

# Image Restoration

**Pengolahan Citra**

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# Review

- We have previously studied **image enhancement** techniques.
- Image enhancement.. aims to improve the image heuristically until we achieve a result that is **visually pleasing**.
- We will now consider **image restoration**
- Image restoration.. attempts to recover a degraded image using **prior knowledge** of the distortion.

# Restoration vs Enhancement

- Image enhancement dilakukan secara heuristic, **trial and error**, sampai diperoleh image yang sudah **baik menurut kita**
- In image restoration usually **we know the model of distortion**



Image Enhancement



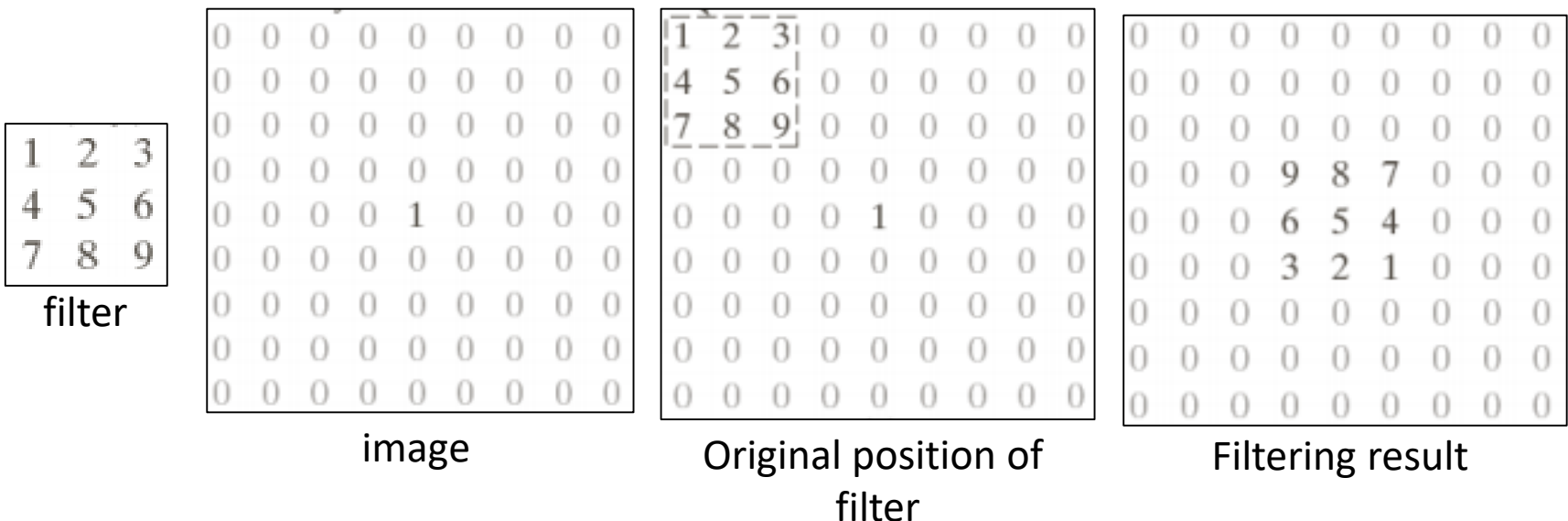
Image Restoration

# Before we continue....

- Convolution vs Correlation
  - Are both used in spatial and frequency filtering
  - Are not the same thing..

# Correlation

- The process of moving a filter mask over the image and computing the sum of products at each location.
  - Slide filter  $g(x)$  by image  $f(x)$

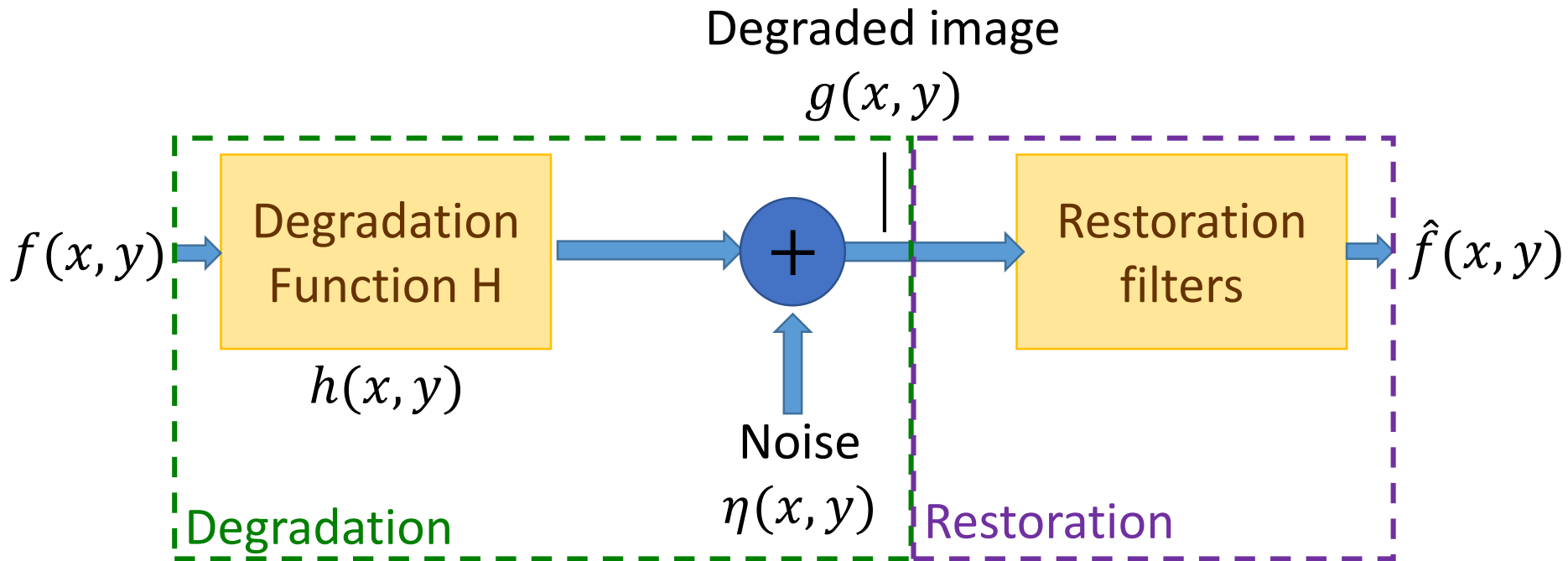


# Convolution

- Rotating a filter mask by 180° and moving it over the image and computing the sum of products at each location.
  - Flip filter  $g(x)$
  - Slide filter  $g(x)$  by image  $f(x)$



# Model of Image Degradation



# Model of Image Degradation (2)

- Spatial domain

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

- If the degradation function  $h$  is linear, and the noise is spatially independent

- Frequency domain

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

- $g/G(x, y)$  : degraded image
- $h/H(x, y)$  : degradation function
- $f/F(x, y)$  : original image
- $\eta/N(x, y)$  : noise



# Which Domain to Use?

	<b>Spatial Restoration</b>	<b>Frequency Restoration</b>
Type of Noise	Additive	Blur
Solution	Spatial Masks	Filter Masks

# Restoring Images based on Noise Model

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

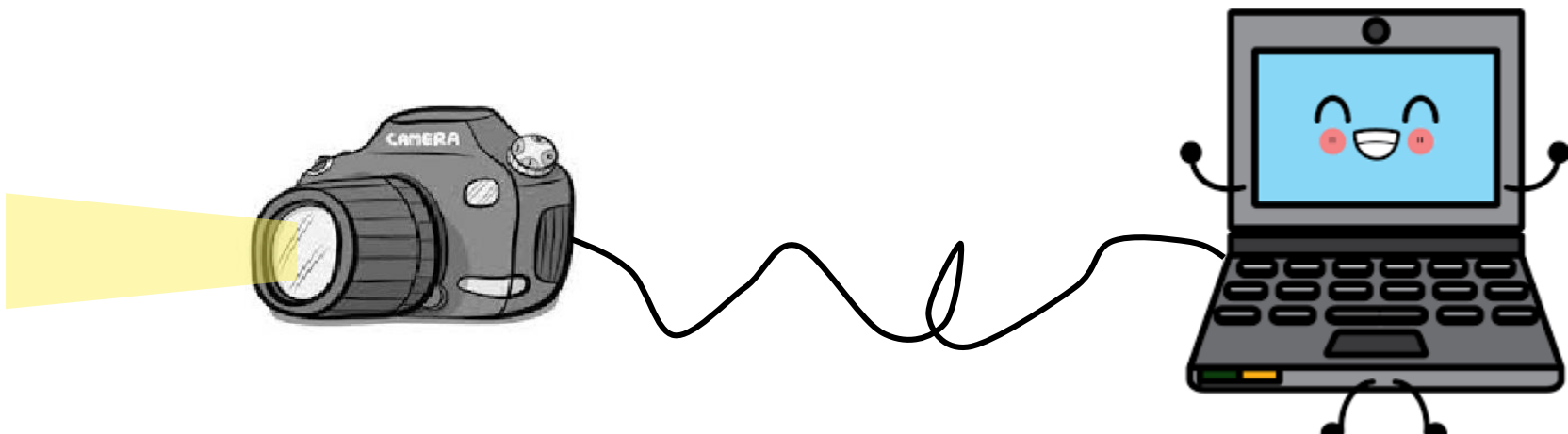
$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

- Recall:
  - Image restoration attempts to recover a degraded image using **prior knowledge** of the distortion.
- Assume:
  - Degradation function can be ignored

We focus on the noise model  $\eta(x, y)$

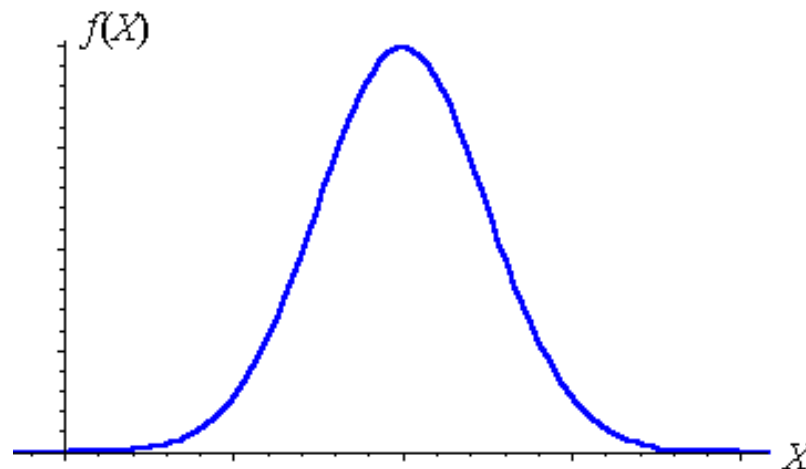
# Noise Model

- Noise in digital images occur during acquisition, such as:
  - Sensor errors: quality of electronics
  - Light levels
  - Sensor temperature
- Or during transmission, such as:
  - Interference in the transmission channel

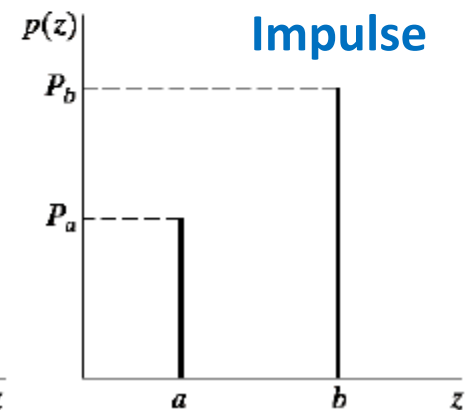
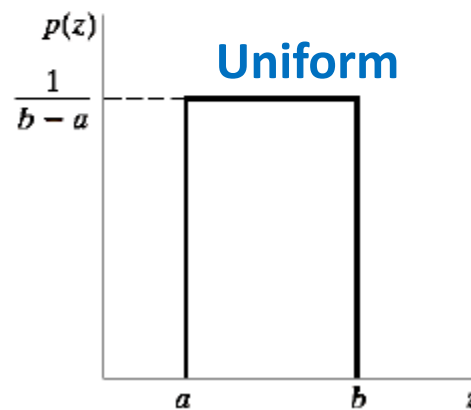
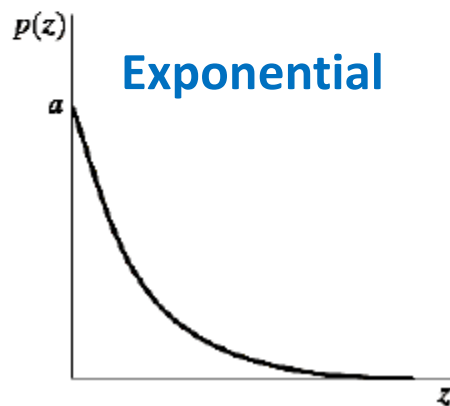
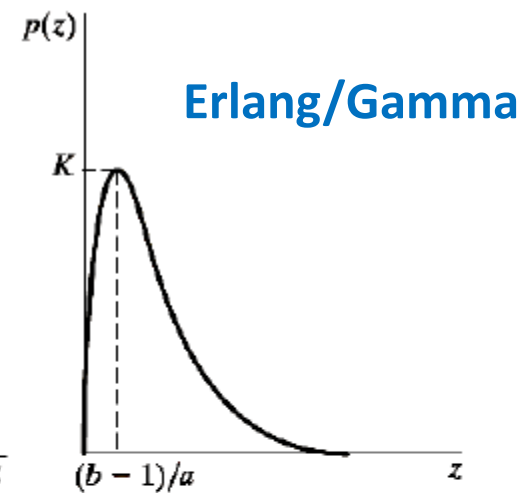
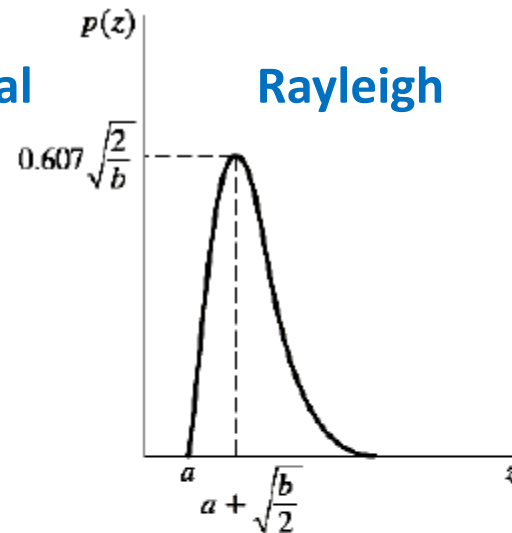
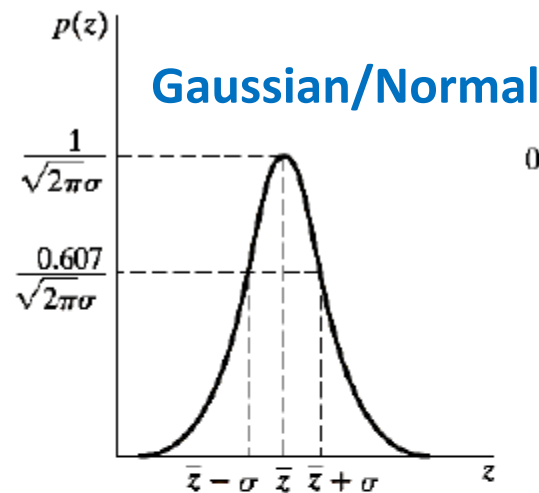


# Spatial Noise

- Can be described statistically as random variables
  - Hence we will consider them as probability density functions (PDF)
  - Probability Density Functions: A function that describes the distribution of a continuous random variable

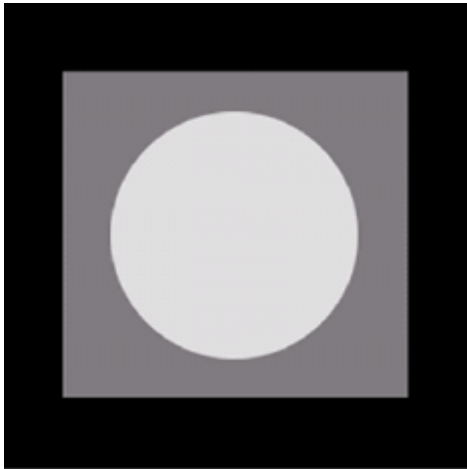


# Common PDFs in Images

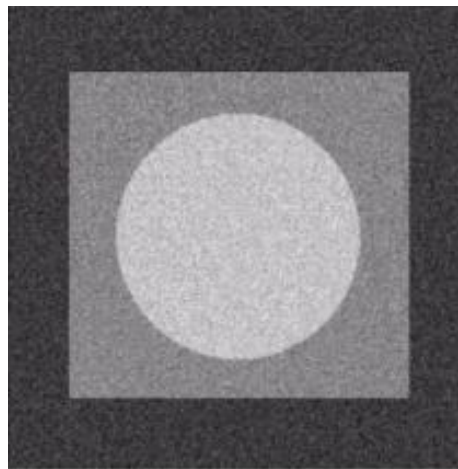


# Gaussian Noise

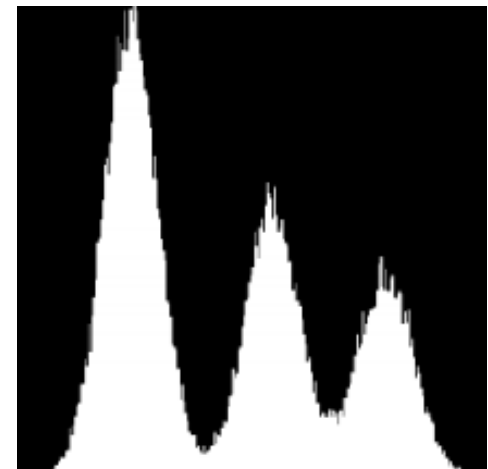
Clear Image



Degraded Image



Histogram of degraded image



- Very commonly used to describe image noise

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$

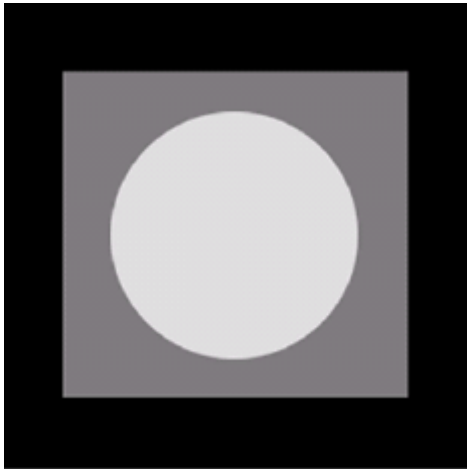
$z$  : intensity

$\bar{z}$ : mean intensity

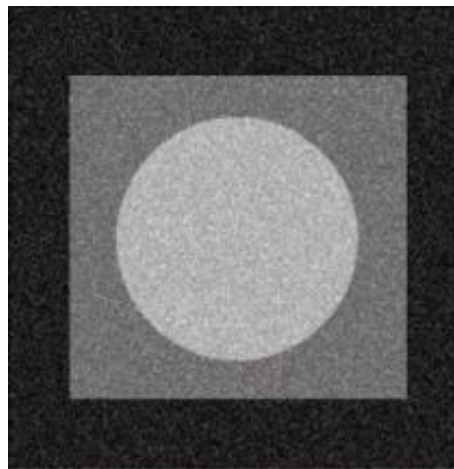
$\sigma$ : standard deviation

# Rayleigh Noise

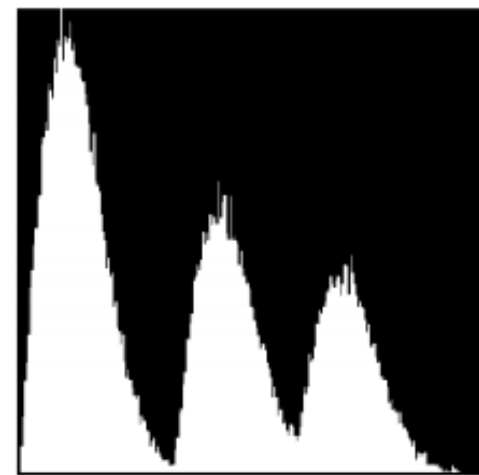
Clear Image



Degraded Image



Histogram of degraded image



$$p(z) = \begin{cases} \frac{2}{b} (z - a) e^{-\frac{(z-a)^2}{b}}, & \text{for } z \geq a \\ 0, & \text{for } z < a \end{cases}$$

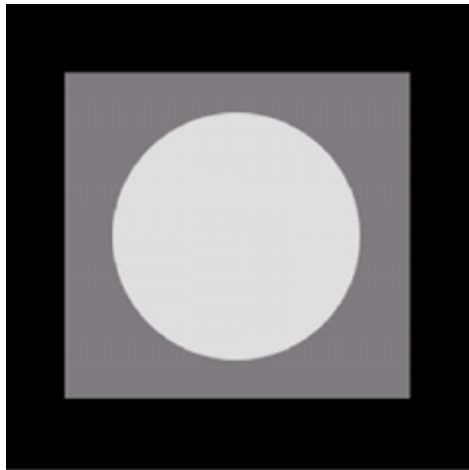
$z$  : intensity

$\bar{z}$  : mean intensity =  $a + \sqrt{\pi b/4}$

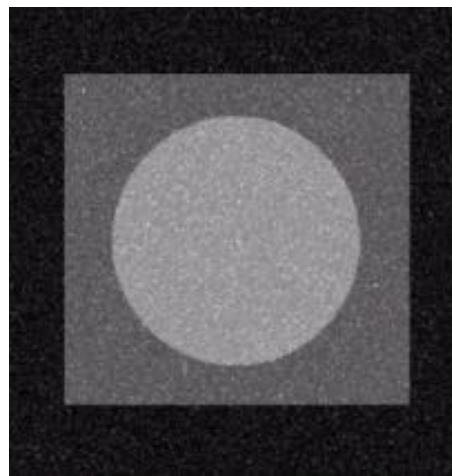
$a, b$  : certain values

# Erlang/Gamma Noise

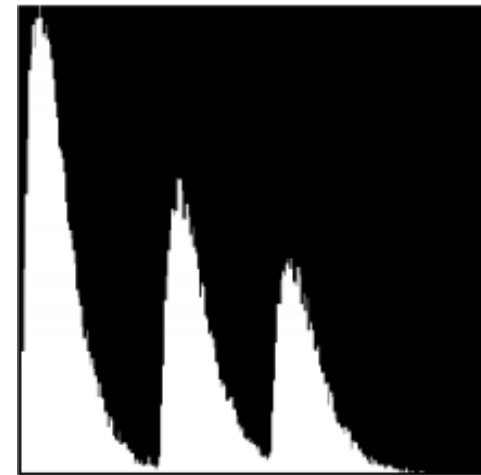
Clear Image



Degraded Image



Histogram of degraded image



- Can be called gamma noise only when the denominator is the gamma function

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!}, & \text{for } z \geq 0 \\ 0, & \text{for } z < 0 \end{cases}$$

$z$  : intensity

$\bar{z}$ : mean intensity =  $\frac{b}{a}$

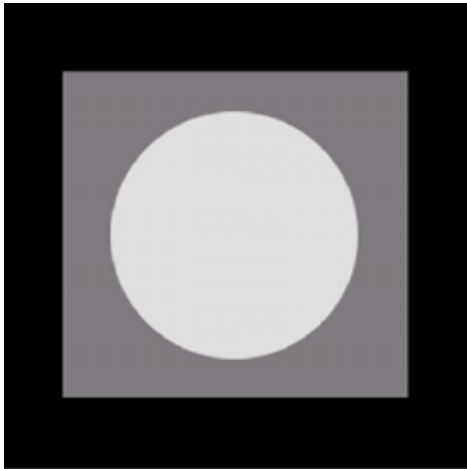
$a, b$  : certain values

$a > 0$ ,  $b$  is a positive integer

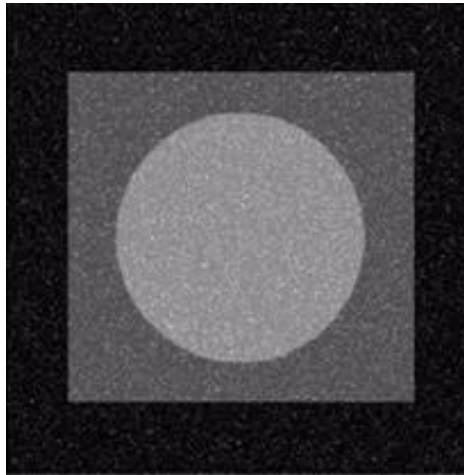


# Exponential Noise

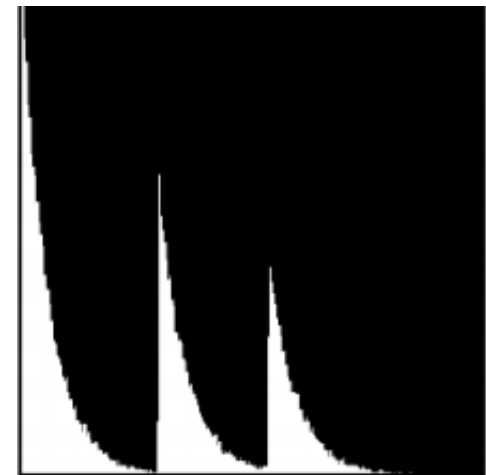
Clear Image



Degraded Image



Histogram of degraded image



- This is a special case of Erlang where  $b = 1$

$$p(z) = \begin{cases} ae^{-az}, & \text{for } z \geq 0 \\ 0, & \text{for } z < 0 \end{cases}$$

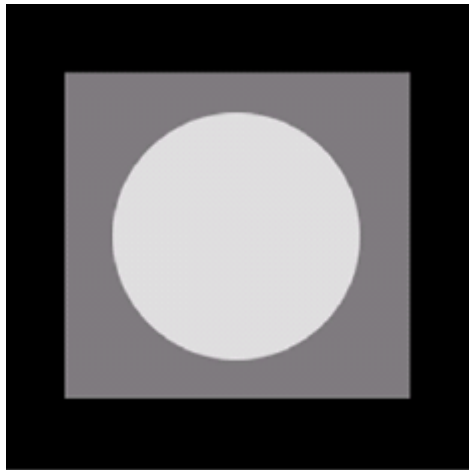
$z$  : intensity

$\bar{z}$ : mean intensity =  $\frac{1}{a}$

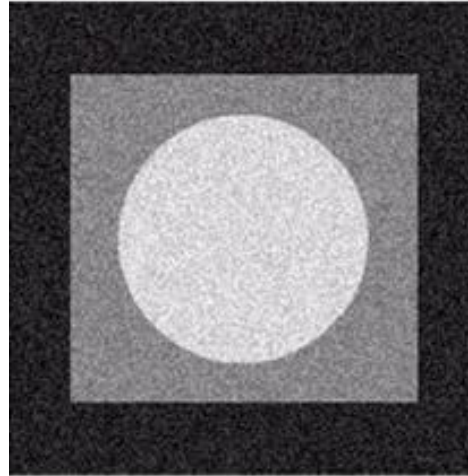
$a$  : certain value

# Uniform Noise

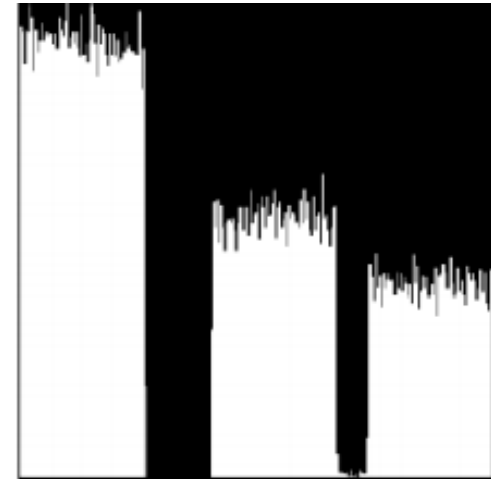
Clear Image



Degraded Image



Histogram of degraded image



- This noise is –as its name says- uniform

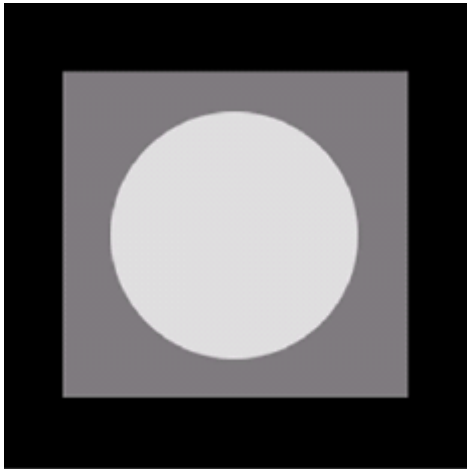
$$p(z) = \begin{cases} \frac{1}{b-a}, & \text{for } a \leq z \leq b \\ 0, & \text{otherwise} \end{cases}$$

$z$  : intensity

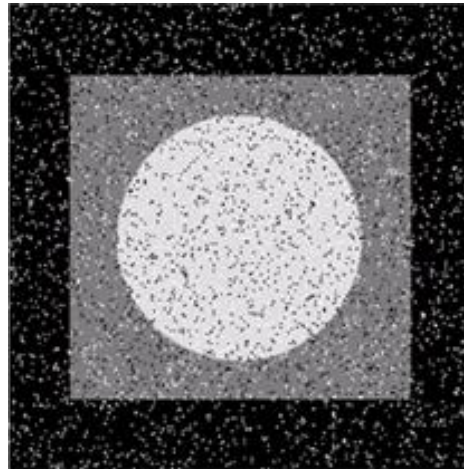
$\bar{z}$ : mean intensity =  $\frac{a+b}{2}$

# Impulse Noise

Clear Image



Degraded Image



Histogram of degraded image



- Only gives peaks of noise: salt-and pepper noise

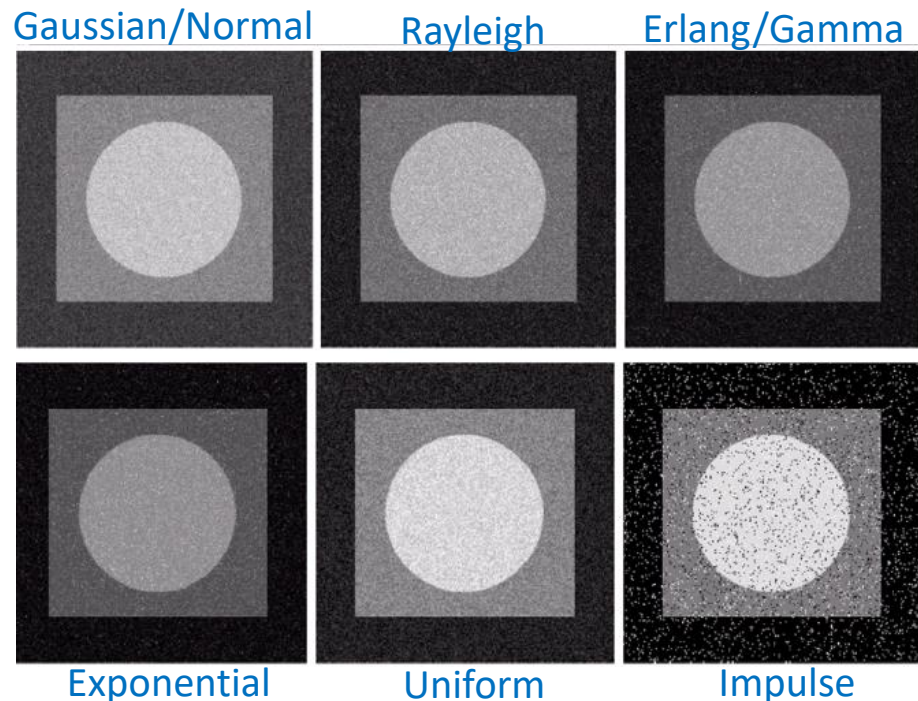
$$p(z) = \begin{cases} P_a, & \text{for } z = a \\ P_b, & \text{for } z = b \\ 0, & \text{otherwise} \end{cases}$$

$a, b$  : certain values

# Statistical Noise

- What do all of the previously described noise have in common?

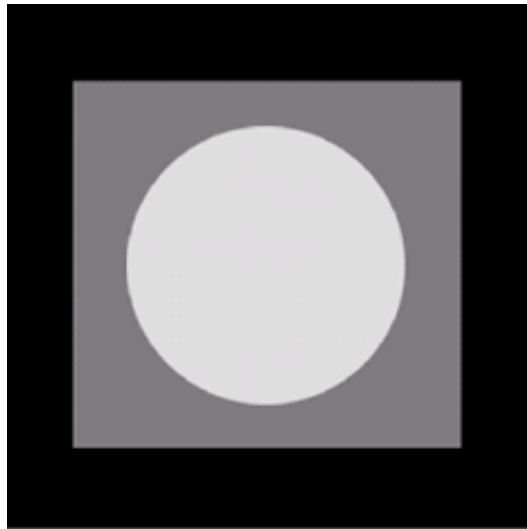
- Gaussian
- Rayleigh
- Erlang
- Exponential
- Uniform
- Impulse



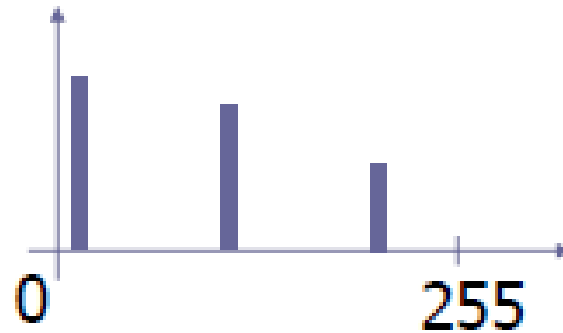
Spatially Independent

# Estimation of Noise

- What does a clear image look like?



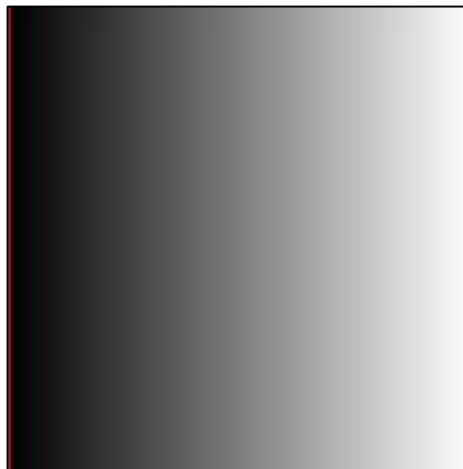
Clear Image



Histogram

# Noise Estimation: If the imaging system is available

- Capture images of a “flat” environment
  - Capture a plain **known** surface
  - We can then see the pattern of the noise



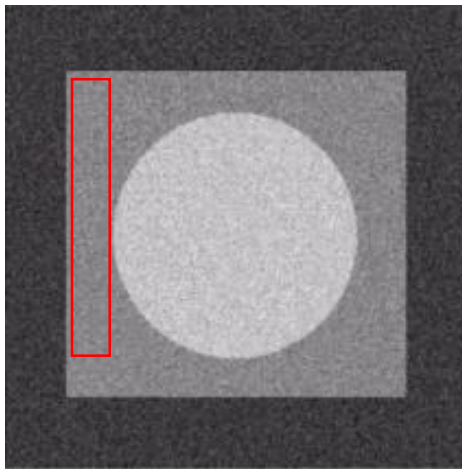
Clear Image



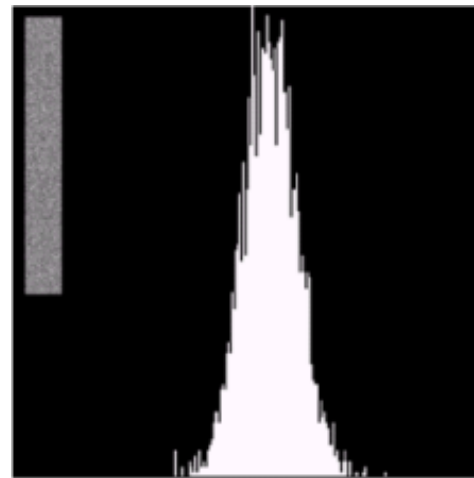
Captured Image

# Estimation of Noise (3)

- If the imaging system is **not** available
  - Use the captured images
  - Take strips of a reasonably constant intensity
  - Observe the histogram of the strips

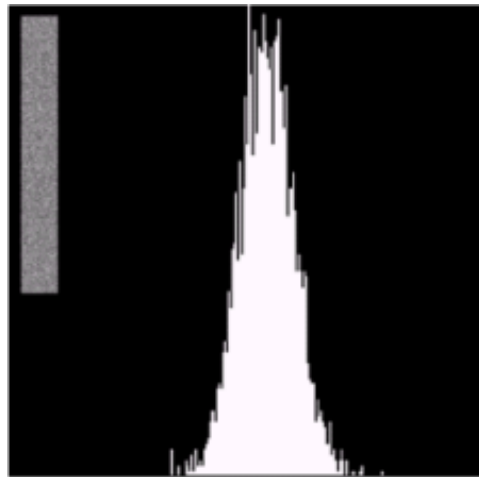


Captured Image



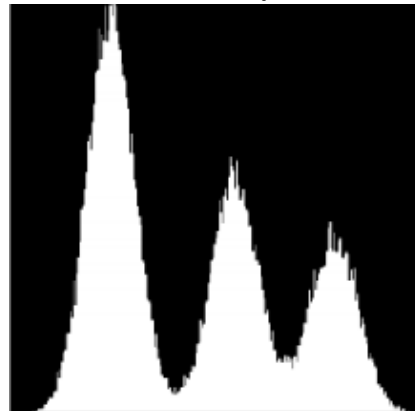
Histogram

# Which noise is this?

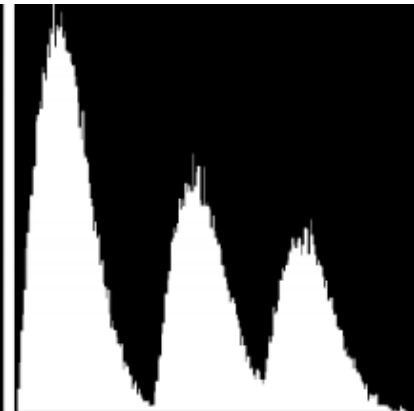


Gaussian/Normal

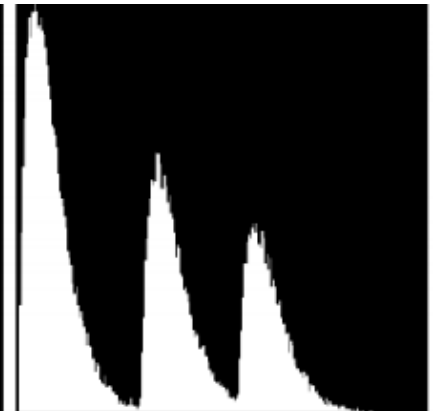
Gaussian/Normal



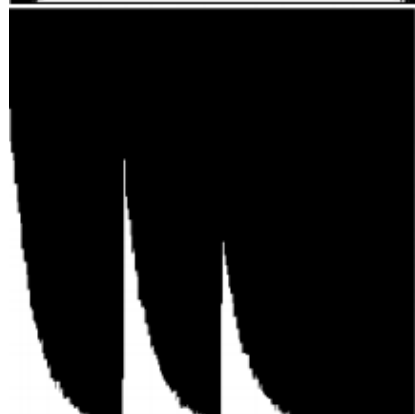
Rayleigh



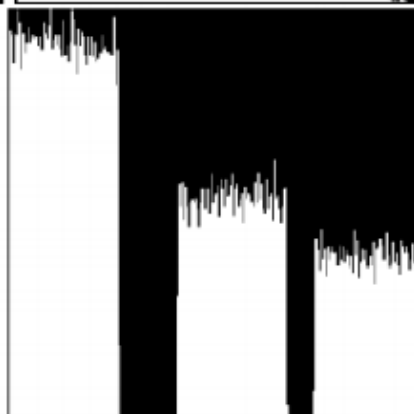
Erlang/Gamma



Exponential



Uniform



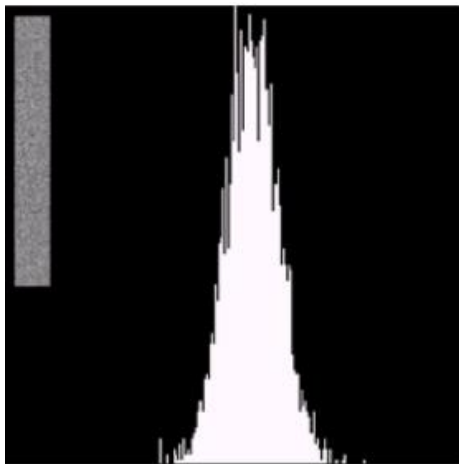
Impulse



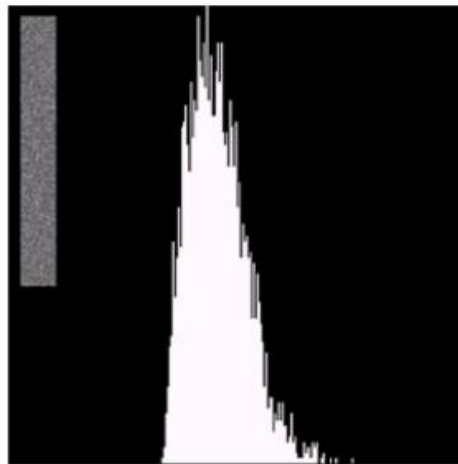


# Estimation of Noise (4)

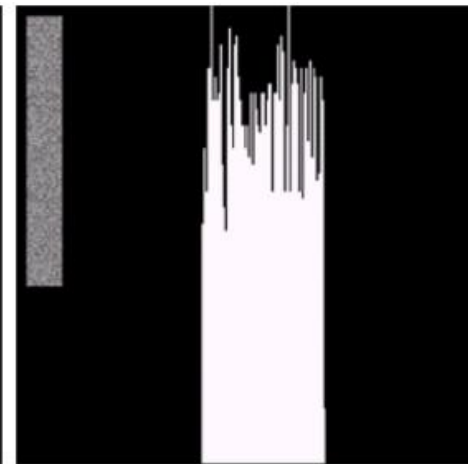
- Indicators:
  - Mean intensity  $\bar{z}$
  - Standard deviation  $\sigma$



Gaussian/Normal



Rayleigh



Uniform

# Eliminating Additive Noise

We have assumed:

- Degradation function can be ignored
- Focus on the additive noise model  $\eta(x, y)$

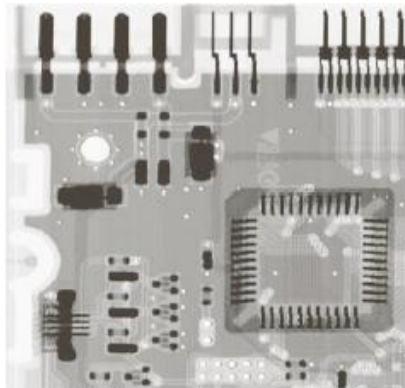
Spatial Filtering

# Arithmetic Mean Filter

- Smoothing and noise reduction
- Loss of detail

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{x,y}} g(s, t)$$

- $m \times n$  image
- $S_{x,y}$  kernel centered at  $(x, y)$



Clear Image

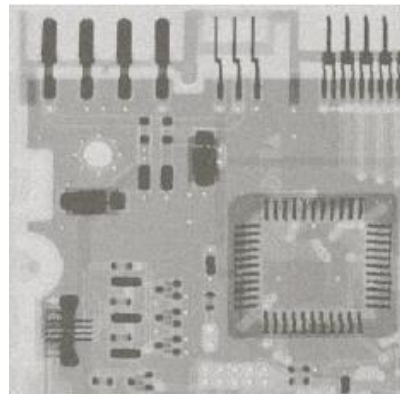
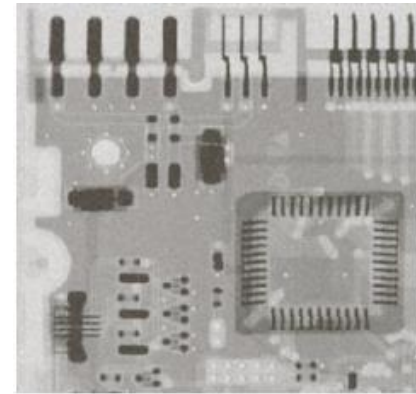


Image w/ Gaussian  
Noise



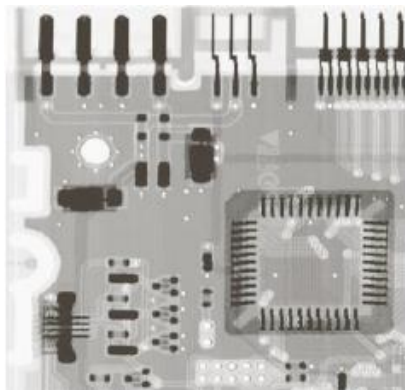
Restored Image with  
Arithmetic Mean Filter

# Geometric Mean Filter

- Smoothing and noise reduction
- Less loss of detail compared to arithmetic mean filter

$$\hat{f}(x, y) = \left[ \prod_{(s,t) \in S_{x,y}} g(s, t) \right]^{1/2}$$

- $m \times n$  image
- $S_{x,y}$  kernel centered at  $(x, y)$



Clear Image

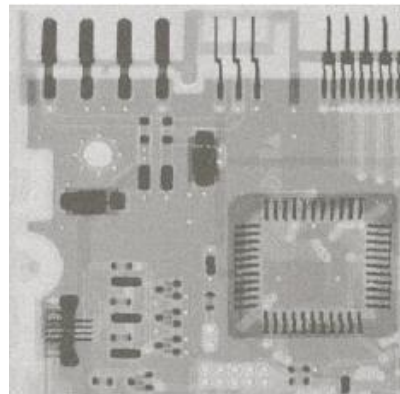
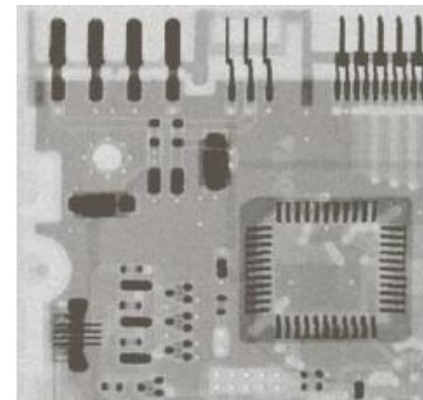


Image w/ Gaussian  
Noise



Restored Image with  
Geometric Mean Filter

# Harmonic Mean Filter

- Works especially well for salt noise (not pepper noise!)
- Works well for Gaussian noise and other noise types

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t) \in S_{x,y}} \frac{1}{g(s, t)}}$$

- $m \times n$  image
- $S_{x,y}$  kernel centered at  $(x, y)$

# Contraharmonic Mean Filter

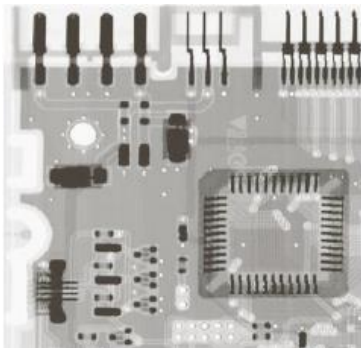
- Perfect for salt-and-pepper noise

$$\hat{f}(x, y) = \frac{\sum_{(s,t) \in S_{x,y}} g(s, t)^{Q+1}}{\sum_{(s,t) \in S_{x,y}} g(s, t)^Q}$$

- $m \times n$  image
- $S_{x,y}$  kernel centered at  $(x, y)$

- $Q$  (filter order)
- $Q > 0$  eliminates pepper,
- $Q < 0$  eliminates salt

# Contraharmonic Mean Filter (2)



Clear Image

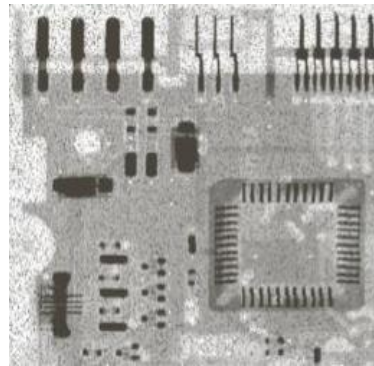


Image w/ Pepper Noise

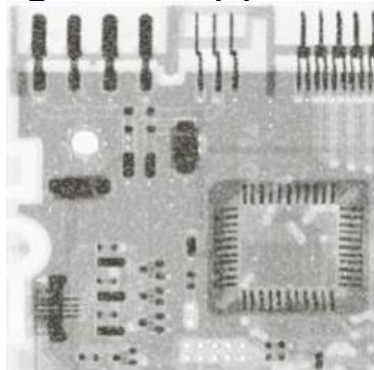
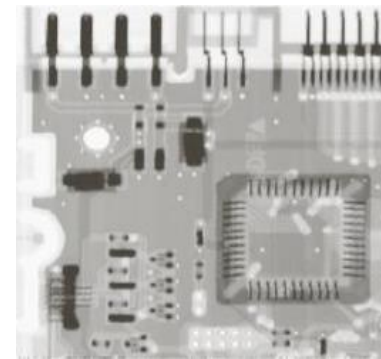
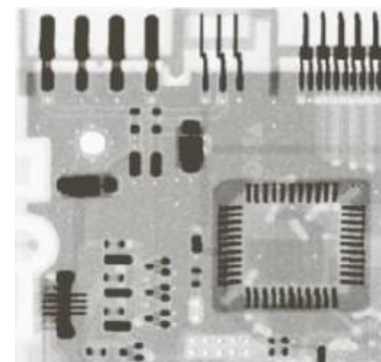


Image w/ Salt Noise



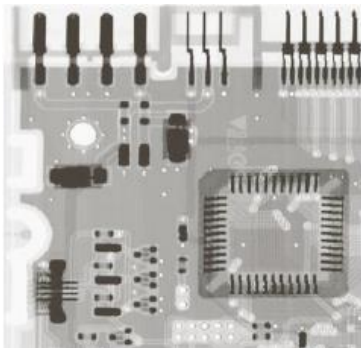
Restored Image ( $Q=1.5$ )



Restored Image ( $Q=-1.5$ )

# Contraharmonic Mean Filter (3)

- The choice of  $Q$  is crucial!



Clear Image

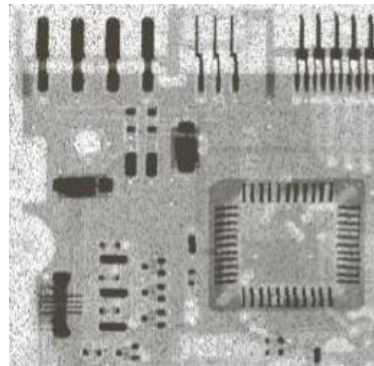
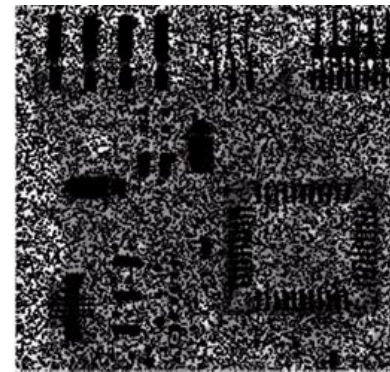


Image w/ Pepper Noise



Restored Image ( $Q = -1.5$ )

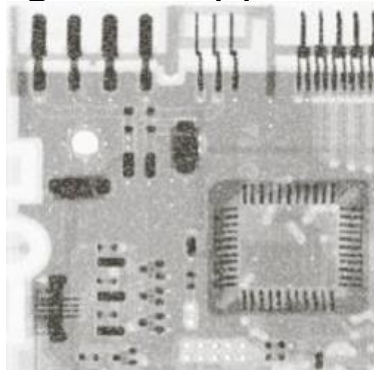
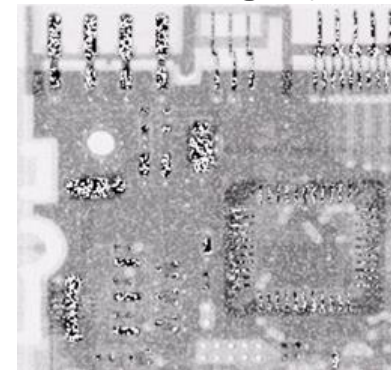


Image w/ Salt Noise



Restored Image ( $Q = 1.5$ )



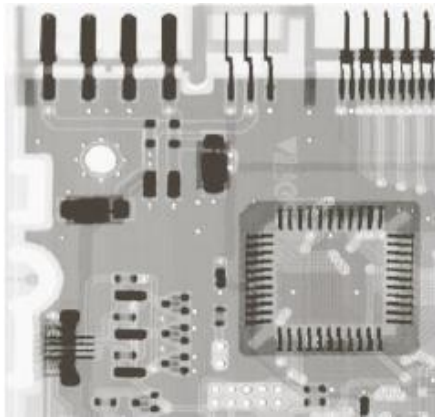
# Order-Statistic Filters

- Spatial filters based on ordering (ranking) the values of the pixels contained in the local image area encompassed by the filter.
  - Median Filters
  - Min/Max
  - Alpha Trimmed

# Order-Statistic Filters (2)

- Median Filters

$$\hat{f}(x, y) = \text{median}_{(s,t) \in S_{x,y}} \{g(s, t)\}$$



Clear Image

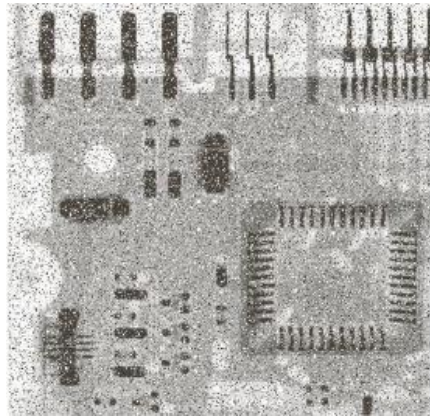
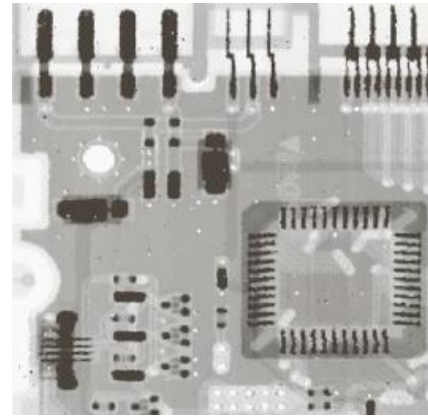
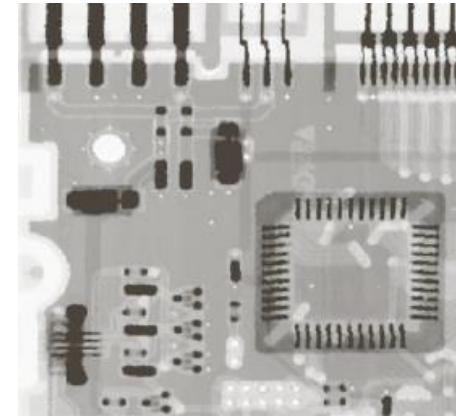


Image w/ Salt and  
Pepper Noise



Restored Image with  
1 pass of Median Filter



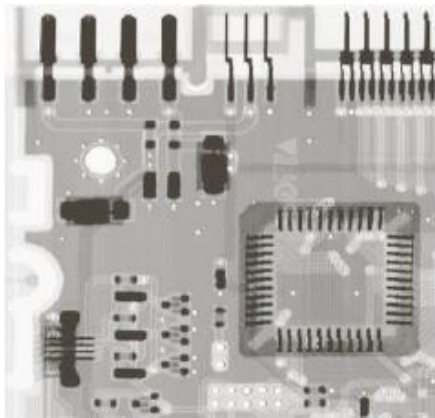
Restored Image with  
3 passes of Median Filter

# Order-Statistic Filters (3)

- Min / Max Filters

$$\hat{f}(x, y) = \min_{(s,t) \in S_{x,y}} \{g(s, t)\}$$

$$\hat{f}(x, y) = \max_{(s,t) \in S_{x,y}} \{g(s, t)\}$$



Clear Image

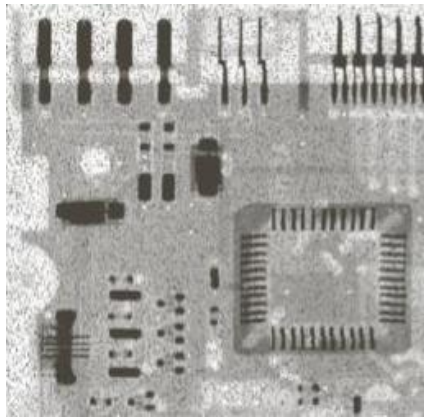
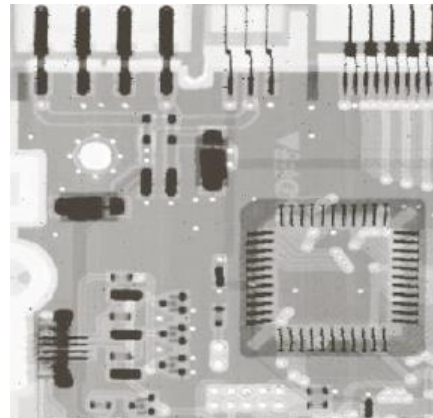
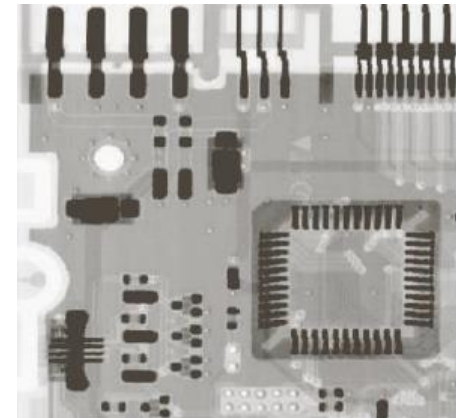


Image w/ Pepper Noise



Restored Image with  
Max Filter

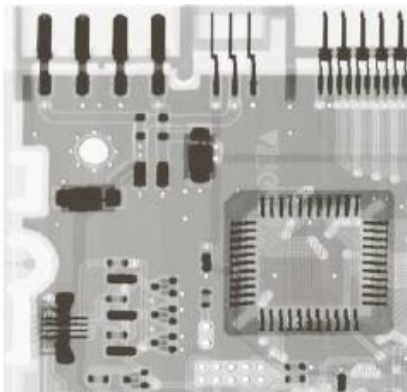


Restored Image with  
Min Filter

# Order-Statistic Filters (4)

- Alpha Trimmed Mean Filter

$$\hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s,t) \in S_{x,y}} g(s, t) \quad d : \text{between } 0 \text{ and } mn - 1$$



Clear Image

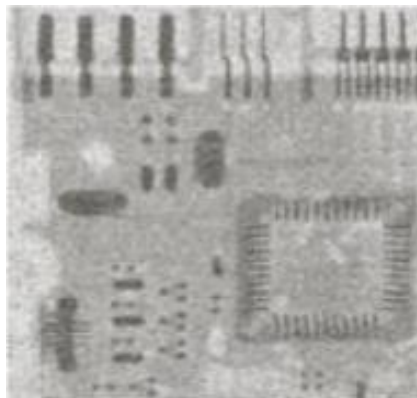
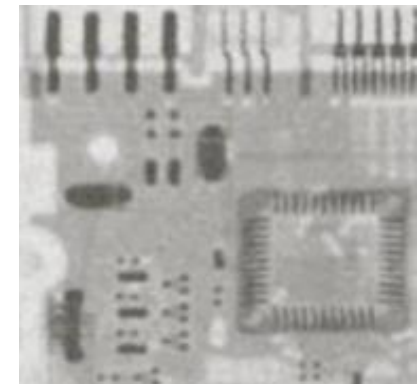
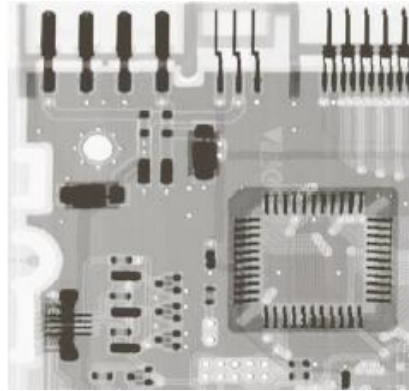


Image w/ Salt and Pepper Noise



Restored Image with  
Alpha-Trimmed Filter

# Image Restoration with Spatial Filters



Clear Image

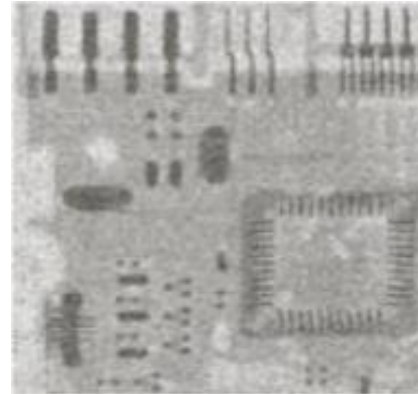
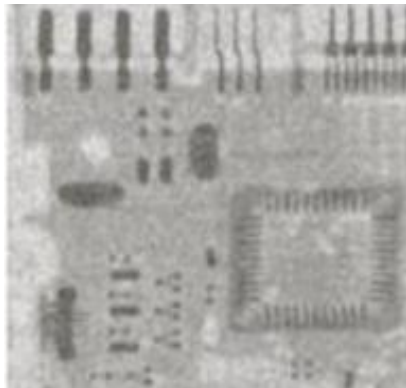
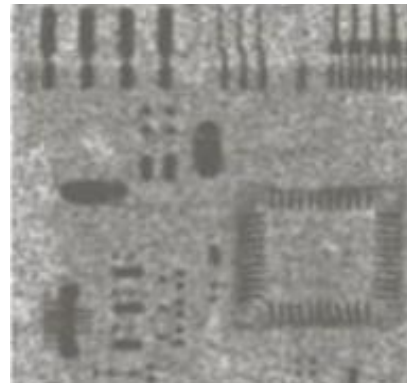


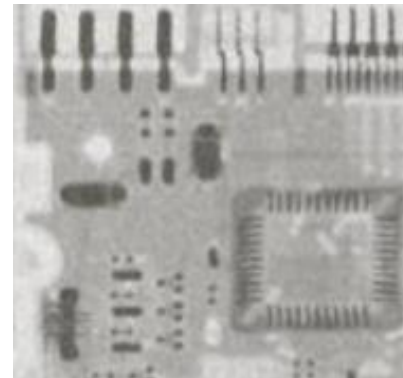
Image w/ Salt and Pepper Noise



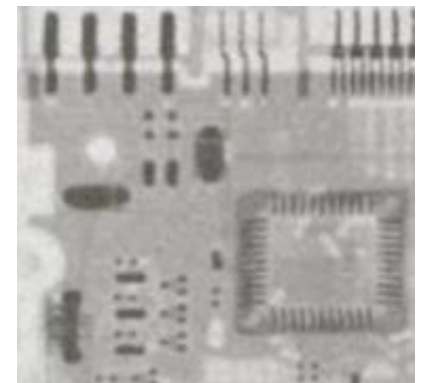
Restored Image with  
Arithmetic Mean Filter



Restored Image with  
Geometric Mean Filter



Restored Image with  
Median Filter



Restored Image with  
Alpha-Trimmed Filter

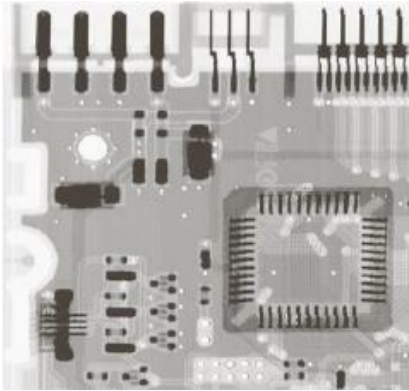
# Adaptive Local Noise Reduction Filter

- Changes its behavior based on the statistical characteristics of the image inside the filter region defined by the neighborhood.
  - Simplest statistical measurement
    - Mean and variance
  - Known parameters on local region  $S_{x,y}$ 
    - $g(x, y)$ : noisy image pixel value
    - $\sigma_\eta^2$ : noise variance (assume known a prior)
    - $m_L$ : local mean
    - $\sigma_L^2$ : local variance

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2} [g(x, y) - m_L]$$



# Adaptive Local Noise Reduction Filter (2)



Clear Image

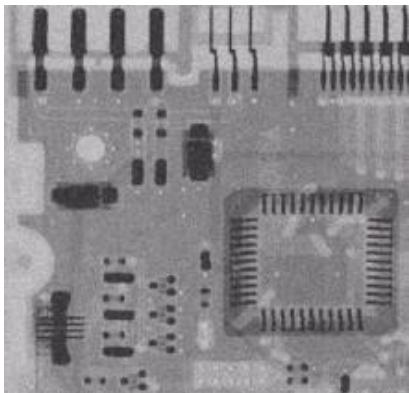
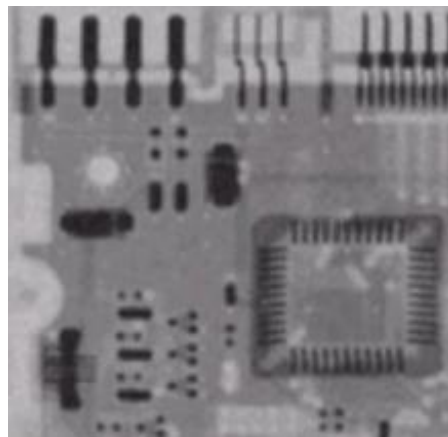
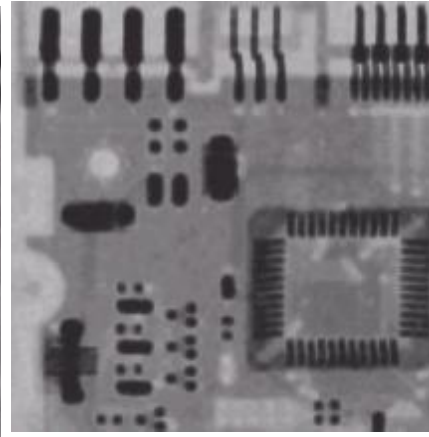


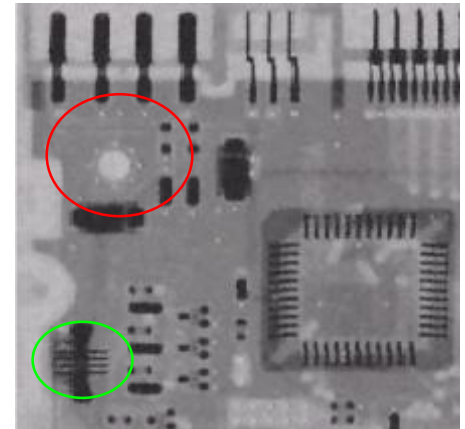
Image w/ Gaussian Noise



Restored Image with  
Arithmetic Mean Filter



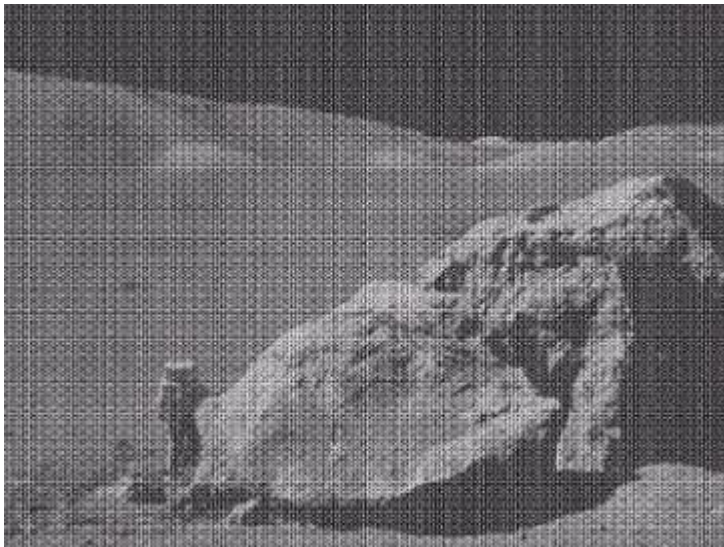
Restored Image with  
Geometric Mean Filter



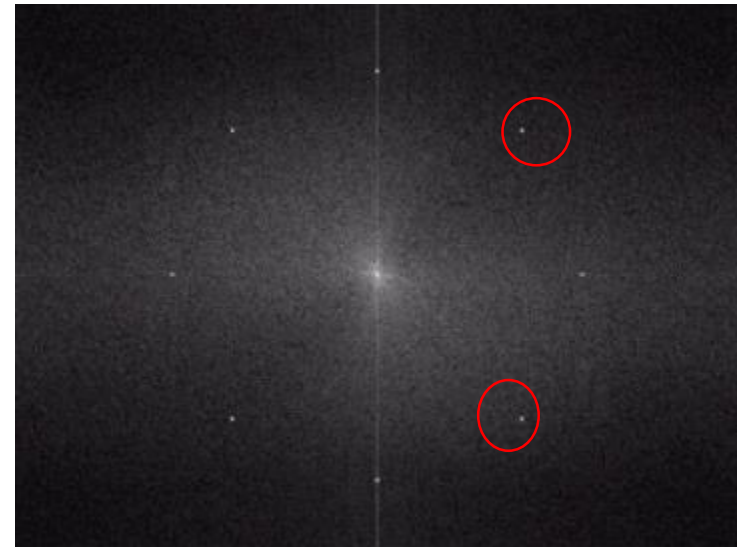
Restored Image with  
Adaptive Local Filter

# Periodic Noise

- Unlike previously, periodic noise is spatially dependent
- Can be handled well by frequency domain filtering



Spatial Image

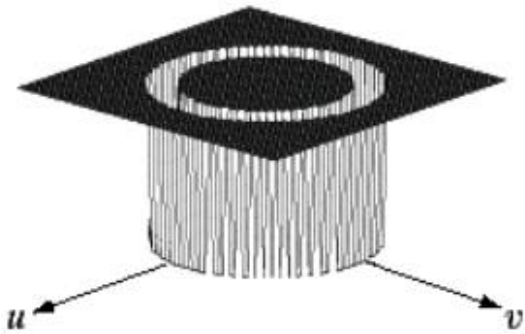


Fourier Transform

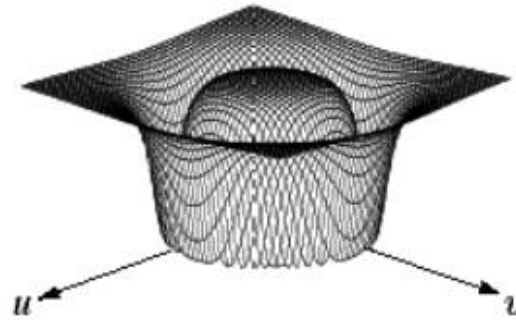
Band Reject Filters



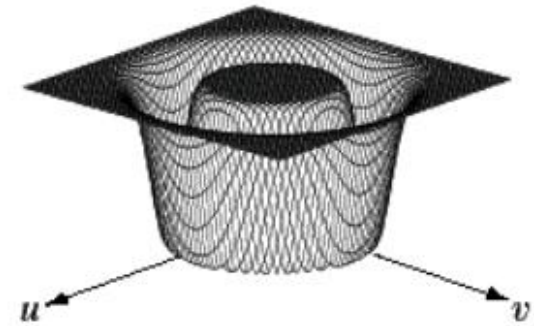
# Band Reject Filters



Ideal Bandreject Filter

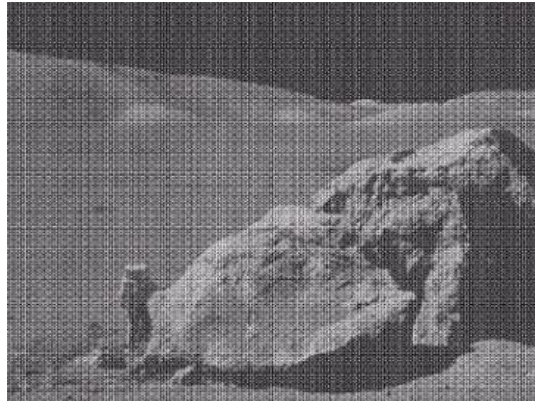


Butterworth  
Bandreject Filter

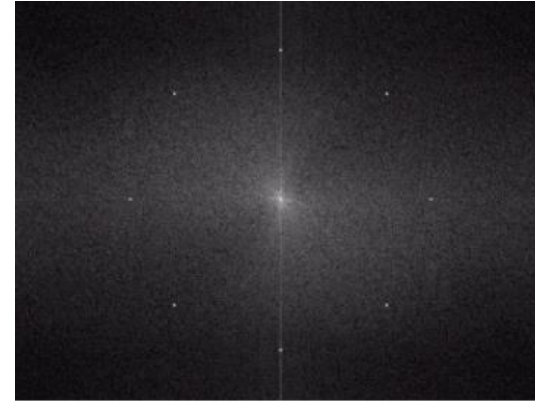


Gaussian  
Bandreject Filter

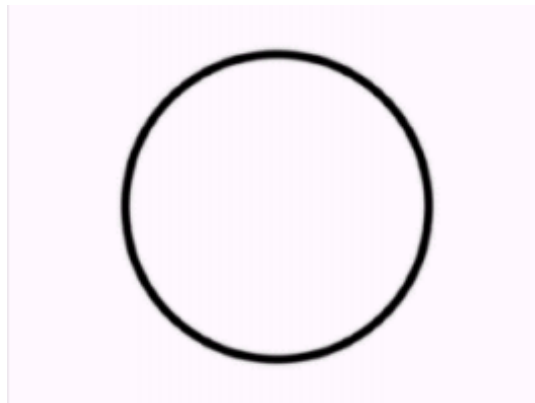
# Band Reject Filters (2)



Spatial Image



Fourier Transform



Band Reject Filter

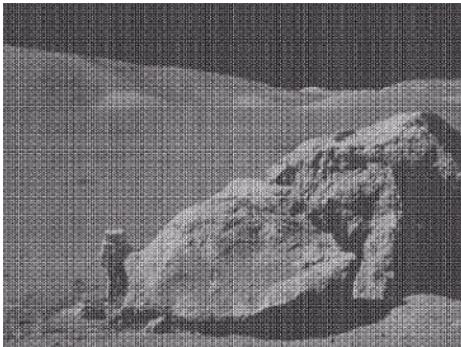


Restored Image

# Band Pass Filters

- The opposite of Band Reject Filters

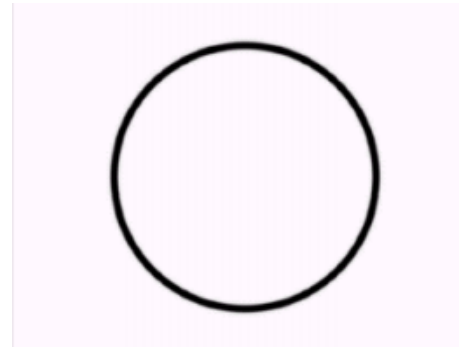
$$H_{BP}(u, v) = 1 - H_{BR}(u, v)$$



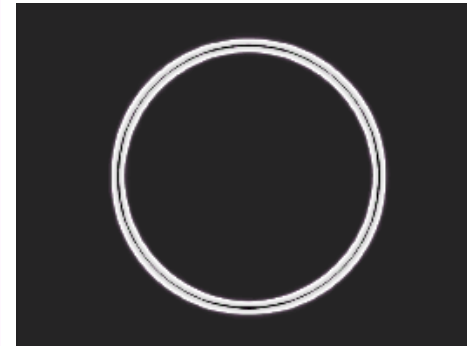
Spatial Image



Fourier Transform



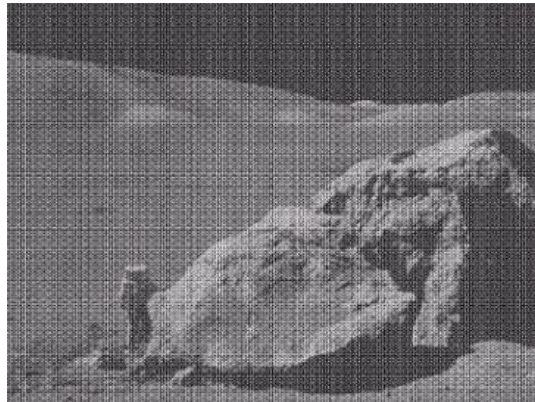
Band Reject Filter



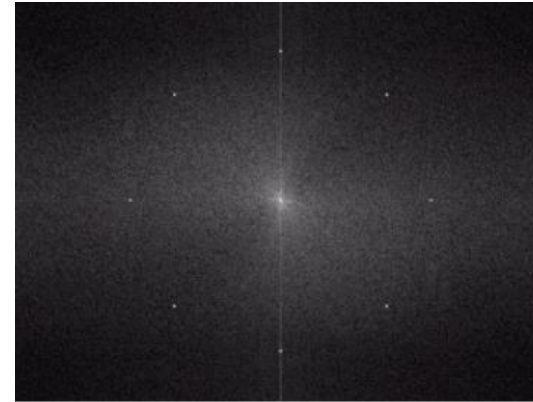
Band Pass Filter

- What will I obtain if I filter the image with the band pass filter?

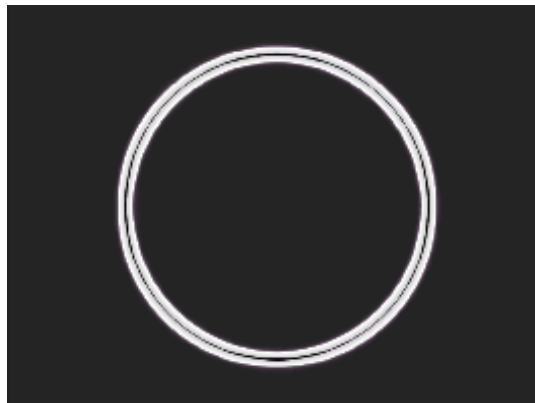
# Band Pass Filters (2)



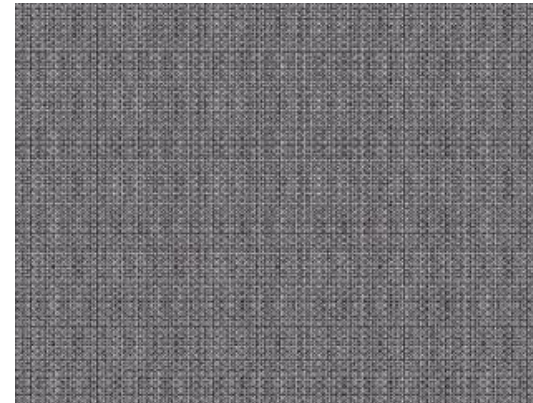
Spatial Image



Fourier Transform



Band Pass Filter



Noise Pattern of Image

# Restoring Images based on Degradation Model

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

- Previously we assumed:
  - Degradation function can be ignored

We focus now on the degradation model  $H(u, v)$

# Estimating the Degradation Function

- Given a degraded image without any knowledge of the degradation.
- Assume a **linear, spatially independent** process.
- What can we do?
  - Observation
  - Experimentation
  - Mathematical Modeling.

# Estimation By Image Observation

- Take a local window in the degraded image  $G_s(u, v)$   
→ simple structure, strong signal
- Estimate the original image in the window  $\hat{F}_s(u, v)$
- Use the degraded and the estimated “clear” window

$$H(u, v) = \frac{G_s(u, v)}{\hat{F}_s(u, v)}$$

# Estimation By Image Observation (2)

- Example:
  - We have a blurred image
  - Take a small rectangular section of background
  - Attempt to enhance the image → any ideas how?
- Thus, we approximate the degradation by assuming position invariance

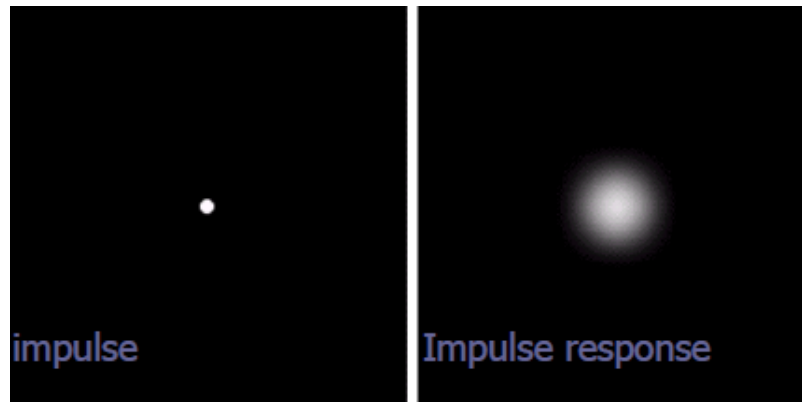
$$H(u, v) = \frac{G_s(u, v)}{\hat{F}_s(u, v)}$$



# Estimation By Experimentation

- If we can obtain similar equipment to that used in the image acquisition.
  - Capture an image of a single impulse
  - The fourier transform of a single impulse is a constant, A
- Thus, we can estimate

$$H(u, v) = \frac{G(u, v)}{A}$$



# Estimation By Mathematical Modelling

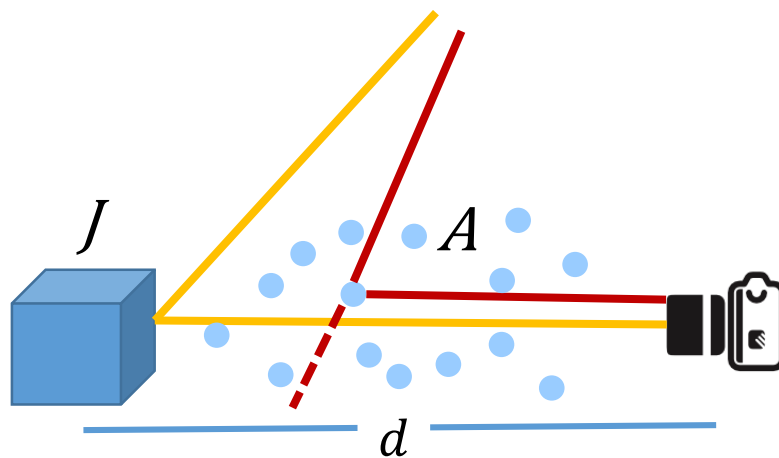
- Can be used to estimate environmental effects to image capture
- Example in the frequency domain: Atmospheric effects

$$H(u, v) = e^{-k(u^2+v^2)^{\frac{5}{6}}}$$



# Estimation By Mathematical Modelling (2)

- Example in the spatial domain: Photometric scattering effects



$$I = JT + A(1 - T)$$

- Transmission

- Beer-lambert law  $T = e^{-cd}$

$c$  : Attenuation coefficient of media  
 $d$  : Distance of object and camera  
 $J$  : Original intensity of scene  
 $A$  : Airlight (The color of surrounding media, composed solely of scattering effects.)

# Inverse Filtering

- Using the previous estimation methods, we obtain

$$\hat{F}(u, v) = \frac{G(u, v)}{H(u, v)}$$

- Back to our degradation model

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

- We obtain

$$\hat{F}(u, v)H(u, v) = H(u, v)F(u, v) + N(u, v)$$

$$\hat{F}(u, v) = F(u, v) + \frac{N(u, v)}{H(u, v)}$$

We still need  $N(u, v)$  .

If  $H(u, v)$  is close to 0,  $N(u, v)$  will dominate the estimate  $\hat{F}(u, v)$ .

# Inverse Filtering(2)

- To avoid near-zero values:
  - Limit filter frequencies near the origin
  - Manual adjustments to restoration
  - Experimental approaches to set parameters

# Wiener Filter

- N. Wiener (1942)
- Also known as:
  - Minimum mean square filter
  - Least square error filter
- What does it do:
  - Considers both degradation function and statistical noise
  - The objective is to find the estimate  $\hat{f}$  for a clear image  $f$  with the mean square error between them is minimized

# Wiener Filter (2)

- The objective is to find the estimate  $\hat{f}$  for a clear image  $f$  with the mean square error between them is minimized

$$\hat{F}(u, v) = \left[ \frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + \frac{S_\eta(u, v)}{S_f(u, v)}} \right] G(u, v)$$

$H(u, v)$ : degradation function

$H^*(u, v)$ : complex conjugate of  $H(u, v)$

$|H(u, v)|^2$ :  $H^*(u, v) H(u, v)$

$S_\eta(u, v)$  : power spectrum of noise

$S_f(u, v)$ : power spectrum of the original image

Usually unknown

# Wiener Filter (3)

- Constant  $K$  as an estimate

$$\hat{F}(u, v) = \left[ \frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + K} \right] G(u, v)$$

$H(u, v)$ : degradation function

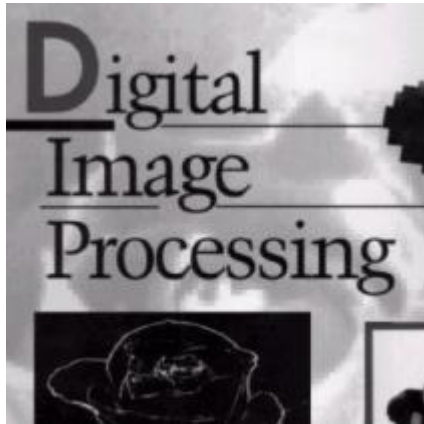
$H^*(u, v)$ : complex conjugate of  $H(u, v)$

$|H(u, v)|^2$ :  $H^*(u, v) H(u, v)$

$K$ : constant chosen for the best visual result



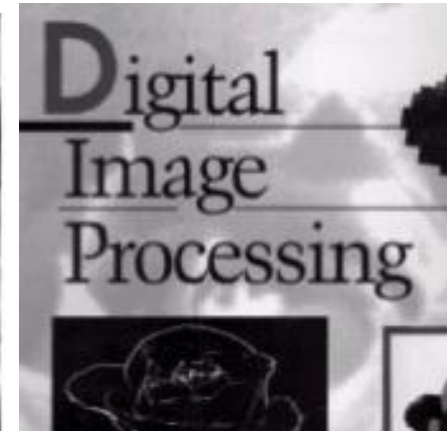
# Wiener Filter Results



Clear Image



Image w/ Motion Blur  
and Additive Noise



Wiener Filter Results  
with various  
noise variance

# Additional Reading

- Inverse Filtering and its applications
- Wiener Filter and its applications
- Constrained Least Squares Filtering
- Image Quality Measures

# Image Quality

Pedersen, Marius & Yngve Hardeberg, Jon. (2012). Full-Reference Image Quality Metrics: Classification and Evaluation. Foundations and Trends in Computer Graphics and Vision.

AHMED, ISMAIL T., Chen Soong Der, and BARAA TAREQ HAMMAD. "A Survey of Recent Approaches on No-Reference Image Quality Assessment with Multiscale Geometric Analysis Transforms."

# Image Quality

- Image Quality
  - The level of accuracy in which digital systems display the signals that form an image.
  - The weighted combination of all of the visually significant attributes of an image.
- Image Quality Methods, Image Quality Metrics, Image Quality Assessment
- Ways to assess image quality
  - Subjective methods
  - Objective methods

# Image Quality Metrics (1)

- Objective methods:
  - **Full-reference (FR) methods** – The image quality is computed by comparing it with a reference image that is assumed to have perfect quality
  - **Reduced-reference (RR) methods** – The image quality is measured based on a features extracted from the reference image.
  - **No-reference (NR) methods** – The image quality is measured without any reference to the original one.

# Image Quality Metrics

- Based on the information used:
  - **Pixel difference-based** which uses the difference of value of each pixel.
  - **Correlation-based measures:** correlation of pixels or vector angular directions.
  - **Edge-based measures:** displacement of edge positions or their consistency across resolution levels.
  - **Spectral distance-based measures:** the Fourier magnitude and/or phase spectral discrepancy on a block basis.
  - **Context-based measures:** based on various functionals of the multidimensional context probability.
  - **HVS-based measures**, based on the human visual system (HVS) weighted spectral distortion measures or (dis)similarity criteria used in image-based browsing functions.

# A Non-Exhaustive List of Image Quality Measures

- Full/Reduced Reference
  - Signal to Noise Ratio
  - Mean Square Error / Root Mean Square Error
  - Structural Similarity (SSIM)
  - $\Delta E_{ab}^*$ ,  $\Delta E_{94}^*$ , etc: color difference methods
  - Adaptive Image Difference
  - Universal Image Quality Index
- No-Reference
  - Blind Image Spatial Quality Evaluator (BRISQUE)
  - Natural Image Quality Evaluator (NIQE)
  - Perception based Image Quality Evaluator (PIQE)