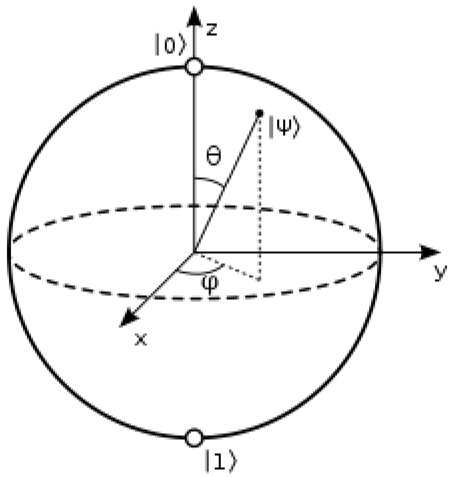
Intro to Quantum Computing

Quantum Circuits



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- But in much (much…) more detail.
- Also, Preskill's Lecture notes (Chapter 5 http://theory.caltech.edu/~preskill/ph219/chap5_15.pdf) contain a very thorough discussion and we will borrow some of the exposition from him

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- Quantum Mechanics is a *Physical Theory*
- Just like *Logic….*
- The rules of QM can also be interpreted as *executable operations in a programmable machine.*
- Just like a classical computing machine is a bunch of logical circuits, a quantum computing machine is a bunch of *quantum logical circuits (gates)*

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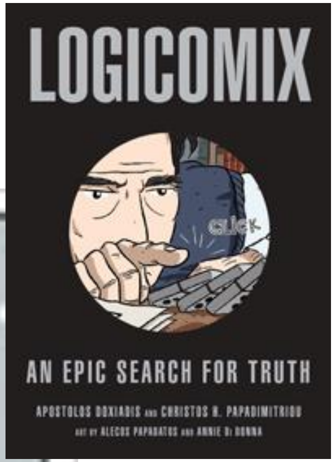

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- Let’s just make the experiments and see what we get…

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With probability 0.25 proposition (B or A) is false!

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Heisenberg's Uncertainty Principle (once again…)

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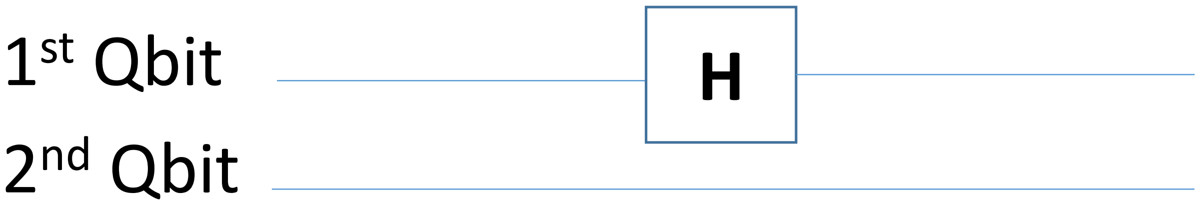

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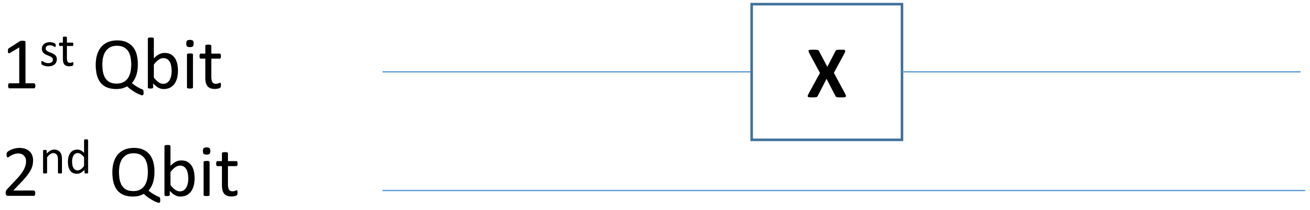

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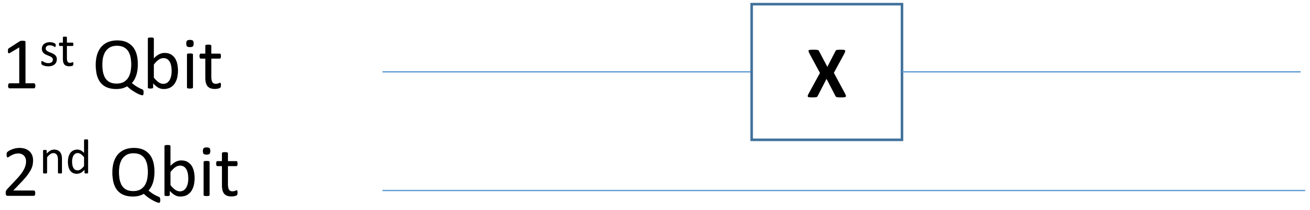

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- **Etc.**

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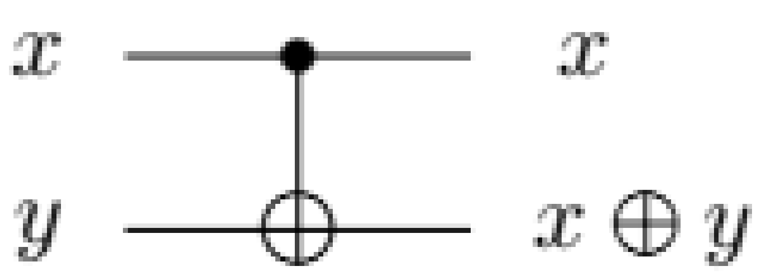

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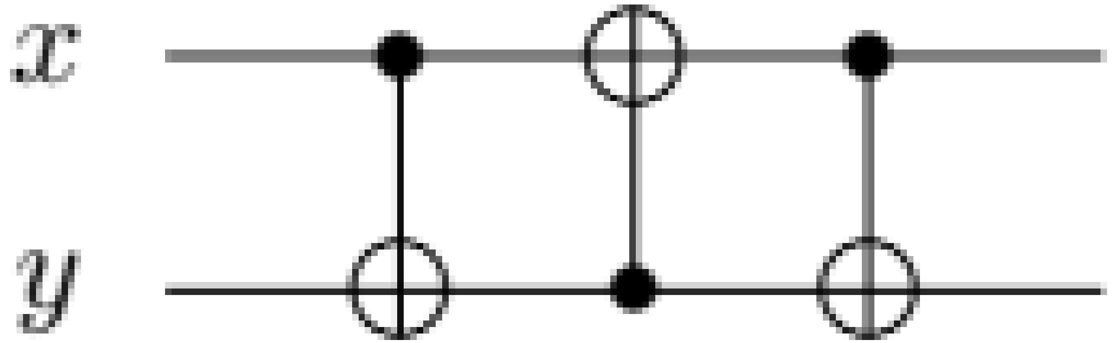
- **Question:** *What does the following circuit do?*



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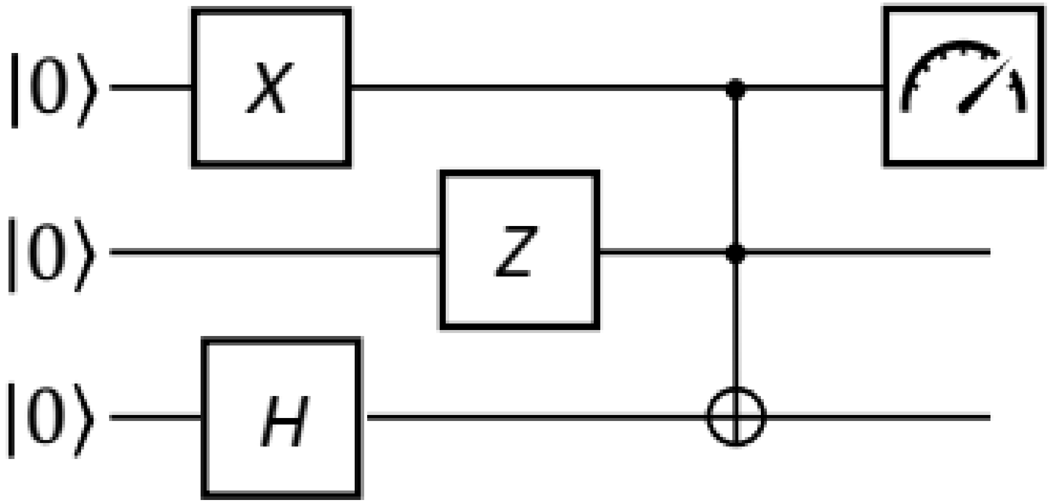

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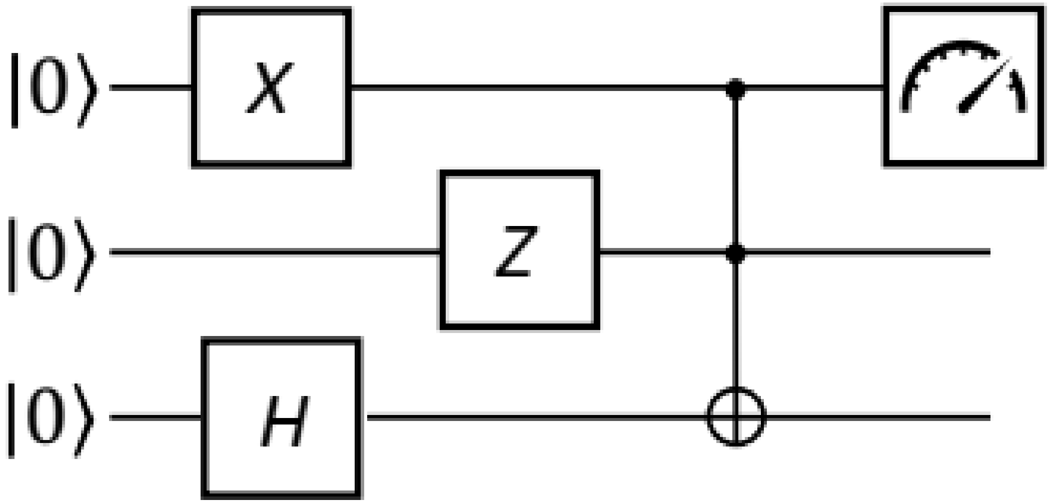

Measurement (of the value of the 1st Qbit)!

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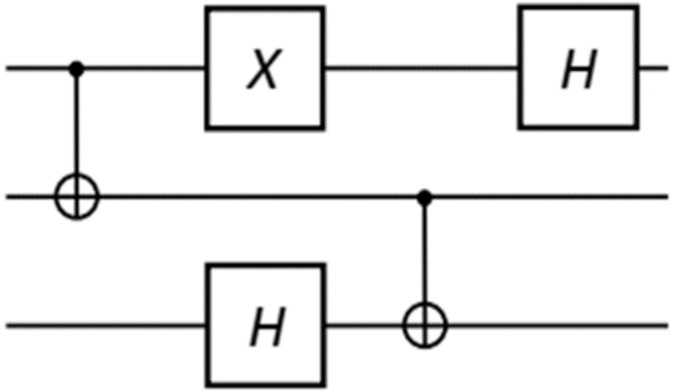
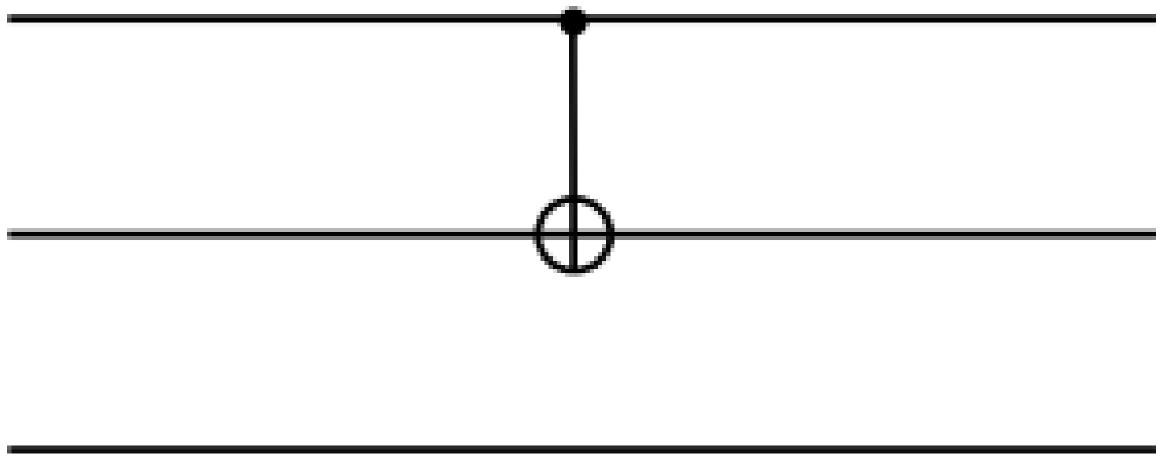
- **Question**: *What is the corresponding matrix product of the circuit?*



More on the CNOT circuit



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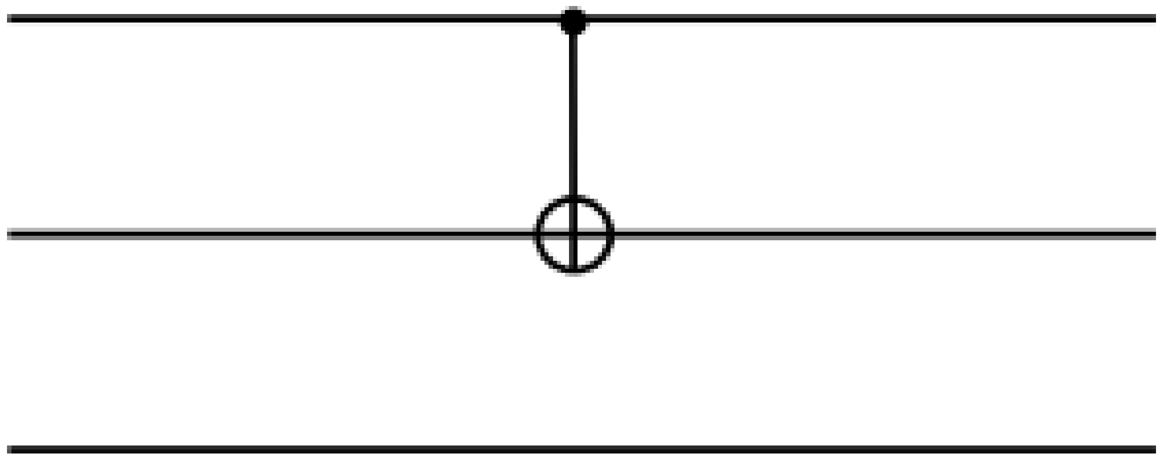

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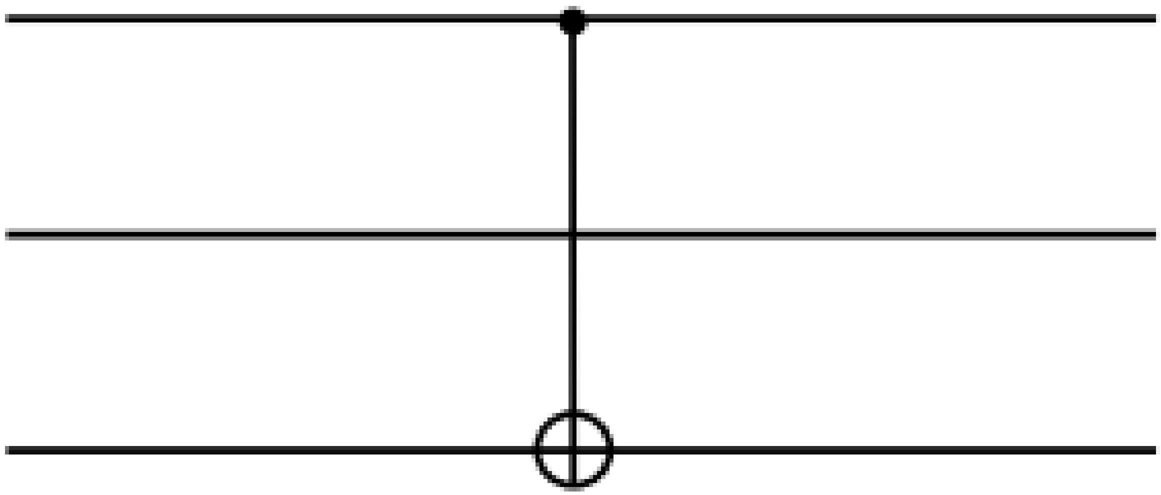

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- $(\boldsymbol{I_2} \otimes \boldsymbol{SWAP}) \times (\boldsymbol{CNOT} \otimes \boldsymbol{I_2}) \times (\boldsymbol{I_2} \otimes \boldsymbol{SWAP})$

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- **Gottesman-Kill Theorem**: Any quantum circuit consisting of **X,Y,Z,H,CNOT** can be *efficiently* simulated by a classical machine.

Exercise to think about

- Construct a circuit that given two Qbits, both prepared in the $|\mathbf{0}\rangle$ state, outputs the Bell state $\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|\mathbf{00}\rangle + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|\mathbf{11}\rangle$