Intro to Quantum Computing 2020

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Exercise Set II

Exercise 1. *Suppose that a given two-Qbit system is in the state* $0.8|00\rangle + 0.6|11\rangle$. *A Pauli* $\mathbf{X}$ *gate is applied to the second Qbit. After that, a measurement is performed on both Qbits in the standard basis.*

▷ *What are the probabilities of the possible measurement outcomes?*

Exercise 2. *Let* A *and* B *be* 2×2 *matrices, and let* $|\psi\rangle$ *and* $|\phi\rangle$ *be two single Qbit states.*

▷ *Show that:*

$$(A \otimes B)(|\psi\rangle \otimes |\phi\rangle) = (A|\psi\rangle) \otimes (B|\phi\rangle).$$

Exercise 3. *Give the matrix form for the following two-Qbit gate:*

The first Qbit is the control and the second Qbit be the target – if the first (control) Qbit equals to zero, then the second (target) Qbit is unchanged; if the first (control) Qbit equals one, then a Hadamard gate is perform ed on the second (target) Qbit.

Exercise 4. *Show that the 2 circuits in Figure 1 are equivalent.*

Exercise 5. *What is wrong Alice's approach?*
Alice has a Qbit $\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. *She wants to initialise her Qbit to* $|0\rangle$. *She knows that when measuring* $|\psi\rangle$, *then with probability* $1/2$ *she get the measurement outcome 0 and the Qbit will be in state* $|0\rangle$. *Thus she repeatedly measures her Qbit in the standard basis until she gets the outcome 0. Since the probability is* $1/2$ *each time she measures, the expected number of measurements until she gets her* $|0\rangle$*-initialised Qbit is 2.*

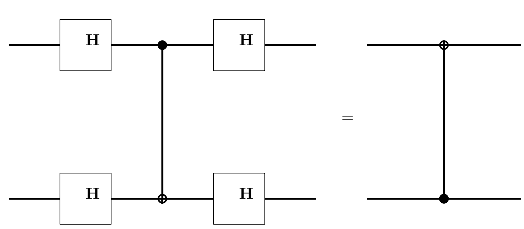

Figure 1: Circuit for exercise 4.

Exercise 6. *Consider a two-Qbit system where the first Qbit is initialized in the state* $|0\rangle$ *and the second Qbit is initialized in the state* $|1\rangle$*. We perform a Hadamard operator on the first Qbit, and then we perform a CNOT operator, with the first Qbit as the control bit and the second Qbit as the target Qbit. Then, we perform a measurement on both QBits.*

- *Draw the quantum circuit diagram that corresponds to the previous description.*
- *What are the possible outcomes of the measurements?*
- *With what probabilities do they occur?*