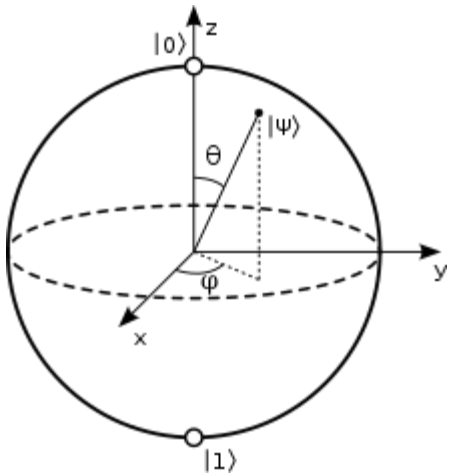
Intro to Quantum Computing

Recap...and Beyond



QC in a Nutshell

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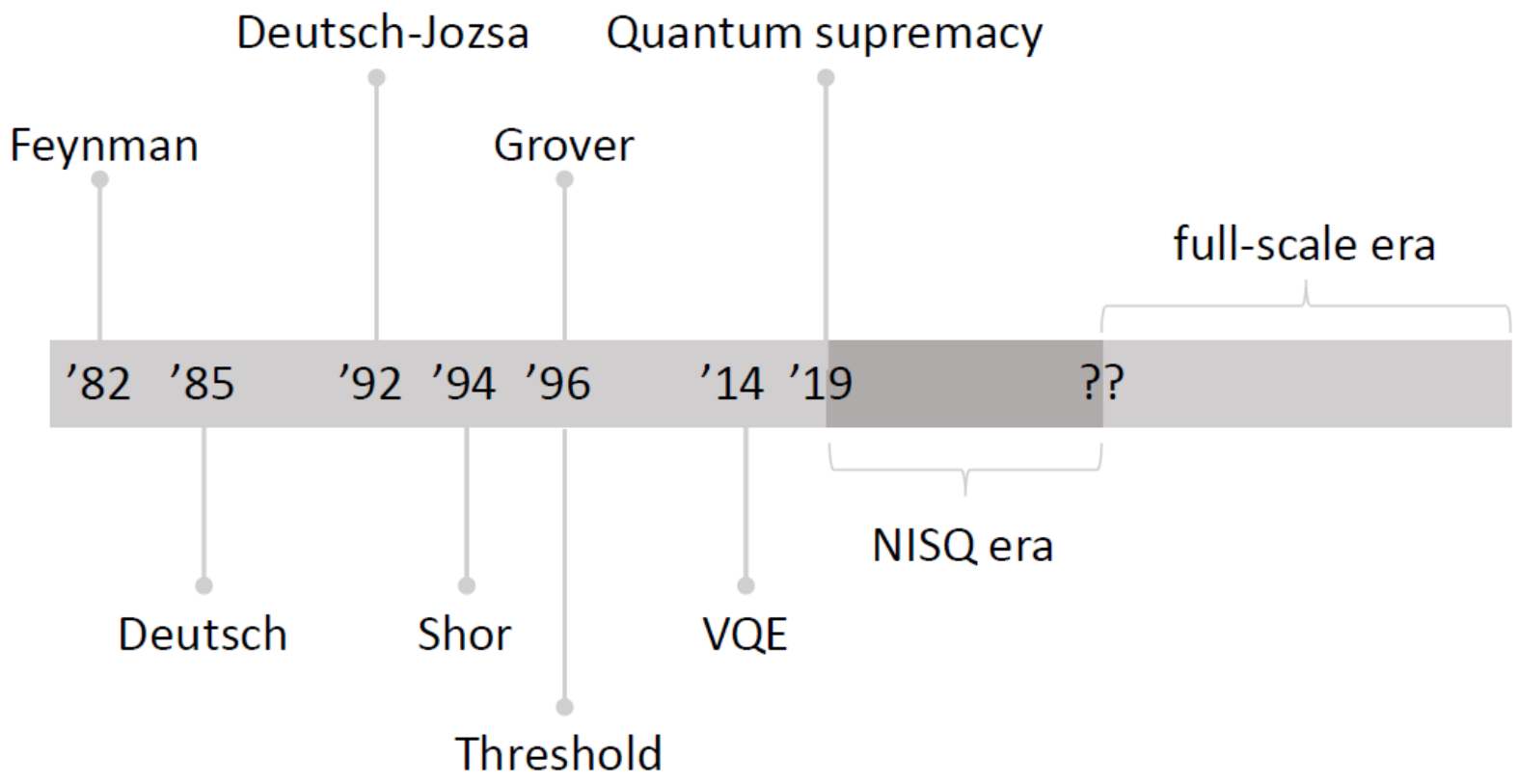
- Quantum algorithm zoo: *quantumalgorithmzoo.org*

Timeline


Deutsch-Jozsa
Quantum supremacy
Feynman
Grover
full-scale era
'82
'85
'92
'94
'96
'14
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??
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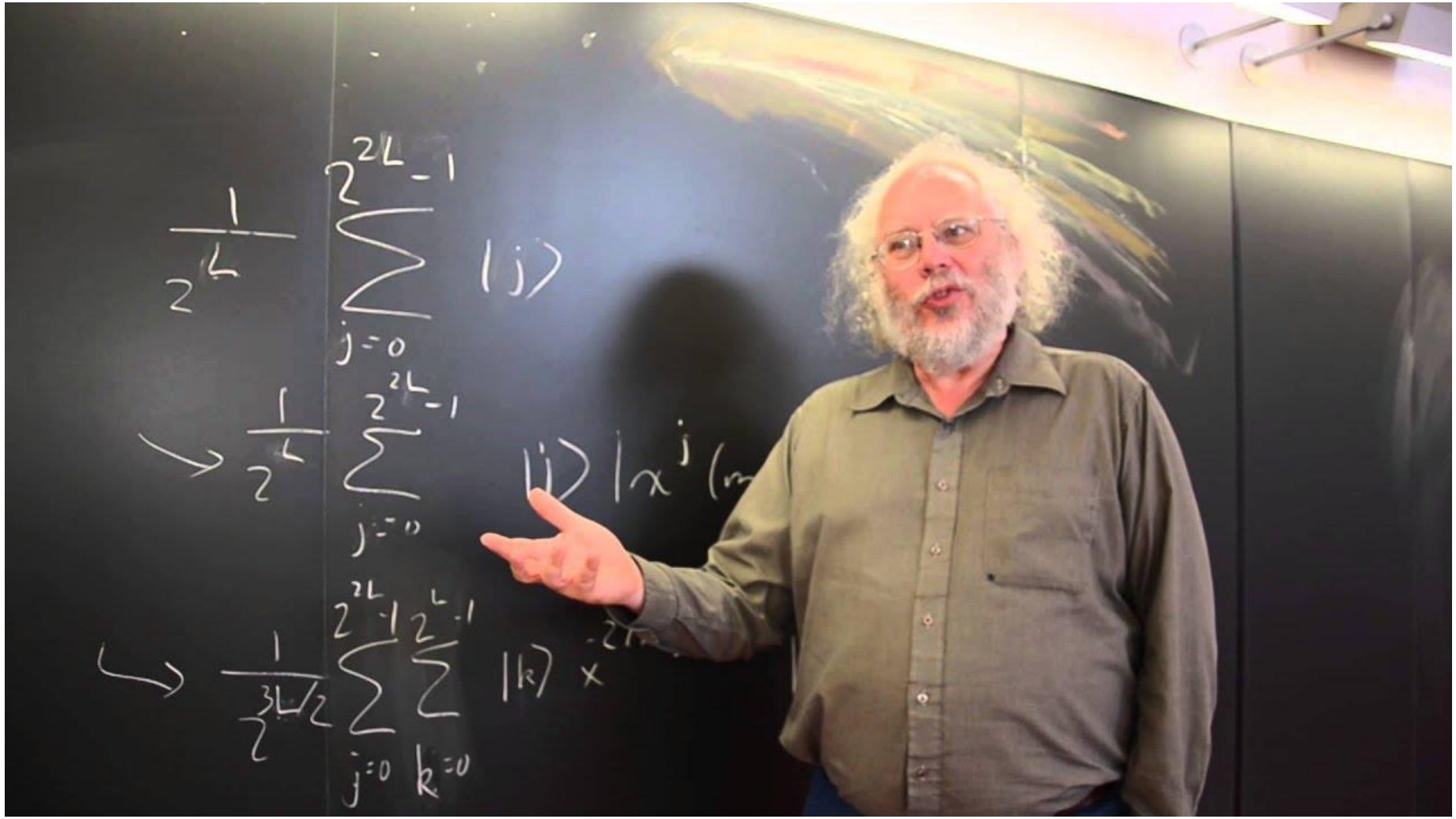
- Much of the current research on QM is on finding real-world applications of quantum computing using NISQs.
- Not always possible! Many (most!) QC algorithms *do* require noise-free, full-scale quantum machines!

Some Quantum Algorithms

Algorithm	**Function**	**Speed-up**	**Era**
Shor	factoring	super-polynomial	full-scale
Grover	search	polynomial	full-scale
HHL	linear algebra	super-polynomial	full-scale
QPE	chemistry	super-polynomial	full-scale
VQE	chemistry	heuristic*	NISQ
Annealing	optimisation	heuristic	NISQ
QAOA	optimisation	heuristic	NISQ

Shor's Algorithm



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- And probably won't ever be discovered…

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- *Do you see why?*

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- *Exponential* running time!!
- *The running time is exponential in n=log(N), which is the number of digits needed to specify N*
- Best classical algorithm $\boldsymbol{O((N)^{\frac{1}{3}})}$

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- Based on *Quantum Fourier Transform / Quantum Phase Estimation*

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- Classical Fourier Transform would need $\boldsymbol{O(n2^n)}$ such operations.

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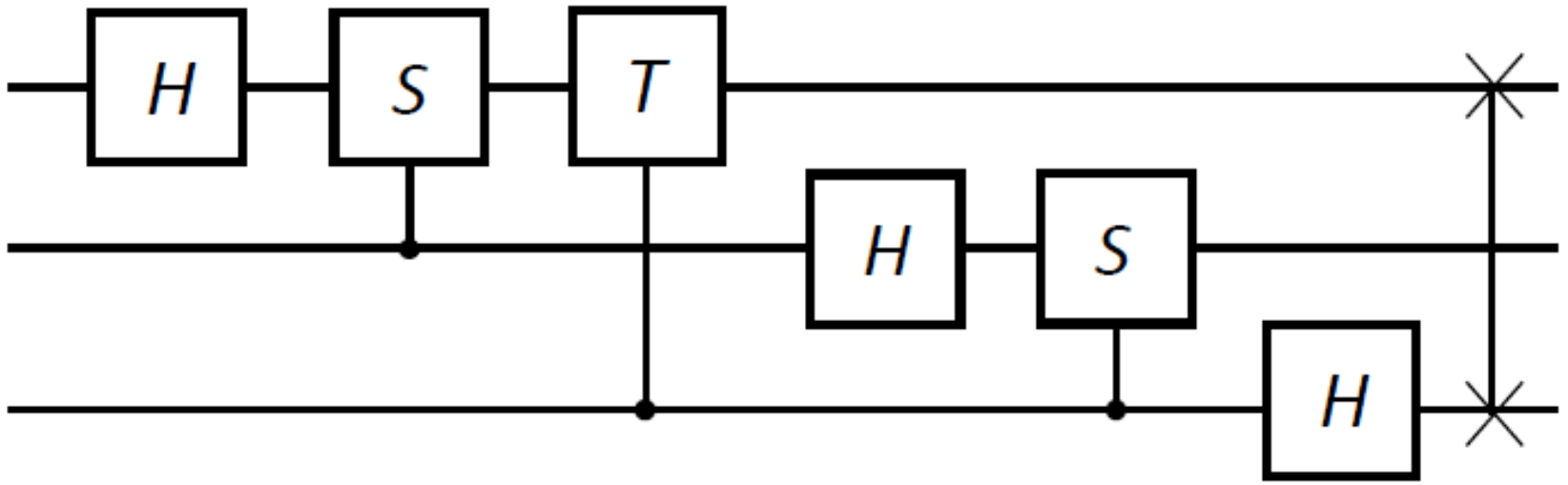

- *S,T* are phase change matrices (map $|\mathbf{0}\rangle \rightarrow |\mathbf{0}\rangle$ and $|\mathbf{1}\rangle \rightarrow e^{i\phi}|\mathbf{1}\rangle$).
- The last gate performs a SWAP.

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- We can find $\boldsymbol{r}$ ***efficiently*** by *Quantum Fourier Transform*!

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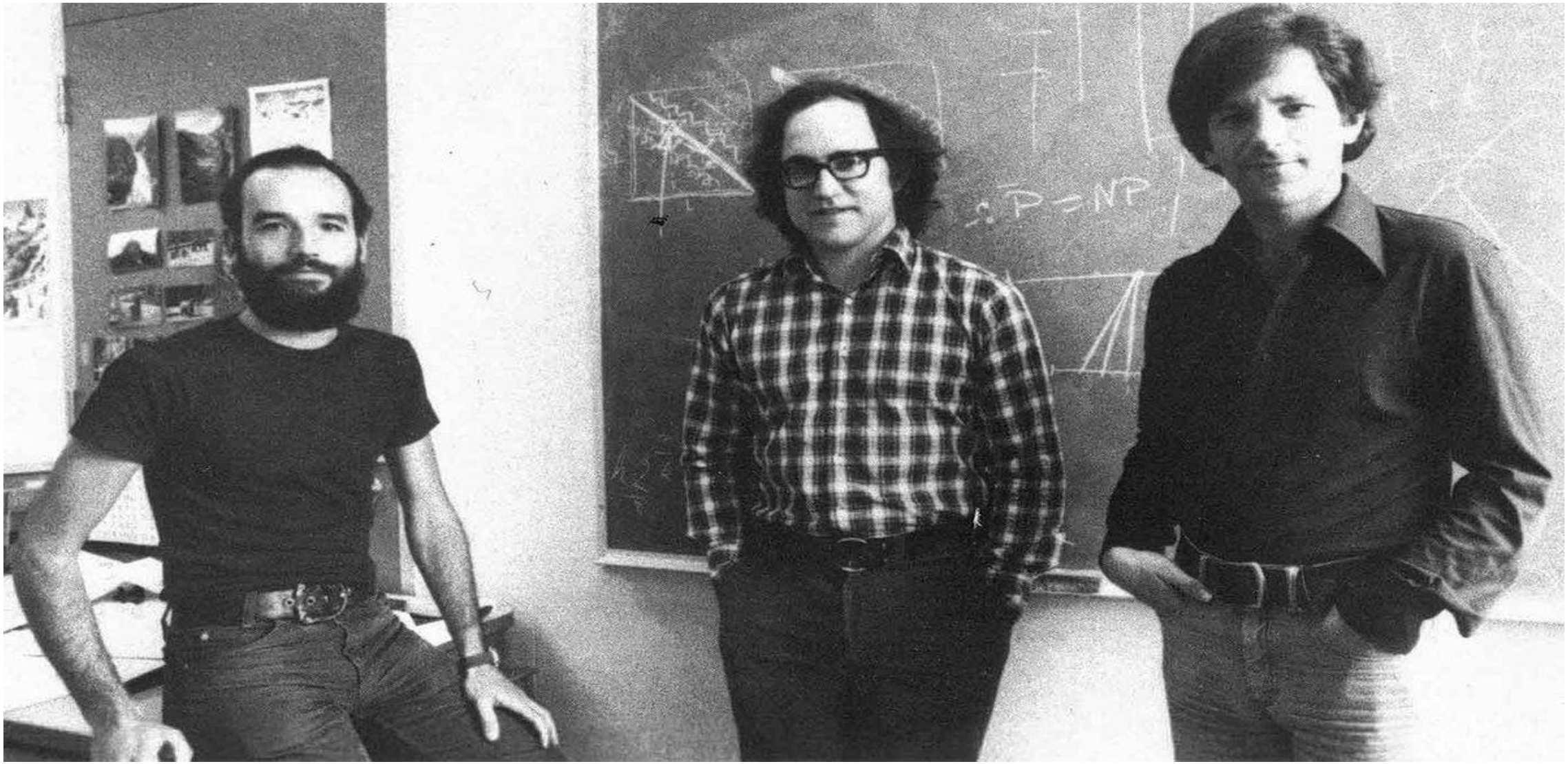

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- Everyone can encrypt messages with $\boldsymbol{N}$.
- Nobody can decrypt the encrypted messages **unless** they know $\boldsymbol{p}, \boldsymbol{q}$!

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- But still, we should start thinking about Quantum-resistance Crypto protocols.

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- (ii) a quantum channel of communication (to send Qbits).
- Alice and Bob have access and can measure Qbits in states $|\mathbf{0}\rangle, |\mathbf{1}\rangle, |+\rangle, |-\rangle$

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- If Bob was correct, they keep the Qbit and append it the current key.
- Otherwise, they discard the bit.

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- So, on average, Alice and Bob will discard 3/4 of their bits.

BB84 Quantum Protocol for QKD

- On average, Bob will be right half of the time and wrong half of the time in his guess.
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- And also, anyone trying to listen to their communication channel will lead to detectable changes!
- So, this results to a secure key distribution!

BB84 Quantum Protocol for QKD: Example

Alice's Bit	Alice basis	QBit	Bob's Basis	Bob's Bit

BB84 Quantum Protocol for QKD: Example

Alice's Bit	Alice basis	QBit	Bob's Basis	Bob's Bit
0				

BB84 Quantum Protocol for QKD: Example

Alice's Bit	Alice basis	QBit	Bob's Basis	Bob's Bit
0	$\|0\rangle, \|1\rangle$			

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Alice's Bit	Alice basis	QBit	Bob's Basis	Bob's Bit
0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0
1	$\|+\rangle, \|-\rangle$	$\|-\rangle$	$\|+\rangle, \|-\rangle$	1

BB84 Quantum Protocol for QKD: Example

Alice's Bit	Alice basis	QBit	Bob's Basis	Bob's Bit
0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0
1	$\|+\rangle, \|-\rangle$	$\|-\rangle$	$\|+\rangle, \|-\rangle$	1
1	$\|0\rangle, \|1\rangle$	$\|1\rangle$	$\|+\rangle, \|-\rangle$	$\mathbf{0}\ \boldsymbol{or}\ \mathbf{1}$

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1	$\|0\rangle, \|1\rangle$	$\|1\rangle$	$\|+\rangle, \|-\rangle$	**0 *or* 1**
0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0
1	$\|0\rangle, \|1\rangle$	$\|1\rangle$	$\|0\rangle, \|1\rangle$	1
0	$\|+\rangle, \|-\rangle$	$\|+\rangle$	$\|+\rangle, \|-\rangle$	0
1	$\|0\rangle, \|1\rangle$	$\|1\rangle$	$\|+\rangle, \|-\rangle$	**0 *or* 1**
1	$\|+\rangle, \|-\rangle$	$\|-\rangle$	$\|0\rangle, \|1\rangle$	**0 *or* 1**

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- With 50% he makes the wrong measurement, and he sends to Bob the wrong Qbit.
- Let's see how this works out…

BB84 Quantum Protocol for QKD: Example

Alice's Bit	Alice basis	QBit	Charlie's Basis	Charlie's Bit	QBit	Bob's Basis	Bob's Bit

BB84 Quantum Protocol for QKD: Example

Alice's Bit	Alice basis	QBit	Charlie's Basis	Charlie's Bit	QBit	Bob's Basis	Bob's Bit
0	$\|0\rangle, \|1\rangle$	$\|0\rangle$					

BB84 Quantum Protocol for QKD: Example

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BB84 Quantum Protocol for QKD: Example

Alice's Bit	Alice basis	QBit	Charlie's Basis	Charlie's Bit	QBit	Bob's Basis	Bob's Bit
0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0			

BB84 Quantum Protocol for QKD: Example

Alice's Bit	Alice basis	QBit	Charlie's Basis	Charlie's Bit	QBit	Bob's Basis	Bob's Bit
0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0	$\|0\rangle$		

BB84 Quantum Protocol for QKD: Example

Alice's Bit	Alice basis	QBit	Charlie's Basis	Charlie's Bit	QBit	Bob's Basis	Bob's Bit
0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0	$\|0\rangle$	$\|0\rangle, \|1\rangle$	

BB84 Quantum Protocol for QKD: Example

Alice's Bit	Alice basis	QBit	Charlie's Basis	Charlie's Bit	QBit	Bob's Basis	Bob's Bit
0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0

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Alice's Bit	Alice basis	QBit	Charlie's Basis	Charlie's Bit	QBit	Bob's Basis	Bob's Bit
0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0
1	$\|+\rangle, \|-\rangle$	$\|-\rangle$					

BB84 Quantum Protocol for QKD: Example

Alice's Bit	Alice basis	QBit	Charlie's Basis	Charlie's Bit	QBit	Bob's Basis	Bob's Bit
0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0
1	$\|+\rangle, \|-\rangle$	$\|-\rangle$	$\|0\rangle, \|1\rangle$	0 or 1			

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0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0
1	$\|+\rangle, \|-\rangle$	$\|-\rangle$	$\|0\rangle, \|1\rangle$	0 or 1	$\{\|0\rangle, \|1\rangle\}$		

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0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0
1	$\|+\rangle, \|-\rangle$	$\|-\rangle$	$\|0\rangle, \|1\rangle$	0 or 1	$\{\|0\rangle, \|1\rangle\}$	$\|+\rangle, \|-\rangle$	

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0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0
1	$\|+\rangle, \|-\rangle$	$\|-\rangle$	$\|0\rangle, \|1\rangle$	0 or 1	$\{\|0\rangle, \|1\rangle\}$	$\|+\rangle, \|-\rangle$	$0\ or\ 1$

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Alice's Bit	Alice basis	QBit	Charlie's Basis	Charlie's Bit	QBit	Bob's Basis	Bob's Bit
0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0	$\|0\rangle$	$\|0\rangle, \|1\rangle$	0
1	$\|+\rangle, \|-\rangle$	$\|-\rangle$	$\|0\rangle, \|1\rangle$	0 or 1	$\{\|0\rangle, \|1\rangle\}$	$\|+\rangle, \|-\rangle$	$0\ or\ 1$
1	$\|0\rangle, \|1\rangle$	$\|1\rangle$	$\|+\rangle, \|-\rangle$	0 or 1	$\{\|+\rangle, \|-\rangle\}$	$\|+\rangle, \|-\rangle$	$0\ or\ 1$
0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|+\rangle, \|-\rangle$	0 or 1	$\{\|+\rangle, \|-\rangle\}$	$\|0\rangle, \|1\rangle$	$0\ or\ 1$
1	$\|0\rangle, \|1\rangle$	$\|1\rangle$	$\|0\rangle, \|1\rangle$	1	$\|1\rangle$	$\|0\rangle, \|1\rangle$	1
0	$\|+\rangle, \|-\rangle$	$\|+\rangle$	$\|+\rangle, \|-\rangle$	0	$\|+\rangle$	$\|+\rangle, \|-\rangle$	1
1	$\|0\rangle, \|1\rangle$	$\|1\rangle$	$\|0\rangle, \|1\rangle$	1	$\|1\rangle$	$\|+\rangle, \|-\rangle$	$0\ or\ 1$
1	$\|+\rangle, \|-\rangle$	$\|-\rangle$	$\|0\rangle, \|1\rangle$	0 or 1	$\{\|0\rangle, \|1\rangle\}$	$\|0\rangle, \|1\rangle$	$0\ or\ 1$

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1	$\|+\rangle, \|-\rangle$	$\|-\rangle$	$\|0\rangle, \|1\rangle$	0 or 1	$\{\|0\rangle, \|1\rangle\}$	$\|+\rangle, \|-\rangle$	$0\ or\ 1$
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0	$\|0\rangle, \|1\rangle$	$\|0\rangle$	$\|+\rangle, \|-\rangle$	0 or 1	$\{\|+\rangle, \|-\rangle\}$	$\|0\rangle, \|1\rangle$	$0\ or\ 1$
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0	$\|+\rangle, \|-\rangle$	$\|+\rangle$	$\|+\rangle, \|-\rangle$	0	$\|+\rangle$	$\|+\rangle, \|-\rangle$	0
1	$\|0\rangle, \|1\rangle$	$\|1\rangle$	$\|0\rangle, \|1\rangle$	1	$\|1\rangle$	$\|+\rangle, \|-\rangle$	$0\ or\ 1$
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And that was it…

- Thank you all!
- Hope you enjoyed and you learned something interesting ☺
- Best of luck on your exam(s) and everything else ☺