Exercises

1	2	3	4	5	6

Surname, First name

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
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KEN3241 Introduction to Quantum Computing
Final Exam KEN3241

Program: Bachelor Data Science and Artificial Intelligence
Course code: KEN3241
Examiner(s): Dibenedetto Domenica
Date/time:
Format: Closed book exam
Allowed aids: Calculator from the DACS list of allowed calculators and tables of basic quantum operators
Instructions to students:

- The exam consists of 6 questions.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded! As a last resort if you run out of space, use the extra answer space at the end of the exam.
- In no circumstance write on or near the QR code at the bottom of the page!
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

Exercise 1

20p **1** Consider a two-Qbit system:
$|\psi_1\rangle = \alpha|0\rangle + \beta|1\rangle$ and $|\psi_2\rangle = \gamma|0\rangle + \delta|1\rangle$
Of course, we have that $\alpha^2 + \beta^2 = \gamma^2 + \delta^2 = 1$.
We perform a Hadamard operator on the first Qbit, and then we perform two CNOT operators, with the first Qbit as the control bit and the second Qbit as the target Qbit. Then, we perform a Hadamard operator on the first Qbit.

1. (5 points) Draw the quantum circuit diagram that corresponds to the previous description.
2. (10 points) What is the output of the circuit? In particular, describe the new quantum states $|\psi'_1\rangle$, $|\psi'_2\rangle$ and the composite state of the two Qbits.
3. (5 points) How would your answer on part (2) if between the two CNOT gates we had a single Pauli-X operator acting on the first Qbit?

Exercise 2

20p **2** Assume we have three Qbits A (Alice's), B (Bob's) and C (Charlie's) in the composite state:

$|\psi\rangle = \frac{2}{\sqrt{5}}|000\rangle - \frac{1}{\sqrt{5}}|101\rangle$

1. (10 points) Are the three Qbits in an entangled or in a non-entangled state? If they are in an entangled state, justify fully your answer. If they are in a non-entangled state, provide the individual quantum states of each Qbit.
2. (10 points) Assume that Alice measures her Qbit in the standard $\{|0\rangle, |1\rangle\}$ basis. What does she observe? What is the new state of the system?

Exercise 3

20p **3**

Remember the Quantum Teleportation protocol: Alice would like to send to Bob a Qbit $|\psi\rangle = a_0|0\rangle + a_1|1\rangle$ she can achieve this by sending to Bob only two classical bits of information. The teleportation protocol requires that Alice and Bob initially share the Bell state $\frac{1}{2}(|00\rangle + |11\rangle) = |b_{00}\rangle$.
The composite state is denoted by the three Qbits arranged in that order: $|\psi\rangle|b_{00}\rangle$. Alice manipulates her Qbits (apply a CNOT, and then an H on her 1st Qbit) and makes a joint measurement of her Qbit $|\psi\rangle$ and her share of the Bell state. She then sends the results of her measurements (classical bits a, b) to Bob over a classical channel. The values of a, b are used by Bob in order to control the operation Bob performs on his Qbit (his share of the Bell state). After Bob performs the required operation (which is either a Pauli-X or a Pauli-Z), his Qbit is left in state $|\psi\rangle$.
Question: What would happen if Alice and Bob share not the Bell state $|b_{00}\rangle$, but the $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$ instead? Could Bob somehow make appropriate measurements and still retrieve $|\psi\rangle$?
Justify your answer.

Exercise 4

20p **4** Explain Grover's algorithm in the context of an unsorted database search. Provide a concise description of the key quantum operations involved, and discuss how it achieves a quadratic speedup compared to classical algorithms. In your explanation, address the role of the oracle and the Grover diffusion operator.

Exercise 5

10p **5** A Qbit is prepared in one of the two following states: either (1) in state $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ or in (2) $\frac{1}{2}|+\rangle + \frac{\sqrt{3}}{2}|-\rangle$

1. (5 points) Is there an experiment (measurement) where you can distinguish with absolute certainty whether the Qbit is in the state (1) vs in state (2)
2. (5 points) Suppose we have a Qbit described by the state (1), what are the probabilities that we will measure $|+\rangle$ or $|-\rangle$ if we perform the measurement in the $\{|+\rangle, |-\rangle\}$ basis?

Exercise 6

10p **6** Show that a bitflip operation, preceded and followed by Hadamard transforms, equals a phase flip operation: HXH = Z.

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