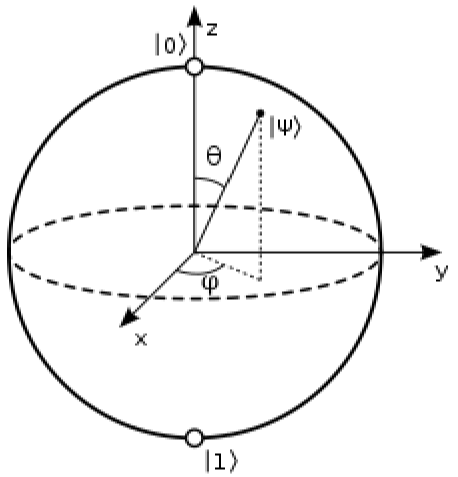
Intro to Quantum Computing

No Cloning and Quantum Teleportation



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- ***And measurement, definitely doesn't help!***

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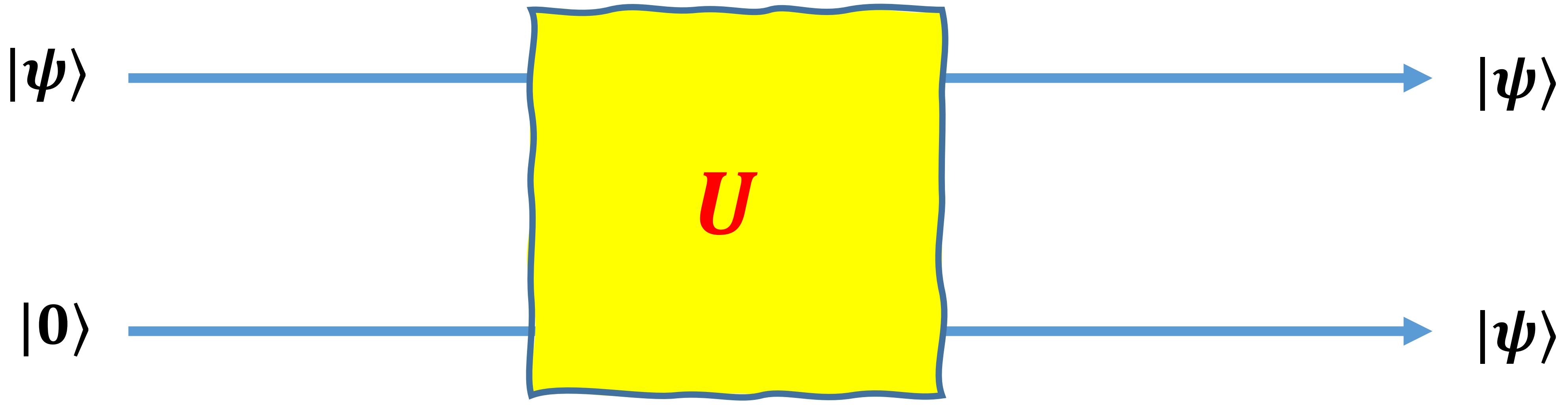

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- Which means that under $\boldsymbol{U}$ we have $|\mathbf{00}\rangle \rightarrow |\mathbf{00}\rangle$ ***and*** $|\mathbf{10}\rangle \rightarrow |\mathbf{11}\rangle$

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- $\Rightarrow$ *we cannot clone an unknown quantum state* $|\boldsymbol{\psi}\rangle$

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How can she possibly teleport $|\psi\rangle$ to Bob using only Classical Information?

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- Bob will immediately deduce what $\boldsymbol{a}, \boldsymbol{b}$ are…

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- *Alice has successfully teleported her unknown Qbit to Bob.*
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- In the process, she destroys her Qbit, but we do not care: we only care that she teleports it somehow to Bob.

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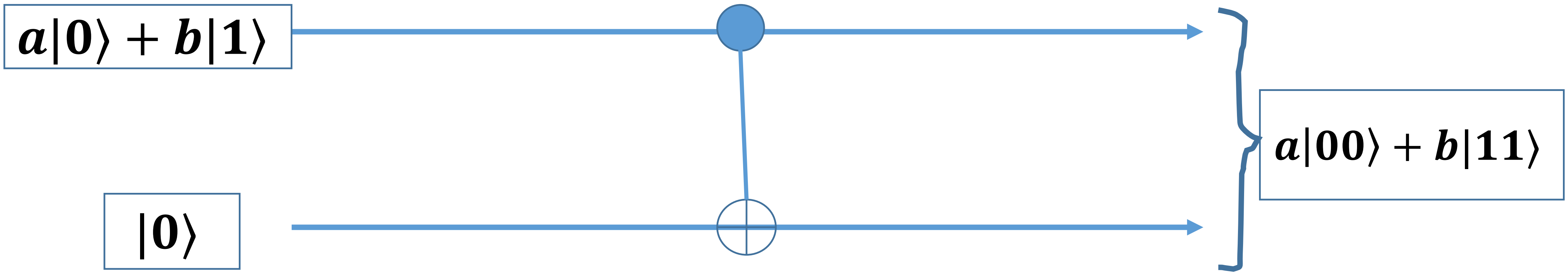

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- ***This is much better!***

First Approach

- Here is a better idea.
- Alice measures in the $|+\rangle, |-\rangle$ basis.
- We need to rewrite Alice's Qbit in the $+, -$ basis:

$$\boldsymbol{a}|\mathbf{00}\rangle + \boldsymbol{b}|\mathbf{11}\rangle = \boldsymbol{a}\left(\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|+\rangle + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|-\rangle\right) \otimes |\mathbf{0}\rangle + \boldsymbol{b}\left(\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|+\rangle - \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|-\rangle\right) \otimes |\mathbf{1}\rangle$$

- Now we make the calculations…

$$\boldsymbol{a}|\mathbf{00}\rangle + \boldsymbol{b}|\mathbf{11}\rangle = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|+\rangle[\boldsymbol{a}|\mathbf{0}\rangle + \boldsymbol{b}|\mathbf{1}\rangle] + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|-\rangle[\boldsymbol{a}|\mathbf{0}\rangle - \boldsymbol{b}|\mathbf{1}\rangle]$$

- ***This is much better!***
- If Alice measures $+$ (i.e., 0), Bob's bit is in $|\boldsymbol{\psi}\rangle$!

First Approach

- But what if Alice measures – (i.e., 1)?

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- Then Bob knows his Qbit is in state $\boldsymbol{a}|\mathbf{0}\rangle - \boldsymbol{b}|\mathbf{1}\rangle$

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- $\boldsymbol{Z(a|0\rangle - b|1\rangle) = a|0\rangle + b|1\rangle}$
- **And that is it!**

First Approach: The algorithm

- Alice has $|\boldsymbol{\psi}\rangle = \boldsymbol{a}|\mathbf{0}\rangle + \boldsymbol{b}|\mathbf{1}\rangle$. Bob has $|\mathbf{0}\rangle$**.**
- They share a CNOT gate with which they entangle their Qbits in the composite state $|\boldsymbol{\psi}'\rangle = \boldsymbol{a}|\mathbf{00}\rangle + \boldsymbol{b}|\mathbf{11}\rangle$.
- Alice measures her Qbit in the $|+\rangle, |-\rangle$ basis.
- She calls and tells Bob if she observed 0 or 1.
- If she observed 0, Bob's Qbit is in $|\boldsymbol{\psi}\rangle$.
- If she observes 1, Bob apply $\boldsymbol{Z}$ to his Qbit and now this is again in $|\boldsymbol{\psi}\rangle$.

First Approach: The algorithm

But how they can create their entangled state $|\psi'\rangle = a|00\rangle + b|11\rangle$ without any means of Quantum communications (i.e., the CNOT gate) between them?

Exercise to think about (from last lecture)

- Construct a circuit that given two Qbits, both prepared in the $|\mathbf{0}\rangle$ state, outputs the Bell state $\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|\mathbf{00}\rangle + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|\mathbf{11}\rangle$

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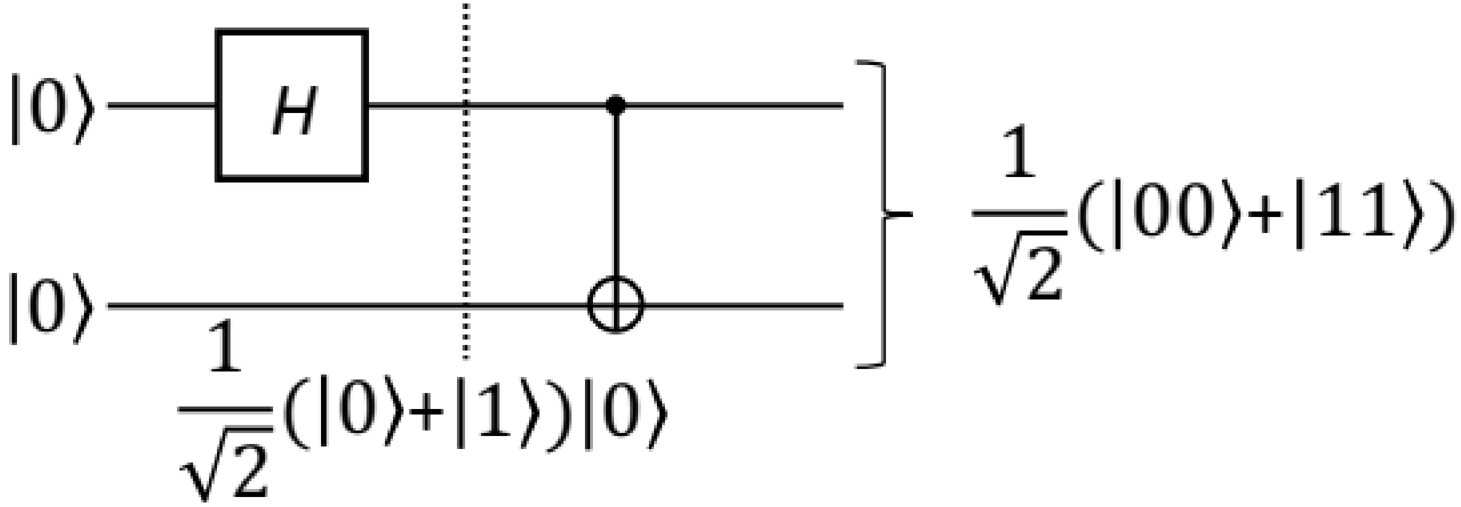

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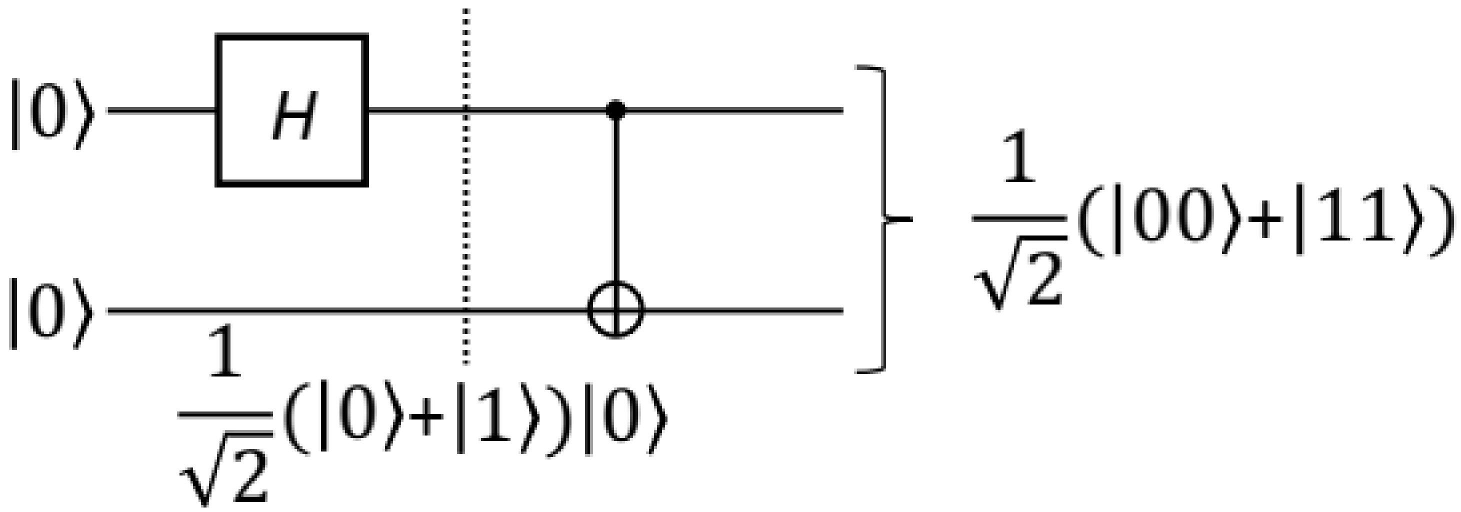

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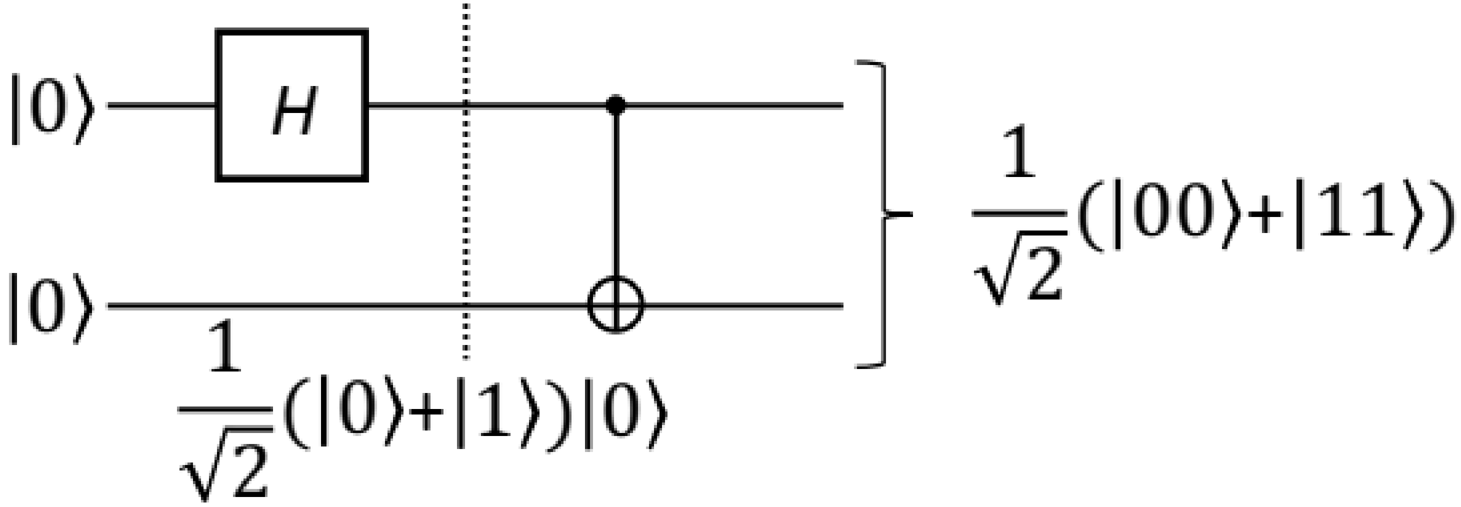

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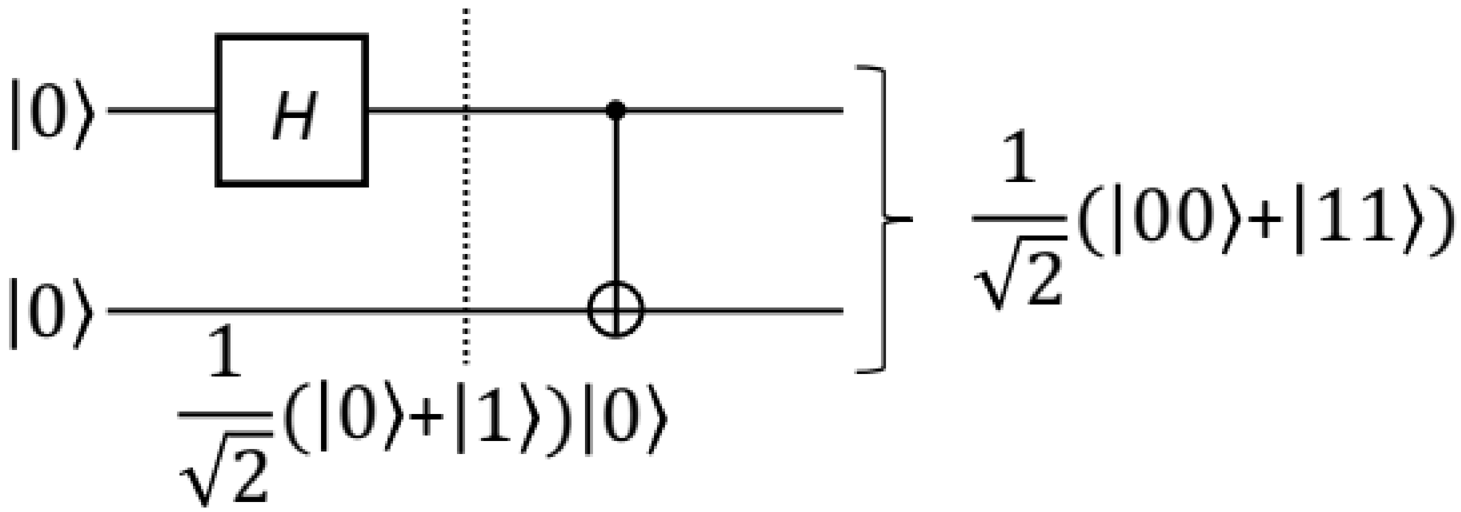

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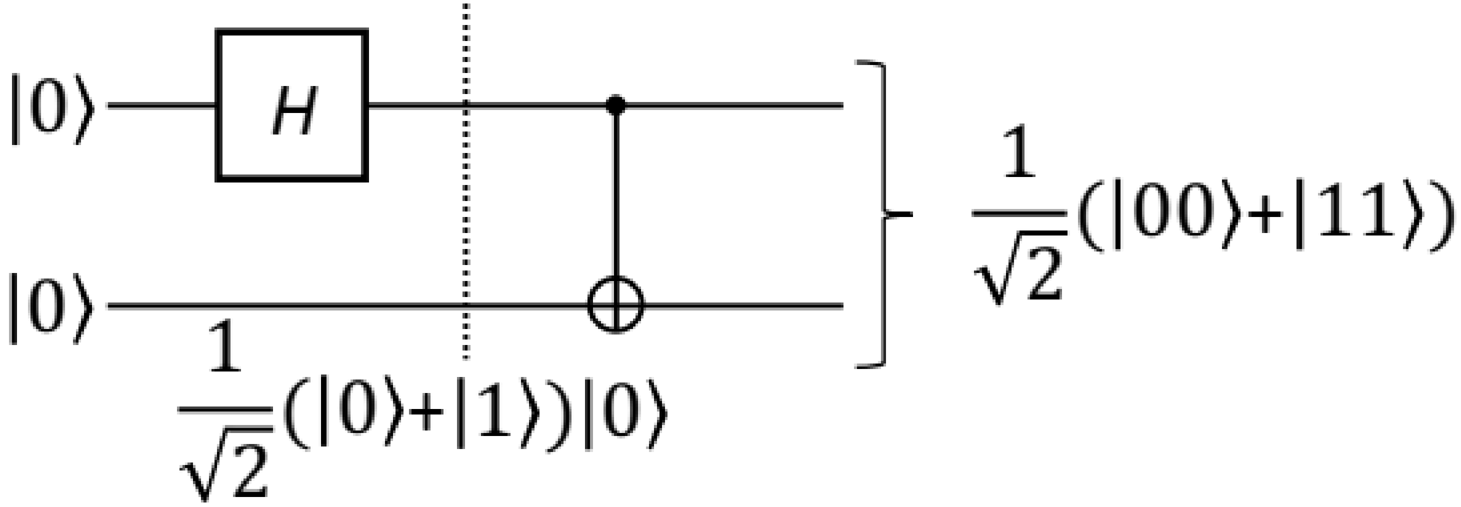

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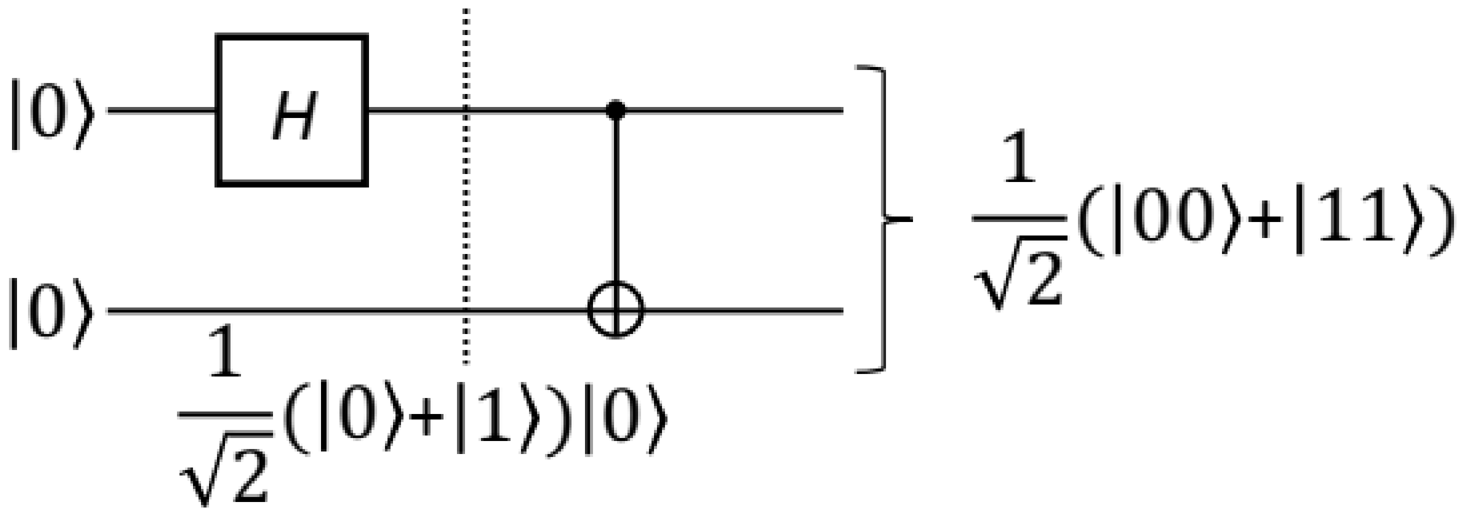

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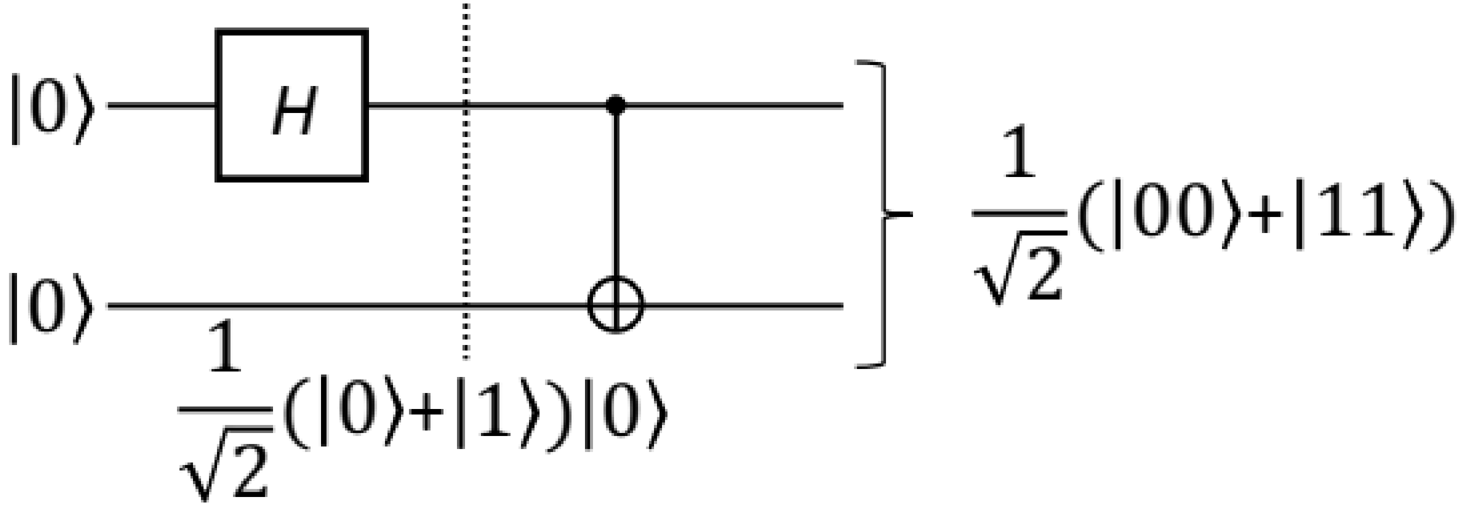

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- They form an ***orthonormal*** basis for the 4-dimensional vector space describing composite systems of two Qbits

Second approach: no Quantum channel

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- Alice applies a CNOT to her Qbit $|\psi\rangle$ and her Qbit from the entangled state $|\boldsymbol{\Phi}^+\rangle$.
- No problem with that, she can do it.

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Exercise to think about

- Construct the circuit for the Quantum Teleportation protocol.