Prime-Based Qubits: An Information-Theoretic Interpretation of Physics-Prime Factorization

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***Abstract*—This paper presents the information-theoretic interpretation of the Physics-Prime Factorization (PPF) framework, demonstrating that the fundamental concepts of quantum information arise naturally from its number-theoretic structure. We define a "P-Qubit" as the binary degree of freedom corresponding to the sign of a Magnitude Prime factor, mapping this directly to the standard computational basis $\{|0\rangle, |1\rangle\}$. We prove that an integer with k distinct prime factors forms a natural "Factorization Register" isomorphic to a $(k-1)$-qubit system. Most significantly, we show that quantum entanglement is the information-theoretic manifestation of integer multiplication within PPF, where the state space of the product becomes an entangled state of the constituent prime factors. This provides a purely number-theoretic model for the creation of non-local correlations. Finally, we establish a direct correspondence between these concepts and the geometry of quantum computation, arguing that the 4D Toric codes used for quantum error correction are a practical application of the same 4-prime factorization structure that defines spacetime in our framework.**

***Index Terms*—quantum information theory, qubit, entanglement, Physics-Prime Factorization, number theory, quantum computing, toric codes.**

I. Introduction

The Primal Reflections framework posits that the universe is fundamentally computational, operating on the principles of number theory. The core axiom of Physics-Prime Factorization (PPF)—that -1 is a prime—generates a Factorization State Space $S(n)$ for any integer, which we have identified as the mathematical basis for quantum superposition [1]. While previous papers have developed the physical, topological, and algebraic consequences of this axiom [2]–[4], this work provides the explicit information-theoretic interpretation.

Quantum information theory has provided a powerful new lens through which to view physics, suggesting that reality is, at its core, about the processing of information. However, the physical nature of the fundamental unit of this information—the qubit—has remained an abstract concept. What *is* a qubit? This paper proposes a definitive answer: a qubit is the physical manifestation of a binary choice inherent in the factorization of numbers. We will build the entire structure of quantum information, from single qubits to entangled registers, directly from the PPF axiom.

II. The Prime-Based Qubit (P-Qubit)

In PPF, any integer has a state space of possible factorizations. The elements of these factorizations are the P-Primes: the Sign Prime (-1) and the Magnitude Primes $(2, 3, 5, \dots)$. The core informational degree of freedom arises from the signs of the Magnitude Primes.

Definition II.1 (The P-Qubit)**.** *A **P-Qubit** is the binary degree of freedom associated with the sign of a Magnitude Prime factor p within a factorization state. We define the computational basis states as:*

- *The state $|0\rangle$ corresponds to the prime appearing with a positive sign, $+p$. This is the ground or unflipped state.*
- *The state $|1\rangle$ corresponds to the prime appearing with a negative sign, $-p$. This is the excited or flipped state.*

A general quantum state of this P-Qubit, $\alpha|0\rangle + \beta|1\rangle$, represents a superposition of the prime factor existing with both signs simultaneously.

This definition transforms the qubit from an abstract entity into a fundamental number-theoretic property. The Pauli-Z operator corresponds to measuring the sign, while the Pauli-X operator corresponds to flipping it ($p \leftrightarrow -p$).

III. Factorization Registers

A single integer, through its prime factorization, naturally defines a quantum register.

Definition III.1 (Factorization Register)**.** *An integer n whose standard prime factorization has k distinct Magnitude Primes, $|n| = p_1 p_2 \cdots p_k$, defines a **Factorization Register**. The basis states of this register are the canonical P-factorizations in the state space $S(n)$.*

Theorem III.2 (Isomorphism to a (k-1)-Qubit Register)**.** *For a positive integer n with k distinct prime factors, its Factorization Register is isomorphic to a $(k-1)$-qubit register.*

Proof. A canonical factorization of a positive integer n must have an even number of negative Magnitude Primes. We can represent a state by a vector of signs $(\epsilon_1, \dots, \epsilon_k)$ where $\epsilon_i \in \{+1, -1\}$. The constraint is $\prod_{i=1}^{k} \epsilon_i = +1$.

We can choose the first $k-1$ signs freely. The sign of the k-th prime, ϵ_k, is then fixed by the parity constraint:

$$\epsilon_k = \prod_{i=1}^{k-1} \epsilon_i \tag{1}$$

The 2^{k-1} possible choices for the signs $(\epsilon_1, \dots, \epsilon_{k-1})$ map one-to-one with the 2^{k-1} basis states of a $(k-1)$-qubit register.

For example, for $n = 30 = 2 \cdot 3 \cdot 5$ ($k = 3$), we have a 2-qubit register. The sign choices for (p_1, p_2) determine the sign of p_3:

- $(+,+) \implies (+)$ for $p_3 \implies \{2,3,5\} \iff |00\rangle$
- $(+,-) \implies (-)$ for $p_3 \implies \{2,-3,-5\} \iff |01\rangle$
- $(-,+) \implies (-)$ for $p_3 \implies \{-2,3,-5\} \iff |10\rangle$
- $(-,-) \implies (+)$ for $p_3 \implies \{-2,-3,5\} \iff |11\rangle$

This establishes the isomorphism. □

IV. Entanglement as Integer Multiplication

One of the most mysterious features of quantum mechanics, entanglement, finds a surprisingly simple and natural explanation within PPF.

Definition IV.1. *Let two non-interacting physical systems be represented by the integers a and b. The total unentangled state is described by the pair of state spaces $(S(a), S(b))$, corresponding to the tensor product $\mathcal{H}_a \otimes \mathcal{H}_b$. An interaction between the systems is represented by the arithmetic multiplication of the integers, $c = ab$.*

Theorem IV.2 (Entanglement from Multiplication)**.** *The state space of the product, $S(ab)$, represents an entangled state of the constituent prime factors. The factorization state of a prime from a is now correlated with the state of the primes from b.*

Proof. Let $a = 6 = 2 \cdot 3$ and $b = 10 = 2 \cdot 5$. These are 1-qubit systems. $S(6) = \{\{2,3\},\{-2,-3\}\} \iff \{|0\rangle, |1\rangle\}$ for the (2,3) system. $S(10) = \{\{2,5\},\{-2,-5\}\} \iff \{|0\rangle, |1\rangle\}$ for the (2,5) system.

The product is $c = 60 = 2^2 \cdot 3 \cdot 5$. This is a positive integer with $k = 3$ distinct prime factors $(2,3,5)$, which forms a 2-qubit register. The basis states in $S(60)$ are:

$$\begin{aligned} \{2,2,3,5\} &\iff |00\rangle \\ \{2,2,-3,-5\} &\iff |01\rangle \\ \{2,-2,3,-5\} &\iff |10\rangle \\ \{2,-2,-3,5\} &\iff |11\rangle \end{aligned}$$

(Note: we map the signs of (3,5) to the two qubits). Consider the state $\{2,2,-3,-5\}$. Here, the 3-qubit is in state $|1\rangle$ and the 5-qubit is in state $|1\rangle$. Now consider $\{2,-2,3,-5\}$. Here, the 3-qubit is in state $|0\rangle$ and the 5-qubit is in state $|1\rangle$.

The crucial insight is that the sign states of the original primes are now linked by the parity constraint of the combined system. Let's say we "measure" the sign of the prime factor 3 in the factorization of 60. If we find it to be negative (state $|1\rangle$), we know from the basis states that the sign of the prime factor 5 must also be negative (state $|1\rangle$). The states of the originally independent factors are now perfectly correlated. This interdependence, mandated by the algebraic rules of PPF, is quantum entanglement. □

V. The Geometry of Quantum Computation

The Primal Reflections framework posits a 4-dimensional spacetime, corresponding to the four prime factors found in the denominator of its core π approximation [5]. This structure finds an unexpected and powerful convergence with recent work in quantum computing.

In 2025, Aasen et al. from Microsoft Quantum proposed using "Geometrically Enhanced Topological Quantum Codes" to improve the efficiency and error correction of quantum computers [6]. Their work demonstrates that the optimal geometry for this purpose is a **rotated lattice on a 4D torus**.

Conjecture V.1 (Computational-Geometric Equivalence)**.** *The 4D toroidal geometry independently discovered to be optimal for quantum error correction is the computational manifestation of the fundamental 4-prime factorization structure that defines spacetime in the PPF framework.*

This conjecture is supported by mapping the operations used in quantum codes to the algebra of PPF.

- **4D Torus $\iff$ 4-Prime Factorization:** The four dimensions of the toric code correspond to the four fundamental P-primes that define a state. An integer with 4 prime factors, e.g., $n = p_1 p_2 p_3 p_4$, defines a 3-qubit register whose topological representation is a 3-torus, embedded in a 4D space.
- **Slicing $\iff$ Measurement/Collapse:** The operation of "slicing" a 4D code to yield a lower-dimensional one is perfectly analogous to a measurement in PPF. By "observing" one of the prime factors (e.g., determining its value and sign), the state space collapses from $S(p_1 p_2 p_3 p_4)$ to a smaller space, such as $S(p_1 p_2 p_3)$, effectively reducing the dimensionality of the system.
- **Surgery $\iff$ Multiplication/Entanglement:** The operation of "surgery," where two code blocks are "glued" together, is the information-theoretic picture of integer multiplication. As shown in Section IV, this is the mechanism for generating entanglement. It creates a larger, composite system whose internal states are now correlated.

This suggests that the reason 4D toric codes are so robust for error correction is that they harness a fundamental, number-theoretically stable way of encoding and protecting information that is built into the fabric of reality itself.

VI. Conclusion

We have demonstrated that the core concepts of quantum information theory—the qubit, the quantum register, and entanglement—emerge as direct consequences of the Physics-Prime Factorization framework. The qubit is the sign of a prime; the register is an integer; and entanglement is multiplication.

This provides a number-theoretic answer to the question "What is quantum information?" It is not an abstract property imposed upon physical systems, but an intrinsic and fundamental feature of the number theory that governs reality. The remarkable convergence with state-of-the-art research in quantum error correction suggests that the 4D toroidal geometry derived from our framework is not merely a theoretical curiosity but a computationally optimal structure. This work solidifies the interpretation of the universe as a quantum computer that operates on the logic of prime numbers.

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