Elements on Basic Case Studies - Consumption

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Income and Substitution effect

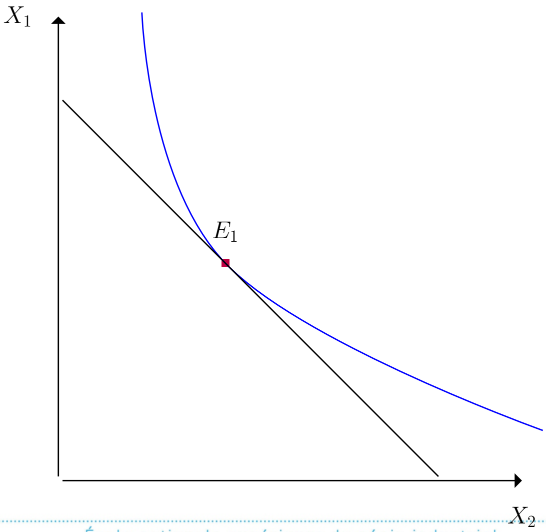
In case of Normal good :

If the X_1 price increases then the demand for X_1 will decrease due to :

- and Substitution effect arises upon the relative price variation
- Income effect appends upon the budget constraint

These two effects are negative

Initial situation



Substitution effect

- Assuming the compensatory variation of income :
  an income variation compensating the change in price in order to keep utility constant
- A new bundle is chosen at the new price ratio $\frac{p_2}{p_1}$ (E_2),
- located on the same indifference curve (same utility level)

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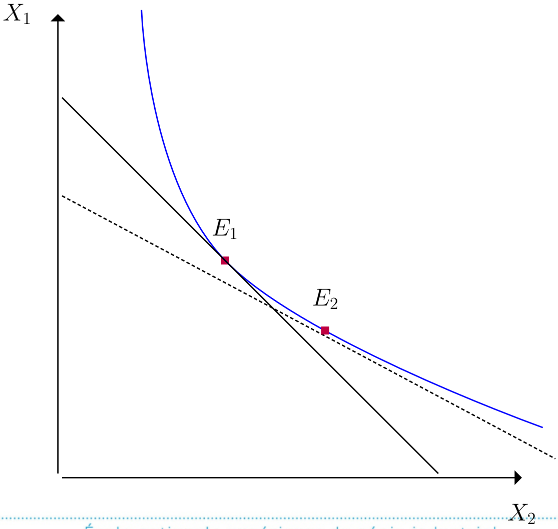
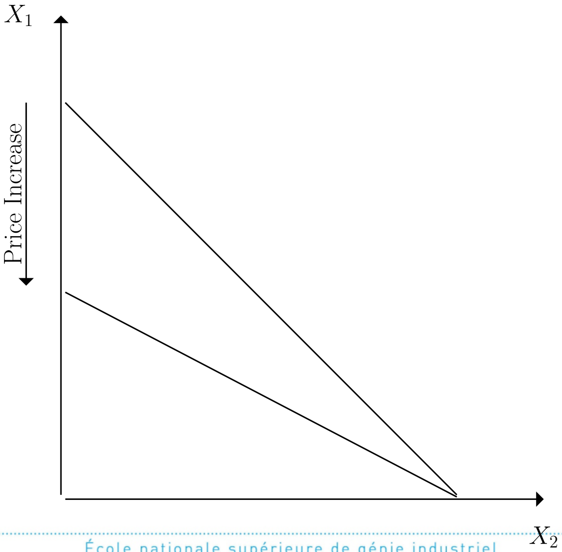
Initial situation



Income effect

- In the same time, the increase of price reduces available income for consumption
- The budget constraint is modified :

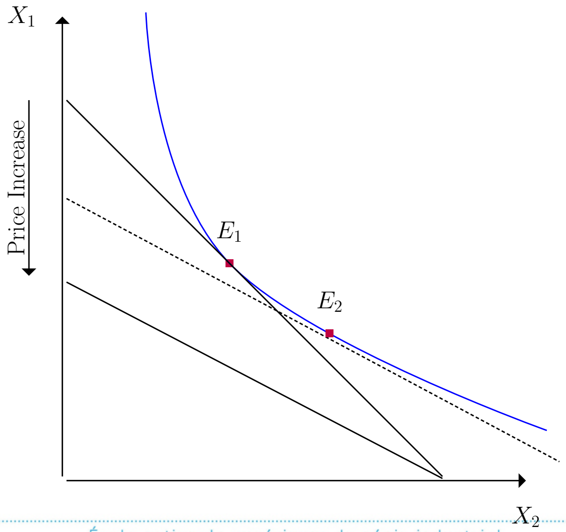
Income effect



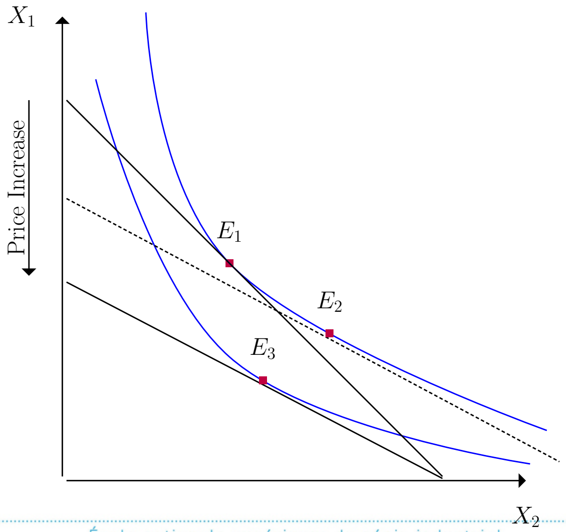
Income effect

- In the same time, the increase of price reduces available income for consumption
- The budget constraint is modified
- A new choice comes (E_3)

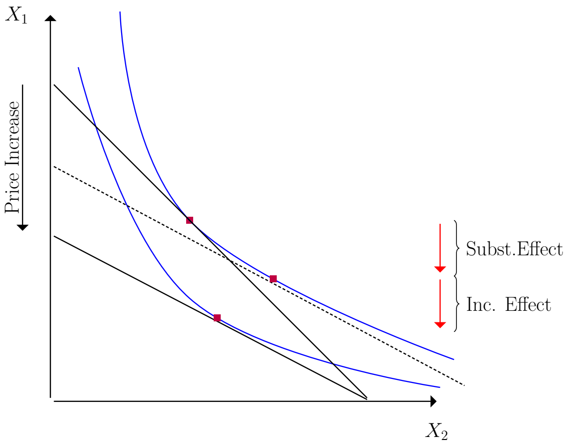
Income and Substitution effect



Income and Substitution effect



Income and Substitution effect



Income and Substitution effect

In the normal good case

- the two effects on X_1 demand are in the same direction

Inferior Good

In the Inferior good case

- Substitution effect and Income effect are in opposite direction
- Substitution effect (-) dominates income effect (+)
- The demand decreases with price increase

Giffen Good

Some conditions must be met

1. The Giffen good is an inferior good associated to a large income effect
2. Substitution effect is small
3. the share of income devoted to the Giffen good is large

In the Giffen good case,

- Substitution effect and Income effect are in opposite direction
- Income effect (+) dominates substitution effect (-)
- The demand decreases with price increase

Conclusion (1)

Type of Good	Substitution Effect	Income Effect	Total Effect
Normal	Increase	Increase	Increase
Inferior (but not Giffen)	Increase	Decrease	Increase
Giffen	Increase	Decrease	Decrease

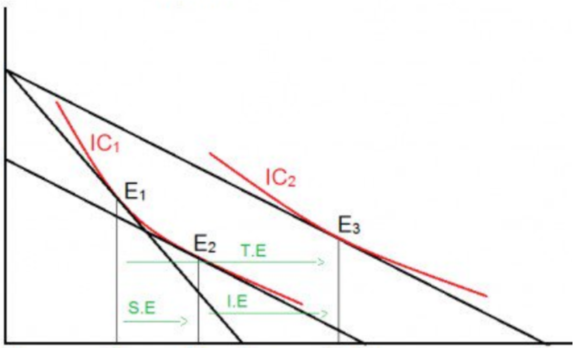
Conclusion (2)

Finally three demand theorems to which the Marshalian law of demand is special case :

1. demand varies inversely with price decrease if income effect is positive or null
2. demand varies inversely with price decrease if income effect is negative, but lower than substitution effect (on abs. value)
3. demand varies with price decrease if income effect is negative, but greater than substitution effect (on abs. value)
4. for 1. and 2. Marshalian law holds : normal and inferior goods.

Exercises

We can discuss some graphics from the internet



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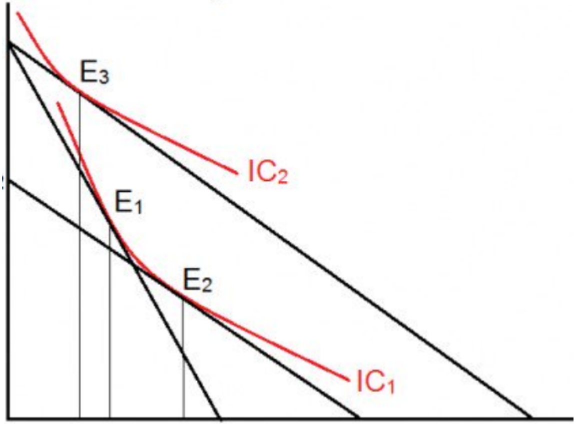

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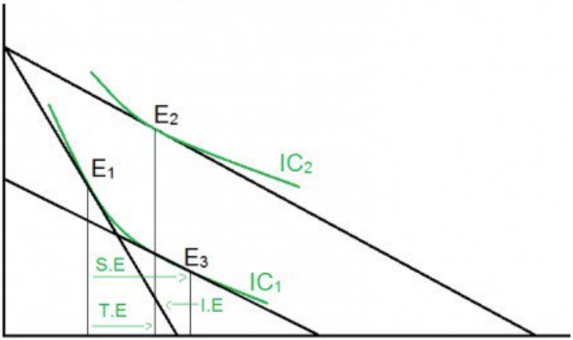

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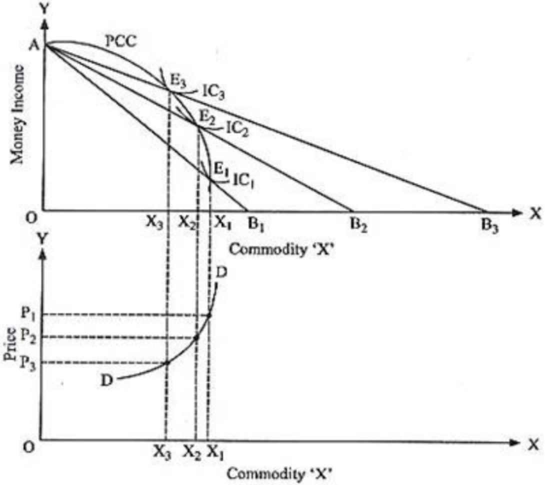


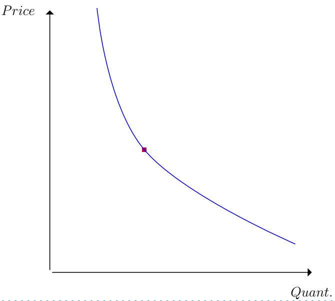
Two Definitions (1)

- General meaning of Elasticity :
  Elasticity measures the sensitivity of a variable with respect to another variable.
- we can found two definitions of elasticity
- one general economic elasticity precising a general trend between to variables
- one precise economic elasticity measure precising the sensitivity of a function

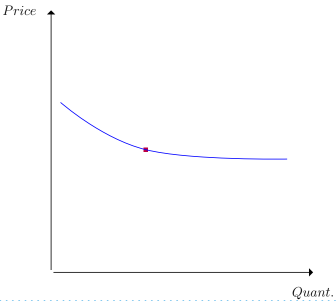
General Elasticity

Elasticity can be used to describe and compare two relations.

For example, the elasticity of demand (or supply) on a market can be elastic or inelastic :



Elastic Demand



Inelastic Demand

General Elasticity (1)

- We can distinguish elastic vs inelastic cases
- In the relation of Q with respect to *price* :

ε	Description	1% of price $\Rightarrow$ % Q
0	Perfect inelasticity : Vertical line	0% $\Rightarrow$ constat Q
$]0;1[$	Inelastic	$< 1\%$
1	Unitary elasticity	$= 1\%$
> 1	Elastic	$> 1\%$
∞	Perfect elasticity : horizontal line	$\infty\%$: $Q \longrightarrow 0$

General Elasticity (2)

Depending on elasticities of demand and supply :

- Market equilibrium varies slowly or not in case of taxes, floor/cap prices, etc.
- Pricing strategy of the firm differs depending on demand sensitivity

Measure of Elasticity Definition (1)

Economic Elasticity

Elasticity measures the sensitivity of a variable with respect to another variable.

- This sensitivity is measured in relative terms (in %) (by opposition to absolute term (in unit)
- Classic interpretation : For a given elasticity measure $\varepsilon_{Y|X}$: if X varies by 1%, Y will varies by $\varepsilon_{Y|X}$ %.

Measure of Elasticity Definition (2)

General Definition

$$\varepsilon_{f(a)}(a) = \frac{a}{f(a)} f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \frac{a}{f(a)}$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{f(a)} \frac{a}{x - a} = \lim_{x \to a} \frac{1 - \frac{f(x)}{f(a)}}{1 - \frac{x}{a}}$$

$$= \frac{\%\Delta f(a)}{\%\Delta a} = \frac{d \log f(x)}{d \log x}$$

- ratio of relative change (%) in the function output $f(x)$

Measure of Elasticity Definition (3)

- to relative change in its input x
- for infinitesimal change from point $(a, f(a))$

Measure of Elasticity Definition (4)

Taking the example of the study of the sensitivity of demand function ($x(I, p_i, p_j)$) on market with respect to prices changes, we can define Direct price elasticity $\varepsilon_{x_i|p_i}$ and Cross price elasticity $\varepsilon_{x_i|p_j}$.

Measure of Elasticity Definition (5)

Direct price elasticity

- Percent change in demand x_i resulting form a 1 % change in price p_i

$$\varepsilon_{x_i|p_i} = \frac{\text{% of change in} x_i}{\text{% of change in} p_i} = \frac{\Delta x_i / x_i}{\Delta p_i / p_i} = \frac{\Delta x_i}{\Delta p_i} \Big/ \frac{x_i}{p_i}$$

- Asymptotically

$$\varepsilon_{x_i p_i} = \frac{p_i}{x_i(I, p_i, p_j)} \frac{\partial x_i(I, p_i, p_j)}{\partial p_i}$$

Measure of Elasticity Definition (6)

Cross price elasticity

- Percent change in demand x_i resulting form a 1 % change in price p_j

$$\varepsilon_{x_i|p_j} = \frac{p_j}{x_i(I, p_i, p_j)} \frac{\partial x_i(I, p_i, p_j)}{\partial p_j}$$

Some remarks :

- Elasticity level depends on the point at which it is evaluated : see the ratio $\frac{p_j}{x_i}$
- We can define an Income elasticity of demand

Exercises (1)

Exercises

Characterize elasticity $\varepsilon_{y|x}$ of the two following forms :

- Linear form : $y = f(x) = -10x + 1000$
- Cobb-Douglas form : $y = f(x) = Cx^{\alpha}$, with $C > 0$ a constant.

Exercises (2)

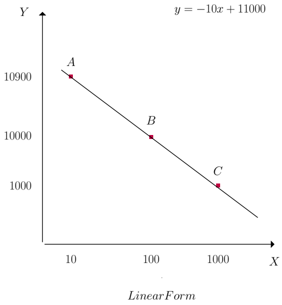


FIGURE: Elasticity of linear form

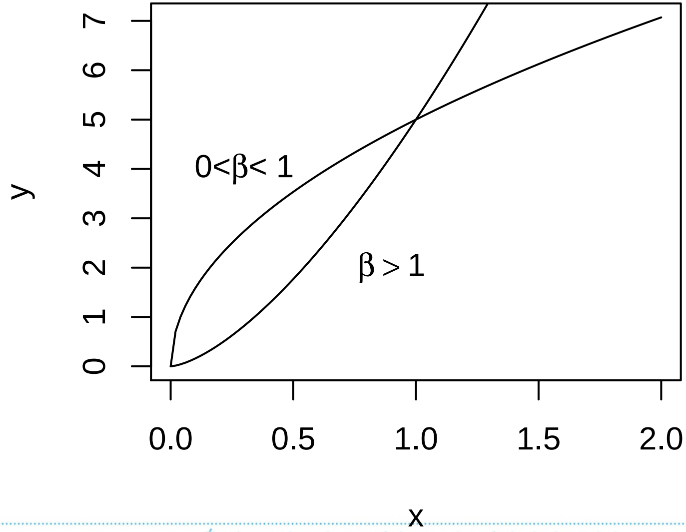
Inmpact of $+1\%$ of x :

- In point A : $\Delta x = 0.1$;
  $\Delta y = -1$, $y = 10899$
  % change $\%y = 1/11000$
  and $\varepsilon_{y|x} = \frac{1}{11000}$
- In point B : $\Delta x = 1$;
  $\Delta y = -10$, $y = 9990$
  % change $\%y = 1/1000$ and
  $\varepsilon_{y|x} = \frac{1}{1000}$
- In point C : $\Delta x = 10$;
  $\Delta y = -100$, $y = 900$
  % change of $\%y = 1/10$ and
  $\varepsilon_{y|x} = 10\%$

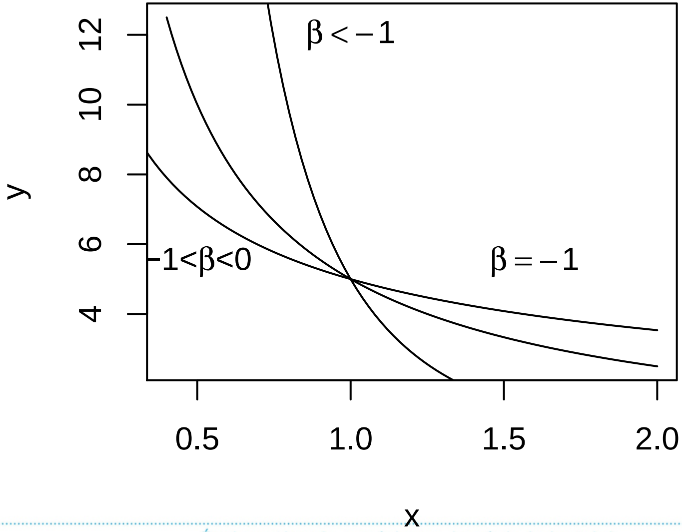
Exercises (3)

The Cobb-Douglas form : $y = Cx^{\alpha}$

- Elasticity can be calculted using $\varepsilon_{f(x)|x} = \frac{\%\Delta f(a)}{\%\Delta a} = \frac{d \log f(x)}{d \log x}$
- $\log f(x) = K + \alpha \log(x)$
- Hence, $\varepsilon_{f(x)|x} = \alpha$
- The Cobb-Douglas function is a constant elasticity function,
- which representation is a curve



0<β< 1
β > 1
y
x
0
1
2
3
4
5
6
7
0.0
0.5
1.0
1.5
2.0


β < −1
−1<β<0
β = −1
12
10
8
6
4
y
0.5
1.0
1.5
2.0
x

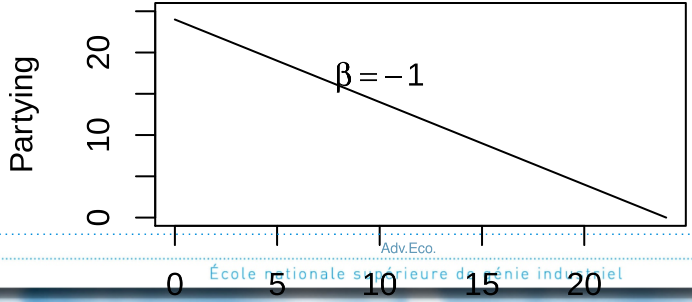
Two Perspectives (1)

In this case two economic notions can be discussed :

- Daily allocation of time problem
- Intertemporal choice between short and long terms

Allocation of time

First model 24h a day have to be spend in activities : *studying* vs *partying*

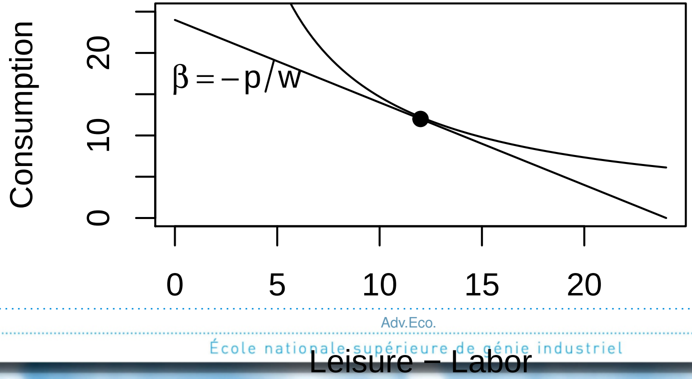


Allocation of time (1)

Second model

- In short term : Labor-Leisure trade-off determine the consumption level through income
- Leisure is 24- Labor
- Hourly wage is w
- General consumption price is p

Allocation of time (2)



Allocation of time

Third model : Intertemporal model Consumption and savings

- Two periods : t_1 and t_2
- Consumption in t_1 : x_1 and t_2 : x_2
- Income in t_1 : y_1 and t_2 : y_2
- s is saving from t_1 to t_2, r : interest rate
- $$\max_{x_1, x_2} U(x_1, x_2) \text{s.t.}$$
  $$x_1 + s \leq y_1 \text{ and } x_2 \leq y_2 + s(1+r)$$
- intertemporal Budget Constraint :
  $$x_1 + \frac{x_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

Allocation of time

Fourth model : Modigliani model

- $\max_{c_t} U(c_t)(1+\gamma)^{-1}$ s.t.

$$\sum_t c_t(1+r)^{-1} \leq \sum_t y_t(1+r)^{-1} + w_0$$

- where γ is the rate of time preference

Allocation of time

Fifth model : Becker model

- Household produces a composite good Z_i
- f the production function of the composite good : $Z_i = f(X_i, T_i)$
- X_i and T_i are consumption goods and time dedicated to the composite good Z_i
- W : the work time and w is the wage rate
- τ is available time
- $\max_{X,T} U(Z_1, \ldots, Z_n)$ s.t.

$$\sum_i p_i X_i \leq I + wW$$

and

$$\sum_i T_i = \tau + W$$

Lagrangian multipliers are shadow costs