Quantifying Uncertainty
Advanced Artificial Intelligence: Workshop

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Overview

- Events
- Marginal Distributions
- Conditional Probabilities
- Normalisation
- Inference by Enumeration
- Bayes Rule

Events

- Somebody throws two dice. What is the $P(\text{dice roll} = 11)$?

- Given the following probability distribution $P(X, Y)$:

X	Y	P
x	y	0.2
x	¬y	0.3
¬x	y	0.4
¬x	¬y	0.1

  - What is the value of $P(x \wedge y)$?
  - What is the value of $P(x)$?
  - What is the value of $P(x \vee y)$?

Marginal Distributions

Given the following probability distribution $P(X, Y)$:

X	Y	P(X,Y)
x	y	0.2
x	¬y	0.3
¬x	y	0.4
¬x	¬y	0.1

What are the values of $P(X)$

X	P(X)
x	
¬x	

and $P(Y)$?

Y	P(Y)
y	
¬y	

Conditional Probabilities

Given the following probability distribution $P(X, Y)$:

X	Y	P
x	y	0.2
x	¬y	0.3
¬x	y	0.4
¬x	¬y	0.1

- What is the value of $P(x|y)$?
- What is the value of $P(\neg x|y)$?
- What is the value of $P(\neg y|x)$?

Normalisation

Given the following probability distribution $P(T, W)$:

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

- What is the value of α in $P(W|T{=}cold) = \alpha P(W, T{=}cold)$?

Inference by Enumeration

Given the following probability distribution $P(S, T, W)$:

S	T	W	P
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20

- What is the value of $P(sun)$?
- What is the value of $P(sun|winter)$?
- What is the value of $P(sun|winter, hot)$?

Bayes Rule: Exercises (1/2)

- Pat goes in for a routine health check and takes some tests. One test for a **rare genetic disease** comes back positive. The disease (d) is potentially fatal.
- She asks around and learns that rare means: $P(d)$=1/10000
- The test (t) is very accurate: $P(t|d)$ =0.99, $P(\neg t|\neg d)$ =0.95
- Pat wants to know the probability that she has the disease, i.e. $P(d|t)$ =?

Bayes Rule: Exercises (2/2) [1]

- Consider the following fictitious scientific information. Doctors find that people with the Kreuzfeld-Jacob disease (KJ) almost invariably ate hamburgers, thus $P(HamburgerEater|KJ) = 0.9$.

- The probability of an individual having KJ is rather low, about $1/100,000$.

- Assuming eating logs of hamburgers is rather widespread, say $P(HamburgerEater) = 0.5$, what is the probability that a Hamburger Eater will have the KJ disease, i.e. $P(KJ|HamburgerEater) = ?$

[1] Excercise from Barber, D. (2012), "Bayesian Reasoning and Machine Learning", Cambridge University Press