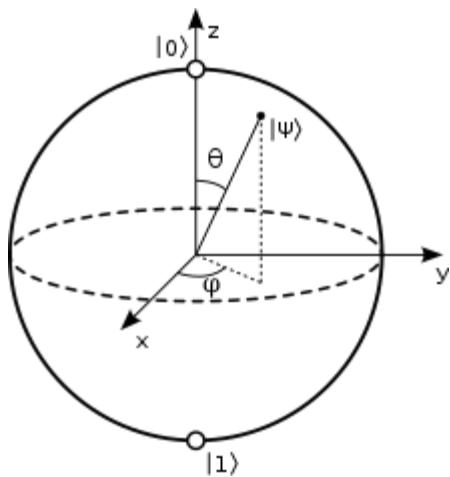


# Intro to Quantum Computing

Entanglement



# Recall: Composition of Quantum Systems

# Recall: Composition of Quantum Systems

- **Postulate 4**

The state space of a composite physical system  $|\psi\rangle$  is the tensor product of the state spaces of the constituent components (physical systems)

# Recall: Composition of Quantum Systems

- **Postulate 4**

The state space of a composite physical system  $|\psi\rangle$  is the tensor product of the state spaces of the constituent components (physical systems), i.e.,  $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle$

# Recall: Composition of Quantum Systems

- **Postulate 4**

The state space of a composite physical system  $|\psi\rangle$  is the tensor product of the state spaces of the constituent components (physical systems), i.e.,  $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle$

i.e., if  $|\psi\rangle$  is a 2-qbit system consisting of  $|q_1\rangle, |q_2\rangle$  then  $|\psi\rangle = |q_1\rangle \otimes |q_2\rangle$

# Recall: Composition of Quantum Systems

- **Postulate 4**

The state space of a composite physical system  $|\psi\rangle$  is the tensor product of the state spaces of the constituent components (physical systems), i.e.,  $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle$

i.e., if  $|\psi\rangle$  is a 2-qbit system consisting of  $|q_1\rangle, |q_2\rangle$  then  $|\psi\rangle = |q_1\rangle \otimes |q_2\rangle$

- From now on, when we multiply two states, we mean take the tensor product

# Recall: Composition of Quantum Systems

- **Postulate 4**

The state space of a composite physical system  $|\psi\rangle$  is the tensor product of the state spaces of the constituent components (physical systems), i.e.,  $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle$

i.e., if  $|\psi\rangle$  is a 2-qbit system consisting of  $|q_1\rangle, |q_2\rangle$  then  $|\psi\rangle = |q_1\rangle \otimes |q_2\rangle$

- From now on, when we multiply two states, we mean take the tensor product
- $|q_1\rangle \otimes |q_2\rangle = |q_1\rangle|q_2\rangle$

# Composite Systems of two Qbits

- Assume we have two Qbits (polarized photons, spins of electrons whatever...).

# Composite Systems of two Qbits

- Assume we have two Qbits (polarized photons, spins of electrons whatever...).
- State of the first Qbit

$$|\psi_1\rangle = a_0|0\rangle + a_1|1\rangle$$

# Composite Systems of two Qbits

- Assume we have two Qbits (polarized photons, spins of electrons whatever...).
- State of the first Qbit

$$|\psi_1\rangle = a_0|0\rangle + a_1|1\rangle$$

- State of the second Qbit

$$|\psi_2\rangle = b_0|0\rangle + b_1|1\rangle$$

# Composite Systems of two Qbits

- Assume we have two Qbits (polarized photons, spins of electrons whatever...).
- State of the first Qbit

$$|\psi_1\rangle = a_0|0\rangle + a_1|1\rangle$$

- State of the second Qbit

$$|\psi_2\rangle = b_0|0\rangle + b_1|1\rangle$$

- What we want to know:

# Composite Systems of two Qbits

- Assume we have two Qbits (polarized photons, spins of electrons whatever...).
- State of the first Qbit

$$|\psi_1\rangle = a_0|0\rangle + a_1|1\rangle$$

- State of the second Qbit

$$|\psi_2\rangle = b_0|0\rangle + b_1|1\rangle$$

- What we want to know:

***The state of the composite system consisting of the two Qbits***

# Composite Systems

- Simply take the state of the first system and multiply (tensor) it by the state of the second system:

# Composite Systems

- Simply take the state of the first system and multiply (tensor) it by the state of the second system:

$$(|\psi_1\rangle)(|\psi_2\rangle) = (a_0|0\rangle + a_1|1\rangle)(b_0|0\rangle + b_1|1\rangle)$$

# Composite Systems

- Simply take the state of the first system and multiply (tensor) it by the state of the second system:

$$\begin{aligned}(|\psi_1\rangle)(|\psi_2\rangle) &= (a_0|0\rangle + a_1|1\rangle)(b_0|0\rangle + b_1|1\rangle) \\ &= a_0b_0|0\rangle|0\rangle + a_0b_1|0\rangle|1\rangle + a_1b_0|1\rangle|0\rangle + a_1b_1|1\rangle|1\rangle \end{aligned}$$

# Composite Systems

- Simply take the state of the first system and multiply (tensor) it by the state of the second system:

$$\begin{aligned}(|\psi_1\rangle)(|\psi_2\rangle) &= (a_0|0\rangle + a_1|1\rangle)(b_0|0\rangle + b_1|1\rangle) \\ &= a_0b_0|0\rangle|0\rangle + a_0b_1|0\rangle|1\rangle + a_1b_0|1\rangle|0\rangle + a_1b_1|1\rangle|1\rangle \end{aligned}$$

- Let  $|s_1\rangle|s_2\rangle = |s_1s_2\rangle$ . Then the above becomes

# Composite Systems

- Simply take the state of the first system and multiply (tensor) it by the state of the second system:

$$\begin{aligned}(|\psi_1\rangle)(|\psi_2\rangle) &= (a_0|0\rangle + a_1|1\rangle)(b_0|0\rangle + b_1|1\rangle) \\ &= a_0b_0|0\rangle|0\rangle + a_0b_1|0\rangle|1\rangle + a_1b_0|1\rangle|0\rangle + a_1b_1|1\rangle|1\rangle \end{aligned}$$

- Let  $|s_1\rangle|s_2\rangle = |s_1s_2\rangle$ . Then the above becomes

$$|\psi_1\psi_2\rangle = a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle$$

# Composite Systems

- For example, suppose  $|\psi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

# Composite Systems

- For example, suppose  $|\psi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
- And  $|\psi_2\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

# Composite Systems

- For example, suppose  $|\psi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
- And  $|\psi_2\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$
- Then

$$|\psi_1\psi_2\rangle = \frac{1}{2\sqrt{2}}|00\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|01\rangle + \frac{1}{2\sqrt{2}}|10\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|11\rangle$$

# Composite Systems

- For example, suppose  $|\psi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
- And  $|\psi_2\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$
- Then

$$|\psi_1\psi_2\rangle = \frac{1}{2\sqrt{2}}|00\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|01\rangle + \frac{1}{2\sqrt{2}}|10\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|11\rangle$$

- *Observe that the sum of the squares of the amplitudes sum up to 1 so this is a valid quantum state!*

# Composite Systems

- For example, suppose  $|\psi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
- And  $|\psi_2\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$
- Then

$$|\psi_1\psi_2\rangle = \frac{1}{2\sqrt{2}}|00\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|01\rangle + \frac{1}{2\sqrt{2}}|10\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|11\rangle$$

- *Observe that the sum of the squares of the amplitudes sum up to 1 so this is a valid quantum state!*
- *Do you see why?*

# Composite Systems

- Now consider the following question:

# Composite Systems

- Now consider the following question:
- Suppose that we are given the state of the composite system  
 $|\psi_1\psi_2\rangle = a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle.$

# Composite Systems

- Now consider the following question:
- Suppose that we are given the state of the composite system  
 $|\psi_1\psi_2\rangle = a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle$ .
- *Can you figure out what  $|\psi_1\rangle$ ,  $|\psi_2\rangle$  are?*

# Composite Systems

- Now consider the following question:
- Suppose that we are given the state of the composite system  $|\psi_1\psi_2\rangle = a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle$ .
- *Can you figure out what  $|\psi_1\rangle$ ,  $|\psi_2\rangle$  are?*
- In the case of the previous example, that's easy.

# Composite Systems

- Now consider the following question:
- Suppose that we are given the state of the composite system  $|\psi_1\psi_2\rangle = a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle$ .
- *Can you figure out what  $|\psi_1\rangle$ ,  $|\psi_2\rangle$  are?*
- In the case of the previous example, that's easy.

***But can we do it in general?***

# Composite Systems

- The answer is a big **NO.**

# Composite Systems

- The answer is a big **NO.**
- In general there are states of the composite quantum system (consisting of two qbits, or more) *that cannot be factored into two individual quantum states.*

# Composite Systems

- The answer is a big **NO.**
- In general there are states of the composite quantum system (consisting of two qbits, or more) *that cannot be factored into two individual quantum states.*
- These states are called *entangled states.*

# Composite Systems

- Consider the following simple composite state of two qbits.

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

# Composite Systems

- Consider the following simple composite state of two qbits.

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- (btw this is called Bell's state)

# Composite Systems

- Consider the following simple composite state of two qbits.

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- (btw this is called Bell's state)
- Let's try to decompose it into  $|\psi_1\rangle|\psi_2\rangle = |\psi_1\psi_2\rangle$ .

# Composite Systems

- Consider the following simple composite state of two qbits.

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- (btw this is called Bell's state)
- Let's try to decompose it into  $|\psi_1\rangle|\psi_2\rangle = |\psi_1\psi_2\rangle$ .
- And see what happens....😊

# Composite Systems

- Consider the following simple composite state of two qbits.

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- (btw this is called Bell's state)
- Let's try to decompose it into  $|\psi_1\rangle|\psi_2\rangle = |\psi_1\psi_2\rangle$ .
- And see what happens....😊
- Sometimes quantum systems they get close and they interact with each other.

# Composite Systems

- Consider the following simple composite state of two qbits.

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- (btw this is called Bell's state)
- Let's try to decompose it into  $|\psi_1\rangle|\psi_2\rangle = |\psi_1\psi_2\rangle$ .
- And see what happens....😊
- Sometimes quantum systems they get close and they interact with each other.
- They can get into a state where none of them can be described individually.

# Composite Systems

- Consider the following simple composite state of two qbits.

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- (btw this is called Bell's state)
- Let's try to decompose it into  $|\psi_1\rangle|\psi_2\rangle = |\psi_1\psi_2\rangle$ .
- And see what happens....😊
- Sometimes quantum systems they get close and they interact with each other.
- They can get into a state where none of them can be described individually.
- They are *entangled*.

# Quantum Entanglement

- Once two quantum systems (Qbits) are entangled (we will see how we can entangle them later during the course) *they remain entangled.*

# Quantum Entanglement

- Once two quantum systems (Qbits) are entangled (we will see how we can entangle them later during the course) *they remain entangled.*
- Even if we separate them by millions of light years.

# Quantum Entanglement

- Once two quantum systems (Qbits) are entangled (we will see how we can entangle them later during the course) *they remain entangled.*
- Even if we separate them by millions of light years.
- So, let's say we have two Qbits  $|\psi_1\rangle, |\psi_2\rangle$  and we somehow entangle them.

# Quantum Entanglement

- Once two quantum systems (Qbits) are entangled (we will see how we can entangle them later during the course) *they remain entangled.*
- Even if we separate them by millions of light years.
- So, let's say we have two Qbits  $|\psi_1\rangle, |\psi_2\rangle$  and we somehow entangle them.
- The entangled state is described by the previous Bell state

$$|\psi_1\psi_2\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

# Quantum Entanglement

- Once two quantum systems (Qbits) are entangled (we will see how we can entangle them later during the course) *they remain entangled.*
- Even if we separate them by millions of light years.
- So, let's say we have two Qbits  $|\psi_1\rangle, |\psi_2\rangle$  and we somehow entangle them.
- The entangled state is described by the previous Bell state
$$|\psi_1\psi_2\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$
- **And now let's separate them!**

# Quantum Entanglement

- Take the first Qbit and measure it.

# Quantum Entanglement

- Take the first Qbit and measure it.
- *What is the probability that we will see 0 or 1?*

# Quantum Entanglement

- Take the first Qbit and measure it.
- *What is the probability that we will see 0 or 1?*
- *What is the new state of  $|\psi_1\psi_2\rangle$  after we measure the 1<sup>st</sup> Qbit?*

# Quantum Entanglement

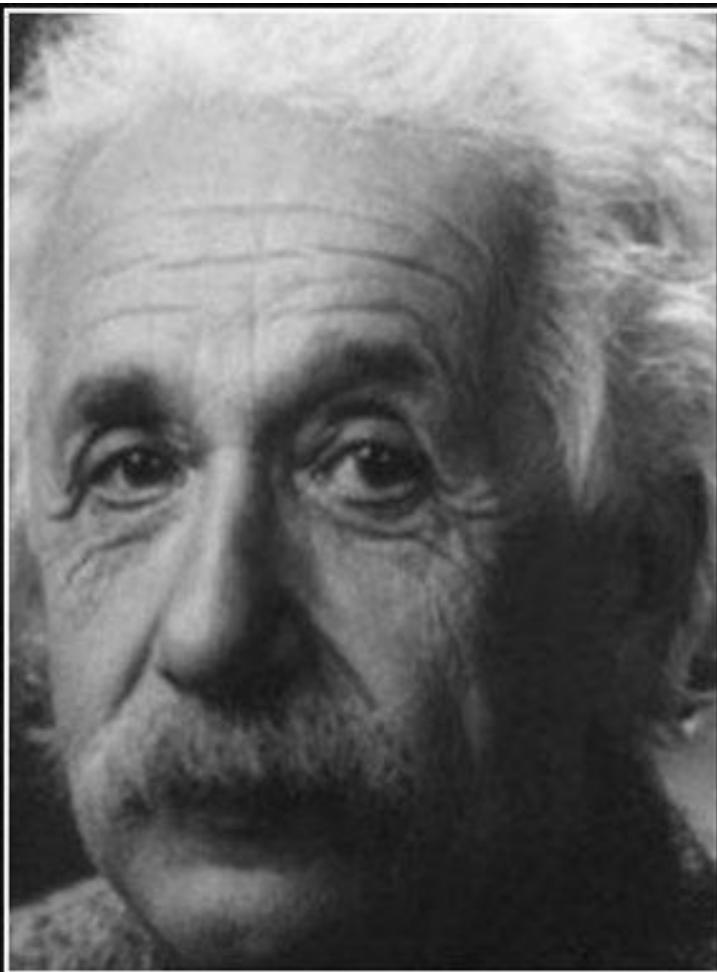
- Take the first Qbit and measure it.
- *What is the probability that we will see 0 or 1?*
- *What is the new state of  $|\psi_1\psi_2\rangle$  after we measure the 1<sup>st</sup> Qbit?*
- *What about the 2<sup>nd</sup> Qbit?*

# Quantum Entanglement

- Take the first Qbit and measure it.
- *What is the probability that we will see 0 or 1?*
- *What is the new state of  $|\psi_1\psi_2\rangle$  after we measure the 1<sup>st</sup> Qbit?*
- *What about the 2<sup>nd</sup> Qbit?*

***Does this mean we have exchanged information faster than the speed of light?***

# Spooky Action at a Distance



*I cannot seriously believe in it  
[quantum theory] because the theory  
cannot be reconciled with the idea  
that physics should represent a reality  
in time and space, free from spooky  
actions at a distance [spukhafte  
Fernwirkungen]*

*~Albert Einstein*