1 For each one of the following operations, write down the matrix form that describes the mapping induced by applying this operation on the *second* of a three Qbit system $|q_1\rangle|q_2\rangle|q_3\rangle = |q_1q_2q_3\rangle$:

- Hadamard $\mathbf{H}$,
- NOT (negation, or flip),
- controlled-NOT (3rd qbit is the control),
- rotation clockwise by $\pi/4$.

2. (25 points) We have a Qbit in the state

$$|\psi\rangle = \frac{1}{5}\Big(3|\mathbf{0}\rangle - 4|\mathbf{1}\rangle\Big).$$

We apply to this Qbit the Hadamard $\mathbf{H}$ operator and immediately after the Pauli-$\mathbf{X}$ operator which transforms its state $|\psi\rangle$ to a new quantum state $|\psi'\rangle$.

1. What is $|\psi'\rangle$? (10 points)

Assume that the Qbit corresponds to the polarization of a photon. As we know, $|\mathbf{0}\rangle$ corresponds to a photon being polarized horizontally and $|\mathbf{1}\rangle$ corresponds to a photon polarized vertically.

We rotate our polaroid filter, which is initially oriented in the *vertical direction*, by 30^o *clockwise* and we send our photon (described by the transformed $|\psi'\rangle$ above) through the rotated polaroid filter.

- What is the probability that the photon will pass through the rotated polaroid filter? (10 points)
- What is the probability that the photon will pass through a second (subsequent) filter which is rotated 60^o *counterclockwise* from the vertical direction? (5 points)

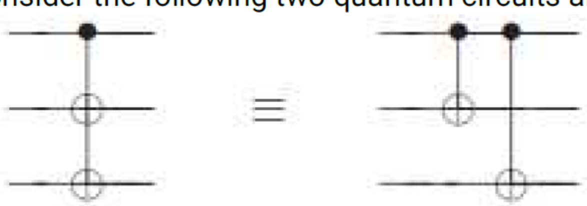
  (I.e., we now try to pass the photon described by $|\psi'\rangle$ through two polaroid filters: one rotated 30^o clockwise and the second rotated 60^o counterclockwise, both with reference to the vertically oriented filter).

You might find the following quantities useful: $\sin 30^o = \cos 60^o = \frac{1}{2}$ *and* $\sin 60^o = \cos 30^o = \frac{\sqrt{3}}{2}$.

3a Express the arbitrary 1-Qbit state $a|0\rangle + b|1\rangle$ in the $|+\rangle, |-\rangle$ basis.

3b Alice and Bob share the state $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. Bob measures his Qbit and he observes outcome. What would Alice observe if she decided to measure her Qbit in the $|+\rangle, |-\rangle$ basis?

3e Consider the following two quantum circuits and prove that they are equivalent:



4. (15 points) Describe a four-Qbit quantum circuit (i.e., it takes as input 4 Qbits) that on input $|\mathbf{0000}\rangle$ outputs $\frac{1}{\sqrt{2}}|\mathbf{0000}\rangle - \frac{1}{\sqrt{2}}|\mathbf{1111}\rangle$.