 17
6 23
10 27
i, 6
23 29
27 33
b, 8
0 8
0 8
d, 13
8 21
8 21
j, 12
21 33
21 33
c, 5
0 5
7 12
e, 9
5 14
12 21


CPM Example
• Float
f, 15
3
6 21
9 24
h, 9
3
21 30
24 33
a, 6
3
0 6
3 9
g, 17
4
6 23
10 27
i, 6
4
23 29
27 33
b, 8
0
0 8
0 8
d, 13
0
8 21
8 21
j, 12
0
21 33
21 33
c, 5
7
0 5
7 12
e, 9
7
5 14
12 21

Critical Path

- The longest path in the network
- Defines the shortest time project can be completed
- Critical path activity delay project delay
- Represented with solid line →

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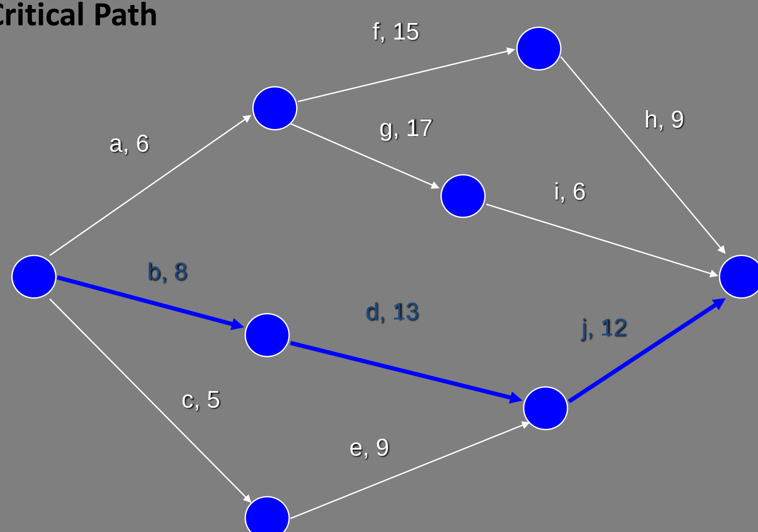
CPM Example

- **Critical Path**



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Example – 2 (AON)

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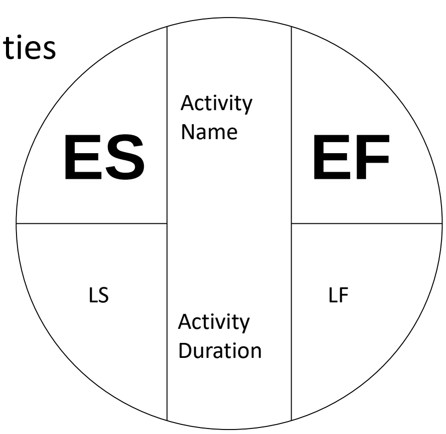
Earliest Start and Earliest Finish

- Begin at starting event and work forward
- ES is earliest start
  - ES = 0 for starting activities
  - ES = Maximum EF of all predecessors for non-starting activities
- EF is earliest finish
  - EF = ES + Activity time



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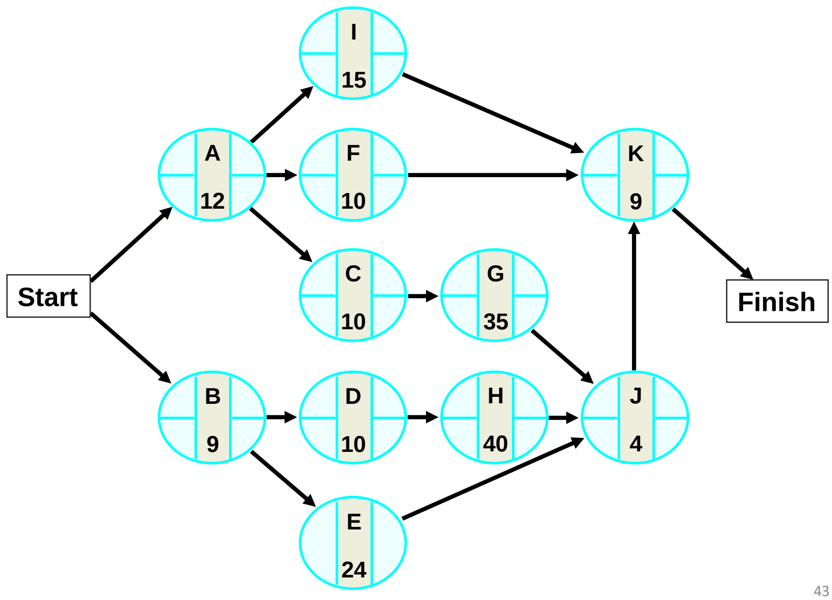
Earliest Start / Earliest Finish



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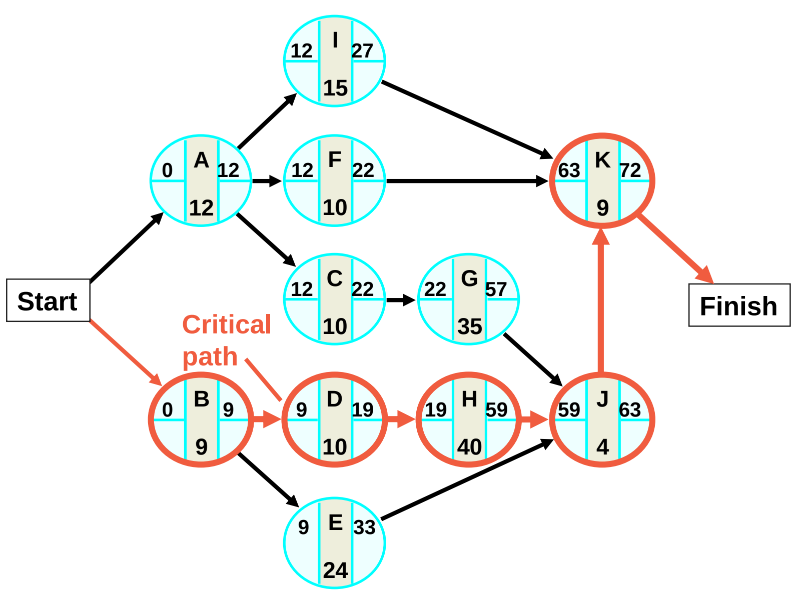
Earliest Start / Earliest Finish



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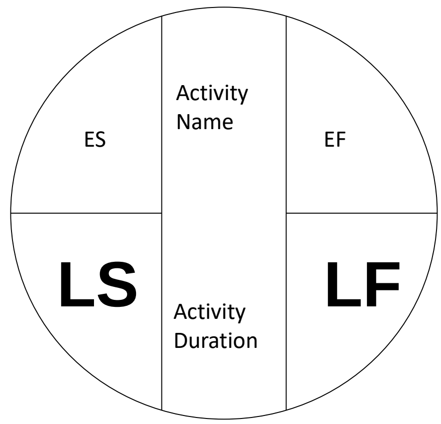
Latest Start and Latest Finish

- Begin at ending event and work backward
- LF is latest finish
  - LF = Maximum EF for ending activities
  - LF = Minimum LS of all successors for non-ending activities
- LS is latest start
  - LS = LF – Activity time



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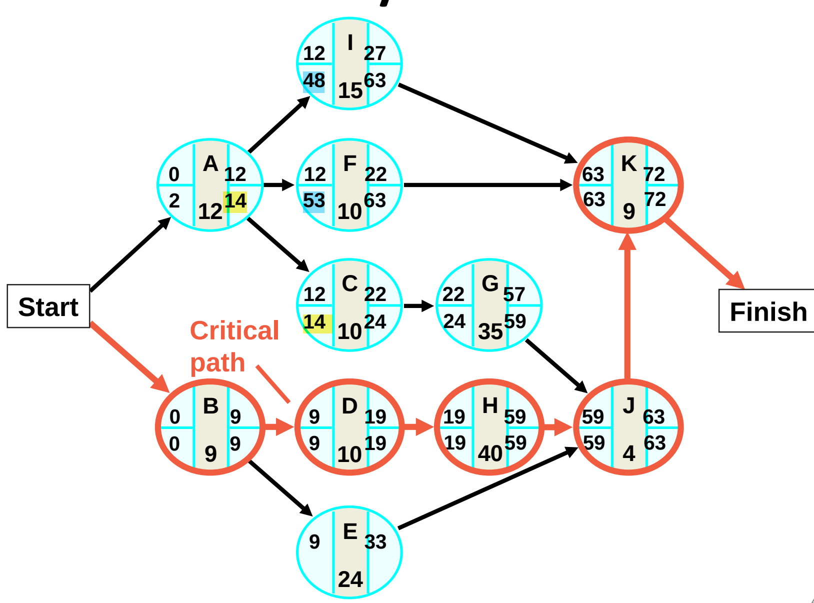
Latest Start / Latest Finish



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Activity Slack Analysis

Slack = *LS* – *ES*

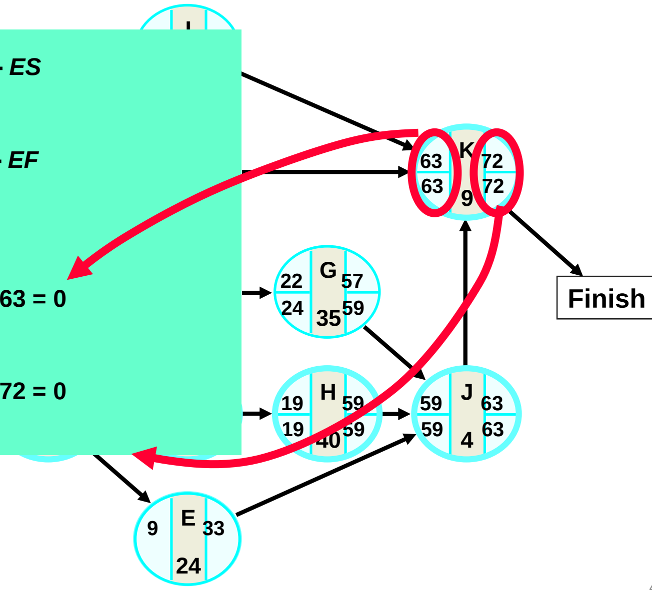
or

Slack = *LF* – *EF*

$Slack_K = 63 - 63 = 0$

or

$Slack_K = 72 - 72 = 0$



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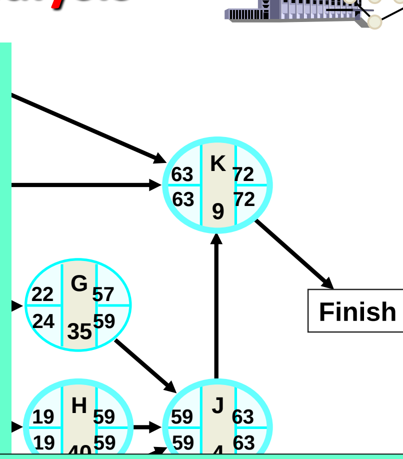
Activity Slack Analysis

Node	Duration	*ES*	*LS*	Slack
A	12	0	2	2
B	9	0	0	0
C	10	12	14	2
D	10	9	9	0
E	24	9	35	26
F	10	12	53	41
G	35	22	24	2
H	40	19	19	0
I	15	12	48	36
J	4	59	59	0
K	6	63	63	0



Activity slack = maximum delay time

Critical path activities have zero slack

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A critical path of the project is a path comprising of critical activities of the project.

It is to be remembered that

- a project can have more than one critical path,
- Any critical path will start at node 1 and will end at node n and that
- The sum of the durations of the activities lying on a critical path is the duration of the project.

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QUIZ 2

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Questions:

1. Develop Network Diagram,
2. Calculate Duration and
3. Calculate Slack

Activity	Description	Immediate Predecessor	Duration (weeks)
A	Develop product specifications	None	4
B	Design manufacturing process	A	6
C	Source & purchase materials	A	3
D	Source & purchase tooling & equipment	B	6
E	Receive & install tooling & equipment	D	14
F	Receive materials	C	5
G	Pilot production run	E & F	2
H	Evaluate product design	G	2
I	Evaluate process performance	G	3
J	Write documentation report	H & I	4
K	Transition to manufacturing	J	2

PERT https://www.youtube.com/watch?v=J1WwNKDdDC0

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PERT

- PERT is based on the assumption that an activity's duration follows a probability distribution instead of being a single value
- Three time estimates are required to compute the parameters of an activity's duration distribution:
  - pessimistic time (t_p) - the time the activity would take if things did not go well
  - most likely time (t_m) - the consensus best estimate of the activity's duration
  - optimistic time (t_o) - the time the activity would take if things did go well

Mean (expected time): $t_e = \dfrac{t_p + 4\,t_m + t_o}{6}$

Variance: $V_t = \sigma^2 = \left(\dfrac{t_p - t_o}{6}\right)^2$

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PERT analysis

- Draw the network.
- Analyze the paths through the network and find the critical path.
- The length of the critical path is the mean of the project duration probability distribution which is assumed to be normal
- The standard deviation of the project duration probability distribution is computed by adding the variances of the critical activities (all of the activities that make up the critical path) and taking the square root of that sum
- Probability computations can now be made using the normal distribution table.

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Probability computation

Determine probability that project is completed within specified time

$$Z = \frac{x - \mu}{\sigma}$$

where $\mu = t_p$ = project mean time

σ = project standard mean time

x = (proposed) specified time

Variance = $[(O-E)^2 + 4(M-E)^2 + (P-E)^2]/6$

S. D. = $\sqrt{[(O-E)^2 + 4(M-E)^2 + (P-E)^2]/6}$

For the example above, the standard deviation is 6.95 days, or about one week.

A simplified, but less accurate method of computing the standard deviation is often used instead:

S.D. = (E - O)/6,

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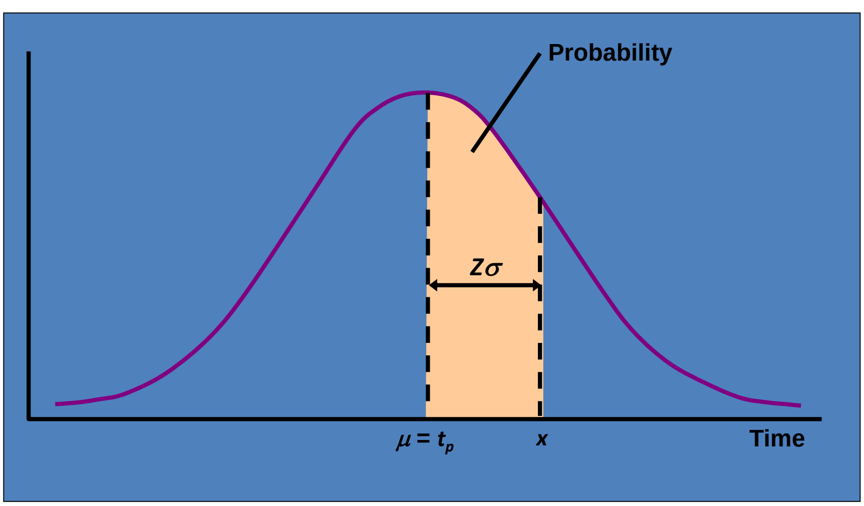
Normal Distribution of Project Time



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PERT Example – 1

- The duration time for each of a Project's activities in a PERT environment are estimated on the basis of *most likely, pessimistic,* and *optimistic* completion times.
- These times can be arrived at in various ways. A number of these ways contain substantial subjective components because it is often the case that little historical information is available to guide those estimates.
- The expected (value) duration time of an individual activity and its variation follow what is called a *beta distribution* and are calculated as follows:

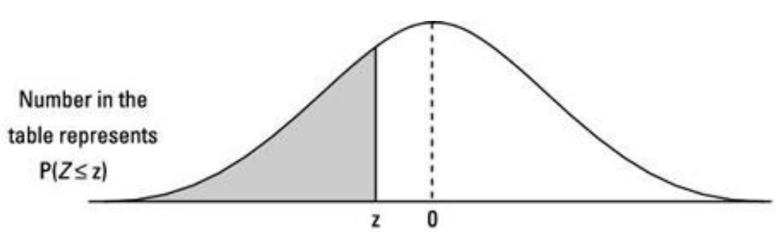
$$E(t_i) = (a + 4m + b) / 6 \quad \text{and} \quad var(t_i) = (b - a)^2 / 36$$

where "a" is the activity's optimistic completion time, "m" is the activity's most likely completion time, and "b" is the activity's most pessimistic completion time.

- **Where**
  - PERT Weighted Average (expected time) (mean)=(O+4M+P)/6
  - Standard Deviation (SIGMA)=(P-O)/6
  - Variance =(Standard Deviation)^2

How to read Z value table

- We use the Z-score table to find a full set of "less-than" probabilities for a wide range of z-values using the z-score formula.



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How to read Z value table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
–3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
–3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
–3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
–3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
–3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
–3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
–3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
–2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
–2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
–2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
–2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
–2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
–2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
–2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
–2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
–2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
–2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
–1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
–1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294

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How to read Z value table

What is $P(Z \leq 1.5)$?
Answer: 0.9332

To find the answer using the *Z*-table, find where the row for $\boxed{1.5\text{ in}}$tersects with the column fo$\boxed{0.00;}$ this value is $\boxed{0.9332.}$

The *Z*-table shows only "less than" probabilities so it gives you exactly what you need for this question.

Note: No probability is exactly at one single point, so:
$P(Z \leq 1.5) = P(Z < 1.5)$

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How to read Z value table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706

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How to read Z value table

What is $P(Z \geq 1.5)$?

Answer: 0.0668
We use the Z-table to find where the row for 1.5 intersects with the column for 0.00, which is 0.9332.

Because the Z-table gives you only "less than" probabilities, subtract $P(Z < 1.5)$ from 1 (remember that the total probability for the normal distribution is 1.00, or 100%):

$$P(Z \geq 1.5) = 1 - P(Z < 1.5)$$
$$= 1 - 0.9332 = 0.0668$$

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How to read Z value table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
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1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706

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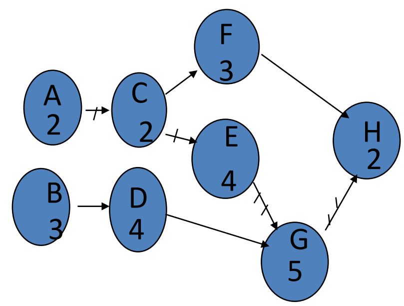
PERT Example – 1

- **Consider the following table of activities;**
- **immediate predecessor(s) (I.P.);**
- **optimistic, most likely,**
- **and pessimistic completion times for General Foundry;**
- **and the E(t_i) and var(t_i) for each activity.**

ACT	I.P.	Optimistic (a)	Most Likely (m)	Pessimistic(b)	E(t_i)	var(t_i)
A	__	1 week	2 weeks	3 weeks	2 wks.	4/36
B	__	2	3	4	3	4/36
C	A	1	2	3	2	4/36
D	B	2	4	6	4	16/36
E	C	1	4	7	4	36/36
F	C	1	2	9	3	64/36
G	D,E	3	4	11	5	64/36
H	F,G	1	2	3	2	4/36

2. PERT Example



2. PERT Example

From the reduced version of General Foundy's PERT table., the $E(t_i)$ for each activity can be entered into the project's network diagram.

ACT	I.P.	(a)	(m)	(b)	$E(t_i)$	$var(t_i)$
A	__	1	2	3	2	4/36
B	__	2	3	4	3	4/36
C	A	1	2	3	2	4/36
D	B	2	4	6	4	16/36
E	C	1	4	7	4	36/36
F	C	1	2	9	3	64/36
G	D,E	3	4	11	5	64/36
H	F,G	1	2	3	2	4/36

Inspection of the network discloses three paths thru the project:

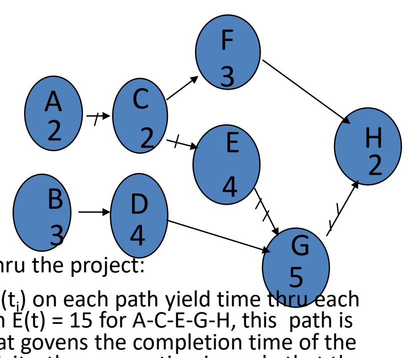
PERT Example – 1

- A-C-F-H; A-C-E-G-H; and B-D-G-H.
- Summing the $E(t_i)$ on each path yield time thru each path of 9, 15, and 14 weeks, respectively.
- With an E(t) = 15 for A-C-E-G-H, this path is defined as the critical path (CP) being the path that governs the completion time of the project .
- Despite the beta distribution of each activity, the assumption is made that the number of activities on the CP is sufficient for it to be normally distributed with a variance equal to the sum of the variances of *its* activities only, var(t) = 112/36 = 3.11.

2. PERT Example

•From the reduced version of General Foundy's PERT table., the $E(t_i)$ for each activity can be entered into the project's network diagram.

ACT	I.P.	(a)	(m)	(b)	$E(t_i)$	$var(t_i)$
A	__	1	2	3	2	4/36
B	__	2	3	4	3	4/36
C	A	1	2	3	2	4/36
D	B	2	4	6	4	16/36
E	C	1	4	7	4	36/36
F	C	1	2	9	3	64/36
G	D,E	3	4	11	5	64/36
H	F,G	1	2	3	2	4/36



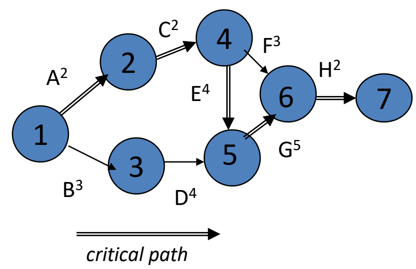
Inspection of the network discloses three paths thru the project:

A-C-F-H; A-C-E-G-H; and B-D-G-H. Summing the $E(t_i)$ on each path yield time thru each path of 9, 15, and 14 weeks, respectively. With an E(t) = 15 for A-C-E-G-H, this path is defined as the critical path (CP) being the path that govens the completion time of the project . Despite the beta distribution of each activity, the assumption is made that the number of activities on the CP is sufficient for it to be normally distributed with a variance equal to the sum of the variances of *its* activities only, var(t) = 112/36 = 3.11.

PERT Example – 1

- From the reduced version of General Foundy's PERT table., the $E(t_i)$ for each activity can be entered into the project's network diagram.

ACT	I.P.	(a)	(m)	(b)	$E(t_i)$	$var(t_i)$
A	__	1	2	3	2	4/36
B	__	2	3	4	3	4/36
C	A	1	2	3	2	4/36
D	B	2	4	6	4	16/36
E	C	1	4	7	4	36/36
F	C	1	2	9	3	64/36
G	D,E	3	4	11	5	64/36
H	F,G	1	2	3	2	4/36



- Inspection of the network discloses three paths thru the project: A-C-F-H; A-C-E-G-H; and B-D-G-H. Summing the $E(t_i)$ on each path yield time thru each path of 9, 15, and 14 weeks, respectively. With an $E(t) = 15$ for A-C-E-G-H, this path is defined as the critical path (CP) being the path that governs the completion time of the project . Despite the beta distribution of each activity, the assumption is made that the number of activities on the CP is sufficient for it to be normally distributed with a variance equal to the sum of the variances of *its* activities only, $var(t) = 112/36 = 3.11$.

PERT Example – 1

- **Given E(t) = 15 weeks and**
- **var(t) = 3.11,**
- **what is the probability of the project requiring**
- **in excess of 16 weeks to complete?**

$$P(X \geq 16) = 1 - P(X \leq 16) = 1 - P(Z \leq (16 - 15) / 1.76 = .57) = 1 - 0.716$$

where the value *1.76* is the square root of 3.11, the standard deviation of the expected completion time of the project.

The assumption being made is that summing up a sufficient number of activities following a beta distribution yield a result which approximates or a approaches a variable which is normally distributed--~N(μ,σ^2)

PERT Example - 2

Activity	Immed. Predec.	Optimistic Time (Hr.)	Most Likely Time (Hr.)	Pessimistic Time (Hr.)
A	--	4	6	8
B	--	1	4.5	5
C	A	3	3	3
D	A	4	5	6
E	A	0.5	1	1.5
F	B,C	3	4	5
G	B,C	1	1.5	5
H	E,F	5	6	7
I	E,F	2	5	8
J	D,H	2.5	2.75	4.5
K	G,I	3	5	7

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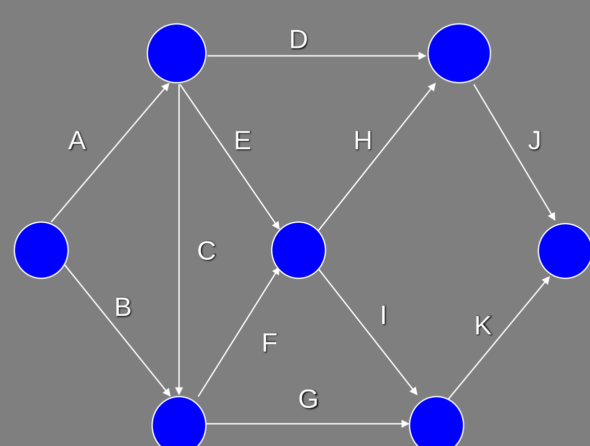
PERT Example

PERT Network



PERT Example

Activity	Expected Time	Variance
A	6	4/9
B	4	4/9
C	3	0
D	5	1/9
E	1	1/36
F	4	1/9
G	2	4/9
H	6	1/9
I	5	1
J	3	1/9
K	5	4/9

PERT Example

Activity	ES	EF	LS	LF	Slack
A	0	6	0	6	0 *critical
B	0	4	5	9	5
C	6	9	6	9	0 *
D	6	11	15	20	9
E	6	7	12	13	6
F	9	13	9	13	0 *
G	9	11	16	18	7
H	13	19	14	20	1
I	13	18	13	18	0 *
J	19	22	20	23	1
K	18	$\boxed{23}$	18	23	0 *

PERT Example

$$V_{path} = V_A + V_C + V_F + V_I + V_K$$
$$= 4/9 + 0 + 1/9 + 1 + 4/9$$
$$= 2$$

$$\sigma_{path} = 1.414$$

$$z = (24 - 23)/\sigma = (24\text{-}23)/1.414 = .71$$

From the Standard Normal Distribution table:

$$P(z \leq .71) = .5 + .2612 = \boxed{.7612}$$

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Difference PERT and CPM

- PERT uses three time estimates
- CPM uses one time estimate
- PERT is probabilistic in nature, based on beta distribution for each activity time and normal distribution for each expected time allowing us to incorporate risk factor
- CPM is deterministic in nature
- PERT is useful for R&D projects where risks in calculating time durations have a high variability
- CPM is used for construction projects
- PERT is used for those projects where percent complete is impossible to determine except at milestone completion
- CPM is used for those projects where percent complete can be determined with reasonable accuracy.
- ***QUESTION: Which of these is suitable for software industry***

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SOLUTION TO QUIZ 2

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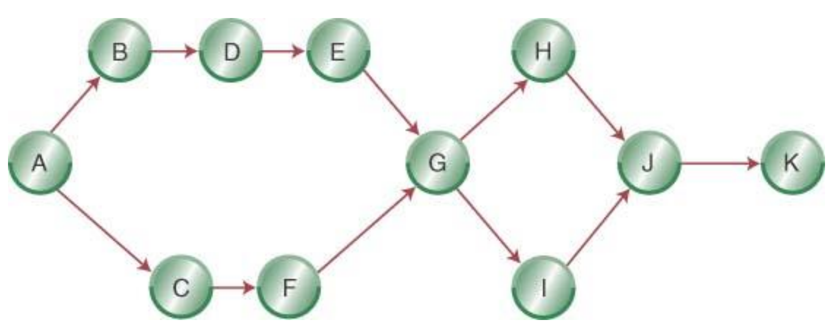
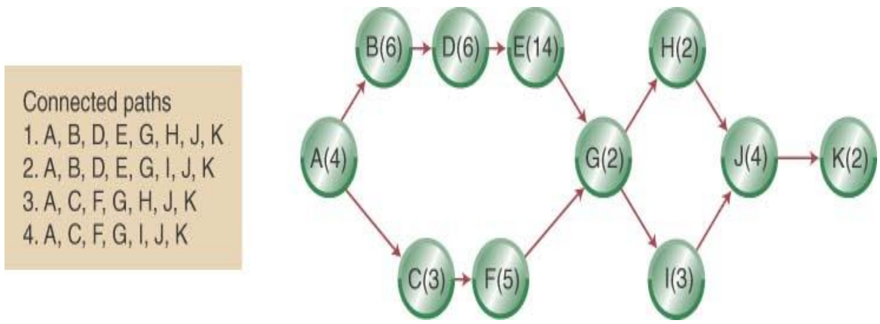
Diagram the Network for Cables By Us



Add Deterministic Time Estimates and Connected Paths



Calculate the Project Completion Times

Paths	Path duration
ABDEGHJK	40
ABDEGIJK	41
ACFGHJK	22
ACFGIJK	23

- The longest path (ABDEGIJK) limits the project’s duration (project cannot finish in less time than its longest path)
- **ABDEGIJK is the project’s critical path**