Formula for Harris corner detector

J.-B. Ryu, C.-G. Lee and H.-H. Park

A new cornerness formula for the well-known Harris corner detector is proposed. The new formula is used to analyse the corner angle range detectable by the Harris detector for some applications and find out the actual corner angle.

Introduction: The Harris corner detector (HCD) has been one of the most successful algorithms in corner detection [1]. We analyse the HCD and derive a new cornerness formula. To show the usage of the new formula, we provide some applications about corner angle detection at L-junctions.

Background: For a given input image I, the HCD first computes the derivatives of x, y axes. Then it performs convolutions of I_x^2, I_y^2 and I_xI_y with a Gaussian window to obtain an autocorrelation matrix M. In the following equations, $\otimes$ and W represent the convolution operator and Gaussian window, respectively:

$$I_x = \frac{\partial I}{\partial x}, \quad I_y = \frac{\partial I}{\partial y} \tag{1}$$

$$\overline{I_x^2} = I_x^2 \otimes W, \quad \overline{I_y^2} = I_y^2 \otimes W, \quad \overline{I_xI_y} = I_xI_y \otimes W \tag{2}$$

$$M = \begin{bmatrix} \overline{I_x^2} & \overline{I_xI_y} \\ \overline{I_xI_y} & \overline{I_y^2} \end{bmatrix} \tag{3}$$

A cornerness value $C(x, y)$ is calculated by the determinant and trace of matrix M:

$$\mathrm{Det}(M) = \alpha\beta = (\overline{I_x^2}\,\overline{I_y^2}) - (\overline{I_xI_y})^2, \quad \mathrm{Tr}(M) = \alpha + \beta = \overline{I_x^2} + \overline{I_y^2} \tag{4}$$

where α, β are eigenvalues of M:

$$C(x, y) = \mathrm{Det}(M) - k \times \mathrm{Tr}^2(M) = (\overline{I_x^2I_y^2} - (\overline{I_xI_y})^2) - k(\overline{I_x^2} + \overline{I_y^2})^2 \tag{5}$$

where k is a constant.

New cornerness formula: Rohr, and Deriche and Giraudon observed that edges are composed by straight lines in all images, and the magnitudes and orientations of gradient vectors on each straight line are constant [2, 3]. Under this observation, an nth order corner is formed by n straight lines, and the differential coefficients of every pixel on the ith straight line can be represented by constant values, i.e. I_{xi} and I_{yi}; see Figs. 1*a* and *b* for examples. However, in reality, the differential coefficients on a straight line cannot be the same. Therefore, we take the average value of all the differential coefficients as the representative differential coefficient.

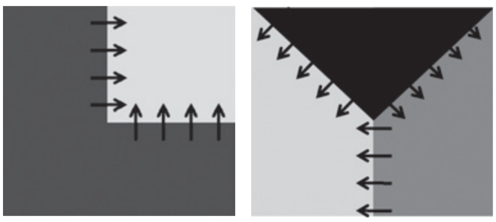

Fig. 1 *Magnitudes and orientations of gradient vectors on edges forming corner*

a L-junction
b Y-junction

The pixels contained in a Gaussian window during the convolution process of (2) have the weights as shown in (6):

$$gauss(x, y) = \frac{1}{2\pi\sigma^2} e^{-\dfrac{x^2+y^2}{2\sigma^2}} \tag{6}$$

Fig. 2 shows the weights of the pixels inside a Gaussian window with $\sigma = 2$ and $r = 4.5$ where the grey coloured pixels comprise two edges forming an L-junction. Let ω_i be the sum of the Gaussian weights on the ith straight line in Fig. 2. In the Figure, the ω_i of each line are $0.028 + 0.025 + 0.017 + 0.009 = 0.079$ and $0.028 + 0.041 + 0.046 + 0.041 + 0.028 + 0.015 + 0.007 = 0.206$, respectively. The convolution in (2) can be presented by the sum of products of the representative derivatives, i.e. I_{xi} and I_{yi}, and ω_i. Therefore, (2) can be transformed into (7). In (7), N represents the number of straight lines in the convolution window:

$$\overline{I_x^2} = \sum_{i=1}^{N} I_{xi}^2 \times \omega_i, \overline{I_y^2} = \sum_{i=1}^{N} I_{yi}^2 \times \omega_i, \overline{I_xI_y} = \sum_{i=1}^{N} I_{xi}I_{yi} \times \omega_i \tag{7}$$

				0.007				
		0.009	0.013	0.015	0.013	0.009		
	0.009	0.017	0.025	0.028	0.025	0.017	0.009	
	0.013	0.025	0.036	0.041	0.036	0.025	0.013	
0.007	0.015	0.028	0.041	0.046	0.041	0.028	0.015	0.007
	0.013	0.025	0.036	0.041	0.036	0.025	0.013	
	0.009	0.017	0.025	0.028	0.025	0.017	0.009	
		0.009	0.013	0.015	0.013	0.009		
				0.007				

Fig. 2 *Gaussian window ($\sigma = 2$, $r = 4.5$)*

The gradient vector of the ith straight line is $G_i = G_{mi}\angle G_{\theta i}$, and the corresponding differential coefficients are presented by the following equations:

$$I_{xi} = G_{mi}\cos(G_{\theta i}) \quad I_{yi} = G_{mi}\sin(G_{\theta i}) \tag{8}$$

To compute the determinant of M, we derive $\overline{I_x^2I_y^2}$ and $\overline{I_xI_y}^2$ by combining (7) and (8) as shown in the following:

$$\begin{aligned} \overline{I_x^2I_y^2} &= \sum_{i=1}^{N} \omega_i G_{mi}^2 \cos^2(G_{\theta i}) \times \sum_{j=1}^{N} \omega_j G_{mj}^2 \sin^2(G_{\theta j}) \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i\omega_j G_{mi}^2 G_{mj}^2 \sin^2(G_{\theta i})\cos^2(G_{\theta j}) \end{aligned} \tag{9}$$

$$\begin{aligned} \overline{I_xI_y}^2 &= \left(\sum_{i=1}^{N} \omega_i G_{mi}^2 \cos(G_{\theta i})\sin(G_{\theta i})\right)^2 \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i\omega_j G_{mi}^2 G_{mj}^2 s_i s_j c_i c_j \end{aligned} \tag{10}$$

where $c_i = \cos(G_{\theta i})$, $s_i = \sin(G_{\theta i})$

By subtracting (10) from (9), we can eliminate the terms $\omega_i^2 G_{mi}^4 c_i^2 s_i^2$ and have the following equation:

$$\begin{aligned} \mathrm{Det(M)} &= \overline{I_x^2I_y^2} - \overline{I_xI_y}^2 = \omega_1\omega_2 G_{m1}^2 G_{m2}^2 \\ &\quad \times (s_1^2c_2^2 + c_1^2s_2^2 - 2s_1s_2c_1c_2) + \ldots. \\ &= \omega_1\omega_2 G_{m1}^2 G_{m2}^2 (s_1c_2 - c_1s_2)^2 + \ldots. \\ &= \omega_1\omega_2 G_{m1}^2 G_{m2}^2 \sin^2(G_{\theta 1} - G_{\theta 2}) + \ldots. \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \omega_i\omega_j G_{mi}^2 G_{mj}^2 \sin^2(G_{\theta i} - G_{\theta j}) \end{aligned} \tag{11}$$

The trace is also simplified by (7) and (8) as shown in the following:

$$\mathrm{Tr}(M) = \overline{I_x^2} + \overline{I_y^2} = \sum_{i=1}^{N} \omega_i G_{mi}^2 \tag{12}$$

Therefore, the new cornerness formula is as follows:

$$\begin{aligned} C(x, y) &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \omega_i\omega_j G_{mi}^2 G_{mj}^2 \sin^2(G_{\theta i} - G_{\theta j}) \\ &\quad - k\left(\sum_{i=1}^{N} \omega_i G_{mi}^2\right)^2 \end{aligned} \tag{13}$$

The second-order corner, i.e. L-junction, is most popular. By replacing N with 2 in (13), we derive the following cornerness formula for the second-order corner:

$$C(x, y) = \omega_1\omega_2 G_{m1}^2 G_{m2}^2 \sin^2(G_{\theta 1} - G_{\theta 2}) - k(\omega_1 G_{m1}^2 + \omega_2 G_{m2}^2)^2 \quad (14)$$

As shown in Fig. 1*a*, $G_{m1} = G_{m2} = G_m$ holds in the second-order corner; hence we can simplify it further as follows:

$$C(x, y) = G_m^4(\omega_1\omega_2 \sin^2(G_{\theta 1} - G_{\theta 2}) - k(\omega_1 + \omega_2)^2) \quad (15)$$

We can infer that $\omega_1 = \omega_2 = \omega$ should hold to maximise $C(x, y)$ in (15). Therefore (16), which has the local maximum cornerness value, is derived:

$$C(x, y) = G_m^4\omega^2(\sin^2(G_{\theta 1} - G_{\theta 2}) - 4k) \quad (16)$$

Applications: Our work can have many applications. Among them, we provide two applications as examples. The detection capability of the HCD can be measured by using (16) on various k values. Some of the literature, such as [3], states that k is set to 0.04 in many practical cases. Note that since $C(x, y) > 0$ in (16), $\sin^2(G_{\theta 1} - G_{\theta 2}) > 0.16$ should hold in order to detect a corner in such a setting. Consequently, it is inferred that the corner angle should have the value between 23.57° and 156.43° to be detectable by the HCD.

The range of the corner angle ϕ of an L-junction which is detectable by the HCD is presented in Fig. 3 where the x-axis represents the value of k and the y-axis represents the corner angle ϕ. The dark region indicates the detectable area. The bandwidth of the detectable corner angles becomes narrower as k increases in general.

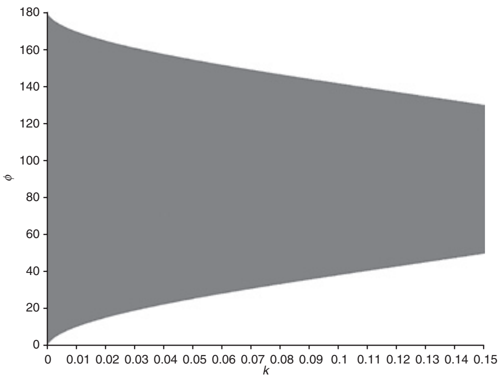

Fig. 3 *Range of corner angle ϕ detectable by k*

As another application, we can calculate the corner angle at an L-junction using the new cornerness formula. We can compute the cornerness value twice, i.e. $C_1(x, y)$ and $C_2(x, y)$ for the same corner for different k_1 and k_2 values using the original formula (5). Each cornerness value can be rewritten using the new cornerness formula (16) as follows:

$$C_1(x, y) = G_m^4\omega^2(\sin^2(G_{\theta 1} - G_{\theta 2}) - 4k_1) \quad (17)$$

$$C_2(x, y) = G_m^4\omega^2(\sin^2(G_{\theta 1} - G_{\theta 2}) - 4k_2) \quad (18)$$

From the above equations we can remove G_m^4 and ω^2 by the following equations and get the absolute value of the corner angle:

$$A = \frac{k_1 C_2(x, y) - k_2 C_1(x, y)}{C_2(x, y) - C_1(x, y)} = \frac{\sin^2(G_{\theta 1} - G_{\theta 2})}{4} \quad (19)$$

$$\phi = G_{\theta 1} - G_{\theta 2} = \sin^{-1}(\pm\sqrt{4A}) \quad (20)$$

From the above result, we can calculate the corner angle at the same time as the normal Harris corner detection without additional algorithms.

Conclusions: We have derived a new cornerness formula for the Harris corner detector. As applications of the new formula, we have analysed the corner detectable angle ranges depending on the k value in the cornerness formula, and we have suggested a simple and fast method to calculate the angle of the detected corner at an L-junction during the normal Harris corner detection process without additional algorithms.

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