

# Mathematical Morphology

**Pengolahan Citra**

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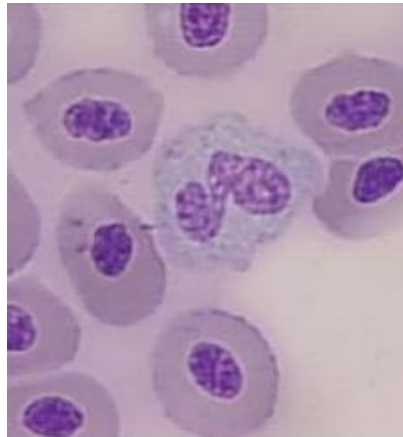
Fakultas Ilmu Komputer Universitas Indonesia

# Morphology

- [Biology] the form and structure of animals and plants.
- [Linguistics] the study of the forms of words.

The study of the forms of things, in particular.

- What about the form and structure of an image?



Shape

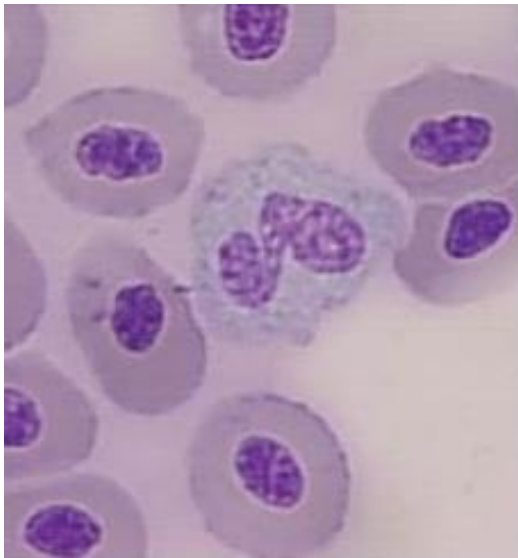
Boundaries

# Morphology for Image Processing

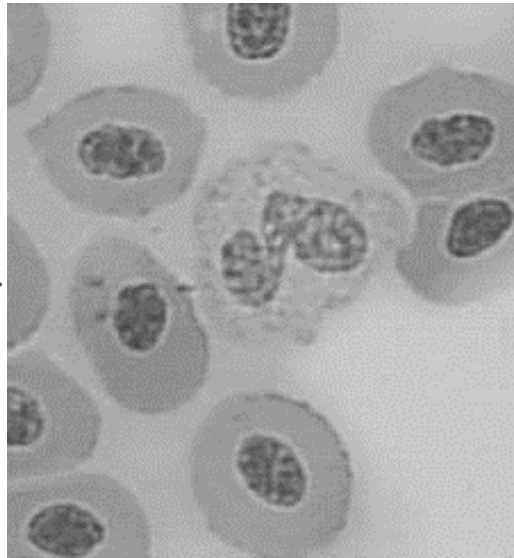
- Mathematical morphology can be used as a tool for extracting image components that are useful in the representation and description of region shape
  - (boundaries, etc.)
- We will use morphological techniques for:
  - image pre-processing
  - intermediate and post-processing
    - segmentation or classification

# Detecting Objects

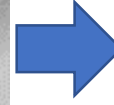
- How many bacteria are in this image?



Original Image



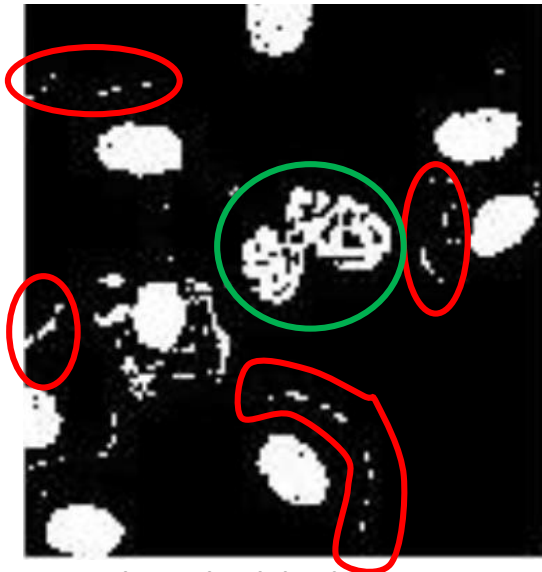
Grayscale Image



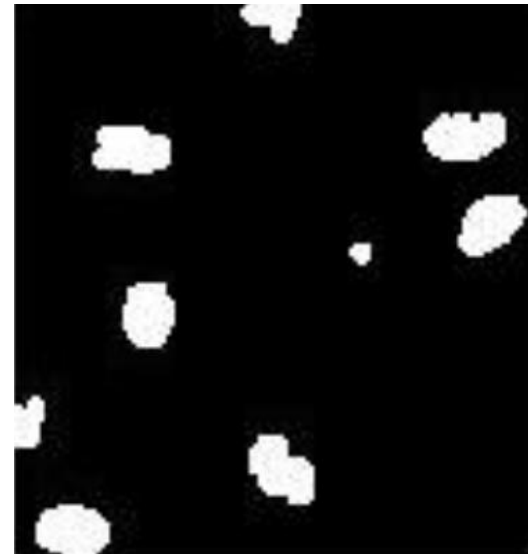
Thresholded Image

# Detecting Objects (2)

- How many bacteria are in this image?

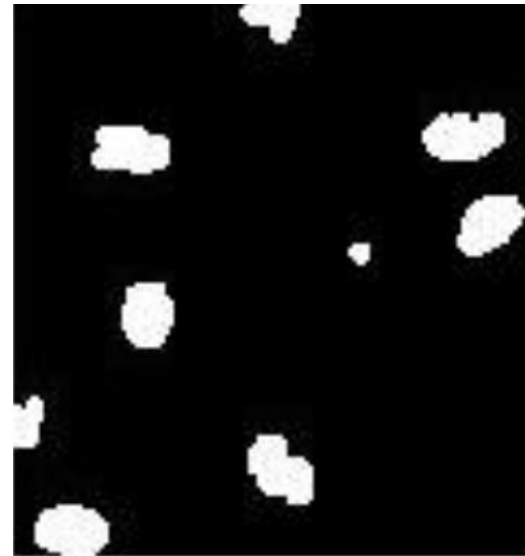


Thresholded Image



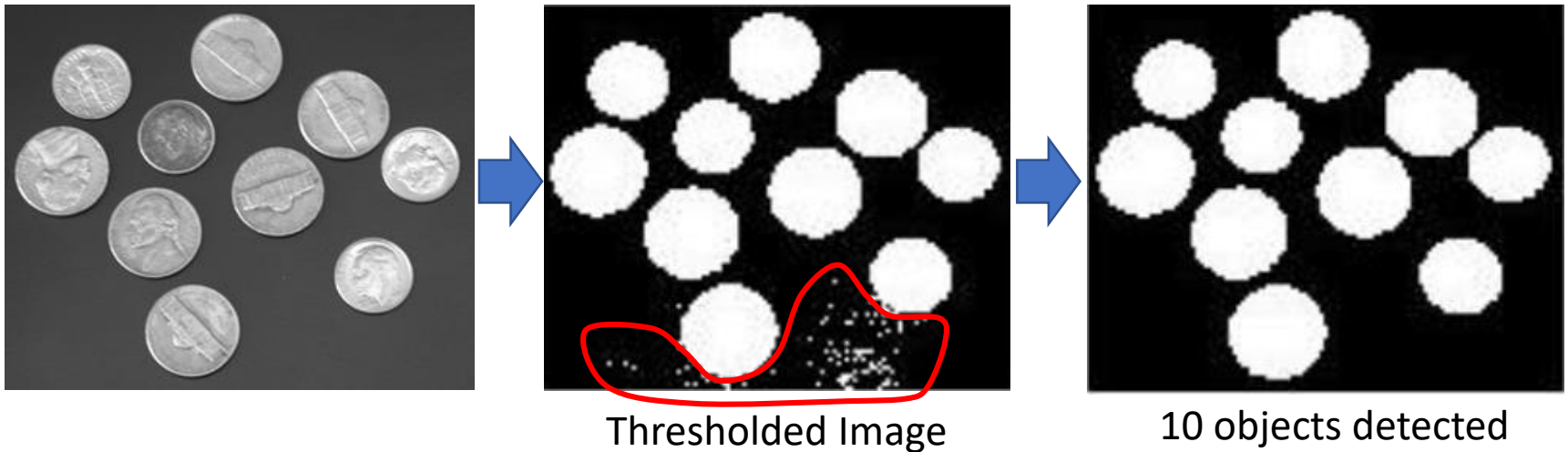
9 objects detected

# Detecting Objects (3)



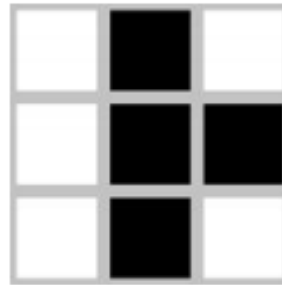
- Too small objects → need to remove them
- 1 object is separated into multiple parts → need to connect these parts

# Another Example

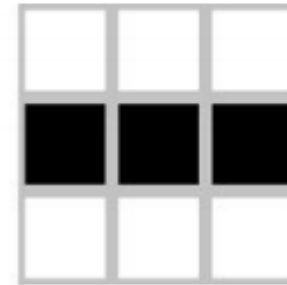


# Basic Set Operations

Input Images

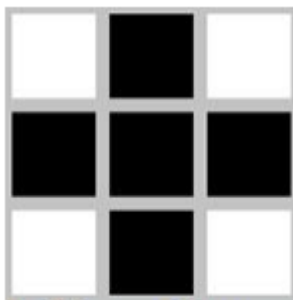


A



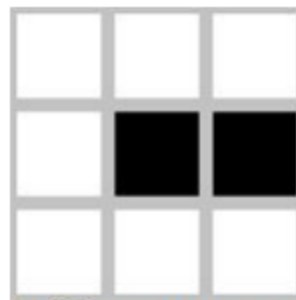
B

Union



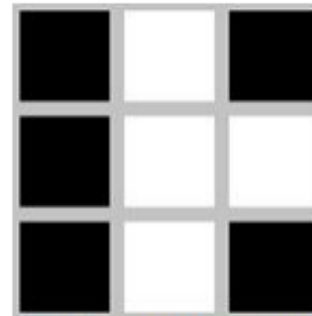
$C = A \cup B$

Intersection



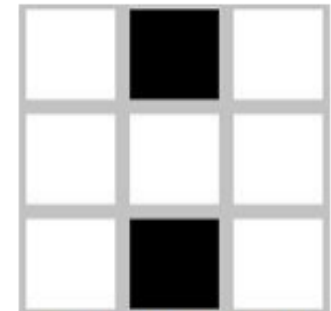
$C = A \cap B$

Complement



$C = A^c$

Difference



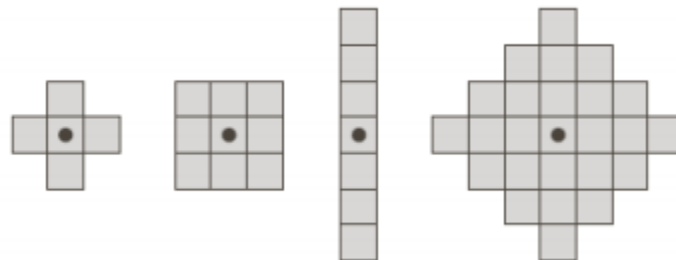
$C = A \setminus B$

Can't we do this also with a larger number of pixels?



# Structural Elements

- Morphology formulates operations based on *structural elements* (SE/ strel)
- *Structural elements*: Small sets or subimages to probe an image for properties / forms of interest
- A structural element is much smaller than the image.
- The shape of the structured element is arbitrary, as long as it can be represented as a binary image of a given size.
  - Ex: buildings may have an elongated-square-shape strel



# Effects of Different Structural Elements



# Erosion

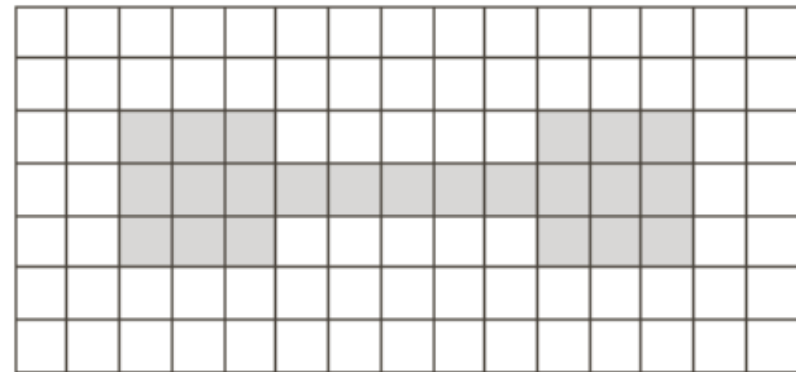
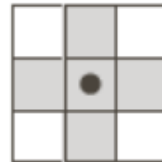
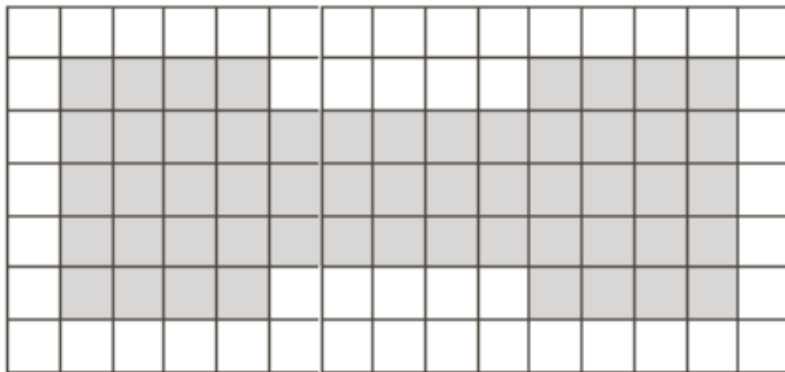
- With  $A$  and  $B$  sets in  $Z^2$ , the erosion of  $A$  by  $B$  is

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

- The structural element, SE, is applied to all pixels of the image
- The pixel is turned on if the entire structural element falls with foreground area (pixels).
- $B$  must be contained in  $A$ , hence  $B$  doesn't share components with the background

$$A \ominus B = \{z | (B)_z \cap A^c = \emptyset\}$$

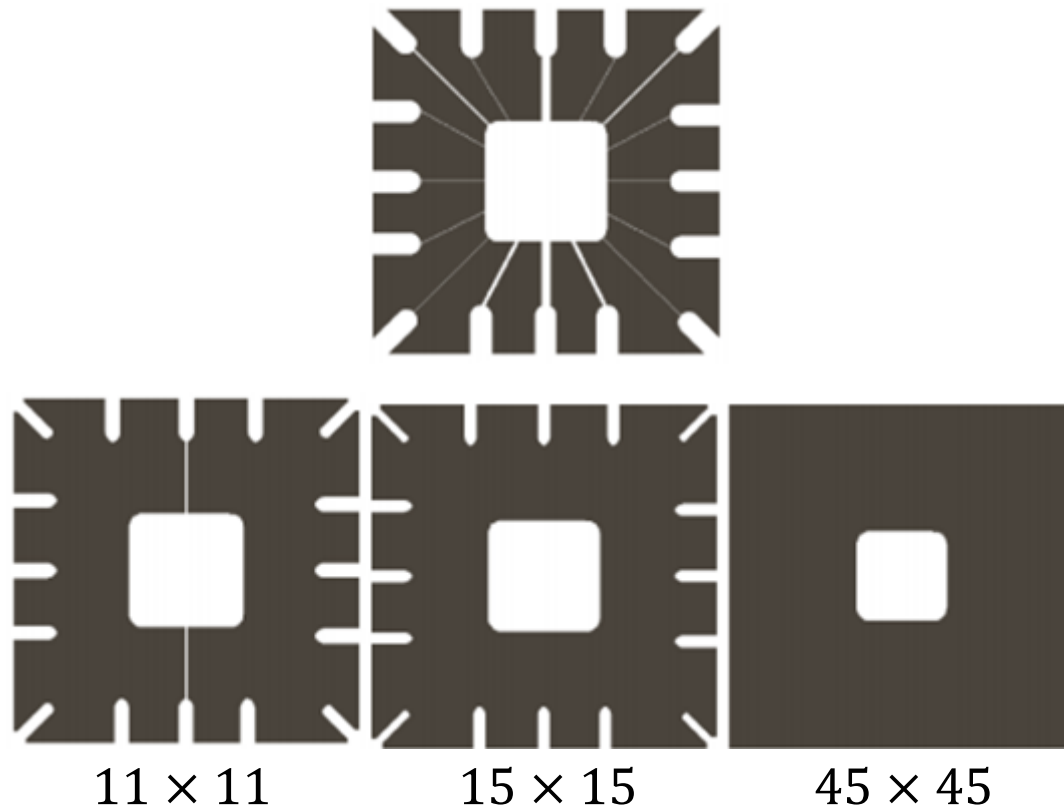
# Erosion (2)



- What does erosion do?
  - Erodes the image based on the given SE
  - Makes the region smaller based on the given SE
  - Deletes smaller regions insignificant to the SE

# Erosion (3)

- Erosion by a square strel



# Dilation

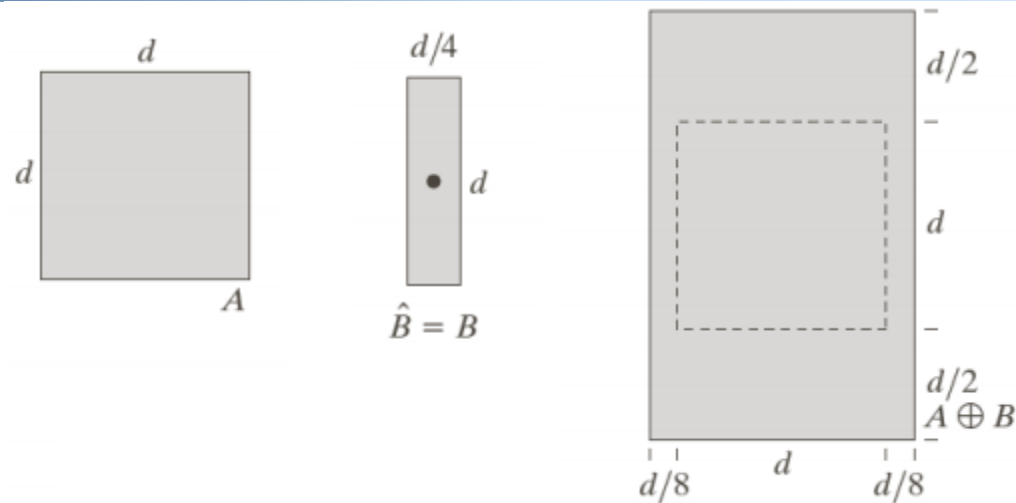
- With  $A$  and  $B$  sets in  $Z^2$ , the dilation of  $A$  by  $B$  is

$$A \oplus B = \{z | [(\hat{B})_z \cap A] \subseteq A\}$$

- The structural element, SE, is applied to all pixels of the image
- The pixel is turned on if at least 1 pixel of the structural element falls with foreground area (pixels).

$$A \ominus B = \{z | (\hat{B})_z \cap A^c \neq \emptyset\}$$

# Dilation (2)

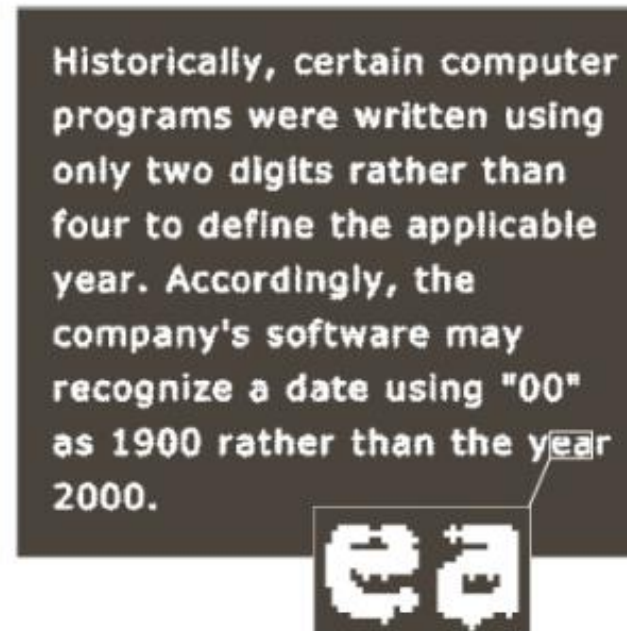
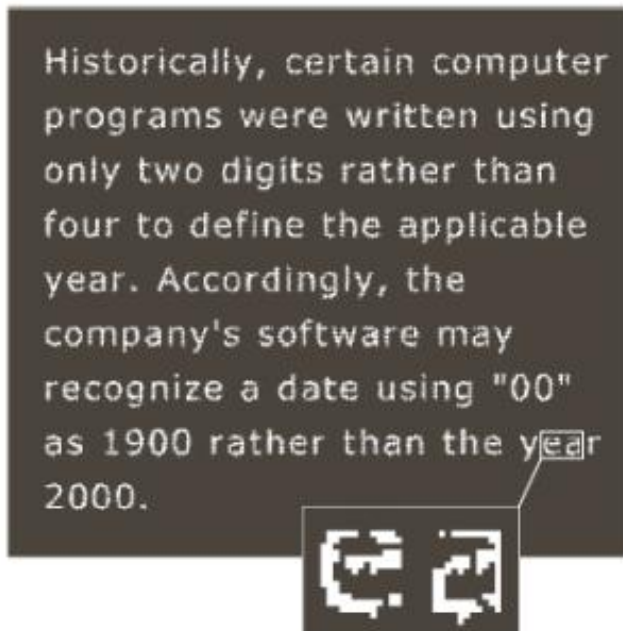


- What does dilation do?
  - Dilates the image based on the given SE
  - Makes the region larger based on the given SE
  - Merge smaller regions whose distance is insignificant to the SE

# Dilation (3)

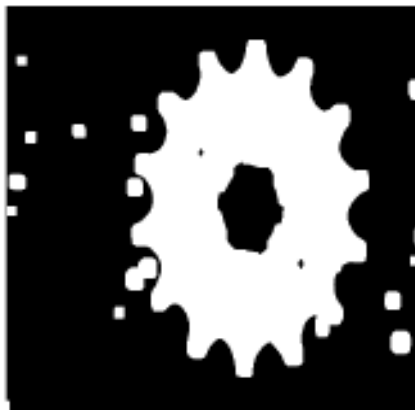
- Dilation by a strel

0	1	0
1	1	1
0	1	0

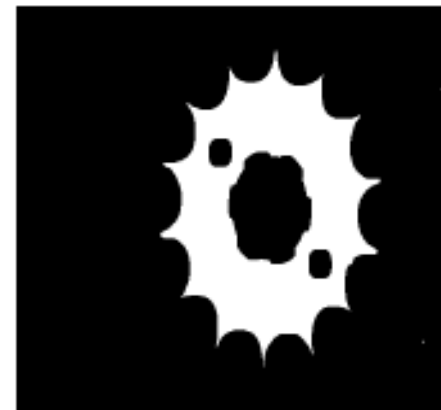




# Erosion vs Dilation



dilation



erosion

# Image Enhancement

- Just as one would use both a low pass filter (smoothing) to eliminate 'salt and pepper' noise and **also** a high pass filter to enhance edges (sharpening)
- We can also do this with morphology operators
  - Dilation and erosion?
  - Erosion and dilation?

# Opening

$$f \circ b = (f \ominus b) \oplus b$$

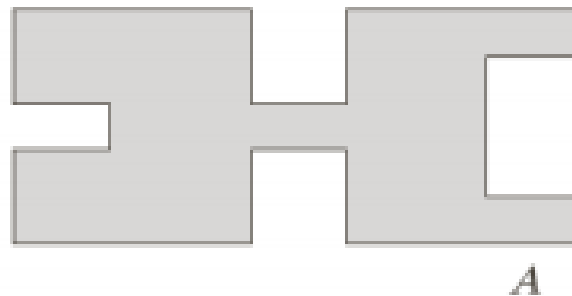
- Erosion followed by a dilation using the same structuring element
- The erosion
  - removes small details
  - darkens the image
- The dilation
  - increases the overall intensity
- The opening-region image result
  - a collection of foreground parts (objects of interest) that fit a particular structuring element

# Closing

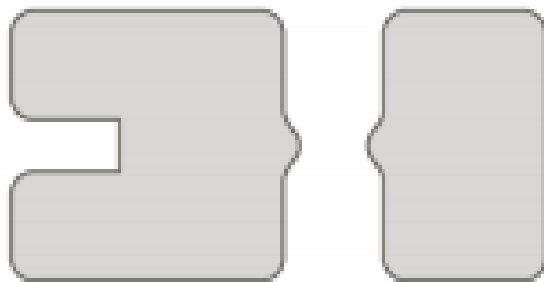
$$f * b = (f \oplus b) \ominus b$$

- Dilation followed by erosion using the same structuring element
- The dilation
  - removes dark details
  - brightens the image,
- The erosion
  - darkens the image
- The closing-region image
  - a collection of background parts that fit a particular structuring element

# Opening and Closing

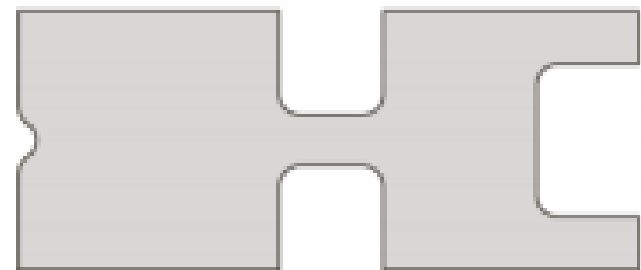


- Using a small circular SE



$$A \circ B = (A \ominus B) \oplus B$$

Opening



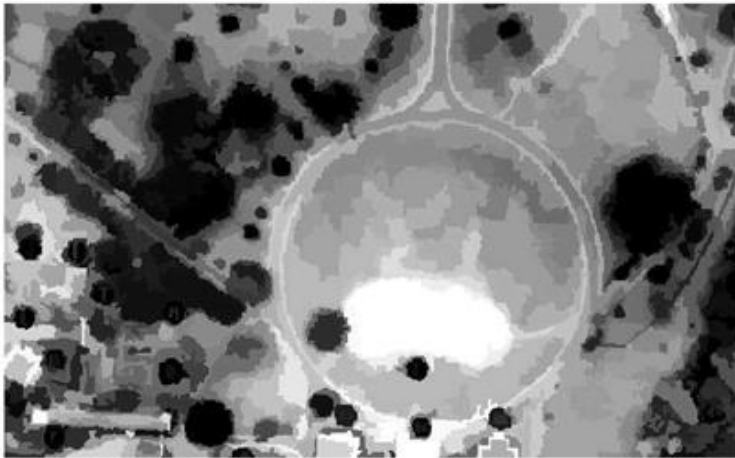
$$A \bullet B = (A \oplus B) \ominus B$$

Closing

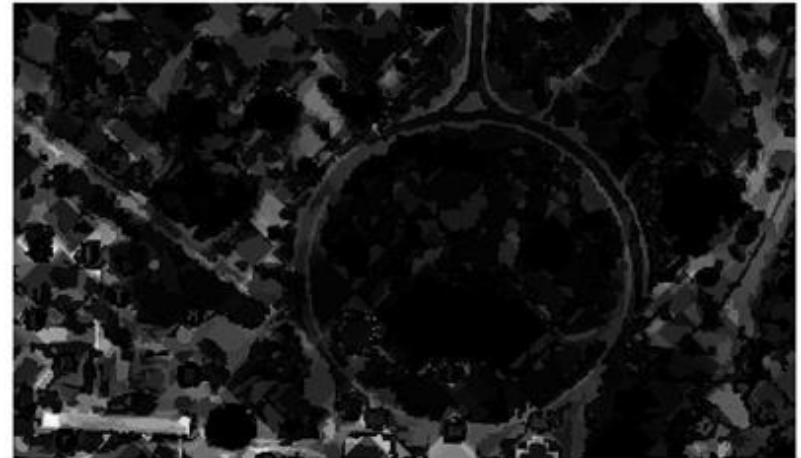
# Top-hat Filtering

$$f - (f \circ b)$$

- Top-hat filtered image:
  - contains the 'peaks' of objects that fit a particular structuring element



Original Image

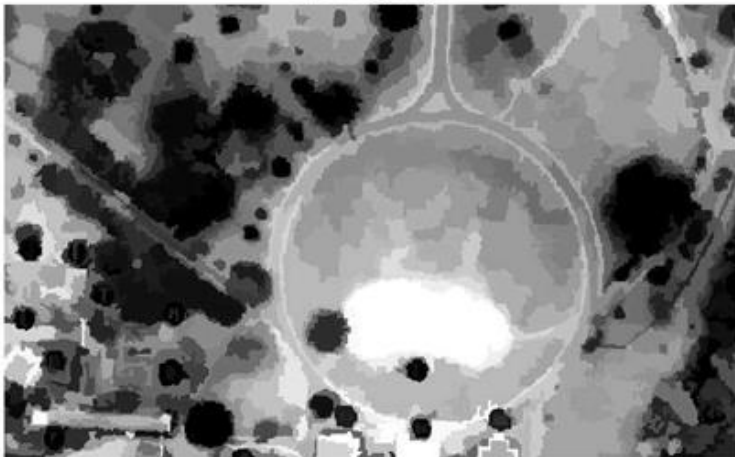


Top-hat filtered image

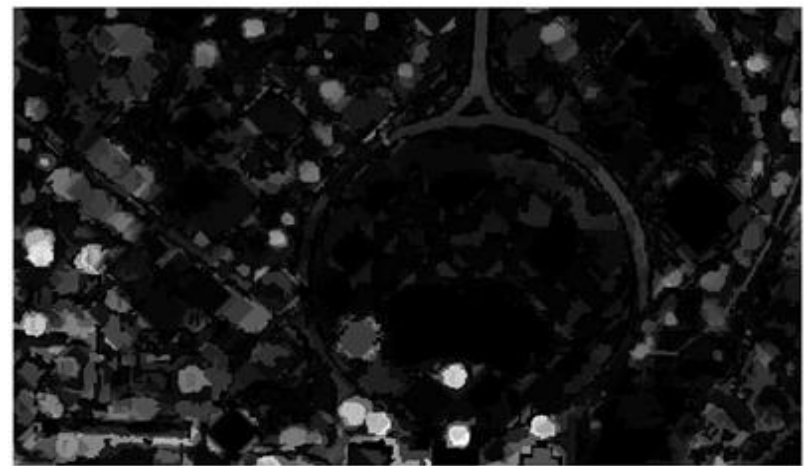
# Bottom-hat Filtering

$$(f * b) - f$$

- Bottom-hat filtering:
  - Bottom-hat filtered image: contains the gaps between the objects of interest



Original Image

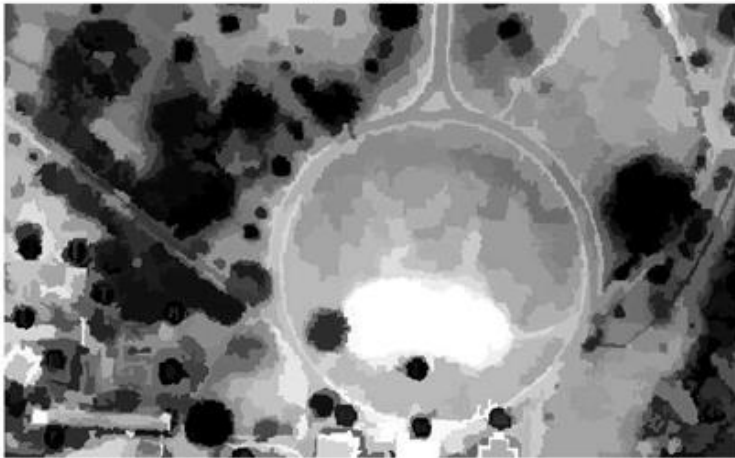


Bottom-hat filtered image

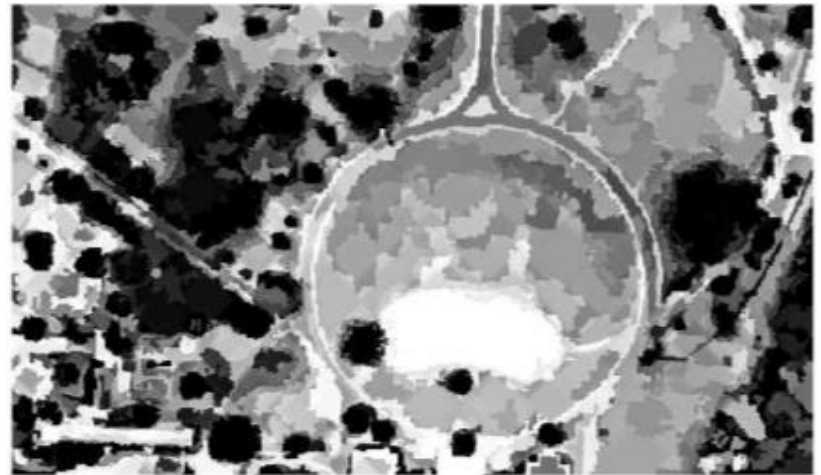
# Contrast Enhancement

$$3f - (f \circ b) - (f * b)$$

- Maximizes the contrast between the objects and the gaps (original + top-hat filtered – bottom-hat filtered)



Original Image



Contrast Enhanced Image



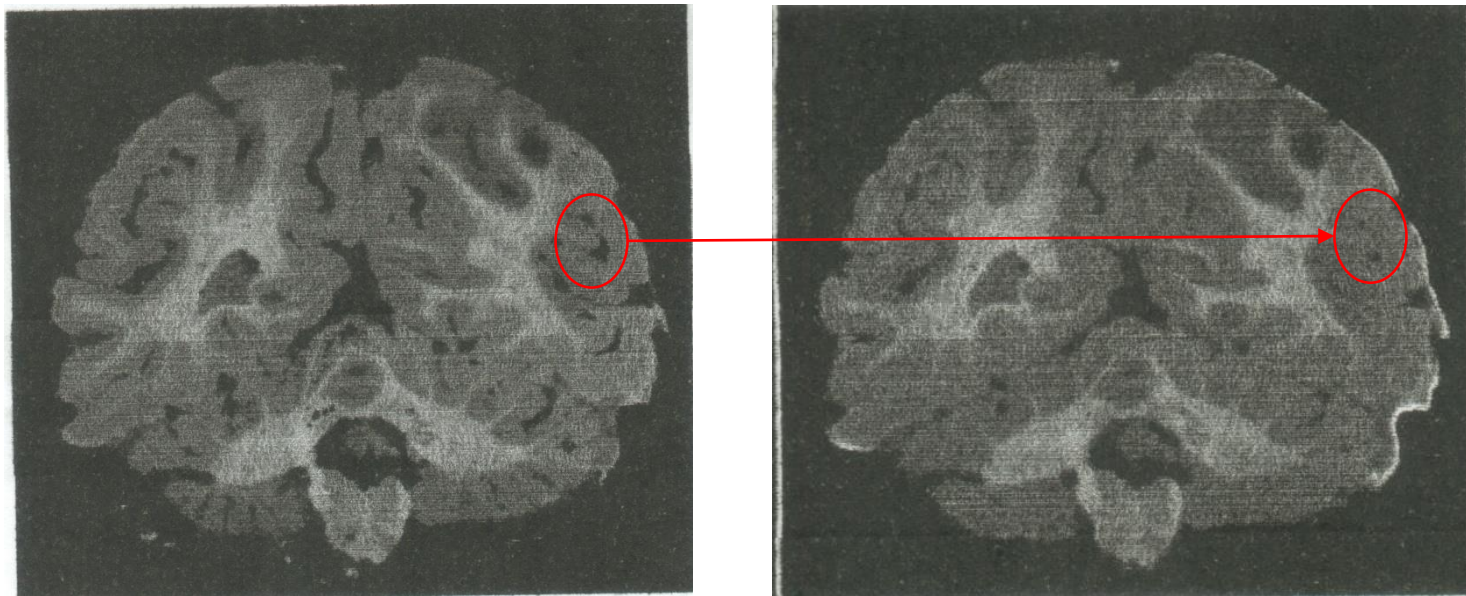
# Boundary extraction

$$\beta(A) = A - (A \ominus B)$$



# Dilation pada Medical Image

- 9x9 circular structuring elements
- Mengisi rongga di dalam brain dan menghaluskan kontur



# Additional Topics

- Region Growing
- Watershed Algorithm