

Binomial Equation

In [S1 Binomial Theorem](#), the binomial expansion given is the one below:

If $n > 0$:

$$(a + b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + \binom{n}{n}a^0 b^n$$

Where $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

This can be expanded to include any rational value of n , as shown below:

If $|ax| < 1$:

$$(1 + ax)^n \equiv 1 + \frac{n}{1!}ax + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \dots$$

When n is not a positive whole number, it will become an infinite series as the numerator is never equal to 0.

As this is an approximation, it only works when $|ax| < 1$.

When x is small, large powers of x are extremely small, so they can be neglected. This means using the first few terms can approximate the original expression.

Note - "The expansion is valid" \equiv "The result will converge"

Code

The code below will take the value of a and n in $(1 + ax)^n$ and the term wanted, then calculate the coefficient of x of each term

```
from fractions import Fraction
from math import factorial

def binomialCoeffCalculator(termNo: int, nVal: Fraction):
    numerator = 1
    for x in range(termNo):
        numerator *= (nVal - x)
    denominator = factorial(termNo)

    return Fraction(numerator, denominator)
```

```
def termCalculator(termNo: int, xVal, nVal: Fraction):  
    return (xVal**termNo)*binomialCoeffCalculator(termNo, nVal)
```