The Spectrum of Integers: A Commutative Algebra for Physics-Prime Factorization

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***Abstract*—The Physics-Prime Factorization (PPF) framework provides a parallel number-theoretic system for modeling quantum phenomena. This paper develops the corresponding commutative algebra for this system. We introduce the "State-Space Ideal," an object that combines the classical principal ideal (n) with the Factorization State Space $S(n)$ of its generator. This allows a distinction between ideals generated by n and $-n$, which are identical as sets but carry different "quantum" information. We define Physics-Prime Ideals (P-Prime Ideals) corresponding to the P-Primes of the PPF system and construct the Spectrum of the Integers, $\mathrm{Spec}_P(\mathbb{Z})$. We prove that this spectrum contains points for all Magnitude Primes and a unique point corresponding to the Sign Prime, -1, which represents the uncollapsed potential of the system. We demonstrate that the multiplication of State-Space Ideals provides a rigorous algebraic description of quantum collapse, where the interaction of two "uncollapsed" ideals (generated by negative integers) yields a "collapsed" ideal (generated by a positive integer) with a constrained state space. This algebraic structure provides a fundamental justification for the retrocausal dynamics of the IOT framework.**

***Index Terms*—commutative algebra, prime ideals, spectrum of a ring, Physics-Prime Factorization, quantum foundations, number theory, state space.**

I. Introduction

The Primal Reflections framework is built upon the axiom that -1 is a prime number, a postulate formalized in the complementary system of Physics-Prime Factorization (PPF) [1]. This axiom gives rise to a Factorization State Space $S(n)$ for each integer, whose combinatorial and topological properties necessitate a toroidal geometry for its physical manifestation [3], [4].

While topology describes the global structure, commutative algebra provides the language for local structure and dynamics. Classical number theory is deeply intertwined with the theory of rings and ideals, particularly in the ring of integers, $\mathbb{Z}$. In this paper, we develop the parallel commutative algebra for PPF. We demonstrate that by enriching the classical concept of an ideal to include the factorization state space of its generator, we can construct an algebraic framework that naturally encodes the concepts of quantum superposition, collapse, and observation. This algebraic structure forms the rigorous foundation for the dynamics of the Involuted Oblate Toroid (IOT) theory [2].

II. State-Space Ideals: Carrying Quantum Information

In classical ring theory, the principal ideal generated by an integer n, denoted (n), is the set of all integer multiples of n. A key feature is that $(n) = (-n)$. The sign of the generator is irrelevant to the ideal itself. This is perfectly suited for classical applications but discards the very information PPF is designed to capture.

Definition II.1 (State-Space Ideal)**.** *A State-Space Ideal $I_S(n)$ is an ordered pair consisting of a principal ideal in $\mathbb{Z}$ and the Factorization State Space of its generator:*

$$I_S(n) = ((n), S(n)) \tag{1}$$

where $(n) = \{kn \mid k \in \mathbb{Z}\}$ and $S(n)$ is the unique set of canonical P-factorizations of n.

This definition immediately introduces a crucial distinction.

Proposition II.2. *For any non-zero integer n, the State-Space Ideals $I_S(n)$ and $I_S(-n)$ are distinct algebraic objects, because $S(n) \neq S(-n)$, even though their underlying sets of multiples are identical.*

Proof. From the definition of PPF [1], the state space $S(-n)$ for a negative integer contains factorizations involving the Sign Prime -1, while the state space $S(n)$ for a positive integer does not. Therefore, their state spaces are different, and the ordered pairs $((n), S(n))$ and $((-n), S(-n))$ are distinct. □

We define the multiplication of two State-Space Ideals as follows:

Definition II.3 (Ideal Multiplication)**.** *The product of two State-Space Ideals is defined as:*

$$I_S(a) \cdot I_S(b) = I_S(ab) = ((ab), S(ab)) \tag{2}$$

The structure of the resulting state space $S(ab)$ is determined by the PPF multiplication rules, which will be shown to model state collapse.

III. The Physics-Prime Spectrum of the Integers

The cornerstone of modern commutative algebra and algebraic geometry is the spectrum of a ring, the set of its prime ideals. We now construct the analogous object for our PPF-based algebra.

Definition III.1 (Physics-Prime (P-Prime) Ideal). *A State-Space Ideal $I_S(p)$ is a **P-Prime Ideal** if its generator p is a P-Prime.*

This leads to two distinct categories of P-Prime Ideals, directly mirroring the two categories of P-Primes.

Definition III.2 (Magnitude and Sign Ideals).
- *A **Magnitude-Prime Ideal** is a P-Prime Ideal $I_S(p)$ where p is a positive Magnitude Prime (e.g., 2, 3, 5, ...).*
- *The **Sign-Prime Ideal** is the P-Prime Ideal $I_S(-1)$ generated by the Sign Prime.*

Remark III.3. *The underlying ideal of the Sign-Prime Ideal is $(-1) = \mathbb{Z}$, the entire ring of integers. In classical algebra, this is the improper ideal and is not considered prime. In the PPF framework, its role as the generator of the uncollapsed state space grants it a special status. It represents the algebraic potential from which all distinctions emerge.*

Definition III.4 (The P-Prime Spectrum). *The Physics-Prime Spectrum of the Integers, denoted $Spec_P(\mathbb{Z})$, is the set of all P-Prime Ideals.*

$$Spec_P(\mathbb{Z}) = \{I_S(-1)\} \cup \{I_S(p) \mid p \in \textit{Magnitude Primes}\} \tag{3}$$

This spectrum forms the fundamental space upon which the algebraic geometry of PPF is built. It consists of a discrete set of points for the Magnitude Primes, plus one special point, the Sign-Prime Ideal, representing the source of all sign information and factorization multiplicity.

IV. The Algebra of Quantum Collapse

The PPF framework maps positive integers to observed, "collapsed" states and negative integers to unobserved, "superposed" states. This physical interpretation is now seen to have a rigorous algebraic foundation in the properties of State-Space Ideals.

Theorem IV.1 (Algebraic State Collapse). *The multiplication of two "uncollapsed" State-Space Ideals, generated by negative integers, results in a "collapsed" State-Space Ideal with a constrained state space.*

Proof. Let $I_S(-a)$ and $I_S(-b)$ be two State-Space Ideals, with $a, b > 0$. Their generators are negative, and their state spaces $S(-a)$ and $S(-b)$ are large, containing factorizations with an odd number of negative factors (or an explicit Sign Prime). Their product is:

$$I_S(-a) \cdot I_S(-b) = I_S(ab) \tag{4}$$

The generator of the product ideal is the positive integer ab. Its state space, $S(ab)$, contains only factorizations with an even number of negative factors. Per PPF, if $S(-a)$ has N_a states and $S(-b)$ has N_b states, the product state space $S(ab)$ has N_{ab} states, where typically $N_{ab} \leq \min(N_a, N_b)$. This represents a reduction in the number of possible factorization "outcomes," which is the algebraic analogue of wave function collapse. □

Corollary IV.2 (Foundation of DLCE). *The multiplication rule for State-Space Ideals provides the fundamental justification for the Doubly Linked Causal Evolution (DLCE) equation. The physical process of an observed "Present" emerging from the interaction of an unobserved "Past" and "Future" is a direct reflection of the algebraic operation $I_S(\textit{past}) \cdot I_S(\textit{future}) = I_S(\textit{present})$. The retrocausal term in the DLCE is thus a requirement for the dynamics to be consistent with the underlying algebra.*

V. Modules and Localization

The framework can be extended to modules and localization, providing deeper insights.

Definition V.1 (State-Space Module). *A State-Space Module M_S over $\mathbb{Z}$ is a classical $\mathbb{Z}$-module M equipped with a map that assigns a Factorization State Space to its elements, compatible with the PPF structure.*

This allows for the study of more complex systems (e.g., vector spaces in quantum mechanics) within the PPF algebraic framework.

Remark V.2 (Localization at the Sign-Prime Ideal). *Localization is a key tool in commutative algebra for studying the local properties of a ring at a prime ideal. Localizing $\mathbb{Z}$ at a Magnitude-Prime Ideal $I_S(p)$ yields the classical ring $\mathbb{Z}_{(p)}$.*

The novel operation is localizing at the Sign-Prime Ideal, $I_S(-1)$. Since the underlying ideal is $\mathbb{Z}$, the classical localization is the zero ring. In our framework, this operation has a physical meaning: it corresponds to the complete collapse of all quantum states. It isolates the "present moment" where all factorization possibilities have resolved into a single, definite reality, and all "quantum" information (the multiplicity of states) is annihilated, leaving only the "classical" outcome.

VI. Conclusion

We have constructed a parallel commutative algebra for the Physics-Prime Factorization system. The introduction of the State-Space Ideal, which weds the classical ideal to the quantum Factorization State Space, provides a rigorous algebraic language for the core concepts of the Primal Reflections framework.

This algebra accomplishes several key goals:

1) It provides a formal distinction between the states of positive and negative integers, corresponding to observed and unobserved physical systems.
2) It defines the P-Prime Spectrum of the Integers, which includes a special point for the Sign Prime, the algebraic source of superposition.
3) It models quantum collapse as a deterministic algebraic operation—the multiplication of State-Space Ideals.
4) It provides a fundamental, axiomatic justification for the retrocausal dynamics of the DLCE equation.

Together with the previously established topological results, this work demonstrates that the geometry and physics of the IOT are not arbitrary postulates but emerge as necessary consequences from the algebraic and topological properties inherent to the PPF axiom.

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