Демидович

5.73 а) $\min X \nexists$

$\max X = 1$

б) $\sup X = 1$

$\inf X = 0$

5.75 $\min X = -1$

$\max X = 1$

$\sup X = 1$

$\inf X = -1$

5.77 $\min X \nexists$

$\max X \nexists$

$\sup X = 0$

$\inf X \nexists$

5.78 $x = \frac{m}{n}, m < n$

$\min X \nexists$

$\max X \nexists$

$\sup X = 1$

$\inf X = 0$

5.213 $x_n = 1 + (-1)^n \frac{1}{n}$

$\{0; 1{,}5; \frac{2}{3}; \frac{5}{4}; \frac{4}{5} \ldots\}$

5.215 $x_n = \frac{3n+5}{2n-3}$

$\{-8; 11; \frac{14}{3}; \frac{17}{5}; \frac{20}{7} \ldots\}$

5.231 $\lim\limits_{n\to\infty} \frac{n-1}{3n} = \frac{1}{3}$

5.238 $\lim\limits_{n\to\infty} (\sqrt{n+2} - \sqrt{n}) = 0$

$| :n$

5.241 $\lim\limits_{n\to\infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \ldots + \frac{n-1}{n^2}\right) = \lim\limits_{n\to\infty} \frac{1}{n} = 0$

5.247 $x_n = 2^{\sqrt{n}}$ - б.б

5.248 $x_n = n^{(-1)^n}$ - не б.б.

$\log_{10} 10 = 1$

5.102 $f(-1) = \lg(-1)^2 = 0$

$10^1 = 10$

$f(-0{,}001) = \lg(-0{,}001)^2 = \lg\left(+\frac{1}{1000000}\right) = -6$ $\left(\frac{1}{1000}\right)^2$

$f(100) = \lg(100)^2 = 4$

$f(x) = \lg x^2$

5.103 $f(x)=\begin{cases} 1+x, & -\infty < x \le 0 \\ 2^x, & 0 < x < \infty \end{cases}$

$f(-2) = 1-2 = -1$

$f(-1) = 1-1 = 0$

$f(0) = 1$

$f(1) = 2$

$f(2) = 2^2 = 4$

5.106 $y = \ln(x+3)$

$D(y) = (-3; \infty)$

$E(y) = (-\infty, \infty)$

5.108 $y = \sqrt{\sin\sqrt{x}}$

$D(y) = [0; \infty)$

$E(y) = [0; 1]$

5.115 $y = e^{x^2-2}$

$D(y) = (-\infty; \infty)$

$E(y) = [\frac{1}{e^2}; \infty)$

5.116 $y = x^2$, $F = [-1, 2]$

$F(-1) = 1$

$F(2) = 4$

$G = [1; 4]$

5.120 $y = \log_3 x$, $F = (3, 27)$

$F(3) = \log_3 3 = 1$

$F(27) = \log_3 27 = 3$

$G = (1, 3)$

5.134 $f(x) = x^4 + 5x^2$ четная

5.137 $f(x) = \frac{e^x + 1}{e^x - 1}$ ~~ни четная, ни нечетная~~

ни четная, ни нечетная

$f(2) = \frac{e^2 + 1}{e^2 - 1}$

$f(-2) = \frac{e^{-2} + 1}{e^{-2} - 1} = \frac{\frac{1}{e^2} + 1}{\frac{1}{e^2} - 1} \cdot e^2 =$

$f(-2) = -\frac{e^2 + 1}{e^2 - 1} = \frac{-1 - e^2}{1 - e^2} \Rightarrow f(-x) \neq -f(x)$

5.139 $f(x) = \lg \frac{1+x}{1-x}$ ни четная, ни нечетная

$f(-2) = \lg \frac{-1}{3} = \lg -\frac{1}{3}$ $\varnothing$

5.142 $f(x) = \cos^2 2x$

$T = \frac{2\pi}{2^2} = \frac{\pi}{2}$

5.143 $f(x) = x \sin x$

$\nexists T$, т.к. x сдвиг

5.147 $y = ax + b$ $\quad$ $x = ay + b$

$y^{-1} = \frac{x - b}{a}$, $a \neq 0$

5.149 $y = \cos 2x$ $\quad$ $x = \cos 2y$

$2y = \arccos x$

$y = \frac{\arccos x}{2}$

$y^{-1} = \frac{\arccos x}{2}$

5.152 $y = \frac{1-x}{1+x}$ $\quad$ $x = \frac{1-y}{1+y}$

$y^{-1} = \frac{1-x}{1+x}$, $x \neq -1$

5.137 $f(x)=\frac{e^x+1}{e^x-1}$

$$f(-x)=\frac{e^{-x}+1}{e^{-x}-1}=\left(\frac{\frac{1}{e^x}+1}{\frac{1}{e^x}-1}\right)\cdot e^x=\frac{1+e^x}{1-e^x}=$$

$$=-\frac{e^x+1}{e^x-1}$$ => нечетная

5.139 $f(x)=\lg\frac{1+x}{1-x}$

$$f(-x)=\lg\frac{1-x}{1+x}=-\lg\frac{x-1}{x+1}=\cancel{\lg\frac{x-1}{-x-1}}$$ =>

ни четная, ни нечетная

5.141 $f(x)=5\cos 7x$

$T=\frac{2\pi}{a}$ при cos и sin

$\cancel{f(x+T)=f(x)}$

$\cancel{f\left(x+\frac{2\pi}{7}\right)=5\cos}$

$T=\frac{2\pi}{7}$

$(g\circ f)(x) = g(f(x)) = \begin{cases} g(0) \\ g(x) \end{cases} = \begin{cases} 0 \\ -x^2 \end{cases} = g(x)$

5.164 $f(x) = |x|$

$g(x) = x^2$

$h(x) = \sqrt{x}$

$\cancel{f(h) = \sqrt{x}}$

$(h\circ g)(x) = \sqrt{x^2} = |x|$

5.165 $f(x) = \sin(\cos\sqrt{x})$

$g(x) = \cos\sqrt{x}$

$h(x) = \sin x$

$\cancel{(h\circ g)(x) = \sin(\cos\sqrt{x})}$

$\cancel{j(}$

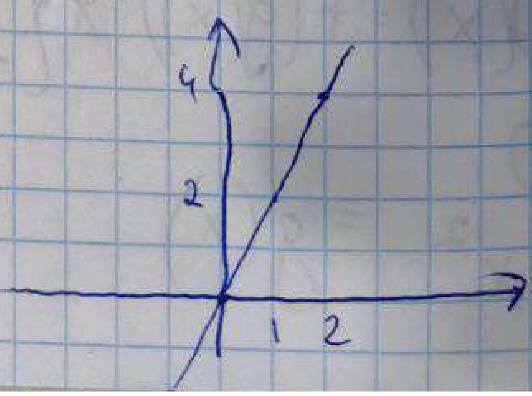
$j(x) = \sqrt{x}$

$(h\circ(g\circ j))(x) = \sin(\cos\sqrt{x})$

5.175 $y = kx + b$

a) $k = 2 \quad b = 0$

$y = 2x$



5.158 $f(x) = x^2$

$g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

5.160 $f(x) = e^x$

$g(x) = \ln x$

$(f \circ g)(x) = f(g(x)) = f(\ln x) = e^{\ln x} = x, \ x > 0$

$(g \circ f)(x) = \cancel{g(e^x)} = g(f(x)) = g(e^x) = \ln e^x = x$

5.162 $f(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ x, & x \in (0; \infty) \end{cases}$

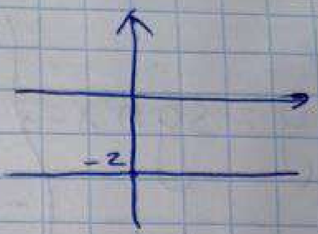
$g(x) = \begin{cases} 0 & , x \in (-\infty, 0] \\ -x^2 & , x \in (0; \infty) \end{cases}$

$(f \circ g)(x) = f(g(x)) = \cancel{f} \begin{cases} f(0) & , x \in (-\infty; 0] \\ f(-x^2) & , x \in (0; \infty) \end{cases} =$

$= \begin{cases} 0 \\ -x^2 \end{cases} = g(x)$

в) $k = 0 \quad b = -2$

$y = -2$

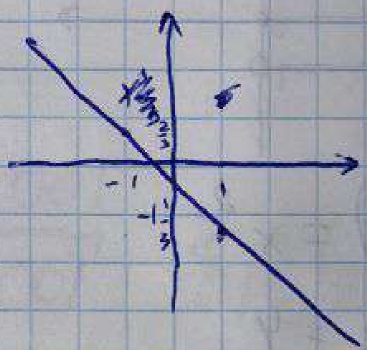


б) $k = -1$,

$b = -\frac{1}{3}$

$y = -x - \frac{1}{3}$

1	-1
$-1\frac{1}{3}$	$\frac{2}{3}$

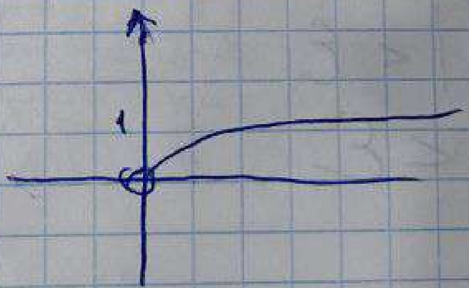


5.183 $y = \log_a (kx + b)$

а) $a = 10 \quad k = 10 \quad b = -1$

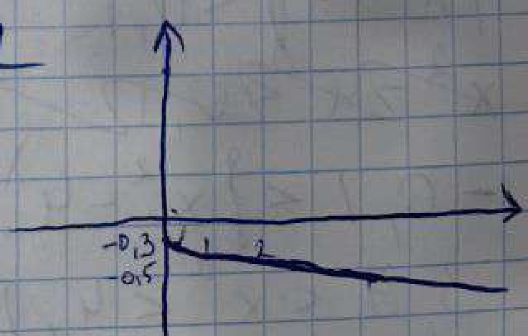
$y = \lg(10x - 1)$

1	2	3
0,95	1,279	1,46



б) $a = \frac{1}{10} \quad k = \frac{1}{2} \quad b = 2$

$y = \log_{\frac{1}{10}} \left(\frac{1}{2}x + 2\right)$



0	1	2	3
-0,3	-0,4	-0,48	-0,56

$\sqrt{3,9} < x < \sqrt{4,1}$ $\quad \delta(\varepsilon) \in (\sqrt{3,9}; \sqrt{4,1})$

$|x^2 - 4| < 0,01$

$-0,01 < x^2 - 4 < 0,01$

$3,99 < x^2 < 4,01$

$\sqrt{3,99} < x < \sqrt{4,01}$ $\quad \delta(\varepsilon) \in (\sqrt{3,99}; \sqrt{4,01})$

$|x^2 - 4| < 0,001$

$3,999 < x^2 < 4,001$

$\sqrt{3,999} < x < \sqrt{4,001}$ $\quad \delta(\varepsilon) \in (\sqrt{3,999}; \sqrt{4,001})$

5.272 $\lim\limits_{x \to 0} \dfrac{x^2 - 2}{3x^2 - 5x + 1} = \dfrac{-2}{1} = -2$

5.277 $\lim\limits_{x \to 1} \dfrac{x^2 - 2x + 1}{x^3 - x} \overset{|:x^3}{=} \cancel{\lim\limits_{x \to 1} \dfrac{\frac{1}{x} - \frac{2}{x^2} + \frac{1}{x^3}}{1 - \frac{1}{x^2}}} = \dfrac{0}{0} =$

$= \cancel{|1 - 2 + 1|}$

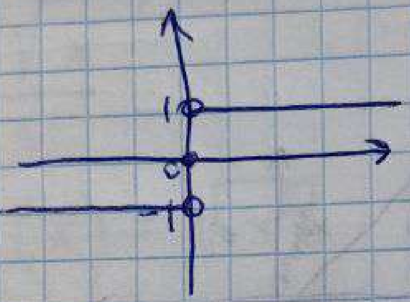
$= \lim\limits_{x \to 1} \dfrac{(x-1)^{\cancel{2}}}{x\cancel{(x-1)}(x+1)} = \dfrac{0}{2} = 0$

5.288 $\lim\limits_{x \to \infty} \dfrac{3x + 1}{5x + \sqrt[3]{x}} = \lim\limits_{x \to \infty} \dfrac{3x + 1}{5x + x^{\frac{1}{3}}} \overset{|:x}{=} \dfrac{\infty}{\infty} =$

$= \lim\limits_{x \to \infty} \dfrac{3 + \frac{1}{x} \to 0}{5 + \frac{x^{\frac{1}{3}}}{x} \to 0} = \dfrac{3}{5}$

5.192 $y = \operatorname{sgn} x = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$

sgn - кусочно-постоянная ф-ция



5.261 $f(x) = x^2$

$x_0 = 2 \quad a = 4$

$\lim\limits_{x \to x_0} f(x) = \lim\limits_{x \to 2} x^2 = 2^2 = 4$

$|f(x) - a| < \varepsilon$

$|x^2 - 4| < \varepsilon$

$|x^2 - 4| < 0{,}1$ *

$\cancel{x^2 - 4 < 0{,}1 \quad x \in (-2;2)}$

$-0{,}1 < x^2 - 4 < 0{,}1 \quad |+4$

$3{,}9 < x^2 < 4{,}1$

5.289 $\lim\limits_{x\to10}\dfrac{\sqrt{x-1}-3}{x-10}=\cancel{\lim\limits_{x\to10}\dfrac{(\sqrt{x-1}-3)(\sqrt{x+1})}{(x-10)(\sqrt{x+1})}}=\dfrac{0}{0}$

$=\cancel{\lim\limits_{x\to10}\dfrac{1}{x\sqrt{x+1}-10\sqrt{x+1}}}$

$=\lim\limits_{x\to10}\dfrac{\sqrt{x-1}-3}{x-1-9}=\lim\limits_{x\to10}\dfrac{\sqrt{t}-3}{t-9}=\lim\limits_{x\to10}\dfrac{\cancel{\sqrt{t}-3}}{\cancel{(\sqrt{t}-3)}(\sqrt{t}+3)}$

$=\lim\limits_{x\to10}\dfrac{1}{\sqrt{t}+3}=\lim\limits_{x\to10}\dfrac{1}{\sqrt{x-1}+3}=\dfrac{1}{6}$

5.303 $\lim\limits_{x\to0}\dfrac{\sin3x}{x}=\lim\limits_{x\to0}\dfrac{3\sin3x}{3x}=3$

5.305 $\lim\limits_{x\to0}x\,\mathrm{ctg}\,\pi x=\lim\limits_{x\to0}x\cdot\dfrac{1}{\mathrm{tg}\,\pi x}=$

$=\lim\limits_{x\to0}\dfrac{x}{\mathrm{tg}\,\pi x}=\lim\limits_{x\to0}\dfrac{\pi x}{\pi\,\mathrm{tg}\,\pi x}=\dfrac{1}{\pi}$

5.320 $\lim\limits_{x\to\infty}\left(\dfrac{x+3}{x-2}\right)^{2x+1}=\lim\limits_{x\to\infty}\left(\dfrac{x-2+5}{x-2}\right)^{2x+1}=$

$=\boxed{\lim\limits_{x\to\infty}\left(1+\dfrac{5}{x-2}\right)^{\frac{x-2}{5}}}^{\frac{5}{x-2}\cdot(2x+1)}=$ lim

$=e^{\lim\limits_{x\to\infty}\frac{5\cdot(2x+1)}{x-2}}=e^{\lim\limits_{x\to\infty}\frac{10x+5}{x-2}}=e^{\lim\limits_{x\to\infty}\frac{10+\frac{5}{x}\to0}{1-\frac{2}{x}\to0}}=e^{10}$

5.329

$=\lim\limits_{x\to0}$

$=\lim\limits_{x\to0}$

$=\dfrac{0}{0}=$

$=\lim\limits_{x\to0}$

$=-$

5.33

$\lim\limits_{x\to3}$

5.34

$$\lim_{x\to-\infty} \operatorname{arctg} x = -\frac{\pi}{2}$$

5.349 $\alpha(x) = \frac{3\sqrt{x^3}}{1-x}$

$$\lim_{x\to 0} \frac{\frac{3\sqrt{x^3}}{1-x}}{x} = \lim_{x\to 0} \frac{3\sqrt{x^3}}{x(1-x)} = \lim_{x\to 0} \frac{3\cdot x^{\frac{3}{2}}}{x(1-x)} =$$

$$= \cancel{\lim_{x\to 0} \frac{3}{\frac{1}{\frac{x}{2}}\left(1-\frac{1}{\frac{x}{2}}\right)}} = \lim_{x\to 0} \frac{3\cdot x^{\frac{1}{2}}}{(1-x)} = 0$$

=> б.м. не можем доказать

5.351 $\alpha(x) = \frac{1-\cos x}{x}$

$$\lim_{x\to 0} \frac{\frac{1-\cos x}{x}}{x} = \lim_{x\to 0} \frac{1-\cos x}{x^2} \Big| \cdot (1+\cos x) = \lim_{x\to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)} =$$

$\beta(x)_{x\to 0} = x$ - б.м.

$$= \lim_{x\to 0} \frac{\sin^2 x}{x^2(1+\cos x)} = \frac{1}{(1+\cos 0)} = \frac{1}{2}$$

т.к. $\frac{1}{2}$ - конечно и $\neq 0$, то $\alpha(x)$ - б.м.

5.329 $\lim\limits_{x\to 0} \frac{e^{ax}-e^{bx}}{x} = \cancel{\lim\limits_{x\to 0} \frac{\frac{1}{e^{bx}} e^{ax} - e^{bx}\cdot\frac{1}{e^{bx}}}{x \cdot \frac{1}{e^{bx}}}}$

$= \cancel{\lim\limits_{x\to 0} \frac{a(e^{ax}-e^{bx})}{ax}}$

$= \cancel{\lim\limits_{x\to 0} \frac{e^{x}(e^{a}-e)}{}}$

$= \frac{0}{0} = \lim\limits_{x\to 0} \frac{e^{ax}\cdot(1-e^{(b-a)x})}{x} =$

$(b-a)x + ax = bx - ax + ax = bx$

$= \lim\limits_{x\to 0} \frac{e^{ax}\cdot(-(e^{(b-a)x}-1))}{x} = \lim\limits_{x\to 0} \frac{-e^{ax}\cdot(e^{(b-a)x}-1)}{x} =$

$= \cancel{e^{ax+bx-ax}} - \lim\limits_{x\to 0} e^{ax} \cdot \lim\limits_{x\to 0} \boxed{\frac{(e^{(b-a)x}-1)}{(b-a)x}}(b-a) =$

$= -\lim\limits_{x\to 0} e^{ax} \cdot (b-a) = (a-b) \cdot 1 = a-b$

5.338 $\lim\limits_{x\to 3+0} \frac{x-3}{|x-3|} = \frac{x-3}{x-3} = 1$

$\lim\limits_{x\to 3-0} \frac{x-3}{|x-3|} = -\frac{x-3}{x-3} = -1$

5.342 $\lim\limits_{x\to +\infty} \operatorname{arctg} x = \frac{\pi}{2}$

5.361 $\sqrt{25,3} \approx 5,03$

5.362 $(1,03)^5 = \cancel{1,0000000243}$

$\approx 1,159$

$$\begin{array}{r} \times 27 \\ 3 \\ \hline 81 \\ \times 3 \\ \hline 243 \end{array}$$

5.387 $f(x) = \dfrac{1}{x^2(x-1)}$

0 и 1

$f(0) \nexists \quad f(1) \nexists$

$\lim\limits_{x \to 0-} f(x) = \lim\limits_{x \to 0-} \dfrac{1}{x^2(x-1)} = -\infty$

$\lim\limits_{x \to 0+} f(x) = \lim\limits_{x \to 0+} \dfrac{1}{x^2(x-1)} = -\infty$

т.к. $\lim = -\infty$, то точка $x = 0$ - разрыв II рода

$\lim\limits_{x \to 1-} f(x) = \lim\limits_{x \to 1-} \dfrac{1}{x^2(x-1)} = -\infty$

$\lim\limits_{x \to 1+} f(x) = \lim\limits_{x \to 1+} \dfrac{1}{x^2(x-1)} = \infty$

т.к. $\lim = \infty$, то точка $x = 1$ - разрыв II рода

5.388 $f(x) = \dfrac{|3x-5|}{3x-5}$

$x \neq \dfrac{5}{3}$

$f(x) = f\left(\dfrac{5}{3}\right) = \dfrac{|5-5|}{0}$ $\nexists$ $\varnothing$

$$\lim_{x\to\frac{5}{3}-} f(x) = \lim_{x\to\frac{5}{3}-} \frac{|3x-5|}{3x-5} = \frac{+0}{-0} = -1$$

$$\lim_{x\to\frac{5}{3}+} f(x) = \lim_{x\to\frac{5}{3}+} \frac{|3x-5|}{3x-5} = \frac{+0}{+0} = +1$$

$$\lim_{x\to\frac{5}{3}-} f(x) \neq \lim_{x\to\frac{5}{3}+} f(x) \Rightarrow$$

$\dfrac{5}{3}$ - точка разрыва I рода, т.к. пределы конечны, типа скачок

5.390 $f(x) = \dfrac{1}{x}\sin x$

$f(0)$ $\varnothing$

$$\lim_{x\to 0-} f(x) = \cancel{\frac{1}{-0}\sin(-0) = -\infty} = \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0+} f(x) = \frac{\sin x}{x} = 1$$

$$= \lim_{x\to 0-} \frac{1}{x} \cdot \lim_{x\to 0-} \ln \left(\frac{1-x+2x}{1-x} \right) =$$

$$= \lim_{x\to 0-} \frac{1}{x} \cdot \lim_{x\to 0-} \ln \left(1 + \frac{2x}{1-x} \right)^{\frac{1-x}{2x} \cdot \frac{2x}{1-x}} =$$

$$= \lim_{x\to 0-} \frac{1}{x} \cdot \lim_{x\to 0} \ln e^{\frac{2x}{1-x}} = \lim_{x\to 0-} \left(\frac{1}{x} \cdot \frac{2x}{1-x} \right) =$$

$$= \lim_{x\to 0-} \frac{2}{1-x} = 2$$

$$\lim_{x\to 0+} \frac{1}{x} \cdot \ln \frac{1+x}{1-x} = \lim_{x\to 0+} \frac{2}{1-x} = 2$$

$\lim\limits_{x\to 0-} f(x) = \lim\limits_{x\to 0+} f(x) \Rightarrow x=0$ – точка разрыва

I порядка, устранимая

$$\lim_{x\to 1-} (f(x)) = \lim_{x\to 1-} \frac{2}{1-x} = \frac{2}{+0} = \infty \Rightarrow$$

$x = 1$ - точка разрыва II род

$$\lim_{x\to -1-} f(x) = \lim_{x\to -1-} \frac{2}{1-x} = \frac{2}{2-} = \nexists \Rightarrow$$

$x = -1$ точка разрыва II рода

$\lim\limits_{x\to 0-}(f(x)) = \lim\limits_{x\to 0+}(f(x)) \Rightarrow 0$ - точка

разрыва I рода, устранимая

5.391 $f(x) = 1 - x\sin\frac{1}{x}$

$f(0)\ \varnothing$

$\lim\limits_{x\to 0-} f(x) = \lim\limits_{x\to 0-} 1 - x\sin\frac{1}{x} = \cancel{\lim\limits_{x\to 0-}} \ 1+0 = 1$

$\lim\limits_{x\to 0+} f(x) = \lim\limits_{x\to 0+} 1 - x\sin\frac{1}{x} = 1 - 0 = 1$

$\lim\limits_{x\to 0-} f(x) = \lim\limits_{x\to 0+} f(x) \Rightarrow 0$ - точка разрыва

I рода, устранимая

5.396 $f(x) = \frac{1}{x} \ln\frac{1+x}{1-x}$ $\cancel{\ }) | -1$

$x = 0$; $x = 1$; $x = -1$ (д/не отриц. логарифме)

$\lim\limits_{x\to 0-} f(x) = \frac{0}{0} = \cancel{\frac{1}{x} \cdot \ln\frac{(1+x)^2 (1-x)}{(1-x)^3 (1+x)}} =$

$= \cancel{\frac{1+x}{x} \ln \frac{1+x}{1}} \frac{\ln(1+x)}{x\ln(1-x)} = \frac{\ln(1+x)}{x(\ln(1-x))} = \cancel{\ \ }$

5.399 $f(x) = \begin{cases} 2^x, & -1 \le x < 1 \\ x-1, & 1 < x \le 4 \\ 1, & x = 1 \end{cases}$

~~$x = -1 \quad x = 1 \quad x = 4$~~

~~$f(-1) = 2^{-1} = \frac{1}{2}$~~

~~$\lim\limits_{x \to -1-} f(x) = 2^{-1} = \frac{1}{2}$ ... $\Rightarrow x = -1$ - ...~~

~~$\lim\limits_{x \to -1+} f(x)$~~

$f(1) = 1$

$\lim\limits_{x \to 1-} f(x) = \lim\limits_{x \to 1-} 2^x = 2$

$\lim\limits_{x \to 1+} f(x) = \lim\limits_{x \to 1+} x - 1 = 0 \Rightarrow$

$x = 1$ - точка разрыва I рода, скачок $= 2$