(2015-16) Contd....

Q:→3 Define & list the properties of autocorrelation. Show that the random process $X(t) = A\cos(\omega_c t + \theta)$, where "$\theta$" is a random variable uniformly distributed in the range $(0, 2\pi)$ is a wide sense stationary process.

Q4:→ A binary source produces 0's & 1's independently with probabilities $P(0) = 0.2$ & $P(1) = 0.8$. The binary data is then transmitted over a noisy channel. The probability of correct reception when a '0' has been transmitted is 0.9 & the probability of erraneous reception when "1" has been transmitted is 0.2.

→ (i) Find the probability of erraneous reception, when a '0' is transmitted & probability of correct reception when a '1' was transmitted.

→ (ii) Find the overall probability of receiving a '0' & a '1'.

→ (iii) If a '1' is received, what is the probability that a '0' was transmitted.

(2014-15)

Q1:→ Write a short note on

(i) CDF, (ii) PDF, (iii) random process

Q2:→ Define mean, variance & standard deviation for random variables.

Also prove the following theorem on variance

(i) $\sigma^2 = E(X^2) - \mu^2$

(ii) $Var(cX) = c^2\, Var(X)$

(iii) $Var(X-Y) = Var(X) + Var(Y)$

Q3:→ The probability density function is given as $f_X(x) = a\, e^{-b|x|}$, where X is a random variable.

Find :

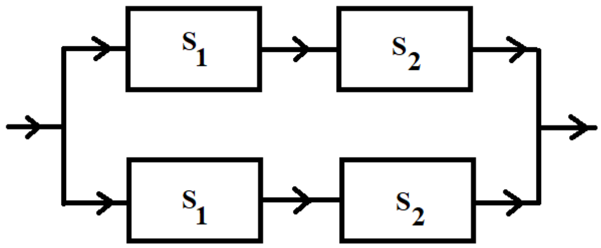
(i) Relationship between a & b

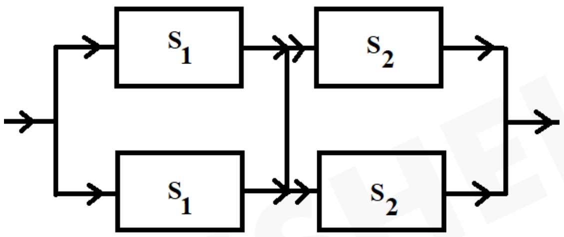
(ii) CDF

(iii) The probability that outcome lies between '1' & '2'.

2013-14

1. **Network reliability improves when redundant links are used. The reliability of the network can be improved building two sub networks in parallel. Thus, if one sub networks fails, the other will still connect. Determine the reliability of the networks given that the failure probabilty of links S1 and S2 each.**





2. **A binary symmetric channel has an error probability Pe. The probability of transmitting 1 is Q. If the receiver detects an incoming digits 1, what is the probability that the originally transmitted digit was**
   a. **1**
   b. **0**

3. **For an R.V . "X" with PDF**

$$P_X(x) = \frac{1}{2 * \sqrt{2\pi}} exp\left(-\frac{x^2}{32}\right) * u(x)$$

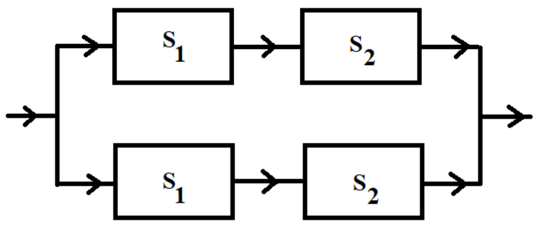
   a. **Sketch PDF and state if this is Gaussian RV**
   b. **Determine $P(x \geq 1)$ *and* $P(1 \leq x \leq 2)$**
   c. **How to generate RV x from another RV.**

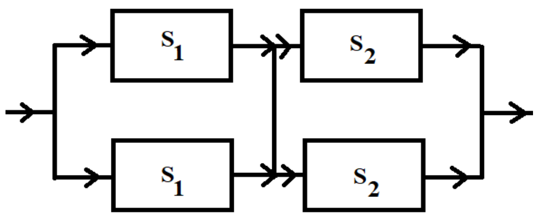
4. **For the Random binary process. Determine $R_x(z)$ and $S_x(f)$, if the probability of transition (from 1 to -1 or vice versa) at each node is 0.5.**

(2012-2013) Set 2

1. **State Bay's Theorem**

2. **Network reliability improves when redundant links are used. The reliability of the network can be improved building two sub networks in parallel. Thus, if one sub networks fails, the other will still connect. Determine the reliability of the networks given that the failure probability of links S1 and S2 each**





2012-2013 Set 2

1. If Z=X+Y-C, where X and Y are the independent random variable with variance σ_X, σ_Y and C is constant. Find the variance of Z.

2. $X(t) = A\cos(\omega_c t + \varphi)$, where A and ω_c are constant, while φ is a random variable with an uniform pdf $f_\varphi(\varphi) = 0.5\pi,\ -\pi < \varphi < \pi$.
   a. Find the mean and autocorrelation function and the PSD of X(t). Show that X(t) is wide sense stationary.
   b. Find the autocorrelation by time averaging
3. X and Y are independent, zero mean, Gaussian random variable σ_x and σ_y. Let $z = \frac{X+Y}{2} \quad and \quad w = \frac{X-Y}{2}$
   a. Find the joint pdf f_{zw}(z,w)
   b. Find the marginal pdf $f_Z(z)$

2011-2012

1. If Z=X+Y-C, where X and Y are the independent random variable with variance σ_X, σ_Y and C is constant. Find the variance of Z.

2. Write a short note on Kraft inequality

2010-2011

1. Define Central Limit Theorem

2009-2010

1. The joint PDF of X and Y is given by $f_{XY}(x,y) = K\, e^{-(\alpha x+\beta y)}u(x)u(y)$, where α and β are positive constants. Determine the value of constant K.
2. Derive Auto correlation and Power spectral density of random process. A pulse train consists of rectangular pulses having an amplitude of 2 volts width which are either 1 micro sec or 2 micro sec with equal probability. The mean time between pulses is 5 micro sec. Find the power spectral density $G_n(f)$ of the pulse train.

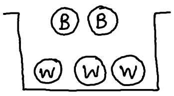
2008-2009

1. Discuss the following
   a. ENTROPY
   b. BAY's rule of probability

2. A binary symmetric channel (BSC) error probability is P_e. The probability of transmitting 1 is Q and that of transmitting '0' is 1-Q. Determine the probabilities of receiving 1 and 0 at the receiver.

Q// An urn contain two black balls & three white balls. Two balls are successive drawn from urn. If X represents the number of white balls, Then find out the PDF & CDF.

Solution :→



$$S = \{ \underset{\downarrow\, x_1}{WW},\ \underset{\downarrow\, x_2}{WB},\ \underset{\downarrow\, x_3}{BW},\ \underset{\downarrow\, x_4}{BB} \}$$

P(X = 0) = probability for no white ball is drawn

$$= \frac{{}^2C_1}{{}^5C_1} \times \frac{{}^1C_1}{{}^4C_1} = \frac{2}{5.4} = \frac{1}{10}$$

↓ Black Ball (First Attempt) ↓ Black Ball (Second Attempt)

P(X = 1) = Probability for first white ball (W B) + Probability for second ball will be white (B W)

$$= \frac{{}^2C_1}{{}^5C_1} \cdot \frac{{}^3C_1}{{}^4C_1} + \frac{{}^3C_1}{{}^5C_1} \cdot \frac{{}^2C_1}{{}^4C_1} = \frac{6}{10}$$

$$= \frac{3}{5}$$

$$P(X=2) = \frac{{}^3C_1}{{}^5C_1} \cdot \frac{{}^2C_1}{{}^4C_1} = \frac{3}{10}$$

Both ball will be white (WW)

x =	2	1	0
$f(x)$ =	$\frac{3}{10}$	$\frac{3}{5}$	$\frac{1}{10}$

$\Rightarrow \sum_x f(x) = 1$ Note

CDF ⇒ $F_X(x) = P(X \le x)$

1. $F_X(-\infty) = 0$

2. $F_X(0) = P(X \le 0) = 0 + \frac{1}{10} = \frac{1}{10}$

3. $F_X(1) = P(X \le 1) = \frac{1}{10} + P(0 < X \le 1)$

$$= \frac{1}{10} + \frac{6}{10} = \frac{7}{10}$$

4. $F_X(2) = P(X \le 2) = F_X(1) + P(1 < X \le 2)$

$$= \frac{7}{10} + \frac{3}{10} = 1$$

Ans

Q// In an experiment a trial consist of four successive with draw of playing cards from a pack of 52 cards. If we define R.V. X as a number of king appearing in a trial. Find $F_X(x)$. {Q.1(a) 2016-17}

Solution :→ X = No. of kings in outcome

No king in outcome

$$\Rightarrow P(X=0) = \frac{{}^{48}C_1}{{}^{52}C_1} \cdot \frac{{}^{47}C_1}{{}^{51}C_1} \cdot \frac{{}^{46}C_1}{{}^{50}C_1} \cdot \frac{{}^{45}C_1}{{}^{49}C_1}$$

$$= \frac{4669920}{6497400}$$

Only One King

$$\Rightarrow P(X=1) = 4 \times \left\{ \frac{{}^4C_1}{{}^{52}C_1} \cdot \frac{{}^{48}C_1}{{}^{51}C_1} \cdot \frac{{}^{47}C_1}{{}^{50}C_1} \cdot \frac{{}^{46}C_1}{{}^{49}C_1} \right\}$$

Cases
$$\begin{bmatrix} K--- \\ -K-- \\ --K- \\ ---K \end{bmatrix}$$

$$= \frac{1660416}{6497400}$$

$$P(X=2) = \left\{ \frac{{}^4C_1}{{}^{52}C_1} \cdot \frac{{}^3C_1}{{}^{51}C_1} \cdot \frac{{}^{48}C_1}{{}^{50}C_1} \cdot \frac{{}^{47}C_1}{{}^{49}C_1} \right\} \times 6$$

$$\begin{bmatrix} KK-- \\ K--K \\ K-K- \\ -KK- \\ --KK \\ -K-K \end{bmatrix}$$

$$= \frac{162432}{6497400}$$

$$P(X=3)=\left\{\frac{^{4}C_1}{^{52}C_1}\cdot\frac{^{3}C_1}{^{51}C_1}\cdot\frac{^{2}C_1}{^{50}C_1}\cdot\frac{^{1}C_1}{^{49}C_1}\right\}\times 4$$

$$=\frac{4608}{6497400}$$

$$P(X=4)=\frac{^{4}C_1\cdot{}^{3}C_1\cdot{}^{2}C_1\cdot{}^{1}C_1}{6497400}=\frac{24}{6497400}$$

Note $\sum_x f(x)=1$

$$\therefore \Rightarrow \frac{4669920+162432+4608+24}{6497400}=1.$$

CDF :- $F_X(x)=P(X\le 0)=\dfrac{4669920}{6497400}$

$$F_X(1)=P(X\le 1)=\frac{4669920}{6497400}+\frac{1660416}{6497400}$$

$$=\frac{6330336}{6497400}$$

$$F_X(2)=P(X\le 2)=\frac{6330336+162432}{6497400}$$

$$=\frac{6492768}{6497400}$$

$$F_X(3)=P(X\le 3)=\frac{6492768+4608}{6497400}$$

$$=\frac{6497376}{6497400}$$

$$F_X(4)=P(X\le 4)=\frac{6497376}{6497400}+\frac{24}{6497400}$$

$$=1 \quad \text{Ans}$$

Q :- {(2014-15) Q.2(c)}
The probability density function is given as

$f_X(x)=a\,e^{-b|x|}$; where X is a random variable.

Find :- (i) Relationship between a & b {2017-18}

(ii) CDF

(iii) The probability that outcome lies between 1 & 2.

Solution :- $f_X(x)=a\,e^{-b|x|}=\begin{cases} a e^{-bx} ; & 0\le x \\ a e^{bx} ; & x\le 0\end{cases}$

we know that for a continuous R.V.

$$\int_{-\infty}^{\infty} f(x)\,dx=1$$

$$\int_{-\infty}^{0} a e^{bx}\,dx+\int_{0}^{\infty} a e^{-bx}\,dx=1$$

$$\Rightarrow \frac{a}{b}\left[e^{0}-0\right]-\frac{a}{b}\left[0-e^{0}\right]=1 \Rightarrow \frac{2a}{b}=1 \Rightarrow \boxed{2a=b}$$

② CDF $\Rightarrow F_X(x)=P(X\le x)$

$$F_X(0)=P(X\le 0)=\int_{-\infty}^{x} a e^{bx}\,dx=\frac{a}{b}\left[e^{bx}\right]$$

$$F_X(\infty)=P(X\le\infty)=\int_{-\infty}^{0} a e^{+bx}\,dx+\int_{0}^{x} a e^{-bx}\,dx$$

$$=\frac{a}{b}\left[1-0\right]-\frac{a}{b}\left[e^{-bx}-1\right]$$

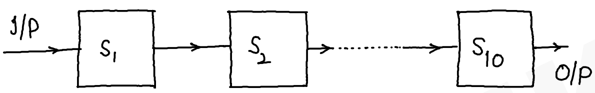
$$=\frac{a}{b}+\frac{a}{b}-\frac{a}{b}e^{-bx}=\frac{a}{b}\left[2-e^{-bx}\right] \quad \text{Ans}$$

$$F_X(x)=\begin{cases}\frac{a}{b}e^{bx} ; & x\le 0\\ \frac{a}{b}\left[2-e^{-bx}\right] ; & 0\le x\end{cases}$$

Q A system consist of 10 subsystems $S_1, S_2, \ldots S_{10}$ in cascade. The probability of failure of any one of the subsystem is 0.01

a) What is the probability of failure of the system {considering none of the system fails}

b) Reliability of a system is the probability of not failing. If the system reliability is required to be 0.99, what must be the failure probability of each subsystem?



Solution :-

Failure of 1 subsystem = (0.01)

Success probability of subsystem

$= (1 - 0.01)$

Now, Success probability of whole system

$$= \underset{1^{st}}{(1-0.01)} \times \underset{2^{nd}}{(1-0.01)} \times \ldots \underset{10^{th}}{(1-0.01)}$$

$$= (1-0.01)^{10} = .904$$

Whole system's failure probability

$= 1 - (.904)$

$= 0.0956$

(b) Reliability ⇒ Success

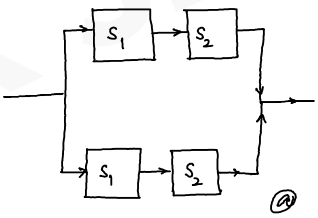
Reliability $\equiv 0.99$, System failure $= 1 - 0.99$
$= 0.01$

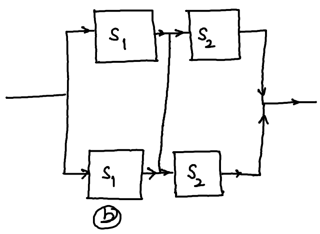
Probability of failure of any subsystem then

$$0.99 = (1-p)^{10} \Rightarrow$$

$$p = 0.0010045$$

Q Network reliability improves when redundant link are used. The reliability of the network can be improved building two sub network in parallel. Thus, if one subnetwork fails, the other will still connect. Determine the reliability of the network given that the failure probability of links S_1 & S_2 is p each.





(2013-14), (2012-13)

Solution :-

let P be the probability of failure of a subsystem (S_1 or S_2)

For the system in Fig(a):

System fails if upper & lower branches fail simultaneously. The probability of any branch not failing is

$(1-P)(1-P) = (1-P)^2$. Hence the probability of any branch failing is $1-(1-P)^2$.

(P.T.O)

Q the probability of failure

is $Q = \left[1-(1-p)^2\right]\left[1-(1-p)^2\right]$

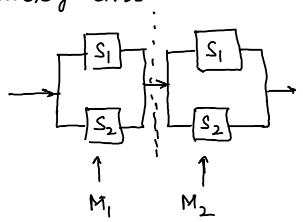
$= \{\cancel{1}-\cancel{1}-p^2+2p\}\{\cancel{1}-\cancel{1}-p^2+2p\}$

$= (p^2-2p)^2$

if $p << 1 \Rightarrow Q \cong 4p^2$

For System (b)

This system is a cascade of two subsystems M_1 & M_2.



Let Q_{M_1} is the probability of failure of M_1.

$$Q_{M_1} = Q_{M_2} = p^2$$

System do not fail if neither M_1 nor M_2 fails.

Hence probability of not failing is $(1-p^2)(1-p^2)$.

Therefore, the probability of system failing

$$Q_M = 1-\{(1-p^2)(1-p^2)\} = 2p^2-p^4 \cong 2p^2, \quad p<<1$$

Hence the system in fig. (a) has twice the probability of failure of the system in fig (b).