Probabilistic Modeling and Bayesian Networks

Introduction

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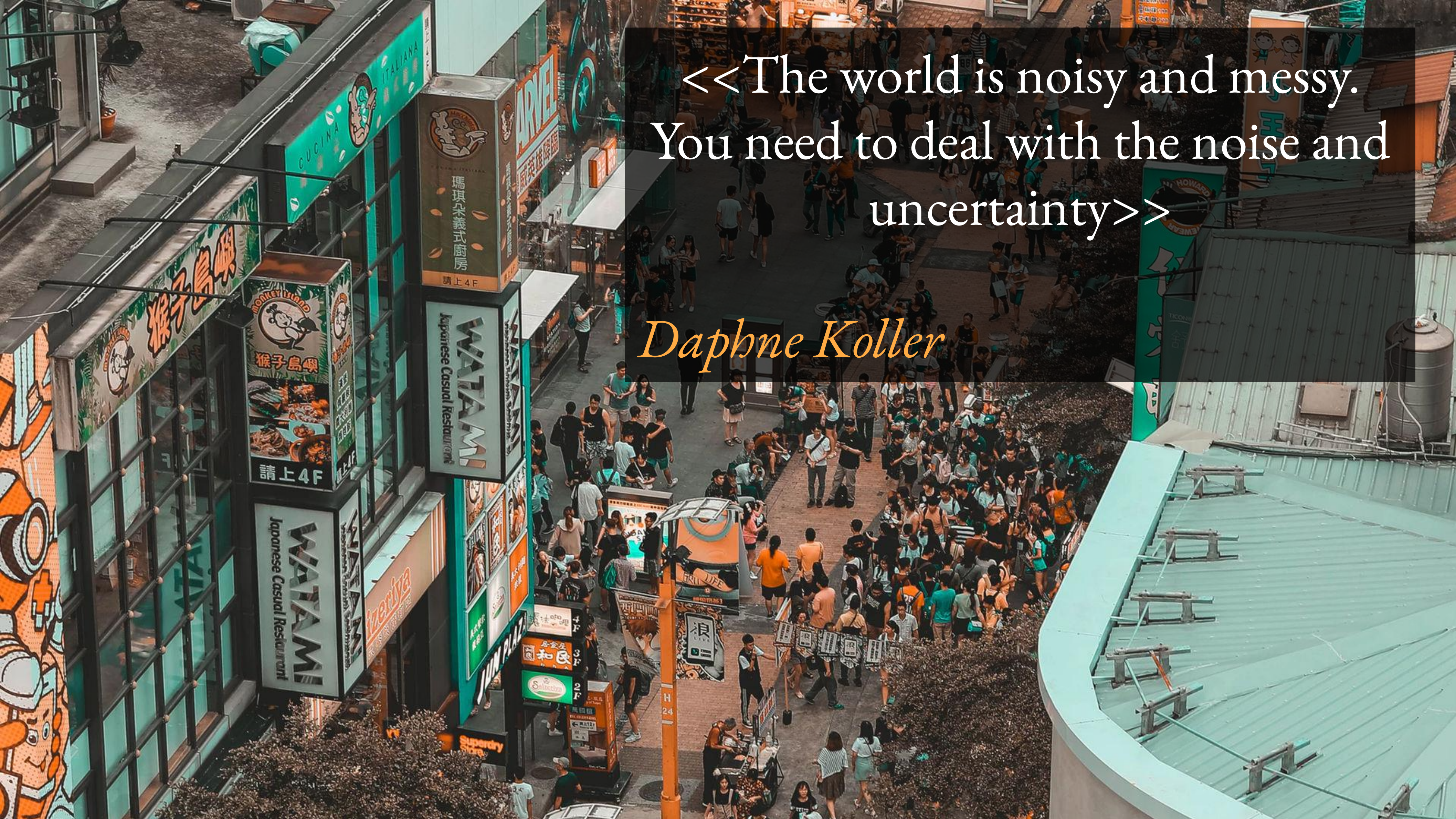
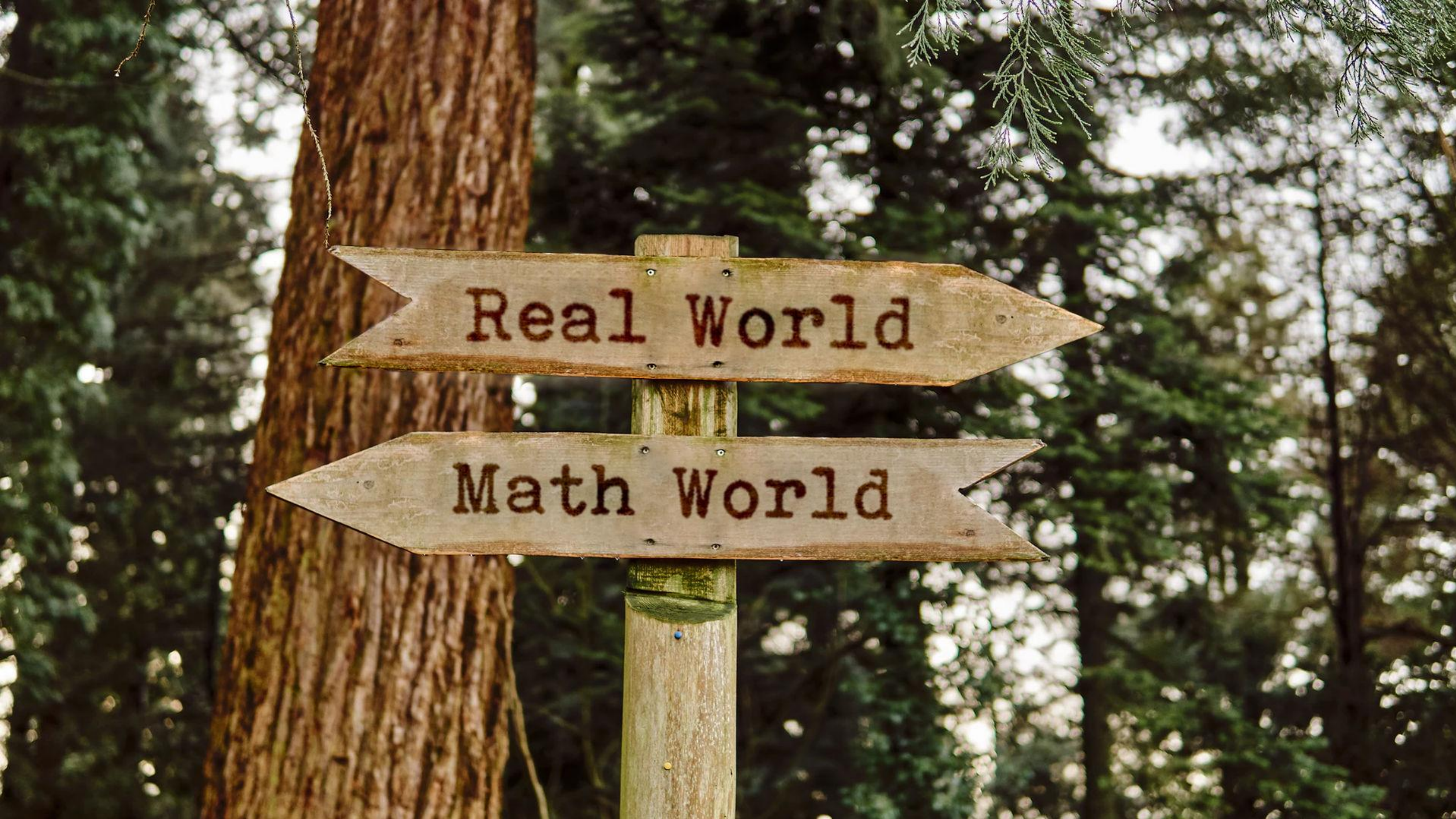


<<The world is noisy and messy. You need to deal with the noise and uncertainty>>

Daphne Koller


Parmesan Pepper

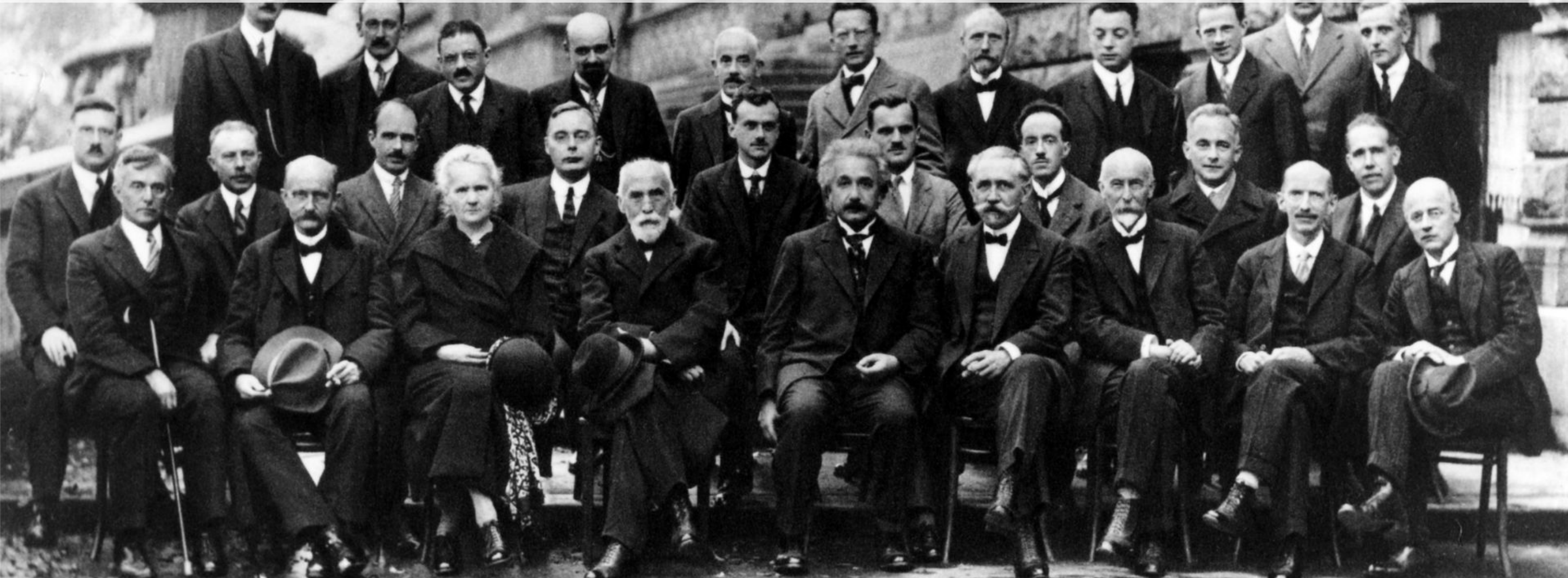

Real World
Math World

What is Probability?

and uncertainty?

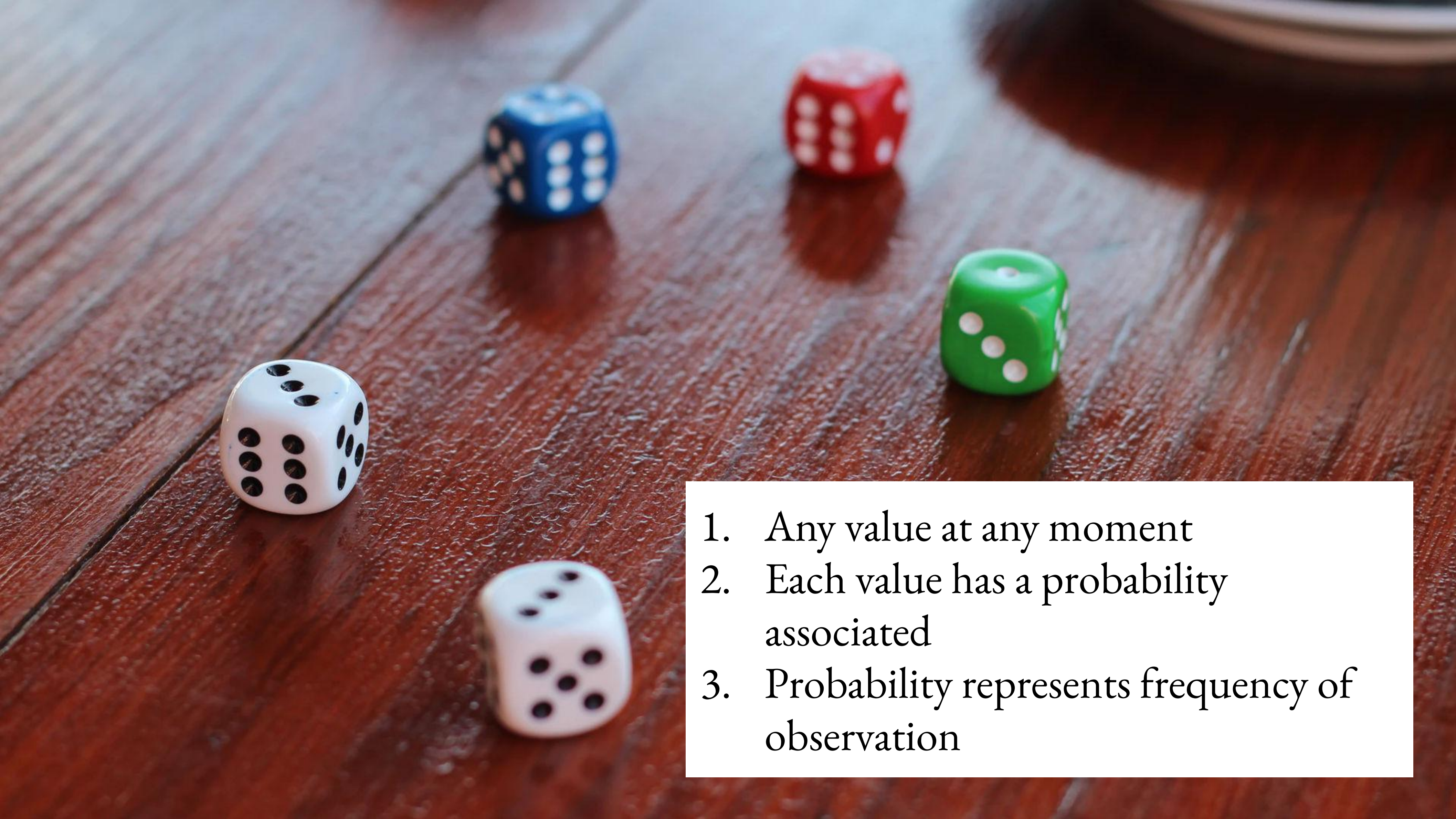
≪As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality≫

Albert Einstein



Random Variables

Basic Concepts



1. Any value at any moment
2. Each value has a probability associated
3. Probability represents frequency of observation

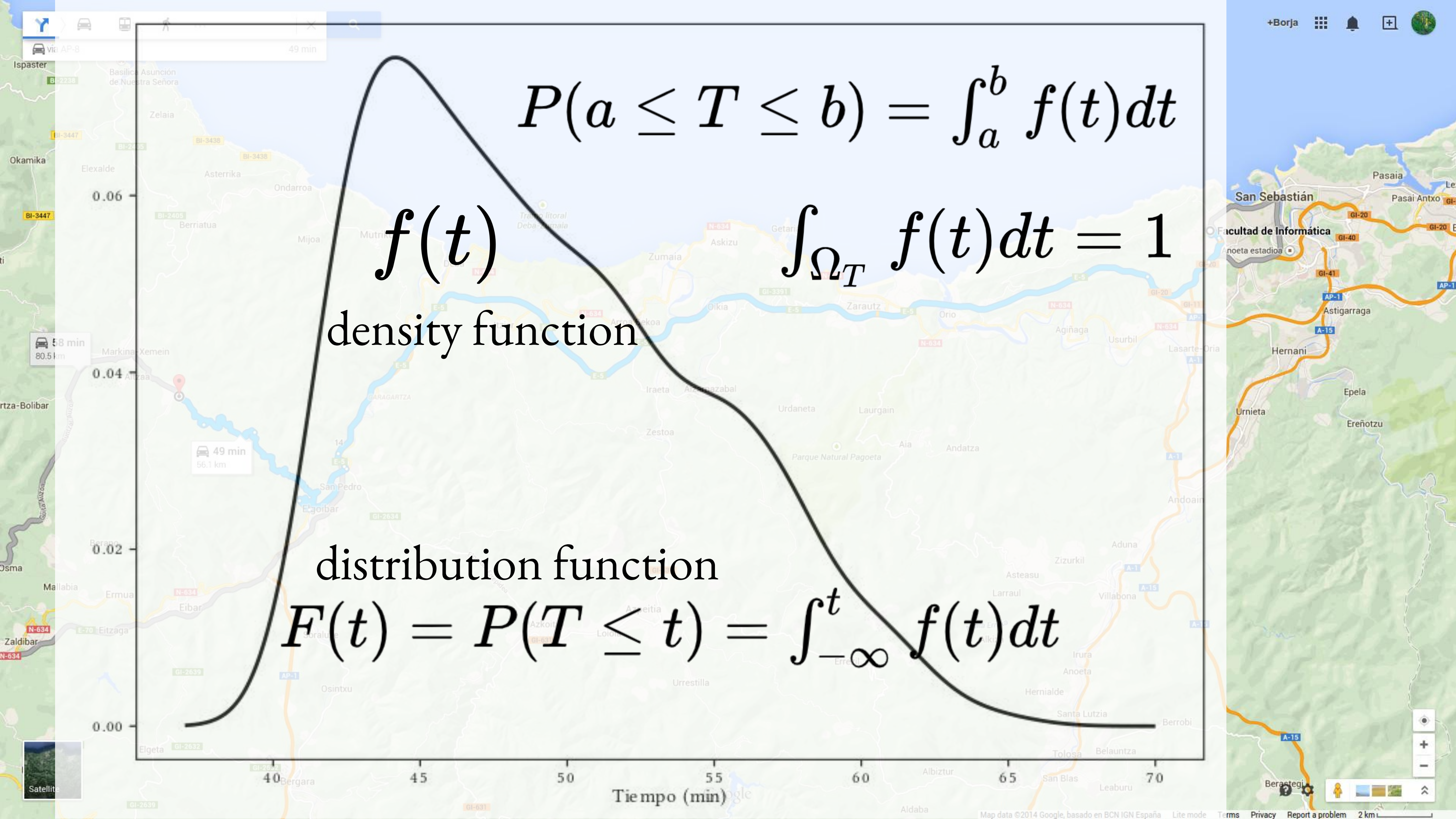
What's the probability of getting ⚅ (P(X=6))?

What's the probability of getting an even number? And ANY number?

$$0 \leq P(X) \leq 1$$

$$P(X = x_i or X = x_j) = P(X = x_i) + P(X = x_j)$$

$$P(X \in \Omega_X) = \sum_{x \in \Omega_X} P(X = x) = 1$$



$P(a \leq T \leq b) = \int_a^b f(t)dt$
$f(t)$
$\int_{\Omega_T} f(t)dt = 1$
density function
distribution function
$F(t) = P(T \leq t) = \int_{-\infty}^t f(t)dt$
0.00
0.02
0.04
0.06
40
45
50
55
60
65
70
Tiempo (min)

SUMMARY

- A random variable can take any value at any moment
- Each value has an associated probability
- Random variables can be ...

DISCRETE VARIABLES

Probability mass function, P(X)

$$0 \leq P(X) \leq 1$$

$$P(X \in A) = \sum_{x \in A} P(X = x)$$

$$\sum_{x \in \Omega_X} P(X = x) = 1$$

CONTINUOUS VARIABLES

Probability density function, f(X)

$$f(X) \geq 0$$

$$P(a \leq X \leq b) = \int_a^b f(X) dX$$

$$\int_{\Omega_X} f(X) dX = 1$$

Random Variables

Vectors of Variables

JOINT PROBABILITY

$$P(X, Y)$$

MARGINAL PROBABILITY

$$P(X) \qquad P(Y)$$

$X \in \{H, T\}$

$Y \in \{1, 2, 3, 4, 5, 6\}$

Provided that I have the joint probability, what's the probability of Y=6?

MARGINALIZATION

$$P(Y = y) = \sum_{x \in \Omega_X} P(X = x, Y = y)$$

$$P(Y = 6) = P(Y = 6, X = H) + P(Y = 6, X = T)$$

1. Toss the coin
2. If the result is R, roll the red dice; otherwise, roll the green dice

* The green dice is a regular dice, but the red dice only has even numbers

What's the probability of Red and 1? and Red and 2?

CONDITIONAL PROBABILITIES

$$P(Y = 1 | X = R) = 0$$
$$P(Y = 2 | X = R) = 0.33$$

$$P(X, Y) = P(X) P(Y|X)$$
$$P(X, Y) = P(Y) P(X|Y)$$

GENERALIZATION: CHAIN RULE

$$P(X_1, \ldots, X_n) = P(X_1) \cdot P(X_2|X_1) \cdot \ldots \cdot P(X_n|X_1, \ldots, X_{n-1})$$

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i|X_1, \ldots, X_{i-1})$$

BAYES' THEOREM

$$P(X|Y) = \frac{P(X,Y)}{P(Y)} = \frac{P(Y|X)P(X)}{P(Y)}$$

INDEPENDENCE

$$P(Y = 3|X = H)?$$

$$P(Y = 3|X = T)?$$

X and Y are independent if:

$$P(X, Y) = P(X)P(Y)$$

$$P(X|Y) = P(X); P(Y|X) = P(Y)$$

CONDITIONAL INDEPENDENCE

X and Y are conditionally independent given Z if:

$$P(X|Y,Z) = P(X|Z); P(Y|X,Z) = P(Y|Z)$$

$$P(X,Y|Z) = P(X|Z)P(Y|Z)$$

SUMMARY

Types of probability

$$P(X, Y) \qquad P(X) \qquad P(X|Y)$$

Chain Rule

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | X_1, \ldots, X_{i-1})$$

Bayes' Theorem

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

Independency

$$P(X, Y) = P(X)P(Y) \qquad P(X|Y) = P(X)$$

EXERCISE

$P(Y\|X)$	1	2	3	4	5
R	0	0.4	0	0.3	0
G	0.1	0.2	0.1	0.2	0.1

	$P(X)$
R	0.6
G	0.4

$$P(Y = 6|X = R)$$
$$P(Y = 6)$$
$$P(Y = 2, X = G)$$
$$P(X = G|Y = 5)$$

Are X and Y independent?

MONTY HALL PARADOX

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