

Intro to Quantum Computing 2021

Exercise Set I

Exercise 1. Consider the following states for a Qbit corresponding to the basis: $\{|\rightarrow\rangle, |\leftarrow\rangle\}$:

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \quad \text{and} \quad |\leftarrow\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle.$$

Compute the following quantities (numbers/vectors or matrices) using braket notation and matrix notation in each case.

1. $\langle 1| |\rightarrow\rangle = \langle 1| |\rightarrow\rangle$.
2. $|0\rangle \langle \rightarrow |$.
3. $\langle \rightarrow | \leftarrow \rangle$.
4. $|\rightarrow\rangle \langle \leftarrow |$.
5. $|\rightarrow\rangle \otimes |\leftarrow\rangle$.
6. $(X \otimes I_2)(|\rightarrow\rangle \otimes |\leftarrow\rangle)$ where X is the 2×2 Pauli matrix and I_2 is the 2×2 identity matrix.
7. $(X \otimes Y)(|\rightarrow\rangle \otimes |\leftarrow\rangle)$ where Y is the second Pauli matrix.

Exercise 2. Consider the following four 2-Qbit states:

$$\begin{aligned} |T_1\rangle &= \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle \\ |T_2\rangle &= \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle. \end{aligned}$$

1. Show that neither state can be written as a tensor product of two single Qbit states $|q_1\rangle \otimes |q_2\rangle$.
2. Show that $(U \otimes U)|T_2\rangle = |T_2\rangle$ for every 1-Qbit unitary operator U (i.e., U is a 2×2 matrix satisfying the properties of every unitary matrix: its transpose is equal to its inverse).

Exercise 3. Suppose we have a Qbit described by the state

$$|\psi\rangle = \sqrt{\frac{2}{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle.$$

- What are the probabilities that we will measure $|\rightarrow\rangle$ or $|\leftarrow\rangle$ as they are defined in Exercise 1, if we perform the measurement in this new basis?