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CMP9132M

Advanced Artificial Intelligence

Lecture 5: “Markov Models”

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Static vs. Dynamic Models

- So far you considered static worlds, where the random variables were fixed, not changing over time
- From now on we investigate **dynamic** worlds, where random variables can change between time t and $t+1$
- We assume however that the underlying process obeys to static rules (**stationary process**)

Weather example

- Let's assume a discrete random variable w_t containing the weather' state at time t
- Three possible values of w_t :

  Rainy, Sunny, Foggy

- Question examples:
  - Given that today is Sunny, what is the probability that tomorrow is Sunny and the day after is Rainy?
  - What is the probability that today is Rainy given yesterday and the day before were Sunny?

N-order Markov Models

- In theory, the more I know about the past, the more accurate my estimate of today's weather will be

$$P(w_t \mid w_{t-1}, w_{t-2}, w_{t-3}, w_{t-4}, \ldots)$$

- In practice, however, we use approximations that use only N events in the past

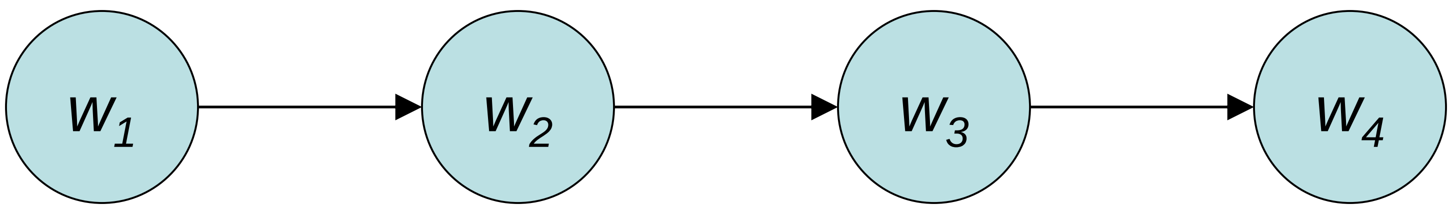
$$P(w_t \mid w_{t-1}, w_{t-2}, w_{t-3}, w_{t-4}, \ldots) \approx P(w_t \mid w_{t-1}, \ldots, w_{t-N})$$

- In this case we talk about **N-order Markov assumption**, i.e. the probability of the current event w_t depends only on the previous N events $w_{t-1}, \ldots, w_{t-N}$

First-order Markov Model

- For N = 1 we have a 1st-order MM:

$$P(w_t \mid w_{t-1}, w_{t-2}, w_{t-3}, w_{t-4}, \ldots) = P(w_t \mid w_{t-1})$$



- This is the most common choice (the one we will use from now on…)

Joint Probability of Markov Model

- Applying iteratively the product rule, the joint probability of $w_1, \ldots, w_T$ can be written as

$$P(w_1,\ldots,w_T) = P(w_1)P(w_2|w_1)P(w_3|w_2,\cancel{w_1})\ldots P(w_T|w_{T-1},\cancel{\ldots,w_1})$$

- With the Markov assumption:

$$P(w_1,\ldots,w_T) = P(w_1)P(w_2|w_1)P(w_3|w_2)\ldots P(w_T|w_{T-1})$$

$$\boxed{P(w_1,\ldots,w_T) = P(w_1)\,\Pi_{i=2..T}\,P(w_i \mid w_{i-1})}$$

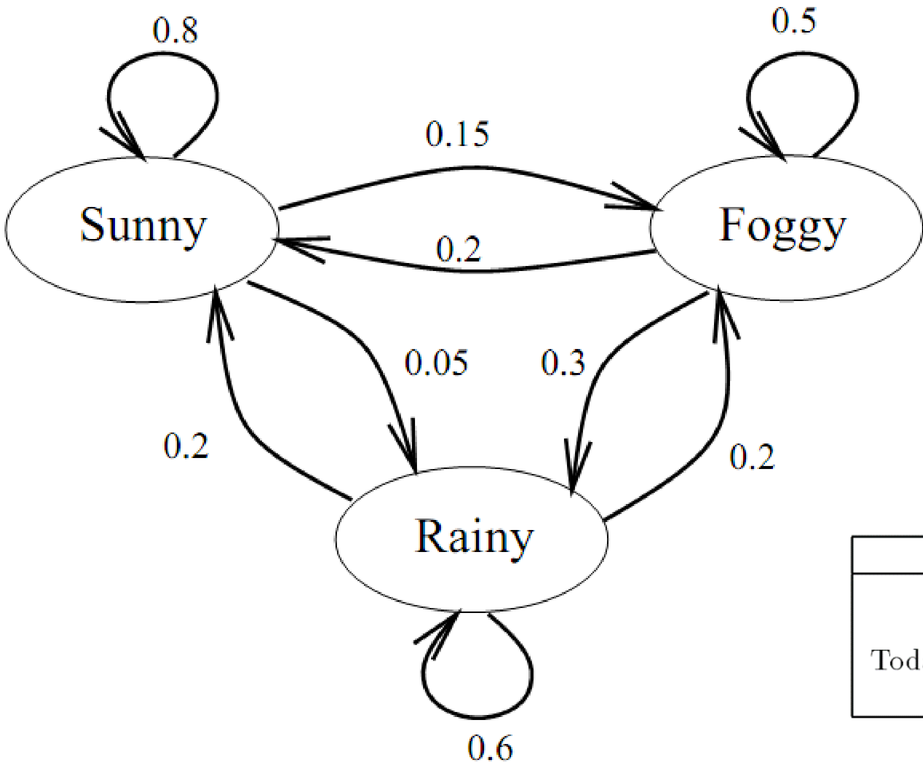
State Transition

- Table of conditional probabilities $P(w_t|w_{t-1})$
  - probability of weather tomorrow (w_t) given today's weather (w_{t-1})

		Tomorrow's Weather		
		Sunny	Rainy	Foggy
Today's Weather	Sunny	0.8	0.05	0.15
	Rainy	0.2	0.6	0.2
	Foggy	0.2	0.3	0.5

- e.g. $P(w_t\text{=Rainy}|w_{t-1}\text{=Sunny}) = 0.05$

State Transition



		Tomorrow's Weather		
		Sunny	Rainy	Foggy
Today's Weather	Sunny	0.8	0.05	0.15
	Rainy	0.2	0.6	0.2
	Foggy	0.2	0.3	0.5

- This represents the possible transitions from one state of the weather variable at time *t-1* to another one at time *t*.
- NOTE: this is NOT a graphical model!

Weather example

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- Given that today is Sunny, what is the probability that tomorrow is Sunny and the day after is Rainy?

- Suppose that today is t=1, tomorrow is t=2, etc.

$$P(w_3\text{=Rainy}, w_2\text{=Sunny} \mid w_1\text{=Sunny}) = ?$$

Weather example

- We want to decompose the previous expression into something that contains our known terms $P(w_t|w_{t-1})$

$P(w_3, w_2 \mid w_1) =$

$= P(w_3, w_2, w_1) / P(w_1)$

$= \cancel{P(w_1)}\, P(w_2 \mid w_1)\, P(w_3 \mid w_2, w_1) / \cancel{P(w_1)}$

$= P(w_2 \mid w_1)\, P(w_3 \mid w_2, \cancel{w_1})$ (Markov assumption)

(with values)

		Tomorrow's Weather		
		Sunny	Rainy	Foggy
Today's Weather	Sunny	0.8	0.05	0.15
	Rainy	0.2	0.6	0.2
	Foggy	0.2	0.3	0.5

$P(w_3\text{=Rainy}, w_2\text{=Sunny} \mid w_1\text{=Sunny})$

$= P(w_2\text{=Sunny} \mid w_1\text{=Sunny})\, P(w_3\text{=Rainy} \mid w_2\text{=Sunny})$

$= 0.8 * 0.05$

$= 0.04$

Conclusions

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- Suggested reading
  - Tutorial on Blackboard (try the exercises!)
  - Chapter 15 of Russell & Norvig
- Next week
  - Hidden Mark Models
- **Starting from next week, workshops are mainly dedicated to Assignment 1**
- Any question?