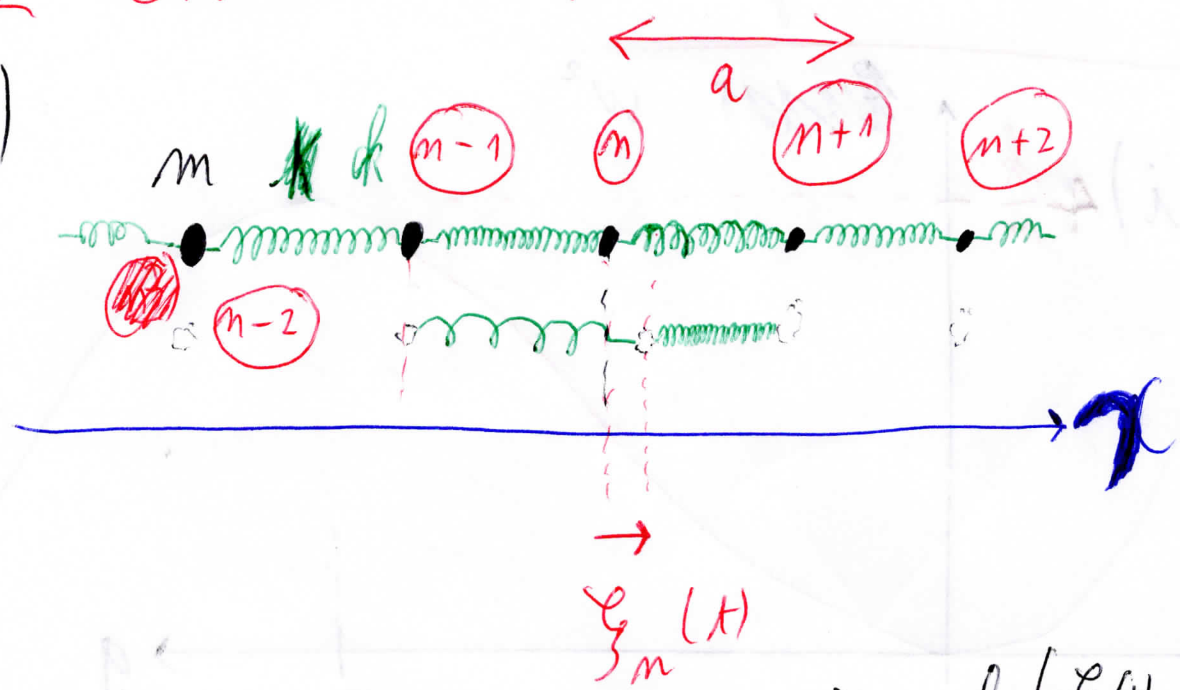
I Chaîne d'atomes

1)



Système : { atome n } : $\sum \vec{F} = k\left(\xi_{n+1}(t) - \xi_n(t)\right)$

$$- k\left(\xi_n(t) - \xi_{n-1}(t)\right)$$

$$= k\left[\xi_{n+1}(t) + \xi_{n-1}(t) - 2\xi_n(t)\right].$$

$$\rightsquigarrow \boxed{\frac{d^2\xi_n}{dt^2}(t) + \frac{2k}{m}\xi_n(t) = \frac{k}{m}\left(\xi_{n+1}(t) + \xi_{n-1}(t)\right)}$$

2) $\underline{\xi_n}(t) = A e^{j(\omega t - qna)}$ (onde)

$q = 2\pi \cdot \frac{1}{\lambda}$ → une longueur d'onde

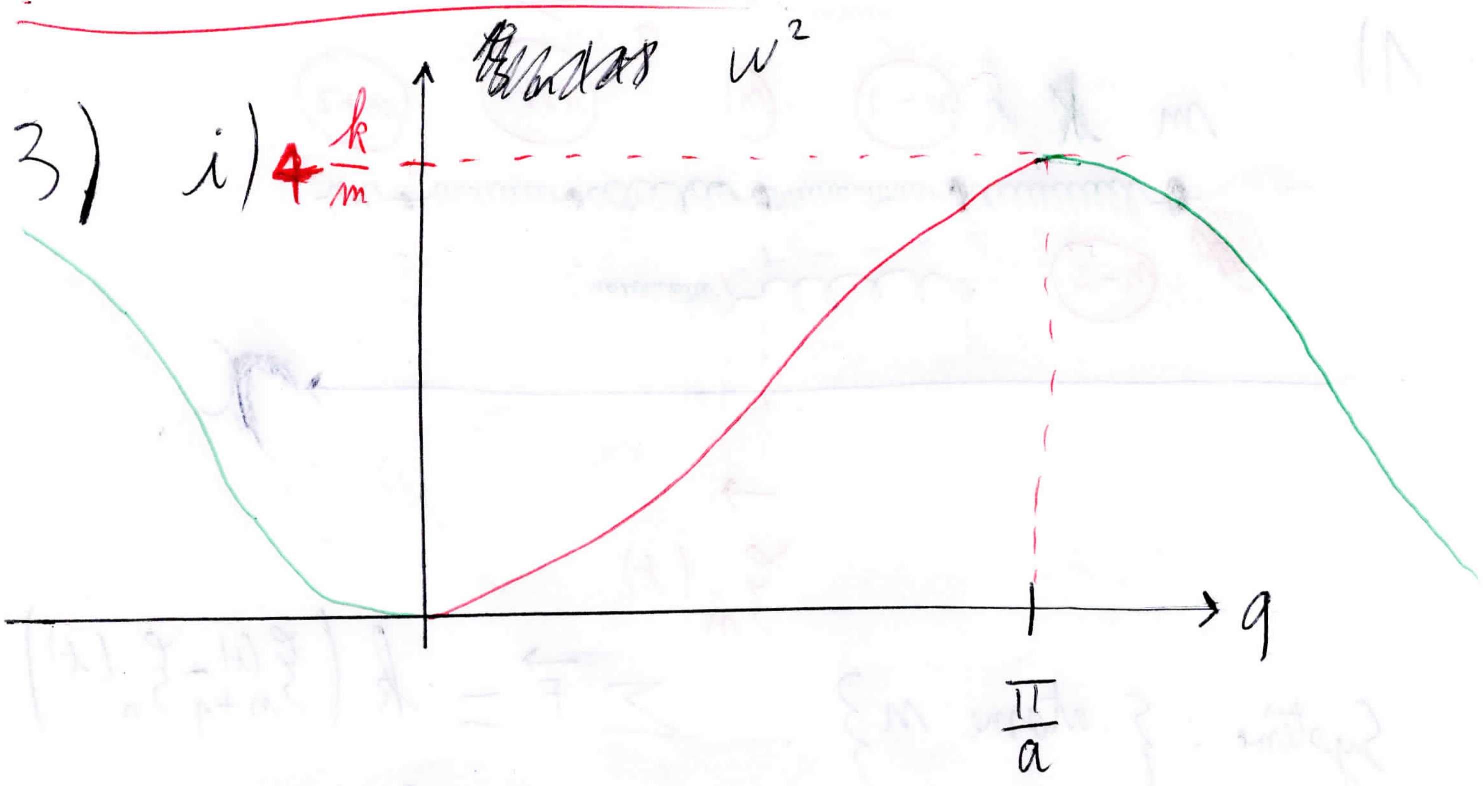
$$\frac{d^2 \underline{\xi_n}(t)}{dt^2} = -\omega^2 \underline{\xi_n}(t), \quad \underline{\xi_{n+1}}(t) = e^{-qa}\underline{\xi_n}(t), \quad \underline{\xi_{n-1}}(t) = e^{qa}\underline{\xi_n}(t)$$

$$\rightsquigarrow \frac{2k}{m} - \omega^2 = 2\frac{k}{m}\cos(qa)$$

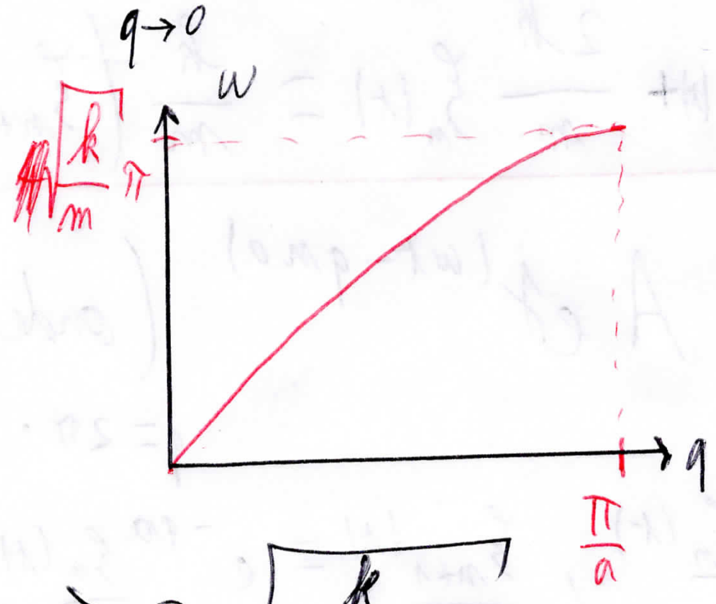
$$\rightsquigarrow \boxed{\omega^2 = 2\frac{k}{m}\left(1 - \cos(qa)\right)}$$

Relation de dispersion

3) i)



ii) $\omega^2 \underset{q\to 0}{\simeq} \frac{k}{m}(qa)^2$, $\omega \sim \sqrt{\frac{k}{m}}\,aq \sim q$



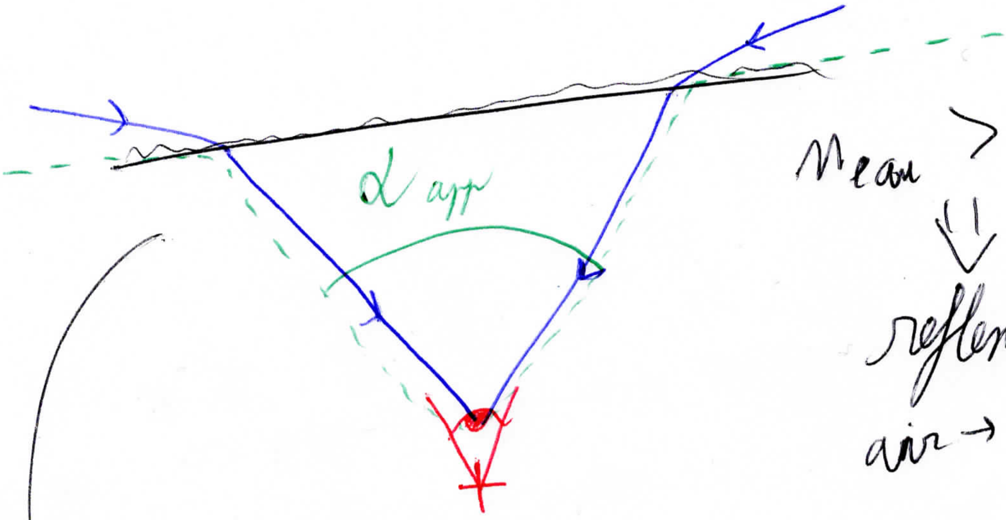
4. $\omega > 2\sqrt{\frac{k}{m}}$: on n'a pas $e^{j(\omega t + qna)}$

: plus d'onde possible. → q en fait $\in \mathbb{C}$

→ atténuation $e^{-\frac{na}{\delta}}$: onde évanescente

II

1)

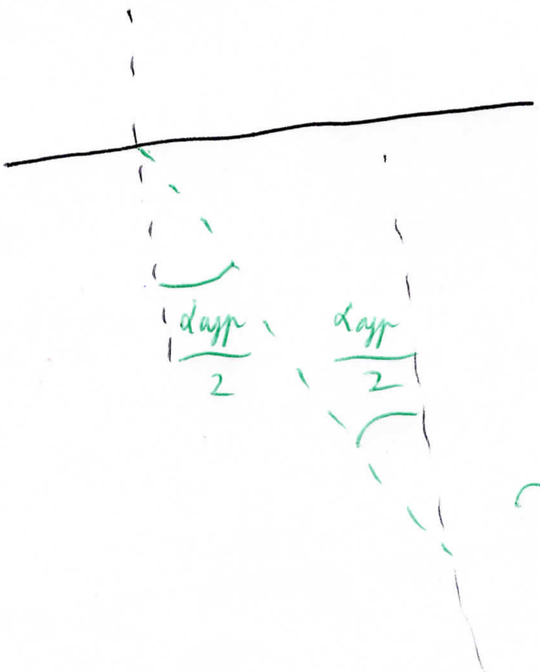


$n_{eau} > n_{air}$

$\Downarrow$

reflexion totale

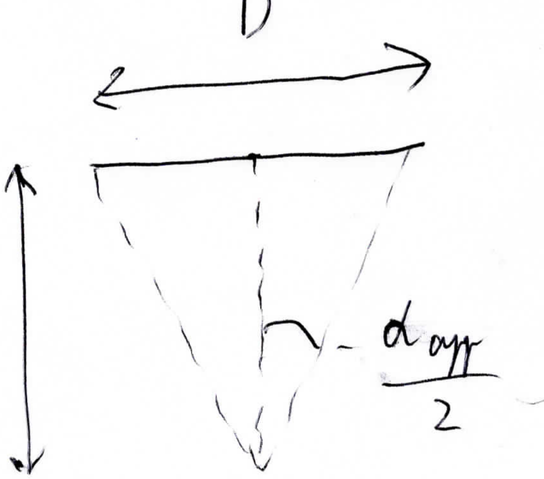
air → eau

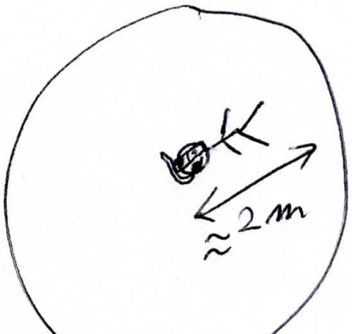


$$n_{eau} \sin\left(\frac{\alpha_{app}}{2}\right) = n_{air}$$

$$\leadsto \alpha_{app} = 2 \arcsin\left(\frac{n_{air}}{n_{eau}}\right)$$

2)





ok ok ok ... 8 fois

$$D \approx 8 \cdot 2\,m$$

$$\approx 16\,m$$

$$H = \frac{D}{2\tan\left(\frac{\alpha_{app}}{2}\right)}$$

$$\leadsto H \approx 18\,m.$$