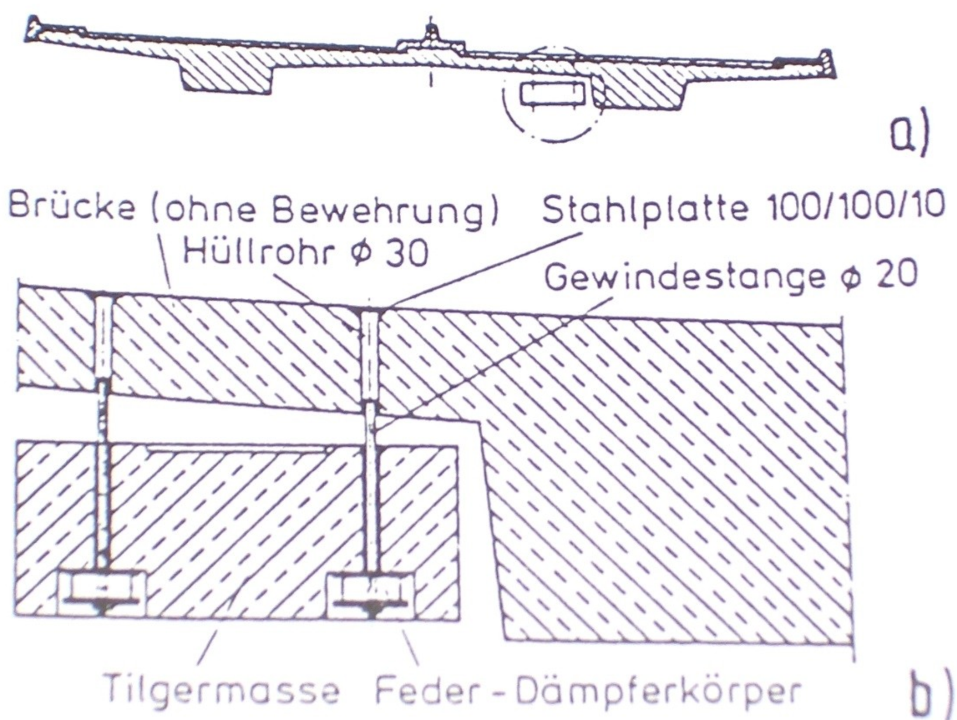
Tuned Mass Dampers

a mass that is connected to a structure by a spring and a damping element without any other support,in order to reduce vibration of the structure.



Tuned mass dampers are mainly used in the following applications:

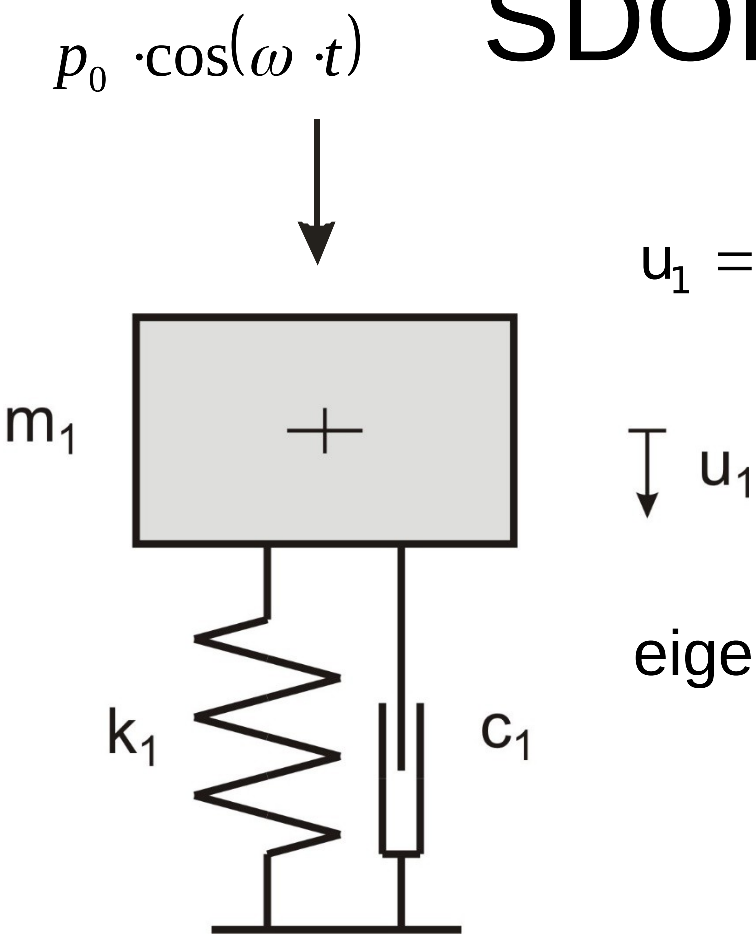
tall and slender free-standing structures (bridges, pylons of bridges, chimneys, TV towers) which tend to be excited dangerously in one of their mode shapes by wind,

<u>stairs, spectator stands, pedestrian bridges</u> excited by marching or jumping people. These vibrations are usually not dangerous for the structure itself, but may become very unpleasant for the people,

<u>steel structures</u> **like factory floors excited in one of their natural frequencies by machines , such as screens, centrifuges, fans etc.,**

<u>**ships**</u> **exited in one of their natural frequencies by the main engines or even by ship motion.**

SDOF System



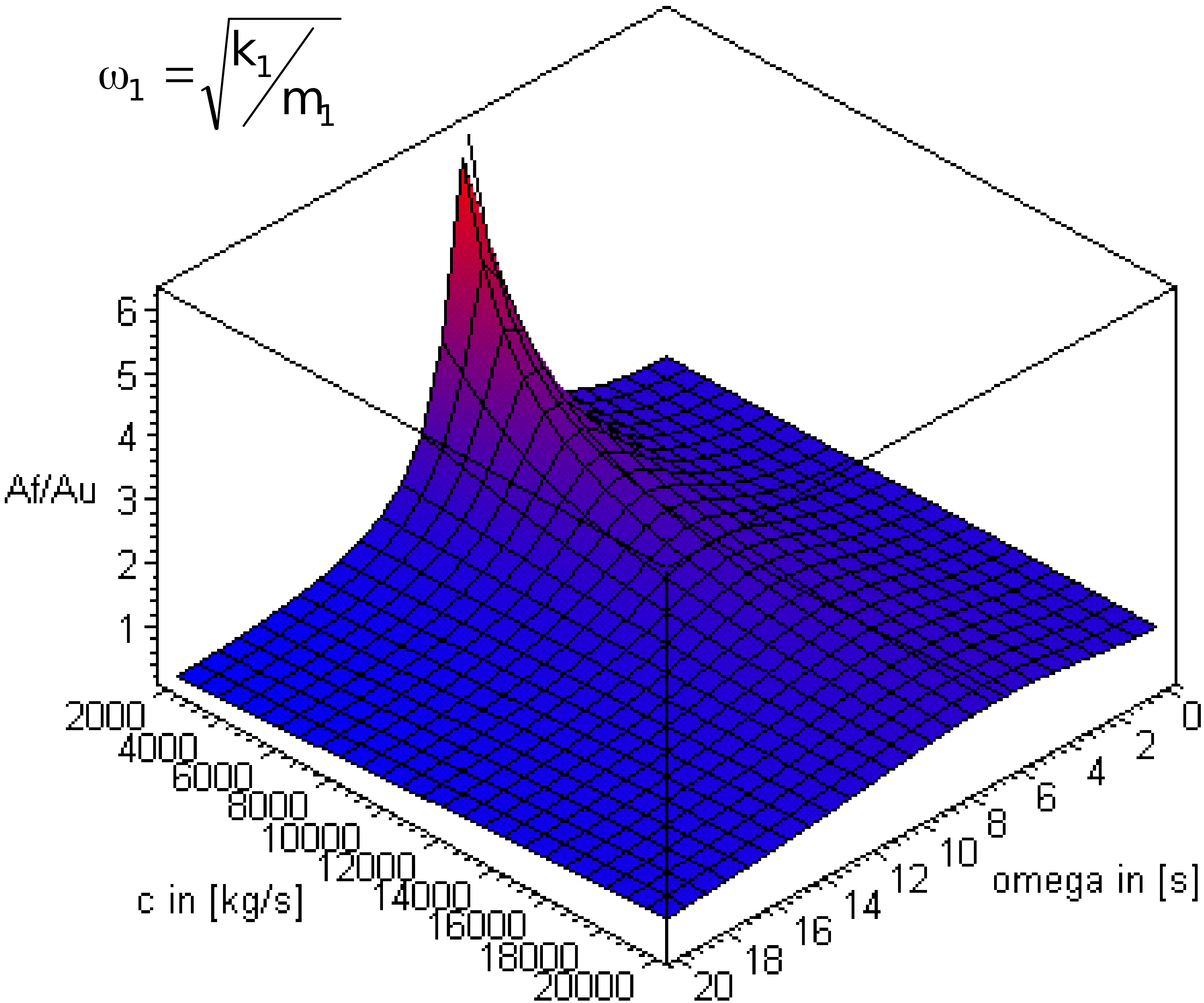
$$u_1 = \frac{p_0}{k_1} \cdot \frac{1}{\sqrt{(1-\beta^2)^2 + (2 \cdot \xi \cdot \beta)^2}} \cdot \sin(\omega \cdot t + \alpha)$$

eigenfrequency: $\omega_1 = \sqrt{k_1 / m_1}$

damping ratio of Lehr: $\xi = \frac{c_1}{2 \cdot m_1 \cdot \omega_1}$

Amplitudenverhältnis in Abhängigkeit von Frequenz und Dämpfung

$$\omega_1 = \sqrt{k_1 / m_1}$$



- Thin structures with low damping have a high peak in their amplification if the frequency of excitation is similar to eigenfrequency
- → High dynamic forces and deformations

Solutions:

- **Strengthen the structure to get a higher eigenfrequency**
- **Application of dampers**
- **Application of tuned mass dampers**

- **Strengthen the structure to get a higher eigenfrequency**

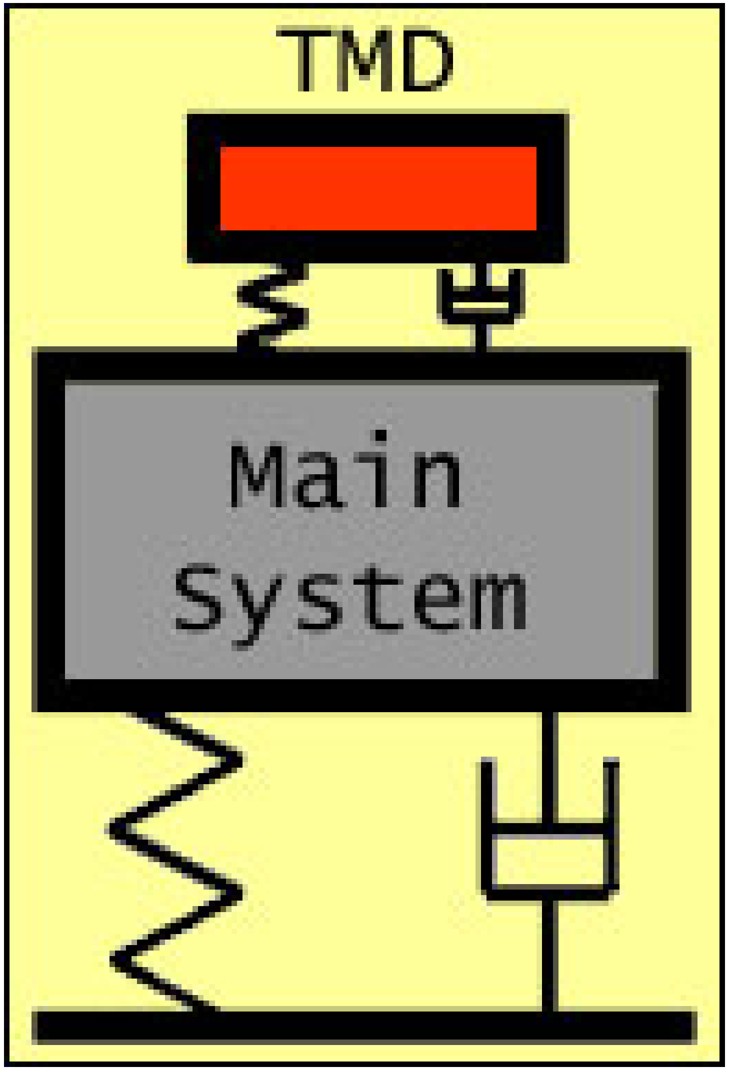
Eigenfrequency of a beam: $f_1 = \frac{\pi^2}{2} \cdot \frac{\pi}{L^2} \sqrt{\frac{EI}{m}}$

Doubling the stiffness only leads to multiplication of the eigenfrequency by about 1.4.

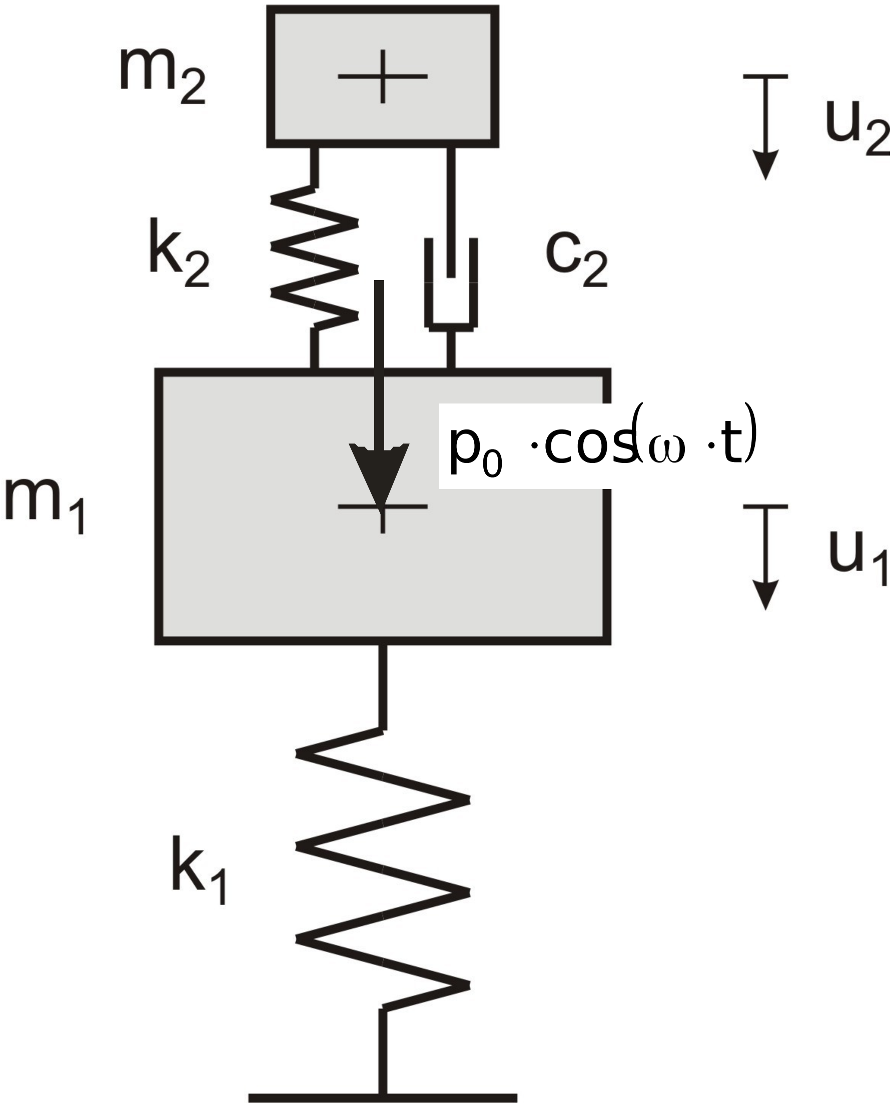
Most dangerous eigenfrequencies for human excitation: 1.8 - 2.4 Hz

•**Application of dampers**

• **Application of tuned mass dampers**



2 DOF System



differential equations:

$$m_1 \cdot \ddot{u}_1 + k_1 \cdot u_1 + k_2 \cdot (u_1 - u_2)$$
$$+ c_2 \cdot (\dot{u}_1 - \dot{u}_2) = p_0 \cdot \cos(\omega \cdot t)$$

$$m_2 \cdot \ddot{u}_2 + k_2 \cdot (u_2 - u_1) + c_2 \cdot (\dot{u}_2 - \dot{u}_1) = 0$$

solution:

$$u_1 = C_1 \cdot \cos(\omega \cdot t) + C_2 \cdot \sin(\omega \cdot t) \qquad u_{1,max} = \sqrt{C_1^2 + C_2^2}$$
$$u_2 = C_3 \cdot \cos(\omega \cdot t) + C_4 \cdot \sin(\omega \cdot t) \qquad u_{2,max} = \sqrt{C_3^2 + C_4^2}$$

linear equation system by derivation of the solution and application to the differential equations:

$$
\begin{aligned}
&C_1(-m_1\omega^2+k_1+k_2) + C_2(c\,\omega) + C_3(-k_2) + C_4(-c\,\omega) = p_o \\
&C_1(-c\,\omega) + C_2(-m_1\omega^2+k_1+k_2) + C_3(c\,\omega) + C_4(-k_2) = 0 \\
&C_1(-k_2) + C_2(-c\,\omega) + C_3(-m_2\omega^2+k_2) + C_4(c\,\omega) = 0 \\
&C_1(c\,\omega) + C_2(-k_2) + C_3(-c\,\omega) + C_4(-m_2\omega^2+k_2) = 0
\end{aligned}
$$

$$C := \left[p0\,(m1\,\omega^2 + k1 + k2) \quad -p0\,c\,\omega \quad -p0\,k2 \quad p0\,c\,\omega \right]$$

$$\frac{u_{1,\max}}{u_{1,stat}} = \sqrt{\frac{4 \cdot \xi^2 \cdot \beta^2 + \left(\beta^2 - \alpha^2\right)^2}{4 \cdot \xi^2 \cdot \beta^2 \left(\beta^2 - 1 + \mu \cdot \beta^2\right)^2 \left[\mu \cdot \alpha^2 \cdot \beta^2 - \left(\beta^2 - 1\right) \cdot \left(\beta^2 - \alpha^2\right)\right]^2}}$$

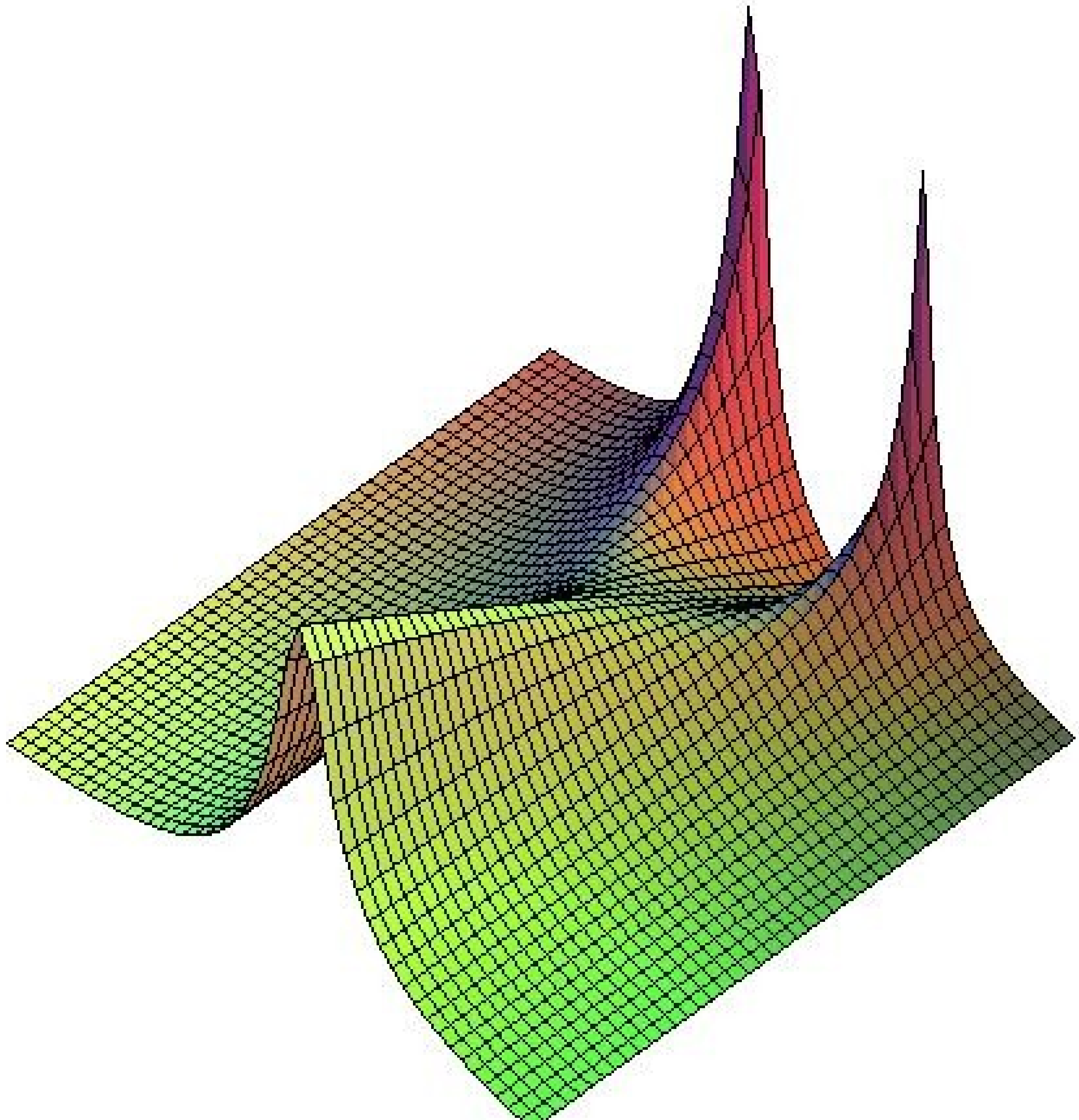
static deformation: $u_{1,stat} = \frac{p_0}{k_1}$

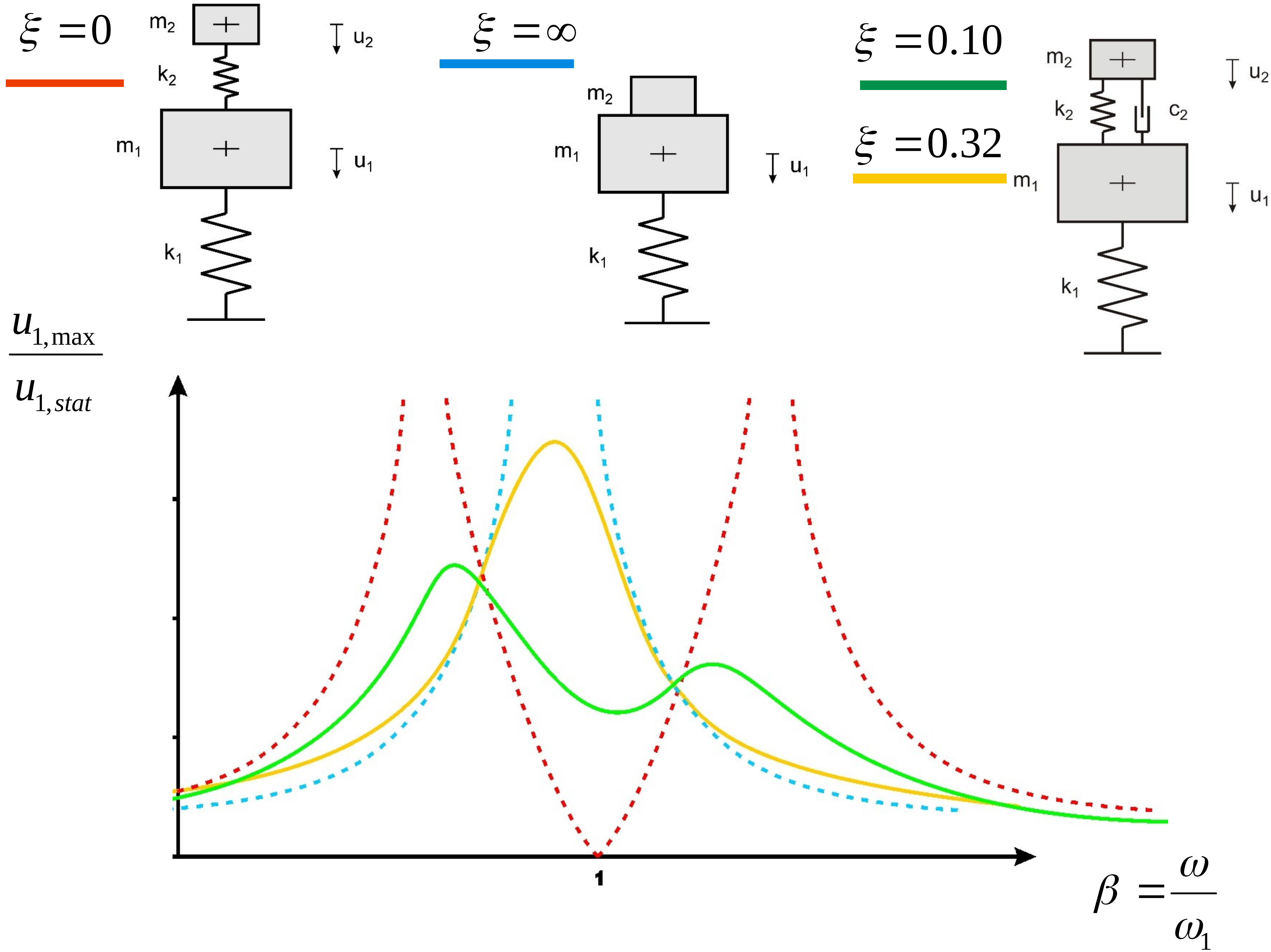
eigenfrequencies: $\omega_1 = \sqrt{\frac{k_1}{m_1}}$ $\omega_2 = \sqrt{\frac{k_2}{m_2}}$

ratio of frequencies: $\alpha = \frac{\omega_2}{\omega_1}$ $\beta = \frac{\omega}{\omega_1}$

Damping ratio of Lehr: $\xi = \frac{c_2}{2 \cdot m_2 \cdot \omega_2}$

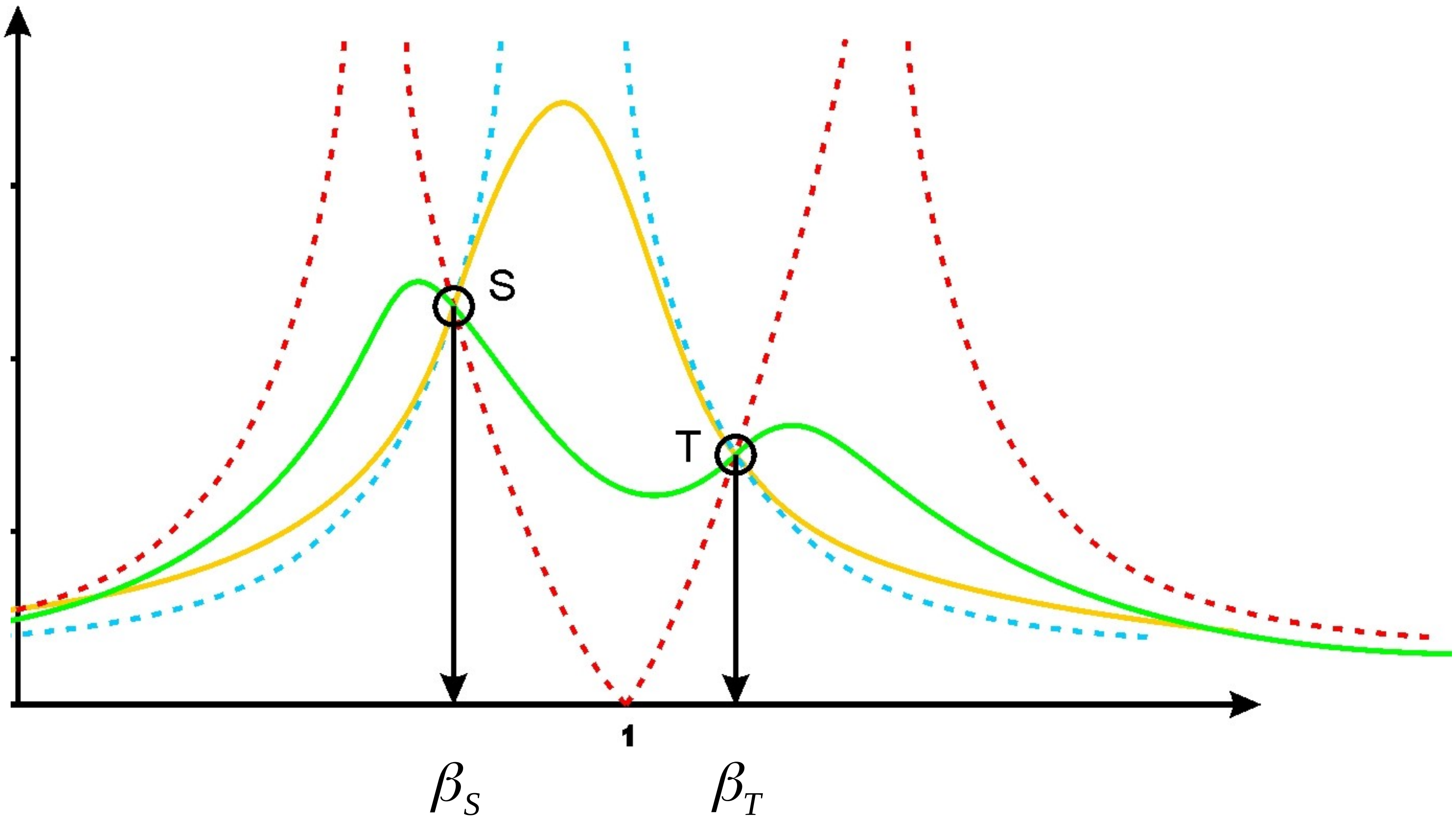
mass ratio: $\mu = \frac{m_2}{m_1}$

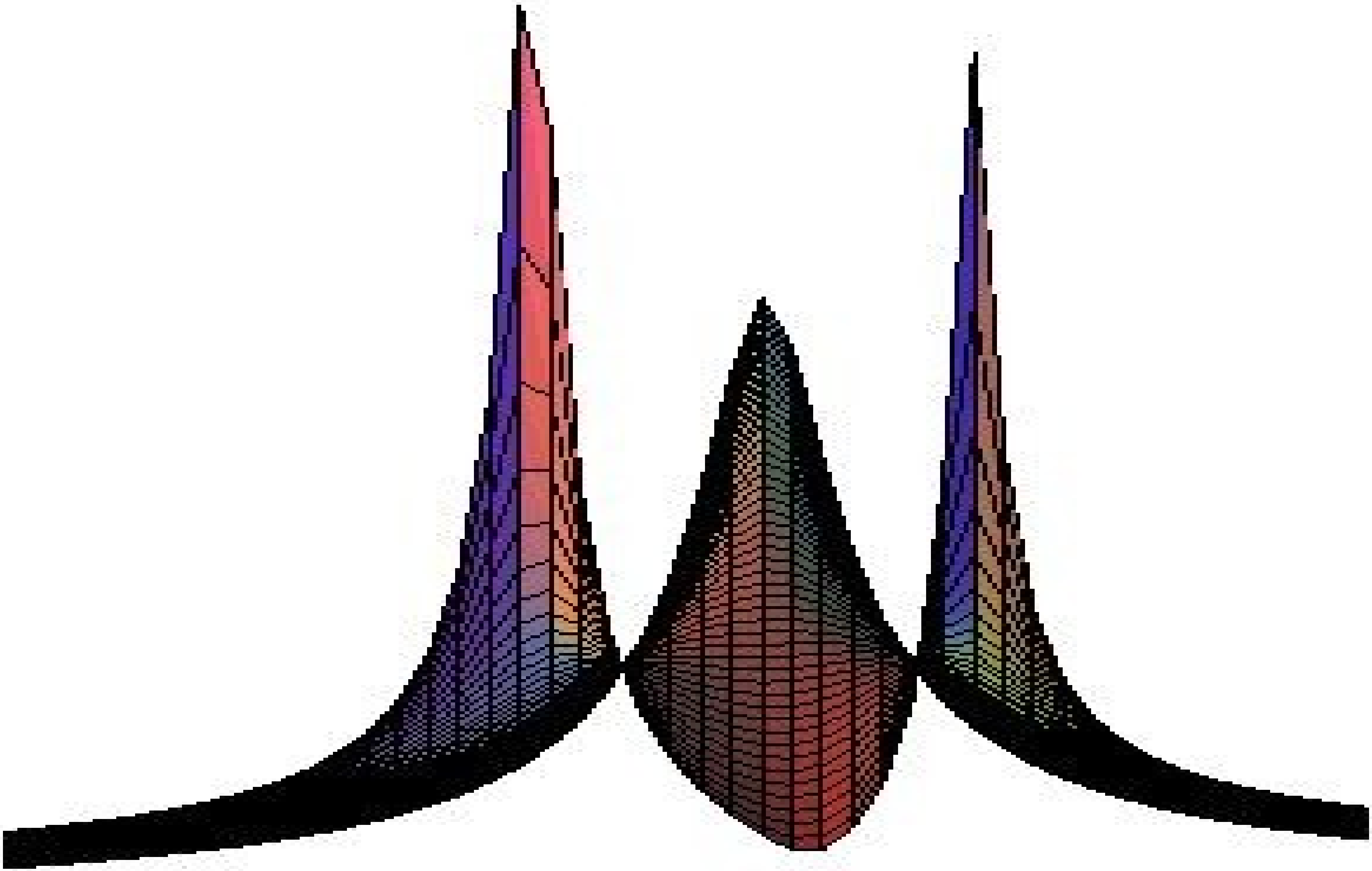




$\xi = 0$
$\xi = \infty$
$\xi = 0.10$
$\xi = 0.32$
m2
m1
k2
k1
c2
u2
u1
$\frac{u_{1,\max}}{u_{1,stat}}$
1
$\beta = \frac{\omega}{\omega_1}$

All lines meet in the points S and T





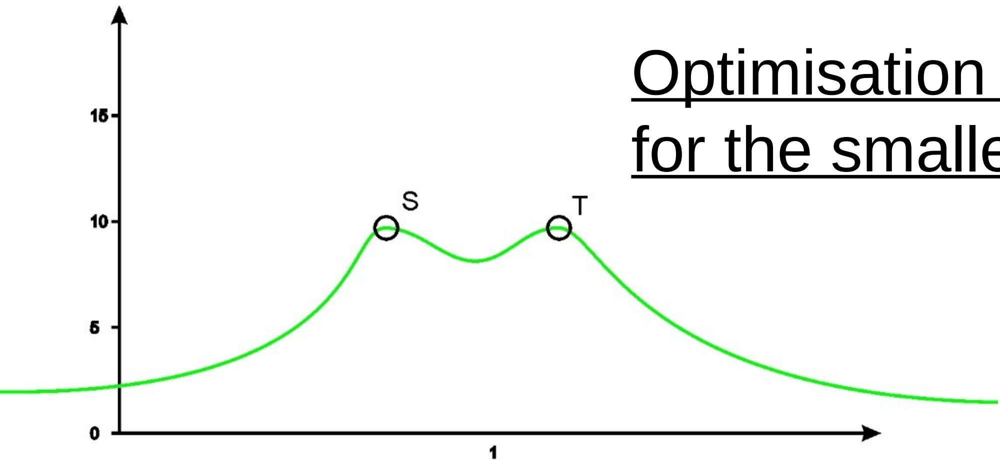
$$\frac{u_1(\xi=0)}{u_{1,stat}}=\frac{u_1(\xi=\infty)}{u_{1,stat}} \quad \rightarrow \quad \frac{\beta^2-\alpha^2}{\mu\alpha^2\beta^2-\left(\beta^2-1\right)\left(\beta^2-\alpha^2\right)}=\frac{1}{\beta^2(1+\mu)-1}$$

$$\rightarrow \quad \beta^4-2\beta^2\frac{1+\alpha^2+\mu\alpha^2}{2+\mu}+\frac{2\alpha^2}{2+\mu}=0$$

$$\rightarrow \quad \beta_{S,T}{}^2=\frac{1+\alpha^2+\mu\cdot\alpha^2}{2+\mu}\pm\sqrt{\left(\frac{1+\alpha^2\cdot(1+\mu)}{2+\mu}\right)^2-\frac{2\cdot\alpha^2}{2+\mu}}$$

$$\text{in} \rightarrow \frac{u1}{u1,stat}\left(\xi=\infty\right): \qquad u_{1,S}=\frac{u_{1,stat}}{\beta_S{}^2-1+\mu\cdot\beta_S{}^2}$$

$$u_{1,T}=\frac{u_{1,stat}}{\beta_T{}^2-1+\mu\cdot\beta_T{}^2}$$



Optimisation of TMD for the smallest deformation:

$$u_{1,S} = u_{1,T}$$

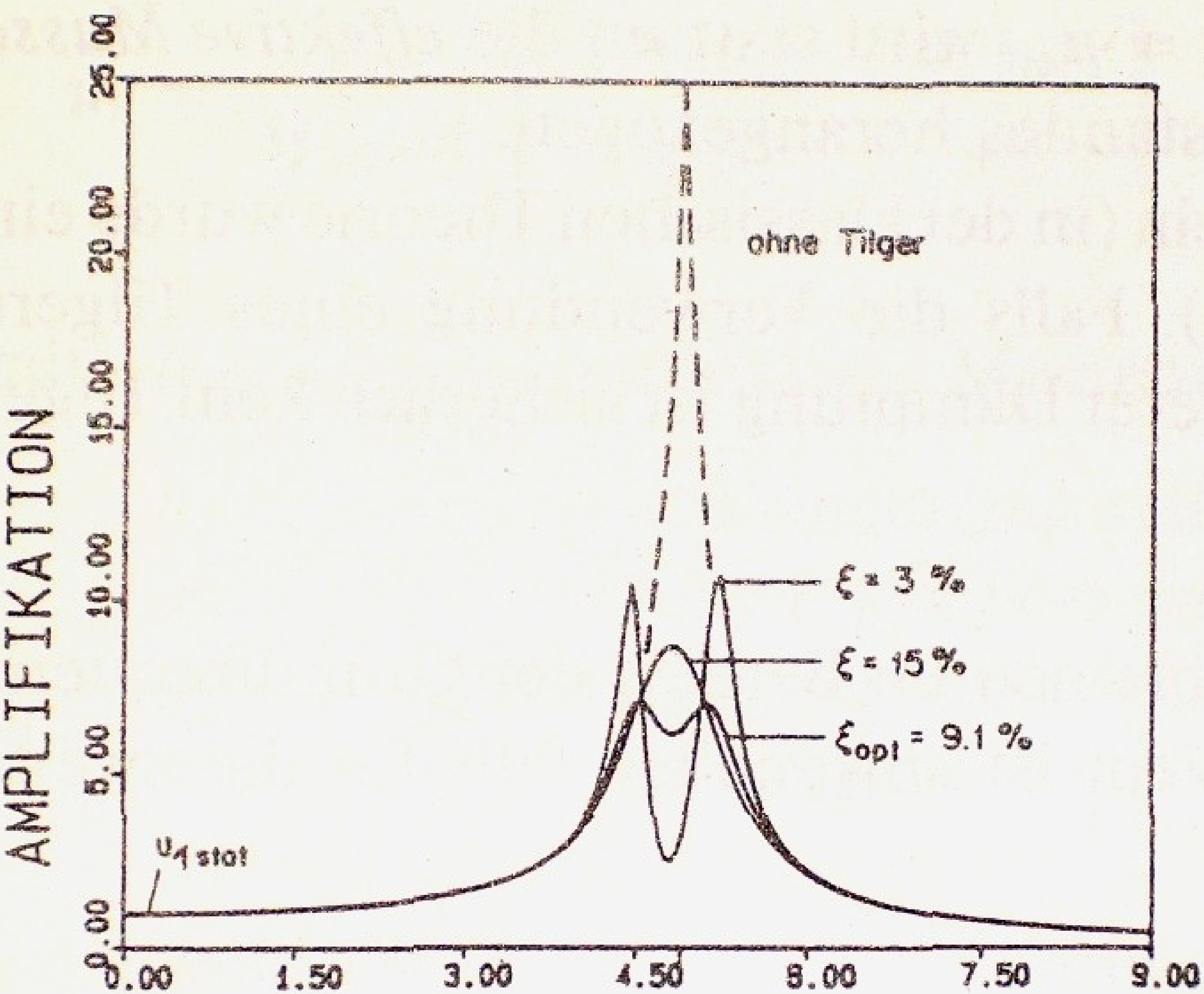
$$\rightarrow \quad \beta_S^2 + \beta_T^2 = \frac{2}{1+\mu}$$

$$\beta_S^2 + \beta_T^2 = 2\frac{1+\alpha^2+\mu\alpha^2}{2+\mu}$$

Optimal ratio of eigenfrequencies: $\alpha = \dfrac{1}{1+\mu}$ → Optimal spring constant k_2

Minimize $u_{1,S} = u_{1,T} = u_{1,stat} \cdot \sqrt{\dfrac{3\cdot\mu}{8\cdot(1+\mu)^3}}$

→ Optimal damping constant: $\xi_{opt} = \sqrt{\dfrac{3\cdot\mu}{8\cdot(1+\mu)^3}}$



AMPLIFIKATION
0.00
5.00
10.00
15.00
20.00
25.00
0.00
1.50
3.00
4.50
6.00
7.50
9.00
ohne Tilger
ξ = 3 %
ξ = 15 %
ξopt = 9.1 %
u1 stat

Ratio of masses: μ

The higher the mass of the TMD is, the better is the damping.
Useful: from 0.02 (low effect) up to 0.1 (often constructive limit)

Ratio of frequencies: α

0.98 - 0.86

Damping Ratio of Lehr: ξ

0.08 - 0.20

<u>**Adjustment:**</u>

- Different Assumptions of Youngs Modulus and Weights

- Increased Main Mass caused by the load

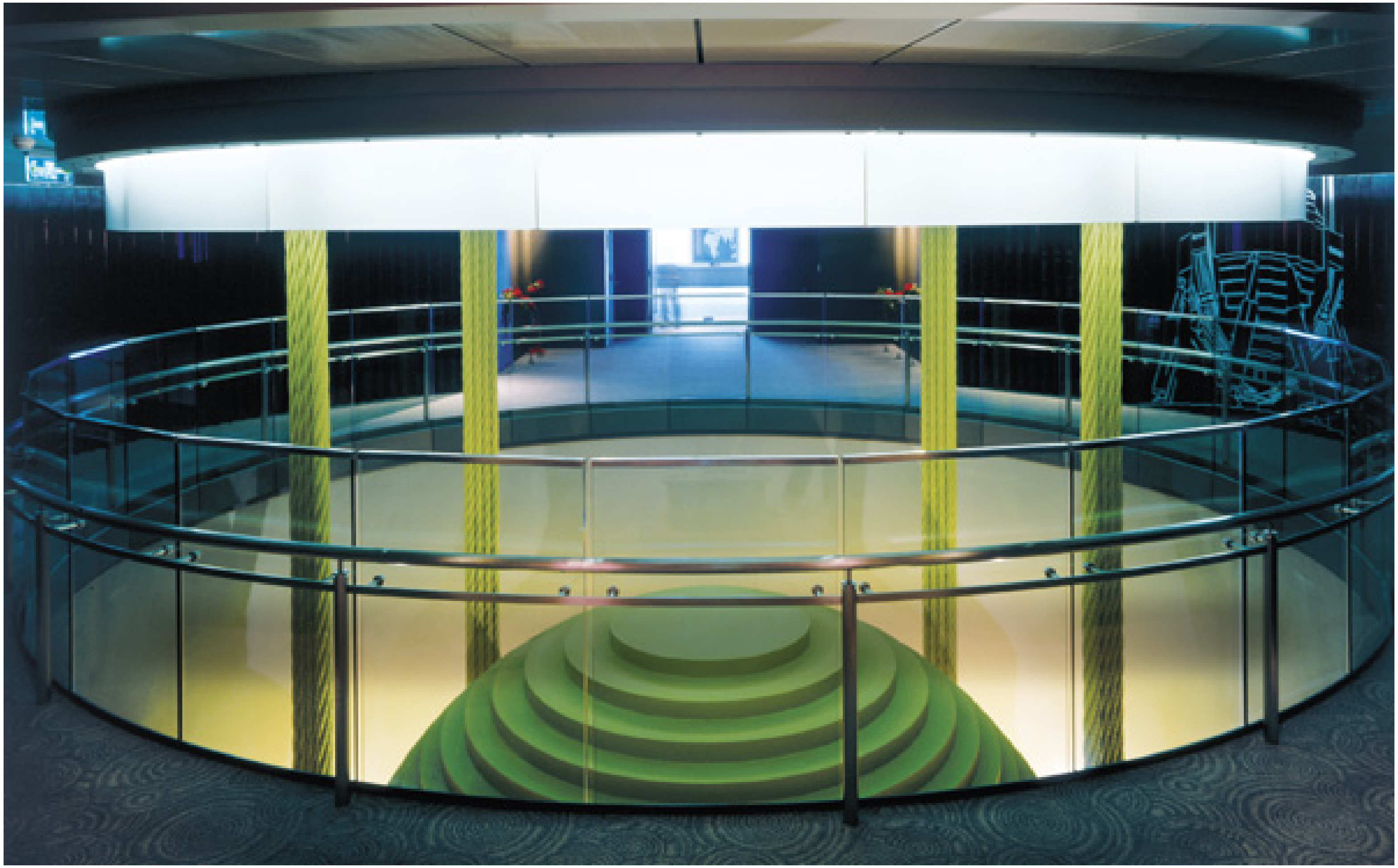
Problem:

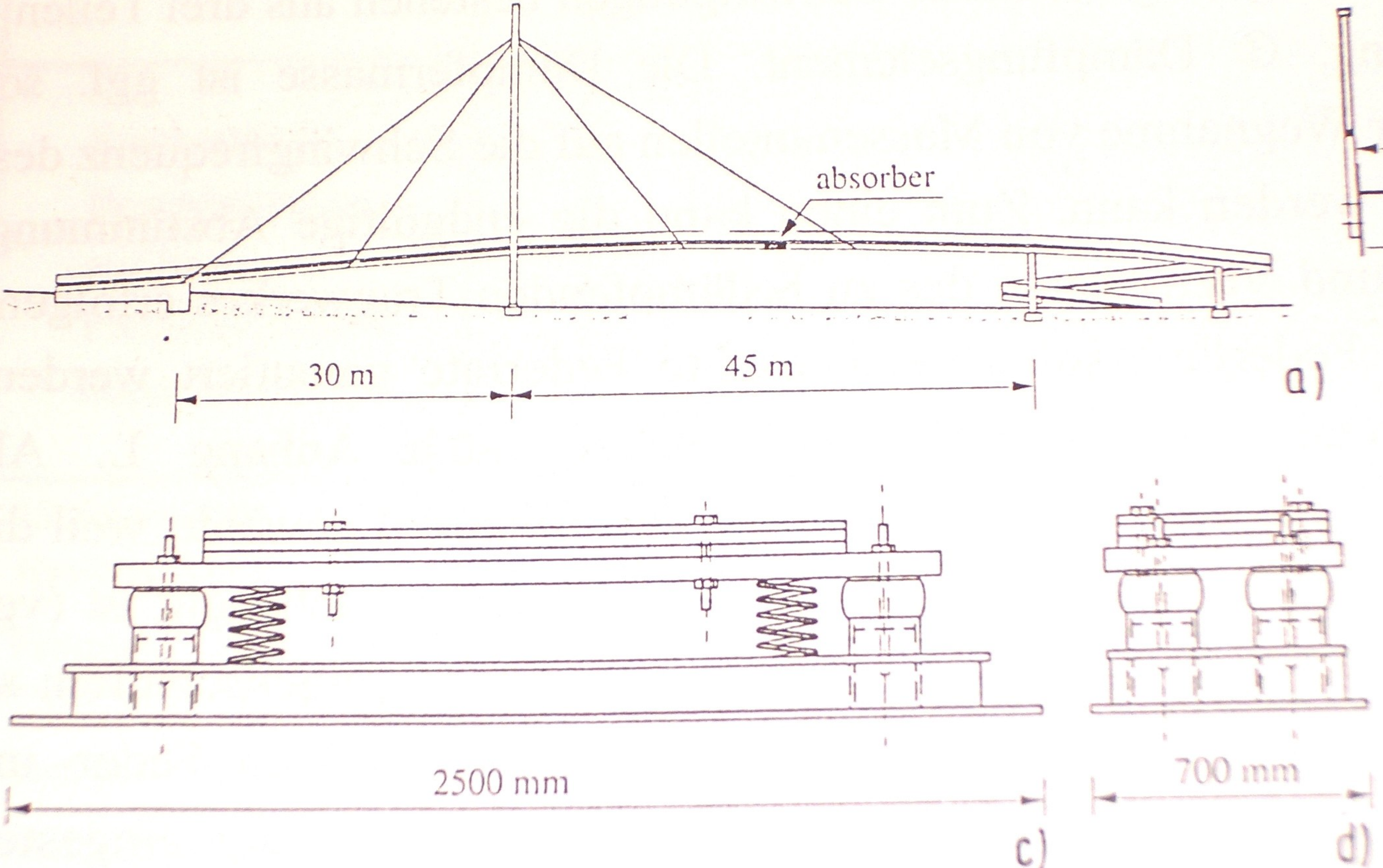
- **Large displacement of the damper mass**

$\rightarrow$ Plastic deformation of the spring

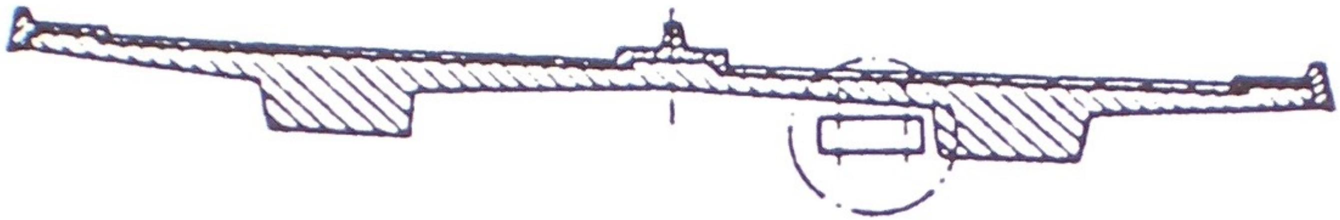
$\rightarrow$ Exceeding the limit of deformation

Realization

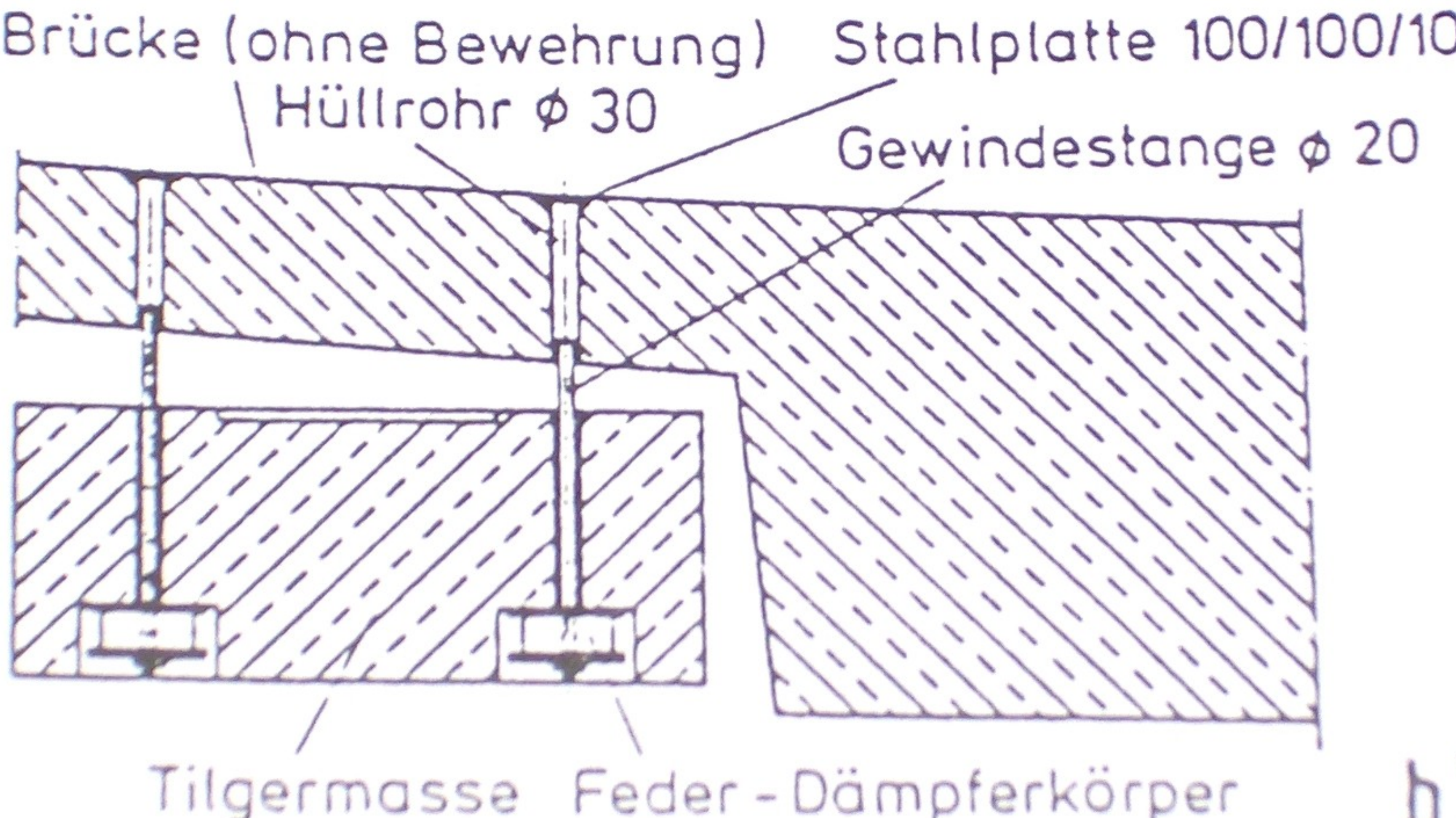



absorber
30 m
45 m
a)
2500 mm
c)
700 mm
d)

damping of torsional oscillation

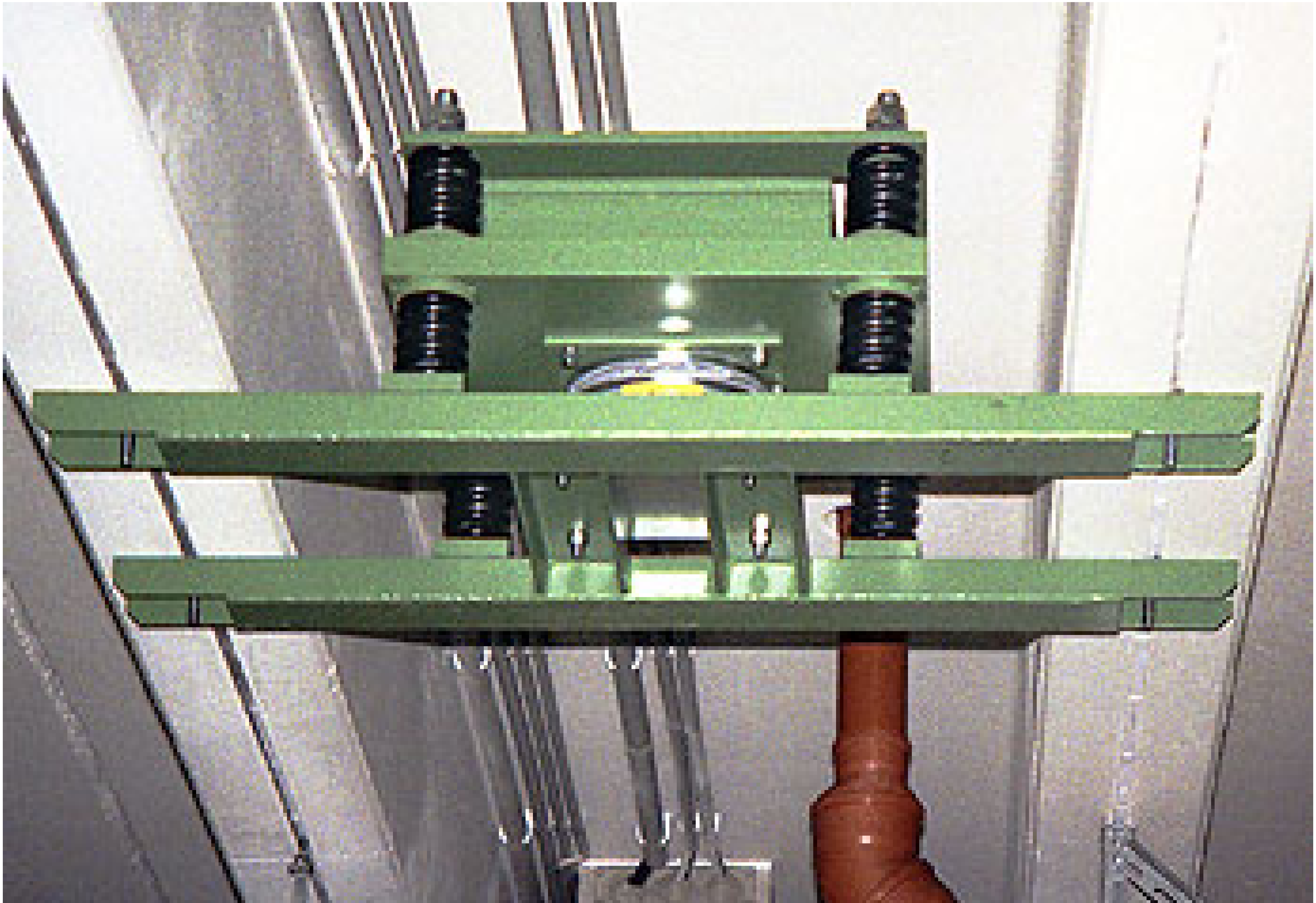


a)



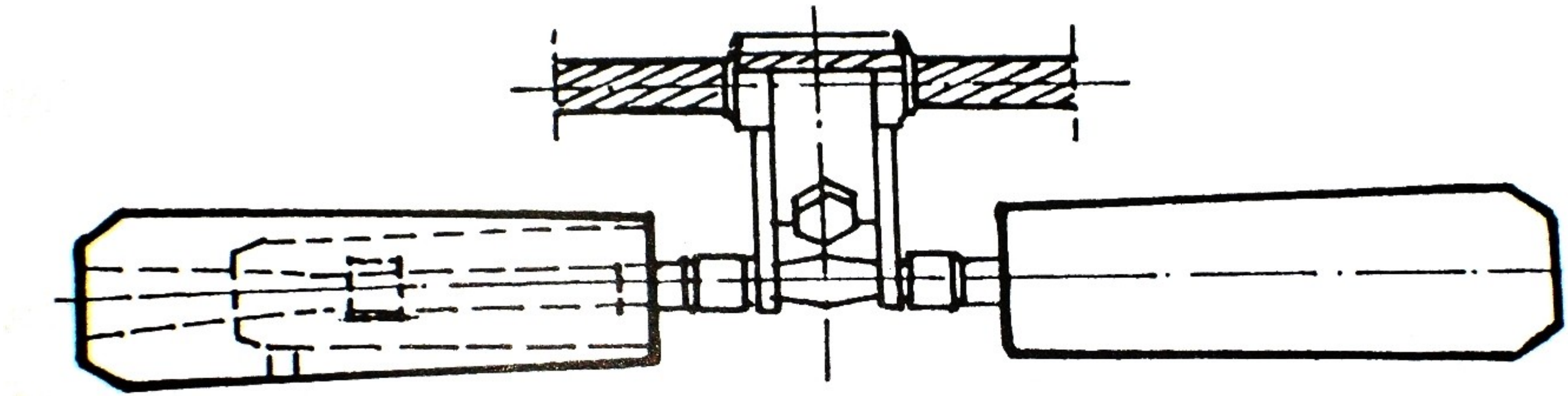
b)

400 kg - 14 Hz

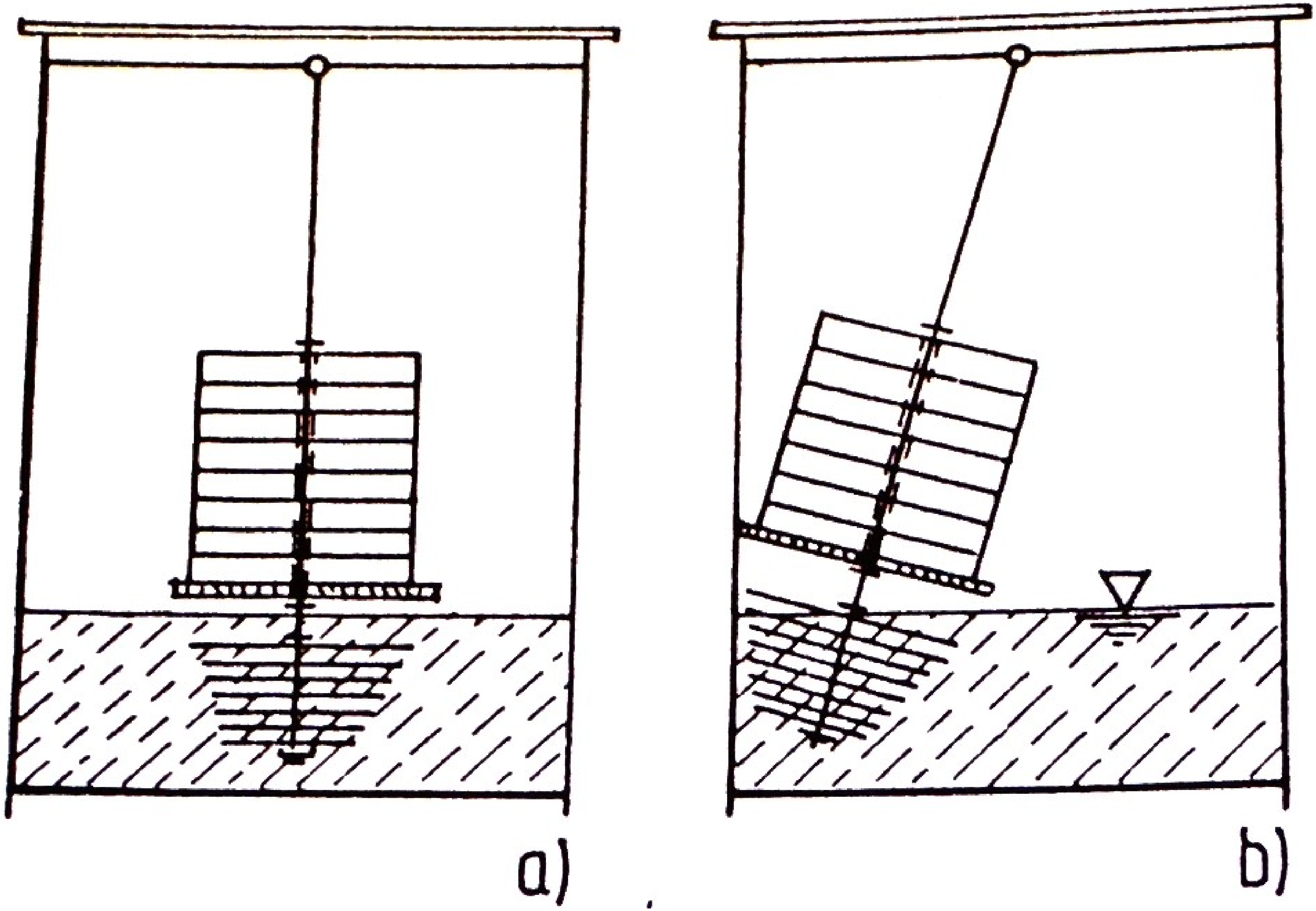


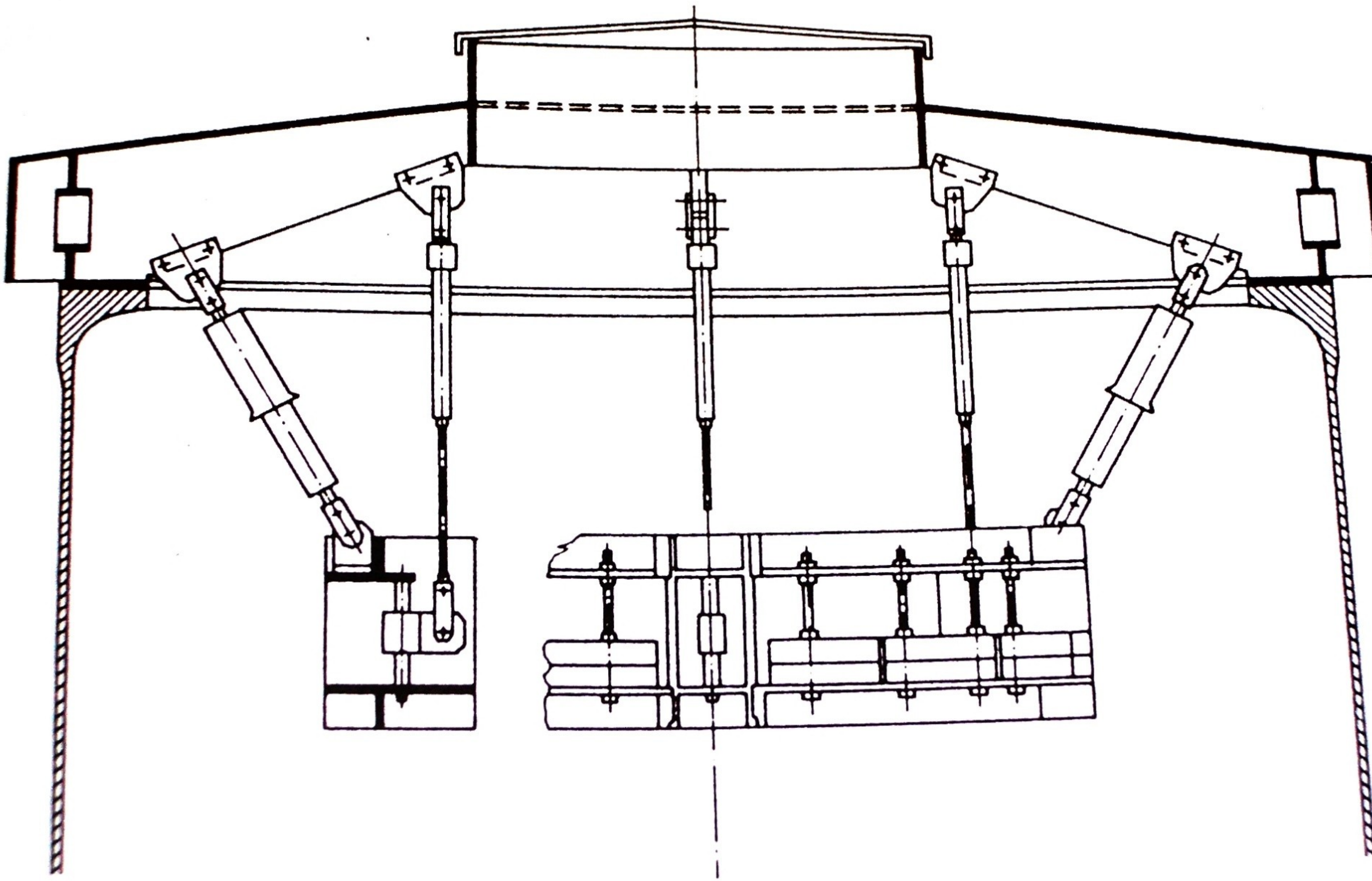
Millennium Bridge

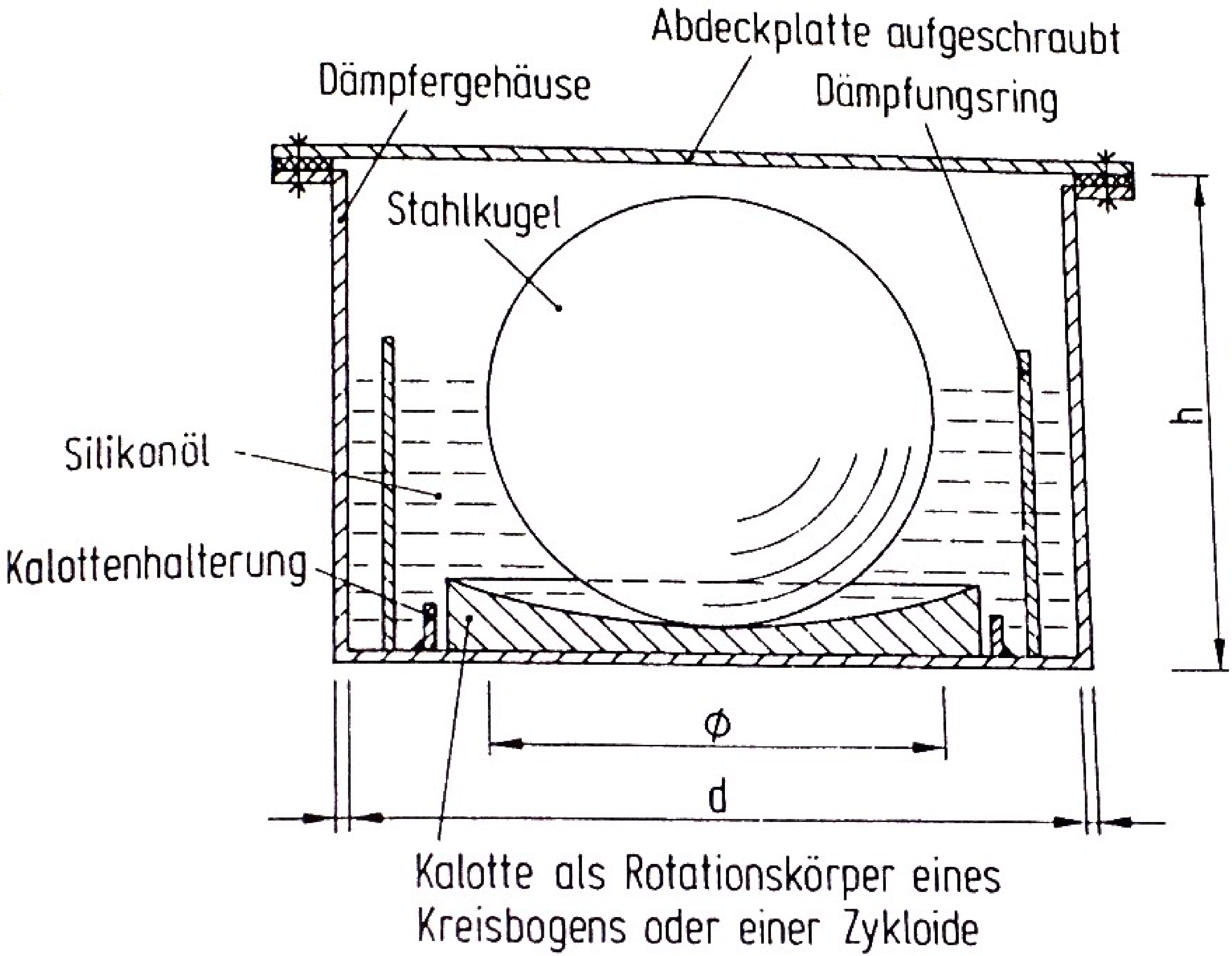
Mass damper on an electricity cable



Pendular dampers






Abdeckplatte aufgeschraubt
Dämpfergehäuse
Dämpfungsring
Stahlkugel
Silikonöl
Kalottenhalterung
h
ϕ
d
Kalotte als Rotationskörper eines
Kreisbogens oder einer Zykloide