

# Intro to Quantum Computing 2021

## Exercise Set I

**Exercise 1.** Consider the following states for a Qbit corresponding to the basis:  $\{|\rightarrow\rangle, |\leftarrow\rangle\}$ :

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \quad \text{and} \quad |\leftarrow\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle.$$

Compute the following quantities (numbers/vectors or matrices) using bracket notation and matrix notation in each case.

1.  $\langle 1 | |\rightarrow\rangle = \langle 1 | \rightarrow\rangle$ .
2.  $|0\rangle\langle\rightarrow|$ .
3.  $\langle\rightarrow| \leftarrow\rangle$ .
4.  $|\rightarrow\rangle\langle\leftarrow|$ .
5.  $|\rightarrow\rangle \otimes |\leftarrow\rangle$ .
6.  $(X \otimes I_2)(|\rightarrow\rangle \otimes |\leftarrow\rangle)$  where  $X$  is the  $2 \times 2$  Pauli matrix and  $I_2$  is the  $2 \times 2$  identity matrix.
7.  $(X \otimes Y)(|\rightarrow\rangle \otimes |\leftarrow\rangle)$  where  $Y$  is the second Pauli matrix.

**Exercise 2.** Consider the following four 2-Qbit states:

$$|T_1\rangle = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

$$|T_2\rangle = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle.$$

1. Show that the neither state can be written as a tensor product of two single Qbit states  $|q_1\rangle \otimes |q_2\rangle$ .
2. Show that  $(U \otimes U)|T_2\rangle = |T_2\rangle$  for every 1-Qbit unitary operator  $U$  (i.e.,  $U$  is a  $2 \times 2$  matrix satisfying the properties of every unitary matrix: its transpose is equal to its inverse).

**Exercise 3.** Suppose we have a Qbit described by the state

$$|\psi\rangle = \sqrt{\frac{2}{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle.$$

- What are the probabilities that we will measure  $|\rightarrow\rangle$  or  $|\leftarrow\rangle$  as they are defined in Exercise 1, if we perform the measurement in this new basis?