Basic Economics for Industrial Engineering

The Firm's Costs

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Introduction

Basic Problem of the Firm

1. Defining the production level and the factors need: *production function*
2. Measuring *costs*

(Usefull handbooks: Varian 2006, Stiglitz and Walsh (2004), Frank (2008), Krugman and Wells (2005))

The Input-Output Relationship or Production Function

Economics of the firm - Producer model

Producer model

- We assume a firm on a competitive market
  Atomicity, price taker, perfect information, homogeneous product, available inputs
- The firm's choice is not the price (imposed by the market), BUT the quantities to produce given the market price, the technology and the factors prices.
- The firm's strategy is to choose the quantities to produce such that it maximises the profit (and then reduces the costs)

The goals of production economics are twofold:

1. to provide guidance to individual farmers in using their resources most efficiently, and
2. to facilitate the most efficient use of resources from the standpoint of the consuming economy

Definitions

Definitions

- Profit firm = Revenu - Total cost of production
  $\pi(q) = R(q) - CT(q)$
- Revenu is the value os sales : $R(q) = p \times q$, where p is the given market price.
- Total cost of production is the sum of fixed and variable costs : $CT(q) = CF + CV(q)$
- Variable costs are linked to factor of production (or inputs):$x_1, ..., x_k$
- Production q and the inputs $x_1, ..., x_k$ are linked by the production function: $q = f(x_1, x_2)$ (in the 2-inputs case).

Given the assumptions of the model:

- Maximising profit is equivalent to minimising the total cost
- The firm's levers are:
  - Produced quantity q
  - Input mix (x_1, x_2) (taking into account all possible substitutions between inputs allowed by the technology of production)

Production and Productivity

- The production function (and indeed all representations of technology) is a **purely technical relationship** that is void of economic content.
- to **portray** and to **predict** economic behavior accurately
- without direct examination of the production function
- Economists are **not engineers** and have no insights into why technologies take on any particular shape.
- Economists are only interested in those properties that make the production function consistent with optimizing behavior.

Example: One factor Production case

- Let assume a **Silk Producer** that own a fixed number of fields (or an factory owning a fixed number of machines).
- He has to determine the **number of workers**.
- The only input in this production case is the only variable factor: labor.

Definition: Production Function

The Production Function describes the relationship between inputs x_i and production (y).
It gives the maximum quantity of products (y_m) achievable using given quantities of inputs (x_i).
The production function $f()$ is a function:

$$f : \mathcal{R}_n^+ \longrightarrow \mathcal{R}_m^+$$

from n inputs into m outputs.

Adjustment in the production function

- long run: the shortest period of time required to alter the amounts of all inputs used in a production process.
- short run the longest period of time during which at least one of the inputs used in a production process cannot be varied.
- variable input an input that can be varied in the short run.
- fixed input an input that cannot vary in the short run.

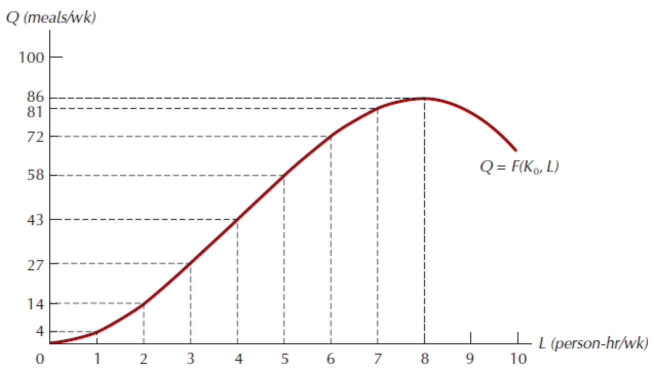
Production in the Short Run

- Hence, a common form is $y = f(x)$
- where y is the scalar (single) output and x is a *vector* of inputs.
- In the single output case, we examine the relationship between y and x_i given that all the other factors of production are held constant.

$$y = f(x_i | x_{-i})$$

Total, Marginal and Average Products

Increasing and decreasing returns short-run production function



Law of diminishing returns

if other inputs are fixed, the increase in output from an increase in the variable input must eventually decline

Lets identify three primary relationships:

- Total Product: which is the production function.
- Average Product: defined as the average output per unit of input.
  Mathematically, $AP = \frac{y}{x} = \frac{f(x)}{x}$

- Marginal Product: defined as the rate of change in the total product at a specific input level.
  Mathematically, $MP = \frac{dTP}{dx} = \frac{dy}{dx} = \frac{df(x)}{dx} = f'(x)$
  We will note MP_{x_i} the marginal product of the factor x_i.

  It gives the increase in output y due to an increase of the input x_i, **everything kept equal** (assuming the other factor kept constant).
  The MP corresponds to the slope of the PT curve.

Example: One input production case

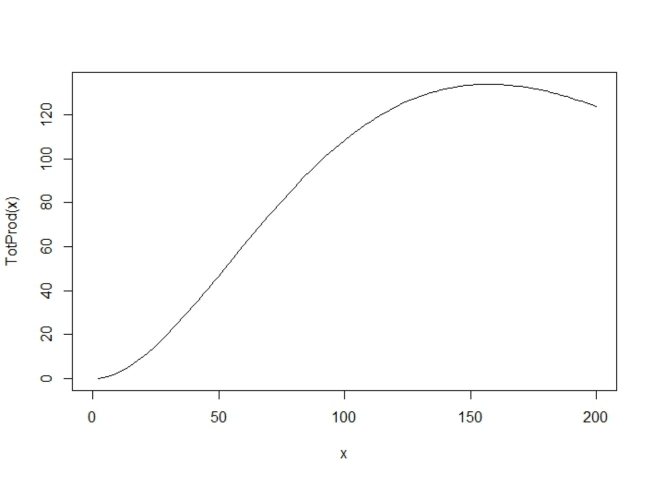
Assume the following quantities of labor used and associated silk produced (in tonnes):

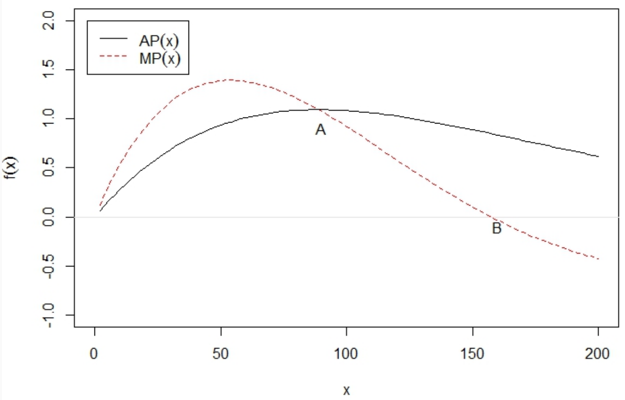
Nb 1000 H	Silk (tonnes)	MP
5	95	
6	120	
7	140	
8	155	
9	165	
10	170	
11	170	

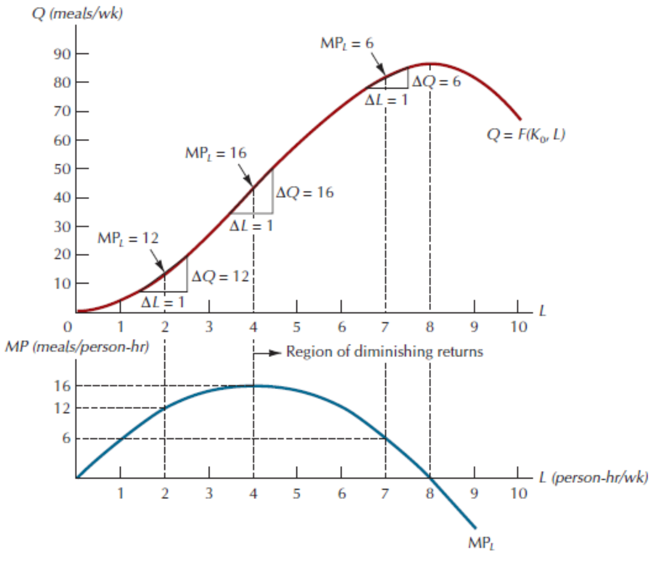
Question: $\backslash$ Define and calculate the *Marginal Product* of the labor input (x_L).

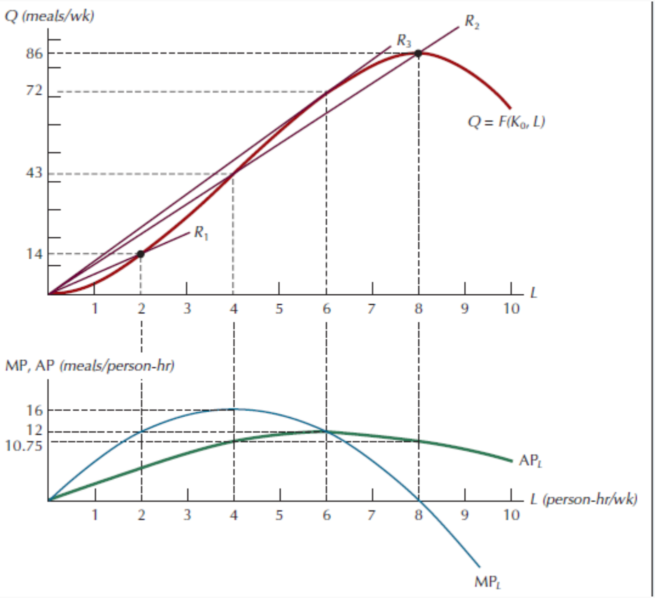
Nb 1000 (H)	Coton (tonnes)	MP
5	95	-
6	120	25
7	140	20
8	155	15
9	165	10
10	170	5
11	170	0

TP is increasing with the input x. $\backslash$ MP is positive and decreasing. This illustrates the common assumption of **decreasing returns of the factor**.


TotProd(x)
0
20
40
60
80
100
120
0
50
100
150
200
x


AP(x)
MP(x)
A
B
f(x)
x
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
0
50
100
150
200


Q (meals/wk)
90
80
70
60
50
40
30
20
10
MP_L = 6
ΔQ = 6
ΔL = 1
Q = F(K_0, L)
MP_L = 16
ΔQ = 16
ΔL = 1
MP_L = 12
ΔQ = 12
ΔL = 1
L
0 1 2 3 4 5 6 7 8 9 10
MP (meals/person-hr)
Region of diminishing returns
16
12
6
L (person-hr/wk)
1 2 3 4 5 6 7 8 9 10
MP_L



Q (meals/wk)
86
72
43
14
R_2
R_3
R_1
$Q = F(K_0, L)$
L
1
2
3
4
5
6
7
8
9
10
MP, AP (meals/person-hr)
16
12
10.75
AP_L
L (person-hr/wk)
MP_L

Read through the following scenario carefully and try to answer the question posed at the end:

> *Suppose you own a fishing fleet consisting of a given number of boats, and can send your boats in whatever numbers you wish to either of two ends of an extremely wide lake, east or west. Under your current allocation of boats, the ones fishing at the east end return daily with 100 pounds of fish each, while those in the west return daily with 120 pounds each. The fish populations at each end of the lake are completely independent, and your current yields can be sustained indefinitely. Should you alter your current allocation of boats?*

Should you move one of your boats from the east end to the west end?

Number of boats	East end			West end		
	AP	TP	MP	AP	TP	MP
0	0	0		0	0	
			100			130
1	100	100		130	130	
			100			110
2	100	200		120	240	
			100			90
3	100	300		110	330	
			100			70
4	100	400		100	400	

Optimizing Production

- The general rule for allocating an input efficiently: allocate the next unit of the input to the production activity where its marginal product is highest
- allocate the resource so that its marginal product is the same in every activity

Definition: Return of a Factor

We qualify of increasing return of a factor (resp. constant / decreasing) the situations in which a supplementary unit of factor used leads to a higher (equivalent/ smaller) increase of the output than the preceding input unit in the process.

It corresponds to the analysis of the first derivative of the MP or of the second derivative of the TP, with respect to the x_i input, all other inputs kept constant.

Definition: return to scale

- Study of the scale of production may indicate specific properties of the production, when all the inputs are **varying by the same factor**.
- Return to scale are said to be **increasing** (resp. constant / decreasing) when the increase of all the inputs by the same factor leads to an increase of output of a **higher factor** (resp. same / smaller factor).
- The return to scale notion is equivalent to the study of the *degree of homogeneity* of the production function.

It is important to bear in mind that decreasing returns to scale have nothing whatsoever to do with the law of diminishing returns

Elasticity of Production

Elasticities are often used in economics to produce a unit-free indicator of the shape of a function.

Definition: Factor Elasticity

The factor elasticity evaluates the sensitivity of the output, the production function, to a change in an input. Thsese two quantities are expressed in relative terms, hence in percentage of variation of output and input.

$$E = \frac{\%\Delta y}{\%\Delta x} = \frac{\frac{dy}{y}}{\frac{dx}{x}} = \frac{dy}{dy}\frac{x}{y} = \frac{MP}{AP}$$

Average and Marginal notions are linked by specific mathematical relation:

$$MP(x) = \frac{dTP(x)}{dx} = \frac{dx \times AP(x)}{dx} = AP(x) + x \times \frac{dAP(x)}{dx}$$

Thus it follows:

- $AP(x)$ is maximized when : $\frac{dAP(x)}{dx} = 0 \Rightarrow MP(x) = AP(x)$.
- $\frac{dAP(x)}{dx} > 0 \Rightarrow MP(x) > AP(x) \Rightarrow E(x) > 1$.
- $\frac{dAP(x)}{dx} = 0 \Rightarrow MP(x) = AP(x) \Rightarrow E(x) = 1$
- $\frac{dAP(x)}{dx} < 0 \Rightarrow MP(x) < AP(x) \Rightarrow E(x) < 1$
- If $E(x) = 0 \Leftrightarrow MP(x) = 0$: the TP is maximized.
- If $E(x) < 0 \Leftrightarrow MP(x) < 0$.

We can see the three stages/levels of factor elasticity on the TP curve.

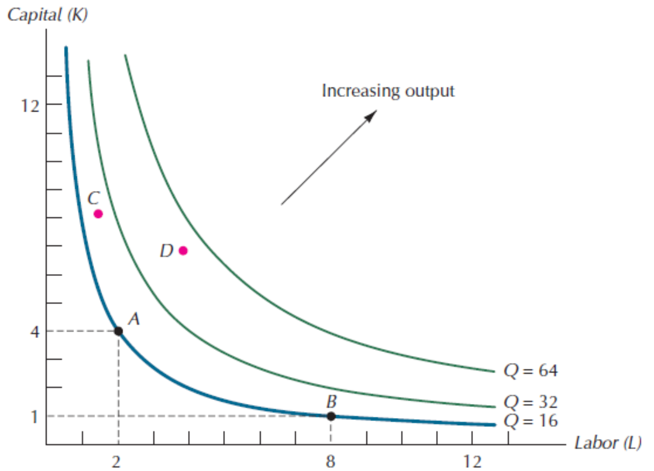
Production in the Long Run

Suppose a production function with 2 inputs in the long run.

$$Q = F(K, L) = 2KL$$

Isoquants describe all possible combinations of K and L for a given productino level Q :

$$K = \frac{Q}{2L}$$



Capital (K)
Increasing output
12
4
1
C
D
A
B
Q = 64
Q = 32
Q = 16
2
8
12
Labor (L)

Isoquants, Isoclines, and Ridgelines

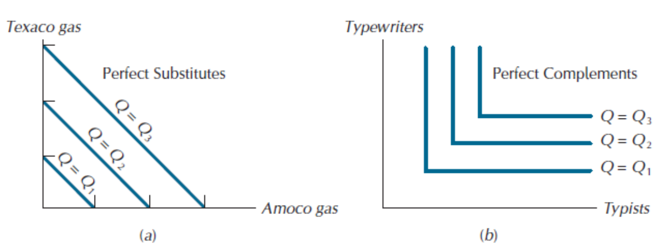
- In the case of 2 inputs production function ($y = f(x_1; x_2)$), it is possible to define isoquants: the inputs mixes that yield the same output level.

$$x_2 = f^*(x_1; y)$$

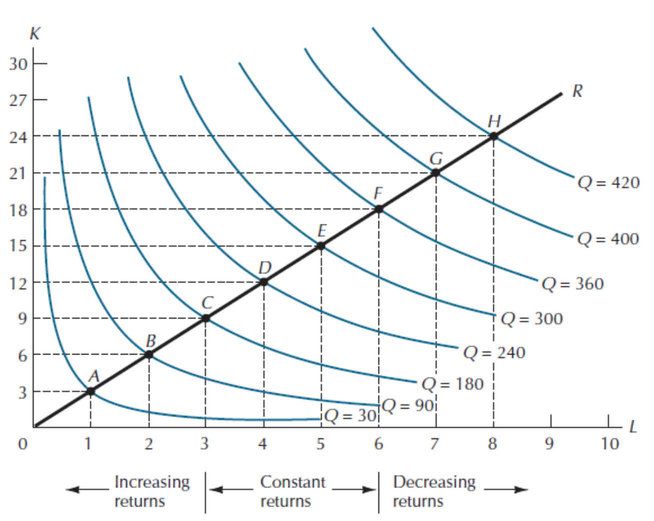
- The isoquants are useful in defining the Marginal rate of technical substitution or the rate at which one input must be traded for the other input, without altering total output.

- $$dy = \frac{\partial f(x_1, x_2)}{\partial x_1} dx_1 + \frac{\partial f(x_1, x_2)}{\partial x_2} dx_2 = 0$$

- or simply: $MRTS = |\frac{\Delta x_2}{\Delta x_1}| = |\frac{MP_{x_1}}{MP_{x_2}}|$



Texaco gas
Perfect Substitutes
$Q = Q_3$
$Q = Q_2$
$Q = Q_1$
Amoco gas
(a)
Typewriters
Perfect Complements
$Q = Q_3$
$Q = Q_2$
$Q = Q_1$
Typists
(b)


K
30
27
24
21
18
15
12
9
6
3
0
1
2
3
4
5
6
7
8
9
10
L
R
A
B
C
D
E
F
G
H
Q = 420
Q = 400
Q = 360
Q = 300
Q = 240
Q = 180
Q = 90
Q = 30
Increasing returns
Constant returns
Decreasing returns

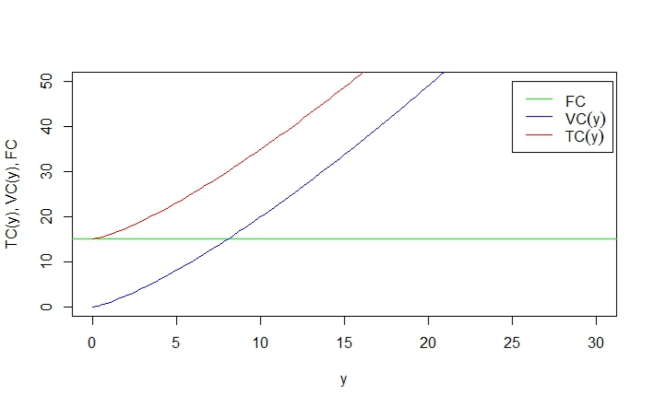
Cost of Production

Different Types of Production Costs

- Prices and quantities used of the inputs determine the production cost.
- The production function gives information on the multiple production alternatives (input mixes) to produce a given amount of output.
- The optimal strategy of the firm is to choose the input mix that minimize the cost of production for a given output level, given the inputs prices (relative prices).
- What types of costs can enter the firm's decision?

We distinguish:

- The Fixed Costs (FC): Cost that are independant from the quantity produced.
- The Variable Costs ($VC(y)$): Cost that depend on the quantity y.
- The Total Cost (TC) : $TC(y) = VC(y) + FC$



FC
VC(y)
TC(y)
TC(y), VC(y), FC
0
10
20
30
40
50
0
5
10
15
20
25
30
y

Marginal Analysis of the Firm

The firm decision considers:

- The Marginal Cost (MC)
- The Total revenue (TR) and Marginal revenue (MR)
- The Total profit (π) and marginal profit (π_m)

Definitions

Marginal Cost: is the incremental cost induced by the next (or the last) output unit produced. $MC(y)$ is defined by:

- in discrete case: $MC(Y) = \frac{\Delta TC}{\Delta Y}$
- in continuous case: $MC(y) = \frac{\delta TC(y)}{\delta y}$

- $MC(y)$ indicates the total cost *shape* $(TC(y))$
- and indicates the *nature of returns to scale*.
- In the one-input case: $MC(y)$ indicates also the return to factor.

Definitions

Marginal Revenue $MR(y)$: is the incremental revenue due to the production (and the sell) of the last unit (or the next unit y.

- If the firm is price-taker : $RT(y) = p \times y$
  Hence $MR(y) = p$.

Definitions

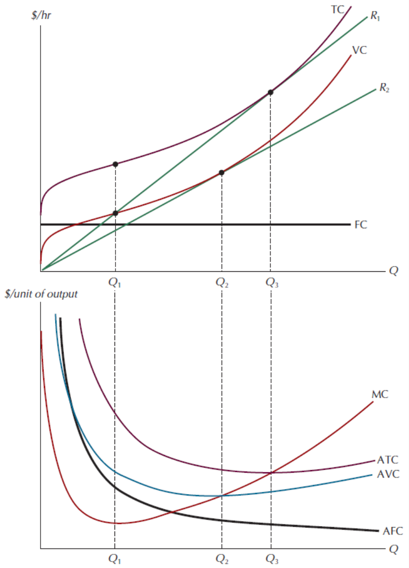
Marginal Profit $M\pi(y)$: is the incremental profit associated to the last unit produced (and sold).

- It comes $\pi(y) = TR(y) - TC(y)$
- Hence $M\pi(y) = MR(y) - MC(y)$
- We deduce the decision rule: an option is chosen only if its Marginal Revenue is greater or equal to its Marginal Cost.
- And the maximum profit quantities y^* is achieved when: $MR(y) = MC(y)$

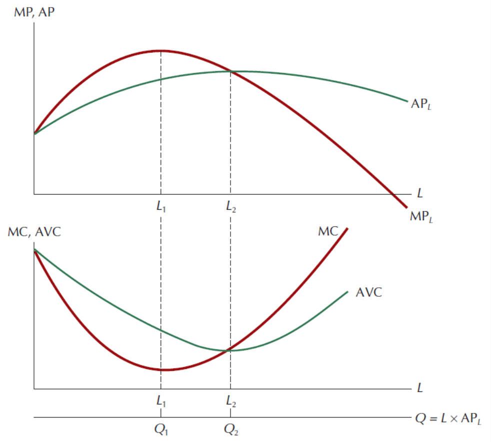
Average and Marginal Costs

- If $TC(y) = FC + VC(y)$, then $ATC = \frac{TC(y)}{y}$.
- $AC(y)$ is U shaped.
- $ATC = \frac{VC(y)}{y} + \frac{FC}{y} = AVC + AFC$
  Lets identify this form, its impact on Marg. Cost and on the nature of returns.
- two opposing effects on average total cost-the "spreading effect" and the "diminishing returns effect"

The Marginal, Average Total, Average Variable, and Average Fixed Cost Curves



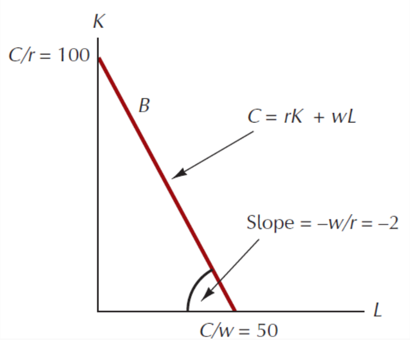
The Relationship among MP, AP, MC, and AVC



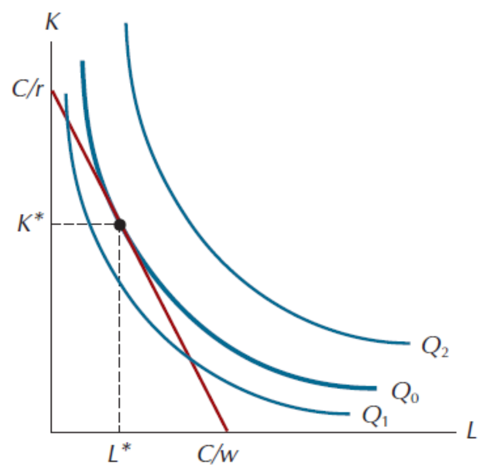
Choosing the Optimal Input Combination

Suppose only two inputs, capital (K) and labor (L), whose prices, measured in dollars per unit of input per day, are $r = 2$ and $w = 4$, respectively.

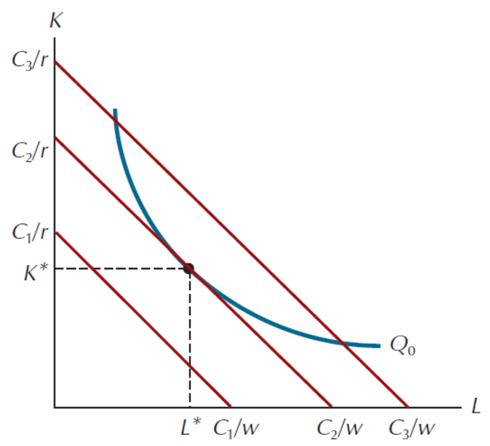
What different combinations of inputs can this firm purchase for a total expenditure of $C = \$200/day$?



The Maximum Output for a Given Expenditure



The Minimum Cost for a Given Level of Output



Choosing the Optimal Input Combination

Minimum cost occurs at a point of tangency with the isocost line (whose slope is $-w/r$), it follows that

$$\frac{MP_{L*}}{MP_{K*}} = \frac{w}{r}$$

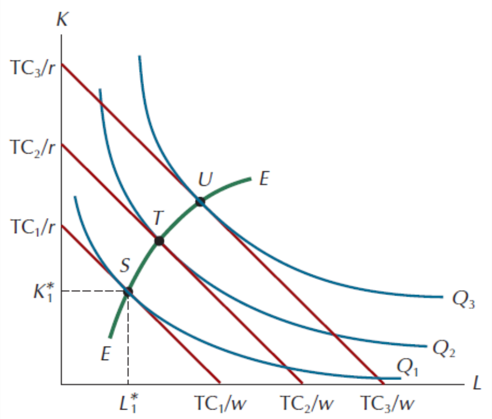
where K^* and L^* again denote the minimum-cost values of K and L.

$$\frac{MP_{L*}}{w} = \frac{MP_{K*}}{r}$$

Costs are at a minimum, when the extra output we get from the last dollar spent on an input must be the same for all inputs.

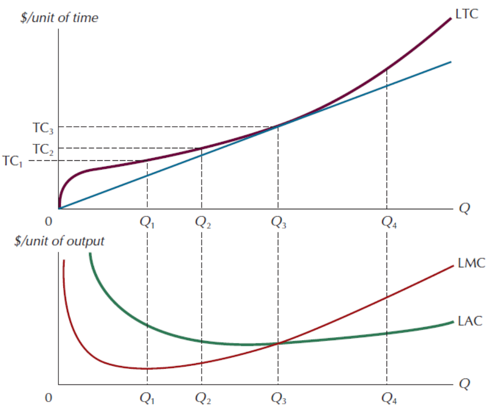
Relationship Between Optimal Input Choice and Long-run Costs

Output Expansion Path: \ Set of cost-minimizing input bundles when the input price ratio is fixed, and the output level increases.



The Long-Run Total, Average, and Marginal Cost Curves

Using the input prices, we can determine the LRTC

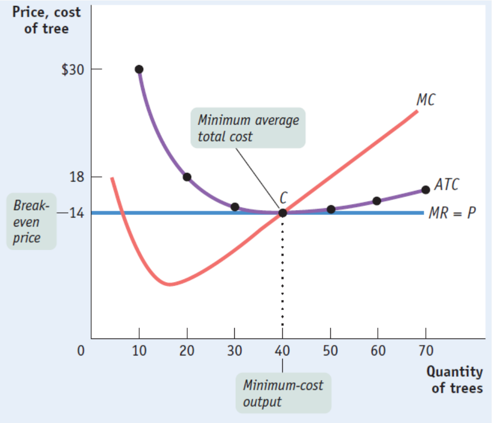


When Is Production Profitable?

- economic profit: the measure of profit based on the opportunity cost of resources used in the business. The firm's total cost incorporates the implicit cost-the benefits forgone in the next best use of the firm's resources (as well as the explicit cost in the form of actual cash outlays).
- In contrast, accounting profit is profit calculated using only the explicit costs incurred by the firm.
- firm's decision to produce or not, to stay in business or to close down permanently, should be based on economic profit, not accounting profit.

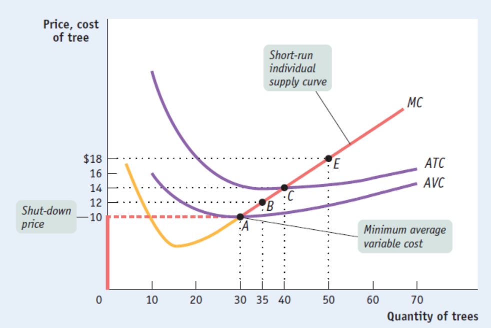
Minimum efficient scale

Minimum efficient scale: the level of production required for LRAC to reach its minimum level.



The Firm's Supply Curve

- Firm produces when price are greater than minimum AVC c_{min}
- For prices greater than c_{min}, the firm produces up to the point where price equals marginal cost (firm's supply curve coincides with the marginal cost curve)

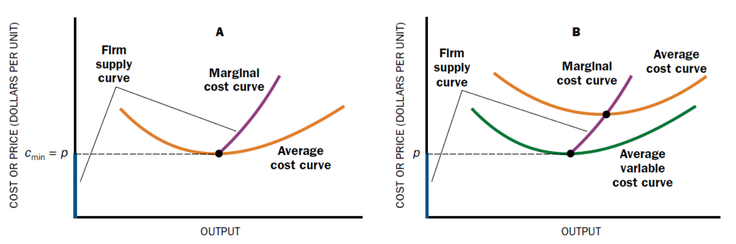


But what if the market price lies between the shut-down price (A) and the break-even price (C) : that is, between minimum average variable cost and minimum average total cost?

- If a firm in this situation shuts down, it would incur no variable cost but would incur the full fixed cost.
- As a result, shutting down generates an even greater loss than continuing to operate.
- it can cover its variable cost per unit and at least some of its fixed cost

The Firm's Supply Curve

Without or with sunk cost



Summary of the Perfectly Competitive Firm's Profitability and Production Conditions

TABLE 12-4 Summary of the Perfectly Competitive Firm's Profitability and Production Conditions

Profitability condition (minimum *ATC* = break-even price)	Result
$P >$ minimum *ATC*	Firm profitable. Entry into industry in the long run.
$P =$ minimum *ATC*	Firm breaks even. No entry into or exit from industry in the long run.
$P <$ minimum *ATC*	Firm unprofitable. Exit from industry in the long run.
Production condition (minimum *AVC* = shut-down price)	**Result**
$P >$ minimum *AVC*	Firm produces in the short run. If $P <$ minimum *ATC*, firm covers variable cost and some but not all of fixed cost. If $P >$ minimum *ATC*, firm covers all variable cost and fixed cost.
$P =$ minimum *AVC*	Firm indifferent between producing in the short run or not. Just covers variable cost.
$P <$ minimum *AVC*	Firm shuts down in the short run. Does not cover variable cost.

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