1. BINOMIAL DISTRIBUTION {Bernoulli Trial}

A random variable X is called a binomial R.V. with parameters (n, p) if its PDF (or PMF) is given by

$$\boxed{f_X(x) = P(X=x) = \binom{n}{x} p^x (1-p)^{n-x} \quad x = 0,1,2,3,\ldots n} \quad (40)$$

where $0 \leq P \leq 1$

$$\binom{n}{k} = {}^nC_k = \frac{\lfloor n}{\lfloor n-k \lfloor k}$$

which is a binomial coefficient.

The corresponding CDF of X is

$$F_X(x) = P(X \leq x)$$

$$= \sum_{k=0}^{m} \binom{n}{k} p^k (1-p)^{n-k} \quad (41)$$

$$m \leq x \leq m+1$$

The mean and variance of the binomial r.v. X are

$$\boxed{\text{Mean} : \mu_X = np}, \quad \boxed{\text{Variance} : \sigma_X^2 = np(1-p)} \quad (42)$$

The binomial r.v. X is an integer-valued discrete random variable associated with repeated trials of an experiment.

$p \rightarrow$ probability of Success

$1-p = q \rightarrow$ probability of failure

Q, If 4% of the total item made by factory are defective, find the probability that less than 2 item are defective in a sample of 50 items

Solu:-

$$f(x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}$$

$n = 50$, $p = 4\% = .04$, $q = 0.96$

$$P(X \leq 2) = P(X=0) + P(X=1)$$

$$= {}^{50}C_0 \, (0.04)^0 (0.96)^{50} + {}^{50}C_1 \cdot (0.04)^1 \cdot (0.96)^{50}$$

$$\approx 0.406 \quad \underline{\text{Ans}}$$

variables.

$$\lim_{n\to\infty} P\left[\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\frac{x_i-\mu}{\sigma}\leq x\right] = \int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\vartheta^2/2}\,d\vartheta$$

or equivalently

L.H.S.

$$\Rightarrow \lim_{n\to\infty} P\left(\frac{1}{\sqrt{n}}\left\{\frac{x_1-\mu}{\sigma}+\frac{x_2-\mu}{\sigma}+\frac{x_3-\mu}{\sigma}+\cdots\cdots\right\}\leq x\right)$$

$$\Rightarrow \lim_{n\to\infty} P\left[\frac{1}{\sqrt{n}}\left\{\frac{(x_1+x_2+x_3+\cdots\cdots+x_n)-n\mu}{\sigma}\right\}\leq x\right]$$

$$\Rightarrow \lim_{n\to\infty} P\left[\frac{1}{\sqrt{n}}\left\{\frac{n\frac{(x_1+x_2+x_3+\cdots+x_n)}{n}-n\mu}{\sigma}\right\}\leq x\right]$$

$$\Rightarrow \lim_{n\to\infty} P\left[\frac{1}{\sqrt{n}}\,n.\left\{\frac{\tilde{x}_n-\mu}{\sigma}\right\}\leq x\right]$$

where $\tilde{x}_n = \dfrac{x_1+x_2+x_3+\cdots\cdots+x_n}{n}$

$$\Rightarrow \lim_{n\to\infty} P\left[\frac{\tilde{x}_n-\mu}{\sigma/\sqrt{n}}\leq x\right]$$

Now consider R.H.S.

$$\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\vartheta^2/2}\,d\vartheta = 1-Q(x)$$

$\left\{ \because\right.$ from equ (48), we know that $F_X(z) = \int_{-\infty}^{z}\frac{1}{\sqrt{2\pi}}e^{-\vartheta^2/2}\,d\vartheta = 1-Q(z)$

Now R.H.S. = L.H.S

$$\Rightarrow \lim_{n\to\infty} P\left[\frac{\tilde{x}_n-\mu}{\sigma/\sqrt{n}}\leq x\right] = 1-Q(x)$$

→ Note

$$\Rightarrow \lim_{n\to\infty}\left\{1-P\left[\frac{\tilde{x}_n-\mu}{\sigma/\sqrt{n}}>x\right]\right\} = 1-Q(x)$$

$$\Rightarrow 1-\lim_{n\to\infty} P\left[\frac{\tilde{x}_n-\mu}{\sigma/\sqrt{n}}>x\right] = 1-Q(x)$$

$$\Rightarrow \boxed{\lim_{n\to\infty} P\left[\frac{x_n-\mu}{\sigma/\sqrt{n}}>x\right] = Q(x)}$$

Ex:→

Consider a communication system that transmits a data packet of 1024 bits. Each bit can be in error with probability of 10^{-2}. Find the (approximate) probability that more than 30 of the 1024 bits are in error.

Solution :→ Define a Random Variable X_i such that $X_i = 1$ if the i_{th} bit is in error and $X_i = 0$ if not.

Hence $\vartheta = \sum_{i=1}^{1024} x_i$; where ϑ is the number of errors in the data packet.

We have to find $P(\vartheta > 30)$

Given $P(X_i=1) = 10^{-2}$ & $P(X_i=0) = 1-10^{-2}$,

$$P(\vartheta>30) = \sum_{m=31}^{1024}\binom{1024}{m}.(10^{-2})^m.(1-10^{-2})^{1024-m}$$

To solve the above expression, too much time is required

Application of Central Limit Theorem :-

Mean $\overline{X_i} = 10^{-2}\times(1)+(1-10^{-2})\times(0) = 10^{-2}$

$\overline{X_i^2} = 10^{-2}\times(1)^2+(1-10^{-2})\times(0) = 10^{-2}$

$$\sigma_i^2 = Var(X_i) = E[(X_i-\mu)^2] = E[X_i^2]-\mu^2 = \overline{X_i^2}-(X_i)^2$$

$$= 10^{-2}-(10^{-2})^2 = .01-.0001 = 0.0099$$

Based on the central limit theorem

$\vartheta = \sum_i x_i$ is approximately Gaussian with mean of $1024.10^{-2} = 10.24$ and variance $1024\times 0.0099 = 10.1376$

$y = \dfrac{\vartheta-1024}{\sqrt{10\ 1376}}$ is a standard Gaussian with zero mean & unit variance.

Contd....

$$P(v > 30) = P\left(y > \frac{30 - 10.24}{\sqrt{10.1376}}\right)$$

$$= P(y > 6.20611)$$

$$= Q(6.20611) \simeq 1.925 \times 10^{-10}$$

Answer

Q. Over a certain binary communication channel, the symbol 0 is transmitted with probability 0.4 & 1 is transmitted with probability 0.6.

It is given that $P(\epsilon|0) = 10^{-6}$ and $P(\epsilon|1) = 10^{-4}$, where $P(\epsilon|x_i)$ is the probability of detecting the error given that x_i is transmitted.

Determine $P(\epsilon)$, the error probability of the channel.

Solution:- If $P(\epsilon, x_i)$ is the joint probability that x_i is transmitted & is detected wrongly, then the total probability theorem yields

$$P(\epsilon) = \sum_i P(\epsilon|x_i) \cdot P(x_i)$$

$$= P_X(0)\, P(\epsilon|0) + P_X(1) \cdot P(\epsilon|1)$$

$$= 0.4\,(10^{-6}) + (0.6) \cdot 10^{-4}$$

$$= 0.604 \times 10^{-4}$$

Note :- $P(\epsilon|0) = 10^{-6}$ means that on the average, one out of 1 million received 0s will be detected erroneously. Similarly, $P(\epsilon|1) = 10^{-4}$ means that on the average, 1 out of 10,000 received 1s will be in error.

$P(\epsilon) = 0.604 \times 10^{-4}$ indicates that on the average, one out of $1/0.604 \times 10^{-4} \simeq 16556$ digits (regardless they are 1s or 0s) will be received in error.

RANDOM PROCESS

Random Process is a function of time, we can find the averages over some period of time, 'T'.

The calculation of average & variance in time are different from the calculation of the stastics.

Let us consider an experiment of measuring the temproture of a room. Let there be a collection of thermometers. Each thermometer reading is a random variable which can take on any value from sample space S. Also, at different time the reading of thermometers may be different.

Thus the room tempreture is a function of both the sample space 'S' & time 't'.

"

A random process is defined as the ensemble (collection) of time functions together with a probability rule.

$X(t, S)$ represent an ensemble of time functions, where t & S are variables.

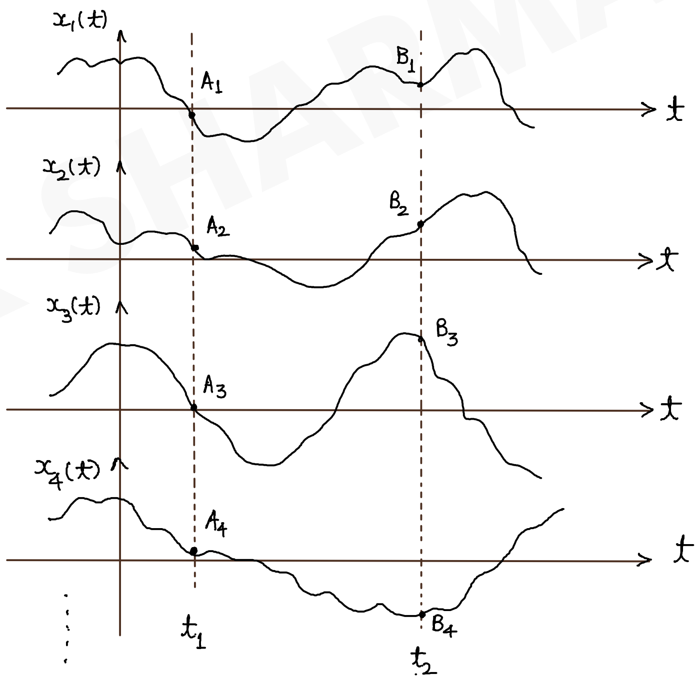
$X(t, S) \longrightarrow$ Random Process

Let $x_1(t)$ is an outcome of experiment 1

$x_2(t)$ is an outcome of experiment 2

Sample Functions

$x_n(t)$ is an outcome of experiment n.



Determine the statistics of this Random Process.

$\underline{\text{Random Variable}}$ X
$\hookrightarrow X(t_1, S) = X(t_1) = [A_1, A_2, A_3, \ldots, A_n]$

$\mu_{X(t_1)} = E[X(t_1)] =$ Ensemble Average

Ensemble average is a function of time.

Similarly

$X(t_2, S) = X(t_2) = [B_1, B_2, B_3, \ldots, B_m]$

3) Determination of statistics of Random process, can also be calculated by measuring the mean of $x_1(t)$

$$\langle x_1(t)\rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x_1(t)\,dt \longrightarrow (56)$$

Similarly

$$\langle x_2(t)\rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x_2(t)\,dt \longrightarrow (57)$$

and so on.

The mean or expected value of this time average is given as

$$\langle X(t)\rangle = E[\langle X(t)\rangle]$$

Note :→

If the mean values of all sample functions are same, then the random process will be considered as Regular Random Process.

In this case

$$\langle x_1(t)\rangle = \langle x_2(t)\rangle = \cdots\cdots = \langle x_n(t)\rangle$$

For Some processes, ensemble averages is independent of time i.e.

$$E[X(t_1)] = E[X(t_2)] = \cdots\cdots = \text{Constant}$$

Such process are consider as Stationary Random Process

ERGODIC PROCESS :-

When ensemble average is equal to the time average, then the process is known as ERGODIC PROCESS in restricted sense.

When all statistical ensemble properties are equal to the statistical time properties, then the process is known as a ergodic process in strict sense.

When we say ergodic process, then it meant that the ergodic process in strict sense.

Markov's Process :-

Many times a given random variable x_n is statistically dependent upon some finite number of previously occurring R.V. s, Thus if

$$f_c\left(x_n / x_{n-1}, x_{n-2}, \ldots\ldots\right) = f_{ck}\left(x_n / x_{n-1}, x_{n-2}, \ldots x_{n-k}\right)$$

then we say that x_n is a k^{th} order Markov Process.

Autocorrelation of $X(t)$ is defined by

$$R_{XX}(t_1, t_2) = E[X(t_1)\,X(t_2)] \longrightarrow (55)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1\, x_2\, f_X(x_1, x_2; t_1, t_2)\, dx_1 dx_2 \longrightarrow (56)$$

The auto covariance of X(t) is defined by

$$E[X(t_1)\ldots X(t_n)] = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} x_1 \ldots x_n f_X(x_1,\ldots x_n;\, t_1,\ldots t_n)\, dx_1\ldots dx_n \rightarrow (57)$$

Power Spectral Densities

$$PSD = F.T.[\text{autocorrelation}]$$

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\cdot e^{-j\omega\tau}\, d\tau$$

$$R_{XX}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, e^{j\omega\tau}\, d\omega$$

Properties :- (1) $S_{XX}(\omega)$ is real & $S_{XX}(\omega) \geq 0$

(2) $S_{XX}(-\omega) = S_{XX}(\omega)$

(3) $\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, d\omega = R_{XX}(0) = E[X^2(t)]$

(58)

Noise Process is stationary process because its statistics (Mean, mean square values, etc) do not change with time.

Q) Write a short note on Classification of Random Process

Solution :-

1) Stationary and Non Stationary Random Process

A random process whose statistical characteristics do not change with time is classified as a stationary random process.

(1) ie process will appear to be the same all over the time interval

(2) in other words PDF's of x at t_1 and $t_2 = t_1 + t_0$ must be same.

(3) or $p_X(x;t)$ is independent of t. Thus the first order $p_X(x;t) = p_X(t)$

(4) Imp For a stationary random process the autocorrelation function $R_X(t_1, t_2)$ must depend on t_1 and t_2 only through the difference $t_2 - t_1$.

$$R_X(t_1, t_2) = R_X(t_2 - t_1)$$

(5) For a stationary process, the joint PDF PDF for x_1 & x_2 must also depend only on $t_2 - t_1$. Thus

$$p_X(x_2, x_1;\, t_1, t_2) = p_X(x_1, x_2;\, t_1 - t, t_2 - t) \quad \forall t$$

$$= p_X(x_1, x_2;\, 0, t_2 - t_1) \quad \forall\, t = t_1$$

The random process $x(t)$ representing the temperature of the city is an example of a Non stationary Process.

Wide Sense (or weakly) Stationary Process

A process that is not stationary in the strict sense.

Wide-Sense stationary process have a mean value & an autocorrelation function that are independent of the shift of time origin.

This means $\overline{x(t)} = \text{Constant}$

& $R_x(t_1, t_2) = R_x(\tau) \qquad \tau = t_2 - t_1$

Q/ Show that the random process $x(t) = A\cos(\omega_c t + \phi)$, where ϕ is a R.V. uniformly distributed in the range $(0, 2\pi)$, is a wide sense stationary process

Solution:- In question it is given that ϕ is a R.V., which is uniformly distributed over $[0, 2\pi]$, i.e.

$$f_\phi(\phi) = \begin{cases} \frac{1}{2\pi} & ; 0 \le \phi \le 2\pi \\ 0 & ; \text{otherwise} \end{cases}$$

The given process would be wide sense stationary iff its ensemble average is constant & autocorrelation depend on the difference of the time interval.

So we have to calculate $\overline{x(t)} = E[x(t)]$

$$E[x(t)] = A\, E[\cos(\omega_c t + \phi)]$$

$$= A \int_0^{2\pi} \cos(\omega_c t + \phi) \cdot f_\phi(\phi)\, d\phi$$

$$= A \frac{1}{2\pi} \int_0^{2\pi} \cos(\omega_c t + \phi)\, d\phi = 0$$

hence $E[x(t)] = \overline{x(t)} = 0 \longrightarrow (a)$

Now using equ. (55)

$$R_x(t_1, t_2) = E[X(t_1) \cdot X(t_2)]$$

$$= E[A^2 \cos(\omega_c t_1 + \phi) \cdot \cos(\omega_c t_2 + \phi)]$$

$$= \frac{A^2}{2} \cdot E[\cos[\omega_c(t_2 - t_1)] + \cos[\omega_c(t_2 + t_1) + 2\phi]\}$$

$$= \frac{A^2}{2} E[\cos(\omega_c(t_2 - t_1))] + \frac{A^2}{2} E[\cos(\omega_c(t_2 + t_1) + 2\phi)]$$

$$= \frac{A^2}{2} \cos(\omega_c(t_2 - t_1)) + \underbrace{\frac{A^2}{2} E[\cos(\omega_c(t_1 + t_2) + 2\phi)]}_{B}$$

$$B = \frac{A^2}{2} \cdot \frac{1}{2\pi} \int_0^{2\pi} \cos(\omega_c(t_1 + t_2) + 2\theta)\, d\theta$$

$$= 0$$

$$R_x(t_1, t_2) = \frac{A^2}{2} [\cos(\omega_c(t_2 - t_1))]$$

or $R_x(t_1, t_2) = \frac{A^2}{2} \cos \omega_c \tau, \quad \because \tau = t_2 - t_1 \longrightarrow (b)$

From equation a & b it is clear that the random process $x(t)$ is a wide sense stationary process.

Ergodic wide-sense stationary Process

(1) Let $\langle x(t)\rangle$ is the time average of a sample function $x(t_1, S)$, of a random process $X(t, S)$.

$$\langle x(t)\rangle = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt$$

Let ensemble average of this process is $E[X(t_1, S)]$.

if $E[X(t_1, S)] = \langle x(t)\rangle$

Then process is an ergodic process.

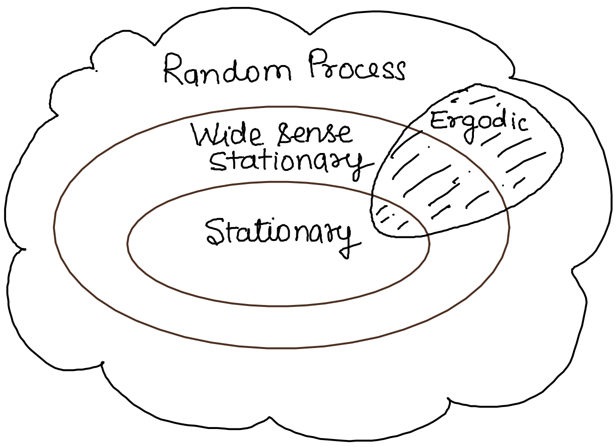
(2) Similarly, if auto correlation of sample function is same as autocorrelation of process.

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\cdot x(t+\tau)\,dt$$

For ergodic (wide sense) stationary process

$$E[x(t)] = \langle x(t)\rangle$$

& $R_x(\tau) = R_X(\tau)$



Lathi P(447)

Consider a communication system that transmits a data packet of 1024 bits. Each bit can be in error with probability of 10^{-2}. Find the (approximate) probability that more than 30 of the 1024 are in error.

Solution :- Let X be a R.V. for bits in error.

$$P(X>30) = 1 - P(X \leq 70)$$

$$= \sum_{m=31}^{1024} \binom{1024}{m} (10^{-2})^m (1-10^{-2})^{1024-m}$$

Ans

$$\simeq 1.925\times 10^{-10}$$

PSD of a Random Process :-
Wiener-Khintchine Theorem

PSD will be calculated, keep in mind that process must be stationary.

We know that autocorrelation and PSD are fourier transform pairs, having

$$R_X(\tau) \Longleftrightarrow S_X(f) \longrightarrow (59)$$

and

$$S_X(f) = \lim_{T\to\infty}\left[\frac{\overline{|X_T(f)|^2}}{T}\right] \longrightarrow (60)$$

$$= \lim_{T\to\infty}\left[\frac{E\left[|X_T(f)|^2\right]}{T}\right]$$

Now

$$|X_T(f)|^2 = X_T(-f)\cdot X_T(f) \longrightarrow (61)$$

Now $\rightarrow$ $X_T(f) = \int_{-T/2}^{T/2} x_T(t)\, e^{-j2\pi f t}\, dt$

at $t = t_2$, $\Rightarrow dt = dt_2$

$$X_T(f) = \int_{-T/2}^{T/2} x_T(t_2)\cdot e^{-j2\pi f t_2}\, dt_2 \longrightarrow (62)$$

$$X_T(-f) = \int_{-T/2}^{T/2} x_T(t)\cdot e^{+j2\pi f t}\, dt$$

at $t = t_1$

$$X_T(-f) = \int_{-T/2}^{T/2} x_T(t_1)\cdot e^{j2\pi f t_1}\, dt \longrightarrow (63)$$

Using equ. (61) – (63)

$$|X(f)|^2 = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2} x_1(t)\cdot x_2(t)\cdot e^{-j2\pi(t_2-t_1)f}\, dt_1 dt_2$$

$$S_X(f) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2} \overline{x_1(t)\cdot x_2(t)}\cdot e^{-j2\pi f(t_2-t_1)}\, dt_1 dt_2$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2} R_X(t_2-t_1)\cdot e^{-j2\pi f(t_2-t_1)}\, dt_1 dt_2 \longrightarrow (64)$$

Using equ.(55)

Let $R_X(t_2-t_1)\cdot e^{-j2\pi f(t_2-t_1)} = \phi(t_2-t_1) \longrightarrow (65)$

Then

$$S_X(f) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2}\phi(t_2-t_1)\, dt_1 dt_2 \longrightarrow (65)$$

or

$$S_X(f) = \lim \frac{1}{2T}\int_{-T}^{T}\int_{-T}^{T}\phi(t_2-t_1)\, dt_1 dt_2 \longrightarrow (66)$$

The above integration is a volume under the surface $\phi(t_2-t_1)$, over the square region in fig shown below

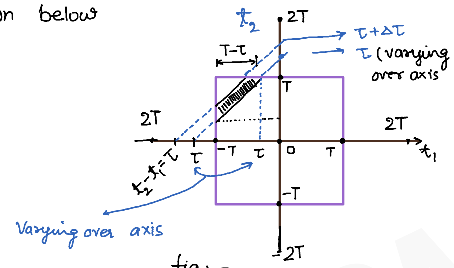


fig:–

Assuming process is stationary in wide sense $t_2 - t_1 = \tau$ constant in the $t_2 - t_1$ plane. By assuming this double integral converted to a single integral.

Let us consider $t_2 - t_1 = \tau$

or $t_2 - t_1 = \tau + \Delta\tau$ if $\Delta\tau \to 0$

Now $\phi(t_2 - t_1) \simeq \phi(\tau)$ over the shaded region whose area is $(T-\tau)\Delta\tau$. Hence, the volume under the surface $\phi(t_2-t_1)$ over the shaded region is $\phi(\tau)(T-\tau)\Delta\tau$. If τ is '–'ive, the volume would be $\phi(\tau)(T+\tau)\Delta\tau$. Hence in general volume over the shaded region is $\phi(\tau)(T-|\tau|)\Delta\tau$.

$$S_X(f) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\phi(\tau)\,(T-|\tau|)\,d\tau = \lim_{T\to\infty}\int_{-T/2}^{T/2}\phi(\tau)\left(1-\frac{|\tau|}{T}\right)d\tau$$

$$= \int_{-\infty}^{\infty}\phi(\tau)\,d\tau$$

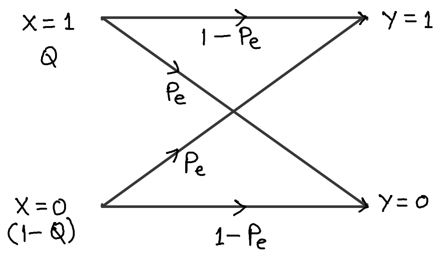
$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau)\, e^{-j2\pi f\tau}\cdot d\tau$$

Thus $S_X(f) \Longleftrightarrow R_X(\tau)$

Imp

Q. A binary symmetric channel (BSC) error probability is P_e. The probability of transmitting 1 is Q, and that of transmitting 0 is $1-Q$. (As shown in Fig.

Determine the probabilities of receiving 1 and 0 at the receiver.



Solution :- If X & Y are the transmitted and received digit respectively, then for a BSC (Binary Symmetric Channel)

$$P_{Y/X}(0|1) = P_e$$

$Y/X \rightarrow$ y is receiving while x is transmitting.
$0|1 \rightarrow$ 0 is receiving while 1 is transmitted
i.e This is the error Probability P_e

Similarly $P_{Y/X}(1/0) = P_e = P_{Y/X}(0|1)$

Also given

$P_X(1) = Q$ {Probability of transmitting 1 is Q}

$P_X(0) = 1-Q$ {Probability of transmitting 0 is $1-Q$}

We need to find $P_Y(1)$ and $P_Y(0)$.

Probability of Receiving 1 will be

$P_Y(1)$ = Probability that '0' is transmitted and '1' is received + Probability that '1' is trans. & '1' is received.

$$P_Y(1) = P_{Y/X}(1|0) \cdot P_X(0) + P_X(1)\, P_{Y/X}(1/1)$$

$$= P_e \cdot (1-Q) + Q \cdot (1-P_e) \longrightarrow (69)$$

Similarly

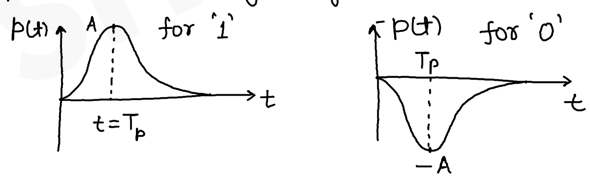
Probability of receiving '0'

$$P_Y(0) = P_{Y/X}(0|1) \cdot P(1) + P_{Y/X}(0|0) \cdot P(0)$$

$$P_Y(0) = P_e\, Q + (1-P_e)(1-Q) \longrightarrow (70)$$

Imp

Q. Over a certain binary channel, message m=0 & 1 are transmitted with equal probability by using a pulse shown in figure given below



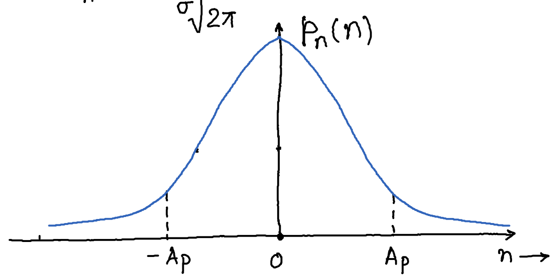
Find out the probability of error if the noise n(t) in the channel is $N(0; \sigma_n^2)$

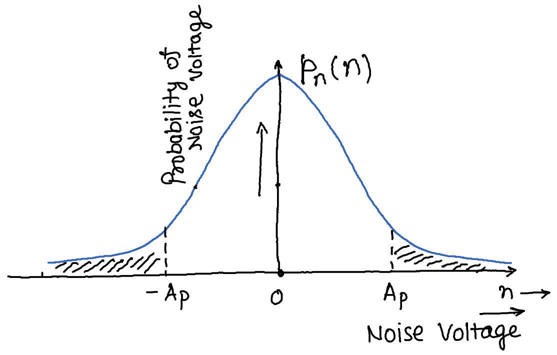
Solution :-

As shown in Fig. the peak value of pulse is A at $t = T_p$ for '1', & peak value of pulse is $-A$ at $t = T_p$ for '0'

The pdf of noise n(t) will be given by

$$P_n(n) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{n^2}{2\sigma^2}}$$, as shown





As seen, because of the symmetry of the curve, the optimum detection threshold is zero; that is the received pulse is detected as a '1' or a '0', depending on whether the sample value is '+'ive or '−'ive.

Note that noise amplitudes are ranging from $-\infty$ to $+\infty$.

The chance or Probability of error arises only when if

	Correct	Wrong (Error)
if '1' is transmitted →	$A_p + n > 0$	$A_p + n < 0$ $\Rightarrow n < -A_p$ i.e. $P(\epsilon\vert 1) = P(n < -A_p)$
if '0' is transmitted →	$-A_p + n < 0$	$-A_p + n > 0$ $n > A_p$ i.e. $P(\epsilon\vert 0) = P(n > A_p)$

Now for '0'

$$P(\epsilon|0) = P(n > A_p)$$

Now From equ. (48)

$$P(X \le x) = 1 - Q\left(\frac{x-\mu}{\sigma}\right) \quad \text{or}$$

$$P(X > x) = Q\left(\frac{x-\mu}{\sigma}\right)$$

$$= \int_A^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{n^2}{2\sigma^2}} dn = Q\left(\frac{A-0}{\sigma}\right) = Q\left(\frac{A}{\sigma}\right)$$

Similarly,

$$P(\epsilon|1) = P(n < -A_p)$$

$$= \int_{-\infty}^{-A_p} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{n^2}{2\sigma^2}} dn$$

$$= Q\left(\frac{A_p - 0}{\sigma}\right) = Q\left(\frac{A_p}{\sigma}\right)$$

Probability of Error

$$= P(1/0) \cdot P(0) + P(0/1) P(1)$$

$$= P\left(\frac{A_p}{\sigma}\right) \cdot \left(\frac{1}{2}\right) + P\left(\frac{A_p}{\sigma}\right)\left(\frac{1}{2}\right)$$

$$\boxed{P_e = P\left(\frac{A_p}{\sigma}\right)} \longrightarrow (71)$$

(2017-18)

Q1. Differentiate between a random variable & random process? A random variable has an exponential probability density function given by $f(x) = ae^{-b|x|}$ where 'a' & 'b' are constants. Find relationship between 'a' & 'b'.

Solution :-

As per the property of PDF for continuous random variable

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} a e^{-b|x|}\,dx = 1$$

$$\int_{-\infty}^{0} a e^{bx}\,dx + \int_{0}^{\infty} a \cdot e^{-bx}\,dx = 1$$

$$\frac{a}{b} e^{bx}\Big|_{-\infty}^{0} - \frac{a}{b} e^{-bx}\Big|_{0}^{\infty} = 1$$

$$\frac{a}{b}\left[1-0\right] - \frac{a}{b}\left[0-1\right] = 1$$

$$\frac{a}{b} + \frac{a}{b} = 1 \Rightarrow \frac{2a}{b} = 1$$

$$\boxed{a = \frac{b}{2}}$$ Ans

Q2:→ Write a short note on

(i) Gaussian random variable

(ii) Power spectral density for wide sense stationary random process

2016-2017

Q1:- In an experiment a trial consists of four successive with draw of playing cards from a pack of 52 cards. If we define R.V. "X" as a number of king appearing in a trial. Find $F_X(x)$.

Q2:→ Calculate auto-correlation for white noise.

Q3:→ A binary communication channel, the receiver detects the pulse with an error probability P_e. What is the probability that out of 100 received digits, no more than four digits are in error.

Q4:→ Explain Central Limit Theorem.

2015 - 16

Q1:→ Define CDF.

Q.2:→ Why the Gaussian distribution is widely used in communication? In an experiment, a trial consist of 4 successive tosses of a coin. If we define a Random Variable X as the number of heads appearing in a trial, Determine $P_X(x)$ & $F_X(x)$.