Position Control of a DC Motor Using Feedback Linearization and LuGre Based Friction Extended State Observer

This project implements feedback linearization and a simple PI controller for precise position control of a brushed DC motor. Additionally, a LuGre-based observer is used to estimate and compensate for friction and external disturbances, enhancing control performance.

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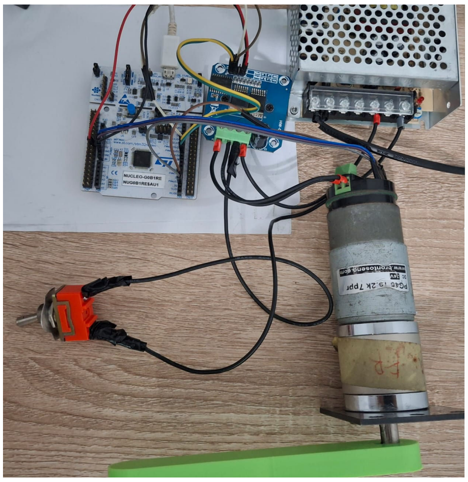
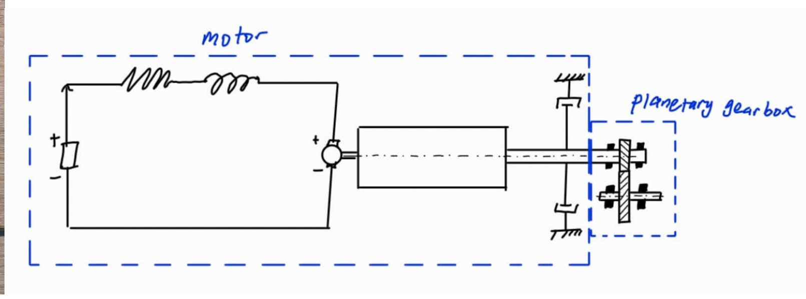
Research Equipment





In this research, i use a 70 Watt (output power) DC motor PG45 Brontoseno, BTS7960 motor driver, power supply (24 V and 5 A), and an STM32 Nucleo G0B1RE microcontroller

BDC Motor Modelling

Documentation on modelling and parameter identification can be accessed through the following link below

https://github.com/IrfNyafi/Modelling-and-Parameter-Identification-of-Brushed-DC-Motor.git

$$\frac{d\omega(t)}{\mathrm{dt}} = \rho(t)(-0.1628\,|\omega\,(t)| + 11.04\,) - 0.7184\,\omega(t) - 167037.21858901123\,z - 4569.25\,\frac{\mathrm{dz}}{\mathrm{dt}}$$

$$\frac{\mathrm{dz}(t)}{\mathrm{dt}} = \omega(t) - \frac{1336.29774871209|\omega(t)|\,z}{0.1668 + 0.29525698321793\,e^{-\left(\frac{\omega}{0.5617977287214991}\right)^2}}$$

- $\rho(t)$ is the average voltage define as percentage duty cycle times the input supply voltage
- $z(t)$ is the internal LuGre friction state
- $\omega(t)$ is the motor speed

Non-linear Luenberger Observer

let us consider the following model

$$\frac{d\omega(t)}{\mathrm{dt}} = \rho(t)(a_1 \,|\omega\,(t)| + a_2) - \beta\omega(t) - \frac{\sigma_0}{J}z - \frac{\sigma_1}{J}\frac{\mathrm{dz}}{\mathrm{dt}} + f(t)$$

$$\frac{\mathrm{dz}(t)}{\mathrm{dt}} = \omega(t) - \frac{\sigma_0|\omega(t)|\, z}{T_c + \Delta T e^{-\left(\frac{\omega}{\omega_s}\right)^2}}$$

$f(t)$ is generalized disturbance which is the combination of internal disturbance (arises from modeling errors) and external disturbance (comes from external torque exerted by the environoment). By the above model, we design the observer as follow

$$\frac{d\,\hat{z}(t)}{\mathrm{dt}} = \omega(t) - \sigma_0\,\frac{|\omega(t)|}{h(\omega\,)}\,\hat{z}(t) + K_1(\omega(t) - \,\hat{\omega}\,(t))$$

$$\frac{d\,\hat{\omega}\,(t)}{\mathrm{dt}} = -\frac{\sigma_o}{J}\hat{z} - \frac{\sigma_1}{J}\,\frac{d\hat{z}(t)}{\mathrm{dt}} - \frac{F_v}{J}\,\omega(t) + \,\hat{f}(t) + u(t) + \frac{K_2}{J}(\omega(t) - \,\hat{\omega}\,(t))$$

$$\frac{d\,\hat{f}(t)}{\mathrm{dt}} = K_3\,(\omega(t) - \,\hat{\omega}\,(t))$$

Documentation of the observer can be found here, https://github.com/IrfNyafi/Lugre-Friction-Model-Based-Observer-For-Friction-Compensation-of-DC-Motor-Motion-Control.git

Control System Law: Modified PID Controller Using Feedback Linearization

let us define

$$u(t) = \rho(t)(a_1\,|\omega\,(t)| + a_2) - \beta\omega(t) - \frac{\sigma_0}{J}z - \frac{\sigma_1}{J}\frac{\mathrm{dz}}{\mathrm{dt}} + f(t)$$

then our dynamic model can be written as second order dynamic model

$$\frac{d\dot{\theta}(t)}{\mathrm{dt}} = u(t)$$

define the error dynamics

$$e(t) = \theta_d - \theta(t)$$

$$\dot{e}(t) = \dot{\theta}_d(t) - \dot{\theta}(t)$$

$$\ddot{e}(t) = \ddot{\theta}_d - \ddot{\theta}(t) = \ddot{\theta}_d - u(t)$$

we design the control input law $u(t)$ using simple proportional integral controller. The writer choose the control input to be

$$u(t) = \ddot{\theta}_d + K_p\,e(t) + K_i\int_{t_0}^{\tau} e(\tau)\,d\tau + K_d\frac{\mathrm{de}(t)}{\mathrm{dt}}$$

substitute the proposed control law to the error dynamics, we will get

$$\ddot{e}(t) = \dot{\omega}_d(t) - \dot{\omega}(t) = \dot{\omega}_d(t) - \dot{\omega}_d(t) - K_p\, e(t) - K_i \int_{t_0}^{\tau} e(\tau)\, d\tau - K_d \frac{\mathrm{de}(t)}{\mathrm{dt}}$$

$$\ddot{e}(t) + K_p\, e(t) + K_i \int_{t_0}^{\tau} e(\tau)\, d\tau + K_d \frac{\mathrm{de}(t)}{\mathrm{dt}} = 0$$

similar dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t}\ddot{e}(t) + K_d\, \ddot{e}(t) + K_p\, \dot{e}(t) + K_i\, e(t) = 0$$

based on that, the gain K_p and $K_i$is chosen to be Hurwitz to ensure stability. We can also use pole placement technique to make tuning easier. Let us define P_1, P_2, and P_3 as the desire error dynamic poles. Construct the desire characteristic equation

$$(s - P_1)(s - P_2)(s - P_3) = 0$$

$$s^3 - (P_1 + P_2 + P_3)s^2 + (P_1P_2 + P_1P_3 + P_2P_3)s - P_1P_2P_3 = 0$$

thus, we get $K_p = (P_1P_2 + P_1P_3 + P_2P_3)$, $K_i = -P_1P_2P_3$, and $K_d = -(P_1 + P_2 + P_3)$.

We conclude the control law as the following:

$$u(t) = \dot{\omega}_d(t) + K_p\, e(t) + K_i \int_{t_0}^{\tau} e(\tau)\, d\tau + K_d \frac{\mathrm{de}(t)}{\mathrm{dt}} = \rho(t)(a_1\, |\omega\,(t)| + a_2) - \beta\omega(t) - \frac{\sigma_0}{J}\hat{z} - \frac{\sigma_1}{J}\frac{d\hat{z}}{\mathrm{dt}} + \hat{f}(t)$$

$$\rho(t) = \frac{\dot{\omega}_d(t) + K_p\, e(t) + K_i \int_{t_0}^{\tau} e(\tau)\, d\tau + K_d \frac{\mathrm{de}(t)}{\mathrm{dt}} + \beta\omega(t) + \frac{\sigma_0}{J}\hat{z} + \frac{\sigma_1}{J}\frac{d\hat{z}}{\mathrm{dt}} - \hat{f}(t)}{(a_1\, |\omega\,(t)| + a_2)}$$

the values of $\hat{z}$, $\frac{d\hat{z}}{\mathrm{dt}}$, and $\hat{f}(t)$ are obtained from the nonlinear Luenberger observer

Impedance Control

for this case, we can design

$$u(t) = \ddot{\theta}_d + \frac{1}{M_d}\left(K_{\mathrm{sd}}\, e(t) + C_d \frac{\mathrm{de}(t)}{\mathrm{dt}}\right)$$

where

- M_d is the desire apparent inersia
- C_d is the desire apparent damping
- K_{sd} is the desire apparent stiffness
- we define, $e(t) = \theta_d(t) - \theta(t)$

remember, our system is

$$\frac{d\dot{\theta}(t)}{\mathrm{dt}} = u(t)$$

substitute the propose control input $u(t)$ to the system dynamic

$$\frac{d\dot{\theta}(t)}{\mathrm{dt}} = u(t) = \ddot{\theta}_d + \frac{1}{M_d}\left(K_{\mathrm{sd}}\, e(t) + C_d \frac{\mathrm{de}(t)}{\mathrm{dt}}\right)$$

$$-\left(\ddot{\theta}_d - \ddot{\theta}\right) = \frac{1}{M_d}\left(K_{\mathrm{sd}}\, e(t) + C_d \frac{\mathrm{de}(t)}{\mathrm{dt}}\right)$$

$$M_d\left(\ddot{\theta}_d - \ddot{\theta}\right) + C_d\left(\dot{\theta}_d - \dot{\theta}\right) + K_{\mathrm{sd}}\left(\theta_d - \theta\right) = 0$$

We conclude the control law as the following:

$$u(t) = \dot{\omega}_d(t) + K_p\, e(t) + K_d \frac{\mathrm{de}(t)}{\mathrm{dt}} = \rho(t)(a_1 \left|\omega\,(t)\right| + a_2) - \beta\omega(t) - \frac{\sigma_0}{J}\hat{z} - \frac{\sigma_1}{J}\frac{d\hat{z}}{\mathrm{dt}} + \hat{f}(t)$$

$$\rho(t) = \frac{\dot{\omega}_d(t) + K_p\, e(t) + K_d \frac{\mathrm{de}(t)}{\mathrm{dt}} + \beta\omega(t) + \frac{\sigma_0}{J}\hat{z} + \frac{\sigma_1}{J}\frac{d\hat{z}}{\mathrm{dt}} - \hat{f}(t)}{(a_1 \left|\omega\,(t)\right| + a_2)}$$

the values of $\hat{z}$, $\frac{d\hat{z}}{\mathrm{dt}}$, and $\hat{f}(t)$ are obtained from the nonlinear Luenberger observer