Econométrie 3

Multiple Choices Modelling

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Multiple Choice Model

Choice between multiple alternatives (1)

Choice between multiple alternatives

- Utility is defined for all $i \in \mathcal{C}_n$: $U_{in} = V_{in} + \varepsilon_{in}$
- We need to define
  - Choice set : $\mathcal{C}_i n$
  - Error term : ε_{in}
  - Systematic part : V_{in}

Choice set (1)

Choice set

- How to model "awareness" ?
- What does "long distance" exactly mean ?
- What does "unreachable" exactly mean ?

We assume here deterministic rules

- Car is available if *n* has a driver license and a car is available in the household
- Walking is available if trip length is shorter than 4km.

Error term (1)

Main assumption

ε_{in} are

- extreme value $EV(0, \mu)$,
- independent and
- identically distributed (i.i.d.).

Comments

- Independence : across *i* and *n*.
- Identical distribution : same scale parameter μ across *i* and *n*.
- Scale must be normalized : $\mu = 1$.

Derivation of the logit model (1)

Assumptions

- $C_n = \{1, \ldots, J_n\}$
- $U_{in} = V_{in} + \varepsilon_{in}$
- $\varepsilon_{in} \sim EV(0, \mu)$
- ε_{in} i.i.d.

Choice model

$$P(i|\mathcal{C}_n) = Pr(V_{in} + \varepsilon_{in} \geq max_{j=1,\ldots,J_n} V_{jn} + \varepsilon_{jn})$$

Assume without loss of generality that $i = 1$

$$P(1|\mathcal{C}_n) = P(V_{1n} + \varepsilon_{1n} \geq max_{j=2,\ldots,J_n} V_{jn} + \varepsilon_{jn})$$

Derivation of the logit model (2)

Composite alternative

- Define a composite alternative : "anything but alternative one"
- Associated utility :

$$U^* = max_{j=2,\dots,J_n}(V_{jn} + \varepsilon_{jn})$$

- From a property of the EV distribution

$$U^* \sim EV\left(\frac{1}{\mu} ln \sum_{j=2}^{J_n} exp\{\mu V_{jn}\}, \mu\right)$$

Derivation of the logit model (3)

Composite alternative

From another property of the EV distribution

$$U^* = V^* + \varepsilon^*$$

where

$$V^* = \frac{1}{\mu} ln \sum_{j=2}^{J_n} exp\{\mu V_{jn}\}$$

and

$$\varepsilon^* \sim EV(0, \mu)$$

Derivation of the logit model (4)

Binary choice

$$P(1|\mathcal{C}_n) = P(V_{1n} + \varepsilon_{1n} \geq max_{j=2,\ldots,J_n} V_{jn} + \varepsilon_{jn})$$

$$P(V_{1n} + \varepsilon_{1n} \geq V^* + \varepsilon^*)$$

ε_{1n} and ε^* are both $EV(0,\mu)$.

Binary logit

$$P(1|\mathcal{C}_n) = \frac{exp\{\mu V_{1n}\}}{exp\{\mu V_{1n}\} + exp\{\mu V^*\}}$$

where

$$V^* = \frac{1}{\mu} ln \sum_{j=2}^{J_n} exp\{\mu V_{jn}\}$$

Derivation of the logit model (5)

We have

$$exp\{\mu V^*\} = exp\{ln \sum_{j=2}^{J_n} exp\{\mu V_{jn}\}\}$$

and

$$\begin{aligned} P(1|\mathcal{C}_n) &= \frac{exp\{\mu V_{1n}\}}{exp\{\mu V_{1n}\} + exp\{\mu V^*\}} \\ &= \frac{exp\{\mu V_{1n}\}}{exp\{\mu V_{1n}\} + \sum_{j=2}^{J_n} exp\{\mu V_{jn}\}} \\ &= \frac{exp\{\mu V_{1n}\}}{\sum_{j=1}^{J_n} exp\{\mu V_{jn}\}} \end{aligned}$$

Derivation of the logit model (1)

Assumptions

- The scale parameter μ is not identifiable : $\mu = 1$.
- Warning : not identifiable $\neq$ not existing

Derivation of the logit model (2)

$\mu \to 0$, that is variance goes to infinity

$$lim_{\mu \to 0} P(i|\mathcal{C}_n) = \frac{1}{J_n} \quad \forall i \in \mathcal{C}_n$$

$\mu \to +\infty$, that is variance goes to zero

$$\begin{aligned} lim_{\mu \to +\infty} P(i|\mathcal{C}_n) &= lim_{\mu \to +\infty} \frac{1}{1 + \sum_{j \neq i} exp\{\mu(V_{jn} - V_{in})\}} \\ &= 1 \text{if} V_{in} > max_{j \neq i} V_{jn} \\ &= 0 \text{if} V_{in} < max_{j \neq i} V_{jn} \end{aligned}$$

Systematic part of the utility function

Systematic part of the utility function (1)

$$V_{in} = V(z_{in}, S_n)$$

- z_{in} is a vector of attributes of alternative i for individual n
- S_n is a vector of socio-economic characteristics of n

Functional form : linear utility

$$x_{in} = (z_{in}, S_n)$$

Linear-in-parameters utility functions

$$V_{in} = V(z_{in}, S_n) = V(x_{in}) = \sum_k \beta_k (x_{in})_k$$

Explanatory variables : alternatives attributes (1)

Alt. Attributes : Numerical and continuous

$$(z_{in})_k \in \mathbb{R}, \forall i, n, k$$

Associated with a specific unit

Examples

- Auto in-vehicle time (in min.)
- Auto out-of-pocket cost (in cents)
- Walking time to the bus stop (in min.)

Explanatory variables : alternatives attributes (2)

V_{in} is unitless

- Therefore, β depends on the unit of the associated attribute
- Example : consider two specifications

$$V_{in} = \beta_1 TT_{in} + \dots \qquad \text{or} \qquad V_{in} = \beta_1' TT_{in}' + \dots$$

- If TT_{in} is a number of minutes, the unit of β_1 is 1/min
- If TT_{in}' is a number of hours, the unit of β_1' is 1/hour
- Both models are equivalent, but the estimated value of the coefficient will be different

$$\beta_1' TT_{in}' = \beta_1 TT_{in} \Rightarrow \frac{TT_{in}'}{TT_{in}} = \frac{\beta_1}{\beta_1'} = 60$$

Explanatory variables : alternatives attributes (3)

Generic and alternative specific parameters

Modeling assumption : a minute has/has not the same marginal utility

$$V_{auto} = \beta_1 TT_{auto} + \dots \qquad \text{and} \qquad V_{bus} = \beta_1 TT_{bus} + \dots$$

or

$$V_{auto} = \beta_1 TT_{auto} + \dots \qquad \text{or} \qquad V_{bus} = \beta_2 TT_{bus} + \dots$$

Explanatory variables : socio-eco. characteristics (1)

Socio-eco. Characteristics : Numerical and continuous

- $(S_n)_k \in \mathbb{R}, \forall n, k$
- Associated with a specific unit
- Examples : Annual income ; Age (in years) ; etc

S_n do not depend on alternative i

Explanatory variables : socio-eco. characteristics (2)

Numerical and continuous

They are constant between alternative,

- hence they cannot be associated with the same effect in all utility functions

$$\begin{aligned} V1 &= \beta_1 x_{11} + \beta_2 income + \dots & & V_1' = \beta_1 x_{11} \\ V2 &= \beta_1 x_{21} + \beta_2 income + \dots & \Leftrightarrow \quad & V_2' = \beta_1 x_{21} \\ V3 &= \beta_1 x_{31} + \beta_2 income + \dots & & V_3' = \beta_1 x_{31} \end{aligned}$$

- In general : alternative specific characteristics

$$\begin{aligned} V_1 &= \beta_1 x_{11} + \beta_2 income + \beta_4 age \\ V_2 &= \beta_1 x_{21} + \beta_3 income + \beta_5 age \\ V_3 &= \beta_1 x_{31} \end{aligned}$$

Discrete variables (1)

Discrete variables

- Mainly used to capture qualitative attributes or characteristics
  Level of comfort for the train, Color, etc ; Sex, Education, etc.
- Carrefull model specification :
  - If a qualitative attribute has n levels, we introduce $n-1$ dummies variables (0/1) in the model
  - Introduce a 0/1 attribute for all levels except the base case

Discrete variables (2)

Discrete variables

- For example :

  Base : level 1

$$V_{in} = \cdots + 0 z_{i,lev1} + \beta_{lev2} z_{i,lev2} + \beta_{lev3} z_{i,lev3}$$

  - β_{lev2} : difference of utility between level 2 and reference level (level 1)
  - β_{lev3} : difference of utility between level 3 and reference level (level 1)
  - if levels are ordered, signs are in coherence

- **Base** : level 2

$$V_{in} = \cdots + \beta_{lev1} z_{i,lev1} + 0 z_{i,lev2} + \beta_{lev3} z_{i,lev3}$$

Nonlinear transformations of the variables (1)

Nonlinear transformations of the variables

- Introduction of variable in linear additive utiliy function leads to implicit behavioural assumptions
- Example with travel time
  Behavioral assumption : One more minute of travel is not perceived the same way for short trips as for long trips

Nonlinear transformations of the variables (2)

- Assumption 1 : the marginal impact of travel time is constant

$$V_i = \beta_T time_i + \ldots$$

- Assumption 2 : the marginal impact of travel time decreases with travel time

$$V_i = \beta_T ln(time_i) \ldots$$

- It is still a linear-in-parameters form
- The unit, the value, and the interpretation of β_T are different

Nonlinear transformations of the variables (3)

- Data can be preprocessed to account for nonlinearities

$$V_{in} = V(h(z_{in}, S_n)) = \sum_k \beta_k (h(z_{in}, S_n))_k$$

- It is linear-in-parameter, even with h nonlinear.

Specifications

- Categories with constants (inferior solution)
- Piecewise linear specification (spline)
- Numerous tranforms (Box-Cox transforms)

Categories with constant (1)

Nonlinear transformations of the variables

Same specification as for discrete variables

$$V_i = \beta_{T1} x_{T1} + \beta_{T2} x_{T2} + \beta_{T3} x_{T3} + \beta_{T4} x_{T4} + \dots$$

with

- $x_{T1} = 1$ if $TT_i \in [0; 90[$, 0 otherwise
- $x_{T2} = 1$ if $TT_i \in [90; 180[$, 0 otherwise
- $x_{T3} = 1$ if $TT_i \in [180; 270[$, 0 otherwise
- $x_{T4} = 1$ if $TT_i \in [270; +\infty[$, 0 otherwise
- One β must be normalized to 0.

Categories with constant (2)

Drawbacks

- No sensitivity to travel time within the intervals
- Discontinuous utility function (jumps)
- Need for many small intervals
- Results may vary significantly with the definition of the intervals

Appropriate when

- Categories have been used in the survey (income, age)
- Definition of categories is natural (weekday)

Piecewise linear specification (1)

Features

- Capture the sensitivity within the intervals
- Enforce continuity of the utility function

$$V_i = \beta_{T1} x_{T1} + \beta_{T2} x_{T2} + \beta_{T3} x_{T3} + \beta_{T4} x_{T4} + \ldots$$

where x_{Ti} is t in four classes of range 90 $(0; 90; 180; 270)$

$$x_{T1} = \begin{cases} t & \text{if } t < 90 \\ 90 & \text{otherwise} \end{cases} \qquad x_{T2} = \begin{cases} 0 & \text{if } t < 90 \\ t - 90 & \text{if } 90 \leq t < 180 \\ 90 & \text{otherwise} \end{cases}$$

$$x_{T3} = \begin{cases} 0 & \text{if } t < 180 \\ t - 180 & \text{if } 180 \leq t < 270 \\ 90 & \text{otherwise} \end{cases} \qquad x_{T4} = \begin{cases} 0 & \text{if } t < 270 \\ t - 270 & \text{otherwise} \end{cases}$$

Piecewise linear specification (2)

Coding :

$$x_{T1} = min(t, 90)$$
$$x_{T2} = max(0, min(t - 90, 90))$$
$$x_{T3} = max(0, min(t - 180, 90))$$
$$x_{T4} = max(0, t - 270)$$

t	TT1	TT2	TT3	TT4
40	40	0	0	0
100	90	10	0	0
200	90	90	20	0
300	90	90	90	30

Box-Cox transforms (1)

Box-Cox transforms

Other power transforms are possible :

- Box-Cox, Box-Tukey, Taylor expansion,...

Interactions (1)

Motivation

- All individuals in a population are not alike
- Socio-economic characteristics define segments in the population
- How to capture heterogeneity ?

The $h()$ transform can interact attributes and characteristics : cost/income, distance/time,...

$$V_{in} = V(h(z_{in}, S_n)) = \sum_k \beta_k (h(z_{in}, S_n))_k$$

Segmentation (1)

The population is divided into a finite number of segments

- Each individual belongs to exactly one segment

Specification

$$\beta_{M,m} TT_{M,m} + \beta_{M,s} TT_{M,s} + \beta_{M,p} TT_{M,p}$$

$$+\beta_{F,m} TT_{F,} m + \beta_{F,s} TT_{F,s} + \beta_{F,p} TT_{F,p}$$

$TT_i = TT$ if indiv. belongs to segment i , and 0 otherwise

- For a given individual, exactly one of these terms is non zero.
- The number of segments grows exponentially with the number of variables.

Heteroscedasticity (1)

Heteroscedasticity

- Logit is homoscedastic : ε_{in} i.i.d. across both i and n.
- But for ex. : people have different level of knowledge
- Different sources of data

How can we specify the model in order to use logit ?

Heteroscedasticity (2)

Assumption

- Variance of error terms is different across two different groups :

$$U_{in1} = V_{in1} + \varepsilon_{in1}$$

$$U_{in2} = V_{in2} + \varepsilon_{in2}$$

and $Var(\varepsilon_{in2}) = \alpha^2 Var(\varepsilon_{in1})$

- Solution : include scale parameters

$$\alpha U_{in1} = \alpha V_{in1} + \alpha\varepsilon_{in1} = \alpha V_{in1} + \varepsilon'_{in1}$$

$$U_{in2} = V_{in2} + \varepsilon_{in2} = V_{in2} + \varepsilon'_{in2}$$

where ε'_{in1} and ε'_{in2} i.i.d.

Heteroscedasticity (3)

Assumption

- $\alpha V_{in1} = \sum_j \alpha\beta_j x_{jin1}$ is not linear-in-parameters
- Normalization : a different scale parameter can be estimated for each segment of the population, except one that must be normalized.

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Relaxing the independence assumption

The independence from irrelevant alternatives (IIA) Property (1)

The IIA Property

- For any two alternatives i and k, the ratio of the logit probabilities is

$$\frac{P(i|\mathcal{C})}{P(k|\mathcal{C})} = \frac{\frac{exp\{V_{in}\}}{\sum_{j=1}^{J_n} exp\{V_{jn}\}}}{\frac{exp\{V_{kn}\}}{\sum_{j=1}^{J_n} exp\{V_{jn}\}}} = \frac{exp\{V_{in}\}}{exp\{V_{kn}\}} = exp\{V_{in} - V_{kn}\}$$

- these relative odds are independent from ’irrelevant’ alternatives

Blue Bus Red Bus Paradox (1)

Imagine a mode choice for commuting : car or blue bus.

- Two alternatives : car and bus.
- There are red buses and blue buses.
- Car and bus travel times are equal : T.
- Only travel time is considered in the utility function.

Blue Bus Red Bus Paradox (2)

Imagine a mode choice for commuting : car or blue bus.

- For simplicity we assume that the representative utility of the two modes are the same

$$U_{car} = \beta_T T + \varepsilon_{car} \qquad U_{bus} = \beta_T T + \varepsilon_{bus}$$

- Then
$P(\text{car}|\{\text{car, bus}\}) = P(\text{bus}|\{\text{car, bus}\}) = \frac{exp\{\beta_T T\}}{exp\{\beta_T T\}+exp\{\beta_T T\}} = \frac{1}{2}$
- Then $\frac{P(\text{car})}{P(\text{bus})} = 1$

Blue Bus Red Bus Paradox (3)

introduction of a red bus

- Suppose introduction of a red bus (considered to be exactly like the blue bus)

$$U_{car} = \beta_T T + \varepsilon_{car} \qquad U_{bluebus} = \beta_T T + \varepsilon_{bluebus}$$

$$U_{redbus} = \beta_T T + \varepsilon_{redbus}$$

- Then

$$P(\text{car}|\{\text{car, blue bus, red bus}\}) = \frac{1}{3}$$
$$P(\text{blue bus}|\{\text{car, blue bus, red bus}\}) = \frac{1}{3}$$
$$P(\text{red bus}|\{\text{car, blue bus, red bus}\}) = \frac{1}{3}$$

- Then $\frac{P(\text{car})}{P(\text{bus})} \neq 1$

Blue Bus Red Bus Paradox (1)

introduction of a red bus

- With IIA : the logit model predicts :
  $P(\text{red bus}) = P(\text{blue bus}) = P(\text{car}) = 1/3$
- $\frac{P(\text{red bus})}{P(\text{blue bus})} = 1$ and $\frac{P(\text{car})}{P(\text{blue bus})} = 1$
- we would expect :$P(\text{red bus}) = P(\text{blue bus}) = 1/4$ and $P(\text{car}) = 1/2$
- Logit overestimates $P(\text{bus})$ and underestimates $P(\text{car})$

Explaining the paradox

- Shared attributes are captured by the error terms ($\varepsilon_{bluebus}\varepsilon_{redbus}$) : fare, comfort, etc.
- Logit model assumes that $\varepsilon_{bluebus}$ and ε_{redbus} are independent.

Interpreting the paradox (1)

Proportional substitution

- The cross-elasticities of logit probabilities :
  Consider the effect of changing an attribute of alt. j on the probabilities for all the other alternatives.
- Deriving $P(i|\mathcal{C})$:

$$E_{iz_{nj}} = \frac{\partial ln(Pi|\mathcal{C})}{\partial ln(z_{nj})} = -\beta_z z_{nj} P(j|\mathcal{C})$$

- This cross-elasticity is the same for all i.

Interpreting the paradox (2)

- Improvement in one alternative draws proportionaly the other alternatives.
- If one probability drops of x % then all probabilities drop of x %, except for alternative j.
- Behavioral implication of IIA :
  If an alternative is removed, probability is spread equally across the remaining alternatives.

Discrete Choice Model Extensions

Discrete Choice Model Extensions (1)

Discrete Choice Model Extensions

- Heteroscedasticity and other forms of heterogeneity
  - Across individuals
  - Across alternatives
- Panel data (Repeated measures)
  - Random and fixed effects models
  - Building into a multinomial logit model
- The nested logit model
- Latent class model
- Mixed logit, error components and multinomial probit models

Ordinal Logistic Regression

Background

Ordinal Response Y

- represents levels of a standard measurement scale
- constructed by specifying a hierarchy of separate endpoints
- rank-based methods to continuous responses
- do not assume a spacing between levels of Y
- ordinal models use only the rankordering of values of Y
- same β and P-values result from an analysis of Y having levels 0, 1, 2 and levels recoded 0, 1, 20

Proportional Odds Model

***PO* model :**

- for a response variable having levels $j = 1, 2, \ldots, k$:

$$P(Y \geq j|X) = \frac{1}{1 + exp[-(\alpha_j + X\beta)]}$$

- k intercepts (αs)
- For fixed j, the model logistic model for the event $Y \geq j$
- common vector of β connecting probabilities for varying j (parsimonious modeling of the distribution of Y)
- implicit assumption : the β are independent of j, the cutoff level for Y
- the log odds that $Y \geq j$ is linearly related to each X and that there is no interaction between the Xs

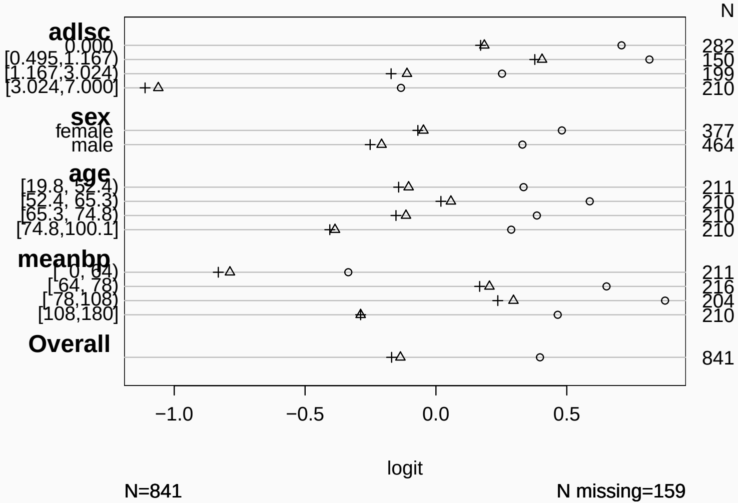
***PO* model :**

- a single odds ratio apply equally to all events $Y \geq j$, $j = 1, 2, \ldots, k$
- If *linearity* and *additivity* hold, the $X_m + 1 : X_m$ odds ratio for $Y \geq j$ is $exp(\beta_m)$, whatever the cutoff j
- BUT, proportional hazards assumption is frequently violated

Assessment of Model Fit (1)

Determining whether the PO assumption is likely

- comparing means of $X|Y$ with and without assuming PO,
- stratifying on each predictor and computing the logits of all proportions of the form $Y \geq j$, $j = 1, 2, \ldots, k$.
  When PO holds, the differences in logits between different values of j should be the same at all levels of X, because the model dictates that $logit(Y \geq j|X) - logit(Y \geq i|X) = \alpha_j - \alpha_i$, for any constant X.


N
adlsc
0.000
[0.495,1.167)
[1.167,3.024)
[3.024,7.000]
sex
female
male
age
[19.8, 52.4)
[52.4, 65.3)
[65.3, 74.8)
[74.8,100.1]
meanbp
[0, 64)
[64, 78)
[78,108)
[108,180]
Overall
282
150
199
210
377
464
211
210
210
210
211
216
204
210
841
−1.0
−0.5
0.0
0.5
logit
N=841
N missing=159

- Checking PO assumption separately for a series of predictors. The circle, triangle, and plus sign correspond to $Y \geq 1, 2, 3$, respectively
- PO is checked by examining the vertical constancy of distances between any two of these three symbols.

Continuation Ratio Model

***CR* model :**

- PO model is based on *cumulative probabilities*
  CR model is based on *conditional probabilities*
- the forward CR model :

$$P(Y = j | Y \geq j, X) = \frac{1}{1 + exp[-(\theta_j + X\gamma)]}$$

$$logit(Y = 0 | Y \geq 0, X) = logit(Y = 0 | X) = \theta_0 + X\gamma$$

$$logit(Y = 1 | Y \geq 1, X) = \theta_1 + X\gamma$$

...

$$logit(Y = k-1 | Y \geq k-1, X) = \theta_{k-1} + X\gamma$$

Exercise (1)

Exercise MROZ - Woolridge

- Reproduce results from the textbook

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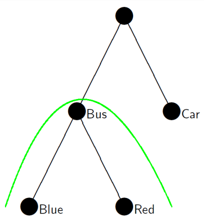
Nested Logit

What about the choice between bus and car ?



$$U_{car} = \beta T + \varepsilon_{car}$$
$$U_{bus} = V_{bus} + \varepsilon_{bus}$$
$$\text{with } V_{bus} = V_{bus}(V_{bluebus}, V_{redbus})$$
$$\varepsilon_{bus} = ?$$

- Use a logit model at the higher level.
- Define V_{bus} as the expected maximum utility of red bus and blue bus
- Within a branch

Capturing the correlation (2)

  - Identical variances (IIA applies)
  - Covariance (all same) = variance at higher level
- Branches have different variances (scale factors)
- Nested logit probabilities : Generalized Extreme Value
- $P(Alt, Branch) = P(Branch) \times P(Alt|Branch)$

Definition

For a set of alternative $\mathcal{C}$, $U_{k|b}$ defines utility of alt. k in branch b

$$U_{k|b} = \alpha_{k|b} + \beta' x_{k|b}$$

and at branch level

$$U_b = \gamma_b' z_i$$

Capturing the correlation (3)

- Leaf probability : $P(k|b) = \frac{exp\{\alpha_{k|b}+\beta'_{k|b}x_{k|b}\}}{\sum_{m=1}^{K|b} exp\{\alpha_{m|b}+\beta'_{m|b}x_{k|b}\}}$
- Inclusive value for branch b : $IV(b) = ln(\sum_{m=1}^{K|b} exp\{\alpha_{m|b} + \beta'_{m|b}x_{k|b}\})$
- Branch level probability :

$$P(b) = \frac{exp\{\lambda_b(\gamma'_b z_i + IV(b))\}}{\sum_{q=1}^{Q} exp\{\lambda_q(\gamma'_q z_i + IV(q))\}}$$

- MNL results if the inclusive parameters, λ_q are all equal to one.

Nested logit version 2 (1)

The nested logit model is obtained

by assuming that the vector of unobserved utility, ε_n has cumulative distribution :

$$exp\{-\sum_{k=1}^{K}(\sum_{j\in B_k} exp\{\varepsilon_{nj}/\lambda_k\})^{\lambda_k}\}$$

- This distribution is a type of GEV distribution.
- A generalization of the distribution that gives rise to the logit model.
- For logit, each ε_{nj} is independent with a univariate extreme value distribution.

Nested logit version 2 (2)

- For this GEV, the marginal distribution of each ε_{nj} is univariate extreme value.
- However, the ε_{nj} are correlated within nests.
  - For any two alternatives j and m in nest B_k, ε_{nj} is correlated with ε_{nm}.
  - For any two alternatives in different nests, the unobserved portion of utility is still uncorrelated : $Cov(\varepsilon_{nj}, \varepsilon_{nm}) = 0$ for any $j \in B_k$ and $m \in B_l$ with $l \neq k$.
- The parameter λ_k is a measure of the degree of independence in unobserved utility among the alternatives in nest k.
- A higher value of λ_k means greater independence and less correlation.

Nested logit version 2 (3)

- $\lambda_k = 1$ leads to the logit model

Choice probability for alternative $i \in B_k$

$$P_{ni} = \frac{e^{V_{ni}/\lambda_k} \left(\sum_{j \in B_n} e^{V_{nj}/\lambda_k}\right)^{\lambda_k - 1}}{\sum_{l=1}^{K} \left(\sum_{j \in B_n} e^{V_{nj}/\lambda_k}\right)^{\lambda_k}}$$

Mixed Logit

Mixed Logit

Mixed logit model advantages

ML resolves the three limitations of standard logit by allowing :

1. random taste variation
   - some parameters are assumed to be randomly distributed
2. unrestricted substitution patterns
   - relaxes IIA hypothesis of MNL by allowing ε to be correlated,
   - while maintaining the assumption of identical distribution *(Greene, 2008)* .
3. correlation in unobserved factors over time
   - takes into account the repeated nature of the choices made by respondents. (Var-Cov matrix form)

(McFadden and Train, 2000) : any discrete choice model can be approximated by a mixed logit model, including the multinomial logit.

Mixed Logit (1)

Mixed logit and error component models

- Basic form of the MNL model, with alternative specific constants α_{ij} and attributes x_{ij} :

$$P(j) = \frac{exp\{\alpha_{ij} + \beta_i' x_{ij}\}}{\sum_{q=1}^{J_i} exp\{\alpha_{iq} + \beta_i' x_{iq}\}}$$

- The random parameters model emerges as the form of the individual specific parameter vector, β_i, is developed.
- The most familiar, simplest version of the model :

$$\beta_{ik} = \beta_k + \sigma_k \nu_{ik}$$

$$\alpha_{ij} = \alpha_j + \sigma_j \nu_{ij}$$

Mixed Logit (1)

Mixed logit and error component models

- Introducing random taste heterogeneity in the parameters the probability that an alternative j is chosen by an individual i, is the integral of the following probability over all possible values of β :

$$P_i(j) = \int P_i(j|x_{ij}, \beta) f(\beta|\Omega) d\beta$$

  where $f(\beta|\Omega)$ is the conditional density function of the β parameters and Ω is a vector of parameters of the distribution of β.

  - error components (EC) version, $U_{ij} = \alpha' x_{ij} + \theta_i' z_{ij} \varepsilon_{ik}$
  - random coefficients (RC) structure $U_{ij} = \theta_{ij}' x_{ij} + \varepsilon_{ij}$

Mixed Logit (1)

- where β_k is the population mean
- ν_{ik} is the individual specific heterogeneity, with zero mean and s.d. one
- σ_k is s.d. of the distribution of β_{ik} around β_k
- The choice-specific constants, α_{ij}, and the elements of β_i are distributed randomly across individuals with fixed means
- Normal distribution or other may be specified.
  for ex : $\beta_{ik} \sim \mathcal{N}(b, W)$ or $ln(\beta) \sim \mathcal{N}(b, W)$

Discret Choice and Stated Preferences

Many Data about Preference

Data about Preference

Relying on what consumers say they will do or observing what they actually do.

- Revealed Preference (RP) data : *Market data*

  (Samuelson, 1948) demonstrate that under some conditions, system demand equations can be estimated consistently...
- Stated Preference data : mainly hypothetical choice situation surveys.

Many Data about Preference

SP data pro

- Estimate demand for new products, or with new attributes or new features

  the 3.5” disk example
- Explanatory variables have little variability in the marketplace

  RP are of limited use to model behavioral change in response to variables change.
- Explanatory variables are higly collinear in the marketplace

  Attributes of products should become negatively correlated in competitive market

  Technological constraints may leads correlation betweeen attributes
- Observational data cannot satisfy model assumptions and/or contain statistical ’nasties’

  SP data can be designed to eliminate such problem

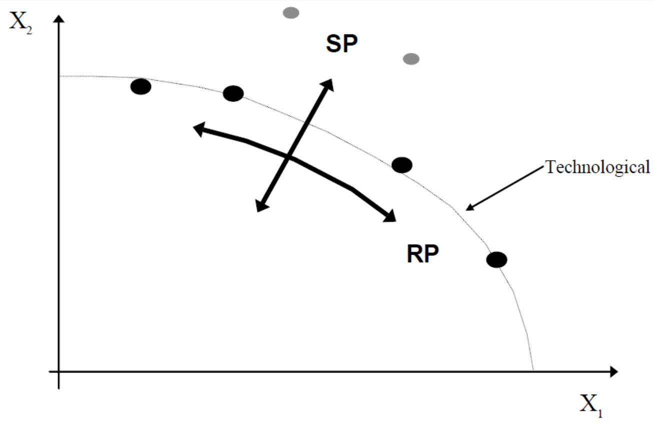


FIGURE 1 – RP and SP studying techno. frontier *(Louviere et al., 2000)*

Many Data about Preference (1)

RP and SP Advantages

RP data

- observation of market (current equilibrium)
- inherent relationships between attributes (techno. constraints)
- only existing alternatives are observable
- embody market and personal constraints on the decision maker
- have high reliability and face validity
- mainly one observation per individual at each observation point.

Many Data about Preference (2)

SP data

- hypothetical or virtual decision contexts
- experimentalist controls relationships between attributes (mapping of utility functions outside the existing technologies)
- can include existing and/or proposed and/or generic (i.e., unbranded or unlabelled) choice alternatives,
- cannot (or hardly) represent changes in market and personal constraints effectively,
- seem to be reliable when respondents understand, are committed to and can respond to tasks,
- (usually) yield multiple observations per respondent at each observation point.

Preference data and RUT (1)

"[...]measurement in the absence of theory is at best uninterpretable, and at worst meaningless." *(Louviere et al., 2000)*

Measurement in choices and preference modelling

- of preferences and choice
- of attributes
- of consumers or decision-making units
- of decision environments.

Preference data and RUT (1)

Measurement of choices and preference

- measures called dominance measures
- one or more objects are indistinguishable in degree of preference from one another (i.e., equal) or are more/less preferred to one another
- Many types of dominance measures are (or can be transformed to be) consistent with RUT

Some examples

- Discrete choice of one option from a set of competing ones. Most preferred option no relative preferences among the non-chosen.

Preference data and RUT (2)

- Yes/No : ‘Yes, I like this option’ and ‘No, I do not like this option’.
- Complete ranking of options from most to least preferred.
- Degrees of preference for each option. rate on a scale or psychometric methods
- Allocation of some of resources, such as money, time, trips, chips...

Discrete Choice (1)

Discrete Choice

- one option among competing (exclusive) options
- options differ in attributes
- 'weakly ordered' data : complete preference ordering cannot be determined
- to know complete ordering or change in response : need of more responses (same and/or other individuals)
- see example

Discrete Choice

auto > bus, train, ferry, carpool

Table 2.1. *Discrete choice of commuting option*

'Brands' for journey to work	Consumer chooses
Take bus	
Take train	
Take ferry	
Drive own auto	✓
Carpool	

FIGURE 2 – *(Louviere et al., 2000)*

Yes / No response (1)

Yes / No response

- binary discrete response
- categorises into two groups
- 'weakly ordered' data to be completed
- example can be obtained by asking to consider the 5 options or by 5 days observation

Yes / No response

auto > bus, train, ferry,

carpool > bus, train, ferry

Table 2.2. *Acceptance or rejection of commuting options*

'Brands' for journey to work	Consumer will consider (y/n)
Take bus	no
Take train	no
Take ferry	no
Drive own auto	yes
Carpool	yes

FIGURE 3 – *(Louviere et al., 2000)*

Complete ranking of options (1)

Complete ranking

- some issues : task difficulty ; response reliability ; realibility and validity for option that would never be chosen or option that are not well known
- best ranked option may be not chosen
- 'strongly ordered' data, but *(Louviere et al., 2000)* advise not to use them
- example : asking or observing

Complete ranking

auto > carpool > ferry > train > bus

Table 2.3. *Complete preference ranking of commuting options*

'Brands' for journey to work	Ranking by likelihood of use
Take bus	5
Take train	4
Take ferry	3
Drive own auto	1
Carpool	2

FIGURE 4 – *(Louviere et al., 2000)*

Degrees of preference (1)

Degrees of preference

- Most popular : category rating scale
- Report preference differences or ratios, or...
- Rating leads to ordinal information
- but cardinal interpretation have strong behavioral assumptions
- example : asking

Degrees of preference

auto > carpool > ferry > train = bus

Table 2.4. *Scale rating of commuting options*

'Brands' for journey to work	Consumer likelihood to use (0–10)
Take bus	4
Take train	4
Take ferry	6
Drive own auto	10
Carpool	7

FIGURE 5 – *(Louviere et al., 2000)*

Ressource allocation (1)

Ressource allocation

- ’stronger order’
- behavioral assumptions relative to the resource, the ’numeraire’

Data provided (1)

Implied choice data provided by each response

- For each choice set faced by each consumer, one must identify the chosen and rejected option
- Analogue to RP data : one *observes* some preference (choice) from a sample of individuals, *measures* (observes) attributes associated with choice alternatives, *measures* characteristics of the individual choosers

Data provided

Table 2.5. *Creating choice sets and coding choices from response data*

Implied choice set	Alternative	Implied choice
Discrete choice		
1	Auto	1
1	Bus	0
1	Train	0
1	Ferry	0
1	Carpool	0
Yes/No		
1	Auto	1
1	Bus	0
1	Train	0
1	Ferry	0
1	Carpool	0
2	Auto	0
2	Bus	0
2	Train	0
2	Ferry	0
2	Carpool	1

FIGURE 6 – *(Louviere et al., 2000)*

Data provided

Complete ranking		
1	Auto	1
1	Bus	0
1	Train	0
1	Ferry	0
1	Carpool	0
2	Bus	0
2	Train	0
2	Ferry	0
2	Carpool	1
3	Bus	0
3	Train	0
3	Ferry	1
4	Bus	0
4	Train	1
Rating		
1	Auto	1
1	Bus	0
1	Train	0
1	Ferry	0

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