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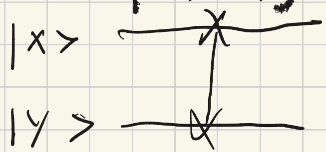

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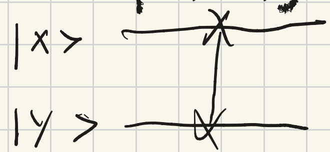

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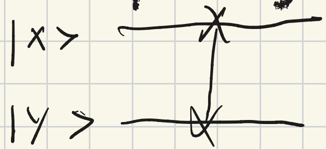

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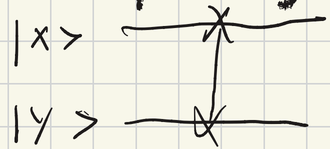

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Unitary:

$$U^{-1} = U^{\dagger} \Rightarrow U^{\dagger}U = \mathbb{1}$$

$$U^{\dagger} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow U^{\dagger}U = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

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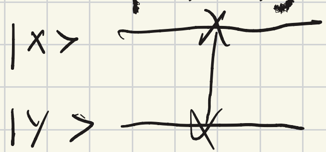

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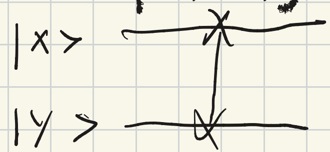

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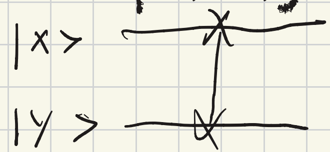

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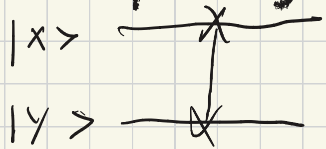

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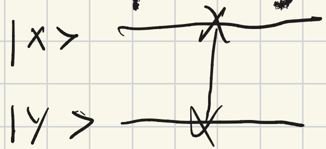

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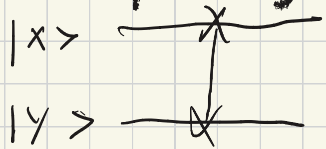

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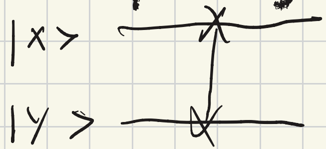

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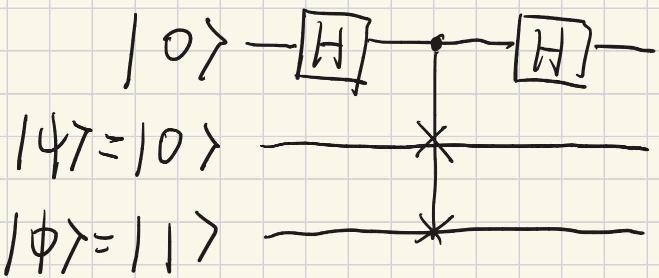

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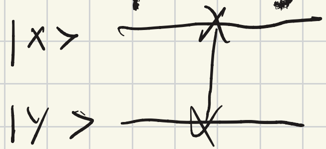

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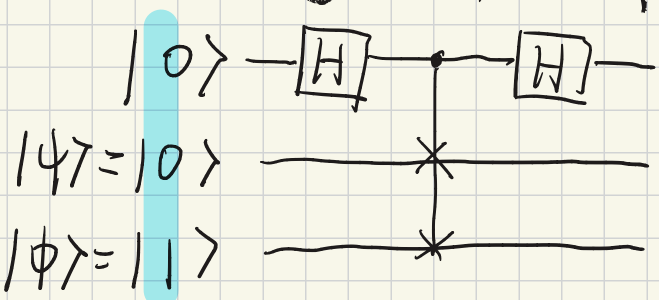

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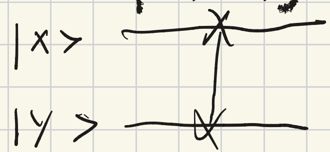

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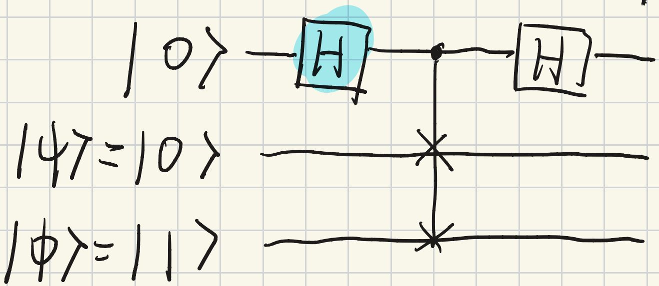

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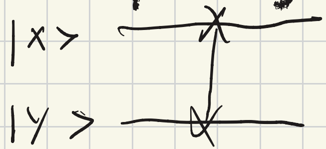

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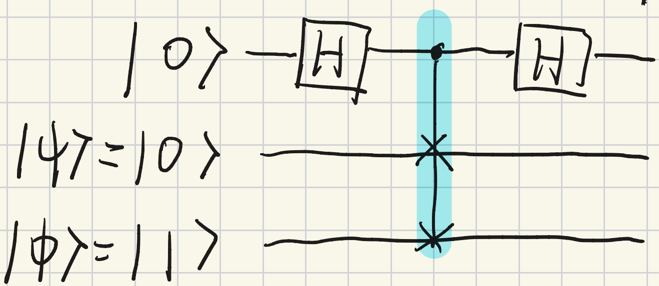

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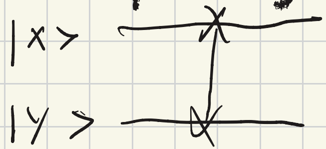

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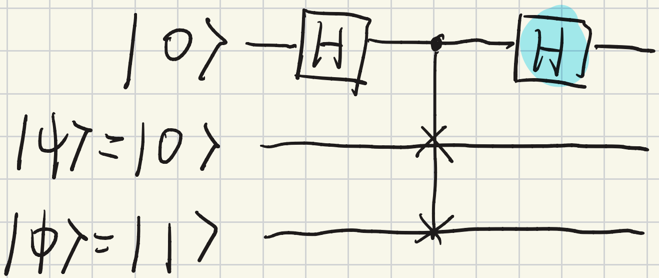

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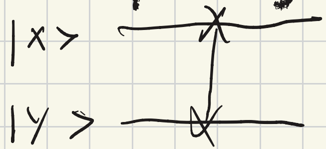

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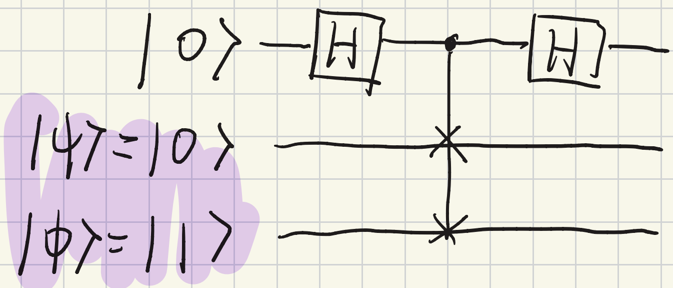

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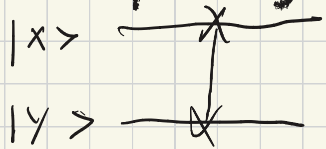

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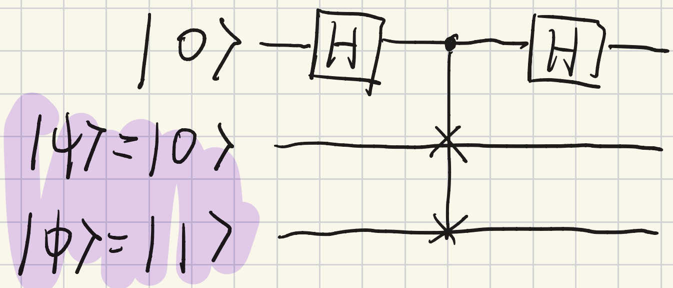

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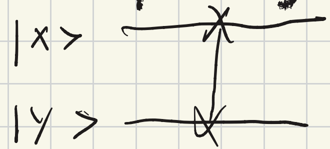

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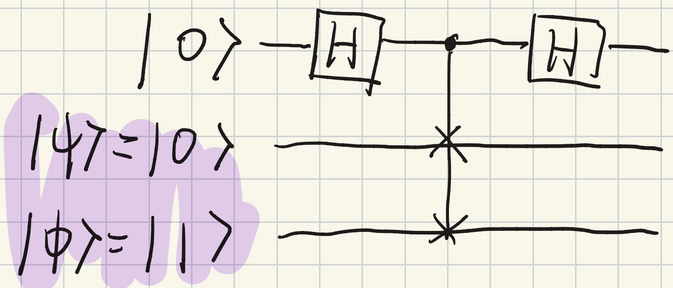

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With prob $1/2$ measure the 1st Qubit being $|1\rangle$

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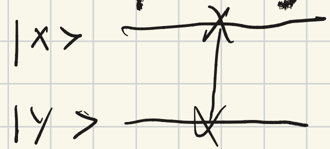

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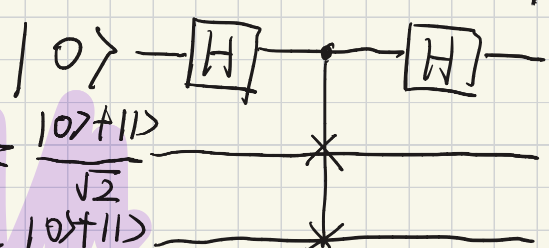

$|\psi\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

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Clearly, $|\psi\rangle = |\phi\rangle$, so $|\psi\rangle|\phi\rangle - |\phi\rangle|\psi\rangle = 0$

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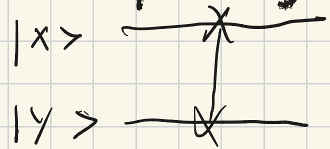

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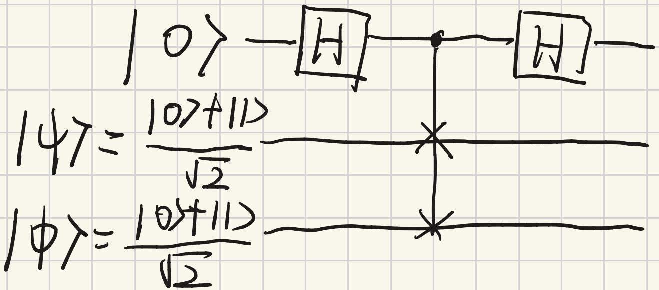

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So the prob of measuring #1 qubit in $|1\rangle$ is 0

Ex2 construct a quantum circuit that compute the parity of three qubits.

We want

Start with $|y_1\rangle|y_2\rangle|y_3\rangle|0\rangle \rightarrow |y_1\rangle|y_2\rangle|y_3\rangle|y_1 \oplus y_2 \oplus y_3\rangle$

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XOR

0	0	0
0	1	1
1	0	1
1	1	0

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Recall that CNON generates the transformation XOR of $|x\rangle|y\rangle \rightarrow |x\rangle|x \oplus y\rangle$

0	0	0
0	1	1
1	0	1
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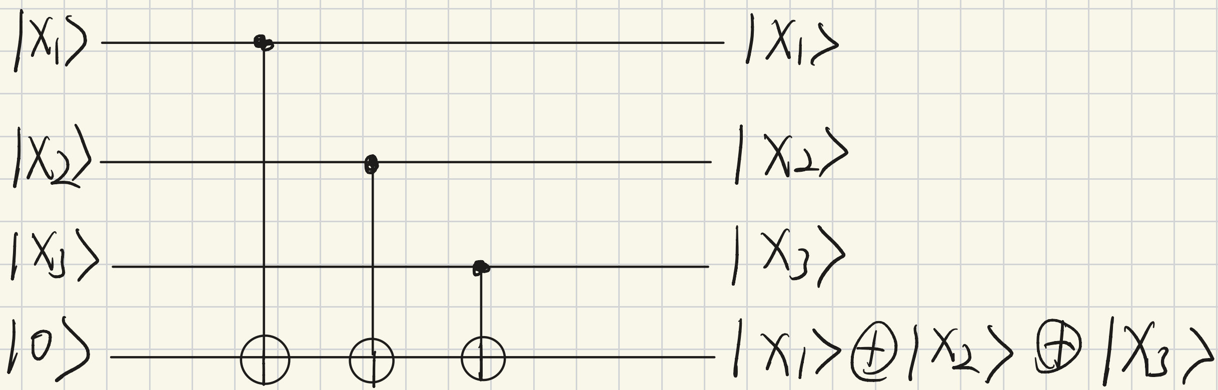
Recall that CNON generates the transformation XOR of $|x\rangle|4\rangle \rightarrow |x\rangle|x \oplus 4\rangle$

0	0	0
0	1	1
1	0	1
1	1	0



Ex3 show that CZ gate can be implemented using a CNOT gate and Hadamard gates

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$$\begin{cases} Z|0\rangle = |0\rangle \\ Z|1\rangle = -|1\rangle \end{cases} \qquad Z|+\rangle = |-\rangle \quad Z|-\rangle = |+\rangle$$

$$\begin{cases} H|0\rangle = |+\rangle \\ H|1\rangle = |-\rangle \end{cases} \qquad H|+\rangle = |0\rangle \quad H|-\rangle = |1\rangle$$

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So $Z|0\rangle = H|+\rangle$

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$= HX|+\rangle$

Ex3 show that CZ gate can be implemented using a CNOT gate and Hadamard gates

$$\begin{cases} Z|0\rangle = |0\rangle \\ Z|1\rangle = -|1\rangle \end{cases} \qquad Z|+\rangle = |-\rangle$$
$$Z|-\rangle = |+\rangle$$

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$= HXH|0\rangle$ $\Rightarrow$ $HXH = Z$

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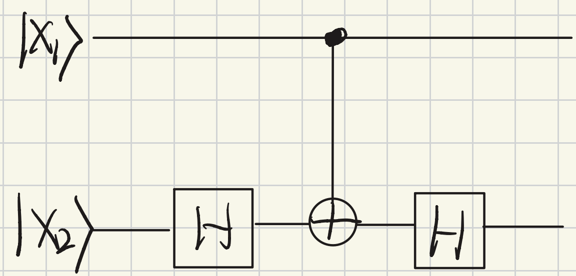

$$\begin{cases} H|0\rangle = |+\rangle \\ H|1\rangle = |-\rangle \end{cases} \qquad \begin{aligned} H|+\rangle &= |0\rangle \\ H|-\rangle &= |1\rangle \end{aligned}$$

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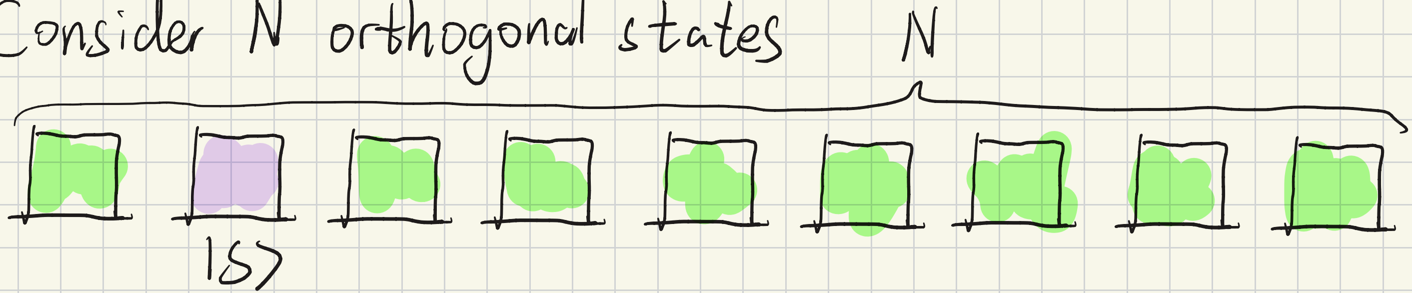
Grover's algo

Consider N orthogonal states



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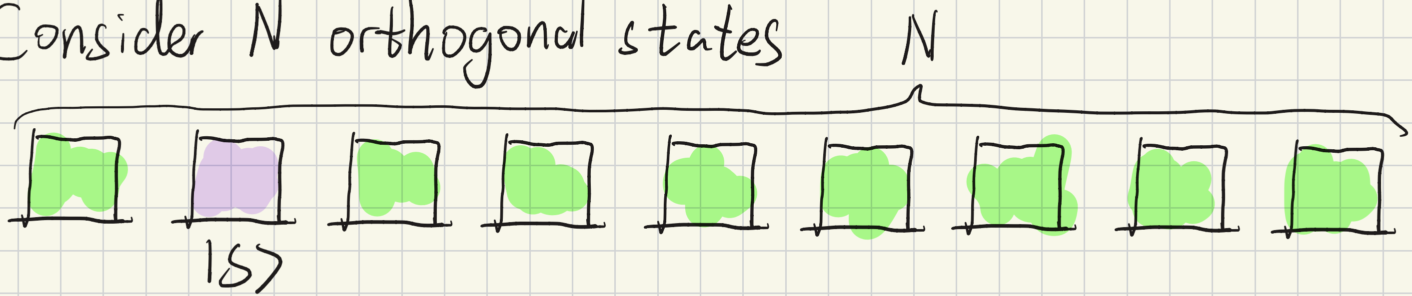

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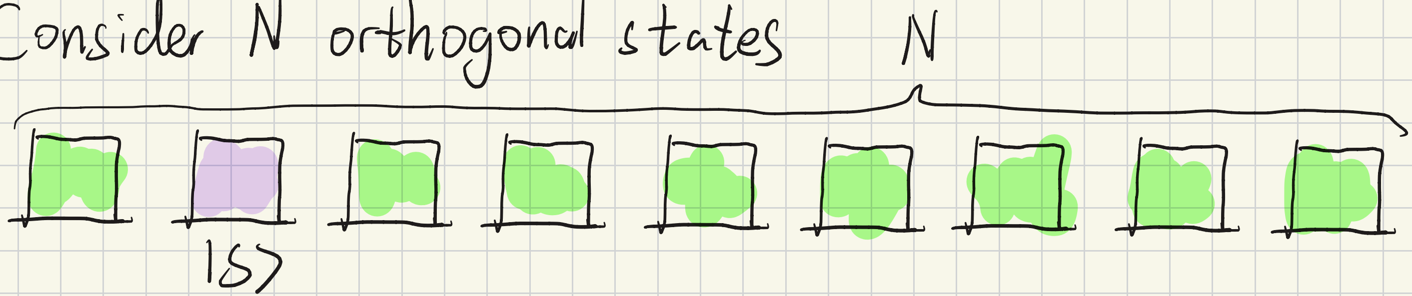

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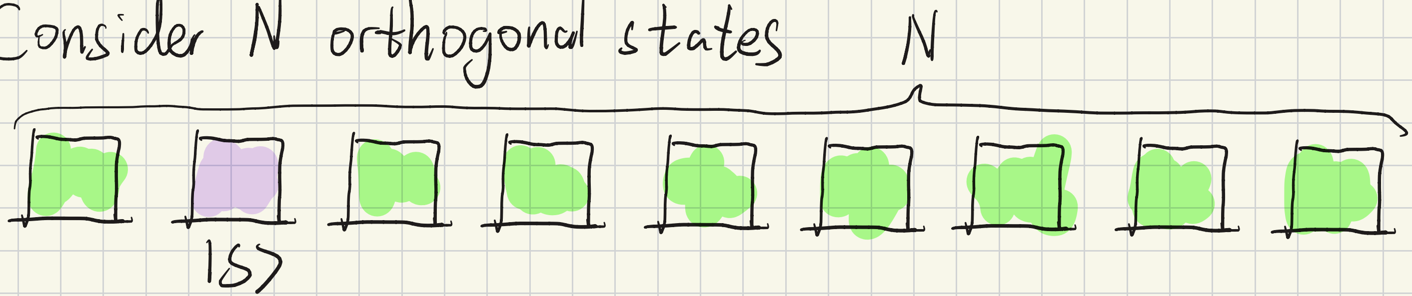

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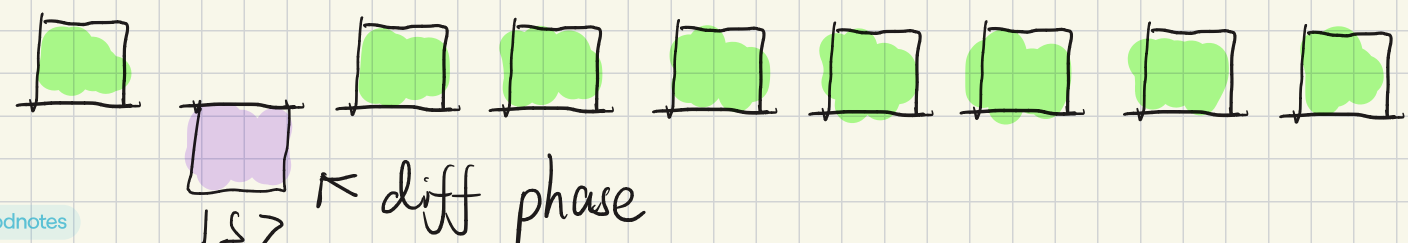
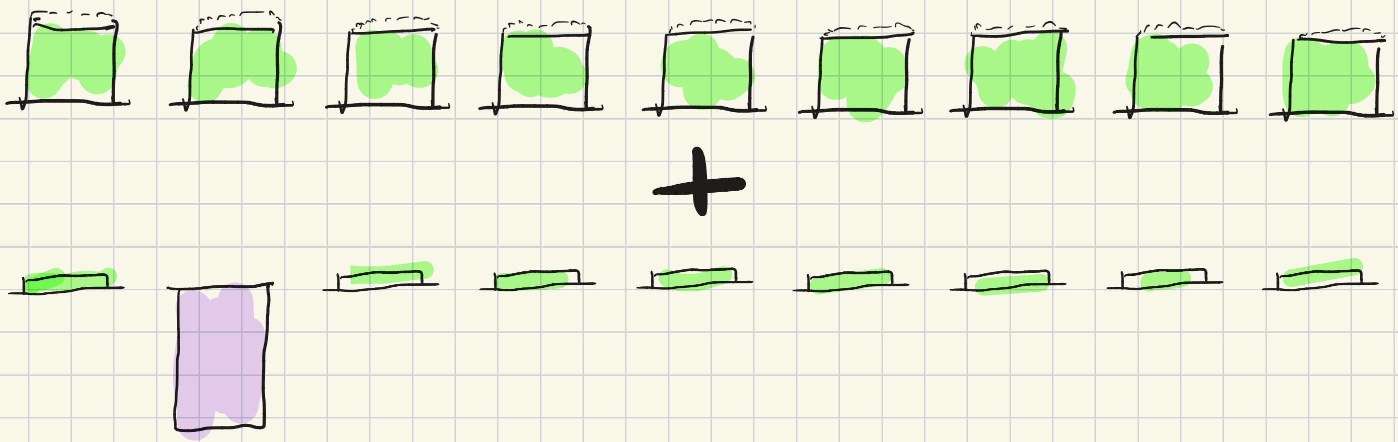


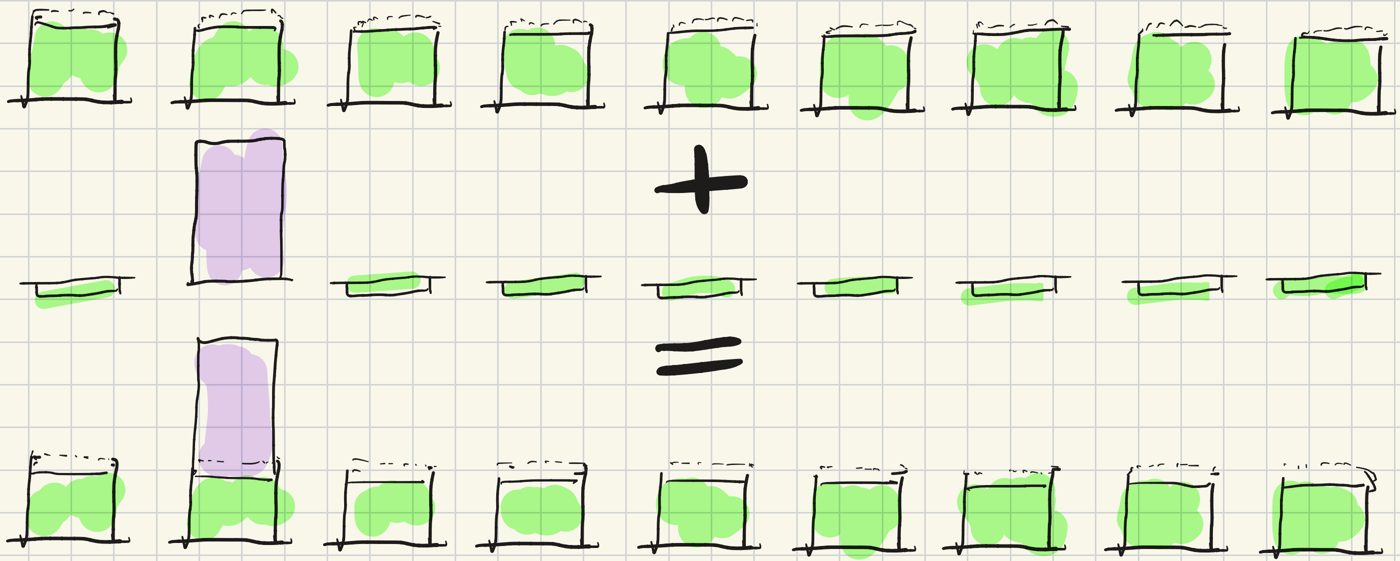
Grover's algo

We can split 2nd-state-phase-flipped N states as a part proportional to the uniform superposition $|\psi\rangle$ (top) and a part orthogonal to $|\psi\rangle$ (bottom)



Grover's algo

Suppose we have an operator D which flips the part orthogonal to the uniform state.



A little of the amplitude from all the other states is "stolen" and pumped into the solution state.

Grover's algo

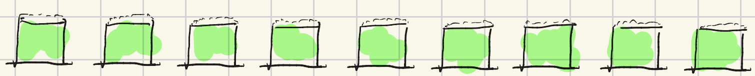
The operator D is called the diffusion operator, as it helps to diffuse amplitude around.

The oracle that does the phase flip can be written as

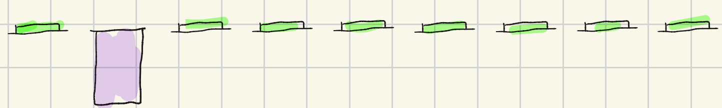
$$U_f = \mathbb{I} - 2|s\rangle\langle s|$$

Easy to check that
$$\begin{cases} U_f|s\rangle = \mathbb{I}|s\rangle - 2|s\rangle\langle s|s\rangle = -|s\rangle \\ U_f|x\rangle = \mathbb{I}|x\rangle - 2|s\rangle\langle s|x\rangle = |x\rangle, \quad x \neq s \end{cases}$$

A good guess of D is $D = 2|\psi\rangle\langle\psi| - \mathbb{I}$, where $|\psi\rangle$ is the uniform superposition.

$|\psi\rangle =$ 

$\langle\psi|\phi\rangle = 0$

$|\phi\rangle =$ 

Grover's algo

The combination $G = DU_f$ is called the **Grover operator**.

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Now consider large N and a state $|v\rangle$ whose overlap with the uniform state $\langle\psi|v\rangle \simeq 1$ and whose overlap with the solution state is $\langle s|v\rangle = c/\sqrt{N}$. When $N \to \infty$, $|s\rangle$ & $|v\rangle$ are "$\perp$".

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$$U_f|v\rangle = \mathbb{I}|v\rangle - 2|s\rangle\langle s|v\rangle = |v\rangle - \frac{2c}{\sqrt{N}}|s\rangle$$

$$DU_f|v\rangle = (2|\psi\rangle\langle\psi| - \mathbb{I})\left(|v\rangle - \frac{2c}{\sqrt{N}}|s\rangle\right)$$

$$= 2\underbrace{|\psi\rangle - |v\rangle}_{|\psi\rangle \simeq |v\rangle} - \frac{4c}{\sqrt{N}}|\psi\rangle\underbrace{\langle\psi|s\rangle}_{0} + \frac{2c}{\sqrt{N}}|s\rangle$$

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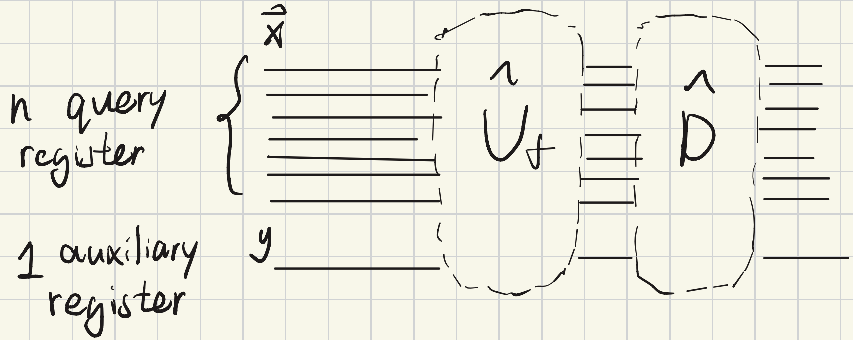
We transfer a chunk of $\frac{2c}{\sqrt{N}}$ into the solution state!

We needs s steps to get an amplitude $\mathcal{O}(1) \Rightarrow s \cdot \mathcal{O}\left(\frac{1}{\sqrt{N}}\right) = \mathcal{O}(1) \Rightarrow s = \mathcal{O}(\sqrt{N})$

Grover's algo

The oracle adds a phase of -1 when it acts on $|s\rangle$



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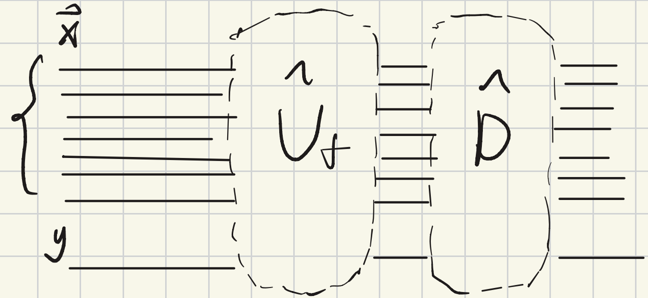
The strategy is to use the aux qubit to shift the phase to query register

phase kickback trick

n query register

1 auxiliary register



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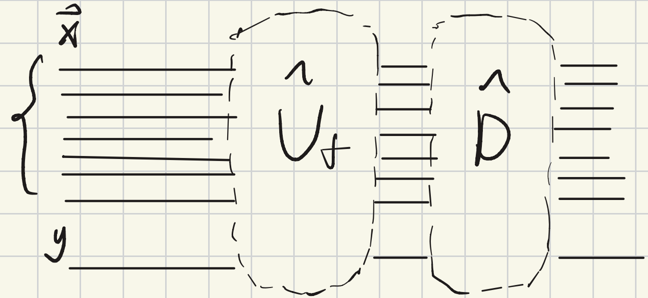

$f: \{0,1\}^n \to \{0,1\}$

We use XOR to achieve this $U|\vec{x}, y\rangle = |\vec{x}, \underline{y \oplus f(\vec{x})}\rangle$

$$y \oplus f(\vec{x}) = \begin{cases} y, & f(\vec{x}) = 0 \\ \text{flips } y, & f(\vec{x}) = 1 \end{cases}$$

Grover's algo

Choose $|y\rangle = |-\rangle$

$$U_f(|\vec{x}\rangle \otimes |-\rangle) = \frac{U_f|\vec{x}0\rangle - U|\vec{x}1\rangle}{\sqrt{2}}$$

$$= \frac{|\vec{x}, 0 \oplus f(x)\rangle - |\vec{x}, 1 \oplus f(x)\rangle}{\sqrt{2}}$$

$$= (-1)^{f(x)} |\vec{x}\rangle \otimes |-\rangle$$

XOR

0	0	0
0	1	1
1	0	1
1	1	0

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XOR

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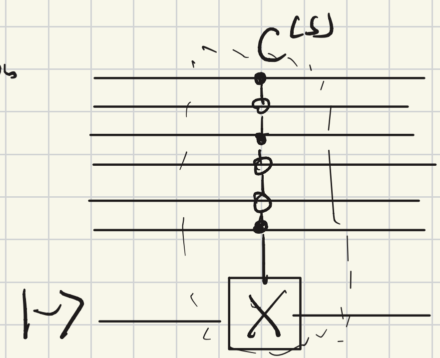
Implementation of U_f is through multi-controlled X gate



$$C^{(s)}X(|\vec{s}\rangle \otimes |-\rangle) = |\vec{s}\rangle \otimes X|-\rangle$$
$$= -|\vec{s}\rangle \otimes |-\rangle$$

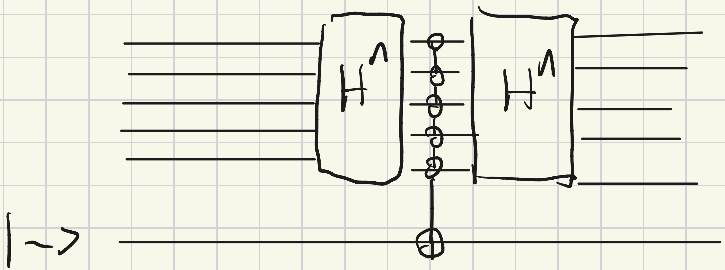
For $\vec{x} \neq s$

$$C^{s}X(|\vec{x}\rangle \otimes |-\rangle) = |\vec{x}\rangle \otimes |-\rangle$$

Grover's algo

Diffusion operator flips $|\psi\rangle$ and leave any orthogonal states alone.

$$D = 2|\psi\rangle\langle\psi| - \mathbb{I}$$



$$|\psi\rangle = H^{\otimes n}|0\rangle \qquad H^{\otimes n} H^{\otimes n} = \mathbb{I}$$

$$|\psi\rangle \xrightarrow{H^{\otimes n}} H^{\otimes n} \cdot H^{\otimes n} |0\rangle = |0\rangle$$

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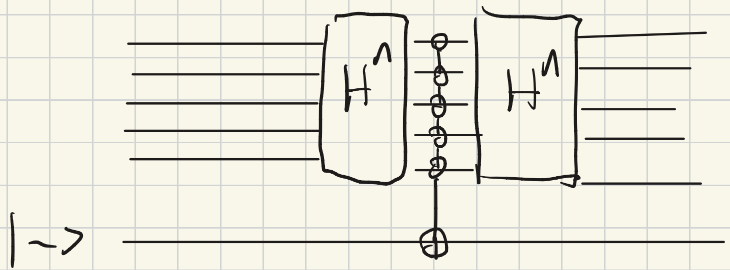

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The aux $|-\rangle$ produces a sign flip

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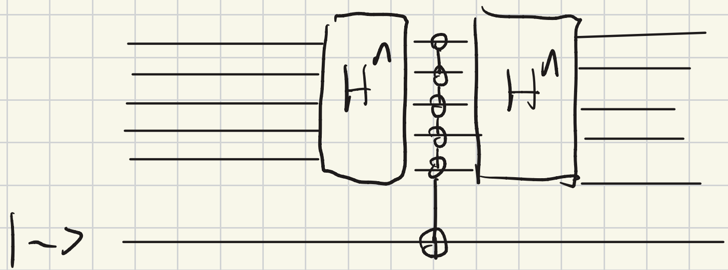

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A state ($|\psi_\perp\rangle$) $\perp$ to $|\psi\rangle$ remains $\perp$ after $H^{\otimes n}$

Ex4 consider Grover's problem in the 4-element set $\{0,1\}$, where there is a uniquely marked element, say $x_0 = 10$.

1. Write down the matrix for the oracle U_f for this value of x_0 w.r.t. the computational basis $\{|0\rangle, |1\rangle\}$.

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The oracle U_f does the phase-flip:

$$U_f |x\rangle = (-1)^{[x = x_0]} |x\rangle \Rightarrow U_f = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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2. Write down the matrix for the diffusion matrix w.r.t. the computational basis $\{|0\rangle,|1\rangle\}$.

Start in $|00\rangle$ and apply Hadamard on both qubits:

$$|\psi\rangle = H^{\otimes 2}|00\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

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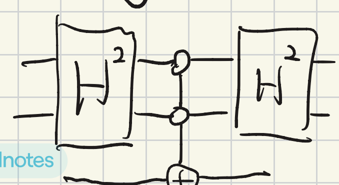
$$\|$$

Or using the circuit



$$D = H^{\otimes 2}(2|0\rangle\langle 0| - \mathbb{I})H^{\otimes 2}$$

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3. Multiply out all matrices and vectors in one step gover algo to verify the claim that the algo find marked element with certainty

Start in $|00\rangle$. Apply $H^{\otimes 2}$ to create uniform state

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Oracle:

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Find marked element with prob 1.

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One more Grover step:

$$U_f|10\rangle = -|10\rangle = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$D\begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

Made with Goodnotes

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$$D\begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$ ← The prob of marked element $|10\rangle$ is $1/4$

Ex5

1. Show that $H^{\otimes n}(|0^{\otimes n}\rangle\langle 0^{\otimes n}| - \mathbb{I})H^{\otimes n} = 2|u\rangle\langle u| - \mathbb{I}$

where $u = H^{\otimes n}|0^{\otimes n}\rangle = \frac{1}{\sqrt{n}}\sum_{x\in\{0,1\}^n}|x\rangle$

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Three generic facts:

① $HH = \mathbb{I}$

② $U|\psi\rangle\langle\psi|U^\dagger = |U\psi\rangle\langle U\psi|$

③ $U\mathbb{I}U^\dagger = \mathbb{I}$

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So $H^{\otimes n}(|0^{\otimes n}\rangle\langle 0^{\otimes n}| - \mathbb{I})H^{\otimes n} = H^{\otimes n}|0^{\otimes n}\rangle\langle 0^{\otimes n}|H^{\otimes n} - H^{\otimes n}\mathbb{I}H^{\otimes n}$

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Three generic facts:

① $HH = \mathbb{I}$

② $U|\psi\rangle\langle\psi|U^{\dagger} = |U\psi\rangle\langle U\psi|$

③ $U\mathbb{I}U^{\dagger} = \mathbb{I}$

So $H^{\otimes n}(2|0^{\otimes n}\rangle\langle 0^{\otimes n}| - \mathbb{I})H^{\otimes n} = 2\overbrace{H^{\otimes n}|0^{\otimes n}\rangle}^{|u\rangle}\overbrace{\langle 0^{\otimes n}|H^{\otimes n}}^{\langle u|} - \overbrace{H^{\otimes n}\mathbb{I}H^{\otimes n}}^{\mathbb{I}}$

$= 2|u\rangle\langle u| - \mathbb{I}$

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$R^2 = (I - 2\Psi)^2 = I - 4\Psi + 4\Psi^2 = I$ Trivially unitary

Ex5 imagine we have 8 items, two of which, x1, x2, are marked (I.e. $f(x1)=f(x2)=1$ and $f(y)=0$, for all $y!=x1,x2$). Show that for one oracle call is enough to solve the problem.

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One Grover step is $G = (2|\psi\rangle\langle\psi| - \mathbb{I})(\mathbb{I} - 2|s\rangle\langle s|)$

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So $(2|\psi\rangle\langle\psi| - \mathbb{I})\left(-\frac{1}{2}|s\rangle + \frac{\sqrt{3}}{2}|u\rangle\right)$

$= \quad |\psi\rangle - \left(-\frac{1}{2}|s\rangle + \frac{\sqrt{3}}{2}|u\rangle\right)$

$= |s\rangle$

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$\theta = \frac{\pi}{6}$

Grover performs a rotation by 2θ

$$\theta + 2\theta = \frac{\pi}{2}$$