ROB 599: Sequential Decision Making in Robotics, Winter 2021
HW #2: Optimization, Deep Learning, and Informative Path Planning
Due: 2/15/21 (midnight)

Questions

1. Performance Guarantees (20 pts)

a. You are given the problem of placing a limited number of sensors (up to K) to maximize a submodular objective function [1]. **What is the performance guarantee relative to optimal for the greedy solution (i.e., selecting the best location for sensor one, placing the sensor, selecting the best location for the second sensor given the first is already placed, etc.)? Explain what this means in your own words (1-2 sentences).**

b. Assume you must generate a trajectory (with path constraints) to maximize a submodular objective function [2]. The vehicle can only take K samples along the trajectory. You devise the following algorithm: greedily select the best locations (as above) and connect them using an approximation to TSP. Assume that the optimal solution maximizes the total quantity F(P)/C(P), where F(P) is the submodular function evaluation for path P, and C(P) is the time cost of executing trajectory P. **What, if any, is the performance guarantee for this algorithm? Can you give an example where this approach performs poorly?**

Programming Assignment (Do not start at last minute!) (60 points)

The zip file "Homework2.zip" contains helper functions and data for this homework. Download it in addition to this word document. **You should use Python for this programming assignment to interface with Pytorch.** The 'readme' file in the zip package provides more information on installing and running the code.

Your robot starts at (0,0) and must navigate to a specific corner of the map. The goal depends on which world you occupy. The worlds are constructed from the MNIST dataset, a hand drawn number dataset consisting of numbers 0-9. Each world is a 28x28 grid.
If the MNIST digit is:

- 0-2: the goal is at (0,27)
- 3-5: the goal is at (27,27)
- 6-9: the goal is at (27,0)

The robot can travel one space at a time and can observe the value of the spaces it has visited and spaces one step away. It costs -1 reward to move and -400 reward if you travel to the wrong goal. Reaching the correct goal provides +100 reward, at which point the mission ends. You will design algorithms to minimize your cost to reach the correct goal. We have provided two trained neural networks to help. One network provides an estimate of what the world looks like given what you have currently observed. The other network takes this estimate and provides an estimate of what digit the world belongs to.

Step 1 (30 points): This assignment deals with a discrete version of the problem. The robot can only select waypoints that are on the grid (e.g., (1.5,1.5) is not a valid waypoint). Keep in mind that subsequent waypoints should always be distance 1 or 0 (if remaining stationary) from each other.

i. Implement a greedy solver for the discrete problem that can only move in the 4 cardinal directions (N,W,S,E). The robot should look one step ahead and move to the location that gains the maximal information based on the neural network prediction (you should determine how to calculate the maximal information). Hint: consider the values of the pixels in the prediction image.

The robot must (at a point of your choosing) stop greedily gathering information and move to a corner to terminate the mission (if correct) or incur the appropriate penalty (if incorrect). If an incorrect corner is chosen, the robot should move to another corner (or continue gathering information) until it reaches the correct corner. Hint: consider the softmax output values from the network as part of this decision.

How did you calculate the information quality from the neural network prediction? How do you decide when to move to a corner and stop moving around greedily? Show two example trajectories on different digits. Provide your average reward over 10 trials using your greedy algorithm.

Step 2 (30 points):

i. Develop and implement your own algorithm with the goal of outperforming the greedy solver. You should use a more sophisticated informative path planning technique (e.g. sampling-based solver or branch and bound). **Describe your algorithm formally. Report your average reward over 10 trials. Also report the average time it took to find this solution. You should terminate your algorithm after 15 minutes of run time (i.e. solutions that take more than 15 minutes to find are considered invalid).**

Discussion (20 points): Address the following questions.

1) Consider solving this informative path planning problem by formulating it as a mixed integer program, using a linear relaxation to find a solution, and then rounding to the closest feasible path. What would be some advantages and disadvantages of this approach versus the ones you implemented? Hint: consider the integrality gap, and also consider how you would represent the objective function and constraints using a MIP.

2) When designing your algorithm in Step 2, what design criteria did you use to determine what to implement? Compare and contrast how your algorithm meets these design criteria relative to other algorithms taught in this course (use at least two additional examples from the course lectures or reading group discussions, and do not compare to a linear relaxation as in the previous discussion question).

3) The neural network provides you with a prediction of the correct digit but not an uncertainty on that prediction. If you had access to an uncertainty estimate, how might you use this to modify your algorithm from Step 2(ii)? Describe qualitatively in words; you do not need to specify the new algorithm formally.

4) Consider the case where you have multiple robots communicating with each other and performing cooperative information gathering in this scenario. What additional design considerations would there be in this case? Describe qualitatively how you would modify your algorithm to coordinate multiple robots to efficiently gather information.

References

[1] Andreas Krause, Carlos Guestrin, "Submodularity and its Applications in Optimized Information Gathering," In ACM Transactions on Intelligent Systems and Technology, vol. 2, no. 4, 2011.

[2] Singh, Amarjeet, et al. “Efficient planning of informative paths for multiple robots,” IJCAI, vol. 7, 2007.