PART

2

THE THEORY OF CONSUMER BEHAVIOR

The next six chapters develop the theory of consumer behavior. Chapter 3 is of special importance, for it lays out the economic theory of how people with limited resources choose between competing alternatives. The methods and tools developed in this chapter recur throughout the remainder of the book, and indeed throughout all of economics. Chapter 4 shows how the theory of rational individual choice can be used to derive individual and market demand curves. Chapter 5 explores numerous applications of rational choice and demand theories, including the theory of choices that involve future consequences.

Chapter 6 shows how the rational choice model can be extended to cover choices that involve uncertainty or incomplete information. Chapter 7 examines the role of unselfish motives in economic and social behavior and shows why honest people often have an economic advantage over people who cheat. Finally, Chapter 8 looks at a variety of circumstances in which ordinary people tend to make irrational choices. Experience shows that being aware of this tendency helps people make better decisions.

CHAPTER

3

RATIONAL CONSUMER CHOICE

You have just cashed your monthly allowance check and are on your way to the local music store to buy an Eric Clapton CD you've been wanting. The price of the disc is $10. In scenario 1 you lose $10 on your way to the store. In scenario 2 you buy the disc and then trip and fall on your way out of the store; the disc shatters as it hits the sidewalk. Try to imagine your frame of mind in each scenario.

a. Would you proceed to buy the disc in scenario 1?

b. Would you return to buy the disc in scenario 2?

These questions[1] were recently put to a large class of undergraduates who had never taken an economics course. In response to the first question, 54 percent answered yes, saying they would buy the disc after losing the $10 bill. But only 32 percent answered yes to the second question—68 percent said they would *not* buy the disc after having broken the first one. There is, of course, no "correct" answer to either question. The events described will have more of an impact, for example, on a poor consumer than on a rich one. Yet a moment's reflection reveals that your behavior in one scenario logically should be exactly the same as in the other. After all, in both scenarios, the only economically relevant change is that you now have $10 less to spend than before. This might well mean that you will want to give up having the disc; or it could mean saving less or giving up some other good that you would have bought. But your choice should not be

[1]These questions are patterned after similar questions posed by decision theorists Daniel Kahneman and Amos Tversky (see Chapter 8).

affected by the particular way you happened to become \$10 poorer. In both scenarios, the cost of the disc is \$10, and the benefit you will receive from listening to it is also the same. You should either buy the disc in both scenarios or not buy it in both. And yet, as noted, many people would choose differently in the two scenarios.

CHAPTER PREVIEW

Our task in this chapter is to set forth the economist's basic model for answering questions such as the ones posed above. This model is known as the theory of *rational consumer choice*. It underlies all individual purchase decisions, which in turn add up to the demand curves we worked with in the preceding chapter.

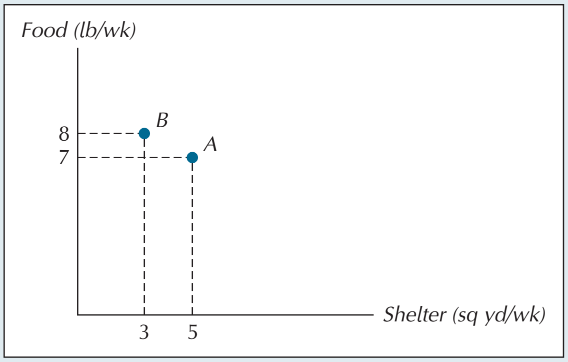
Rational choice theory begins with the assumption that consumers enter the marketplace with well-defined preferences. Taking prices as given, their task is to allocate their incomes to best serve these preferences. Two steps are required to carry out this task. Step 1 is to describe the various combinations of goods the consumer is *able* to buy. These combinations depend on both her income level and the prices of the goods. Step 2 then is to select from among the feasible combinations the particular one that she *prefers* to all others. Analysis of step 2 requires some means of describing her preferences, in particular, a summary of her ranking of the desirability of all feasible combinations. Formal development of these two elements of the theory will occupy our attention throughout this chapter. Because the first element—describing the set of possibilities—is much less abstract than the second, let us begin with it.

THE OPPORTUNITY SET OR BUDGET CONSTRAINT

bundle a particular combination of two or more goods.

For simplicity, we start by considering a world with only two goods,[2] shelter and food. A **bundle** of goods is the term used to describe a particular combination of shelter, measured in square yards per week, and food, measured in pounds per week. Thus, in Figure 3.1, one bundle (bundle *A*) might consist of 5 sq yd/wk of shelter and 7 lb/wk of food, while another (bundle *B*) consists of 3 sq yd/wk of shelter and 8 lb/wk of food. For brevity, we use (5, 7) to denote bundle *A* and (3, 8) to denote bundle *B*. More generally, (S_0, F_0) will denote the bundle with S_0 sq yd/wk of shelter and F_0 lb/wk of food. By convention, the first number of the pair in any bundle represents the good measured along the horizontal axis.

FIGURE 3.1
Two Bundles of Goods
A bundle is a specific combination of goods. Bundle *A* has 5 units of shelter and 7 units of food. Bundle *B* has 3 units of shelter and 8 units of food.



[2]As economists use the term, a "good" may refer to either a product or a service.

Note that the units on both axes are *flows,* which means physical quantities per unit of time—pounds per week, square yards per week. Consumption is always measured as a flow. It is important to keep track of the time dimension because without it there would be no way to evaluate whether a given quantity of consumption was large or small. (Suppose all you know is that your food consumption is 4 lb. If that's how much you eat each day, it's a lot. But if that's all you eat in a month, you're not likely to survive for long.)[3]

Suppose the consumer's income is M = \$100/wk, all of which she spends on some combination of food and shelter. (Note that income is also a flow.) Suppose further that the prices of shelter and food are P_S = \$5/sq yd and P_F = \$10/lb, respectively. If the consumer spent all her income on shelter, she could buy M/P_S = (\$100/wk) ÷ (\$5/sq yd) = 20 sq yd/wk. That is, she could buy the bundle consisting of 20 sq yd/wk of shelter and 0 lb/wk of food, denoted (20, 0). Alternatively, suppose the consumer spent all her income on food. She would then get the bundle consisting of M/P_F = (\$100/wk) ÷ (\$10/lb), which is 10 lb/wk of food and 0 sq yd/wk of shelter, denoted (0, 10).

Note that the units in which consumption goods are measured are subject to the standard rules of arithmetic. For example, when we simplify the expression on the right-hand side of the equation M/P_S = (\$100/wk) ÷ (\$5/sq yd), we are essentially dividing one fraction by another, so we follow the standard rule of inverting the fraction in the denominator and multiplying it by the fraction in the numerator: (sq yd/\$5) × (\$100/wk) = (\$100 × sq yd)/(\$5 × wk). After dividing both the numerator and denominator of the fraction on the right-hand side of this last equation by \$5, we have 20 sq yd/wk, which is the maximum amount of shelter the consumer can buy with an income of \$100/wk. Similarly, M/P_F = (\$100/wk) ÷ (\$10/lb) simplifies to 10 lb/wk, the maximum amount of food the consumer can purchase with an income of \$100/wk.

In Figure 3.2 these polar cases are labeled K and L, respectively. The consumer is also able to purchase any other bundle that lies along the straight line that joins points K and L. [Verify, for example, that the bundle (12, 4) lies on this same line.] This line is called the **budget constraint** and is labeled B in the diagram.

budget constraint the set of all bundles that exactly exhaust the consumer's income at given prices. Also called the *budget line.*

Recall the maxim from high school algebra that the slope of a straight line is its "rise" over its "run" (the change in its vertical position divided by the corresponding change in its horizontal position). Here, note that the slope of the budget constraint

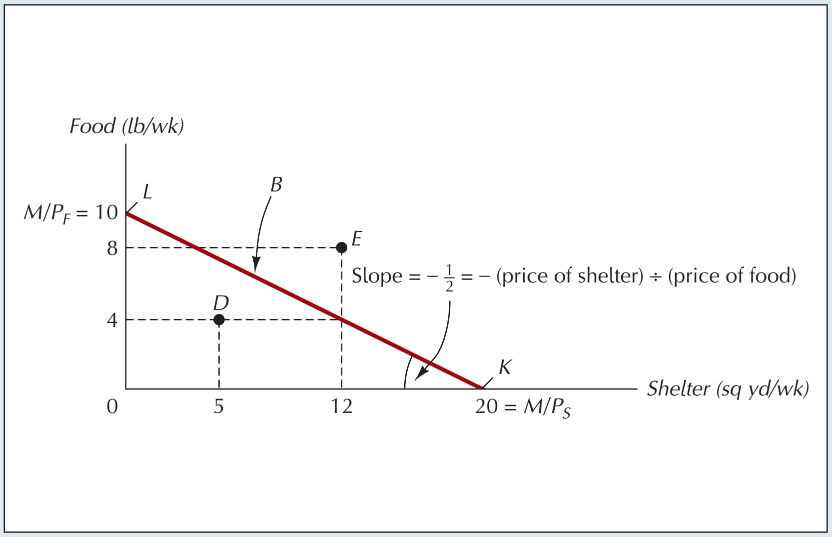


FIGURE 3.2
The Budget Constraint, or Budget Line
Line B describes the set of all bundles the consumer can purchase for given values of income and prices. Its slope is the negative of the price of shelter divided by the price of food. In absolute value, this slope is the opportunity cost of an additional unit of shelter—the number of units of food that must be sacrificed in order to purchase one additional unit of shelter at market prices.

[3]The flow aspect of consumption also helps us alleviate any concern about goods not being divisible. If you consume 1.5 lb/mo, then you consume 18 lb/yr, which is a whole number.

is its vertical intercept (the rise) divided by its horizontal intercept (the corresponding run): $-(10 \text{ lb/wk})/(20 \text{ sq yd/wk}) = -\frac{1}{2}$ lb/sq yd. (Note again how the units obey the standard rules of arithmetic.) The minus sign signifies that the budget line falls as it moves to the right—that it has a negative slope. More generally, if M denotes the consumer's weekly income, and P_S and P_F denote the prices of shelter and food, respectively, the horizontal and vertical intercepts will be given by (M/P_S) and (M/P_F), respectively. Thus the general formula for the slope of the budget constraint is given by $-(M/P_F)/(M/P_S) = -P_S/P_F$, which is simply the negative of the price ratio of the two goods. Given their respective prices, it is the rate at which food can be exchanged for shelter. Thus, in Figure 3.2, 1 lb of food can be exchanged for 2 sq yd of shelter. In the language of opportunity cost from Chapter 1, we would say that the opportunity cost of an additional square yard of shelter is $P_S/P_F = \frac{1}{2}$ lb of food.

In addition to being able to buy any of the bundles along her budget constraint, the consumer is also able to purchase any bundle that lies within the *budget triangle* bounded by it and the two axes. D is one such bundle in Figure 3.2. Bundle D costs \$65/wk, which is well below the consumer's income of \$100/wk. The bundles on or within the budget triangle are also referred to as the *feasible set,* or **affordable set.** Bundles like E that lie outside the budget triangle are said to be *infeasible,* or *unaffordable.* At a cost of \$140/wk, E is simply beyond the consumer's reach.

affordable set bundles on or below the budget constraint; bundles for which the required expenditure at given prices is less than or equal to the income available.

If S and F denote the quantities of shelter and food, respectively, the budget constraint must satisfy the following equation:

$$P_S S + P_F F = M, \tag{3.1}$$

which says simply that the consumer's weekly expenditure on shelter ($P_S S$) plus her weekly expenditure on food ($P_F F$) must add up to her weekly income (M). To express the budget constraint in the manner conventionally used to represent the formula for a straight line, we solve Equation 3.1 for F in terms of S, which yields

$$F = \frac{M}{P_F} - \frac{P_S}{P_F} S \tag{3.2}$$

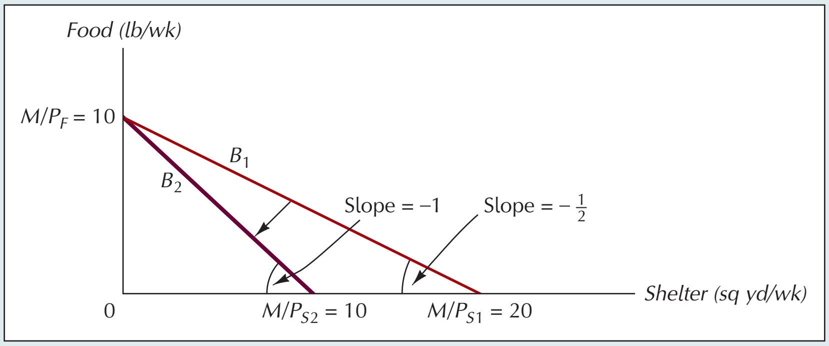
Equation 3.2 is another way of seeing that the vertical intercept of the budget constraint is given by M/P_F and its slope by $-(P_S/P_F)$. The equation for the budget constraint in Figure 3.2 is $F = 10 - \frac{1}{2} S$.

BUDGET SHIFTS DUE TO PRICE OR INCOME CHANGES

Price Changes

The slope and position of the budget constraint are fully determined by the consumer's income and the prices of the respective goods. Change any one of these factors and we have a new budget constraint. Figure 3.3 shows the effect of an increase

FIGURE 3.3
The Effect of a Rise in the Price of Shelter
When shelter goes up in price, the vertical intercept of the budget constraint remains the same. The original budget constraint rotates inward about this intercept.



in the price of shelter from P_{S1} = \$5/sq yd to P_{S2} = \$10. Since both weekly income and the price of food are unchanged, the vertical intercept of the consumer's budget constraint stays the same. The rise in the price of shelter rotates the budget constraint inward about this intercept, as shown in the diagram.

Note in Figure 3.3 that even though the price of food has not changed, the new budget constraint, B_2, curtails not only the amount of shelter the consumer can buy but also the amount of food.[4]

EXERCISE 3.1

Show the effect on the budget constraint B_1 in Figure 3.3 of a fall in the price of shelter from \$5/sq yd to \$4/sq yd.

In Exercise 3.1, you saw that a fall in the price of shelter again leaves the vertical intercept of the budget constraint unchanged. This time the budget constraint rotates outward. Note also in Exercise 3.1 that although the price of food remains unchanged, the new budget constraint enables the consumer to buy bundles that contain not only more shelter but also more food than she could afford on the original budget constraint.

The following exercise illustrates how changing the price of the good on the vertical axis affects the budget constraint.

EXERCISE 3.2

Show the effect on the budget constraint B_1 in Figure 3.3 of a rise in the price of food from \$10/lb to \$20/lb.

When we change the price of only one good, we necessarily change the slope of the budget constraint, $-P_S/P_F$. The same is true if we change both prices by different proportions. But as Exercise 3.3 will illustrate, changing both prices by exactly the same proportion gives rise to a new budget constraint with the same slope as before.

EXERCISE 3.3

Show the effect on the budget constraint B_3 in Figure 3.3 of a rise in the price of food from \$10/lb to \$20/lb and a rise in the price of shelter from \$5/sq yd to \$10/sq yd.

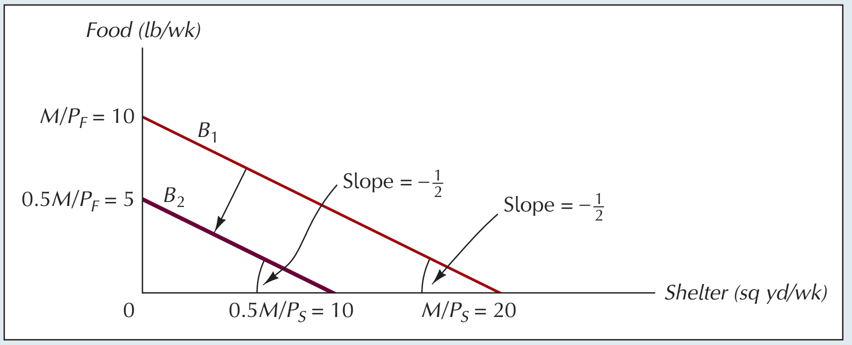
Note from Exercise 3.3 that the effect of doubling the prices of both food and shelter is to shift the budget constraint inward and parallel to the original budget constraint. The important lesson of this exercise is that the slope of a budget constraint tells us only about *relative prices,* nothing about prices in absolute terms. When the prices of food and shelter change in the same proportion, the opportunity cost of shelter in terms of food remains the same as before.

Income Changes

The effect of a change in income is much like the effect of an equal proportional change in all prices. Suppose, for example, that our hypothetical consumer's income is cut by half, from \$100/wk to \$50/wk. The horizontal intercept of the consumer's budget constraint then falls from 20 sq yd/wk to 10 sq yd/wk, and the vertical intercept falls from 10 lb/wk to 5 lb/wk, as shown in Figure 3.4. Thus the new budget, B_2, is parallel to the old, B_1, each with a slope of $-\frac{1}{2}$. In terms of its effect on what the consumer can buy, cutting income by one-half is thus no different from doubling each price. Precisely the same budget constraint results from both changes.

[4]The single exception to this statement involves the vertical intercept (0, 10), which lies on both the original and the new budget constraints.

FIGURE 3.4
The Effect of Cutting Income by Half
Both horizontal and vertical intercepts fall by half. The new budget constraint has the same slope as the old but is closer to the origin.



EXERCISE 3.4

Show the effect on the budget constraint B_1 in Figure 3.4 of an increase in income from \$100/wk to \$120/wk.

Exercise 3.4 illustrates that an increase in income shifts the budget constraint parallel outward. As in the case of an income reduction, the slope of the budget constraint remains the same.

BUDGETS INVOLVING MORE THAN TWO GOODS

In the examples discussed so far, the consumer could buy only two different goods. No consumer faces such narrow options. In its most general form, the consumer budgeting problem can be posed as a choice between not two but N different goods, where N can be an indefinitely large number. With only two goods ($N = 2$), the budget constraint is a straight line, as we just saw. With three goods ($N = 3$), it is a plane. When we have more than three goods, the budget constraint becomes what mathematicians call a *hyperplane,* or *multidimensional plane.* It is difficult to represent this multidimensional case geometrically. We are just not very good at visualizing surfaces that have more than three dimensions.

The nineteenth-century economist Alfred Marshall proposed a disarmingly simple solution to this problem. It is to view the consumer's choice as being one between a particular good—call it X—and an amalgam of other goods, denoted Y. This amalgam is generally called the **composite good.** By convention, the units of the composite good are defined so that its price is \$1 per unit. This convention enables us to think of the composite good as the amount of income the consumer has left over after buying the good X. Equivalently, it is the amount the consumer spends on goods other than X. For the moment, all the examples we consider will be ones in which consumers spend all their incomes. In Chapter 5 we will use the rational choice model to analyze the decision to save.

composite good in a choice between a good X and numerous other goods, the amount of money the consumer spends on those other goods.

To illustrate how the composite good concept is used, suppose the consumer has an income of \$$M$/wk, and the price of X is P_X. The consumer's budget constraint may then be represented as a straight line in the X, Y plane, as shown in Figure 3.5. Because the price of a unit of the composite good is \$1, a consumer who devotes all his income to it will be able to buy M units. All this means is that he will have \$$M$ available to spend on other goods if he buys no X. Alternatively, if he spends his entire income on X, he will be able to purchase the bundle $(M/P_X, 0)$. Since the price of Y is assumed to be \$1/unit, the slope of the budget constraint is simply $-P_X$.

As before, the budget constraint summarizes the various combinations of bundles that exhaust the consumer's income. Thus, the consumer can have X_1 units of X and Y_1 units of the composite good in Figure 3.5, or X_2 and Y_2, or any other combination that lies on the budget constraint.

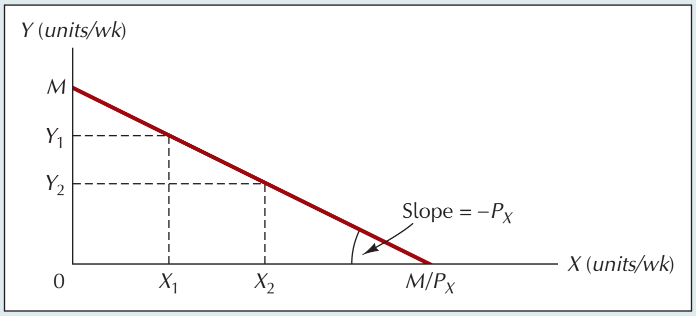


FIGURE 3.5
The Budget Constraint with the Composite Good
The vertical axis measures the amount of money spent each week on all goods other than *X*.

KINKED BUDGET CONSTRAINTS

The budget constraints we have seen so far have been straight lines. When relative prices are constant, the opportunity cost of one good in terms of any other is the same, no matter what bundle of goods we already have. But sometimes budget constraints are kinked lines. To illustrate, consider the following example of quantity discounts.

EXAMPLE 3.1

The Gigawatt Power Company charges $0.10 per kilowatt-hour (kWh) for the first 1000 kWh of power purchased by a residential customer each month, but only $0.05/kWh for all additional kWh. For a residential customer with a monthly income of $400, graph the budget constraint for electric power and the composite good.

If the consumer buys no electric power, he will have $400/mo to spend on other goods. Thus the vertical intercept of his budget constraint is (0, 400). As shown in Figure 3.6, for each of the first 1000 kWh he buys, he must give up $0.10, which means that the slope of his budget constraint starts out at $-\frac{1}{10}$. At 1000 kWh/mo, the price falls to $0.05/kWh, which means that the slope of his budget constraint from that point rightward is only $-\frac{1}{20}$.

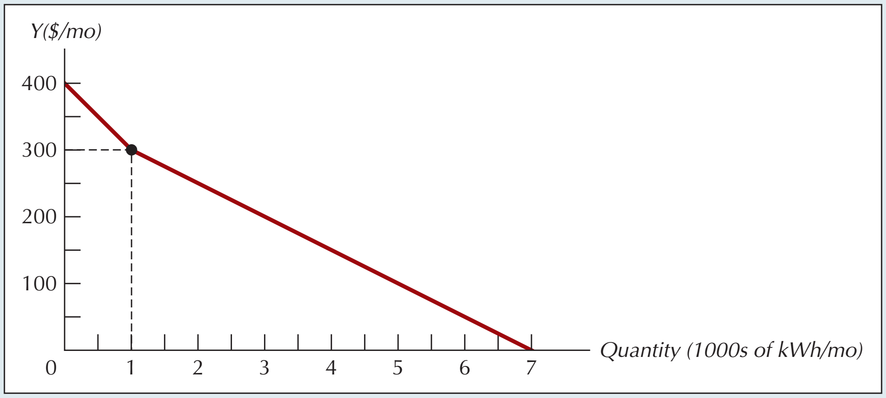


FIGURE 3.6
A Quantity Discount Gives Rise to a Nonlinear Budget Constraint
Once electric power consumption reaches 1000 kWh/mo, the opportunity cost of additional power falls from $0.10/kWh to $0.05/kWh.

Note that along the budget constraint shown in Figure 3.6, the opportunity cost of electricity depends on how much the consumer has already purchased. Consider a consumer who now uses 1020 kWh each month and is trying to decide whether to leave his front porch light on all night, which would result in additional consumption of 20 kWh/mo. Leaving his light on will cost him an extra $1/mo. Had his usual consumption been only 980 kWh/mo, however, the cost of leaving the front porch light on would have been $2/mo. On the basis of this difference, we can predict that people who already use a lot of electricity (more than 1000 kWh/mo) should be more likely than others to leave their porch lights burning at night.

EXERCISE 3.5

Suppose instead Gigawatt Power Company charged $0.05/kWh for the first 1000 kWh of power purchased by a residential consumer each month, but $0.10/kWh each for all additional kilowatt-hours. For a residential consumer with a monthly income of $400, graph the budget constraint for electric power and the composite good. What if the rate jumps to $0.10/kWh for *all* kilowatt-hours if power consumption in a month exceeds 1000 kWh (where the higher rate applies to all, not just the additional, kilowatt-hours)?

IF THE BUDGET CONSTRAINT IS THE SAME, THE DECISION SHOULD BE THE SAME

Even without knowing anything about the consumer's preferences, we can use budgetary information to make certain inferences about how a rational consumer will behave. Suppose, for example, that the consumer's tastes do not change over time and that he is confronted with exactly the same budget constraint in each of two different situations. If he is rational, he should make exactly the same choice in both cases. After all, if the budget constraint is the same as before, the consumer has exactly the same menu of possible bundles available as before; and since we have no reason to believe that his ranking of the desirability of these bundles has changed, the most desirable bundle should also be the same. As the following example makes clear, however, it may not always be immediately apparent that the budget constraints are in fact the same.

EXAMPLE 3.2

On one occasion, Gowdy fills his car's tank with gasoline on the evening before his departure on a fishing trip. He awakens to discover that a thief has siphoned out all but 1 gallon from his 21-gallon tank. On another occasion, he plans to stop for gas on his way out the next morning before he goes fishing. He awakens to discover that he has lost $60 from his wallet. If gasoline sells for $3/gal and the round-trip will consume 5 gallons, how, if at all, should Gowdy's decision about whether to take the fishing trip differ in the two cases? (Assume that, monetary costs aside, the inconvenience of having to refill his tank is negligible.)

Suppose Gowdy's income is $\$M$/mo. Before his loss, his budget constraint is B_1 in Figure 3.7. In both instances described, his budget constraint at the moment he

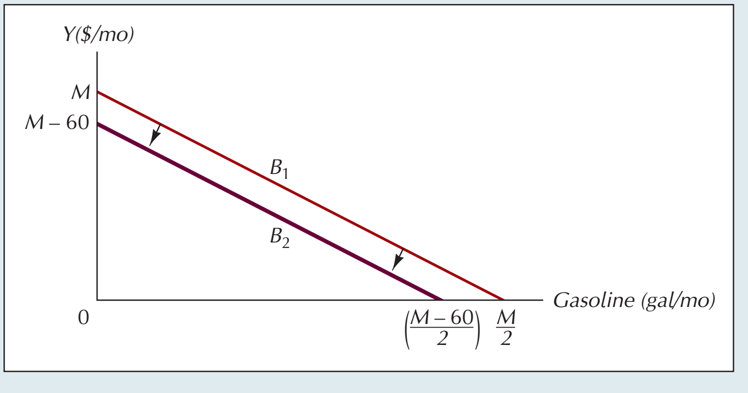


FIGURE 3.7
Budget Constraints Following Theft of Gasoline, Loss of Cash
A theft of $60 worth of gasoline has exactly the same effect on the budget constraint as the loss of $60 in cash. The bundle chosen should therefore be the same, irrespective of the source of the loss.

discovers his loss will shift inward to B_2. If he does not take the trip, he will have $M - \$60$ available to spend on other goods in both cases. And if he does take the trip, he will have to purchase the required gasoline at \$3/gal in both cases. No matter what the source of the loss, the remaining opportunities are exactly the same. If Gowdy's budget is tight, he may decide to cancel his trip. Otherwise, he may go despite the loss. But because his budget constraint and tastes are the same in the lost-cash case as in the stolen-gas case, it would not be rational for him to take the trip in one instance but not in the other.

Note that the situation described in Example 3.2 has the same structure as the one described in the broken-disc example with which we began this chapter. It too is one in which the decision should be the same in both instances because the budget constraint and preferences are the same in each.

Although the rational choice model makes clear that the decisions *should* be the same if the budget constraints and preferences are the same, people sometimes choose differently. The difficulty is often that the way the different situations are described sometimes causes people to overlook the essential similarities between them. For instance, in Example 3.2, many people erroneously conclude that the cost of taking the trip is higher in the stolen-gas case than in the lost-money case, and so they are less likely to take the trip in the former instance. Similarly, many people were less inclined to buy the disc after having broken the first one than after having lost \$10 because they thought, incorrectly, that the disc would cost more under the broken-disc scenario. As we have seen, however, the amount that will be saved by not buying the disc, or by not taking the trip, is exactly the same under each scenario.

To recapitulate briefly, the budget constraint or budget line summarizes the combinations of bundles that the consumer is able to buy. Its position is determined jointly by income and prices. From the set of feasible bundles, the consumer's task is to pick the particular one she likes best. To identify this bundle, we need some means of summarizing the consumer's preferences over all possible bundles she might consume. We now turn to this task.

CONSUMER PREFERENCES

For simplicity, let us again begin by considering a world with only two goods: shelter and food. A **preference ordering** enables the consumer to rank different bundles of goods in terms of their desirability, or order of preference. Consider two bundles, *A* and *B*. For concreteness, suppose that *A* contains 4 sq yd/wk of shelter and 2 lb/wk of food, while *B* has 3 sq yd/wk of shelter and 3 lb/wk of food. Knowing nothing about a consumer's preferences, we can say nothing about which of these bundles he will prefer. *A* has more shelter but less food than *B*. Someone who spends a lot of time at home would probably choose *A*, while someone with a rapid metabolism might be more likely to choose *B*.

preference ordering a ranking of all possible consumption bundles in order of preference.

In general, we assume that for any two such bundles, the consumer is able to make one of three possible statements: (1) *A* is preferred to *B*, (2) *B* is preferred to *A*, or (3) *A* and *B* are equally attractive. The preference ordering enables the consumer to rank different bundles but not to make more precise quantitative statements about their relative desirability. Thus, the consumer might be able to say that he prefers *A* to *B* but not that *A* provides twice as much satisfaction as *B*.

Preference orderings often differ widely among consumers. One person will like Rachmaninoff, another the Red Hot Chili Peppers. Despite these differences, however, most preference orderings share several important features. More specifically, economists generally assume four simple properties of preference orderings. These properties allow us to construct the concise analytical representation of preferences we need for the budget allocation problem.

1. Completeness

A preference ordering is *complete* if it enables the consumer to rank all possible combinations of goods and services. Taken literally, the completeness assumption is never satisfied, for there are many goods we know too little about to be able to evaluate. It is nonetheless a useful simplifying assumption for the analysis of choices among bundles of goods with which consumers are familiar. Its real intent is to rule out instances like the one portrayed in the fable of Buridan's ass. The hungry animal was unable to choose between two bales of hay in front of him and starved to death as a result.

2. More-Is-Better

The more-is-better property means simply that, other things equal, more of a good is preferred to less. We can, of course, think of examples of more of something making us worse off rather than better (as with someone who has overeaten). But these examples usually contemplate some sort of practical difficulty, such as having a self-control problem or being unable to store a good for future use. As long as people can freely store or dispose of goods they don't want, having more of something can't make them worse off.

As an example of the application of the more-is-better assumption, consider two bundles: *A,* which has 12 sq yd/wk of shelter and 10 lb/wk of food, and *B,* which has 12 sq yd/wk of shelter and 11 lb/wk of food. The assumption tells us that *B* is preferred to *A,* because it has more food and no less shelter.

3. Transitivity

If you like steak better than hamburger and hamburger better than hot dogs, you are probably someone who likes steak better than hot dogs. To say that a consumer's preference ordering is *transitive* means that, for any three bundles *A, B,* and *C,* if he prefers *A* to *B* and prefers *B* to *C,* then he always prefers *A* to *C.* For example, suppose *A* is (4, 2), *B* is (3, 3), and *C* is (2, 4). If you prefer (4, 2) over (3, 3) and you prefer (3, 3) over (2, 4), then you must prefer (4, 2) over (2, 4). The preference relationship is thus assumed to be like the relationship used to compare heights of people. If O'Neal is taller than Nowitzki and Nowitzki is taller than Bryant, we know that O'Neal must be taller than Bryant.

Not all comparative relationships are transitive. The relationship "half sibling," for example, is not. I have a half sister who, in turn, has three half sisters of her own. But her half sisters are not my half sisters. A similar nontransitivity is shown by the relationship "defeats in football." Some seasons, Ohio State defeats Michigan, and Michigan defeats Michigan State, but that doesn't tell us that Ohio State will necessarily defeat Michigan State.

Transitivity is a simple consistency property and applies as well to the relation "equally attractive as" and to any combination of it and the "preferred to" relation. For example, if *A* is equally attractive as *B* and *B* is equally attractive as *C,* it follows that *A* is equally attractive as *C.* Similarly, if *A* is preferred to *B* and *B* is equally attractive as *C,* it follows that *A* is preferred to *C.*

The transitivity assumption can be justified as eliminating the potential for a "money pump" problem. To illustrate, suppose you prefer *A* to *B* and *B* to *C,* but you also prefer *C* over *A,* so that your preferences are intransitive. If you start with *C,* you would trade *C* for *B,* trade *B* for *A,* and then trade *A* for *C.* This cycle could continue forever. If in each stage you were charged a tiny fee for the trade, you would eventually transfer all your money to the other trader. Clearly, such preferences are problematic.

As reasonable as the transitivity property sounds, we will see examples in later chapters of behavior that seems inconsistent with it. But it is an accurate description of preferences in most instances. Unless otherwise stated, we will adopt it.

4. Convexity

Mixtures of goods are preferable to extremes. If you are indifferent between two bundles *A* and *B,* your preferences are convex if you prefer a bundle that contains half of *A* and half of *B* (or any other mixture) to either of the original bundles. For example, suppose you are indifferent between $A = (4, 0)$ and $B = (0, 4)$. If your preferences are convex, you will prefer the bundle (2, 2) to each of the more extreme bundles. This property conveys the sense that we like balance in our mix of consumption goods.

INDIFFERENCE CURVES

Let us consider some implications of these assumptions about preference orderings. Most important, they enable us to generate a graphical description of the consumer's preferences. To see how, consider first the bundle *A* in Figure 3.8, which has 12 sq yd/wk of shelter and 10 lb/wk of food. The more-is-better assumption tells us that all bundles to the northeast of *A* are preferred to *A*, and that *A*, in turn, is preferred to all those to the southwest of *A*. Thus, the more-is-better assumption tells us that *Z*, which has 28 sq yd/wk of shelter and 12 lb/wk of food, is preferred to *A* and that *A*, in turn, is preferred to *W*, which has only 6 sq yd/wk of shelter and 4 lb/wk of food.

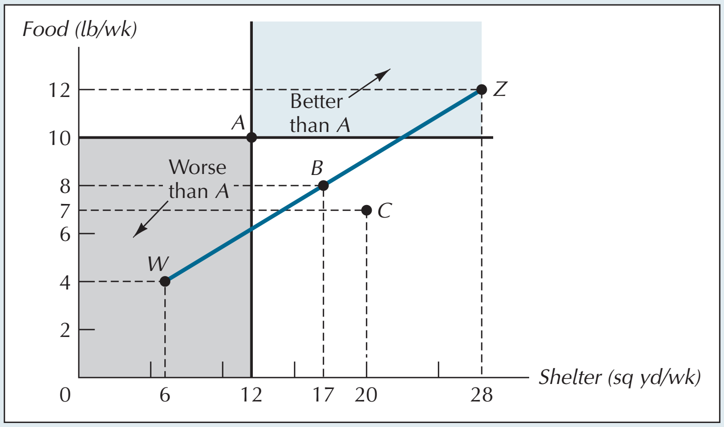


FIGURE 3.8
Generating Equally Preferred Bundles
Z is preferred to *A* because it has more of each good than *A* has. For the same reason, *A* is preferred to *W*. It follows that on the line joining *W* and *Z* there must be a bundle *B* that is equally attractive as *A*. In similar fashion, we can find a bundle *C* that is equally attractive as *B*.

Now consider the set of bundles that lie along the line joining *W* and *Z*. Because *Z* is preferred to *A* and *A* is preferred to *W*, it follows that as we move from *Z* to *W* we must encounter a bundle that is equally attractive as *A*. (The intuition behind this claim is the same as the intuition that tells us that if we climb on any continuous path on a mountainside from one point at 1000 feet above sea level to another at 2000 feet, we must pass through every intermediate altitude along the way.) Let *B* denote the bundle that is equally attractive as *A*, and suppose it contains 17 sq yd/wk of shelter and 8 lb/wk of food. (The exact amounts of each good in *B* will of course depend on the specific consumer whose preferences we are talking about.) The more-is-better assumption also tells us that there will be only one such bundle on the straight line between *W* and *Z*. Points on that line to the northeast of *B* are all better than *B;* those to the southwest of *B* are all worse.

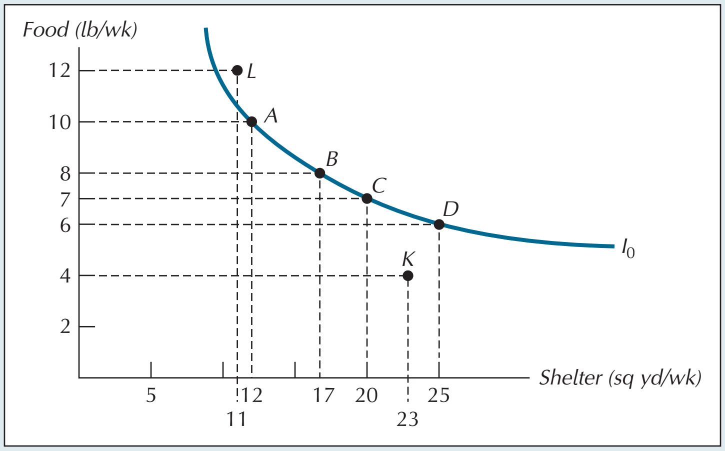
In precisely the same fashion, we can find another point—call it *C*—that is equally attractive as *B*. *C* is shown as the bundle (20, 7), where the specific quantities in *C* again depend on the preferences of the consumer under consideration. By the transitivity assumption, we know that *C* is also equally attractive as *A* (since *C* is equally attractive as *B*, which is equally attractive as *A*).

We can repeat this process as often as we like, and the end result will be an **indifference curve,** a set of bundles all of which are equally attractive as the original bundle *A*, and hence also equally attractive as one another. This set is shown as the curve labeled *I* in Figure 3.9. It is called an indifference curve because the consumer is indifferent among all the bundles that lie along it.

indifference curve a set of bundles among which the consumer is indifferent.

An indifference curve also permits us to compare the satisfaction implicit in bundles that lie along it with those that lie either above or below it. It permits us, for example, to compare bundle *C* (20, 7) to bundle *K* (23, 4), which has less food and more shelter than *C* has. We know that *C* is equally attractive as *D* (25, 6)

FIGURE 3.9
An Indifference Curve
An indifference curve is a set of bundles that the consumer considers equally attractive. Any bundle, such as *L*, that lies above an indifference curve is preferred to any bundle on the indifference curve. Any bundle on the indifference curve, in turn, is preferred to any bundle, such as *K*, that lies below the indifference curve.



because both bundles lie along the same indifference curve. *D,* in turn, is preferred to *K* because of the more-is-better assumption: It has 2 sq yd/wk more shelter and 2 lb/wk more food than *K* has. Transitivity, finally, tells us that since *C* is equally attractive as *D* and *D* is preferred to *K, C* must be preferred to *K*.

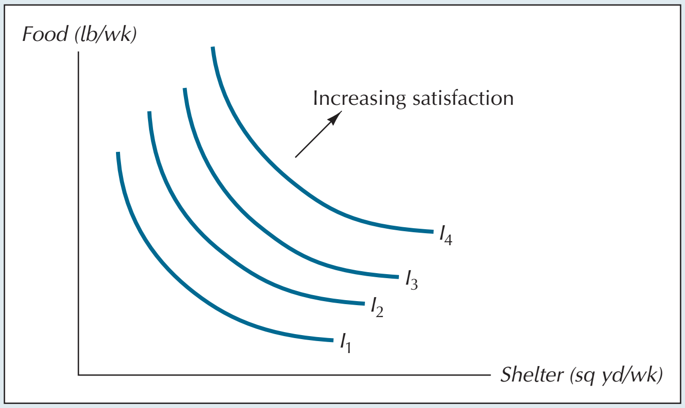
By analogous reasoning, we can say that bundle *L* is preferred to *A*. *In general, bundles that lie above an indifference curve are all preferred to the bundles that lie on it. Similarly, bundles that lie on an indifference curve are all preferred to those that lie below it.*

indifference map a representative sample of the set of a consumer's indifference curves, used as a graphical summary of her preference ordering.

The completeness property of preferences implies that there is an indifference curve that passes through every possible bundle. That being so, we can represent a consumer's preferences with an **indifference map,** an example of which is shown in Figure 3.10. This indifference map shows just four of the infinitely many indifference curves that, taken together, yield a complete description of the consumer's preferences.

The numbers $I_1, \ldots, I_4$ in Figure 3.10 are index values used to denote the order of preference that corresponds to the respective indifference curves. Any index numbers would do equally well provided they satisfied the property $I_1 < I_2 < I_3 < I_4$. In representing the consumer's preferences, what really counts is the *ranking* of the indifference curves, not the particular numerical values we assign to them.[5]

FIGURE 3.10
Part of an Indifference Map
The entire set of a consumer's indifference curves is called the consumer's indifference map. Bundles on any indifference curve are less preferred than bundles on a higher indifference curve, and more preferred than bundles on a lower indifference curve.



[5]For a more complete discussion of this issue, see pp. 87–89 of the appendix to this chapter.

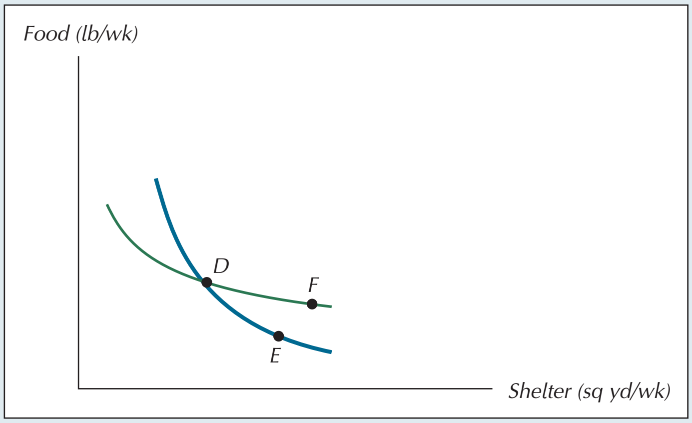


FIGURE 3.11
Why Two Indifference Curves Do Not Cross
If indifference curves were to cross, they would have to violate at least one of the assumed properties of preference orderings.

The four properties of preference orderings imply four important properties of indifference curves and indifference maps:

1. Indifference curves are ubiquitous. Any bundle has an indifference curve passing through it. This property is assured by the completeness property of preferences.
2. Indifference curves are downward-sloping. An upward-sloping indifference curve would violate the more-is-better property by saying a bundle with more of both goods is equivalent to a bundle with less of both.
3. Indifference curves (from the same indifference map) cannot cross. To see why, suppose that two indifference curves did, in fact, cross as in Figure 3.11. The following statements would then have to be true:
   E is equally attractive as *D* (because they each lie on the same indifference curve).
   D is equally attractive as *F* (because they each lie on the same indifference curve).
   E is equally attractive as *F* (by the transitivity assumption).
   But we also know that
   F is preferred to *E* (because more is better).
   Because it is not possible for the statements *E is equally attractive as F* and *F is preferred to E* to be true simultaneously, the assumption that two indifference curves cross thus implies a contradiction. The conclusion is that the original proposition must be true, namely, two indifference curves cannot cross.
4. Indifference curves become less steep as we move downward and to the right along them. As discussed below, this property is implied by the convexity property of preferences.

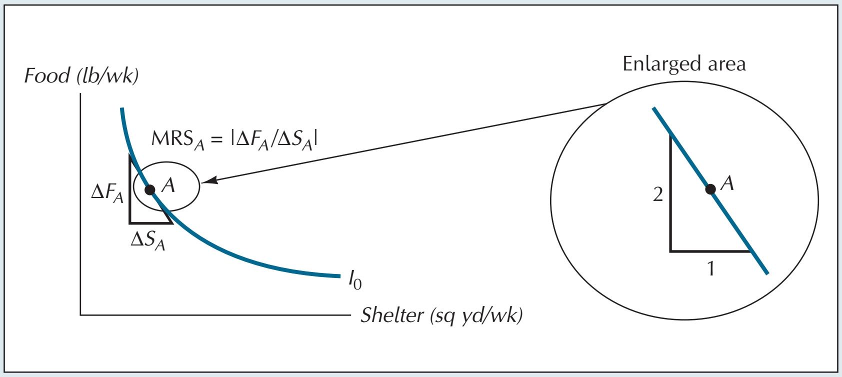
TRADE-OFFS BETWEEN GOODS

An important property of a consumer's preferences is the rate at which he is willing to exchange, or "trade off," one good for another. This rate is represented at any point on an indifference curve by the **marginal rate of substitution (MRS),** which is defined as the absolute value of the slope of the indifference curve at that point. In the left panel of Figure 3.12, for example, the marginal rate of substitution at point *A* is given by the absolute value of the slope of the tangent to the indifference curve at *A*, which is the ratio $\Delta F_A/\Delta S_A$.[6] (The notation ΔF_A means "small change in food from the amount at point *A*.") If we take ΔF_A units of food away from the consumer at point *A*, we have to give him ΔS_A additional units of shelter to make him

marginal rate of substitution (MRS) at any point on an indifference curve, the rate at which the consumer is willing to exchange the good measured along the vertical axis for the good measured along the horizontal axis; equal to the absolute value of the slope of the indifference curve.

[6]More formally, the indifference curve may be expressed as a function $Y = Y(X)$ and the MRS at point *A* is defined as the absolute value of the derivative of the indifference curve at that point: $MRS = |dY(X)/dX|$.

FIGURE 3.12
The Marginal Rate of Substitution
MRS at any point along an indifference curve is defined as the absolute value of the slope of the indifference curve at that point. It is the amount of food the consumer must be given to compensate for the loss of 1 unit of shelter.



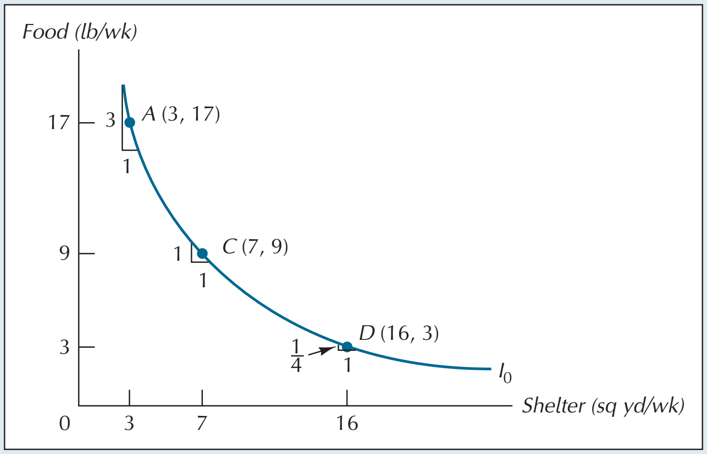
just as well off as before. The right panel of the figure shows an enlargement of the region surrounding bundle *A*. If the marginal rate of substitution at *A* is 2, this means that the consumer must be given 2 lb/wk of food to make up for the loss of 1 sq yd/wk of shelter.

Whereas the slope of the budget constraint tells us the rate at which we can substitute food for shelter without changing total expenditure, the MRS tells us the rate at which we can substitute food for shelter without changing total satisfaction. Put another way, the slope of the budget constraint is the marginal cost of shelter in terms of food, and the MRS is the marginal benefit of shelter in terms of food.

The convexity property of preferences tells us that along any indifference curve, the more a consumer has of one good, the more she must be given of that good before she will be willing to give up a unit of the other good. Stated differently, MRS declines as we move downward to the right along an indifference curve. Indifference curves with diminishing rates of marginal substitution are thus convex—or bowed outward—when viewed from the origin. The indifference curves shown in Figures 3.9, 3.10, and 3.12 have this property, as does the curve shown in Figure 3.13.

In Figure 3.13, note that at bundle *A* food is relatively plentiful and the consumer would be willing to sacrifice 3 lb/wk of it in order to obtain an additional square yard of shelter. Her MRS at *A* is 3. At *C*, the quantities of food and shelter are more balanced, and there she would be willing to give up only 1 lb/wk to obtain an additional square yard of shelter. Her MRS at *C* is 1. Finally, note that food is

FIGURE 3.13
Diminishing Marginal Rate of Substitution
The more food the consumer has, the more she is willing to give up to obtain an additional unit of shelter. The marginal rates of substitution at bundles *A, C,* and *D* are 3, 1, and 1/4, respectively.



relatively scarce at D, and there she would be willing to give up only $\frac{1}{4}$ lb/wk of food to obtain an additional unit of shelter. Her MRS at D is $\frac{1}{4}$.

Intuitively, diminishing MRS means that consumers like variety. We are usually willing to give up goods we already have a lot of to obtain more of those goods we now have only a little of.

USING INDIFFERENCE CURVES TO DESCRIBE PREFERENCES

To get a feel for how indifference maps describe a consumer's preferences, let us see how indifference maps can be used to portray differences in preferences between two consumers. Suppose, for example, that both Tex and Mohan like potatoes but that Mohan likes rice much more than Tex does. This difference in their tastes is captured by the differing slopes of their indifference curves in Figure 3.14. Note in Figure 3.14*a*, which shows Tex's indifference map, that Tex would be willing to exchange 1 lb of potatoes for 1 lb of rice at bundle *A*. But at the corresponding bundle in Figure 3.14*b*, which shows Mohan's indifference map, we see that Mohan would trade 2 lb of potatoes for 1 lb of rice. Their difference in preferences shows up clearly in this difference in their marginal rates of substitution of potatoes for rice.

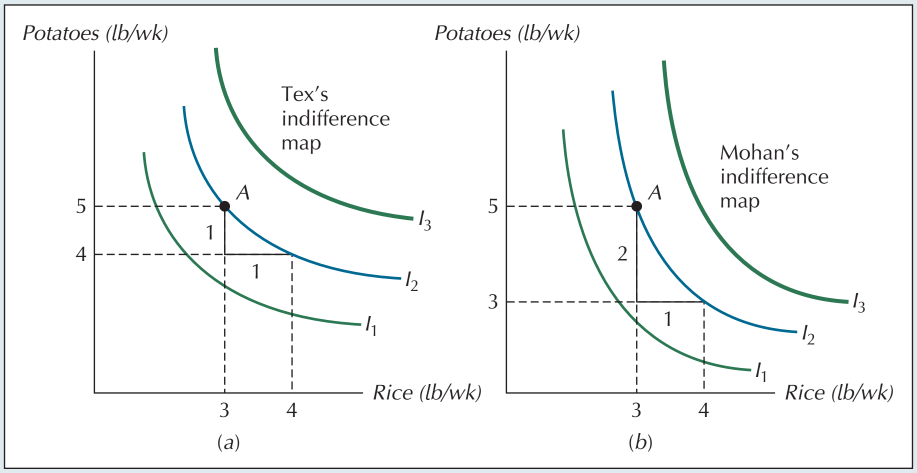


FIGURE 3.14
People with Different Tastes
Relatively speaking, Tex is a potato lover; Mohan, a rice lover. This difference shows up in the fact that at any given bundle Tex's marginal rate of substitution of potatoes for rice is smaller than Mohan's.

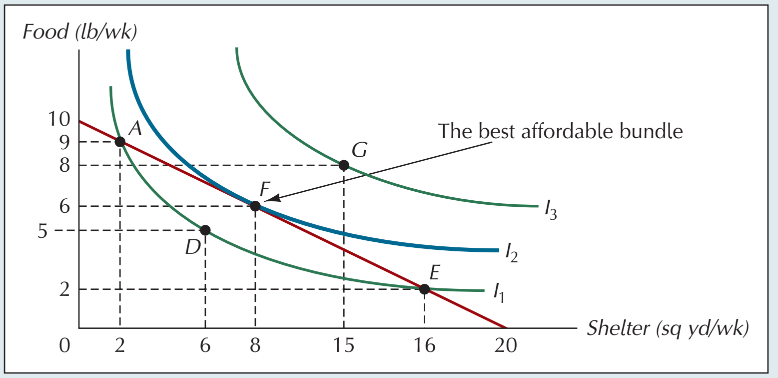
THE BEST FEASIBLE BUNDLE

We now have the tools we need to determine how the consumer should allocate his income between two goods. The indifference map tells us how the various bundles are ranked in order of preference. The budget constraint, in turn, tells us which bundles are affordable. The consumer's task is to put the two together and to choose the most preferred or **best affordable bundle.** (Recall from Chapter 1 that we need not suppose that consumers think explicitly about budget constraints and indifference maps when deciding what to buy. It is sufficient to assume that people make decisions *as if* they were thinking in these terms, just as expert pool players choose between shots as if they knew all the relevant laws of Newtonian physics.)

best affordable bundle the most preferred bundle of those that are affordable.

Let us again consider the choice between food and shelter that confronts a consumer with an income of M = \$100/wk facing prices of P_F = \$10/lb and P_S = \$5/sq yd. Figure 3.15 shows this consumer's budget constraint and part of his indifference map. Of the five labeled bundles—*A, D, E, F,* and *G*—in the diagram, *G* is the most preferred because it lies on the highest indifference curve. *G*, however, is not affordable, nor is any other bundle that lies beyond the budget constraint. The

FIGURE 3.15
The Best Affordable Bundle
The best the consumer can do is to choose the bundle on the budget constraint that lies on the highest attainable indifference curve. Here, that is bundle *F*, which lies at a tangency between the indifference curve and the budget constraint.



more-is-better assumption implies that the best affordable bundle must lie *on* the budget constraint, not inside it. (Any bundle inside the budget constraint would be less preferred than one just slightly to the northeast, which would also be affordable.)

Where exactly is the best affordable bundle located along the budget constraint? We know that it cannot be on an indifference curve that lies partly inside the budget constraint. On the indifference curve I_1, for example, the only points that are even candidates for the best affordable bundle are the two that lie on the budget constraint, namely, *A* and *E*. But *A* cannot be the best affordable bundle because it is equally attractive as *D*, which in turn is less desirable than *F* by the more-is-better assumption. So by transitivity, *A* is less desirable than *F*. For the same reason, *E* cannot be the best affordable bundle.

Since the best affordable bundle cannot lie on an indifference curve that lies partly inside the budget constraint, and since it must lie on the budget constraint itself, we know it has to lie on an indifference curve that intersects the budget constraint only once. In Figure 3.15, that indifference curve is the one labeled I_2, and the best affordable bundle is *F*, which lies at the point of tangency between I_2 and the budget constraint. With an income of \$100/wk and facing prices of \$5/sq yd for shelter and \$10/lb for food, the best this consumer can do is to buy 6 lb/wk of food and 8 sq yd/wk of shelter.

The choice of bundle *F* makes perfect sense on intuitive grounds. The consumer's goal, after all, is to reach the highest indifference curve he can, given his budget constraint. His strategy is to keep moving to higher and higher indifference curves until he reaches the highest one that is still affordable. For indifference maps for which a tangency point exists, as in Figure 3.15, the best bundle will always lie at the point of tangency.

In Figure 3.15, note that the marginal rate of substitution at *F* is exactly the same as the absolute value of the slope of the budget constraint. This will always be so when the best affordable bundle occurs at a point of tangency. The condition that must be satisfied in such cases is therefore

$$MRS = \frac{P_S}{P_F}. \tag{3.3}$$

The right-hand side of Equation 3.3 represents the opportunity cost of shelter in terms of food. Thus, with P_S = \$5/sq yd and P_F = \$10/lb, the opportunity cost of an additional square yard of shelter is $\frac{1}{2}$ lb of food. The left-hand side of Equation 3.3 is $|\Delta F/\Delta S|$, the absolute value of the slope of the indifference curve at the point of tangency. It is the amount of additional food the consumer must be given in order to compensate him fully for the loss of 1 sq yd of shelter. In the language of cost-benefit analysis discussed in Chapter 1, the slope of the budget constraint represents the opportunity cost of shelter in terms of food, while the slope of the indifference curve represents the benefits of consuming shelter as compared with consuming food. Since the slope of the budget constraint is $-\frac{1}{2}$ in this example, the tangency condition tells us that $\frac{1}{2}$ lb of food would be required to compensate for the benefits given up with the loss of 1 sq yd of shelter.

If the consumer were at some bundle on the budget line for which the two slopes are not the same, then it would always be possible for him to purchase a better bundle. To see why, suppose he were at a point where the slope of the indifference curve (in absolute value) is less than the slope of the budget constraint (also in absolute value), as at point *E* in Figure 3.15. Suppose, for instance, that the MRS at *E* is only $\frac{1}{4}$. This tells us that the consumer can be compensated for the loss of 1 sq yd of shelter by being given an additional $\frac{1}{4}$ lb of food. But the slope of the budget constraint tells us that by giving up 1 sq yd of shelter, he can purchase an additional $\frac{1}{2}$ lb of food. Since this is $\frac{1}{4}$ lb more than he needs to remain equally satisfied, he will clearly be better off if he purchases more food and less shelter than at point *E*. The opportunity cost of an additional pound of food is less than the benefit it confers.

EXERCISE 3.6

Suppose that the marginal rate of substitution at point *A* in Figure 3.15 is 1.0. Show that this means the consumer will be better off if he purchases less food and more shelter than at *A*.

CORNER SOLUTIONS

The best affordable bundle need not always occur at a point of tangency. In some cases, there may simply *be* no point of tangency—the MRS may be everywhere greater, or less, than the slope of the budget constraint. In this case we get a **corner solution,** like the one shown in Figure 3.16, where M, P_F, and P_S are again given by \$100/wk, \$10/lb and \$5/sq yd, respectively. The best affordable bundle is the one labeled *A,* and it lies at the upper end of the budget constraint. At *A* the MRS is less than the absolute value of the slope of the budget constraint. For the sake of illustration, suppose the MRS at $A = 0.25$, which means that this consumer would be willing to give up 0.25 lb of food to get an additional square yard of shelter. But at market prices the opportunity cost of an additional square yard of shelter is 0.5 lb of food. He increases his satisfaction by continuing to give up shelter for more food until it is no longer possible to do so. Even though this consumer regards shelter as a desirable commodity, the best he can do is to spend all his income on food. Market prices are such that he would have to give up too much food to make the purchase of even a single unit of shelter worthwhile.

corner solution in a choice between two goods, a case in which the consumer does not consume one of the goods.

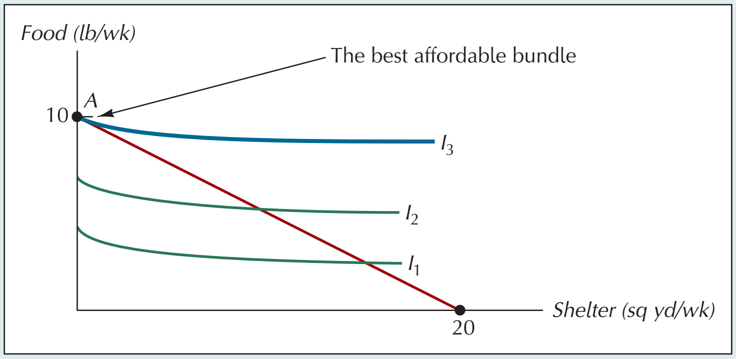


FIGURE 3.16
A Corner Solution
When the MRS of food for shelter is always less than the slope of the budget constraint, the best the consumer can do is to spend all his income on food.

The indifference map in Figure 3.16 satisfies the property of diminishing marginal rate of substitution—moving to the right along any indifference curve, the slope becomes smaller in absolute terms. But because the slopes of the indifference curves start out smaller than the slope of the budget constraint here, the two never reach equality.

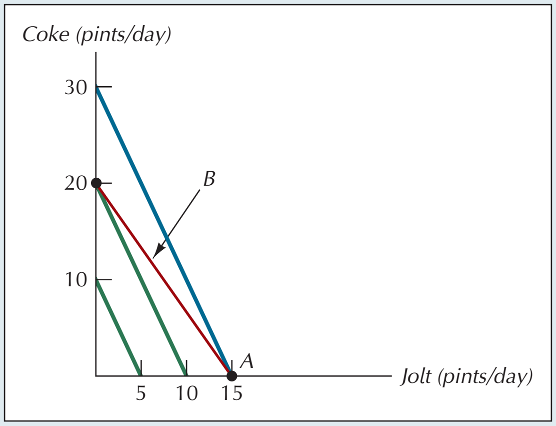
Indifference curves that are not strongly convex are characteristic of goods that are easily substituted for one another. Corner solutions are more likely to occur for such goods, and indeed are almost certain to occur when goods are perfect substitutes. (See Example 3.3.) For such goods, the MRS does not diminish at all; rather, it is everywhere the same. With perfect substitutes, indifference curves are straight lines. If they happen to be steeper than the budget constraint, we get a corner solution on the horizontal axis; if less steep, we get a corner solution on the vertical axis.

EXAMPLE 3.3

Mattingly is a caffeinated-cola drinker who spends his entire soft drink budget on Coca-Cola and Jolt cola and cares only about total caffeine content. If Jolt has twice the caffeine of Coke, and if Jolt costs \$1/pint and Coke costs \$0.75/pint, how will Mattingly spend his soft drink budget of \$15/wk?

For Mattingly, Jolt and Coke are *perfect substitutes,* which means that his indifference curves will be linear. The top line in Figure 3.17 is the set of all possible Coke-Jolt combinations that provide the same satisfaction as the bundle consisting of 0 pints of Jolt per day and 30 pints of Coke per day. Since each pint of Jolt has twice the caffeine of a pint of Coke, all bundles along this line contain precisely the same amount of caffeine. The first green line down is the indifference curve for bundles equivalent to bundle (0, 20); and the second green line down is the indifference curve corresponding to (0, 10). Along each of these indifference curves, the marginal rate of substitution of Coke for Jolt is always $\frac{2}{1}$, that is, 2 pints of Coke for every pint of Jolt.

FIGURE 3.17
Equilibrium with Perfect Substitutes
Here, the MRS of Coke for Jolt is 2 at every point. Whenever the price ratio P_J/P_C is less than 2, a corner solution results in which the consumer buys only Jolt. On the budget constraint *B,* the consumer does best to buy bundle *A.*



In the same diagram, Mattingly's budget constraint is shown as *B*. The slope of his indifference curves is -2; of his budget constraint, $-\frac{4}{3}$. The best affordable bundle is the one labeled *A,* a corner solution in which he spends his entire budget on Jolt. This makes intuitive sense in light of Mattingly's peculiar preferences: he cares only about total caffeine content, and Jolt provides more caffeine per dollar than Coke does. If the Jolt-Coke price ratio, P_J/P_C had been $\frac{3}{1}$ (or any other amount greater than $\frac{2}{1}$), Mattingly would have spent all his income on Coke. That is, we would again have had a corner solution, only this time on the vertical axis. Only if the price ratio had been exactly $\frac{2}{1}$ might we have seen Mattingly spend part of his income on each good. In that case, any combination of Coke and Jolt on his budget constraint would have served equally well.

Most of the time we will deal with problems that have *interior solutions*—that is, with problems where the best affordable bundle will lie at a point of tangency. An interior solution, again, is one where the MRS is exactly the same as the slope of the budget constraint.

EXERCISE 3.7

Suppose Albert always uses exactly two pats of butter on each piece of toast. If toast costs \$0.10/slice and butter costs \$0.20/pat, find Albert's best affordable bundle if he has \$12/mo to spend on toast and butter. Suppose Albert starts to watch his cholesterol and therefore alters his preference to using exactly one pat of butter on each piece of toast. How much toast and butter would Albert then consume each month?

INDIFFERENCE CURVES WHEN THERE ARE MORE THAN TWO GOODS

In the examples thus far, the consumer cared about only two goods. Where there are more than two, we can construct indifference curves by using the same device we used earlier to represent multigood budget constraints. We simply view the consumer's choice as being one between a particular good X and an amalgam of other goods Y, which is again called the composite good. As before, the composite good is the amount of income the consumer has left over after buying the good X.

In the multigood case, we may thus continue to represent the consumer's preferences with an indifference map in the XY plane. Here, the indifference curve tells the rate at which the consumer will exchange the composite good for X. As in the two-good case, equilibrium occurs when the consumer reaches the highest indifference curve attainable on his budget constraint.

AN APPLICATION OF THE RATIONAL CHOICE MODEL

As the following example makes clear, the composite good construct enables us to deal with more general questions than we could in the two-good case.

EXAMPLE 3.4

Is it better to give poor people cash or food stamps?

One objective of the food stamp program is to alleviate hunger. Under the terms of the program, people whose incomes fall below a certain level are eligible to receive a specified quantity of food stamps. For example, a person with an income of \$400/mo might be eligible for \$100/mo worth of stamps. These stamps can then be used to buy \$100/mo worth of food. Any food he buys in excess of \$100/mo he must pay for in cash. Stamps cannot be used to purchase cigarettes, alcohol, and various other items. The government gives food retailers cash for the stamps they accept.

The cost to the government for the consumer in the example given was \$100—the amount it had to reimburse the store for the stamps. Would the consumer have been better off had he instead been given \$100 directly in cash?

We can try to answer this question by investigating which alternative would get him to a higher indifference curve. Suppose Y denotes the composite good and X denotes food. If the consumer's income is \$400/mo and P_X is the price of food, his initial equilibrium is the bundle J in Figure 3.18. The effect of the food stamp program is to increase the total amount of food he can buy each month from

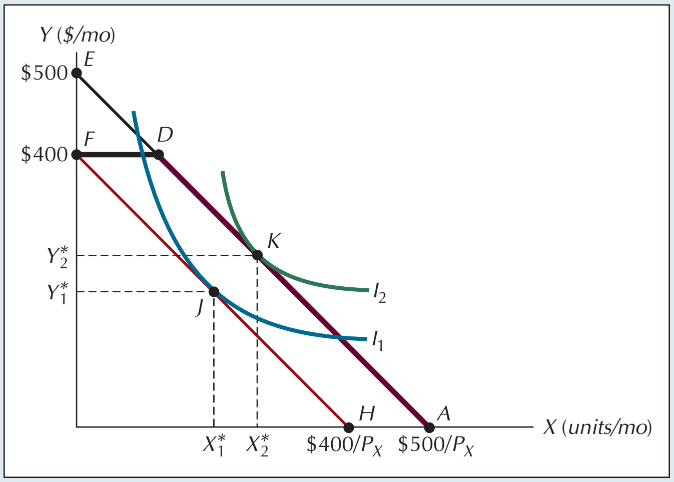
$\$400/P_X$ to $\$500/P_X$. In terms of the maximum amount of food he can buy, the food stamp program is thus exactly the same as a cash grant of \$100.

Where the two alternatives differ is in terms of the maximum amounts of other goods he can buy. With a cash grant of \$100, he has a total monthly income of \$500, and this is, of course, the maximum amount of nonfood goods (the composite good) he can buy. His budget constraint in this case is thus the line labeled *AE* in Figure 3.18.

FIGURE 3.18

Food Stamp Program vs. Cash Grant Program

By comparison with the budget constraint under a cash grant (*AE*), the budget constraint under food stamps (*ADF*) limits the amount that can be spent on nonfood goods. But for the consumer whose indifference map is shown, the equilibrium bundles are the same under both programs.



With the food stamp program, by contrast, the consumer is not able to buy \$500/mo of nonfood goods because his \$100 in food stamps can be used only for food. The maximum amount of nonfood goods he can purchase is \$400. In Figure 3.18, his budget constraint under the food stamp program is labeled *ADF*. For values of *Y* less than \$400, it is thus exactly the same as his budget constraint under the cash grant program. For values of *Y* larger than \$400, however, his budget constraint under the food stamp program is completely flat.

Note that the consumer whose indifference curves are shown in Figure 3.18 buys exactly the same bundle—namely, bundle *K*—under both programs. The effect of the food stamp program here is precisely the same as the effect of the cash grant. In general, this will be true whenever the consumer with a cash grant would have spent more on food anyway than the amount of food stamps he would have received under the food stamp program.

Figure 3.19 depicts a consumer for whom this is *not* the case. With a cash grant, he would choose the bundle *L*, which would put him on a higher indifference curve than he could attain under the food stamp program, which would lead him to buy bundle *D*. Note that bundle *D* contains exactly \$100 worth of food, the amount of food stamps he received. Bundle *L*, by contrast, contains less than \$100 worth of food. Here, the effect of the food stamp program is to cause the recipient to spend more on food than he would have if he had instead been given cash.

The face value of the food stamps most participants receive is smaller than what they would spend on food. For these people, the food stamp program leads, as noted, to exactly the same behavior as a pure cash grant program.

The analysis in Example 3.4 raises the question of why Congress did not just give poor people cash grants in the first place. The ostensible reason is that Congress wanted to help poor people buy food, not luxury items or even cigarettes and alcohol. And yet if most participants would have spent at least as much on food

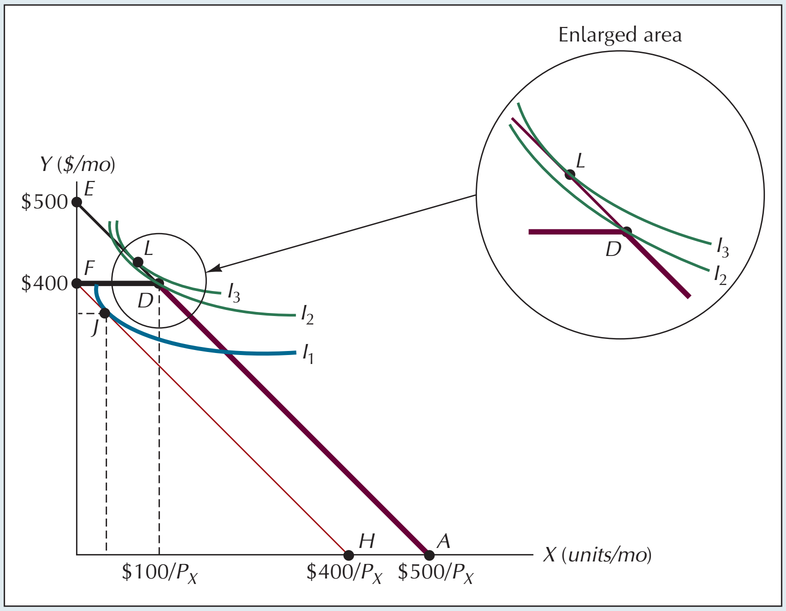


FIGURE 3.19
Where Food Stamps and Cash Grants Yield Different Outcomes
For the consumer whose indifference map is shown, a cash grant would be preferred to food stamps, which force him to devote more to food than he would choose to spend on his own.

as they received in stamps, not being able to use stamps to buy other things is a meaningless restriction. For instance, if someone would have spent $150 on food anyway, getting $100 in food stamps simply lets him take some of the money he would have spent on food and spend it instead on whatever else he chooses.

On purely economic grounds, there is thus a strong case for replacing the food stamp program with a much simpler program of cash grants to the poor. At the very least, this would eliminate the cumbersome step of requiring grocers to redeem their stamps for cash.

As a political matter, however, it is easy to see why Congress might have set things up the way it did. Many taxpayers would be distressed to see their tax dollars used to buy illicit substances. If the food stamp program prevents even a tiny minority of participants from spending more on such goods, it spares many political difficulties.

Example 3.4 calls our attention to a problem that applies not just to the food stamp program but to all other forms of in-kind transfers as well: Although the two forms of transfer are sometimes equivalent, gifts in cash seem clearly superior on those occasions when they differ.

ECONOMIC NATURALIST 3.1

Why do people often give gifts in kind instead of cash?

Occasionally someone receives a gift that is exactly what he would have purchased for himself had he been given an equivalent amount of money. But we are all far too familiar with gifts that miss the mark. Who has never been given an article of clothing that he was embarrassed to wear? The logic of the rational choice model seems to state unequivocally that we could avoid the problem of useless gifts if we followed the simple expedient of giving cash. And yet virtually every society continues to engage in ritualized gift giving.

The fact that this custom has persisted should not be taken as evidence that people are stupid. Rather, it suggests that the rational choice model may fail to capture something important about gift giving. One purpose of a gift is to express affection for the recipient. A thoughtfully chosen gift accomplishes this in a way that cash cannot. Or it may be that some people have difficulty indulging themselves with even small luxuries and would feel compelled to spend cash gifts on

"Sweetheart, perhaps we shouldn't exchange gifts this year."

purely practical items. For these people, a gift provides a way of enjoying a small luxury without having to feel guilty about it.[7] This interpretation is supported by the observation that we rarely give purely practical gifts like plain cotton underwear or laundry detergent.

Whatever the real reasons people may have for giving in kind rather than in cash, it seems safe to assume that we do not do it because it never occurred to us to give cash. On the contrary, occasionally we do give cash gifts, especially to young relatives with low incomes. But even though there are advantages to gifts in cash, people seem clearly reluctant to abandon the practice of giving in kind.

The Appendix to this chapter develops the utility function approach to the consumer budgeting problem. Topics covered include cardinal versus ordinal utility, algebraic construction of indifference curves, and the use of calculus to maximize utility.

SUMMARY

- Our task in this chapter was to set forth the basic model of rational consumer choice. In all its variants, this model takes consumers' preferences as given and assumes they will try to satisfy them in the most efficient way possible.

- The first step in solving the budgeting problem is to identify the set of bundles of goods the consumer is able to buy. The consumer is assumed to have an income level given in advance and to face fixed prices. Prices and income together define the consumer's budget constraint, which, in the simple two-good case, is a downward-sloping line. Its slope, in absolute value, is the ratio of the two prices. It is the set of all possible bundles that the consumer might purchase if he spends his entire income.

[7]For a discussion of this interpretation, see R. Thaler, "Mental Accounting and Consumer Choice," *Marketing Science,* 4, Summer 1985.

- The second step in solving the consumer budgeting problem is to summarize the consumer's preferences. Here, we begin with a preference ordering by which the consumer is able to rank all possible bundles of goods. This ranking scheme is assumed to be complete and transitive and to exhibit the more-is-better property. Preference orderings that satisfy these restrictions give rise to indifference maps, or collections of indifference curves, each of which represents combinations of bundles among which the consumer is indifferent. Preference orderings are also assumed to exhibit a diminishing marginal rate of substitution, which means that, along any indifference curve, the more of a good a consumer has, the more he must be given to induce him to part with a unit of some other good. The diminishing MRS property accounts for the characteristic convex shape of indifference curves.

- The budget constraint tells us what combinations of goods the consumer can afford to buy. To summarize the consumer's preferences over various bundles, we use an indifference map. In most cases, the best affordable bundle occurs at a point of tangency between an indifference curve and the budget constraint. At that point, the marginal rate of substitution is exactly equal to the rate at which the goods can be exchanged for one another at market prices.

QUESTIONS FOR REVIEW

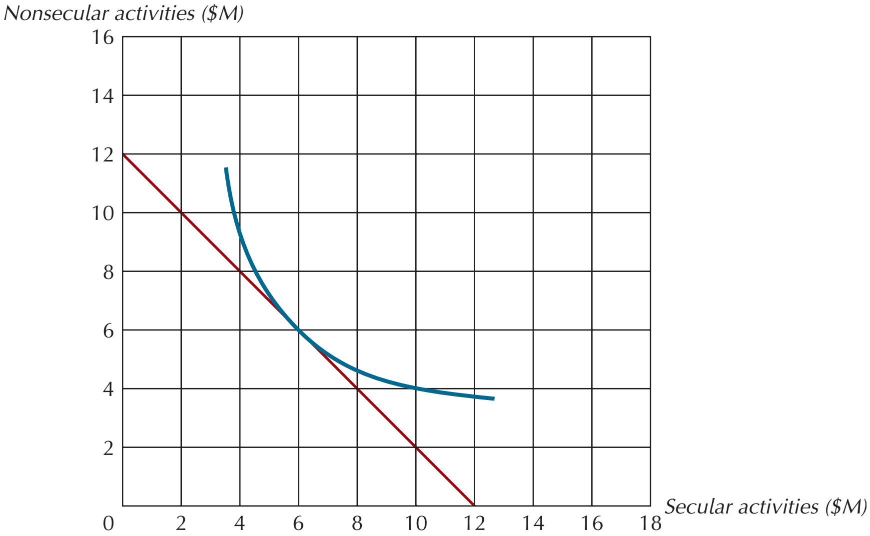
1. If the prices of all products are rising at 20 percent per year and your employer gives you a 20 percent salary increase, are you better off, worse off, or equally well off in comparison with your situation a year ago?
2. *True or false:* If you know the slope of the budget constraint (for two goods), you know the prices of the two goods. Explain.
3. *True or false:* The downward slope of indifference curves is a consequence of the diminishing marginal rate of substitution.
4. Construct an example of a preference ordering over Coke, Diet Coke, and Diet Pepsi that violates the transitivity assumption.
5. Explain in your own words how the slope of an indifference curve provides information about how much a consumer likes one good relative to another.
6. Explain why a consumer will often buy one bundle of goods even though he prefers another.
7. Why are corner solutions especially likely in the case of perfect substitutes?
8. *True or false:* If the indifference curve map is concave to the origin, then the optimal commodity basket must occur at a corner equilibrium, except possibly when there are quantity discounts.
9. If Ralph were given \$10, he would spend none of it on tuna fish. But when asked, he claims to be indifferent between receiving \$10 worth of tuna fish and a \$10 bill. How could this be?

PROBLEMS

1. The Acme Seed Company charges \$2/lb for the first 10 lb you buy of marigold seeds each week and \$1/lb for every pound you buy thereafter. If your income is \$100/wk, draw your budget constraint for the composite good and marigold seeds.
2. Same as Problem 1, except now the price for every pound after 10 lb/wk is \$4/lb.
3. Smith likes cashews better than almonds and likes almonds better than walnuts. He likes pecans equally well as macadamia nuts and prefers macadamia nuts to almonds. Assuming his preferences are transitive, which does he prefer:
   a. Pecans or walnuts?
   b. Macadamia nuts or cashews?
4. Originally P_X is \$120 and P_Y is \$80. *True or false:* If P_X increases by \$18 and P_Y increases by \$12, the new budget line will be shifted inward and parallel to the old budget line. Explain.

5. Martha has \$150 to spend each week and cannot borrow money. She buys Malted Milk Balls and the composite good. Suppose that Malted Milk Balls cost \$2.50 per bag and the composite good costs \$1 per unit.
   a. Sketch Martha's budget constraint.
   b. What is the opportunity cost, in terms of bags of Malted Milk Balls, of an additional unit of the composite good?
6. In Problem 5, suppose that in an inflationary period the price of the composite good increases to \$1.50 per unit, but the price of Malted Milk Balls remains the same.
   a. Sketch the new budget constraint.
   b. What is the opportunity cost of an additional unit of the composite good?
7. In Problem 6, suppose that Martha demands a pay raise to fight the inflation. Her boss raises her salary to \$225/wk.
   a. Sketch the new budget constraint.
   b. What is the opportunity cost of an additional unit of the composite good?
8. Picabo, an aggressive skier, spends her entire income on skis and bindings. She wears out one pair of skis for every pair of bindings she wears out.
   a. Graph Picabo's indifference curves for skis and bindings.
   b. Now draw her indifference curves on the assumption that she is such an aggressive skier that she wears out two pairs of skis for every pair of bindings she wears out.
9. Suppose Picabo in Problem 8 has \$3,600 to spend on skis and bindings each year. Find her best affordable bundle of skis and bindings under both of the preferences described in the previous problem. Skis are \$480/pr and bindings are \$240/pr.
10. For Alexi, coffee and tea are perfect substitutes: One cup of coffee is equivalent to one cup of tea. Suppose Alexi has \$90/mo to spend on these beverages, and coffee costs \$0.90/cup while tea costs \$1.20/cup. Find Alexi's best affordable bundle of tea and coffee. How much could the price of a cup of coffee rise without harming her standard of living?
11. Eve likes apples but doesn't care about pears. If apples and pears are the only two goods available, draw her indifference curves.
12. Koop likes food but dislikes cigarette smoke. The more food he has, the more he would be willing to give up to achieve a given reduction in cigarette smoke. If food and cigarette smoke are the only two goods, draw Koop's indifference curves.
13. If you were president of a conservation group, which rate structure would you prefer the Gigawatt Power Company to use: the one described in Example 3.1, or one in which all power sold for \$0.08/kWh? (Assume that each rate structure would exactly cover the company's costs.)
14. Paula, a former actress, spends all her income attending plays and movies and likes plays exactly three times as much as she likes movies.
   a. Draw her indifference map.
   b. Paula earns \$120/wk. If play tickets cost \$12 each and movie tickets cost \$4 each, show her budget line and highest attainable indifference curve. How many plays will she see?
   c. If play tickets are \$12, movie tickets \$5, how many plays will she attend?
15. For each of the following, sketch:
   a. A typical person's indifference curves between garbage and the composite good.
   b. Indifference curves for the same two commodities for Oscar the Grouch on *Sesame Street,* who loves garbage and has no use for the composite good.
16. Boris budgets \$9/wk for his morning coffee with milk. He likes it only if it is prepared with 4 parts coffee, 1 part milk. Coffee costs \$1/oz, milk \$0.50/oz. How much coffee and how much milk will Boris buy per week? How will your answers change if the price of coffee rises to \$3.25/oz? Show your answers graphically.
17. The federal government wants to support education but must not support religion. To this end, it gives the University of Notre Dame \$2 million with the stipulation that this money be used for secular purposes only. The accompanying graph shows Notre Dame's

pre-federal-gift budget constraint and best attainable indifference curve over secular and nonsecular expenditures. How would the university's welfare differ if the gift came without the secular-use restriction?



18. Continental Long Distance Telephone Service offers an optional package for in-state calling whereby each month the subscriber gets the first 50 min of in-state calls free, the next 100 min at $0.25/min, and any additional time at the normal rate of $0.50/min. Draw the budget constraint for in-state phone calls and the composite good for a subscriber with an income of $400/mo.

19. For the Continental Long Distance subscriber in Problem 18, what is the opportunity cost of making an additional 20 min of calls if he currently makes
    a. 40 min of calls each month?
    b. 140 min of calls each month?

20. You have the option of renting a car on a daily basis for $40/day or on a weekly basis for $200/wk. Draw your budget constraint for a budget of $360/trip.
    a. Find your best affordable bundle if your travel preferences are such that you require exactly $140 worth of other goods for each day of rental car consumption.
    b. Alternatively, suppose you view a day of rental car consumption as a perfect substitute for $35 worth of other goods.

21. Howard said that he was exactly indifferent between consuming four slices of pizza and one beer versus consuming three slices of pizza and two beers. He also said that he prefers a bundle consisting of one slice of pizza and three beers to either of the first two bundles. Do Howard's preferences exhibit diminishing marginal rates of substitution?

22. Your local telephone company has offered you a choice between the following billing plans:
    Plan A: Pay $0.05 per call.
    Plan B: Pay an initial $2/wk, which allows you up to 30 calls per week at no charge. Any calls over 30/wk cost $0.05 per call.
    If your income is $12/wk and the composite good costs $1, graph your budget constraints for the composite good and calls under the two plans.

*23. At your school's fund-raising picnic, you pay for soft drinks with tickets purchased in advance—one ticket per bottle of soft drink. Tickets are available in sets of three types:
    Small: $3 for 3 tickets
    Medium: $4 for 5 tickets
    Large: $5 for 8 tickets
    If the total amount you have to spend is $12 and fractional sets of tickets cannot be bought, graph your budget constraint for soft drinks and the composite good.

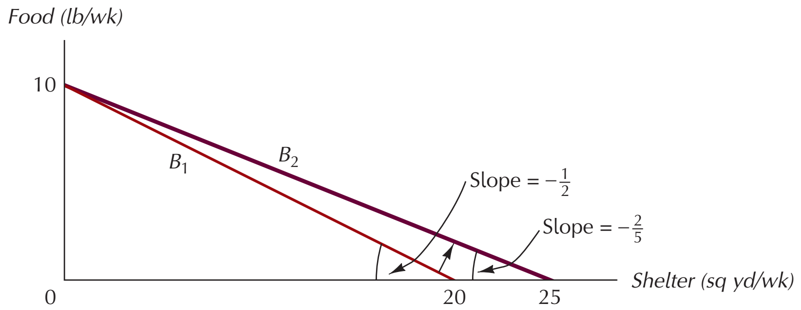
*Problems marked with an asterisk (*) are more difficult.

*24. Consider two Italian restaurants located in identical towns 200 miles apart. The restaurants are identical in every respect but their tipping policies. At one, there is a flat \$15 service charge, but no other tips are accepted. At the other, a 15 percent tip is added to the bill. The average food bill at the first restaurant, exclusive of the service charge, is \$100. How, if at all, do you expect the amount of food eaten in the two restaurants to differ?

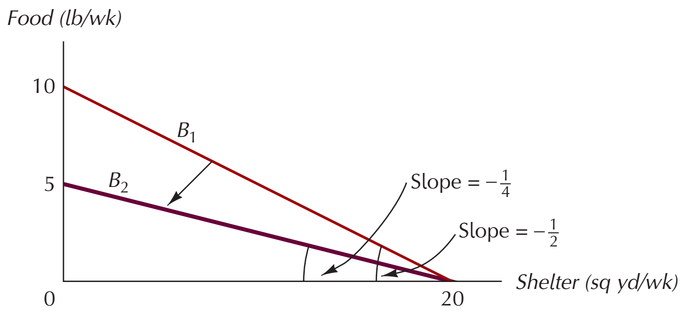
*25. Mr. R. Plane, a retired college administrator, consumes only grapes and the composite good $Y(P_Y = \$1)$. His income consists of \$10,000/yr from social security, plus the proceeds from whatever he sells of the 2000 bushels of grapes he harvests annually from his vineyard. Last year, grapes sold for \$2/bushel, and Plane consumed all 2000 bushels of his grapes in addition to 10,000 units of Y. This year the price of grapes is \$3/bushel, while P_Y remains \$1. If his indifference curves have the conventional shape, will this year's consumption of grapes be greater than, smaller than, or the same as last year's? Will this year's consumption of Y be greater than, smaller than, or the same as last year's? Explain.

ANSWERS TO IN-CHAPTER EXERCISES

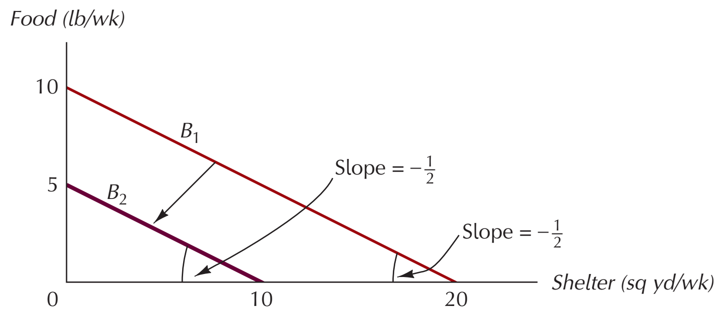
3.1. *Food (lb/wk)*



3.2. *Food (lb/wk)*

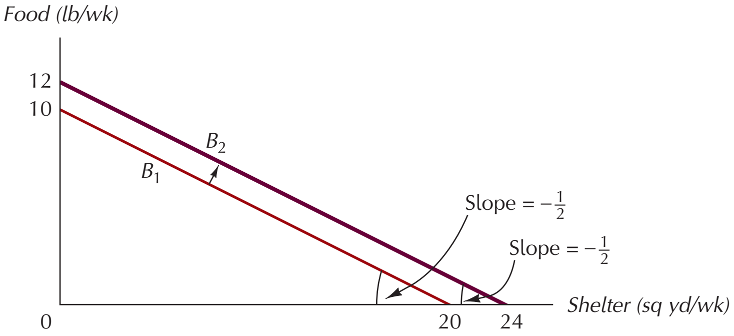


3.3. *Food (lb/wk)*



*Problems marked with an asterisk (*) are more difficult.

3.4. *Food (lb/wk)*



3.5. The budget constraint for a residential consumer with Gigawatt Power Company would be kinked outward, as the initial rate for the first 1000 kWh/mo is lower. For power consumption X up to 1000 kWh/mo, the budget constraint has a slope of the lower rate \$0.05/kWh.

$$Y = 400 - 0.05X \qquad 0 \leq X \leq 1000 \text{ kWh/mo}$$

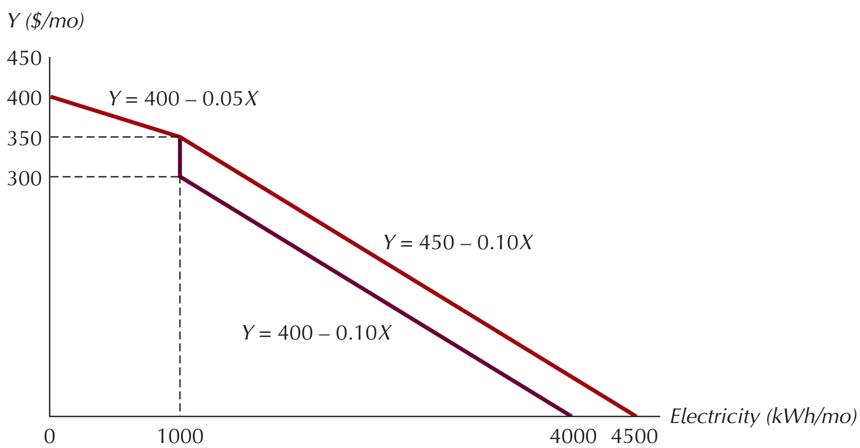
For power consumption X above 1000 kWh/mo, the budget constraint has a slope of the higher rate \$0.10/kWh.

$$Y = 450 - 0.10X \qquad X > 1000 \text{ kWh/mo}$$

The kink occurs when $X = 1000$ kWh/mo, where the level of consumption of other goods is $Y = 400 - 0.05X = 400 - 50 = 350$, or equivalently, $Y = 450 - 0.10X = 450 - 100 = 350$. If the rate were instead \$0.10/kWh for all kWh that exceeded 1000 kWh/mo, then the budget constraint for $X > 1000$ kWh/mo would be

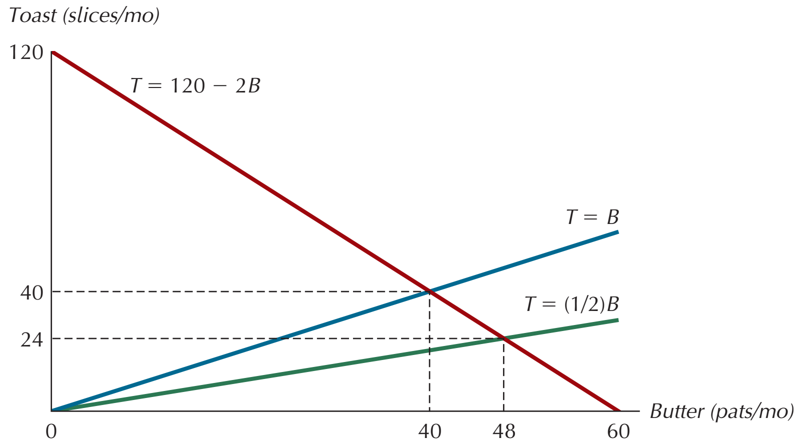
$$Y = 400 - 0.10X \qquad X > 1000 \text{ kWh/mo}$$

and would have a discrete jump from $Y = 350$ to $Y = 300$ at $X = 1000$ kWh/mo.



3.6. At bundle A, the consumer is willing to give up 1 lb of food to get an additional square yard of shelter. But at the market prices it is necessary to give up only $\frac{1}{2}$ lb of food to buy an additional square yard of shelter. It follows that the consumer will be better off than at bundle A if he buys 1 lb less of food and 2 sq yd more of shelter.

3.7. Albert's budget constraint is $T = 120 - 2B$. Albert's initial preferences are for two pats of butter for every slice of toast $B = 2T$. Substituting this equation into his budget constraint yields $T = 120 - 4T$, or $5T = 120$, which solves for $T = 24$ slices of toast, and thus $B = 48$ pats of butter each month. Albert's new preferences are for one pat of butter for every slice of toast $B = T$. Substituting this equation into his budget constraint yields $T = 120 - 2T$, or $3T = 120$, which solves for $T = 40$ slices of toast, and thus $B = 40$ pats of butter each month. Not only has Albert cut the fat, but he is consuming more fiber too!



APPENDIX

3

THE UTILITY FUNCTION APPROACH TO THE CONSUMER BUDGETING PROBLEM

THE UTILITY FUNCTION APPROACH TO CONSUMER CHOICE

Finding the highest attainable indifference curve on a budget constraint is just one way that economists have analyzed the consumer choice problem. For many applications, a second approach is also useful. In this approach we represent the consumer's preferences not with an indifference map but with a *utility function*.

For each possible bundle of goods, a utility function yields a number that represents the amount of satisfaction provided by that bundle. Suppose, for example, that Tom consumes only food and shelter and that his utility function is given by $U(F, S) = FS$, where F denotes the number of pounds of food, S the number of square yards of shelter he consumes per week, and U his satisfaction, measured in "utils" per week.[1] If $F = 4$ lb/wk and $S = 3$ sq yd/wk, Tom will receive 12 utils/wk of utility, just as he would if he consumed 3 lb/wk of food and 4 sq yd/wk of shelter. By contrast, if he consumed 8 lb/wk of food and 6 sq yd/wk of shelter, he would receive 48 utils/wk.

The utility function is analogous to an indifference map in that both provide a complete description of the consumer's preferences. In the indifference curve framework, we can rank any two bundles by seeing which one lies on a higher indifference curve. In the utility-function framework, we can compare any two bundles by seeing which one yields a greater number of utils. Indeed, as the following example illustrates, it is straightforward to use the utility function to construct an indifference map.

EXAMPLE A.3.1

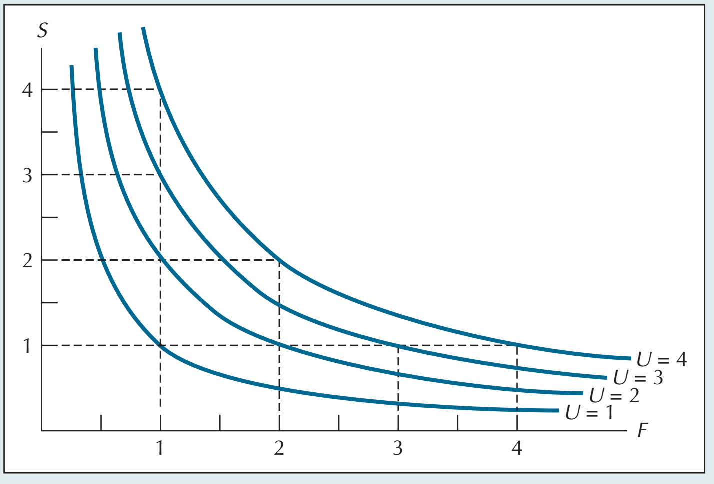
If Tom's utility function is given by $U(F, S) = FS$ graph the indifference curves that correspond to 1, 2, 3, and 4 utils, respectively.

In the language of utility functions, an indifference curve is all combinations of F and S that yield the same level of utility—the same number of utils. Suppose we look at the indifference curve that corresponds to 1 unit of utility—that is, the combinations of bundles for which $FS = 1$. Solving this equation for S, we have

$$S = \frac{1}{F}, \tag{A.3.1}$$

which is the indifference curve labeled $U = 1$ in Figure A.3.1. The indifference curve that corresponds to 2 units of utility is generated by solving $FS = 2$ to get $S = 2/F$, and it is shown by the curve labeled $U = 2$ in Figure A.3.1. In similar fashion, we generate the indifference curves to $U = 3$ and $U = 4$, which are correspondingly labeled in the diagram. More generally, we get the indifference curve corresponding to a utility level of U_0 by solving $FS = U_0$ to get $S = U_0/F$.

FIGURE A.3.1
Indifference Curves for the Utility Function $U = FS$
To get the indifference curve that corresponds to all bundles that yield a utility level of U_0, set $FS = U_0$ and solve for S to get $S = U_0/F$.



In the indifference curve framework, the best attainable bundle is the bundle on the budget constraint that lies on the highest indifference curve. Analogously, the best attainable bundle in the utility-function framework is the bundle on the budget constraint that provides the highest level of utility. In the indifference curve framework, the best attainable bundle occurs at a point of tangency between an indifference

[1]The term "utils" represents an arbitrary unit. As we will see, what is important for consumer choice is not the actual number of utils various bundles provide, but the rankings of the bundles based on their associated utilities.

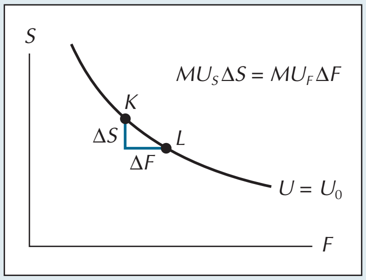


FIGURE A.3.2
Utility Along an Indifference Curve Remains Constant
In moving from K to L, the loss in utility from having less shelter, $MU_s\Delta S$, is exactly offset by the gain in utility from having more food, $MU_F\Delta_F$.

curve and the budget constraint. At the optimal bundle, the slope of the indifference curve, or MRS, equals the slope of the budget constraint. Suppose food and shelter are again our two goods, and P_F and P_S are their respective prices. If $\Delta S/\Delta F$ denotes the slope of the highest attainable indifference curve at the optimal bundle, the tangency condition says that $\Delta S/\Delta F = P_F/P_S$. What is the analogous condition in the utility-function framework?

To answer this question, we must introduce the concept of *marginal utility* (the marginal utility of a good is the rate at which total utility changes with consumption of the good), which is the rate at which total utility changes as the quantities of food and shelter change. More specifically, let MU_F denote the number of additional utils we get for each additional unit of food and MU_S denote the number of additional utils we get for each additional unit of shelter. In Figure A.3.2, note that bundle K has ΔF fewer units of food and ΔS more units of shelter than bundle L. Thus, if we move from bundle K to bundle L, we gain $MU_F\Delta F$ utils from having more food, but we lose $MU_S\Delta S$ utils from having less shelter.

Because K and L both lie on the same indifference curve, we know that both bundles provide the same level of utility. Thus the utility we lose from having less shelter must be exactly offset by the utility we gain from having more food. This tells us that

$$MU_F\Delta F = MU_S\Delta S. \tag{A.3.2}$$

Cross-multiplying terms in Equation A.3.2 gives

$$\frac{MU_F}{MU_S} = \frac{\Delta S}{\Delta F}. \tag{A.3.3}$$

Suppose that the optimal bundle lies between K and L, which are very close together, so that ΔF and ΔS are both very small. As K and L move closer to the optimal bundle, the ratio $\Delta S/\Delta F$ becomes equal to the slope of the indifference curve at that bundle, which Equation A.3.3 tells us is equal to the ratio of the marginal utilities of the two goods. And since the slope of the indifference curve at the optimal bundle is the same as that of the budget constraint, the following condition must also hold for the optimal bundle:

$$\frac{MU_F}{MU_S} = \frac{P_F}{P_S}. \tag{A.3.4}$$

Equation A.3.4 is the condition in the utility-function framework that is analogous to the $MRS = P_F/P_S$ condition in the indifference curve framework.

If we cross-multiply terms in Equation A.3.4, we get an equivalent condition that has a very straightforward intuitive interpretation:

$$\frac{MU_F}{P_F} = \frac{MU_S}{P_S}. \tag{A.3.5}$$

In words, Equation A.3.5 tells us that the ratio of marginal utility to price must be the same for all goods at the optimal bundle. The following examples illustrate why this condition must be satisfied if the consumer has allocated his budget optimally.

EXAMPLE A.3.2

Suppose that the marginal utility of the last dollar John spends on food is greater than the marginal utility of the last dollar he spends on shelter. For example, suppose the prices of food and shelter are $1/lb and $2/sq yd, respectively, and that the corresponding marginal utilities are 6 and 4. Show that John cannot possibly be maximizing his utility.

If John bought 1 sq yd/wk less shelter, he would save $2/wk and would lose 4 utils. But this would enable him to buy 2 lb/wk more food, which would add 12 utils, for a net gain of 8 utils.

Abstracting from the special case of corner solutions, a necessary condition for optimal budget allocation is that the last dollar spent on each commodity yield the same increment in utility.

EXAMPLE A.3.3

Mary has a weekly allowance of $10, all of which she spends on newspapers (N) and magazines (M), whose respective prices are $1 and $2. Her utility from these purchases is given by $U(N) + V(M)$. If the values of $U(N)$ and $V(M)$ are as shown in the table, is Mary a utility maximizer if she buys 4 magazines and 2 newspapers each week? If not, how should she reallocate her allowance?

N	*U*(*N*)	*M*	*V*(*M*)
0	0	0	0
1	12	1	20
2	20	2	32
3	26	3	40
4	30	4	44
5	32	5	46

For Mary to be a utility maximizer, extra utility per dollar must be the same for both the last newspaper and the last magazine she purchased. But since the second newspaper provided 8 additional utils per dollar spent, which is four times the 2 utils per dollar she got from the fourth magazine (4 extra utils at a cost of $2), Mary is not a utility maximizer.

N	*U*(*N*)	*MU*(*N*)	*MU*(*N*)/*PN*	*M*	*U*(*M*)	*MU*(*M*)	*MU*(*M*)/*PM*
0	0			0	0		
		12	12			20	10
1	12			1	20		
		8	8			12	6
2	20			2	32		
		6	6			8	4
3	26			3	40		
		4	4			4	2
4	30			4	44		
		2	2			2	1
5	32			5	46		

To see clearly how she should reallocate her purchases, let us rewrite the table to include the relevant information on marginal utilities. From this table, we see that there are several bundles for which $MU(N)/P_N = MU(M)/P_M$—namely, 3 newspapers and 2 magazines; or 4 newspapers and 3 magazines; or 5 newspapers and 4 magazines. The last of these bundles yields the highest total utility but costs $13, and is hence beyond Mary's budget constraint. The first, which costs only $7, is affordable, but so is the second, which costs exactly $10 and yields higher total utility than the first. With 4 newspapers and 3 magazines, Mary gets 4 utils per dollar from her last purchase in each category. Her total utility is 70 utils, which is 6 more than she got from the original bundle.

In Example A.3.3, note that if all Mary's utility values were doubled, or cut by half, she would still do best to buy 4 newspapers and 3 magazines each week. This illustrates the claim that consumer choice depends not on the absolute number of utils associated with different bundles, but instead on the ordinal ranking of the utility levels associated with different bundles. If we double all the utils associated with various bundles, or cut them by half, the ordinal ranking of the bundles will be preserved, and thus the optimal bundle will remain the same. This will also be true if we take the logarithm of the utility function, the square root of it, or add 5 to it, or transform it in any other way that preserves the ordinal ranking of different bundles.

CARDINAL VERSUS ORDINAL UTILITY

In our discussion about how to represent consumer preferences, we assumed that people are able to rank each possible bundle in order of preference. This is called the *ordinal utility* approach to the consumer budgeting problem. It does not require that people be able to make quantitative statements about how much they like various bundles. Thus it assumes that a consumer will always be able to say whether he prefers *A* to *B*, but that he may not be able to make such statements as "*A* is 6.43 times as good as *B*."

In the nineteenth century, economists commonly assumed that people could make such statements. Today we call theirs the *cardinal utility* approach to the consumer choice problem. In the two-good case, it assumes that the satisfaction provided by any bundle can be assigned a numerical, or cardinal, value by a utility function of the form

$$U = U(X, Y), \tag{A.3.6}$$

where X and Y are the two goods.

In three dimensions, the graph of such a utility function will look something like the one shown in Figure A.3.3. It resembles a mountain, but because of the more-is-better assumption, it is a mountain without a summit. The value on the U axis measures the height of the mountain, which continues to increase the more we have of X or Y.

Suppose in Figure A.3.3 we were to fix utility at some constant amount, say, U_0. That is, suppose we cut the utility mountain with a plane parallel to the XY plane, U_0 units above it. The line labeled JK in Figure A.3.3 represents the intersection of that plane and the surface of the utility mountain. All the bundles of goods that lie on JK provide a utility level of U_0. If we then project the line JK downward onto the XY plane, we have what amounts to the U_0 indifference curve, shown in Figure A.3.4.

Suppose we then intersect the utility mountain with another plane, this time U_1 units above the XY plane. In Figure A.3.3, this second plane intersects the utility mountain along the line labeled LN. It represents the set of all bundles that confer the utility level U_1. Projecting LN down onto the XY plane, we thus get the indifference

FIGURE A.3.3
A Three-Dimensional Utility Surface

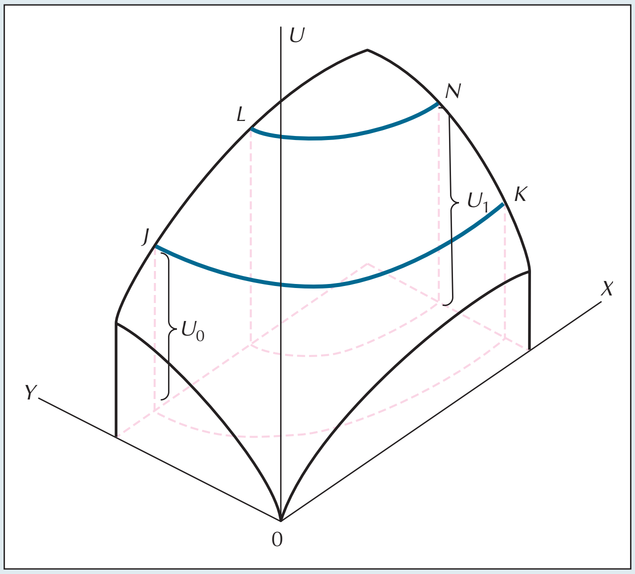
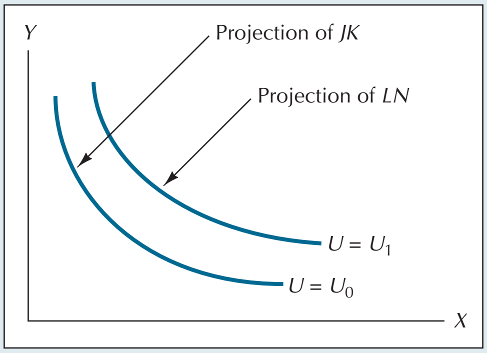


FIGURE A.3.4
Indifference Curves as Projections



curve labeled U_1 in Figure A.3.4. In like fashion, we can generate an entire indifference map corresponding to the cardinal utility function $U(X, Y)$.

Thus we see that it is possible to start with any cardinal utility function and end up with a unique indifference map. *But it is not possible to go in the other direction!* That is, it is not possible to start with an indifference map and work backward to a unique cardinal utility function. The reason is that there will always be infinitely many such utility functions that give rise to precisely the same indifference map.

To see why, just imagine that we took the utility function in Equation A.3.4 and doubled it, so that utility is now given by $V = 2U(X, Y)$. When we graph V as a function of X and Y, the shape of the resulting utility mountain will be much the same as before. The difference will be that the altitude at any X, Y point will be twice what it was before. If we pass a plane $2U_0$ units above the XY plane, it would intersect the new utility mountain in precisely the same manner as the plane U_0 units high did originally. If we then project the resulting intersection down onto the XY plane, it will coincide perfectly with the original U_0 indifference curve.

All we do when we multiply (divide, add to, or subtract from) a cardinal utility function is to relabel the indifference curves to which it gives rise. Indeed, we can make an even more general statement: If $U(X, Y)$ is any cardinal utility function and if V is any increasing function, then $U = U(X, Y)$ and $V = V[U(X, Y)]$ will give rise to precisely the same indifference maps. The special property of an increasing function is that it preserves the rank ordering of the values of the original function. That is, if $U(X_1, Y_1) > U(X_2, Y_2)$, the fact that V is an increasing function assures that $V[U(X_1, Y_1)]$ will be greater than $V[U(X_2, Y_2)]$. And as long as that requirement is met, the two functions will give rise to exactly the same indifference curves.

The concept of the indifference map was first discussed by Francis Edgeworth, who derived it from a cardinal utility function in the manner described above. It took the combined insights of Vilfredo Pareto, Irving Fisher, and John Hicks to establish that Edgeworth's apparatus was not uniquely dependent on a supporting cardinal utility function. As we have seen, the only aspect of a consumer's preferences that matters in the standard budget allocation problem is the shape and location of his indifference curves. Consumer choice turns out to be completely independent of the labels we assign to these indifference curves, provided only that higher curves correspond to higher levels of utility.

Modern economists prefer the ordinal approach because it rests on much weaker assumptions than the cardinal approach. That is, it is much easier to imagine that people can rank different bundles than to suppose that they can make precise quantitative statements about how much satisfaction each provides.

GENERATING INDIFFERENCE CURVES ALGEBRAICALLY

Even if we assume that consumers have only ordinal preference rankings, it will often be convenient to represent those preferences with a cardinal utility index. The advantage is that this procedure provides a compact algebraic way of summarizing all the information that is implicit in the graphical representation of preferences, as we saw in Example A.3.1.

Consider another illustration, this time with a utility function that generates straight-line indifference curves: $U(X, Y) = (\frac{2}{3})X + 2Y$. The bundles of X and Y that yield a utility level of U_0 are again found by solving $U(X, Y) = U_0$ for Y. This time we get $Y = (U_0/2) - (\frac{1}{3})X$. The indifference curves corresponding to $U = 1$, $U = 2$, and $U = 3$ are shown in Figure A.3.5. Note that they are all linear, which tells us that this particular utility function describes a preference ordering in which X and Y are perfect substitutes.

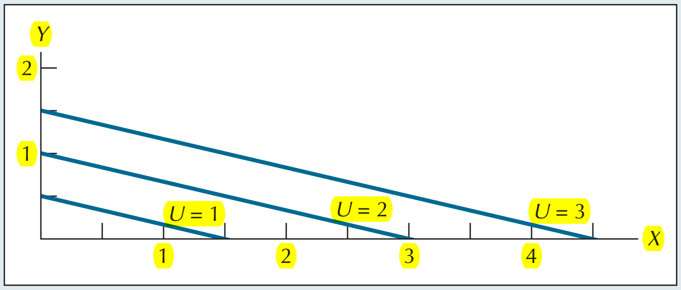


FIGURE A.3.5
Indifference Curves for the Utility Function $U(X, Y) = (\frac{2}{3})X + 2Y$
The indifference curve that corresponds to all bundles yielding a utility level of U_0 is given by $Y = (U_0/2) - (\frac{1}{3})X$.

USING CALCULUS TO MAXIMIZE UTILITY

Students who have had calculus are able to solve the consumer's budget allocation problem without direct recourse to the geometry of indifference maps. Let $U(X, Y)$ be the consumer's utility function; and suppose M, P_X, and P_Y denote income, the price of X, and the price of Y, respectively. Formally, the consumer's allocation problem can be stated as follows:

$$\underset{X,\,Y}{\text{Maximize}}\ U(X, Y) \text{ subject to } P_X X + P_Y Y = M. \tag{A.3.7}$$

The appearance of the terms X and Y below the "maximize" expression indicates that these are the variables whose values the consumer must choose. The price and income values in the budget constraint are given in advance.

THE METHOD OF LAGRANGIAN MULTIPLIERS

As noted earlier, the function $U(X, Y)$ itself has no maximum; it simply keeps on increasing with increases in X or Y. The maximization problem defined in Equation A.3.7 is called a *constrained maximization problem,* which means we want to find the values of X and Y that produce the highest value of U *subject to the constraint that the consumer spend only as much as his income*. We will examine two different approaches to this problem.

One way of making sure that the budget constraint is satisfied is to use the so-called method of *Lagrangian multipliers*. In this method, we begin by transforming the constrained maximization problem in Equation A.3.7 into the following unconstrained maximization problem:

$$\underset{X,\,Y,\,\lambda}{\text{Maximize}}\ \pounds = U(X, Y) - \lambda(P_X X + P_Y Y - M). \tag{A.3.8}$$

The term λ is called a Lagrangian multiplier, and its role is to assure that the budget constraint is satisfied. (How it does this will become clear in a moment.) The first-order conditions for a maximum of £ are obtained by taking the first partial derivatives of £ with respect to X, Y, and λ and setting them equal to zero:

$$\frac{\partial \pounds}{\partial X} = \frac{\partial U}{\partial X} - \lambda P_X = 0, \tag{A.3.9}$$

$$\frac{\partial \pounds}{\partial Y} = \frac{\partial U}{\partial Y} - \lambda P_Y = 0, \tag{A.3.10}$$

and

$$\frac{\partial \pounds}{\partial \lambda} = M - P_X X - P_Y Y = 0 \tag{A.3.11}$$

The next step is to solve Equations A.3.9–A.3.11 for X, Y, and λ. The solutions for X and Y are the only ones we really care about here. The role of the equilibrium value of λ is to guarantee that the budget constraint is satisfied. Note in Equation A.3.11 that setting the first partial derivative of £ with respect to λ equal to zero guarantees this result.

Specific solutions for the utility-maximizing values of X and Y require a specific functional form for the utility function. We will work through an illustrative example in a moment. But first note that an interesting characteristic of the optimal X and Y values can be obtained by dividing Equation A.3.9 by Equation A.3.10 to get

$$\frac{\partial U/\partial X}{\partial U/\partial Y} = \frac{\lambda P_X}{\lambda P_Y} = \frac{P_X}{P_Y}. \tag{A.3.12}$$

Equation A.3.12 is the utility function analog to Equation 3.3 from the text, which says that the optimal values of X and Y must satisfy MRS $= P_X/P_Y$. The terms $\partial U/\partial X$ and $\partial U/\partial Y$ from Equation A.3.12 are called the *marginal utility of* X and the *marginal utility of* Y, respectively. In words, the marginal utility of a good is the extra utility obtained per additional unit of the good consumed. Equation A.3.12 tells us that the ratio of these marginal utilities is simply the marginal rate of substitution of Y for X.

If we rearrange Equation A.3.12 in the form

$$\frac{\partial U/\partial X}{P_X} = \frac{\partial U/\partial Y}{P_Y}, \tag{A.3.13}$$

another interesting property of the optimal values of X and Y emerges. In words, the left-hand side of Equation A.3.13 may be interpreted as the extra utility gained from the last dollar spent on X. Equation A.3.13 is thus the calculus derivation of the result shown earlier in Equation A.3.5.

An Example

To illustrate the Lagrangian method, suppose that $U(X, Y) = XY$ and that $M = 40$, $P_X = 4$, and $P_Y = 2$. Our unconstrained maximization problem would then be written as

$$\underset{X,\,Y,\,\lambda}{\text{Maximize}}\ \pounds = XY - \lambda(4X + 2Y - 40) \tag{A.3.14}$$

The first-order conditions for a maximum of £ are given by

$$\frac{\partial \pounds}{\partial X} = \frac{\partial (XY)}{\partial X} - 4\lambda = Y - 4\lambda = 0, \tag{A.3.15}$$

$$\frac{\partial \pounds}{\partial Y} = \frac{\partial (XY)}{\partial Y} - 2\lambda = X - 2\lambda = 0, \tag{A.3.16}$$

and

$$\frac{\partial \pounds}{\partial \lambda} = 40 - 4X - 2Y = 0. \tag{A.3.17}$$

Dividing Equation A.3.15 by Equation A.3.16 and solving for Y, we get $Y = 2X$; substituting this result into Equation A.3.17 and solving for X, we get $X = 5$, which in turn yields $Y = 2X = 10$. Thus (5, 10) is the utility-maximizing bundle.[2]

[2]Assuming that the second-order conditions for a local maximum are also met.

AN ALTERNATIVE METHOD

There is an alternative way of making sure that the budget constraint is satisfied, one that involves less cumbersome notation than the Lagrangian approach. In this alternative method, we simply solve the budget constraint for Y in terms of X and substitute the result wherever Y appears in the utility function. Utility then becomes a function of X alone, and we can *maximize* it by taking its first derivative with respect to X and equating that to zero.[3] The value of X that solves that equation is the optimal value of X, which can then be substituted back into the budget constraint to find the optimal value of Y.

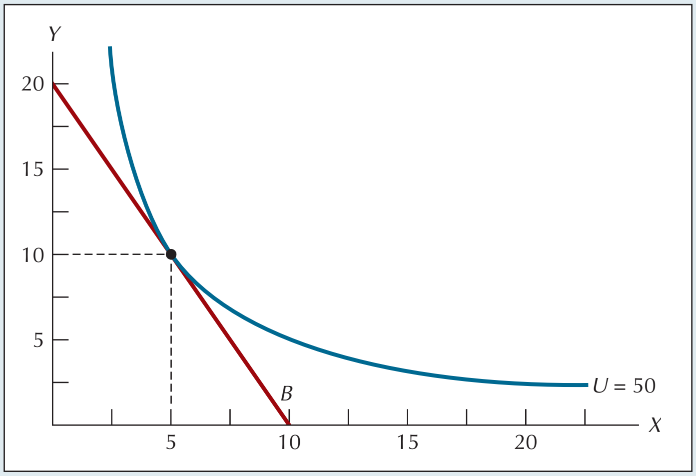
To illustrate, again suppose that $U(X, Y) = XY$, with $M = 40$, $P_X = 4$, and $P_Y = 2$. The budget constraint is then $4X + 2Y = 40$, which solves for $Y = 20 - 2X$. Substituting this expression back into the utility function, we have $U(XY) = X(20 - 2X) = 20X - 2X^2$. Taking the first derivative of U with respect to X and equating the result to zero, we have

$$\frac{dU}{dX} = 20 - 4X = 0. \tag{A.3.18}$$

which solves for $X = 5$. Plugging this value of X back into the budget constraint, we discover that the optimal value of Y is 10. So the optimal bundle is again (5, 10), just as we found using the Lagrangian approach. For these optimal values of X and Y, the consumer will obtain (5)(10) = 50 units of utility.

Both algebraic approaches to the budget allocation problem yield precisely the same result as the graphical approach described in the text. Note in Figure A.3.6 that the $U = 50$ indifference curve is tangent to the budget constraint at the bundle (5, 10).

FIGURE A.3.6
The Optimal Bundle when $U = XY$, $P_x = 4$, $P_y = 2$, and $M = 40$.



A SIMPLIFYING TECHNIQUE

Suppose our constrained maximization problem is of the general form

$$\underset{X,\,Y}{\text{Maximize}}\ U(X, Y) \text{ subject to } P_X X + P_Y Y = M. \tag{A.3.19}$$

[3]Here, the second-order condition for a local maximum is that $d^2U/dX^2 < 0$.

If (X^*, Y^*) is the optimum bundle for this maximization problem, then we know it will also be the optimum bundle for the utility function $V[U(X, Y)]$, where V is any increasing function.[4] This property often enables us to transform a computationally difficult maximization problem into a simple one. By way of illustration, consider the following example:

$$\underset{X,\,Y}{\text{Maximize}}\ X^{1/3}Y^{2/3} \text{ subject to } 4X + 2Y = 24. \tag{A.3.20}$$

First note what happens when we proceed with the untransformed utility function given in Equation A.3.20. Solving the budget constraint for $Y = 12 - 2X$ and substituting back into the utility function, we have $U = X^{1/3}(12 - 2X)^{2/3}$. Calculating dU/dX *is* a bit tedious in this case, but if we carry out each step carefully, we get the following first-order condition:

$$\frac{dU}{dX} = (\tfrac{1}{3})X^{-2/3}(12 - 2X)^{2/3} + X^{1/3}(\tfrac{2}{3})(12 - 2X)^{-1/3}(-2) = 0, \tag{A.3.21}$$

which, after a little more tedious rearrangement, solves for $X = 2$. And from the budget constraint we then get $Y = 8$.

Now suppose we transform the utility function by taking its logarithm:

$$V = \ln[U(X, Y)] = \ln(X^{1/3}Y^{2/3}) = (\tfrac{1}{3})\ln X + (\tfrac{2}{3})\ln Y. \tag{A.3.22}$$

Since the logarithm is an increasing function, when we maximize V subject to the budget constraint, we will get the same answer we got using U. The advantage of the logarithmic transformation here is that the derivative of V is much easier to calculate than the derivative of U. Again, solving the budget constraint for $Y = 12 - 2X$ and substituting the result into V, we have $V = (\tfrac{1}{3})\ln X + (\tfrac{2}{3})\ln(12 - 2X)$. This time the first-order condition follows almost without effort:

$$\frac{dV}{dX} = \frac{\tfrac{1}{3}}{X} - \frac{2(\tfrac{2}{3})}{12 - 2X} = 0, \tag{A.3.23}$$

which solves easily for $X = 2$. Plugging $X = 2$ back into the budget constraint, we again get $Y = 8$.

The best transformation to make will naturally depend on the particular utility function you start with. The logarithmic transformation greatly simplified matters in the example above, but will not necessarily be helpful for other forms of U.

PROBLEMS

1. Tom spends all his \$100 weekly income on two goods, X and Y. His utility function is given by $U(X, Y) = XY$. If $P_X = 4$ and $P_Y = 10$, how much of each good should he buy?
2. Same as Problem 1, except now Tom's utility function is given by $U(X, Y) = X^{1/2}Y^{1/2}$.
3. Note the relationship between your answers in Problems 1 and 2. What accounts for this relationship?
4. Sue consumes only two goods, food and clothing. The marginal utility of the last dollar she spends on food is 12, and the marginal utility of the last dollar she spends on clothing is 9. The price of food is \$1.20/unit, and the price of clothing is \$0.90/unit. Is Sue maximizing her utility?

[4] Again, an increasing function is one for which $V(X_1) > V(X_2)$ whenever $X_1 > X_2$.

5. Albert has a weekly allowance of \$17, all of which he spends on used CDs (C) and movie rentals (M), whose respective prices are \$4 and \$3. His utility from these purchases is given by $U(C) + V(M)$. If the values of $U(C)$ and $V(M)$ are as shown in the table, is Albert a utility maximizer if he buys 2 CDs and rents 3 movies each week? If not, how should be reallocate his allowance?

C	$U(C)$	M	$V(M)$
0	0	0	0
1	12	1	21
2	20	2	33
3	24	3	39
4	28	4	42

CHAPTER

4

INDIVIDUAL AND MARKET DEMAND

A pound of salt costs 30 cents at the grocery store where I shop. My family and I use the same amount of salt at that price as we would if it instead sold for 5 cents/lb or even $10/lb. I also consume about the same amount of salt now as I did as a graduate student, when my income was less than one-tenth as large as it is today.

Salt is an unusual case. The amounts we buy of many other goods are much more sensitive to prices and incomes. Sometimes, for example, my family and I consider spending a sabbatical year in New York City, where housing prices are more than four times what they are in Ithaca. If we ever do go there, we will probably live in an apartment that is less than half the size of our current house.

CHAPTER PREVIEW

Viewed within the framework of the rational choice model, my behavior with respect to salt and housing purchases is perfectly intelligible. Our focus in this chapter is to use the tools from Chapter 3 to shed additional light on why, exactly, the responses of various purchase decisions to changes in income and price differ so widely. In Chapter 3, we saw how changes in prices and incomes affect the budget constraint. Here we will see how changes in the budget constraint affect actual purchase decisions. More specifically, we will use the rational choice model to generate an individual consumer's demand curve for a product and employ our model to construct a relationship that summarizes how individual demands vary with income.

We will see how the total effect of a price change can be decomposed into two separate effects: (1) the substitution effect, which denotes the change in the quantity demanded that results because the price change alters the attractiveness of substitute goods, and (2) the income effect, which denotes the change in quantity demanded that results from the change in purchasing power caused by the price change.

Next we will show how individual demand curves can be added to yield the demand curve for the market as a whole. A central analytical concept we will develop in this chapter is the price elasticity of demand, a measure of the responsiveness of purchase decisions to small changes in price. We will also consider the income elasticity of demand, a measure of the responsiveness of purchase decisions to small changes in income. And we will see that, for some goods, the distribution of income, not just its average value, is an important determinant of market demand.

A final elasticity concept in this chapter is the cross-price elasticity of demand, which is a measure of the responsiveness of the quantity demanded of one good to small changes in the prices of another good. Cross-price elasticity is the criterion by which pairs of goods are classified as being either substitutes or complements.

These analytical constructs provide a deeper understanding of a variety of market behaviors as well as a stronger foundation for intelligent decision and policy analysis.

THE EFFECTS OF CHANGES IN PRICE

THE PRICE-CONSUMPTION CURVE

Recall from Chapter 2 that a market demand curve tells how much of a good the market as a whole wants to purchase at various prices. Suppose we want to generate a demand schedule for a good—say, shelter—not for the market as a whole but for only a single consumer. Holding income, preferences, and the prices of all other goods constant, how will a change in the price of shelter affect the amount of shelter the consumer buys? To answer this question, we begin with this consumer's indifference map, plotting shelter on the horizontal axis and the composite good Y on the vertical axis. Suppose the consumer's income is \$120/wk, and the price of the composite good is again \$1 per unit. The vertical intercept of her budget constraint will then be 120. The horizontal intercept will be $120/P_S$, where P_S denotes the price of shelter. Figure 4.1 shows four budget constraints that correspond to four different prices of shelter, namely, \$24/sq yd, \$12/sq yd, \$6/sq yd, and \$4/sq yd. The corresponding best affordable bundles contain 2.5, 7, 15, and 20 sq yd/wk of shelter, respectively. If we were to repeat this procedure for indefinitely many prices, the resulting points of tangency would trace out the line labeled PCC in Figure 4.1. This line is called the **price-consumption curve**, or **PCC.**

price-consumption curve (PCC) holding income and the price of *Y* constant, the PCC for a good *X* is the set of optimal bundles traced on an indifference map as the price of *X* varies.

For the particular consumer whose indifference map is shown in Figure 4.1, note that each time the price of shelter falls, the budget constraint rotates outward, enabling the consumer to purchase not only more shelter but more of the composite good as well. And each time the price of shelter falls, this consumer chooses a bundle that contains more shelter than in the bundle chosen previously. Note, however, that the amount of money spent on the composite good may either rise or fall when the price of shelter falls. Thus, the amount spent on other goods falls when

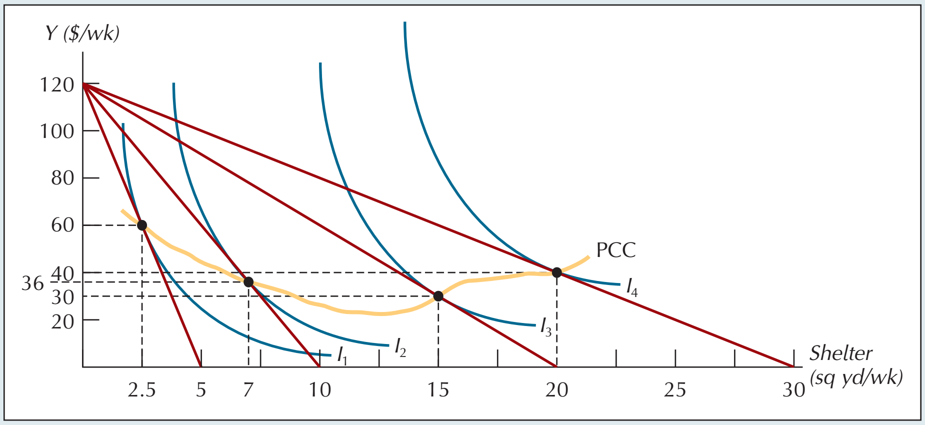


FIGURE 4.1
The Price-Consumption Curve
Holding income and the price of *Y* fixed, we vary the price of shelter. The set of optimal bundles traced out by the various budget lines is called the price-consumption curve, or PCC.

the price of shelter falls from \$24/sq yd to \$12/sq yd but rises when the price of shelter falls from \$6/sq yd to \$4/sq yd. Below, we will see why this is a relatively common purchase pattern.

THE INDIVIDUAL CONSUMER'S DEMAND CURVE

An individual consumer's demand curve is like the market demand curve in that it tells the quantities the consumer will buy at various prices. All the information we need to construct the individual demand curve is contained in the price-consumption curve. The first step in going from the PCC to the individual demand curve is to record the relevant price-quantity combinations from the PCC in Figure 4.1, as in Table 4.1. (Recall from Chapter 3 that the price of shelter along any budget constraint is given by income divided by the horizontal intercept of that budget constraint.)

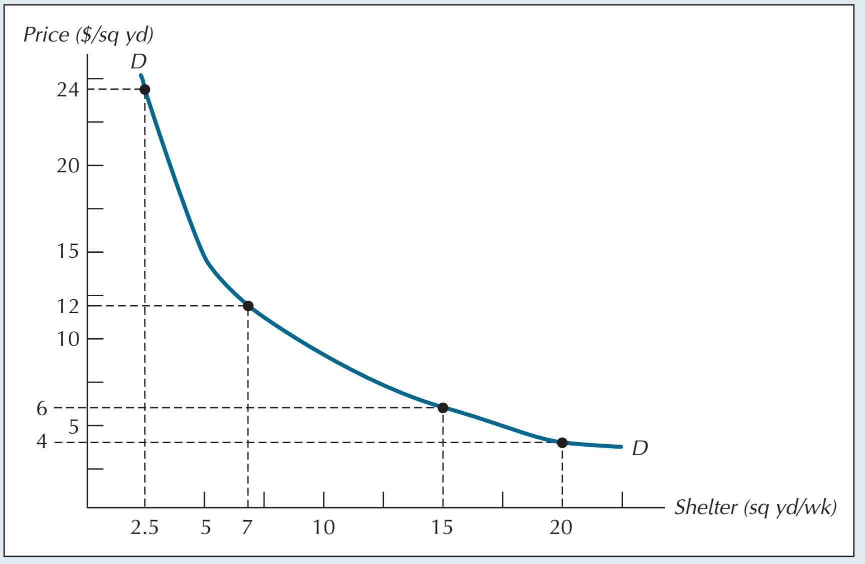
TABLE 4.1
A Demand Schedule

Price of shelter ($/sq yd)	Quantity of shelter demanded (sq yd/wk)
24	2.5
12	7
6	15
4	20

To derive the individual's demand curve for shelter from the PCC in Figure 4.1, begin by recording the quantities of shelter that correspond to the shelter prices on each budget constraint.

The next step is to plot the price-quantity pairs from Table 4.1, with the price of shelter on the vertical axis and the quantity of shelter on the horizontal. With sufficiently many price-quantity pairs, we generate the individual's demand curve, shown as *DD* in Figure 4.2. Note carefully that in moving from the PCC to the individual demand curve, we are moving from a graph in which both axes measure quantities to one in which price is plotted against quantity.

FIGURE 4.2
An Individual Consumer's Demand Curve
Like the market demand curve, the individual demand curve is a relationship that tells how much the consumer wants to purchase at different prices.



THE EFFECTS OF CHANGES IN INCOME

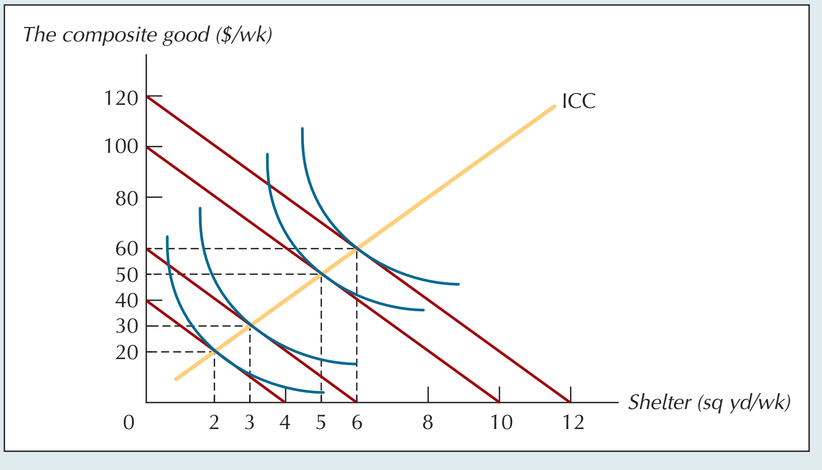
THE INCOME-CONSUMPTION CURVE

income-consumption curve (ICC) holding the prices of *X* and *Y* constant, the ICC for a good *X* is the set of optimal bundles traced on an indifference map as income varies.

The PCC and the individual demand schedule are two different ways of summarizing how a consumer's purchase decisions respond to variations in prices. Analogous devices exist to summarize responses to variations in income. The income analog to the PCC is the **income-consumption curve,** or **ICC.** To generate the PCC for shelter, we held preferences, income, and the price of the composite good constant while tracing out the effects of a change in the price of shelter. In the case of the ICC, we hold preferences and relative prices constant and trace out the effects of changes in income.

In Figure 4.3, for example, we hold the price of the composite good constant at $1 per unit and the price of shelter constant at $10/sq yd and examine what happens when income takes the values $40/wk, $60/wk, $100/wk, and $120/wk. Recall from Chapter 3 that a change in income shifts the budget constraint parallel to itself. As before, to each budget there corresponds a best affordable bundle. The set

FIGURE 4.3
An Income-Consumption Curve
As income increases, the budget constraint moves outward. Holding preferences and relative prices constant, the ICC traces out how these changes in income affect consumption. It is the set of all tangencies as the budget line moves outward.



of best affordable bundles is denoted as ICC in Figure 4.3. For the consumer whose indifference map is shown, the ICC happens to be a straight line, but this need not always be the case.

THE ENGEL CURVE

The analog to the individual demand curve in the income domain is the individual **Engel curve.** It takes the quantities of shelter demanded from the ICC and plots them against the corresponding values of income. Table 4.2 shows the income-shelter pairs for the four budget constraints shown in Figure 4.3. If we were to plot indefinitely many income-consumption pairs for the consumer shown in Figure 4.3, we would trace out the line *EE* shown in Figure 4.4. The Engel curve shown in Figure 4.4 happens to be linear, but Engel curves in general need not be.

Engel curve a curve that plots the relationship between the quantity of *X* consumed and income.

TABLE 4.2
Income and Quantity of Shelter Demanded

Income ($/wk)	Quantity of shelter demanded (sq yd/wk)
40	2
60	3
100	5
120	6

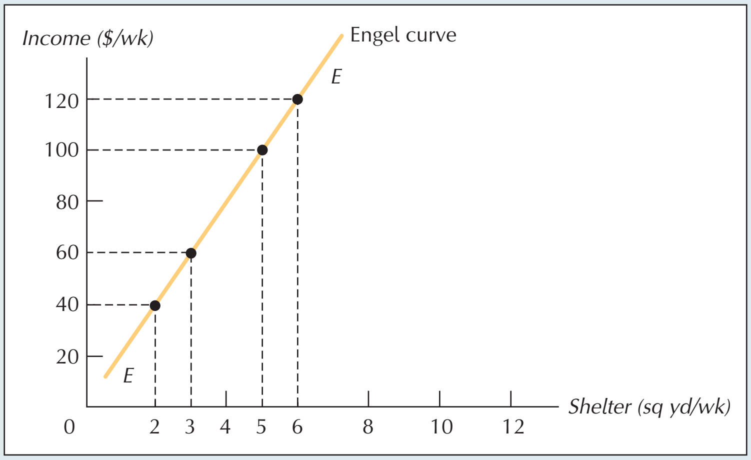
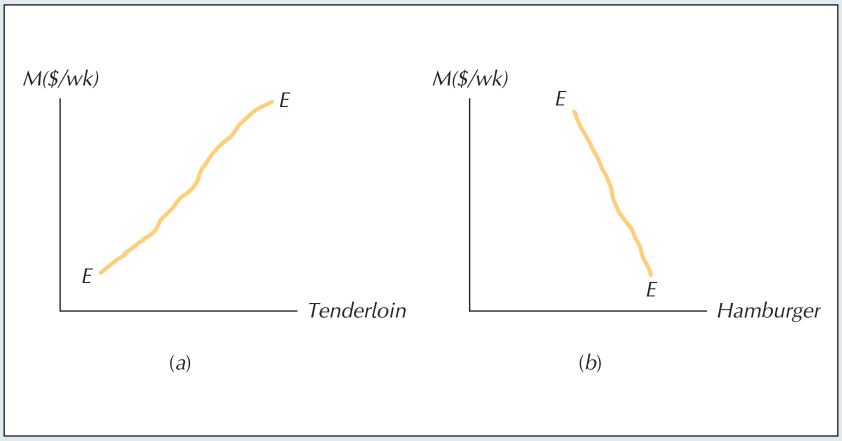


FIGURE 4.4
An Individual Consumer's Engel Curve
Holding preferences and relative prices constant, the Engel curve tells how much shelter the consumer will purchase at various levels of income.

Note carefully the distinction between what we measure on the vertical axis of the ICC and what we measure on the vertical axis of the Engel curve. On the vertical axis of the ICC, we measure the amount the consumer spends each week on all goods other than shelter. On the vertical axis of the Engel curve, by contrast, we measure the consumer's total weekly income.

Note also that, as was true with the PCC and individual demand curves, the ICC and Engel curves contain essentially the same information. The advantage of the Engel curve is that it allows us to see at a glance how the quantity demanded varies with income.

FIGURE 4.5
The Engel Curves for Normal and Inferior Goods
(*a*) This Engel curve is for a normal good. The quantity demanded increases with income. (*b*) This Engel curve for hamburger has the negative slope characteristic of inferior goods. As the consumer's income grows, he switches from hamburger to more desirable cuts of meat.



NORMAL AND INFERIOR GOODS

normal good one whose quantity demanded rises as income rises.

inferior good one whose quantity demanded falls as income rises.

Note that the Engel curve in Figure 4.5*a* is upward-sloping, implying that the more income a consumer has, the more tenderloin steak he will buy each week. Most things we buy have this property, which is the defining characteristic of a **normal good.** Goods that do not have this property are called **inferior goods.** For such goods, an increase in income leads to a reduction in the quantity demanded. Figure 4.5*b* is an example of an Engel curve for an inferior good. The more income a person has, the less hamburger he will buy each week.

Why would someone buy less of a good following an increase in his income? The prototypical inferior good is one with several strongly preferred, but more expensive, substitutes. Supermarkets, for example, generally carry several different grades of ground beef, ranging from hamburger, which has the highest fat content, to ground sirloin, which has the lowest. A consumer trying to restrict the amount of fat in his diet will switch to a leaner grade of meat as soon as he can afford it. For such a consumer, hamburger is an inferior good.

For any consumer who spends all her income, it is a matter of simple arithmetic that not all goods can be inferior. After all, when income rises, it is mathematically impossible to spend less on all goods at once. It follows that the more broadly a good is defined, the less likely it is to be inferior. Thus, while hamburger is an inferior good for many consumers, there are probably very few people for whom "meat" is inferior, and fewer still for whom "food" is inferior.[1]

THE INCOME AND SUBSTITUTION EFFECTS OF A PRICE CHANGE

substitution effect that component of the total effect of a price change that results from the associated change in the relative attractiveness of other goods.

In Chapter 2 we saw that a change in the price of a good affects purchase decisions for two reasons. Consider the effects of a price increase. (The effects of a price reduction will be in the opposite direction.) When the price of a good rises, close substitutes become more attractive than before. For example, when the price of rice increases, wheat becomes more attractive. This is the so-called **substitution effect** of a price increase.

The second effect of a price increase is to reduce the consumer's purchasing power. For a normal good, this will further reduce the amount purchased. But for

[1]Another useful way to partition the set of consumer goods is between so-called *necessities* and *luxuries*. A good is defined as a luxury for a person if he spends a larger proportion of his income on it when his income rises. A necessity, by contrast, is one for which he spends a smaller proportion of his income when his income rises. (More on this distinction follows.)

an inferior good, the effect is just the opposite. The loss in purchasing power, taken by itself, increases the quantity purchased of an inferior good. The change in the quantity purchased attributable to the change in purchasing power is called the **income effect** of the price change.

income effect that component of the total effect of a price change that results from the associated change in real purchasing power.

The *total effect* of the price increase is the sum of the substitution and income effects. The substitution effect always causes the quantity purchased to move in the opposite direction from the change in price—when price goes up, the quantity demanded goes down, and vice versa. The direction of the income effect depends on whether the good is normal or inferior. For normal goods, the income effect works in the same direction as the substitution effect—when price goes up [down], the fall [rise] in purchasing power causes the quantity demanded to fall [rise]. For inferior goods, by contrast, the income and substitution effects work against one another.

The substitution, income, and total effects of a price increase can be seen most clearly when displayed graphically. Let us begin by depicting the total effect. In Figure 4.6, the consumer has an initial income of \$120/wk and the initial price of shelter is \$6/sq yd. This gives rise to the budget constraint labeled B_0, along which the optimal bundle is *A,* which contains 10 sq yd/wk of shelter. Now let the price of shelter increase from \$6/sq yd to \$24/sq yd, resulting in the budget labeled B_1. The new optimal bundle is *D,* which contains 2 sq yd/wk of shelter. The movement from *A* to *D* is called the total effect of the price increase. Naturally, the price increase causes the consumer to end up on a lower indifference curve (I_1) than the one he was able to attain on his original budget (I_0).

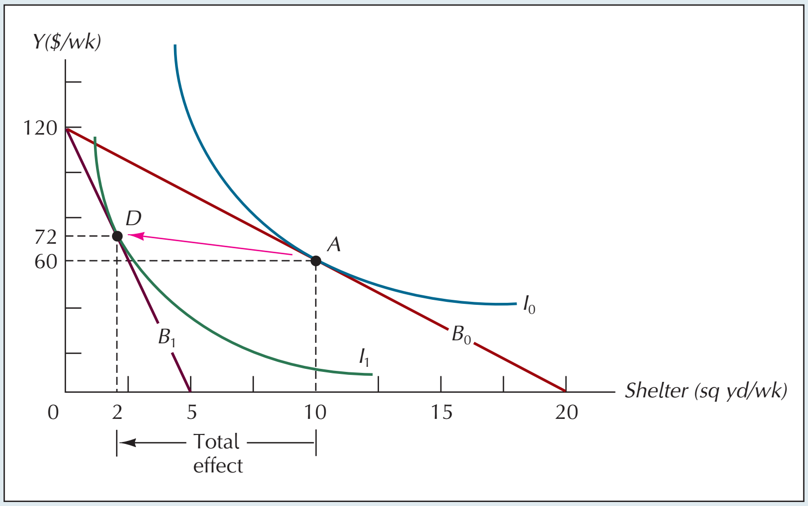
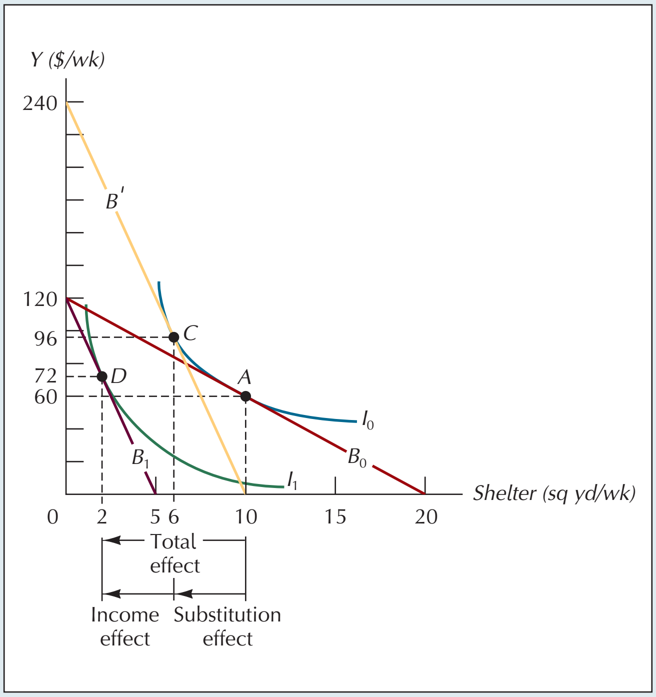


FIGURE 4.6
The Total Effect of a Price Increase
With an income of \$120/wk and a price of shelter of \$6/sq yd, the consumer chooses bundle *A* on the budget constraint B_0. When the price of shelter rises to \$24/sq yd, with income held constant at \$120/wk, the best affordable bundle becomes *D*. The movement from 10 to 2 sq yd/wk of shelter is called the total effect of the price increase.

To decompose the total effect into the income and substitution effects, we begin by asking the following question: How much income would the consumer need to reach his original indifference curve (I_0) after the increase in the price of shelter? Note in Figure 4.7 that the answer is \$240/wk. If the consumer were given a total income of that amount, it would undo the injury caused by the loss in purchasing power resulting from the increase in the price of shelter. The budget constraint labeled B' is purely hypothetical, a device constructed for the purpose at hand. It has the same slope as the new budget constraint (B_1)—namely, −24—and is just far enough out to be tangent to the original indifference curve, I_0. With the budget constraint B', the optimal bundle is *C,* which contains 6 sq yd/wk of shelter. The movement from *A* to *C* gives rise to the substitution effect of the price change—here a reduction of 4 sq yd/wk of shelter and an increase of 36 units/wk of the composite good.

FIGURE 4.7
The Substitution and Income Effects of a Price Change
To get the substitution effect, slide the new budget B_1 outward parallel to itself until it becomes tangent to the original indifference curve, I_0. The movement from *A* to *C* gives rise to the substitution effect, the reduction in shelter due solely to the fact that shelter is now more expensive relative to other goods. The movement from *C* to *D* gives rise to the income effect. It is the reduction in shelter that results from the loss in purchasing power implicit in the price increase.



The hypothetical budget constraint B' tells us that even if the consumer had enough income to reach the same indifference curve as before, the increase in the price of shelter would cause him to reduce his consumption of it in favor of other goods and services. *For consumers whose indifference curves have the conventional convex shape, the substitution effect of a price increase will always reduce consumption of the good whose price increased.*

The income effect stems from the movement from *C* to *D*. The particular good shown in Figure 4.7 happens to be a normal good. The hypothetical movement of the consumer's income from \$240/wk to \$120/wk accentuates the reduction of his consumption of shelter, causing it to fall from 6 sq yd/wk to 2 sq yd/wk.

Whereas the income effect reinforces the substitution effect for normal goods, the two effects tend to offset one another for inferior goods. In Figure 4.8, B_0 depicts the budget constraint for a consumer with an income of \$24/wk who faces a price of hamburger of \$1/lb. On B_0 the best affordable bundle is *A*, which contains 12 lb/wk of hamburger. When the price of hamburger rises to \$2/lb, the resulting budget constraint is B_1 and the best affordable bundle is now *D*, which contains 9 lb/wk of hamburger. The total effect of the price increase is thus to reduce hamburger consumption by 3 lb/wk. Budget constraint B' once again is the hypothetical budget constraint that enables the consumer to reach the original indifference curve at the new price ratio. Note that the substitution effect (the change in hamburger consumption associated with movement from *A* to *C* in Figure 4.8) is to reduce the quantity of hamburger consumed by 4 lb/wk—that is, to reduce it by more than the value of the total effect. The income effect by itself (the change in hamburger consumption associated with the movement from *C* to *D*) actually increases hamburger consumption by 1 lb/wk. The income effect thus works in the opposite direction from the substitution effect for an inferior good such as hamburger.

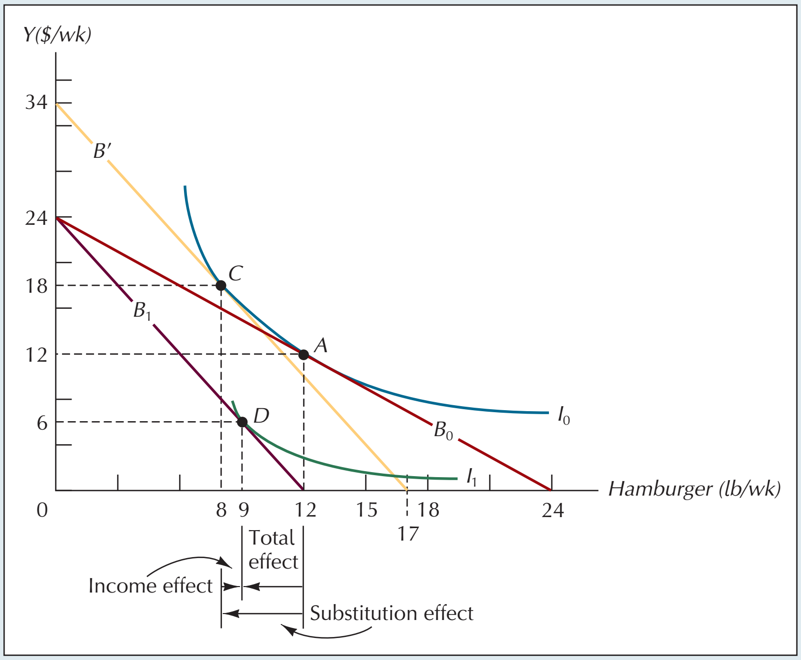


FIGURE 4.8
Income and Substitution Effects for an Inferior Good
By contrast to the case of a normal good, the income effect acts to offset the substitution effect for an inferior good.

GIFFEN GOODS

Giffen good one for which the quantity demanded rises as its price rises.

A **Giffen good** is one for which the total effect of a price increase is to increase, not reduce, the quantity purchased. Since the substitution effect of a price increase is always to reduce the quantity purchased, the Giffen good must be one whose income effect offsets the substitution effect. That is, the Giffen good must be an inferior good—so strongly inferior, in fact, that the income effect is actually larger than the substitution effect.

A much-cited example of a Giffen good was the potato during the Irish potato famine of the nineteenth century. The idea was that potatoes were such a large part of poor people's diets to begin with that an increase in their price had a severe adverse effect on the real value of purchasing power. Having less real income, many families responded by cutting back on meat and other more expensive foods, and buying even more potatoes. (See Figure 4.9.) Or so the story goes.

Modern historians dispute whether the potato ever was really a Giffen good. Whatever the resolution of this dispute, the potato story does illustrate the

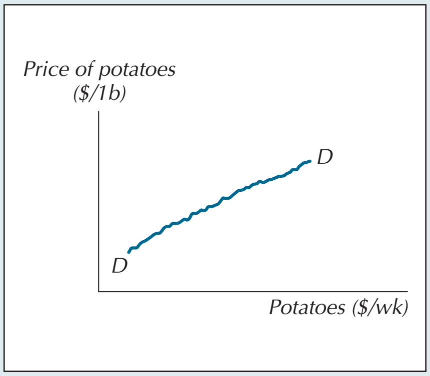


FIGURE 4.9
The Demand Curve for a Giffen Good
If a good is so strongly inferior that the income effect of a price increase dominates the substitution effect, the demand curve for that good will be upward sloping. Giffen goods are a theoretical possibility, but are seldom, if ever, observed in practice.

characteristics that a Giffen good would logically have to possess. First, it would not only have to be inferior, but also have to occupy a large share of the consumer's budget. Otherwise, an increase in its price would not create a significant reduction in real purchasing power. (Doubling the price of keyrings, for example, does not make anyone appreciably poorer.) The second characteristic required of a Giffen good is that it have a relatively small substitution effect, one small enough to be overwhelmed by the income effect.

In practice, it is extremely unlikely that a good will satisfy both requirments. Most goods, after all, account for only a tiny share of the consumer's total expenditures. Moreover, as noted, the more broadly a good is defined, the less likely it is to be inferior. Finally, inferior goods by their very nature tend to be ones for which there are close substitutes. The consumer's tendency to substitute ground sirloin for hamburger, for example, is precisely what makes hamburger an inferior good.

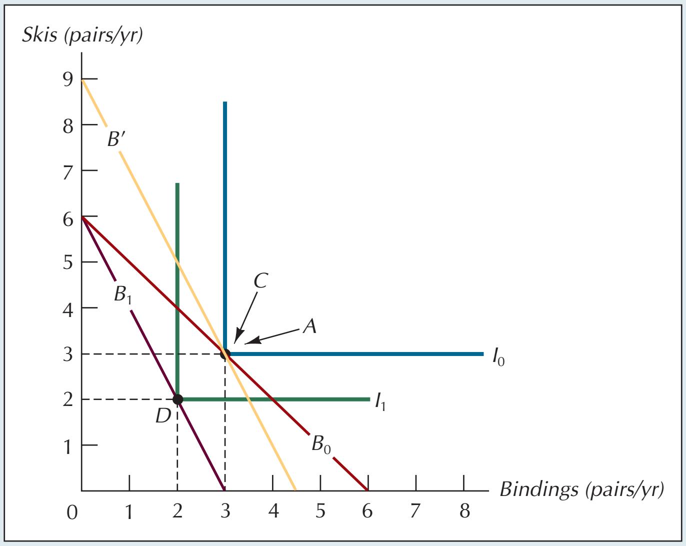
The Giffen good is an intriguing anomaly, chiefly useful for testing students' understanding of the subtleties of income and substitution effects. Unless otherwise stated, all demand curves used in the remainder of this text will be assumed to have the conventional downward slope.

EXAMPLE 4.1

Income and substitution effects for perfect complements. Suppose skis and bindings are perfect, one-for-one complements and Paula spends all her equipment budget of $1200/yr on these two goods. Skis and bindings each cost $200. What will be the income and substitution effects of an increase in the price of bindings to $400 per pair?

Since our goal here is to examine the effect on two specific goods (skis and bindings), we proceed by devoting one axis to each good and dispense with the composite good. On the original budget constraint, B_0, the optimal bundle is denoted A in Figure 4.10. Paula buys three pairs of skis per year and three pairs of bindings. When the price of bindings rises from $200 per pair to $400 per pair, we get the new budget constraint, B_1, and the resulting optimal bundle D, which contains two pairs of skis per year and two pairs of bindings. An equipment budget of $1800/yr is what the consumer would need at the new price to attain the same indifference curve she did originally (I_0). (To get this figure, slide B_1 out until it hits I_0, then calculate the cost of buying the bundle at the vertical intercept—here, nine pairs of skis per year at $200 per pair.) Note that because perfect complements have right-angled indifference curves, the budget

FIGURE 4.10
Income and Substitution Effects for Perfect Complements
For perfect complements, the substitution effect of an increase in the price of bindings (the movement from *A* to *C*) is equal to zero. The income effect (the movement from *A* to *D*) and the total effect are one and the same.



B' results in an optimal bundle C that is exactly the same as the original bundle A. For perfect complements, the substitution effect is zero. So for this case, the total effect of the price increase is exactly the same as the income effect of the price increase.

Example 4.1 tells us that if the price of ski bindings goes up relative to the price of skis, people will not alter the proportion of skis and bindings they purchase. But because the price increase lowers their real purchasing power (that is, because it limits the quantities of both goods they can buy), they will buy fewer units of ski equipment. The income effect thus causes them to lower their consumption of both skis and bindings by the same proportion.

EXERCISE 4.1

Repeat Example 4.1 with the assumption that pairs of skis and pairs of bindings are perfect two-for-one complements. (That is, assume that Paula wears out two pairs of skis for every pair of bindings she wears out.)

EXAMPLE 4.2

Income and substitution effects for perfect substitutes. Suppose Pam considers tea and coffee to be perfect one-for-one substitutes and spends $12/wk on these two beverages. Coffee costs $1/cup, while tea costs $1.20/cup. What will be the income and substitution effects of an increase in the price of coffee to $1.50/cup?

Pam will initially demand 12 cups of coffee per week and no cups of tea (point A in Figure 4.11), since each good contributes equally to her utility but tea is more expensive. When the price of coffee rises, Pam switches to consuming only tea, buying 10 cups per week and no coffee (point D). Pam would need a budget of $14.40/wk to afford 12 cups of tea (point C), which she likes as well as the 12 cups of coffee she originally consumed. The substitution effect is from (12, 0) to (0, 12) and the income effect from (0, 12) to (0, 10), with the total effect from (12, 0) to (0, 10). With perfect substitutes, the substitution effect can be very large: For small price changes (near MRS), consumers may switch from consuming all one good to consuming only the other.

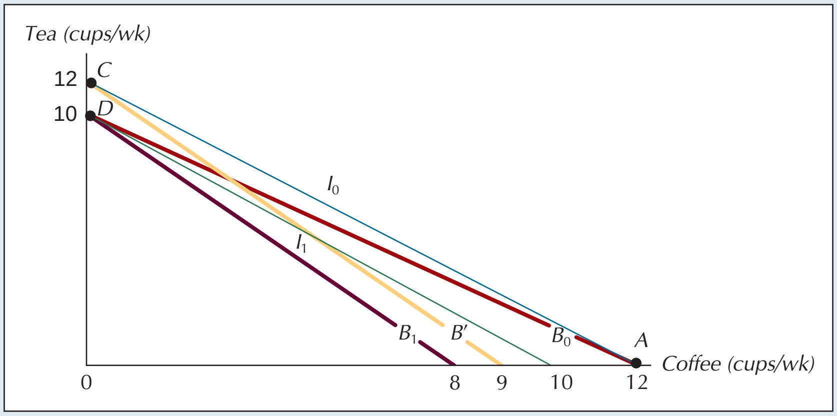


FIGURE 4.11
Income and Substitution Effects for Perfect Substitutes
For perfect substitutes, the substitution effect of an increase in the price of coffee (the movement from A to C) can be very large.

EXERCISE 4.2

Starting from the original price in Example 4.2, what will be the income and substitution effects of an increase in the price of tea to $1.50/cup?

CONSUMER RESPONSIVENESS TO CHANGES IN PRICE

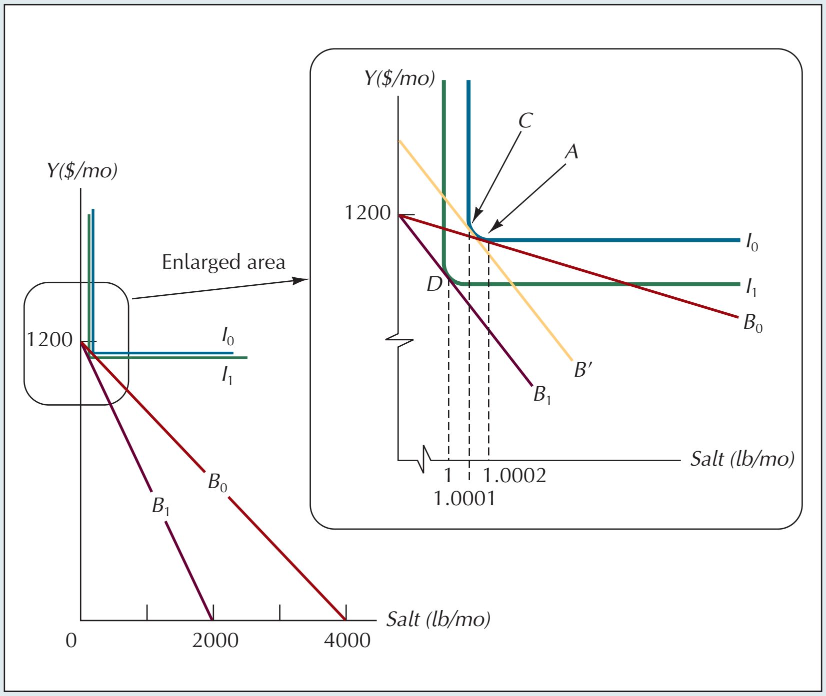
We began this chapter with the observation that for certain goods, such as salt, consumption is highly insensitive to changes in price while for others, such as housing, it is much more sensitive. The principal reason for studying income and substitution effects is that they help us understand such differences.

Consider first the case of salt. When analyzing substitution and income effects, there are two salient features to note about salt. First, for most consumers, it has no close substitutes. If someone were forbidden to shake salt onto his steak, he might respond by shaking a little extra pepper, or even by squeezing some lemon juice onto it. But for most people, these alternatives would fall considerably short of the real thing. Salt's second prominent feature is that it occupies an almost imperceptibly small share of total expenditures. An extremely heavy user of salt might consume a pound every month. If this person's income were $1200/mo, a doubling of the price of salt—say, from $0.30/lb to $0.60/lb—would increase the share of his budget accounted for by salt from 0.00025 to 0.0005. For all practical purposes, the income effect for salt is negligible.

In Figure 4.12, the fact that salt has no close substitutes is represented by indifference curves with a nearly right-angled shape. Salt's negligible budget share is captured by the fact that the cusps of these indifference curves occur at extremely small quantities of salt.

Suppose, as in Figure 4.12, the price of salt is originally $0.30/lb, resulting in the equilibrium bundle *A* in the enlarged region, which contains 1.0002 lb/mo of salt. A price increase to $0.60/lb results in a new equilibrium bundle *D* with 1 lb/mo of salt. The income and substitution effects are measured in terms of the intermediate bundle *C*. Geometrically, the income effect is small because the original tangency occurred so near the vertical intercept of the budget constraint. When we are near the pivot point of the budget constraint, even a very large rotation produces only a small movement. The substitution effect, in turn, is small because of the nearly right-angled shape of the indifference curves.

FIGURE 4.12
Income and Substitution Effects of a Price Increase for Salt
The total effect of a price change will be very small when (1) the original equilibrium bundle lies near the vertical intercept of the budget constraint and (2) the indifference curves have a nearly right-angled shape. The first factor causes the income effect (the reduction in salt consumption associated with the movement from *C* to *D*) to be small; the second factor causes the substitution effect (the reduction in salt consumption associated with the movement from *A* to *C*) to be small.



Let us now contrast salt with housing. The two salient facts about housing are that (1) it accounts for a substantial share of total expenditures (more than 30 percent for many people), and (2) most people have considerable latitude to substitute other goods for housing. Many Manhattanites, for example, can afford to live in apartments larger than the ones they now occupy, yet they prefer to spend what they save in rent on restaurant meals, theater performances, and the like. Another substitution possibility is to consume less conveniently located housing. Someone who works in Manhattan can live near her job and pay high rent; alternatively, she can live in New Jersey or Long Island and pay considerably less. Or she can choose an apartment in a less fashionable neighborhood, or one not quite as close to a convenient subway stop. The point is that there are many different options for housing, and the choice among them depends strongly on income and relative prices.

In Figure 4.13, the consumer's income is \$120/wk and the initial price of shelter is \$0.60/sq yd. The resulting budget constraint is B_0, and the best affordable bundle on it is *A*, which contains 100 sq yd/wk of shelter. An increase in the price of shelter to \$2.40/sq yd causes the quantity demanded to fall to 20sq yd/wk. The smooth convex shape of the indifference curves represents the high degree of substitution possibilities between housing and other goods and accounts for the relatively large substitution effect (the fall in shelter consumption associated with the movement from *A* to *C*). Note also that the original equilibrium bundle, *A*, was far from the vertical pivot point of the budget constraint. By contrast to the case of salt, here the rotation in the budget constraint caused by the price increase produces a large movement in the location of the relevant segment of the new budget constraint. Accordingly, the income effect for shelter (the fall in shelter consumption associated with the movement from *C* to *D*) is much larger than for salt. With both a large substitution and a large income effect working together, the total effect of an increase in the price of shelter (the fall in shelter consumption associated with the movement from *A* to *D*) is very large.

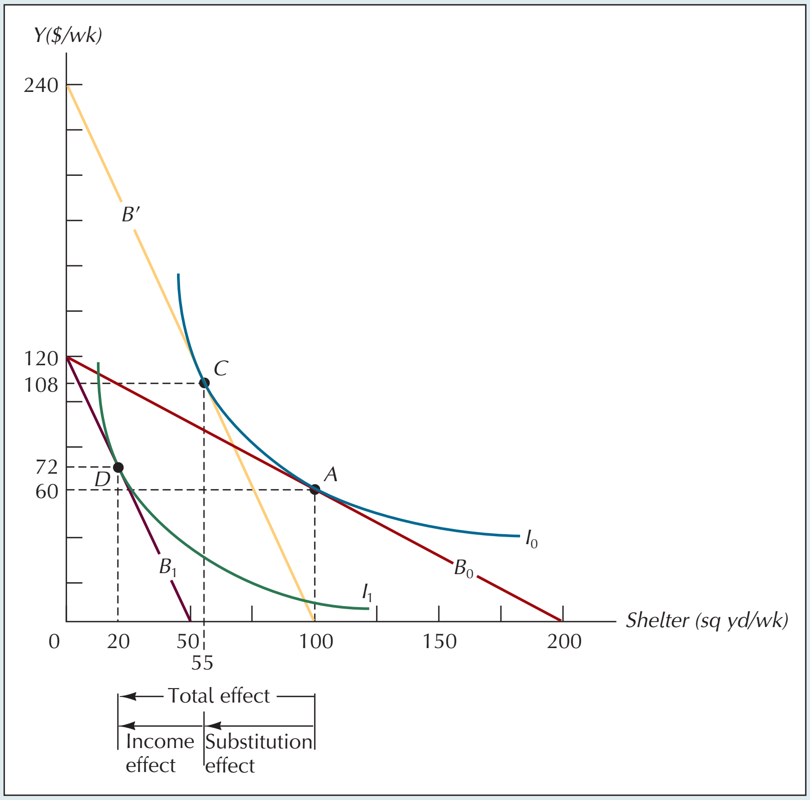


FIGURE 4.13

Income and Substitution Effects for a Price-Sensitive Good

Because shelter occupies a large share of the budget, its income effect tends to be large. And because it is practical to substitute away from shelter, the substitution effect also tends to be large. The quantities demanded of goods with both large substitution and large income effects are highly responsive to changes in price.

EXAMPLE 4.3 **Deriving the individual demand curve for perfect complements. James views car washes and gasoline as perfect complements in a 1-to-10 ratio, requiring one car wash for every 10 gallons of gas. Gas costs \$3/gal, and James has \$144/mo to spend on gas and car washes. (See Figure 4.14.) Construct James's demand curve for car washes by considering his quantity demanded of car washes at various prices (such as \$6, \$18, and \$42; see Figure 4.15).**

FIGURE 4.14
A Price Increase for Car Washes
With \$144/mo, James buys 4 washes/mo when the price is \$6/wash (budget constraint *B*), 3 washes/mo when the price is \$18/wash (budget constraint *B*′), and 2 washes/mo when the price is \$42/wash (budget constraint *B*′′).

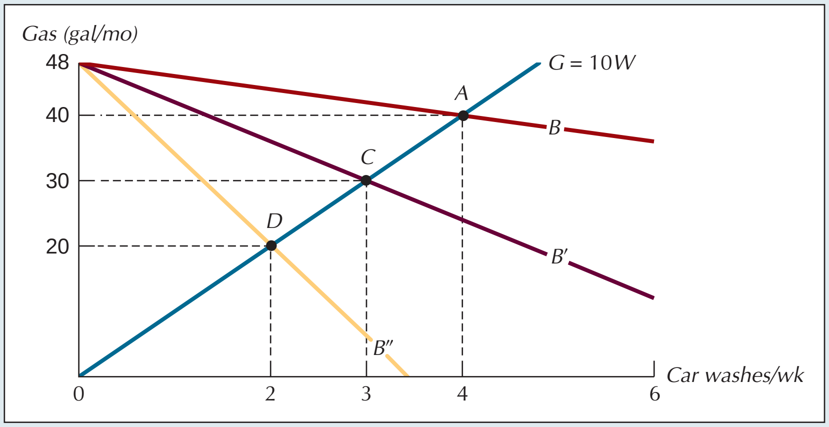
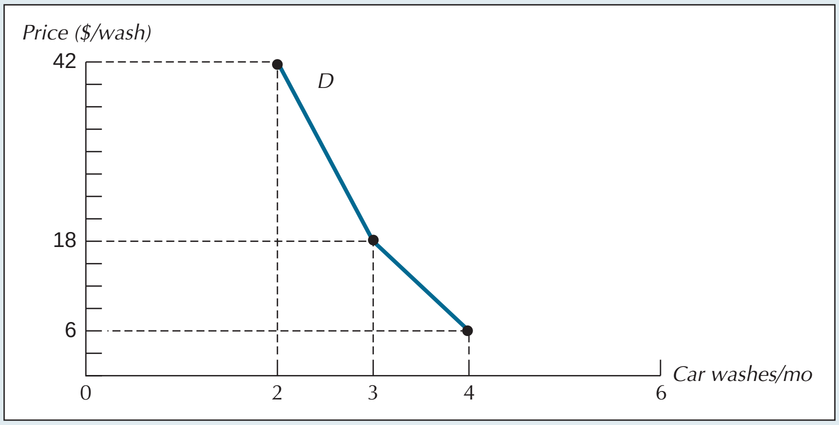


FIGURE 4.15
James's Demand for Car Washes
The quantity of car washes James demands at various prices forms his demand curve for car washes.



James's preferences dictate that his optimal bundle must satisfy $G = 10W$, as his indifference curves are L-shaped. James's budget constraint is $3G + P_W W = 144$, or $G = 48 - \frac{P_W}{3}W$. Substituting $G = 10W$, his budget constraint is $30W + P_W W = 144$, which implies $(30 + P_W)W = 144$. At $P_W = 6$, $W = 4$; at $P_W = 18$, $W = 3$; at $P_W = 42$, $W = 2$, as summarized in Table 4.3.

TABLE 4.3
A Demand Schedule for Car Washes

Price of car wash (\$/wash)	Quantity of car washes demanded (washes/mo)
6	4
18	3
42	2
114	1

MARKET DEMAND: AGGREGATING INDIVIDUAL DEMAND CURVES

Having seen where individual demand curves come from, we are now in a position to see how individual demand curves may be aggregated to form the market demand curve. Consider a market for a good—for the sake of concreteness, again shelter—with only two potential consumers. Given the demand curves for these consumers, how do we generate the market demand curve? In Figure 4.16, D_1 and D_2 represent the individual demand curves for consumers 1 and 2, respectively. To get the market demand curve, we begin by calling out a price—say, \$4/sq yd—and adding the quantities demanded by each consumer at that price. This sum, 6 sq yd/wk + 2 sq yd/wk = 8 sq yd/wk, is the total quantity of shelter demanded at the price \$4/sq yd. We then plot the point (4, 8) as one of the quantity-price pairs on the market demand curve D in the right panel of Figure 4.16. To generate additional points on the market demand curve, we simply repeat this process for other prices. Thus, the price \$8/sq yd corresponds to a quantity of 4 + 0 = 4 sq yd/wk on the market demand curve for shelter. Proceeding in like fashion for additional prices, we trace out the entire market demand curve. Note that for prices above \$8/sq yd, consumer 2 demands no shelter at all, and so the market demand curve for prices above \$8 is identical to the demand curve for consumer 1.

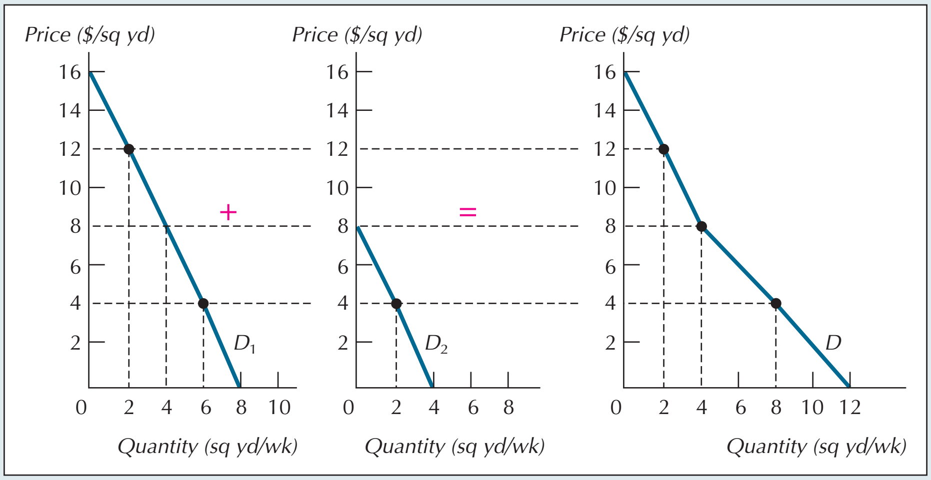


FIGURE 4.16
Generating Market Demand from Individual Demands
The market demand curve (D in the right panel) is the horizontal sum of the individual demand curves, D_1 (left panel) and D_2 (center panel).

The procedure of announcing a price and adding the individual quantities demanded at that price is called *horizontal summation*. It is carried out the same way whether there are only two consumers in the market or many millions. In both large and small markets, the market demand curve is the horizontal summation of the individual demand curves.

In Chapter 2 we saw that it is often easier to generate numerical solutions when demand and supply curves are expressed algebraically rather than geometrically. Similarly, it will often be convenient to aggregate individual demand curves algebraically rather than graphically. When using the algebraic approach, a common error is to add individual demand curves vertically instead of horizontally. A simple example makes this danger clear.

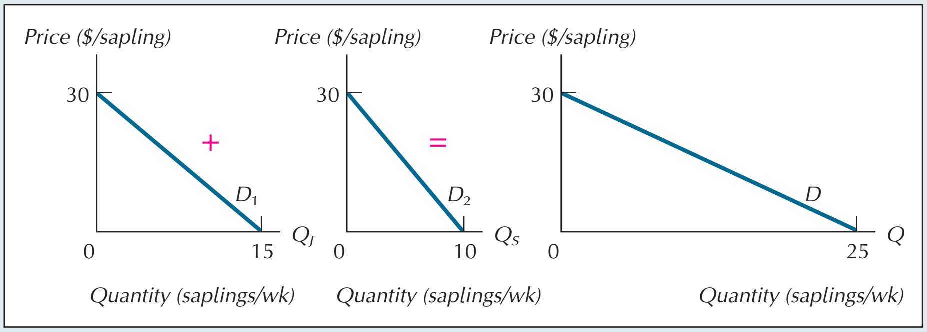
EXAMPLE 4.4

Smith and Jones are the only consumers in the market for beech saplings in a small town in Vermont. Their demand curves are given by $P = 30 - 2Q_J$ and $P = 30 - 3Q_S$ where Q_J and Q_S are the quantities demanded by Jones

and Smith, respectively. What is the market demand curve for beech saplings in their town?

When we add demand curves horizontally, we are adding quantities, not prices. Thus it is necessary first to solve the individual demand equations for the respective quantities in terms of price. This yields $Q_J = 15 - (P/2)$ for Jones, and $Q_S = 10 - (P/3)$ for Smith. If the quantity demanded in the market is denoted by Q, we have $Q = Q_J + Q_S = 15 - (P/2) + 10 - (P/3) = 25 - (5P/6)$. Solving back for P, we get the equation for the market demand curve: $P = 30 - (6Q/5)$. We can easily verify that this is the correct market demand curve by adding the individual demand curves graphically, as in Figure 4.17.

FIGURE 4.17
The Market Demand Curve for Beech Saplings
When adding individual demand curves algebraically, be sure to solve for quantity first before adding.



The common pitfall is to add the demand functions as originally stated and then solve for P in terms of Q. Here, this would yield $P = 30 - (5Q/2)$, which is obviously not the market demand curve we are looking for.

EXERCISE 4.3

Write the individual demand curves for shelter in Figure 4.16 in algebraic form, then add them algebraically to generate the market demand curve for shelter. (*Caution:* Note that the formula for quantity along D_2 is valid only for prices between 0 and 8.)

The horizontal summation of individual consumers' demands into market demand has a simple form when the consumers in the market are all identical. Suppose n consumers each have the demand curve $P = a - bQ_i$. To add up the quantities for the n consumers into market demand, we rearrange the consumer demand curve $P = a - bQ_i$ to express quantity alone on one side $Q_i = a/b - (1/b)\,P$. Then market demand is the sum of the quantities demanded Q_i by each of the n consumers.

$$Q = nQ_i = n\left(\frac{a}{b} - \frac{1}{b}P\right) = \frac{na}{b} - \frac{n}{b}P.$$

We can then rearrange market demand $Q = na/b - n(P/b)$ to get back in the form of price alone on one side $P = a - (b/n)\,Q$. The intuition is that each one unit demanded by the market is $1/n$ unit for each consumer. These calculations suggest a general rule for constructing the market demand curve when consumers are identical. If we have n individual consumer demand curves $P = a - bQ_i$, then the market demand curve is $P = a - (b/n)Q$.

EXAMPLE 4.5

Suppose a market has 10 consumers, each with demand curve $P = 10 - 5Q_i$, where P is the price in dollars per unit and Q_i is the number of units demanded per week by the *i*th consumer (Figure 4.18). Find the market demand curve.

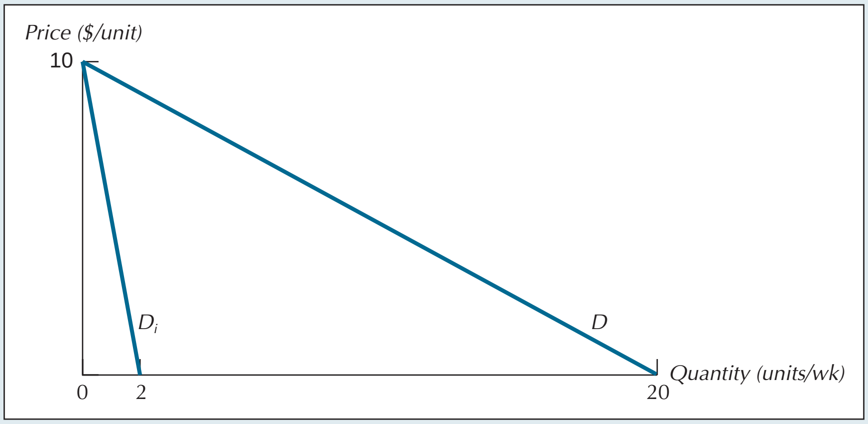


FIGURE 4.18
Market Demand with Identical Consumers
When 10 consumers each have demand curve $P = 10 - 5Q_i$ the market demand curve is the horizontal summation $P = 10 - (\frac{1}{2})\,Q$, with the same price intercept and $\frac{1}{10}$ the slope.

First, we need to rearrange the representative consumer demand curve $P = 10 - 5Q_i$ to have quantity alone on one side:

$$Q_i = 2 - \tfrac{1}{5}P.$$

Then we multiply by the number of consumers, $n = 10$:

$$Q = nQ_i = 10Q_i = 10(2 - \tfrac{1}{5}P) = 20 - 2P.$$

Finally, we rearrange the market demand curve $Q = 20 - 2P$ to have price alone on one side, $P = 10 - (\frac{1}{2})Q$, to return to the slope-intercept form.

EXERCISE 4.4

Suppose a market has 30 consumers, each with demand curve $P = 120 - 60Q_i$, where P is price in dollars per unit and Q_i is the number of units demanded per week by the *i*th consumer. Find the market demand curve.

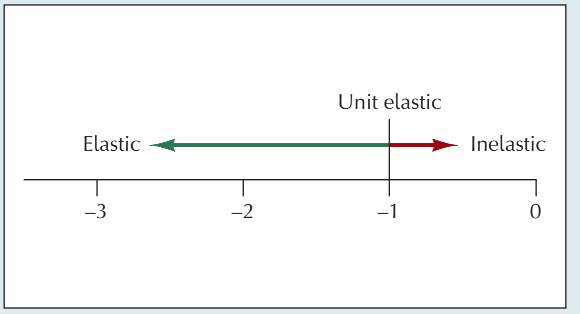
PRICE ELASTICITY OF DEMAND

An analytical tool of central importance is the **price elasticity of demand.** It is a quantitative measure of the responsiveness of purchase decisions to variations in price, and as we will see in both this and later chapters, it is useful for a variety of practical problems. *Price elasticity of demand is defined as the percentage change in the quantity of a good demanded that results from a 1 percent change in price.* For example, if a 1 percent rise in the price of shelter caused a 2 percent reduction in the quantity of shelter demanded, then the price elasticity of demand for shelter would be -2. The price elasticity of demand will always be negative (or zero) because price changes always move in the opposite direction from changes in quantity demanded.

price elasticity of demand the percentage change in the quantity of a good demanded that results from a 1 percent change in its price.

The demand for a good is said to be *elastic* with respect to price if its price elasticity is less than -1. The good shelter mentioned in the preceding paragraph would thus be one for which demand is elastic with respect to price. The demand for a good is *inelastic* with respect to price if its price elasticity is greater than -1 and

FIGURE 4.19
Three Categories of Price Elasticity
With respect to price, the demand for a good is elastic if its price elasticity is less than −1, inelastic if its price elasticity exceeds −1, and unit elastic if its price elasticity is equal to −1.



unit elastic with respect to price if its price elasticity is equal to −1. These definitions are portrayed graphically in Figure 4.19.

When interpreting actual demand data, it is often useful to have a more general definition of price elasticity that can accommodate cases in which the observed change in price does not happen to be 1 percent. Let P be the current price of a good and let Q be the quantity demanded at that price. And let ΔQ be the change in the quantity demanded that occurs in response to a very small change in price, ΔP. The price elasticity of demand at the current price and quantity will then be given by

$$\epsilon = \frac{\Delta Q/P}{\Delta P/P}. \tag{4.1}$$

The numerator on the right side of Equation 4.1 is the proportional change in quantity. The denominator is the proportional change in price. Equation 4.1 is exactly the same as our earlier definition when ΔP happens to be a 1 percent change in current price. The advantage is that the more general definition also works when ΔP is any other small percentage change in current price.

A GEOMETRIC INTERPRETATION OF PRICE ELASTICITY

Another way to interpret Equation 4.1 is to rewrite it as

$$\epsilon = \frac{\Delta Q}{\Delta P}\frac{P}{Q}. \tag{4.2}$$

Equation 4.2 suggests a simple interpretation in terms of the geometry of the market demand curve. When ΔP is small, the ratio $\Delta P/\Delta Q$ is the slope of the demand curve, which means that the ratio $\Delta Q/\Delta P$ is the reciprocal of that slope. Thus the price elasticity of demand may be interpreted as the product of the ratio of price to quantity and the reciprocal of the slope of the demand curve:[2]

$$\epsilon = \frac{P}{Q}\frac{1}{\text{slope}}. \tag{4.3}$$

Equation 4.3 is called the *point-slope method* of calculating price elasticity of demand. By way of illustration, consider the demand curve for shelter shown in Figure 4.20. Because this demand curve is linear, its slope is the same at every point, namely, −2. The reciprocal of this slope is $-\frac{1}{2}$. The price elasticity of demand at point A is therefore given by the ratio of price to quantity at A $(\frac{12}{2})$ multiplied by the reciprocal of the slope at A $(-\frac{1}{2})$, so we have $\epsilon_A = (\frac{12}{2})(-\frac{1}{2}) = -3$.

[2] In calculus terms, price elasticity is defined as $\epsilon = (P/Q)[dQ(P)/dP]$.

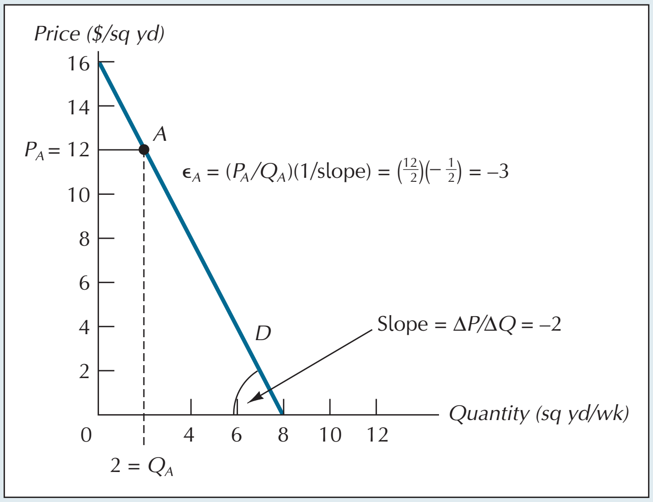


FIGURE 4.20
The Point-Slope Method
The price elasticity of demand at any point is the product of the price-quantity ratio at that point and the reciprocal of the slope of the demand curve at that point. The price elasticity at *A* is thus $(\frac{12}{2})(-\frac{1}{2}) = -3$.

When the market demand curve is linear, as in Figure 4.20, several properties of price elasticity quickly become apparent from this interpretation. The first is that the price elasticity is different at every point along the demand curve. More specifically, we know that the slope of a linear demand curve is constant throughout, which means that the reciprocal of its slope is also constant. The ratio of price to quantity, by contrast, takes a different value at every point along the demand curve. As we approach the vertical intercept, it approaches infinity. It declines steadily as we move downward along the demand curve, finally reaching a value of zero at the horizontal intercept.

A second property of demand elasticity is that it is never positive. As noted earlier, because the slope of the demand curve is always negative, its reciprocal must also be negative; and because the ratio P/Q is always positve, it follows that the price elasticity of demand—which is the product of these two—must always be a negative number (except at the horizontal intercept of the demand curve, where P/Q, and hence elasticity, is zero). For the sake of convenience, however, economists often ignore the negative sign of price elasticity and refer simply to its absolute value. When a good is said to have a "high" price elasticity of demand, this will always mean that its price elasticity is large in absolute value, indicating that the quantity demanded is highly responsive to changes in price. Similarly, a good whose price elasticity is said to be "low" is one for which the absolute value of elasticity is small, indicating that the quantity demanded is relatively unresponsive to changes in price.

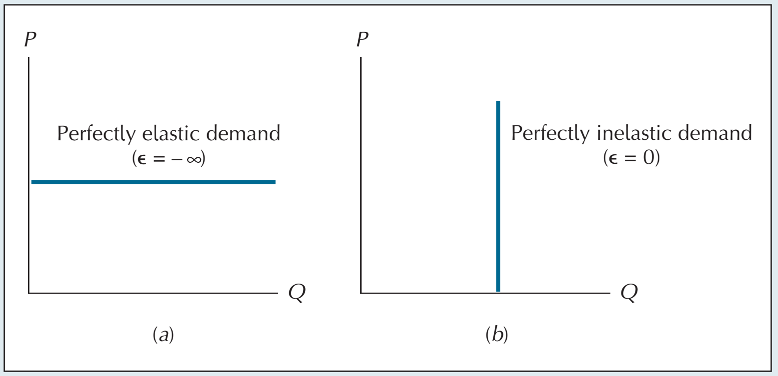
A third property of price elasticity at any point along a straight-line demand curve is that it will be inversely related to the slope of the demand curve. The steeper the demand curve, the less elastic is demand at any point along it. This follows from the fact that the reciprocal of the slope of the demand curve is one of the factors used to compute price elasticity.

EXERCISE 4.5

Use the point-slope method (Equation 4.3) to determine the elasticity of the demand curve $P = 32 - Q$ at the point where $P = 24$.

Two polar cases of demand elasticity are shown in Figure 4.21. In Figure 4.21*a*, the horizontal demand curve, with its slope of zero, has an infinitely high price elasticity at every point. Such demand curves are often called *perfectly elastic* and, as we will see, are especially important in the study of competitive firm

FIGURE 4.21
Two Important Polar Cases
(*a*) The price elasticity of the demand curve is equal to $-\infty$ at every point. Such demand curves are said to be perfectly elastic. (*b*) The price elasticity of the demand curve is equal to 0 at every point. Such demand curves are said to be perfectly inelastic.



behavior. In Figure 4.21*b*, the vertical demand curve has a price elasticity everywhere equal to zero. Such curves are called *perfectly inelastic*.

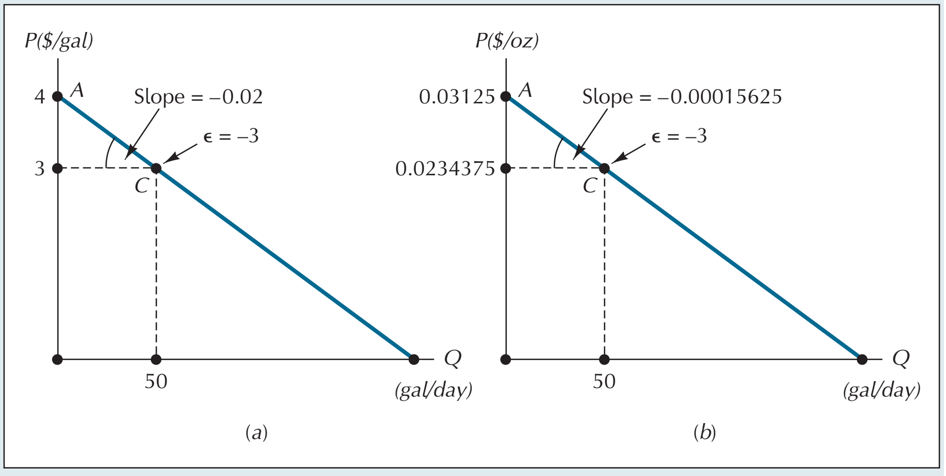
As a practical matter, it would be impossible for any demand curve to be perfectly inelastic at all prices. Beyond some sufficiently high price, income effects must curtail consumption, even for seemingly essential goods with no substitutes, such as surgery for malignant tumors. Even so, the demand curve for many such goods and services will be perfectly inelastic over an extremely broad range of prices (recall the salt example discussed earlier in this chapter).

THE UNIT-FREE PROPERTY OF ELASTICITY

Another way of measuring responsiveness to changes in price is to use the slope of the demand curve. Other things equal, for example, we know that the quantity demanded of a good with a steep demand curve will be less responsive to changes in price than will one with a less steep demand curve.

Since the slope of a demand curve is much simpler to calculate than its elasticity, it may seem natural to ask, "Why bother with elasticity at all?" One reason is that the slope of the demand curve is sensitive to the units we use to measure price and quantity, while elasticity is not. By way of illustration, notice in Figure 4.22*a* that when the price of gasoline is measured in \$/gal, the slope of the demand curve at point C is -0.02. By contrast, in Figure 4.22*b*, where price is measured in \$/oz, the slope at C is -0.00015625. In both cases, however, note that the price elasticity of demand at C is -3. This will be true no matter how we measure price and quantity.

FIGURE 4.22
Elasticity Is Unit-Free
The slope of the demand curve at any point depends on the units in which we measure price and quantity. The slope at point *C* when we measure the price of gasoline in dollars per gallon (*a*) is much larger than when we measure the price in dollars per ounce (*b*). The price elasticity at any point, by contrast, is completely independent of units of measure.



And most people find it much more informative to know that a 1 percent cut in price will lead to a 3 percent increase in the quantity demanded than to know that the slope of the demand curve is -0.00015625.

SOME REPRESENTATIVE ELASTICITY ESTIMATES

As the entries in Table 4.4 show, the price elasticities of demand for different products often differ substantially. The low elasticity for theater and opera performances probably reflects the fact that buyers in this market have much larger than average incomes, so that income effects of price variations are likely to be small. Income effects for green peas are also likely to be small even for low-income consumers, yet the price elasticity of demand for green peas is more than 14 times larger than for theater and opera performances. The difference is that there are many more close substitutes for green peas than for theater and opera performances. Later in this chapter we investigate in greater detail the factors that affect the price elasticity of demand for a product.

TABLE 4.4
Price Elasticity Estimates for Selected Products*

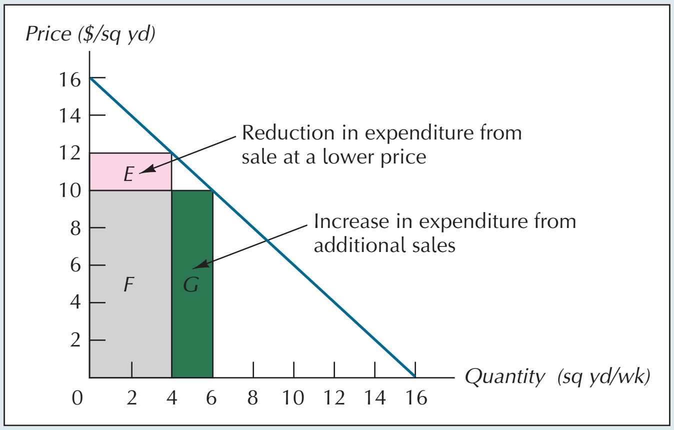
Good or service	Price elasticity
Green peas	−2.8
Air travel (vacation)	−1.9
Frying chickens	−1.8
Beer	−1.2
Marijuana	−1.0
Movies	−0.9
Air travel (nonvacation)	−0.8
Shoes	−0.7
cigarettes	−0.3
Theater, opera	−0.2
Local telephone calls	−0.1

Some of these short-run elasticity estimates represent the midpoint of the corresponding range of estimates. Sources: Fred Nordhauser and Paul L. Farris, "An Estimate of the Short-Run Price Elasticity of Demand for Fryers," *Journal of Farm Economics,* November 1959; H. S. Houthakker and Lester Taylor, *Consumer Demand in the United States: Analyses and Projections,* 2d ed., Cambridge, MA: Harvard University Press, 1970; Charles T. Nisbet and Firouz Vakil, "Some Estimates of Price and Expenditure Elasticities of Demand for Marijuana among UCLA Students," *Review of Economics and Statistics,* November 1972; L. Taylor, "The Demand for Electricity: A Survey," *Bell Journal of Economics,* Spring 1975; K. Elzinga, "The Beer Industry," in Walter Adams (ed.), *The Structure of American Industry,* New York: Macmillan, 1977; Rolla Edward Park, Bruce M. Wetzel, and Bridger Mitchell, *Charging for Local Telephone Calls: Price Elasticity Estimates from the GTE Illinois Experiment,* Santa Monica, CA: Rand Corporation, 1983; Tae H. Oum, W. G. Waters II, and Jong Say Yong, "A Survey of Recent Estimates of Price Elasticities of Demand for Transport," World Bank Infrastructure and Urban Development Department Working Paper 359, January 1990; M. C. Farrelly and J. W. Bray, "Response to Increases in Cigarette Prices by Race/Ethnicity, Income, and Age Groups—United States, 1976–1993," *Journal of the American Medical Association,* 280, 1998.

ELASTICITY AND TOTAL EXPENDITURE

Suppose you are the administrator in charge of setting tolls for the Golden Gate Bridge, which links San Francisco to Marin County. Suppose that with the toll at \$3/trip, 100,000 trips per hour are taken across the bridge. If the price elasticity of demand for trips is -2.0, what will happen to the number of trips taken per

FIGURE 4.23
The Effect on Total Expenditure of a Reduction in Price
When price falls, people spend less on existing units (*E*). But they also buy more units (*G*). Here, *G* is larger than *E,* which means that total expenditure rises.



hour if you raise the toll by 10 percent? With an elasticity of −2.0, a 10 percent increase in price will produce a 20 percent reduction in quantity. Thus the number of trips will fall to 80,000/hr. Total expenditure at the higher toll will be (80,000 trips/hr)($3.30/trip) = $264,000/hr. Note that this is smaller than the total expenditure of $300,000/hr that occurred under the $3 toll.

Now suppose that the price elasticity had been not −2.0 but −0.5. How would the number of trips and total expenditure then be affected by a 10 percent increase in the toll? This time the number of trips will fall by 5 percent to 95,000/hr, which means that total expenditure will rise to (95,000 trips/hr) ($3.30/trip) = $313,500/hr. If your goal as an administrator is to increase the total revenue collected from the bridge toll, you need to know something about the price elasticity of demand before deciding whether to raise the toll or lower it.

This example illustrates the important relationships between price elasticity and total expenditure. The questions we want to be able to answer are of the form, "If the price of a product changes, how will total spending on the product be affected?" and "Will more be spent if we sell more units at a lower price or fewer units at a higher price?" In Figure 4.23, for example, we might want to know how total expenditures for shelter are affected when the price falls from $12/sq yd to $10/sq yd.

The total expenditure, *R,* at any quantity-price pair (*Q, P*) is given by the product

$$R = PQ. \tag{4.4}$$

In Figure 4.23, the total expenditure at the original quantity-price pair is thus ($12/sq yd)(4 sq yd/wk) = $48/wk. Geometrically, it is the sum of the two shaded areas *E* and *F.* Following the price reduction, the new total expenditure is ($10/sq yd)(6 sq yd/wk) = $60/wk, which is the sum of the shaded areas *F* and *G*. These two total expenditures have in common the shaded area *F.* The change in total expenditure is thus the difference in the two shaded areas *E* and *G*. The area *E,* which is ($2/sq yd)(4 sq yd/wk) = $8/wk, may be interpreted as the reduction in expenditure caused by selling the original 4 sq yd/wk at the new, lower price. *G,* in turn, is the increase in expenditure caused by the additional 2 sq yd/wk of sales. This area is given by ($10/sq yd)(2 sq yd/wk) = $20/wk. Whether total expenditure rises or falls thus boils down to whether the gain from additional sales exceeds the loss from lower prices. Here, the gain exceeds the loss by $12, so total expenditure rises by that amount following the price reduction.

If the change in price is small, we can say how total expenditure will move if we know the initial price elasticity of demand. Recall that one way of expressing price elasticity is the percentage change in quantity divided by the corresponding percentage change in price. If the absolute value of that quotient exceeds 1, we know that the percentage change in quantity is larger than the percentage change in price. And when that happens, the increase in expenditure from additional sales will always exceed the reduction from sales of existing units at the lower price. In Figure 4.23, note that the elasticity at the original price of \$12 is 3.0, which confirms our earlier observation that the price reduction led to an increase in total expenditure. Suppose, on the contrary, that price elasticity is less than unity. Then the percentage change in quantity will be smaller than the corresponding percentage change in price, and the additional sales will not compensate for the reduction in expenditure from sales at a lower price. Here, a price reduction will lead to a reduction in total expenditure.

EXERCISE 4.6

For the demand curve in Figure 4.23, what is the price elasticity of demand when *P* = \$4/sq yd? What will happen to total expenditure on shelter when price falls from \$4/sq yd to \$3/sq yd?

The general rule for small price reductions, then, is this: *A price reduction will increase total revenue if and only if the absolute value of the price elasticity of demand is greater than 1.* Parallel reasoning leads to an analogous rule for small price increases: *An increase in price will increase total revenue if and only if the absolute value of the price elasticity is less than 1.* These rules are summarized in the top panel of Figure 4.24, where the point *M* is the midpoint of the demand curve.

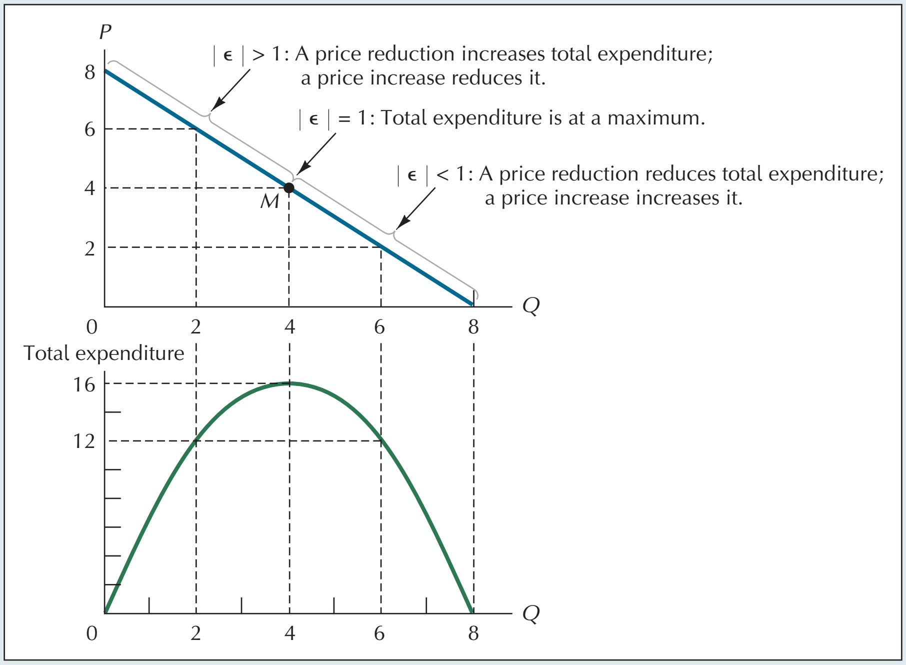


FIGURE 4.24
Demand and Total Expenditure
When demand is elastic, total expenditure changes in the opposite direction from a change in price. When demand is inelastic, total expenditure and price both move in the same direction. At the midpoint of the demand curve (*M*), total expenditure is at a maximum.

The relationship between elasticity and total expenditure is spelled out in greater detail in the relationship between the top and bottom panels of Figure 4.24. The top panel shows a straight-line demand curve. For each quantity, the bottom panel shows the corresponding total expenditure. As indicated in the bottom panel, total expenditure starts at zero when *Q* is zero and increases to its maximum value at the quantity corresponding to the midpoint of the demand curve (point *M* in the top panel). At that quantity, price elasticity is unity. Beyond that quantity, total expenditure declines with output, reaching zero at the horizontal intercept of the demand curve.

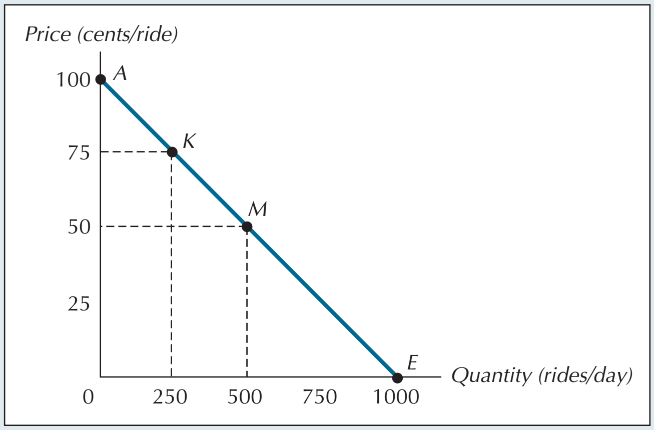
EXAMPLE 4.6

The market demand curve for bus rides in a small community is given by $P = 100 - (Q/10)$, where P is the fare per ride in cents and Q is the number of rides each day. If the price is 50 cents/ride, how much revenue will the transit system collect each day? What is the price elasticity of demand for rides? If the system needs more revenue, should it raise or lower price? How would your answers have differed if the initial price had been not 50 cents/ride but 75?

Total revenue for the bus system is equal to total expenditure by riders, which is the product PQ. First we solve for Q from the demand curve and get $Q = 1000 - 10P$. When P is 50 cents/ride, Q will be 500 rides/day and the resulting total revenue will be \$250/day. To compute the price elasticity of demand, we can use the formula $\epsilon = (P/Q)(1/slope)$. Here the slope is $-\frac{1}{10}$, so $1/\text{slope} = -10$ (see footnote 3). P/Q takes the value $50/500 = \frac{1}{10}$. Price elasticity is thus the product $(-\frac{1}{10})(10) = -1$. With a price elasticity of unity, total revenue attains its maximum value. If the bus company either raises or lowers its price, it will earn less than it does at the current price.

At a price of 50 cents, the company was operating at the midpoint of its demand curve. If the price had instead been 75 cents, it would be operating above the midpoint. More precisely, it would be halfway between the midpoint and the vertical intercept (point K in Figure 4.25). Quantity would be only 250 rides/day, and price elasticity would have been -3 (computed, for example, by multiplying the price-quantity ratio at K, $\frac{3}{10}$, by the reciprocal of the demand curve slope, $-\frac{1}{10}$). Operating at an elastic point on its demand curve, the company could increase total revenue by cutting its price.

FIGURE 4.25
The Demand for Bus Rides
At a price of 50 cents/ride, the bus company is maximizing its total revenues. At a price of 75 cents/ride, demand is elastic with respect to price, and so the company can increase its total revenues by cutting its price.



DETERMINANTS OF PRICE ELASTICITY OF DEMAND

What factors influence the price elasticity of demand for a product? Our earlier discussion of substitution and income effects suggests primary roles for the following factors:

- **Substitution possibilities.** The substitution effect of a price change tends to be small for goods with no close substitutes. Consider, for example, the vaccine against rabies. People who have been bitten by rabid animals have no substitute for this vaccine, so demand for it is highly inelastic. We saw that the same was true for a good such as salt. But consider now the demand for a particular brand of salt, say, Morton's. Despite the advertising claims of salt manufacturers, one brand of salt is a more-or-less perfect substitute for any other.

[3]The slope here is from the formula $P = 100 - (Q/10)$.

Because the substitution effect between specific brands is large, a rise in the price of one brand should sharply curtail the quantity of it demanded. In general, the absolute value of price elasticity will rise with the availability of attractive substitutes.

- **Budget share.** The larger the share of total expenditures accounted for by the product, the more important will be the income effect of a price change. Goods such as salt, rubber bands, cellophane wrap, and a host of others account for such small shares of total expenditures that the income effects of a price change are likely to be negligible. For goods like housing and higher education, by contrast, the income effect of a price increase is likely to be large. In general, the smaller the share of total expenditure accounted for by a good, the less elastic demand will be.
- **Direction of income effect.** A factor closely related to the budget share is the direction—positive or negative—of its income effect. While the budget share tells us whether the income effect of a price change is likely to be large or small, the direction of the income effect tells us whether it will offset or reinforce the substitution effect. Thus, a normal good will have a higher price elasticity than an inferior good, other things equal, because the income effect reinforces the substitution effect for a normal good but offsets it for an inferior good.
- **Time.** Our analysis of individual demand did not focus explicitly on the role of time. But it too has an important effect on responses to changes in prices. Consider the oil price increases of recent years. One possible response is simply to drive less. But many auto trips cannot be abandoned, or even altered, very quickly. A person cannot simply stop going to work, for example. He can cut down on his daily commute by joining a car pool or by purchasing a house closer to where he works. He can also curtail his gasoline consumption by trading in his current car for one that gets better mileage. But all these steps take time, and as a result, the demand for gasoline will be much more elastic in the long run than in the short run.

The short- and long-run effects of a supply shift in the market for gasoline are contrasted in Figure 4.26. The initial equilibrium at *A* is disturbed by a supply reduction from *S* to *S′*. In the short run, the effect is for price to rise to P_{SR} = \$2.80/gal and for quantity to fall to Q_{SR} = 5 million gal/day. The long-run demand curve is more elastic than the short-run demand curve. As consumers have more time to adjust, therefore, price effects tend to moderate while quantity effects tend to become

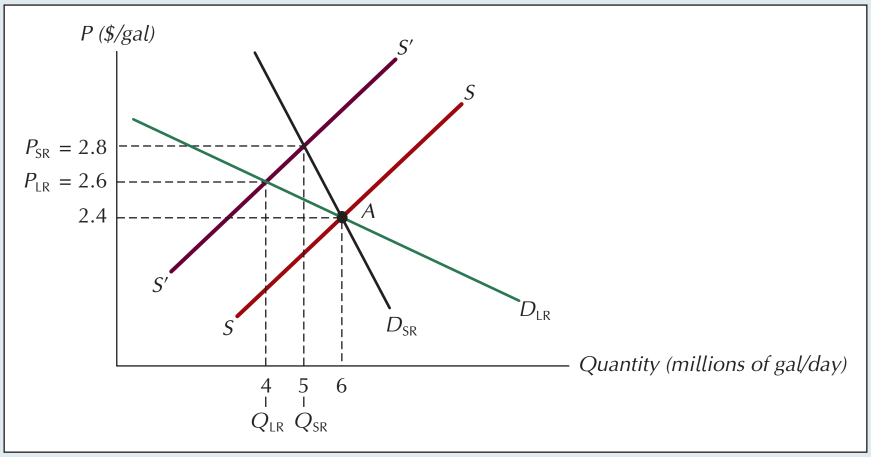


FIGURE 4.26
Price Elasticity Is Greater in the Long Run Than in the Short Run
The more time people have, the more easily they can switch to substitute products. The price effects of supply alterations are therefore always more extreme in the short run than in the long run.

more pronounced. Thus the new long-run equilibrium in Figure 4.26 occurs at a price of P_{LR} = \$2.60/gal and a quantity of Q_{LR} = 4 million gal/day.

We see an extreme illustration of the difference between short- and long-run price elasticity values in the case of natural gas used in households. The price elasticity for this product is only −0.1 in the short run but a whopping −10.7 in the long run![4] This difference reflects the fact that once a consumers have chosen appliances to heat and cook with, they are virtually locked in for the short run. People aren't going to cook their rice for only 10 minutes just because the price of natural gas has gone up. In the long run, however, consumers can and do switch between fuels when there are significant changes in relative prices.

THE DEPENDENCE OF MARKET DEMAND ON INCOME

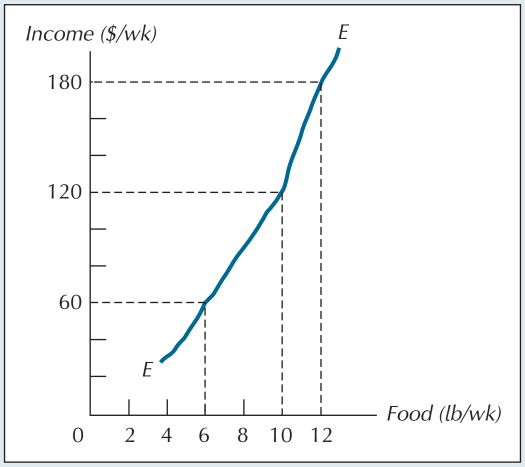
As we have seen, the quantity of a good demanded by any person depends not only on its price but also on the person's income. Since the market demand curve is the horizontal sum of individual demand curves, it too will be influenced by consumer incomes. In some cases, the effect of income on market demand can be accounted for completely if we know only the average income level in the market. This would be the case, for example, if all consumers in the market were alike in terms of preference and all had the same incomes.

In practice, however, a given level of average income in a market will sometimes give rise to different market demands depending on how income is distributed. A simple example helps make this point clear.

EXAMPLE 4.7

Two consumers, *A* and *B*, are in a market for food. Their tastes are identical, and each has the same initial income level, \$120/wk. If their individual Engel curves for food are as given by *EE* in Figure 4.27, how will the market demand curve for food be affected if *A*'s income goes down by 50 percent while *B*'s goes up by 50 percent?

FIGURE 4.27
The Engel Curve for Food of *A* and *B*
When individual Engel curves take the nonlinear form shown, the increase in food consumption that results from a given increase in income will be smaller than the reduction in food consumption that results from an income reduction of the same amount.



The nonlinear shape of the Engel curve pictured in Figure 4.27 is plausible considering that a consumer can eat only so much food. Beyond some point, increases in income should have no appreciable effect on the amount of food consumed. The implication is that *B*'s new income (\$180/wk) will produce an increase in his

[4] H. S. Houthakker and Lester Taylor, *Consumer Demand in the United States: Analyses and Projections,* 2d ed., Cambridge, MA: Harvard University Press, 1970.

consumption (2 lb/wk) that is smaller than the reduction in *A*'s consumption (4 lb/wk) caused by *A*'s new income ($60/wk).

What does all this say about the corresponding individual and market demand curves for food? Identical incomes and tastes give rise to identical individual demand curves, denoted D_A and D_B in Figure 4.28. Adding D_A and D_B horizontally, we get the initial market demand curve, denoted *D*. The nature of the individual Engel curves tells us that *B*'s increase in demand will be smaller than *A*'s reduction in demand following the shift in income distribution. Thus, when we add the new individual demand curves (D'_A and D'_B), we get a new market demand for food (D') that lies to the left of the original demand curve.

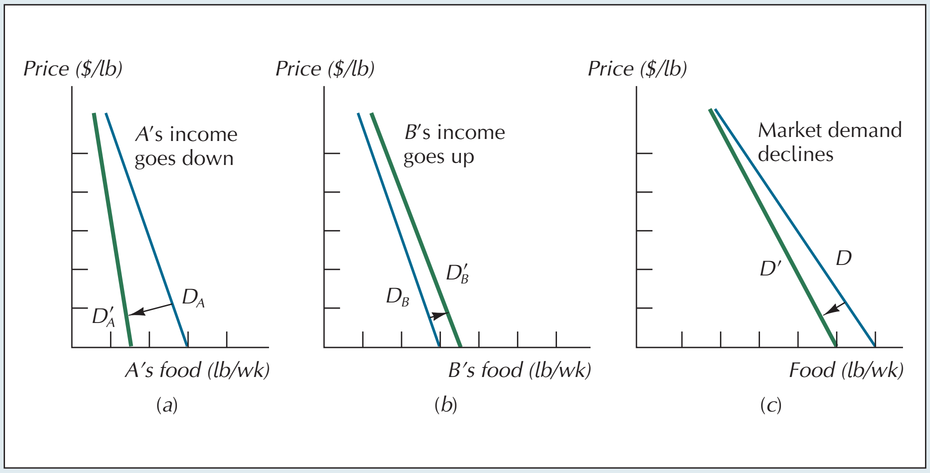


FIGURE 4.28
Market Demand Sometimes Depends on the Distribution of Income
A given increase in income produces a small demand increase for *B* (*b*); an income reduction of the same size produces a larger demand reduction for *A* (*a*). The redistribution from *A* to *B* leaves average income unchanged but reduces market demand (*c*).

The dependence of market demands on the distribution of income is important to bear in mind when the government considers policies to redistribute income. A policy that redistributes income from rich to poor, for example, is likely to increase demand for goods like food and reduce demand for luxury items, such as jewelry and foreign travel.

Demand in many other markets is relatively insensitive to variations in the distribution of income. In particular, the distribution of income is not likely to matter much in markets in which individual demands tend to move roughly in proportion to changes in income.

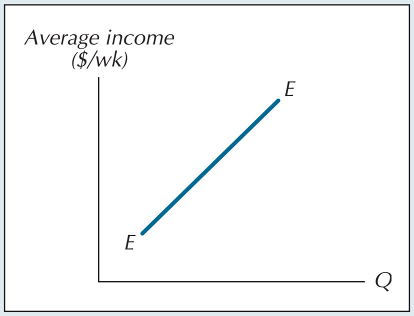
Engel curves at the market level are schedules that relate the quantity demanded to the average income level in the market. The existence of a stable relationship between average income and quantity demanded is by no means certain for any given product because of the distributional complication just discussed. In particular, note that we cannot construct Engel curves at the market level by simply adding individual Engel curves horizontally. Horizontal summation works as a way of generating market demand curves from individual demand curves because all consumers in the market face the same market price for the product. But when incomes differ widely from one consumer to another, it makes no sense to hold income constant and add quantities across consumers.

As a practical matter, however, reasonably stable relationships between various aggregate income measures and quantities demanded in the market may nonetheless exist. Suppose such a relationship exists for the good *X* and is as pictured by *EE* in Figure 4.29, where *Y* denotes the average income level of consumers in the market for *X*, and *Q* denotes the quantity of *X*. This locus is the market analog of the individual Engel curves discussed earlier.

income elasticity of demand the percentage change in the quantity of a good demanded that result from a 1 percent change in income.

If a good exhibits a stable Engel curve, we may then define its **income elasticity of demand,** a formal measure of the responsiveness of purchase decisions to

FIGURE 4.29
An Engel Curve at the Market Level
The market Engel curve tells what quantities will be demanded at various average levels of income.



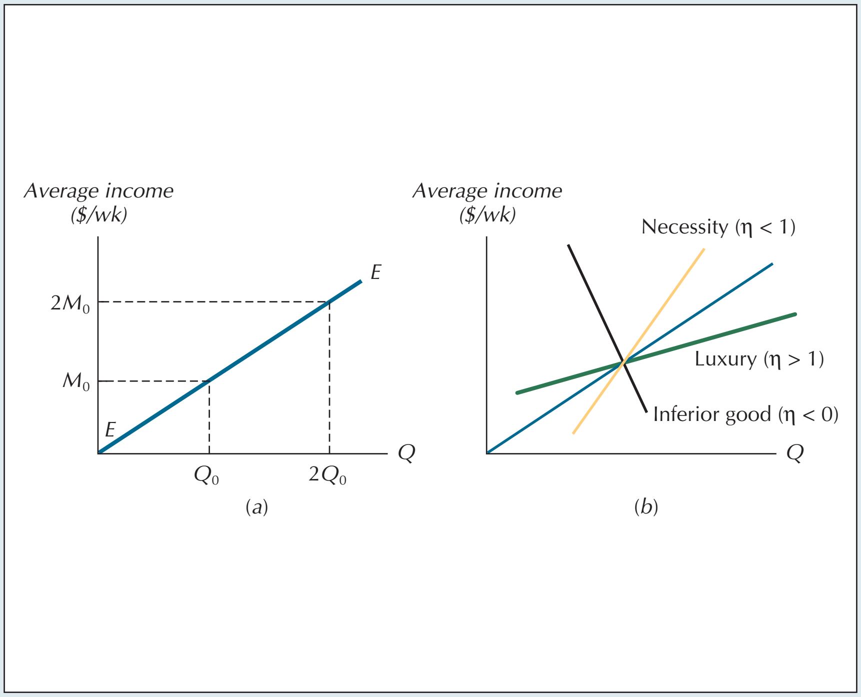
variations in the average market income. Denoted η, it is given by a formula analogous to the one for price elasticity:[5]

$$\eta = \frac{\Delta Q/Q}{\Delta Y/Y}, \tag{4.5}$$

where Y denotes average market income and ΔY is a small change therein.

Goods such as food, for which a change in income produces a less than proportional change in the quantity demanded at any price, thus have an income elasticity less than 1. Such goods are called *necessities,* and their income elasticities must lie in the interval $0 < \eta < 1$. Food is a commonly cited example. *Luxuries* are those goods for which $\eta > 1$. Common examples are expensive jewelry and foreign travel. Inferior goods are those for which $\eta < 0$. Goods for which $\eta = 1$ will have Engel curves that are straight lines through the origin, as pictured by the locus *EE* in Figure 4.30*a*. The

FIGURE 4.30
Engel Curves for Different Types of Goods
(*a*) The good whose Engel curve is shown has an income elasticity of 1. For such goods, a given proportional change in income will produce the same proportional change in quantity demanded. Thus when average income doubles, from M_0 to $2M_0$, the quantity demanded also doubles, from Q_0 to $2Q_0$. (*b*) The Engel curves show that consumption increases more than in proportion to income for a luxury and less than in proportion to income for a necessity, and it falls with income for an inferior good.



[5]In calculus terms, the corresponding formula is $\eta = (Y/Q)\,[dQ(Y)/dY]$.

market Engel curves for luxuries, necessities, and inferior goods, where these exist and are stable, are pictured in Figure 4.30*b*.

The income elasticity formula in Equation 4.5 is easier to interpret geometrically if we rewrite it as

$$\eta = \frac{Y}{Q}\frac{\Delta Q}{\Delta Y}, \tag{4.6}$$

The first factor on the right side of Equation 4.6 is simply the ratio of income to quantity at a point along the Engel curve. It is the slope of the line from the origin (a ray) to that point. The second factor is the reciprocal of the slope of the Engel curve at that point. If the slope of the ray exceeds the slope of the Engel curve, the product of these two factors must be greater than 1 (the luxury case). If the ray is less steep, η will be less than 1 but still positive, provided the slope of the Engel curve is positive (the necessity case). Thus, in distinguishing between the Engel curves for necessities and luxuries, what counts is not the slopes of the Engel curves themselves but how they compare with the slopes of the corresponding rays. Finally, if the slope of the Engel curve is negative, η must be less than zero (the inferior case).

ECONOMIC NATURALIST 4.1

Why has the nature of outdoor cooking appliances changed dramatically in recent decades?

The propane grill I bought during the late 1980s was on a downhill slide for several years. First to go was its ignition button, the crude mechanical spark generator that normally fires up the gas. Lighting the grill suddenly became a delicate operation. I would turn on the gas, wait a few seconds, and then throw a match inside. If I threw it in too soon, it would go out before reaching the burner below. But if I waited too long, it would set off a small explosion. Another problem was that the metal baffle that sat atop the burners had rusted through in the middle. This concentrated an enormous amount of heat over a small area near the center of the cooking surface, but very little elsewhere. I was still able to cook reasonably good chicken and small steaks by quickly rotating pieces in and out of the hot zone. But grilling a big fish filet was impossible.

My grill's various deficiencies could surely be repaired, but I had no idea by whom. And even if I did, the cost would almost surely exceed the $89.95 I originally paid for it. And so, reluctantly, I found myself in the market for a new one.

Courtesy of Robert H. Frank

1989 Sunbeam Grill, $90

© Gary Moss/Jupiter Images

A well-equipped professional grill (Comparable to Viking professional Grill referenced in text)

I was immediately struck by how the menu of available choices had changed in the intervening years. I vaguely remember models from the late 1980s that had built-in storage cabinets and shelf extensions on either side. But even with these options, the most you could have spent was a few hundred dollars. There was nothing—absolutely nothing—like the Viking Professional Grill.

Powered by either natural gas or propane, it comes with an infrared rotisserie that can slowly broil two 20-pound turkeys to perfection as you cook hamburgers for forty guests on its 828-square-inch grilling surface. It has a built-in smoker system that "utilizes its own 5,000-BTU burner and watertight wood chip drawer to season food with rich woodsy flavor." Next to its grilling surface sit two ancillary range-top burners. Unlike the standard burners on your kitchen stove, which generate 7,500 BTUs, these burners generate 15,000 BTUs, a capability that is useful primarily for the flash-stir-frying of some ethnic cuisines and for bringing large cauldrons of water more quickly to a boil. If you have ever longed to throw together a Szechwan pork dish on your backyard patio, or feared getting off to a late start when you have guests about to arrive and 40 ears of corn left to cook, the Viking has the extra power you may need. The entire unit is constructed of gleaming stainless steel, with enamel and brass accents, and with its fold-out workspaces fully extended, it measures more than seven feet across.

The Frontgate catalog's price of the Viking Professional Grill, not including shipping and handling, is $5,000. Other outdoor cooking appliances are now offered that cost more than ten times that amount.

What spawned this dramatic boom in the American outdoor luxury grill market? The short answer is that most of the recent growth in income in the United States occurred among the nation's highest earners. For example, although median after-tax family income grew by only 12.6 percent between 1979 and 2003, the corresponding growth for the top 1 percent of earners was over 200 percent.[6] Still higher up the income ladder, income growth was even more dramatic. Thus CEOs of the largest U.S. corporations, who earned 42 times as much as the average worker in 1980, earned 531 times as much in 2000.[7] Rapid income growth among those with already high incomes spawned increased demand not only for costly outdoor cooking appliances, but for a broad spectrum of other luxury goods as well.

APPLICATION: FORECASTING ECONOMIC TRENDS

If the income elasticity of demand for every good and service were 1, the composition of GNP would be completely stable over time (assuming technology and relative prices remain unchanged). Each year, the proportion of total spending devoted to food, travel, clothing, and indeed to every other consumption category would remain unchanged.

As the entries in Table 4.5 show, however, the income elasticities of different consumption categories differ markedly. And therein lies one of the most important applications of the income elasticity concept, namely, forecasting the composition of future purchase patterns. Ever since the industrial revolution in the West, real purchasing power per capita has grown at roughly 2 percent per year. Our knowledge of income elasticity differences enables us to predict how consumption patterns in the future will differ from the ones we see today.

Thus, a growing share of the consumer's budget will be devoted to goods like restaurant meals and automobiles, whereas ever smaller shares will go to tobacco,

[6]http://www.census.gov/hhes/income/histinc/hϕ3ar.html and Center on Budget and Policy Priorities, "The New, Definitive CBO Data on Income and Tax Trends," September 23, 2003.

[7]*Business Week,* annual executive compensation surveys. See www.inequality.org.

TABLE 4.5
Income Elasticities of Demand for Selected Products*

Good or service	Income elasticity
Automobiles	2.46
Furniture	1.48
Restaurant meals	1.40
Water	1.02
Tobacco	0.64
Gasoline and oil	0.48
Electricity	0.20
Margarine	−0.20
Pork products	−0.20
Public transportation	−0.36

*These estimates come from H. S. Houthakker and Lester Taylor, *Consumer Demand in the United States: Analyses and Projections,* 2d ed., Cambridge, MA: Harvard University Press, 1970; L. Taylor and R. Halvorsen, "Energy Substitution in U.S. Manufacturing," *Review of Economics and Statistics,* November 1977; H. Wold and L. Jureen, *Demand Analysis,* New York: Wiley, 1953.

fuel, and electricity. And if the elasticity estimates are correct, the absolute amounts spent per person on margarine, pork products, and public transportation will be considerably smaller in the future than they are today.

CROSS-PRICE ELASTICITIES OF DEMAND

The quantity of a good purchased in the market depends not only on its price and consumer incomes but also on the prices of related goods. **Cross-price elasticity of demand** is the percentage change in the quantity demanded of one good caused by a 1 percent change in the price of the other. More generally, for any two goods, X and Z, the cross-price elasticity of demand may be defined as follows:[8]

cross-price elasticity of demand the percentage change in the quantity of one good demanded that results from a 1 percent change in the price of the other good.

$$\epsilon_{XZ} = \frac{\Delta Q_X / Q_X}{\Delta P_Z / P_Z}, \tag{4.7}$$

where ΔQ_X is a small change in Q_X, the quantity of X, and ΔP_Z is a small change in P_Z, the price of Z. ϵ_{XZ} measures how the quantity demanded of X responds to a small change in the price of Z.

Unlike the elasticity of demand with respect to a good's own price (the *own-price elasticity*), which is never greater than zero, the cross-price elasticity may be either positive or negative. X and Z are defined as *complements* if $\epsilon_{XZ} < 0$. If $\epsilon_{XZ} > 0$, they are *substitutes*. Thus, a rise in the price of ham will reduce not only the quantity of ham demanded, but also, because ham and eggs are complements, the demand for eggs. A rise in the price of coffee, by contrast, will tend to increase the demand for tea. Estimates of the cross-price elasticity of demand for selected pairs of products are shown in Table 4.6.

[8]In calculus terms, the corresponding expression is given by $\epsilon_{XZ} = (P_Z/Q_X)(dQ_X/dP_Z)$.

TABLE 4.6
Cross-Price Elasticities for Selected Pairs of Products*

Good or service	Good or service with price change	Cross-price elasticity
Butter	Margarine	+0.81
Margarine	Butter	+0.67
Natural gas	Fuel oil	+0.44
Beef	Pork	+0.28
Electricity	Natural gas	+0.20
Entertainment	Food	−0.72
Cereals	Fresh fish	−0.87

*From H. Wold and L. Jureen, *Demand Analysis,* New York: Wiley, 1953; L. Taylor and R. Halvorsen, "Energy Substitution in U.S. Manufacturing," *Review of Economics and Statistics,* November 1977; E. T. Fujii et al., "An Almost Ideal Demand System for Visitor Expenditures," *Journal of Transport Economics and Policy,* 19, May 1985, 161–171; and A. Deaton, "Estimation of Own- and Cross-Price Elasticities from Household Survey Data," *Journal of Econometrics,* 36, 1987: 7–30.

EXERCISE 4.7

Would the cross-price elasticity of demand be positive or negative for the following pairs of goods: (*a*) apples and oranges, (*b*) airline tickets and automobile tires, (*c*) computer hardware and software, (*d*) pens and paper, (*e*) pens and pencils?

SUMMARY

- Our focus in this chapter was on how individual and market demands respond to variations in prices and incomes. To generate a demand curve for an individual consumer for a specific good X, we first trace out the price-consumption curve in the standard indifference curve diagram. The PCC is the line of optimal bundles observed when the price of X varies, with both income and preferences held constant. We then take the relevant price-quantity pairs from the PCC and plot them in a separate diagram to get the individual demand curve.

- The income analog to the PCC is the income-consumption curve, or ICC. It too is constructed using the standard indifference curve diagram. The ICC is the line of optimal bundles traced out when we vary the consumer's income, holding preferences and relative prices constant. The Engel curve is the income analog to the individual demand curve. We generate it by retrieving the relevant income-quantity pairs from the ICC and plotting them in a separate diagram.

- Normal goods are those the consumer buys more of when income increases, and inferior goods are those the consumer buys less of as income rises.

- The total effect of a price change can be decomposed into two separate effects: (1) the substitution effect, which denotes the change in the quantity demanded that results because the price change makes substitute goods seem either more or less attractive, and (2) the income effect, which denotes the change in quantity demanded that results from the change in real purchasing power caused by the price change. The substitution effect always moves in the opposite direction from the movement in price: price increases [reductions] always reduce [increase] the quantity demanded. For normal goods, the income effect also moves in the opposite direction from the price change and thus tends to reinforce the substitution effect. For inferior goods, the income effect moves in the same direction as the price change and thus tends to undercut the substitution effect.

- The fact that the income and substitution effects move in opposite directions for inferior goods suggests the theoretical possibility of a Giffen good, one for which the total effect of a price increase is to increase the quantity demanded. There have been no documented examples of Giffen goods, and in this text we adopt the convention that all goods, unless otherwise stated, are demanded in smaller quantities at higher prices.

- Goods for which purchase decisions respond most strongly to price tend to be ones that have large income and substitution effects that work in the same direction. For example, a normal good that occupies a large share of total expenditures and for which there are many direct or indirect substitutes will tend to respond sharply to changes in price. For many consumers, housing is a prime example of such a good. The goods least responsive to price changes will be those that account for very small budget shares and for which substitution possibilities are very limited. For most people, salt has both of these properties.

- There are two equivalent techniques for generating market demand curves from individual demand curves. The first is to display the individual curves graphically and then add them horizontally. The second is algebraic and proceeds by first solving the individual demand curves for the respective Q values, then adding those values, and finally solving the resulting sum for P.

- A central analytical concept in demand theory is the price elasticity of demand, a measure of the responsiveness of purchase decisions to small changes in price. It is defined as the percentage change in quantity demanded that is caused by a 1 percent change in price. Goods for which the absolute value of elasticity exceeds 1 are said to be elastic; those for which it is less than 1, inelastic; and those for which it is equal to 1, unit elastic.

- Another important relationship is the one between price elasticity and the effect of a price change on total expenditure. When demand is elastic, a price reduction will increase total expenditure; when inelastic, total expenditure falls when the price goes down. When demand is unit elastic, total expenditure is at a maximum.

- The price elasticity of demand depends largely on four factors: substitutability, budget share, direction of income effect, and time. (1) *Substitutability.* The more easily consumers may switch to other goods, the more elastic demand will be. (2) *Budget share.* Goods that account for a large share of total expenditures will tend to have higher price elasticity. (3) *Direction of income effect.* Other factors the same, inferior goods will tend to be less elastic with respect to price than normal goods. (4) *Time.* Habits and existing commitments limit the extent to which consumers can respond to price changes in the short run. Price elasticity of demand will tend to be larger, the more time consumers have to adapt.

- Changes in the average income level in a market generally shift the market demand curve. The income elasticity of demand is defined analogously to price elasticity. It is the percentage change in quantity that results from a 1 percent change in income. Goods whose income elasticity of demand exceeds zero are called normal goods; those for which it is less than zero are called inferior; those for which it exceeds 1 are called luxuries; and those for which it is less than 1 are called necessities. For normal goods, an increase in income shifts market demand to the right; and for inferior goods, an increase in income shifts demand to the left. For some goods, the distribution of income, not just its average value, is an important determinant of market demand.

- The cross-price elasticity of demand is a measure of the responsiveness of the quantity demanded of one good to a small change in the price of another. It is defined as the percentage change in the quantity demanded of one good that results from a 1 percent change in the price of the other. If the cross-price elasticity of demand for X with respect to the price of Z is positive, X and Z are substitutes; and if negative, they are complements. In remembering the formulas for the various elasticities—own price, cross-price, and income—many people find it helpful to note that each is the percentage change in an effect divided by the percentage change in the associated causal factor.

- The Appendix to this chapter examines additional topics in demand theory, including the constant elasticity demand curve and the income-compensated demand curve.

QUESTIONS FOR REVIEW

1. Why does the quantity of salt demanded tend to be unresponsive to changes in its price?
2. Why is the quantity of education demanded in private universities much more responsive than salt is to changes in price?
3. Draw Engel curves for both a normal good and an inferior good.
4. Give two examples of what are, for most students, inferior goods.
5. Can the price-consumption curve for a normal good ever be downward-sloping?
6. To get the market demand curve for a product, why do we add individual demand curves horizontally rather than vertically?
7. Summarize the relationship between price elasticity, changes in price, and changes in total expenditure.
8. Why don't we measure the responsiveness of demand to price changes by the slope of the demand curve instead of using the more complicated expression for elasticity?
9. For a straight-line demand curve, what is the price elasticity at the revenue maximizing point?
10. Do you think a college education at a specific school has a high or low price (tuition) elasticity of demand?
11. How can changes in the distribution of income across consumers affect the market demand for a product?
12. If you expected a long period of declining GNP, what kinds of companies would you invest in?

13. *True or false:* For a budget spent entirely on two goods, an increase in the price of one will necessarily decrease the consumption of both, unless at least one of the goods is inferior. Explain.

14. Mike spends all his income on tennis balls and basketball tickets. His demand curve for tennis balls is elastic. *True or false:* If the price of tennis balls rises, he consumes more tickets. Explain.

15. *True or false:* If each individual in a market has a straight-line demand curve for a good, then the market demand curve for that good must also be a straight line. Explain.

16. Suppose your budget is spent entirely on two goods: bread and butter. If bread is an inferior good, can butter be inferior as well?

PROBLEMS

1. Sam spends \$6/wk on orange juice and apple juice. Orange juice costs \$2/cup while apple juice costs \$1/cup. Sam views 1 cup of orange juice as a perfect substitute for 3 cups of apple juice. Find Sam's optimal consumption bundle of orange juice and apple juice each week. Suppose the price of apple juice rises to \$2/cup, while the price of orange juice remains constant. How much additional income would Sam need to afford his original consumption bundle?

2. Bruce has the same income and faces the same prices as Sam in Problem 1, but he views 1 cup of orange juice as a perfect substitute for 1 cup of apple juice. Find Bruce's optimal consumption bundle. How much additional income would Bruce need to be able to afford his original consumption bundle when the price of apple juice doubles?

3. Maureen has the same income and faces the same prices as Sam and Bruce, but Maureen views 1 cup of orange juice and 1 cup of apple juice as perfect complements. Find Maureen's optimal consumption bundle. How much additional income would Maureen need to afford her original consumption bundle when the price of apple juice doubles?

4. The market for lemonade has 10 potential consumers, each having an individual demand curve $P = 101 - 10Q_i$, where P is price in dollars per cup and Q_i is the number of cups demanded per week by the ith consumer. Find the market demand curve using algebra. Draw an individual demand curve and the market demand curve. What is the quantity demanded by each consumer and in the market as a whole when lemonade is priced at $P = \$1$/cup?

5. a. For the demand curve $P = 60 - 0.5Q$, find the elasticity at $P = 10$.
   b. If the demand curve shifts parallel to the right, what happens to the elasticity at $P = 10$?

6. Consider the demand curve $Q = 100 - 50P$.
   a. Draw the demand curve and indicate which portion of the curve is elastic, which portion is inelastic, and which portion is unit elastic.
   b. Without doing any additional calculation, state at which point of the curve expenditures on the goods are maximized, and then explain the logic behind your answer.

7. Suppose the demand for crossing the Golden Gate Bridge is given by $Q = 10{,}000 - 1000P$.
   a. If the toll (P) is \$3, how much revenue is collected?
   b. What is the price elasticity of demand at this point?
   c. Could the bridge authorities increase their revenues by changing their price?
   d. The Red and White Lines, a ferry service that competes with the Golden Gate Bridge, began operating hovercrafts that made commuting by ferry much more convenient. How would this affect the elasticity of demand for trips across the Golden Gate Bridge?

8. Consumer expenditures on safety are thought to have a positive income elasticity. For example, as incomes rise, people tend to buy safer cars (larger cars with side air bags), they are more likely to fly on trips rather than drive, they are more likely to get regular health tests, and they are more likely to get medical care for any health problems the tests reveal. Is safety a luxury or a necessity?

9. Professors Adams and Brown make up the entire demand side of the market for summer research assistants in the economics department. If Adams's demand curve is

$P = 50 - 2Q_A$ and Brown's is $P = 50 - Q_B$, where Q_A and Q_B are the hours demanded by Adams and Brown, respectively, what is the market demand for research hours in the economics department?

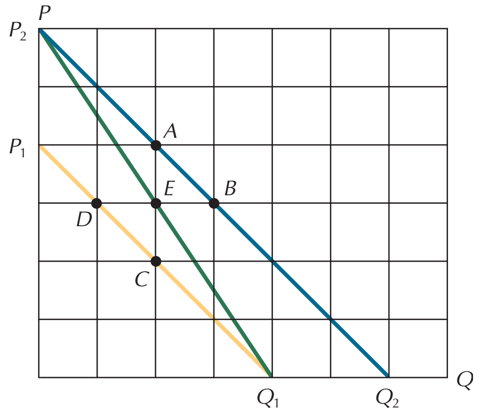
10. Suppose that at a price of \$400, 300 tickets are demanded to fly from Ithaca, New York, to Los Angeles, California. Now the price rises to \$600, and 280 tickets are demanded. Assuming the demand for tickets is linear, find the price elasticities at the quantity-price pairs (300, 400) and (280, 600).

11. The monthly market demand curve for calculators among engineering students is given by $P = 100 - Q$, where P is the price per calculator in dollars and Q is the number of calculators purchased per month. If the price is \$30, how much revenue will calculator makers get each month? Find the price elasticity of demand for calculators. What should calculator makers do to increase revenue?

12. What price maximizes total expenditure along the demand curve $P = 27 - Q^2$?

13. A hot dog vendor faces a daily demand curve of $Q = 1800 - 15P$, where P is the price of a hot dog in cents and Q is the number of hot dogs purchased each day.
    a. If the vendor has been selling 300 hot dogs each day, how much revenue has he been collecting?
    b. What is the price elasticity of demand for hot dogs?
    c. The vendor decides that he wants to generate more revenue. Should he raise or lower the price of his hot dogs?
    d. At what price would he achieve maximum total revenue?

14. Rank the absolute values of the price elasticities of demand at the points A, B, C, D, and E on the following three demand curves.



15. Draw the Engel curves for the following goods: food, Hawaiian vacations, cashews, Kmart brand sneakers (\$4.99/pr).

16. Is the cross-price elasticity of demand positive or negative for the following pairs of items?
    a. Tennis rackets and tennis balls
    b. Peanut butter and jelly
    c. Hot dogs and hamburgers

*17. In 2001, X cost \$3 and sold 400 units. That same year, a related good Y cost \$10 and sold 200 units. In 2002, X still cost \$3 but sold only 300 units, while Y rose in price to \$12 and sold only 150 units. Other things the same, and assuming that the demand for X is a linear function of the price of Y, what was the cross-price elasticity of demand for X with respect to Y in 2001?

*18. Smith cannot tell the difference between rice and wheat and spends all her food budget of \$24/wk on these foodstuffs. If rice costs \$3/lb, draw Smith's price-consumption curve for wheat and the corresponding demand curve.

*19. Repeat the preceding problem on the assumption that rice and wheat are perfect, one-for-one complements.

*Problems marked with an asterisk (*) are more difficult.

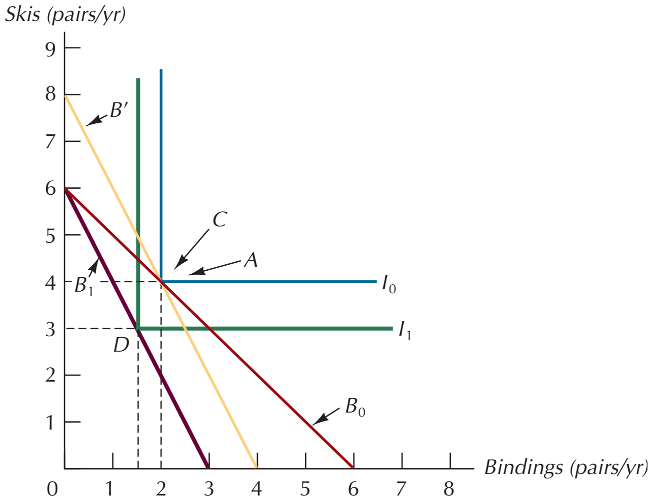
*20. Suppose your local espresso bar makes the following offer: People who supply their own half-pint carton of milk get to buy a cup of cappuccino for only \$1.50 instead of \$2.50. Half-pint cartons of milk can be purchased in the adjacent convenience store for \$0.50. In the wake of this offer, the quantity of cappuccino sold goes up by 60 percent and the convenience store's total revenue from sales of milk exactly doubles.

a. *True or false:* If there is a small, but significant, amount of hassle involved in supplying one's own milk, it follows that absolute value of the price elasticity of demand for cappuccino is 3. Explain.

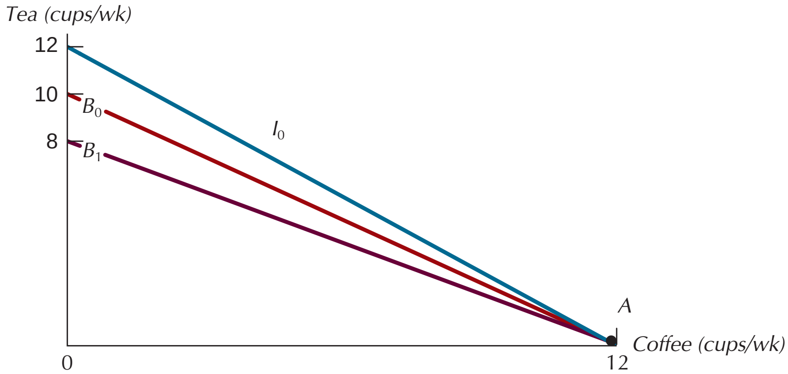
b. *True or false:* It follows that demand for the convenience store's milk is elastic with respect to price. Explain.

ANSWERS TO IN-CHAPTER EXERCISES

4.1. On Paula's original budget, B_0, she consumes at bundle A. On the new budget, B_1, she consumes at bundle D. (To say that D has 1.5 pr of bindings per year means that she consumes 3 pr of bindings every 2 yr.) The substitution effect of the price increase (the movement from A to C) is zero.



4.2. The income effect, substitution effect, and total effects are all zero because the price change does not alter Pam's optimal consumption bundle.



4.3. The formulas for D_1 and D_2 are $P = 16 - 2Q_1$ and $P = 8 - 2Q_2$, respectively. For the region in which $0 \le P \le 8$, we have $Q_1 = 8 - (P/2)$ and $Q_2 = 4 - (P/2)$. Adding, we get $Q_1 + Q_2 = Q = 12 - P$, for $0 \le P \le 8$. For $8 < P \le 16$, the market demand curve is the same as D_1, namely, $P = 16 - 2Q$.

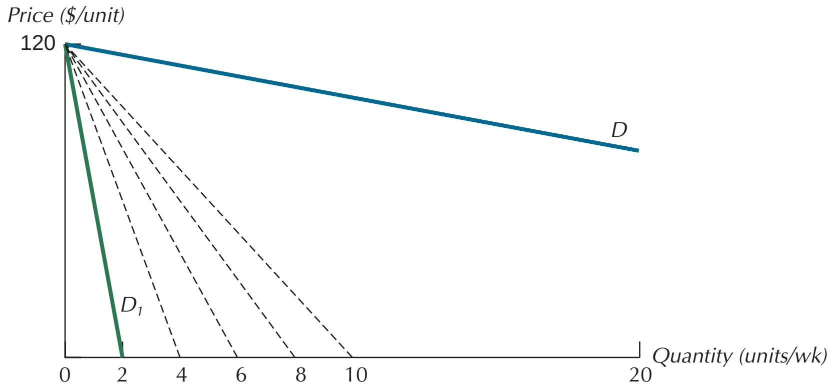
*Problems marked with an asterisk (*) are more difficult.

4.4. First, we need to rearrange the representative consumer demand curve $P = 120 - 60Q_i$ to have quantity alone on one side:

$$Q_i = 2 - \tfrac{1}{60}P.$$

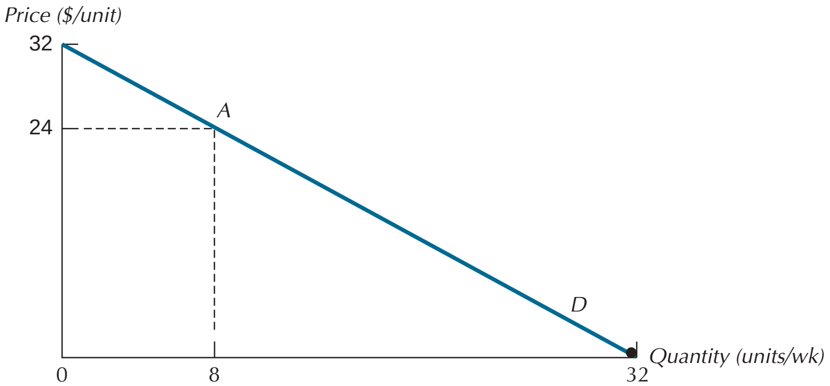
Then we multiply by the number of consumers, $n = 30$,

$$Q = nQ_i = 30Q_i = 30(2 - \tfrac{1}{60}P) = 60 - \tfrac{1}{2}P.$$



Finally, we rearrange the market demand curve $Q = 60 - \frac{1}{2}P$ to have price alone on one side, $P = 120 - 2Q$, to return to the slope-intercept form.

4.5. Since the slope of the demand curve is -1, we have $\epsilon = -P/Q$. At $P = 24$, $Q = 8$, and so $\epsilon = -P/Q = -\frac{24}{8} = -3$.



4.6. Elasticity when $P = \$4$/sq yd is $\frac{1}{3}$, so a price reduction will reduce total expenditure. At $P = 4$, total expenditure is \$48/wk, which is more than the \$39/wk of total expenditure at $P = 3$.

4.7. Substitutes, such as *a, b,* and *e,* have positive cross-price elasticity (an increase in price of one good raises quantity demanded of the other good). Complements, such as *c* and *d,* have negative cross-price elasticity (an increase in price of one good lowers quantity demanded of the other good).