# Breaking points of Early Warning Signals: Robustness in Rate-delayed tipping regime

Atharva Aalok, Induja Pavithran, and R.I. Sujith *

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The dynamical behavior of systems is controlled by certain parameters specific to each system. There exist certain thresholds for these parameters crossing which can lead to sudden changes in the qualitative behavior that a system can show. This sudden change in the dynamical behavior is called tipping. These transitions are, in general, extremely dangerous for the system. Significant efforts are thus being put into developing model-independent measures that can predict upcoming transitions; these measures are called Early Warning Signals (EWSs).

Tipping has been classified by [1] into 3 categories: Bifurcation-induced tipping (B-Tipping), Noise-induced tipping (N-tipping) and Rate-induced tipping (R-tipping). Most systems in nature are non-autonomous, and the parameters controlling a system's behavior are change with time. Eventually, a changing parameter can cross the bifurcation threshold and take the system to an alternate stable state. A time-varying parameter results in a delayed transition from the quasi-static bifurcation point; this is called rate-delayed tipping. In multi-stable systems, the rate of change of the parameter itself is also of prime concern. Starting from a fixed initial condition, the system may track one stable attractor at slow rates but may start tracking a different stable attractor for rates above a threshold rate. This phenomenon is called rate-induced tipping.

In this study, we consider a power system model to study the robustness of Early Warning Signals. First, we identify EWSs that work for the model and then analyze their robustness with the rate of change of parameter as the rate is pushed to the threshold separating rate-delayed and rate-induced regimes. We show measures such as Variance, Skewness, Kurtosis, and AC Lag1 work in the entire rate-delay regime; however, the Hurst exponent slowly loses its prediction capacity as the rate of change of parameter increases.

We also highlight some challenges to true early prediction, such as window size, AC lag, and sampling rate selection, in situations when there is no known model for the system and there is a lack of past data for reference.

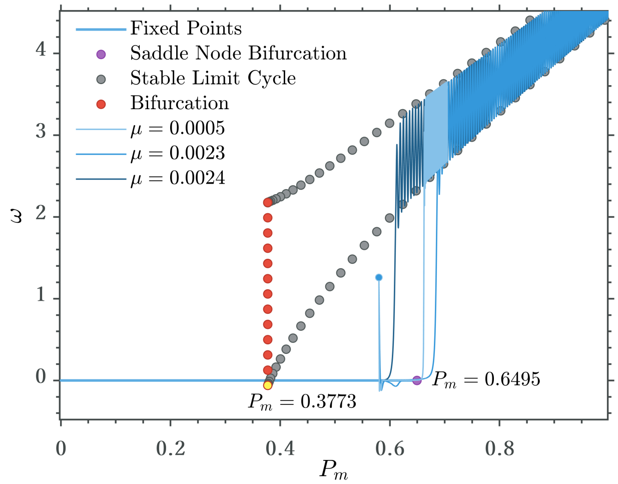


Figure 1: In this figure we show the time series for quasi-static bifurcation, rate-delay tipping and rate-induced tipping on the bifurcation diagram.

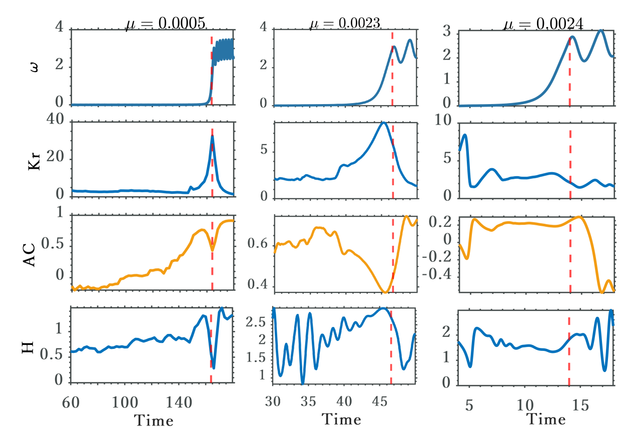


Figure 2: We compare the trends of EWS measures at different rates to find where these measures break. We find that some measures lose their reliability as the rate increases and no measure works after rate-induced tipping.

## References


[1] Peter Ashwin, Sebastian Wieczorek, Renato Vitolo and Peter Cox, *Tipping points in open systems: bifurcation, noise-induced and rate-dependent examples in the climate system.*, Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 370(1962):1166–1184, 2012.


*Atharva Aalok, Induja Pavithran, and R.I. Sujith are with the Indian Institute of Technology Madras (email: ae19b030@smail.iitm.ac.in, indujap2013@gmail.com, sujith@iitm.ac.in)