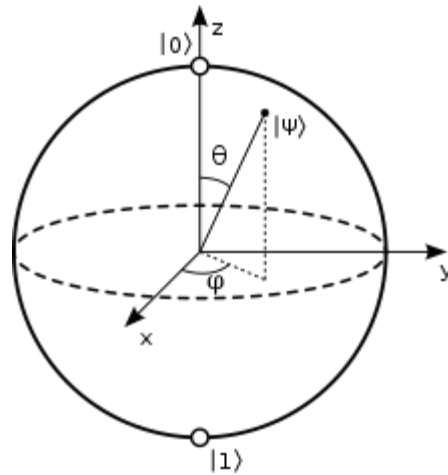


# Intro to Quantum Computing

Entanglement



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- *Do you see why?*

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***But can we do it in general?***

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- **And now let's separate them!**

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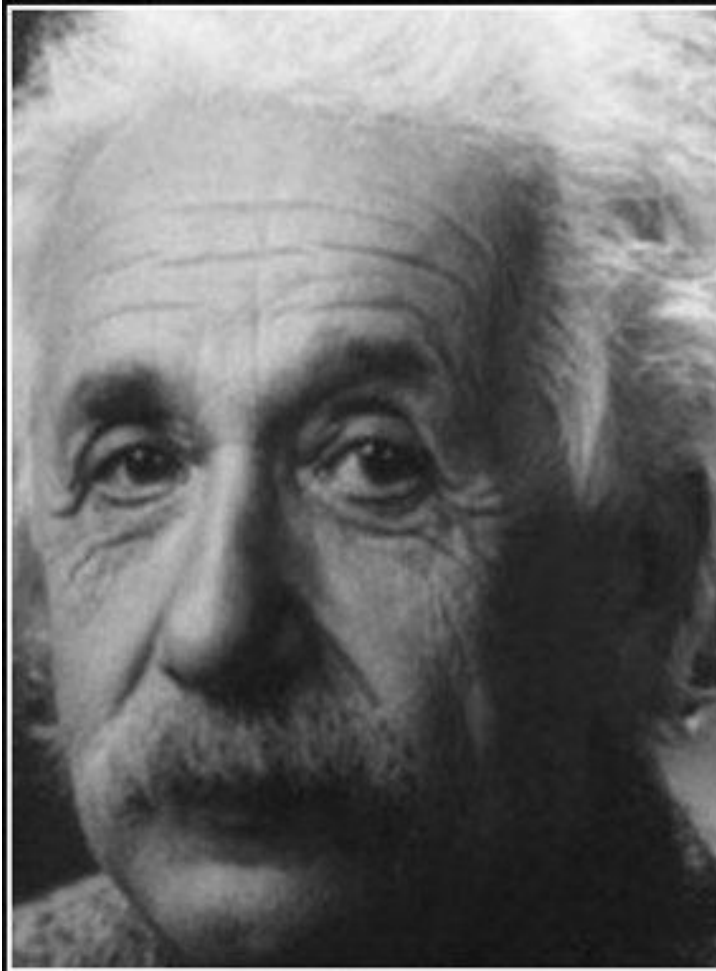
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***Does this mean we have exchanged information faster than the speed of light?***

# Spooky Action at a Distance



*I cannot seriously believe in it  
[quantum theory] because the theory  
cannot be reconciled with the idea  
that physics should represent a reality  
in time and space, free from spooky  
actions at a distance [spukhafte  
Fernwirkungen]*

*~Albert Einstein*