The Leontief Input-Output Model

Text Reference: *Linear Algebra and Its Applications*, 5th Ed. by D. Lay, S. Lay, and J. McDonald **Section 2.6, p. 134**

The purpose of this set of exercises is to provide three more examples of the Leontief Input-Output Model in action. The basic assumptions of the model and the calculations involved are reviewed first. Refer to Section 2.6 of your text for more complete information.

Recall that the input-output model requires that the economy in question be divided into sectors. Each sector produces goods or services except for the **open sector** which only consumes goods and services. A production vector **x** lists the output of each sector. A final demand vector (or bill of final demands) **d** lists the values of the goods and services demanded from the productive sectors by the open sector. As the sectors strive to produce enough goods to meet the final demand vector, they make intermediate demands for the products of each sector. These intermediate demands are described by the **consumption matrix**. This matrix is constructed as follows.

The description of the economy begins with a collection of data called an **input-output table** or (an **exchange table**) for an economy. This table lists the value of the goods produced by each sector and how much of that output is used by each sector. For example, the following table is derived from the table Leontief created for the American economy in 1947. (See References 1 or 2 for the complete table.) For purposes of this example the data from the 42 sectors has been collected into just 3: agriculture, manufacturing, and services. Of course, the open sector is also present.

	Agriculture	Manufacturing	Services	Open Sector
Agriculture	34.69	4.92	5.62	39.24
Manufacturing	5.28	61.82	22.99	60.02
Services	10.45	25.95	42.03	130.65
Total Gross Output	84.56	163.43	219.03	

Table 1: Exchange of Goods and Services in the U.S. for 1947 (in billions of 1947 dollars)

Reading the table is straightforward; for example, in 1947 the agriculture sector spent 84.56 billion dollars for the inputs it needed. These inputs were divided among the sectors as follows: 34.69 billion dollars of agricultural output was consumed by the agriculture sector itself, 5.28 billion dollars of manufacturing output was consumed by the agriculture sector, etc.

To create the consumption matrix from the table, divide each column of the 3x3 table by the Total Gross Output for that sector. The result is Table 2.

	Agriculture	Manufacturing	Services
Agriculture	0.4102	0.0301	0.0257
Manufacturing	0.0624	0.3783	0.1050
Services	0.1236	0.1588	0.1919

Table 2: Inputs Consumed Per Unit of Sector Output

The matrix with entries taken from this table is the consumption matrix C for the economy.

$$c = [0.4102\ 0.0301\ 0.0257;$$
$$0.0624\ 0.3783\ 0.1050;$$
$$0.1236\ 0.1588\ 0.1919]$$

The equilibrium levels of production for each sector may now be calculated. These equilibrium levels are the production levels which will just meet the intermediate demands of the sectors of the economy plus the final demands of each sector. If $\mathbf{x}$ is the desired production vector, $\mathbf{x}$ must satisfy $\mathbf{x} = C\mathbf{x} + \mathbf{d}$. This equation may be solved for $\mathbf{x}$ to find that $\mathbf{x} = (I - C)^{-1}\mathbf{d}$, where I is the identity matrix. In the example,

$$(I-C)^{-1} = \begin{pmatrix} 1.7203 & .1006 & .0678 \\ .2245 & 1.6768 & .2250 \\ .3073 & .3449 & 1.2921 \end{pmatrix} \text{ and thus } \mathbf{x} = (I-C)^{-1}\mathbf{d} = \begin{pmatrix} 82.40 \\ 138.85 \\ 201.57 \end{pmatrix}$$

On MATLAB, this can be found by   inv(eye(3) - C)*d

Question:

1. Suppose the bill of final demand is changed to

$$\mathbf{d1} = \begin{pmatrix} 40.24 \\ 60.02 \\ 130.65 \end{pmatrix}.$$

What is the new equilibrium production vector? Find the difference between this new vector and the old equilibrium vector. How much extra production must each sector provide?

Notice that in the above exercise, the only difference in the old and new demand vectors is the addition of one unit of demand to the agricultural sector. Also notice that the difference in the old and new production vectors is just the first column of the matrix $I - C^{-1}$. This is a valuable interpretation of the entries of $I - C^{-1}$.

Observation: The (i, j) entry in the matrix of $I - C^{-1}$ is the amount by which sector i must change its production level to satisfy an increase of 1 unit in the final demand from sector j.

Question

2. How much would the service production level need to increase if agricultural demand for services increased by 1 unit? How much would the manufacturing production level need to increase in this situation?

Now consider a less abrupt consolidation of the 1947 economic data: the econ-omy is now divided into 25 sectors. These sectors are:

1. Agriculture and Fisheries
2. Food and Kindred Products
3. Textiles and Apparel
4. Lumber, Wood, and Furniture
5. Paper, Printing, and Publishing
6. Chemicals, Petroleum Products, Rubber
7. Leather and Leather Products
8. Stone, Clay and Glass Products
9. Primary Metals
10. Fabricated Metal Products
11. Machinery (non-electric)
12. Electrical Machinery
13. Motor Vehicles
14. Other Transportation Equipment
15. Miscellaneous Manufacturing
16. Coal, Gas, and Electric Power
17. Transportation Services
18. Trade
19. Communications
20. Finance, Insurance, and Real Estate
21. Business Services
22. Personal and Repair Services
23. Miscellaneous Services
24. New Construction and Maintenance
25. Undistributed

The consumption matrix $C1$ and final demand vector $\mathbf{d}1$ for this model can be found in example 3 on the m-file *leontief.m* accompanying this project.

Questions

3. Find the equilibrium production vector for the consumption matrix $C1$ and final demand vector $\mathbf{d1}$.

4. If the final demand for motor vehicles increases by one bill ion dollars, how much will the production of fabricated metal products have to increase to com-pensate?

The input-output model is still used to model economies throughout the world, as well as the global economy itself. The most up-to-date available input-output table for the American economy is the 1998 table. A copy of the entire ta-ble (which is officially called the I-O Use Table) is an appendix to the docu-ment "Annual Input-Output Accounts of the U.S. Economy, 199 8" which can be viewed or downloaded from the Bureau of Economic Analysis website at http://www.bea.gov/bea/an2.htm. This document provides a good overview of different types of tables, as well as some applications of the 1998 table. The table for 1998 divides the economy into nearly 500 sectors, which are then consolidated into over 90 sectors. These categories have been carefully compressed by the author into the 33 listed below.

The resulting table is given in Table 3; the consumption matrix $C2$ and final demand vector **d2** which accompany this project are derived from that data. To see the data in $C2$ in MATLAB, use **format rat** or **format long**.

1. Agriculture, Forestry and Fisheries
2. Mining, Petroleum, and Natural Gas
3. Construction
4. Food and Kindred Products
5. Tobacco Products
6. Textile Products
7. Lumber and Wood Products
8. Furniture and Fixtures
9. Paper Products
10. Printing and Publishing
11. Chemicals, Plastics, Drugs, Paints
12. Petroleum Refining
13. Rubber and Miscellaneous Plastics
14. Footwear and Leather Products
15. Glass, Stone, and Clay Products
16. Metals Manufacturing
17. Fabricated Metal Products
18. Non-Electrical Machinery
19. Electrical Machinery
20. Motor Vehicles
21. Other Transportation Equipment
22. Miscellaneous Manufacturing
23. Transportation Services
24. Communication Services
25. Utilities
26. Trade
27. Finance, Insurance, and Real Estate
28. Personal Professional Services-nonmedical
29. Miscellaneous Services
30. Health Services
31. Educational and Social Services
32. Government
33. Miscellaneous – Imports, Scra p, etc.

Questions

5, Find the equilibrium production vector for the consumption matrix $C2$ and final demand vector **d2**.

6. Which three sectors are most affected by an increase in $1 of final demand for motor vehicles?

7. How many dollars worth of new production is produced **in the entire economy** by an increase in $1 of final demand for motor vehicles? Note : this amount is referred to in economic literature as a "backward linkage."

8. If demands are increased by 10% in all sectors, by what percentage must total production increase to maintain equilibrium?

9. What would it mean for the (i, j) entry in $(I - C)^{-1}$ to be zero?

References

Leontief, W. W. (1951), "Input-Output Economics," *Scientific American* , 15–21, this article explains the author's input-output model, and includes the complete 42-sector exchange table for 1947.

— (1965), "The Structure of the U.S. Economy," *Scientific American* , 25–35, this article contains the 81-sector table mentioned above.

— (1966), *Input-Output Economics*, New York: Oxford University Press, this book contains the full 42-sector exchange table for 1947, as well as an 81-sector table for 1958.

Sect #	1	2	3	4	5	6	7	8	9	10	11
1	68681	78	5860	124826	3182	4088	8698	56	92	99	1207
2	367	31477	7368	188	24	53	3	13	668	0	9388
3	3368	4693	895	2498	126	941	389	306	1786	1177	3604
4	21245	0	0	78643	0	18	0	95	698	0	1971
5	0	0	0	0	3508	0	0	0	0	0	0
6	1529	94	2693	98	8	54013	207	4872	964	138	50
7	529	121	54850	123	2	41	35867	4845	6741	30	46
8	0	0	2219	0	0	0	271	290	0	0	0
9	1836	35	3949	16615	677	333	340	1027	34698	23297	5315
10	57	3	200	957	233	41	19	14	192	16133	681
11	14107	2360	10344	4631	387	15791	1269	651	9864	3142	85977
12	4075	1810	13316	893	44	248	376	174	689	206	2839
13	1484	640	18237	10123	290	1161	942	2539	4278	1620	10249
14	53	1	0	0	0	953	9	57	0	3	0
15	164	498	51373	3565	4	334	1052	331	79	2	1055
16	63	2481	15247	0	0	2	112	2990	545	78	265
17	1048	1557	68795	11387	50	6	2572	4039	656	58	3138
18	1300	4254	10402	560	21	641	441	311	1093	845	1188
19	1284	452	40765	297	47	10	641	27	71	130	290
20	0	0	0	0	0	0	0	0	0	0	0
21	389	175	1603	244	2	23	465	25	32	86	34
22	237	35	5438	50	10	638	85	31	40	832	283
23	7981	3127	18509	14828	438	3268	4479	1369	7966	4407	12129
24	829	345	4082	771	79	344	206	221	409	956	1196
25	3816	9181	2256	5807	103	2721	1318	606	4506	1434	10610
26	13948	3498	81671	30923	920	7755	8037	4365	7986	6058	22446
27	20647	33253	16486	6523	618	2903	1671	1276	2050	8728	6749
28	4722	4836	97144	10982	1146	8450	2517	2165	4460	9334	20655
29	1635	886	6517	16499	4075	2371	1181	1049	2291	4671	13466
30	2459	0	0	0	0	0	0	0	0	0	0
31	182	126	50	308	30	231	194	74	100	186	433
32	164	30	1069	743	85	198	97	123	336	832	472
33	66	1872	6	1033	31	175	15	62	980	480	6900
TGO	283291	147738	1E+06	493690	46203	163259	118243	65889	161487	212238	385970

Table3a: Exchange of Goods and Services in the U.S. for 1998 (in millions of dollars)

Sect #	12	13	14	15	16	17	18	19	20	21	22
1	37	1090	2	65	120	99	122	379	49	192	221
2	59472	32	2	4438	6687	113	55	119	20	370	79
3	1457	1258	63	713	1435	1656	1334	5276	869	2400	1468
4	172	18	851	25	0	0	0	0	0	4	98
5	0	0	0	0	0	0	0	0	0	0	0
6	0	3042	484	201	16	19	289	9	7526	666	1967
7	5	286	0	627	586	157	397	124	7	1138	822
8	0	0	0	0	0	0	0	748	6857	217	88
9	324	3142	48	1732	198	1434	725	3330	87	748	3067
10	9	51	1	11	101	96	35	119	31	82	113
11	3005	35163	374	3396	3295	3459	674	5849	2754	2750	3908
12	15345	256	21	460	714	364	327	534	166	361	288
13	668	9098	197	1032	1028	3355	4287	12528	14478	6724	5371
14	1	0	2370	0	0	7	0	0	3	2	51
15	751	1079	5	10872	2509	779	713	2997	2555	1112	1046
16	12	802	0	735	46771	57336	23615	22167	497	29688	6663
17	299	1582	85	634	3493	21819	15286	19384	26169	18577	7202
18	132	2000	10	565	4830	6176	21673	4790	1494	12658	1865
19	16	197	0	148	1288	1953	5133	127957	15180	13315	18918
20	0	0	0	0	0	0	0	0	75	2299	0
21	24	18	0	73	31	59	104	40	64858	37315	2255
22	30	104	1	75	147	191	256	3217	1875	8595	8566
23	6906	6453	205	6056	9053	5217	3634	6243	5597	5513	2780
24	216	473	24	276	376	695	808	2213	262	627	1060
25	3937	3246	139	3267	7972	3871	2193	4901	885	2951	1729
26	5523	6802	410	3925	15350	13594	13512	39140	17021	15412	11494
27	3779	3243	128	1394	2613	4102	3475	10514	2568	4537	4306
28	3774	5154	179	2883	6319	8512	6481	20459	4810	9741	9129
29	1562	2237	218	1237	2443	3273	3060	9695	17918	5663	5427
30	0	0	0	0	0	0	0	0	0	0	0
31	109	553	3	48	111	325	130	632	348	341	522
32	75	146	23	71	224	266	214	506	285	471	308
33	549	406	5	340	7575	481	671	9146	790	820	831
TGO	133536	160122	8266	90486	189931	250513	222904	482730	229740	308128	230249

Table3b: Exchange of Goods and Services in the U.S. for 1998 (in millions of dollars)

Sect#	23	24	25	26	27	28	29	30	31	32	33
1	15	110	28	1817	11476	1340	7879	2170	923	567	0
2	133	0	52220	31	6	11	8	6	6	3062	0
3	6985	13603	26781	12695	66515	6458	5746	4802	11780	25894	0
4	337	0	0	847	0	206	77652	5322	4234	631	0
5	0	0	0	0	0	0	0	0	0	0	0
6	343	159	3	1209	216	2001	1017	1911	273	356	0
7	330	50	217	3615	318	478	219	252	568	0	0
8	0	0	0	1	2	8	454	3	1	741	0
9	447	513	128	14851	2780	7607	3033	3532	2083	526	0
10	1307	1484	118	9165	6855	10074	3911	3836	13527	1568	0
11	589	234	814	1931	876	4225	2152	32674	834	2835	0
12	14900	293	1881	5598	1042	2491	2387	1050	988	3187	0
13	3414	796	423	6661	1023	3516	7689	8724	1056	186	0
14	5	2	2	263	24	486	105	31	117	92	0
15	125	41	87	441	643	1210	2347	2059	322	883	0
16	508	26	125	347	31	276	32	11	39	73	0
17	2573	1445	488	3112	986	1878	7329	1009	528	560	0
18	1962	767	500	3149	367	3930	1170	182	142	1653	0
19	1569	14513	691	10873	1919	42443	5393	3681	1077	1810	0
20	0	0	0	0	0	0	103	0	0	0	0
21	13449	66	1347	3944	256	1639	25749	503	434	2340	0
22	347	342	737	2000	1989	6917	1904	15868	2069	153	0
23	93427	2376	8489	15621	13947	15053	9724	7356	3014	10731	0
24	6241	53623	924	26371	22228	23399	5965	8370	3495	1066	0
25	5702	1999	28155	26223	16452	12020	15368	12574	4425	11078	0
26	9516	3551	2012	32685	4925	17999	29519	15050	5467	2647	0
27	19668	13119	7498	108418	445681	89791	52151	61523	40287	7945	0
28	35168	33860	12864	133305	147224	225101	46800	74654	37098	11787	0
29	22658	35852	1497	83370	42024	29184	63808	15237	13932	1640	0
30	83	0	0	0	0	4	56	15181	0	0	0
31	910	552	1055	2551	2117	4855	1057	2259	1751	159	0
32	1195	1406	701	11225	28194	10351	3177	7628	3355	3035	0
33	13528	8276	141	7725	8555	4429	887	48	30	1144	0
TGO	534111	393893	322303	2E+06	3E+06	2E+06	832112	764606	323751	221648	990060

Table3c: Exchange of Goods and Services in the U.S. for 1998 (in millions of dollars)

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