Sequences

Sequences can be described in two ways:

- Term-by-term : $u_1 = x, \;\; u_{n+1} = u_n + c$
- Position-to-term : $u_n = an + c$

There are a couple of other descriptors for sequences:

- An increasing sequence is one where : $u_{n+1} > u_n$
- A decreasing sequence is one where : $u_{n+1} < u_n$
- A periodic sequence is one where terms repeat : $u_{n+k} = u_n$ (So k is the period)

There is a key difference between a sequence and a series:

$x_n, x_{(n+1)}, x_{(n+2)} x_{(n+3)}, \ldots \;\;$ is a Sequence

$x_n + x_{(n+1)} + x_{(n+2)} + x_{(n+3)} + \ldots \;\;$ is a Series

There are two types of Sequences in the spec:

- Arithmetic : $a, a + d, a + 2d, a + 3d, \cdots$
- Geometric : $a, ar, ar^2, ar^3, \cdots$

You can see that in arithmetic, a common difference d is added for the next term, and that in geometric, a common ratio r is multiplied for the next term, so we can see a term-by-term rule for both is:

- Arithmetic : $u_{n+1} = u_n + d$
- Geometric : $u_{n+1} = ru_n$

Both of these series have more useful position-to-term rules:

- Arithmetic : $u_n = a + (n - 1)d$
- Geometric : $u_n = ar^n$

Series

As can be inferred above, a series is the sum of each term in a sequence. Series and Sum are denoted by the letter S.

$$S_n \equiv u_1 + u_2 + \cdots + u_{n-1} + u_n \qquad \text{(True for every series)}$$

Formulae

The summation of an arithmetic sequence is:

$$S_n = \frac{n}{2}\Big[2a + (n-1)d\Big]$$

The summation of a geometric sequence is:

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Sum to Infinity

You may notice that in the sum of a geometric sequence, if $|r| < 1$, r^n will be approximately equal to 0, when n is very large. Thus, for a common ratio less than 1:

$$S_\infty = \frac{a}{1-r}, \quad \text{where } |\text{r}|<1$$

Because of this property, we can say a geometric sequence *diverges* when $|r| > 1$, and *converges* when $|r| < 1$.

Sigma Notation

A common method of denoting series is using sigma notation:

$$\sum_{n=\varphi}^{k} u_n \equiv S_k - S_{\varphi-1}$$

- The bottom denotes what term of a series to start at
- The top denotes the last term of the sum
- The right denotes the series