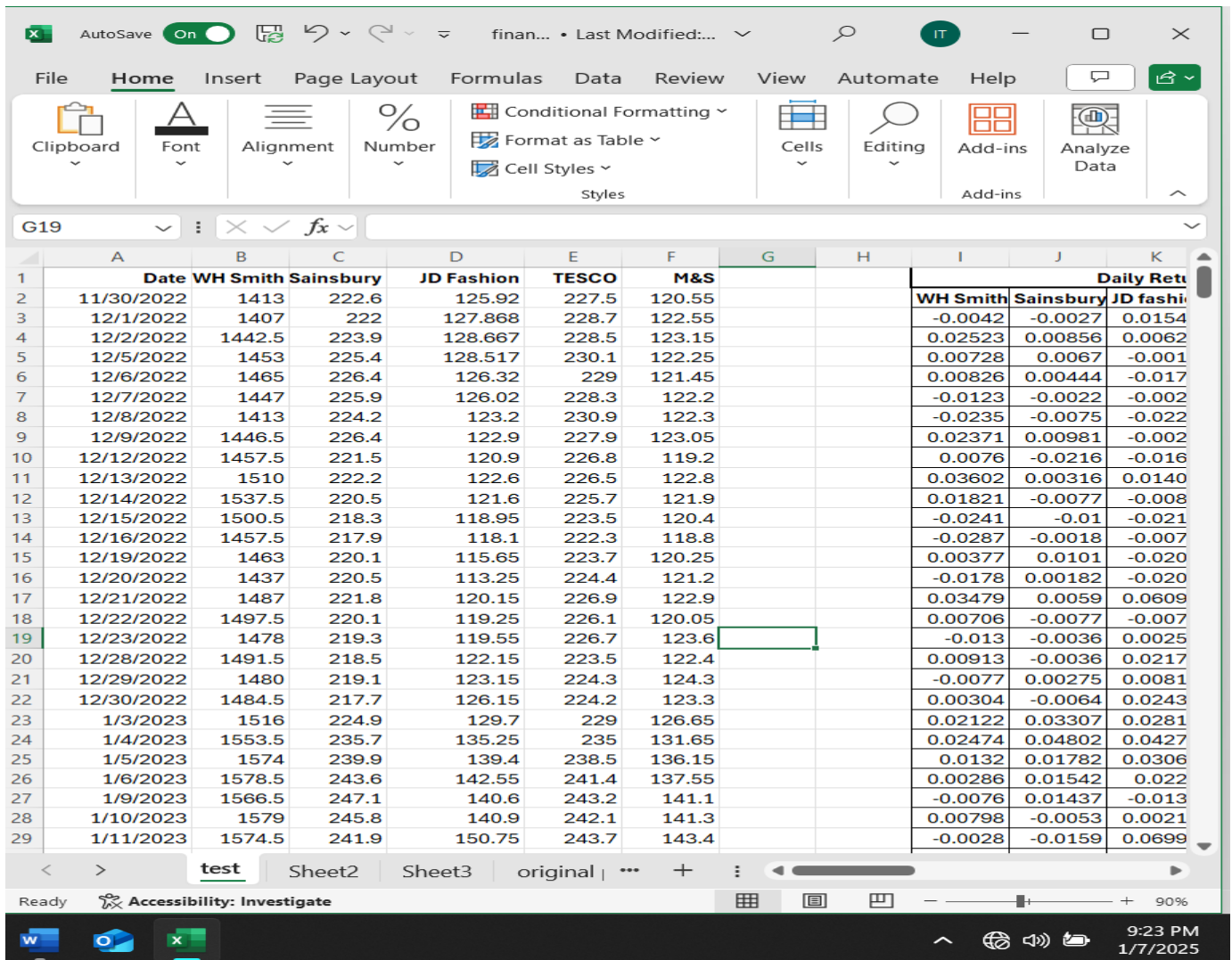


## Introduction:

This report analyses the historical performance and risk metrics of five UK-based companies: Sainsbury, JD Fashion, Tesco, and Marks and Spencer(M&S). The purpose of this analysis is to provide key insight to investors about the market and to give them an idea of which company they should invest in to earn a good return. The selected companies are all retail and consumer-focused sectors providing varying levels of risk and return which offers opportunities for portfolio diversification. As these companies are familiar to most investors having a good reputation in the market makes it relevant to wider audiences and investors. The closing prices for all five companies were sourced from stooq.com for 2 years from 30<sup>th</sup> November 2022 to 30<sup>th</sup> November 2024. These data were used to find portfolio optimization techniques and risk management strategies.

## Data set:

The data set has the closing values of all the firms for a period of 2 years and the data looks like Figure 1.1 in which the first column has all the dates starting from 30<sup>th</sup> November 2022 to 30<sup>th</sup> November 2024. The data are arranged in the order in which the oldest day comes first with the consecutive dates following it.



	A	B	C	D	E	F	G	H	I	J	K
1	Date	WH Smith	Sainsbury	JD Fashion	TESCO	M&S			Daily Return		
2	11/30/2022	1413	222.6	125.92	227.5	120.55			WH Smith	Sainsbury	JD fashi
3	12/1/2022	1407	222	127.868	228.7	122.55			-0.0042	-0.0027	0.0154
4	12/2/2022	1442.5	223.9	128.667	228.5	123.15			0.02523	0.00856	0.0062
5	12/5/2022	1453	225.4	128.517	230.1	122.25			0.00728	0.0067	-0.001
6	12/6/2022	1465	226.4	126.32	229	121.45			0.00826	0.00444	-0.017
7	12/7/2022	1447	225.9	126.02	228.3	122.2			-0.0123	-0.0022	-0.002
8	12/8/2022	1413	224.2	123.2	230.9	122.3			-0.0235	-0.0075	-0.022
9	12/9/2022	1446.5	226.4	122.9	227.9	123.05			0.02371	0.00981	-0.002
10	12/12/2022	1457.5	221.5	120.9	226.8	119.2			0.0076	-0.0216	-0.016
11	12/13/2022	1510	222.2	122.6	226.5	122.8			0.03602	0.00316	0.0140
12	12/14/2022	1537.5	220.5	121.6	225.7	121.9			0.01821	-0.0077	-0.008
13	12/15/2022	1500.5	218.3	118.95	223.5	120.4			-0.0241	-0.01	-0.021
14	12/16/2022	1457.5	217.9	118.1	222.3	118.8			-0.0287	-0.0018	-0.007
15	12/19/2022	1463	220.1	115.65	223.7	120.25			0.00377	0.0101	-0.020
16	12/20/2022	1437	220.5	113.25	224.4	121.2			-0.0178	0.00182	-0.020
17	12/21/2022	1487	221.8	120.15	226.9	122.9			0.03479	0.0059	0.0609
18	12/22/2022	1497.5	220.1	119.25	226.1	120.05			0.00706	-0.0077	-0.007
19	12/23/2022	1478	219.3	119.55	226.7	123.6			-0.013	-0.0036	0.0025
20	12/28/2022	1491.5	218.5	122.15	223.5	122.4			0.00913	-0.0036	0.0217
21	12/29/2022	1480	219.1	123.15	224.3	124.3			-0.0077	0.00275	0.0081
22	12/30/2022	1484.5	217.7	126.15	224.2	123.3			0.00304	-0.0064	0.0243
23	1/3/2023	1516	224.9	129.7	229	126.65			0.02122	0.03307	0.0281
24	1/4/2023	1553.5	235.7	135.25	235	131.65			0.02474	0.04802	0.0427
25	1/5/2023	1574	239.9	139.4	238.5	136.15			0.0132	0.01782	0.0306
26	1/6/2023	1578.5	243.6	142.55	241.4	137.55			0.00286	0.01542	0.022
27	1/9/2023	1566.5	247.1	140.6	243.2	141.1			-0.0076	0.01437	-0.013
28	1/10/2023	1579	245.8	140.9	242.1	141.3			0.00798	-0.0053	0.0021
29	1/11/2023	1574.5	241.9	150.75	243.7	143.4			-0.0028	-0.0159	0.0699

Figure 1.1

There are a total of 506 data to obtain the data to yield positive average returns. Figure 1.2

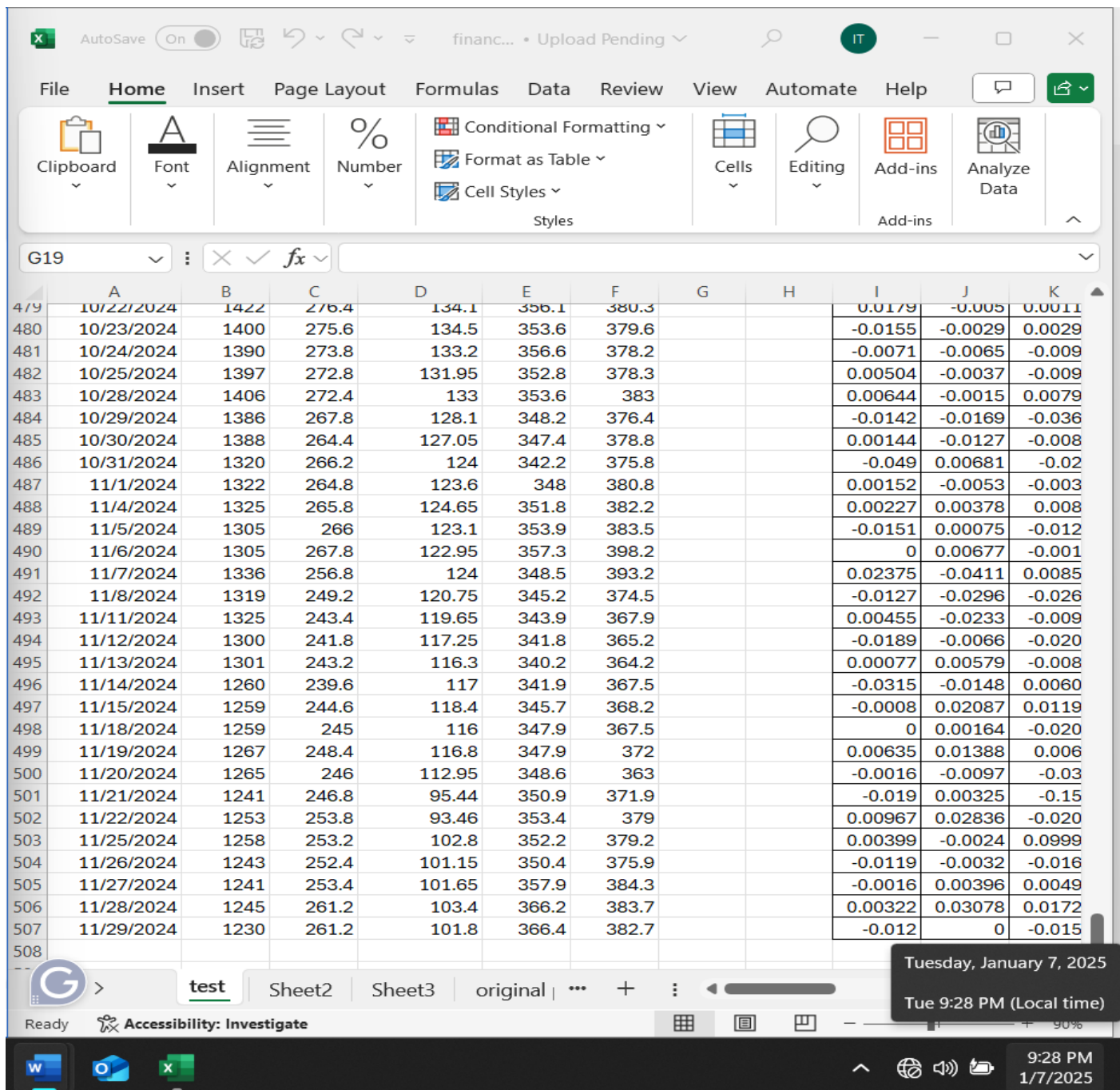


Figure 1.2

Average Daily Returns: After uploading the data into Excel, the next step is to find each day's returns using the formula

$$\text{Daily Returns} = (\text{Present value} - \text{Future Value}) / \text{Future Value}$$

from the original data shown in Figure 1.3. In this picture, the B3 represents the Present Value and the B2 is the future value. The same must be done for each day and separately for each company. This step will be used in calculating the Average Daily Returns and the Standard Deviation for all the companies.

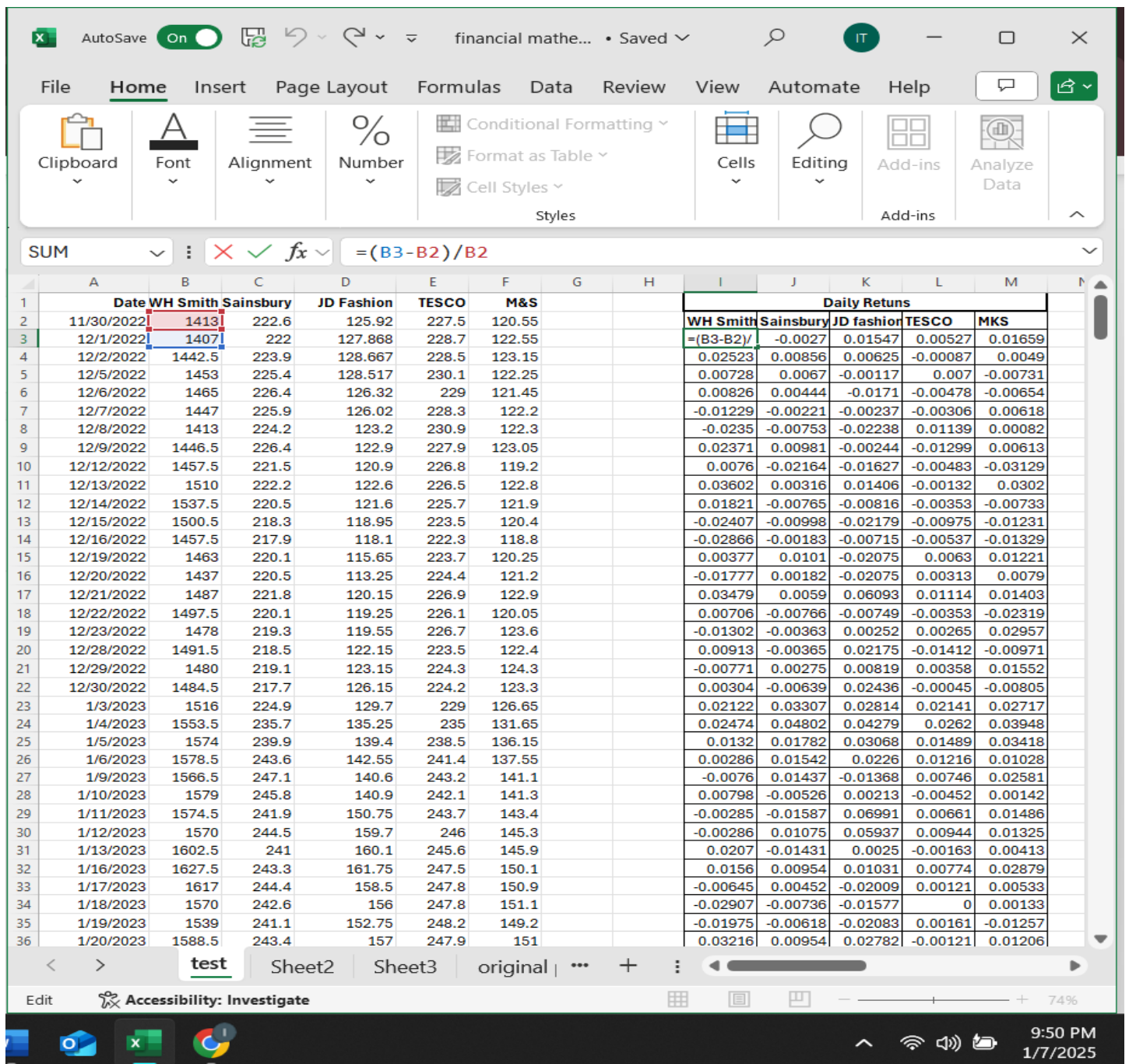


Figure 1.2

### Average Expected Return:

After calculating the daily returns for all the companies, the next step is to find the Average Expected Returns, which is the average of the daily returns of each company. The formula used to calculate the Average expected return is by using the **AVERAGE** function from the Excel. The next step would be to select the daily returns, and it will give an output with the Average Expected Return. Figure 1.3. The same process has to be repeated with each company to obtain the average expected return of each company, which then has to be annualized. To annualize data it has to be multiplied by 252 which is the total number of working days in a year excluding all the bank holidays. The formula for annualized expected return would be

$$\text{Annualised Expected Return} = \text{Average Expected Return} * 252$$

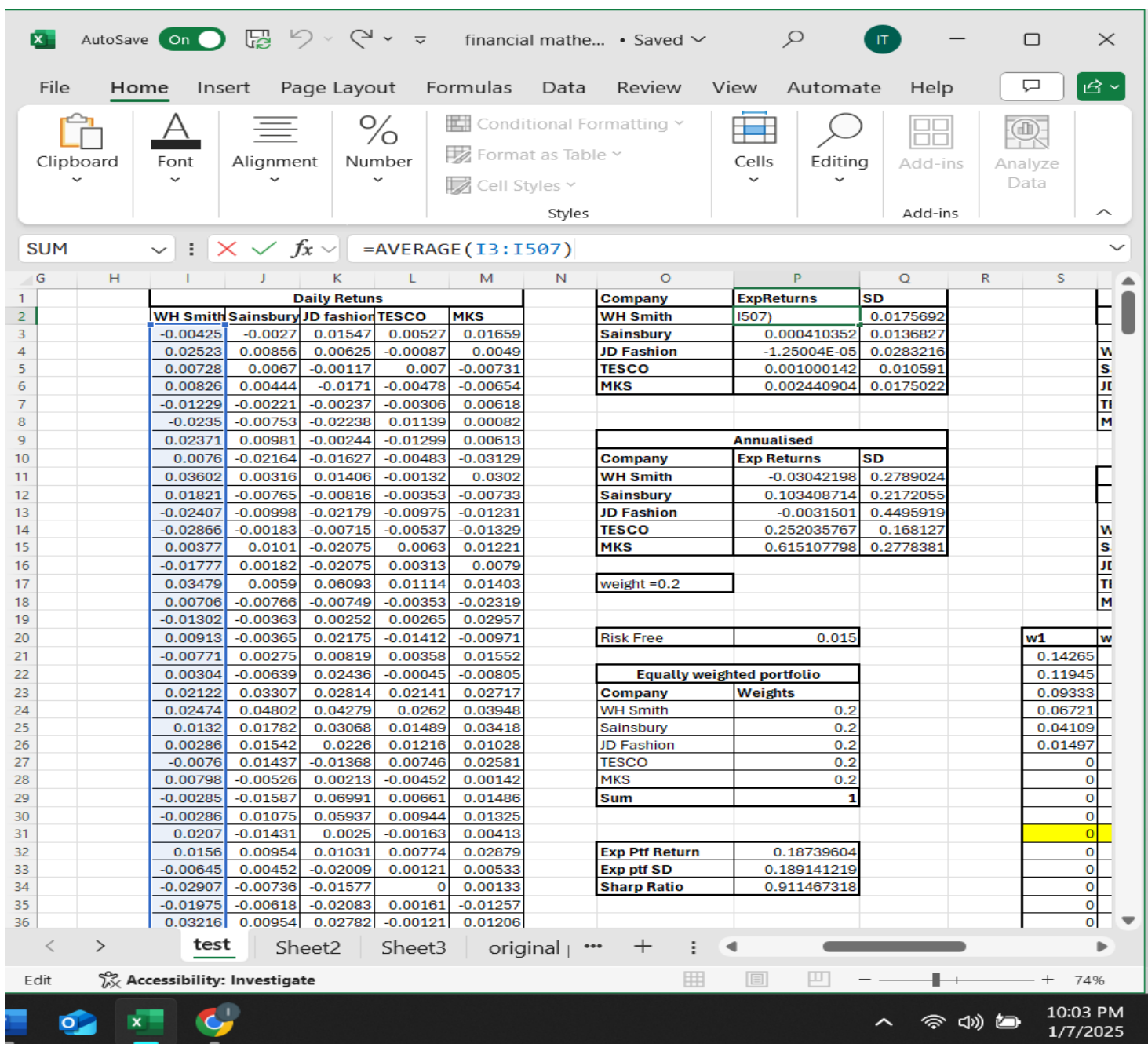


Figure 1.3

## Standard Deviation:

The next step would be to calculate the Standard deviation which is the squared root of the variance. For calculating first, we have to find the variance in Excel by using the function VAR with the values of the respective daily returns of the company. After finding the variance the next step would be to calculate the standard deviation by square rooting the variance by using SQRT of the variance. Figure 1.5. The final formula for finding the standard deviation would be

$$\text{Standard Deviation (SD)} = \text{SQRT}(\text{VAR}(\text{Daily Returns}))$$

After calculating the SD we have to annualize the results to get the Annualised Standard Deviation by multiplying the SD with the squared root of 252. Figure 1.6



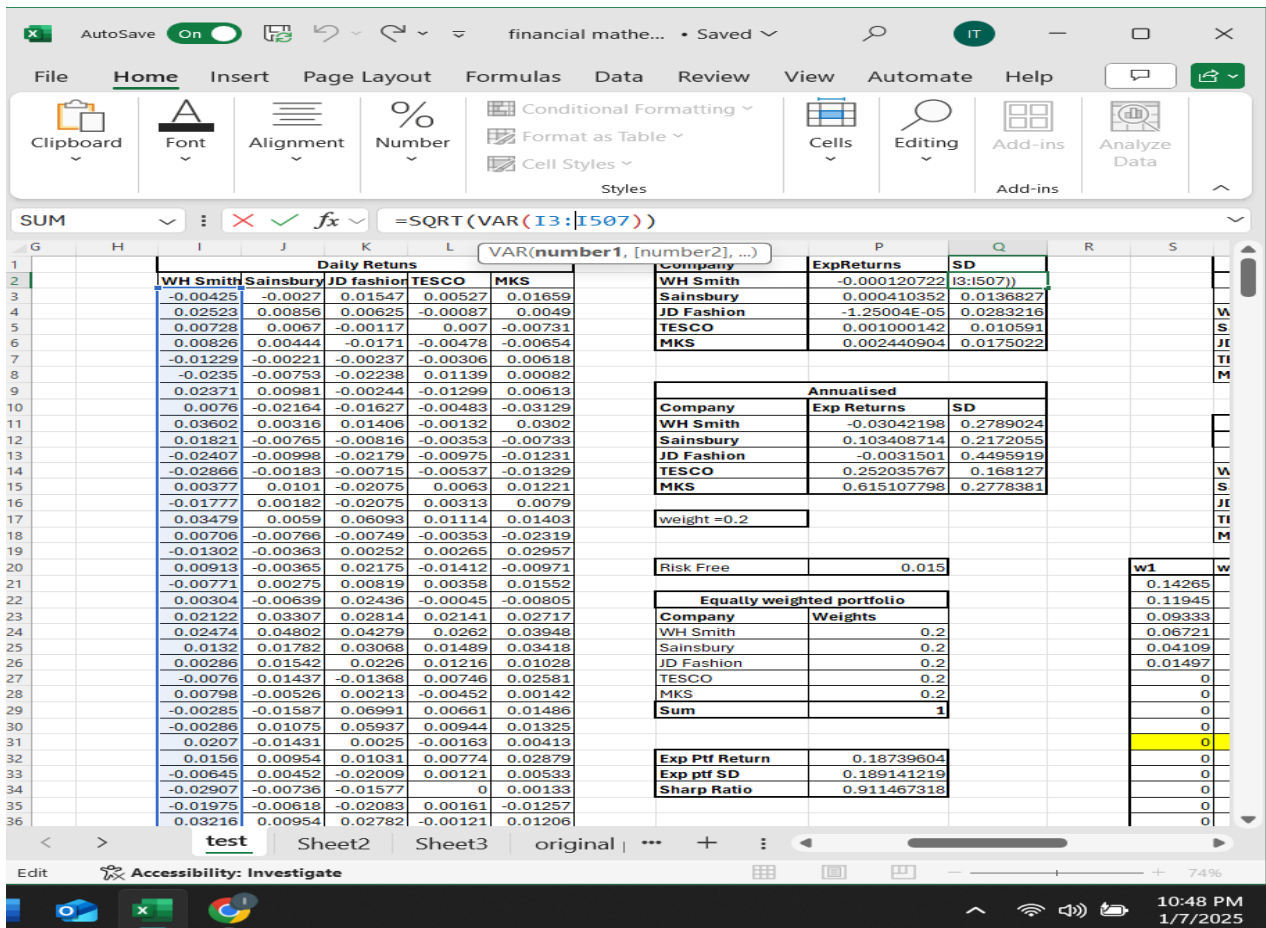


Figure 1.5

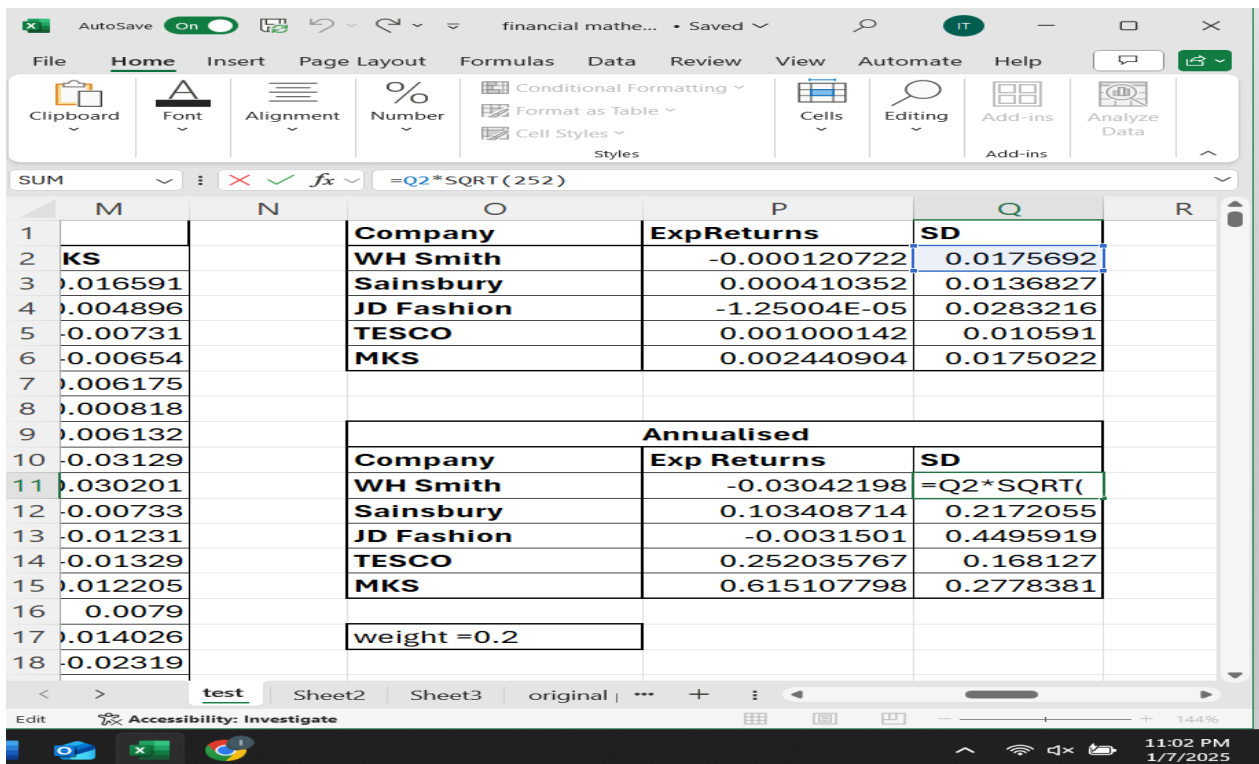


Figure 1.6

## Correlation Matrix:

The correlation matrix has been calculated to find the correlation that the different assets have with each other. This was calculated using the **data analysis** function from the **data** section in Excel. After the selection of the **correlation** function the input should be the daily return of all the companies together will automatically give output in a matrix format. Figure 1.7 The 1s in the matrix format indicated a perfect correlation (where a stock correlates with itself). Since all the values have a positive value this means that all the assets have a positive correlation with each other, this also means that all the assets are moving in the same direction. High positive correlations (0.6) suggest that including both stocks in a portfolio may not provide much diversification benefit, as they are likely to move together. Low or negative correlations can improve diversification, as they indicate that the stocks do not move together, potentially reducing overall portfolio risk.

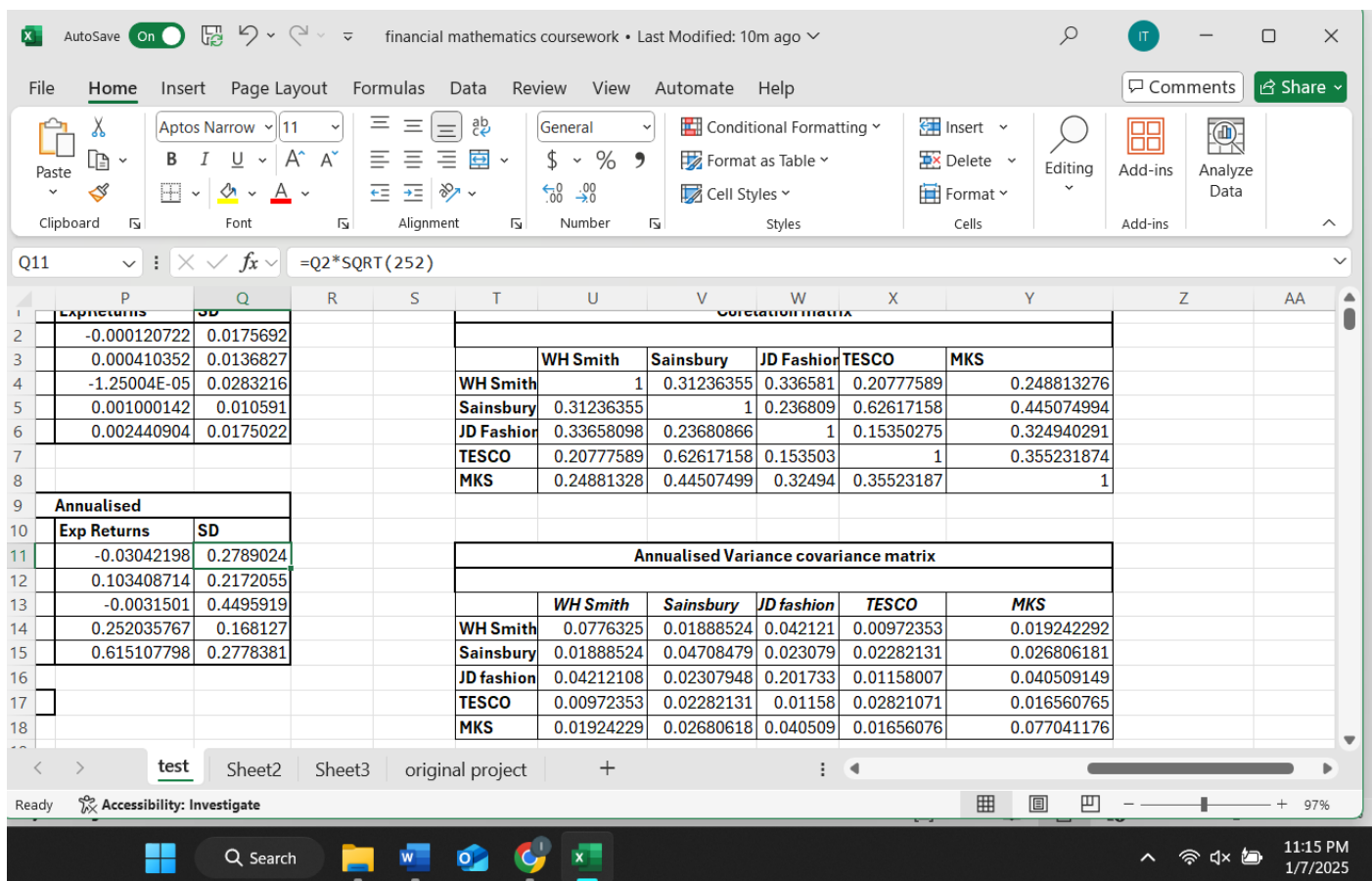


Figure 1.7

**Covariance variance matrix:** The covariance variance matrix is calculated to find the stock movement and the risk or volatility of each asset over the given period. The values tell how each stock deviates from its mean over time. Figure 1.8 By analysing the covariance variance matrix it can be observed that the variance (diagonal elements) of JD Fashion has shown very high volatility (0.2017) indicating that it is very risky followed by WH Smith (0.0776), MKS(0.0770), and Sainsbury and Tesco showed lower volatility which means lower risk. While the covariance of the assets (off-diagonal elements) shows that WH Smith and Sainsbury(0.0189) have positive covariance which means they are likely to move in the same direction, Covariance between JD fashion and Tesco(0.0116) shows less positive covariance which means they will move together but they will not have a strong relationship. Next would be the Sainsbury and MKS(0.0268) suggests that they have a moderate relationship, while

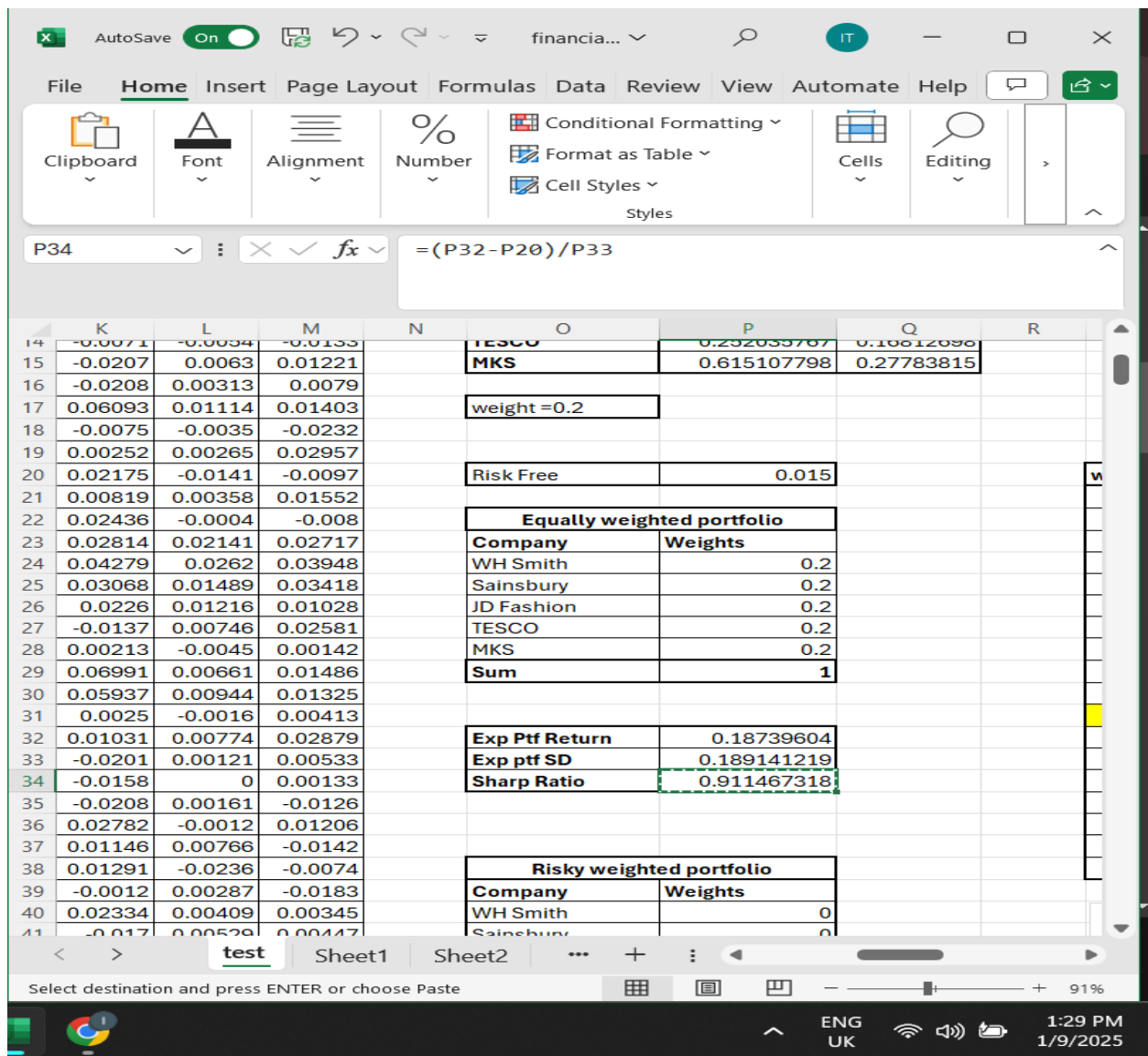
JD Fashion and WH Smith(0.0421) have more positive covariance and have higher chances of moving together. Thus through the covariance variance matrix, it can be said that JD Fashion & WH Smith and Sainsbury & MKS have higher positive covariance and are more likely to move together in the same direction while Tesco& JD Fashion and Tesco and WH Smith have lower positive covariance which suggests that they are less likely to follow each other's movements.

9							
10							
11		<b>Annualised Variance covariance matrix</b>					
12							
13			<b>WH Smith</b>	<b>Sainsbury</b>	<b>JD fashion</b>	<b>TESCO</b>	<b>MKS</b>
14		<b>WH Smith</b>	0.077632501	0.018885237	0.0421211	0.00972353	0.01924229
15		<b>Sainsbury</b>	0.018885237	0.047084787	0.0230795	0.022821315	0.02680618
16		<b>JD fashion</b>	0.042121076	0.023079479	0.2017327	0.011580072	0.04050914
17		<b>TESCO</b>	0.00972353	0.022821315	0.0115801	0.028210709	0.01656076
18		<b>MKS</b>	0.019242292	0.026806181	0.0405091	0.016560765	0.07704117
19							
20	<b>w1</b>	<b>w2</b>	<b>w3</b>	<b>w4</b>	<b>w5</b>	<b>SD</b>	<b>Returns</b>
21	0.14265	0	0.003528662	0.690884083	0.1629362	0.154201796	0.2
22	0.11945	0	0	0.683054408	0.1974936	0.15568356	0.2
23	0.09333	0	0	0.674408573	0.2322589	0.157827633	0.3
24	0.06721	0	0	0.665762964	0.2670242	0.160614774	0.3
25	0.04109	0	0	0.657117296	0.3017895	0.164012202	0.3
26	0.01497	0	0	0.648471681	0.3365548	0.16798289	0.3
27	0	0	0	0.620008638	0.3799914	0.172545943	0.3
28	0	0	0	0.564923149	0.4350768	0.178121185	0.4
29	0	0	0	0.509837671	0.4901623	0.18471588	0.4

Figure 1.8

To find the portfolio risk the first step would be to find the risk-free rate which here is 1.5% as given is the question, which would lead to distributing weights equally to find the equally weighted portfolio moving forward with the Risky weighted portfolio which will give the weights on each assets for a target return

**Equally weighted Portfolio:** To find the equally weighted portfolio, each company should be assigned with equal weights which should sum up to 1 so that the portfolio is equally diversified. Figure 1.8. The weight for each asset is 0.2.



. Figure 1.8.

The expected portfolio return would be calculated with the formulae

Expected portfolio Return= MMULT(transpose(Weights),Annualised expected return)

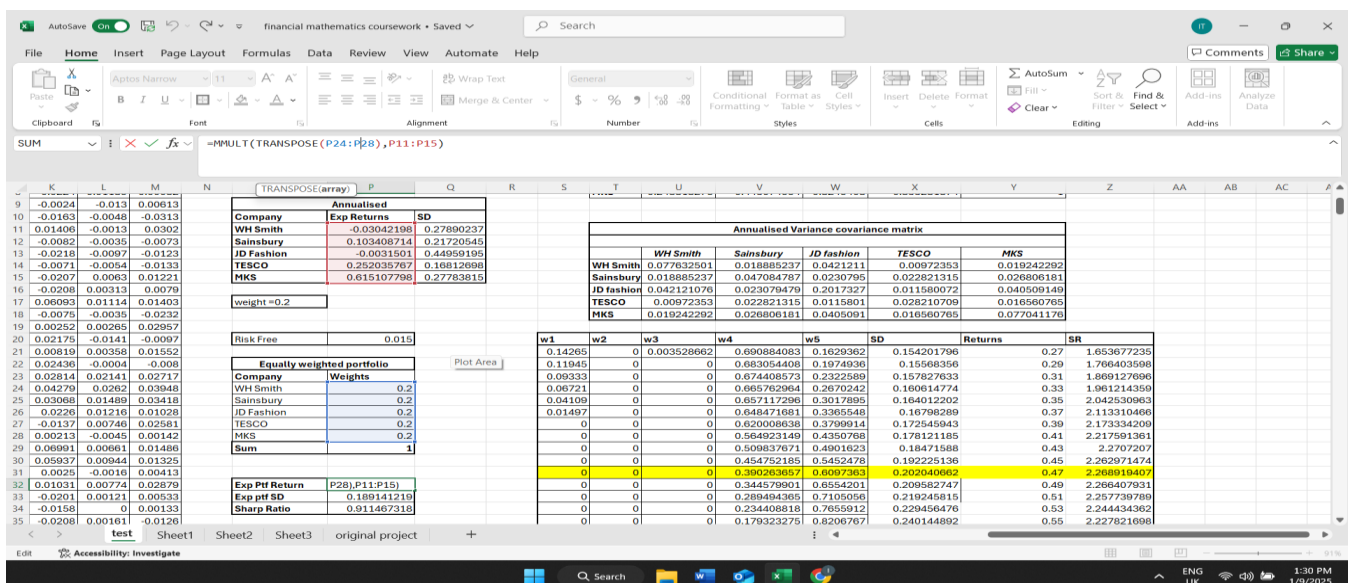


Figure 1.9



This would give us the expected portfolio return which is 0.18739604, (Figure 1.8,)then the SD for the portfolio is calculated using the formula

**SD for the portfolio= Sqrt(MMULT(MMULT(TRANPOSE(Weights),Covariance variance matrix), Weights))**

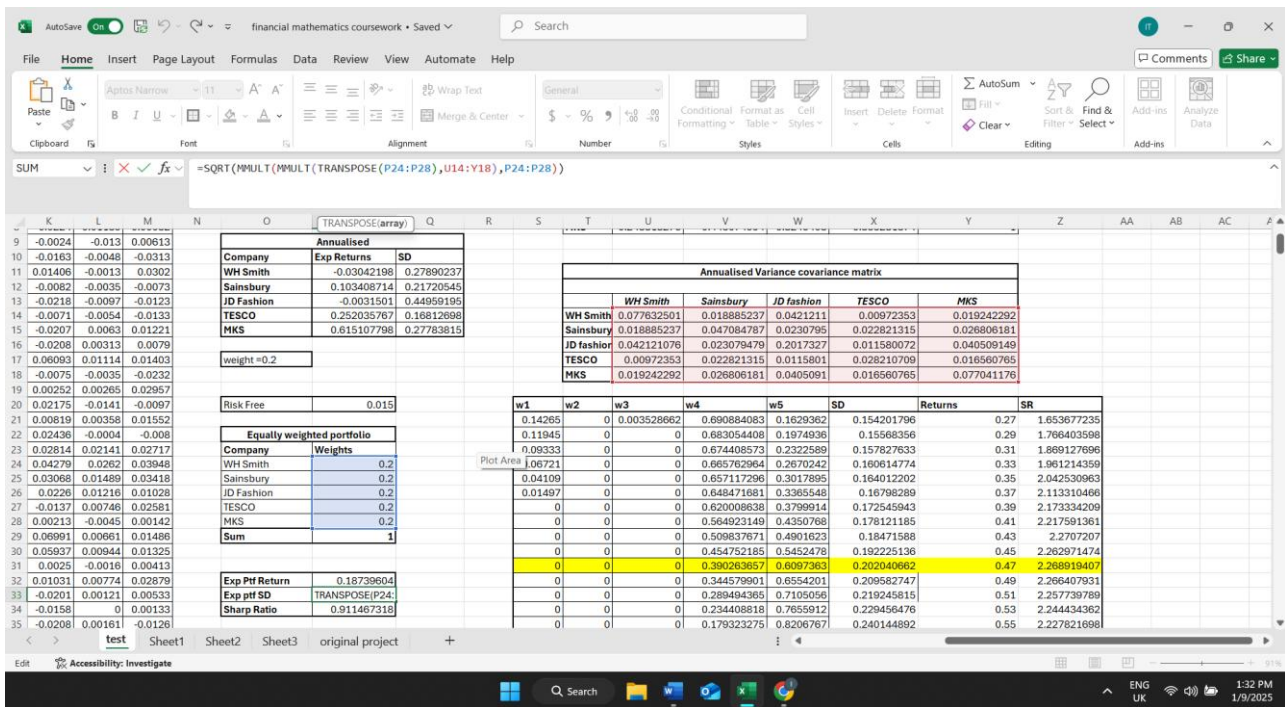


Figure 2.1

This will give the SD for the equally weighted portfolio which would be 0.18914219.(Figure 2.1) Finally, the sharp ratio is calculated by using the formula to give 0.911467318

**Sharp Ratio= (Expected Return-Risk Free Rate)/SD**

This will give the Sharp Ratio for the equally weighted portfolio which is 0.911467318(Figure 2.2)

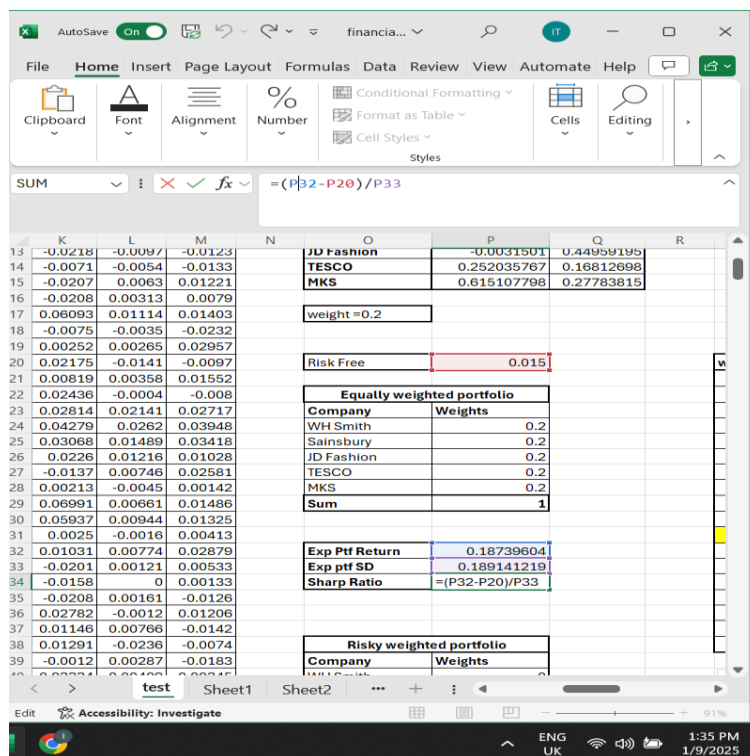


Figure 2.2

**Risky Weighted portfolio:** At first same steps as the equally weighted portfolio should be followed, after calculating the SR then we go ahead to use the solver function in the EXCEL to determine the portfolio risk and the percentage of investment in each asset by updating or increasing the expected return of the risky weighted portfolio. After doing if a couple of times, we get a series of instances with different weights and Sharp ratios. Among the series of instances, the one with the highest sharp ratio is highlighted and is used as the optimal market portfolio risk. So the optimal portfolio risk would be(Figure 2.3):

WH Smith:0

Sainsbury: 0

JD Fashion:0

Tesco:0.39

MKS:0.61

Expected Return: 0.47

SD:0.202040662

Sharp ratio:2.268919407

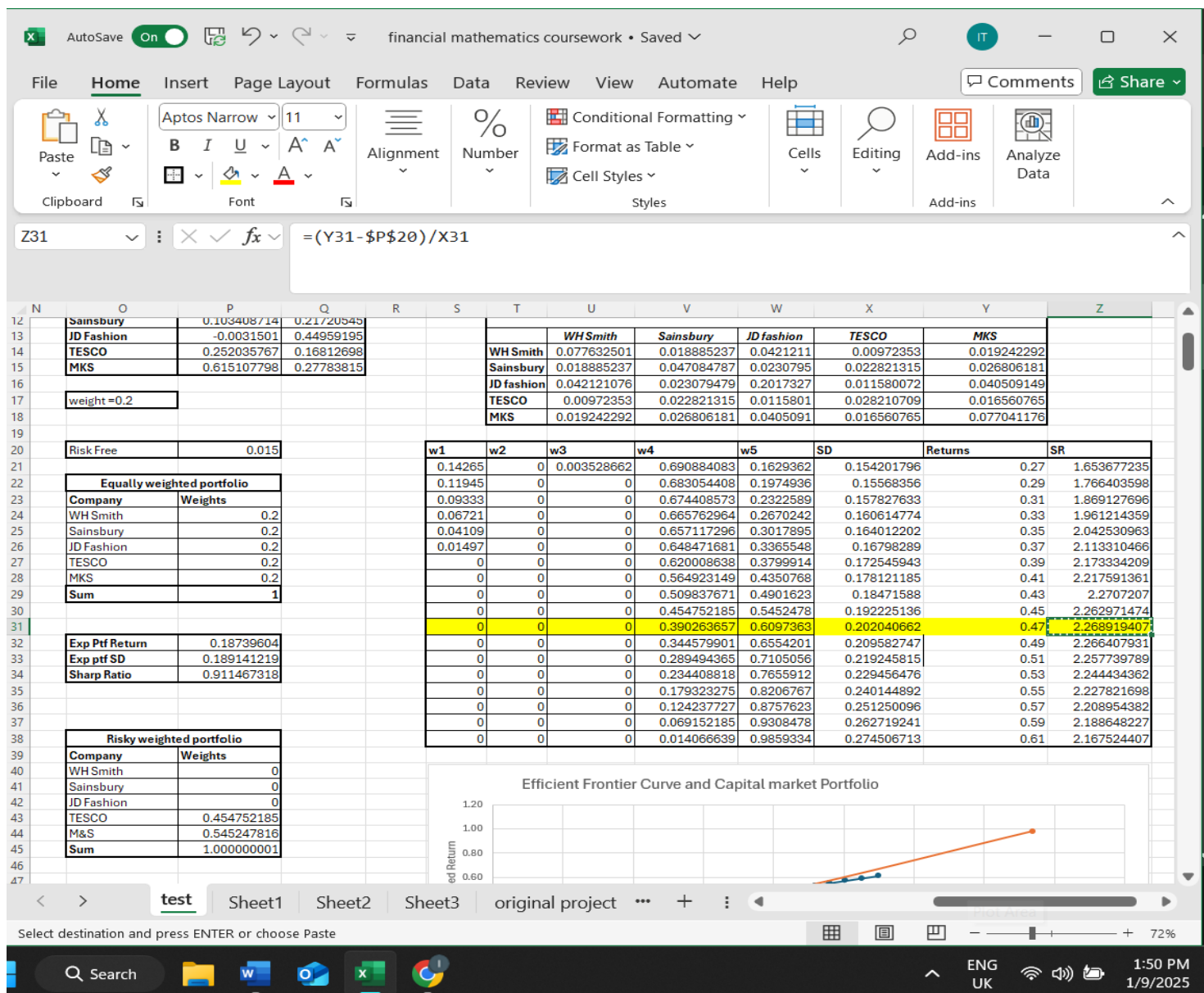


Figure 2.3

**Efficient Frontier Curve:** The efficient frontier curve that has been obtained by using the SD and the expected returns into a graph and choosing a scatter plot to mark the points. It shows the portfolio of assets that give the highest return for a given level of risk. The curve with the blue dots starts steep but flattens as the risk increases. The curve shows the best possible portfolios of assets which means any portfolio below the curve would not be good. The x axis shows the Standard Deviation and the Y axis shows the Expected return from the instances created by the solver. (Figure 2.4)

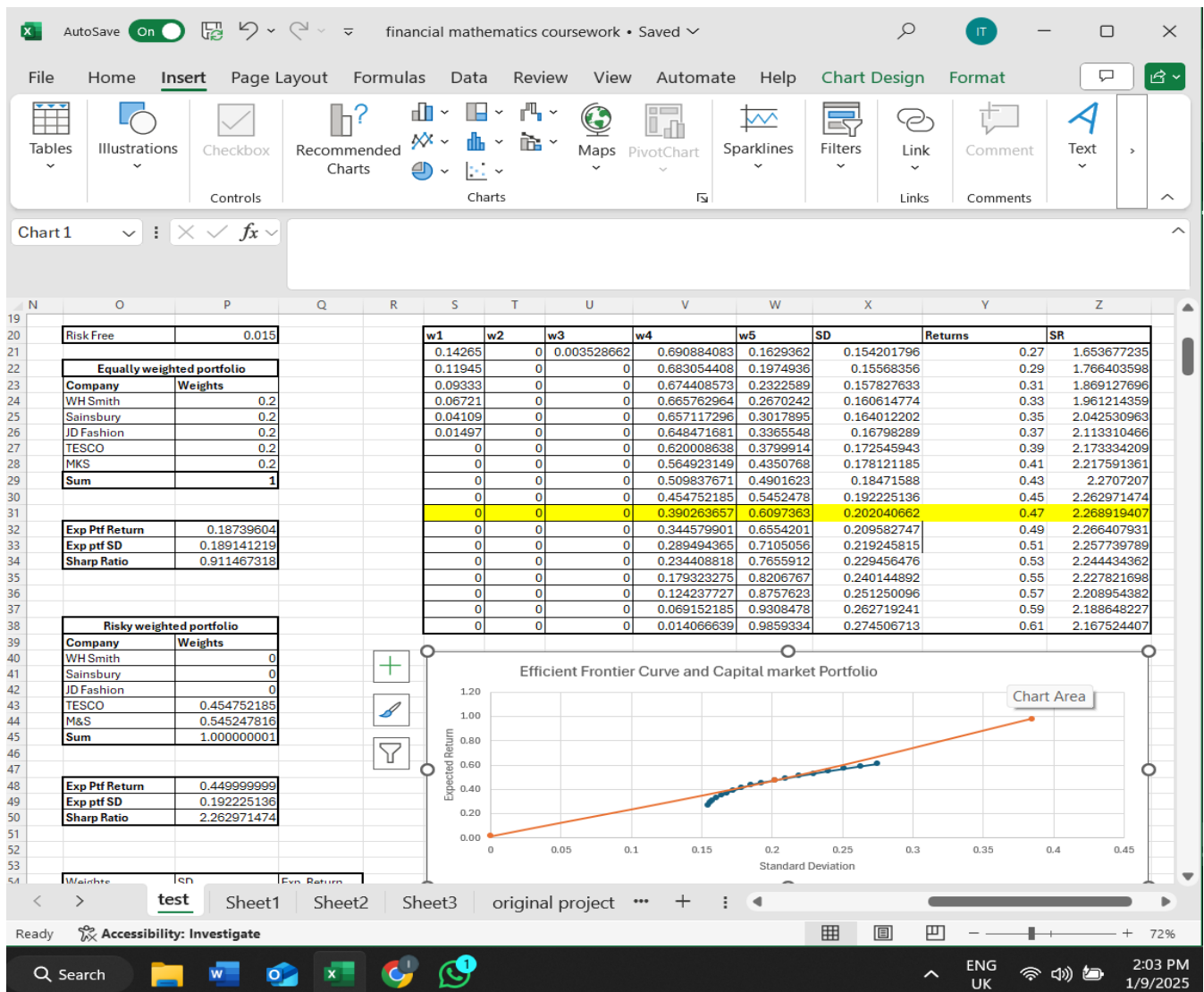


Figure2.4

**Capital Market Line:** The capital market line is calculated to allow investors to combine risk-free investments with their risky portfolio. Here the CML starts with risk-free rate which is 0.015. The orange line represents the CML. Any portfolio on the orange line is better than those on the blue curve because it gives a higher return for the same level of risk.(Figure 2.5). The points for CML are obtained by creating a table of weights starting from 0 to 3 with Standard Deviation and Expected Returns.

The formula to find the Standard Deviation is **Standard Deviation= Weight\*SD of the optimal market portfolio**, then the Expected return for the capital market line is calculated with the formula **Expected Return= Weights \* Expected Return of optimal market portfolio (1-weights) \*risk-free rate**. This has to be done for all the weights used to calculate the Capital market line which will give us the points for plotting it on the efficient frontier graph. (Figure 2.5). The point at which the CML and the efficient frontier curve meet is called the market portfolio, The CML shows the best combinations of risky assets and risk-free investments, offering higher returns for the same risk than the efficient frontier.

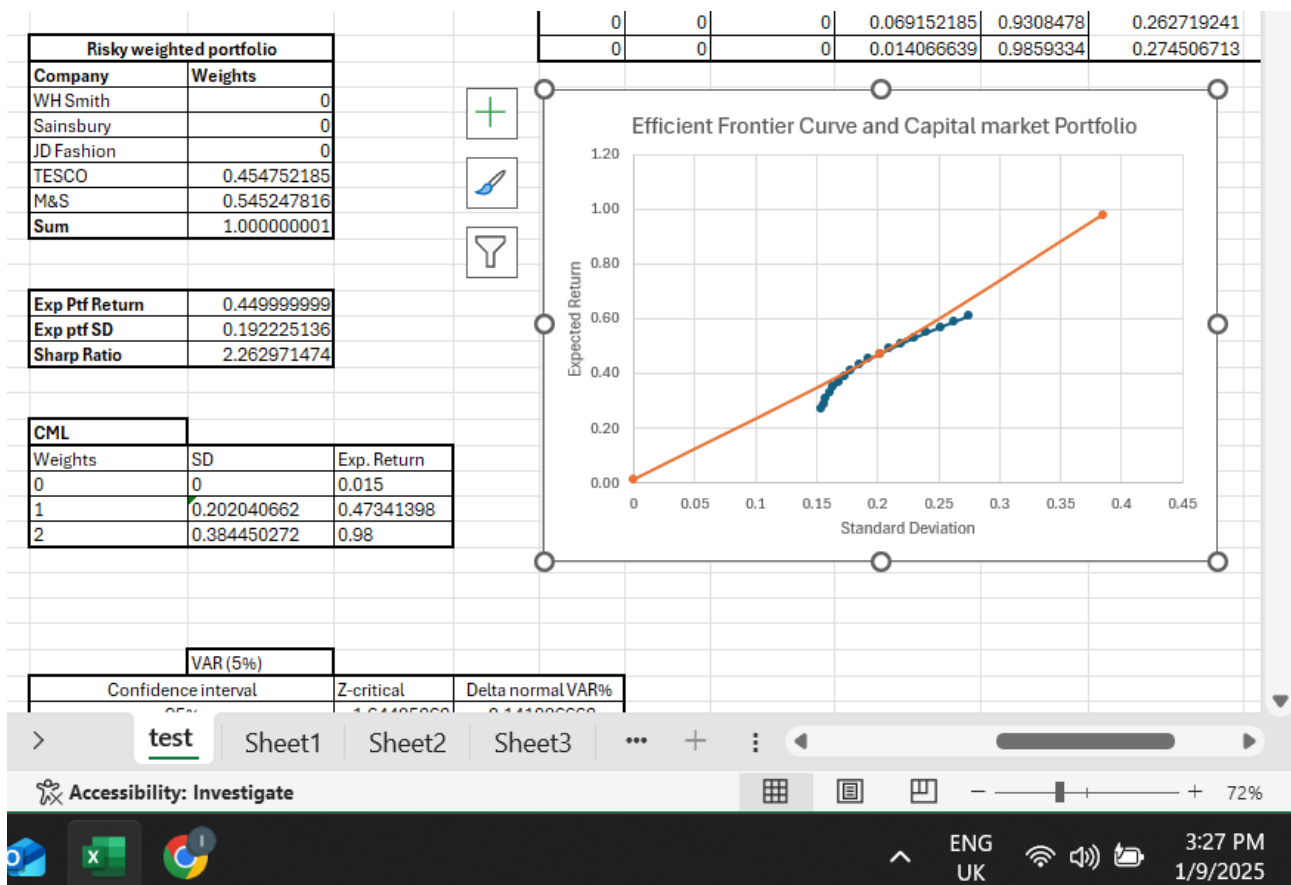


Figure 2.5

**Beta using linear regression:** To calculate the beta first step would be to download the data for the same date range of FTSE 100 to provide as a benchmark to represent the overall market . The daily returns for FTSE 100 has to be calculated in the same way as for others and then use the data analysis function to calculate the beta for each assts individually. The fuction asks for x and y input where the y input would be the daily returns of the Benchmark i.e., FTSE 100 and the x input would be the daily return of a individual asset.The output will have a range of data including the regression statistics, Anova and many more but the beta is found under the co-efficient section marked under X variable 1.Below are the beta for some of the assets used in this coursework.

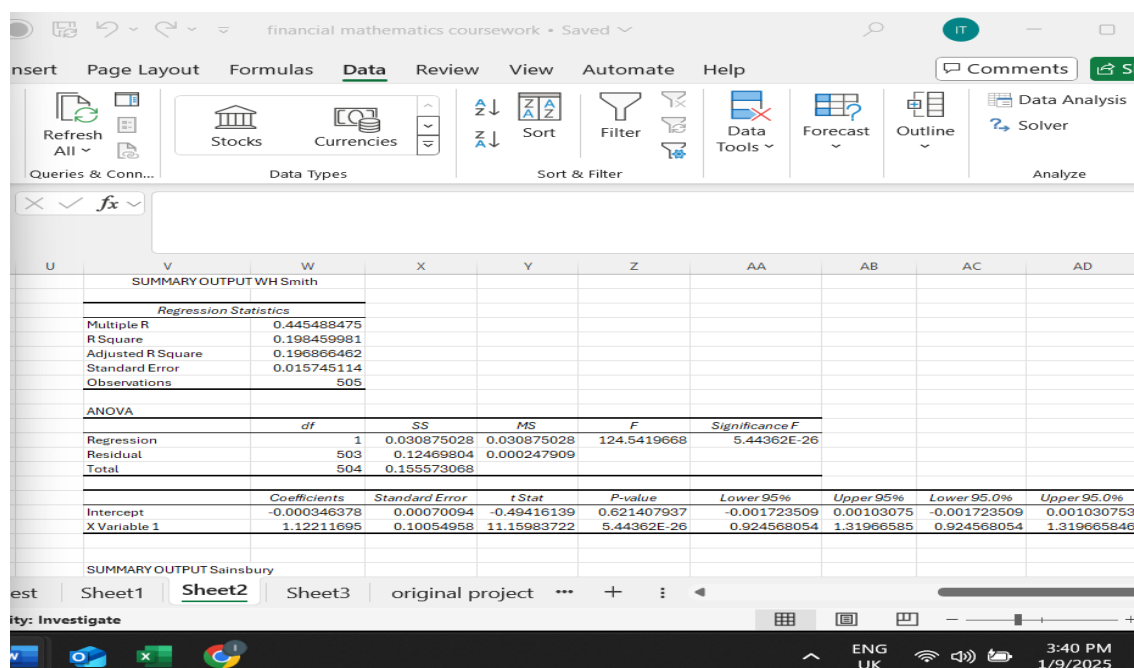


Figure 2.6-Beta for WH Smith



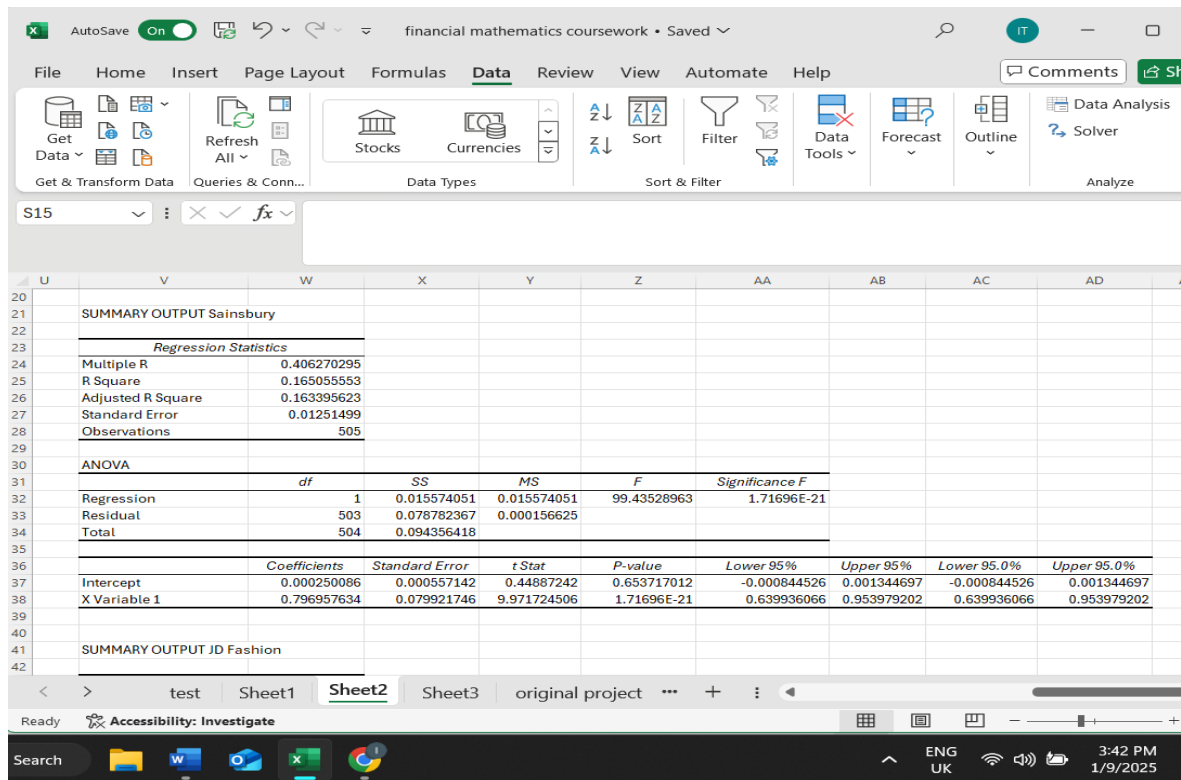


Figure 2.7 Beta for Sainsbury

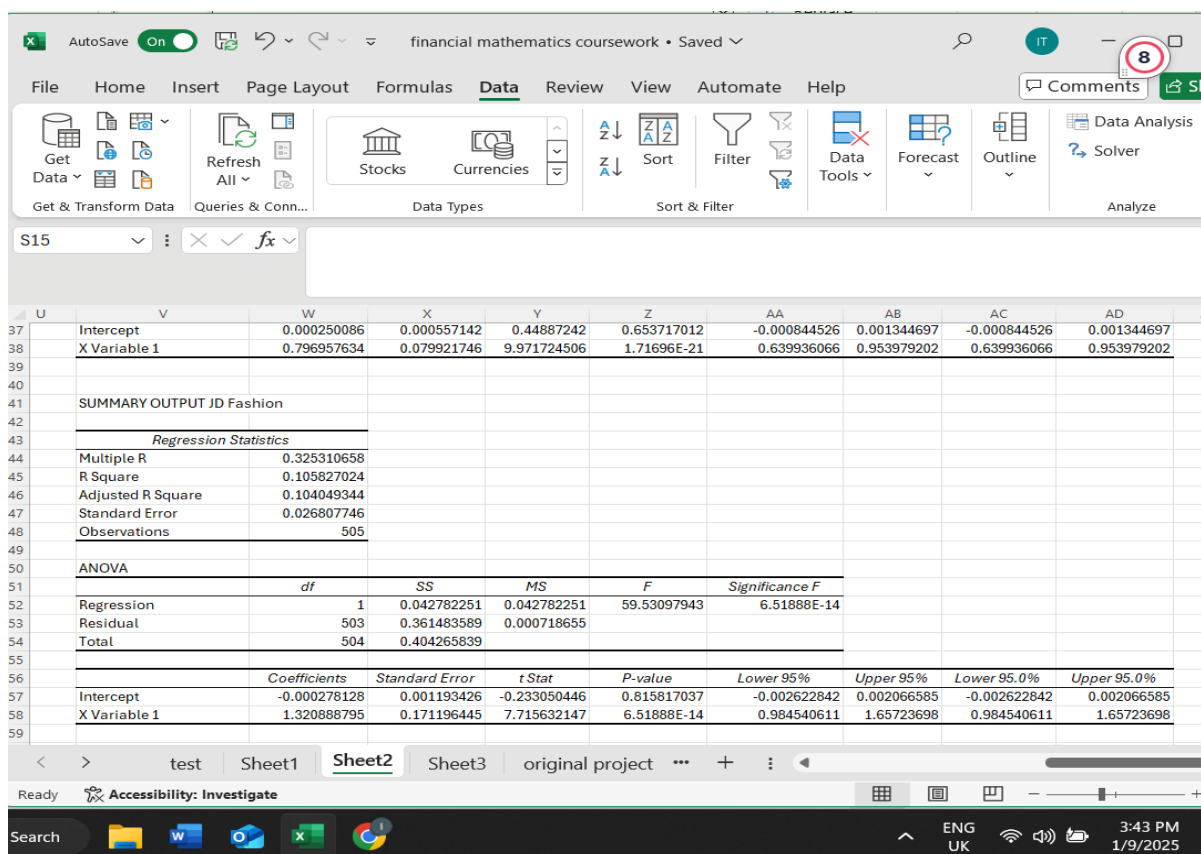


Figure 2.8

After calculating the beta for all 5 assets a table is made to store all the betas together. The assets like WH Smith and JD Fashion are growth-oriented (Beta >1) stocks which shows that it is highly sensitive to the market while Sainsbury, Tesco and M&S are less sensitive (Beta < 0) to the market fluctuations. Figure 2.9

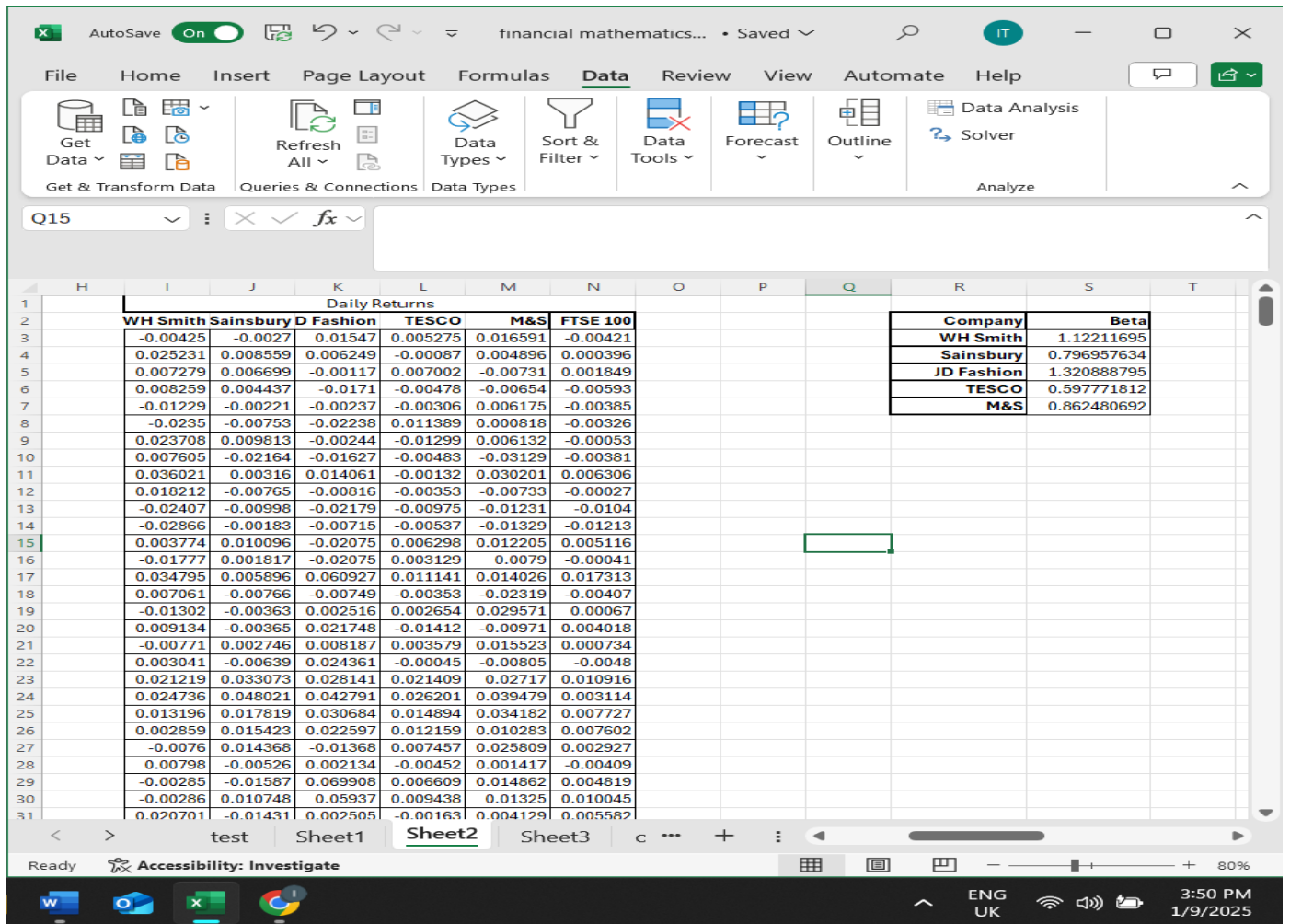


Figure 2.9

**VAR 5% and Asset Contribution:** VAR 5 % means that there is a 95% confidence that the losses will not exceed the value of VAR over a given period under normal market conditions. The formula to find the VAR5% is **VAR(5%)=Expected Return + Z-critical \* SD.**(Figure 3.0) The Z-critical is calculated by using the function **NORM.S.INV** and the formula is **Z- critical = NORM.S.INV(1-Confidence interval)** which gives **-1.6448536**. (Figure 3.1) The value after calculating with this formula is -0.141086 . The negative value shows that at a 5% probability, the portfolio is expected to lose up to 14.11% of its value over time.

After calculating the VAR for the portfolio the next step would be to find the contribution that each asset have on the VAR 5% . To calculate this we need the optimal portfolio SD and expected return and the weights along with the annualised weights of each asset. So the formula to calculate each asset contribution would be

**Contribution of individual asset=Optimal Weight of individual asset\*(SD of individual asset/Optimal Portfolio SD)\*Optimal portfolio Expected return**

The weights of each asset on the VaR would be

weights	Contribution
0	0
0	0
0	0
0.390263657	0.035818695
0.609736343	0.118299017

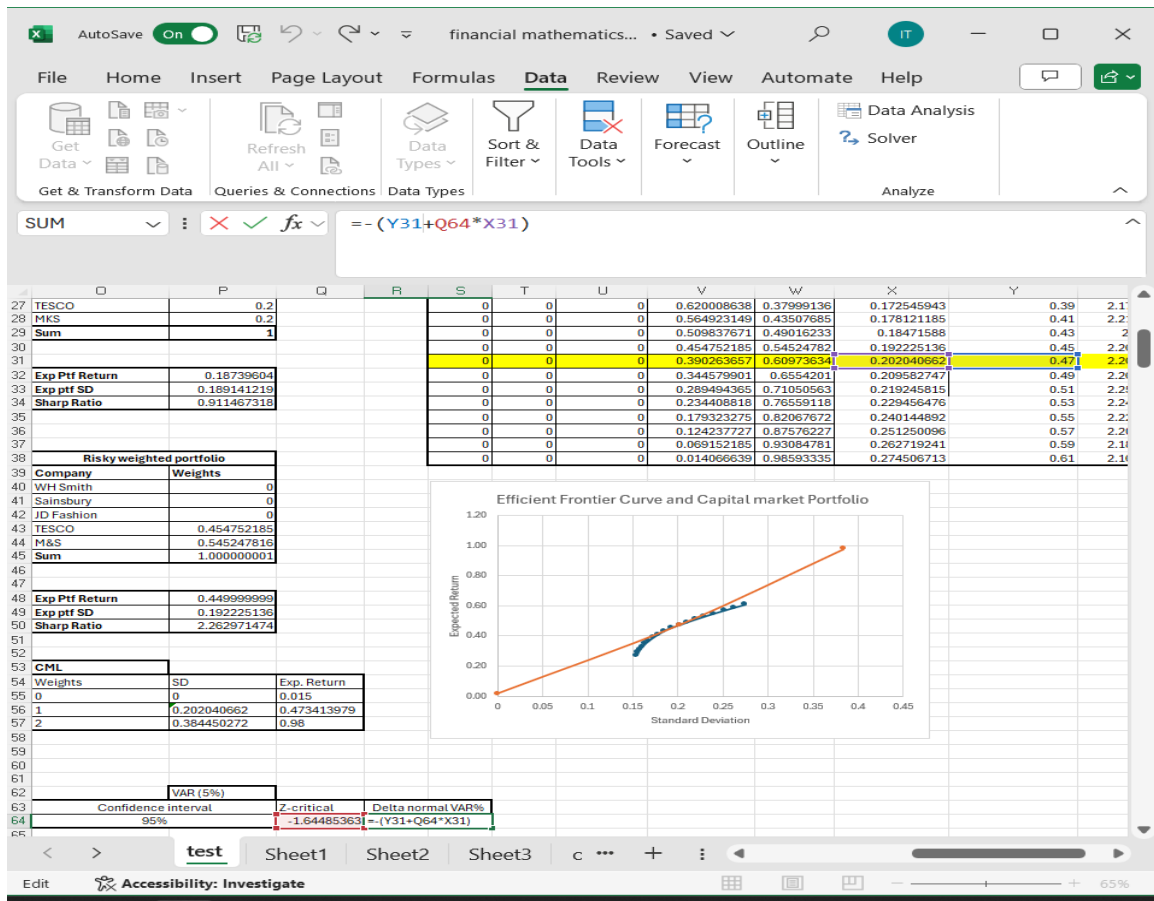


Figure 3.0

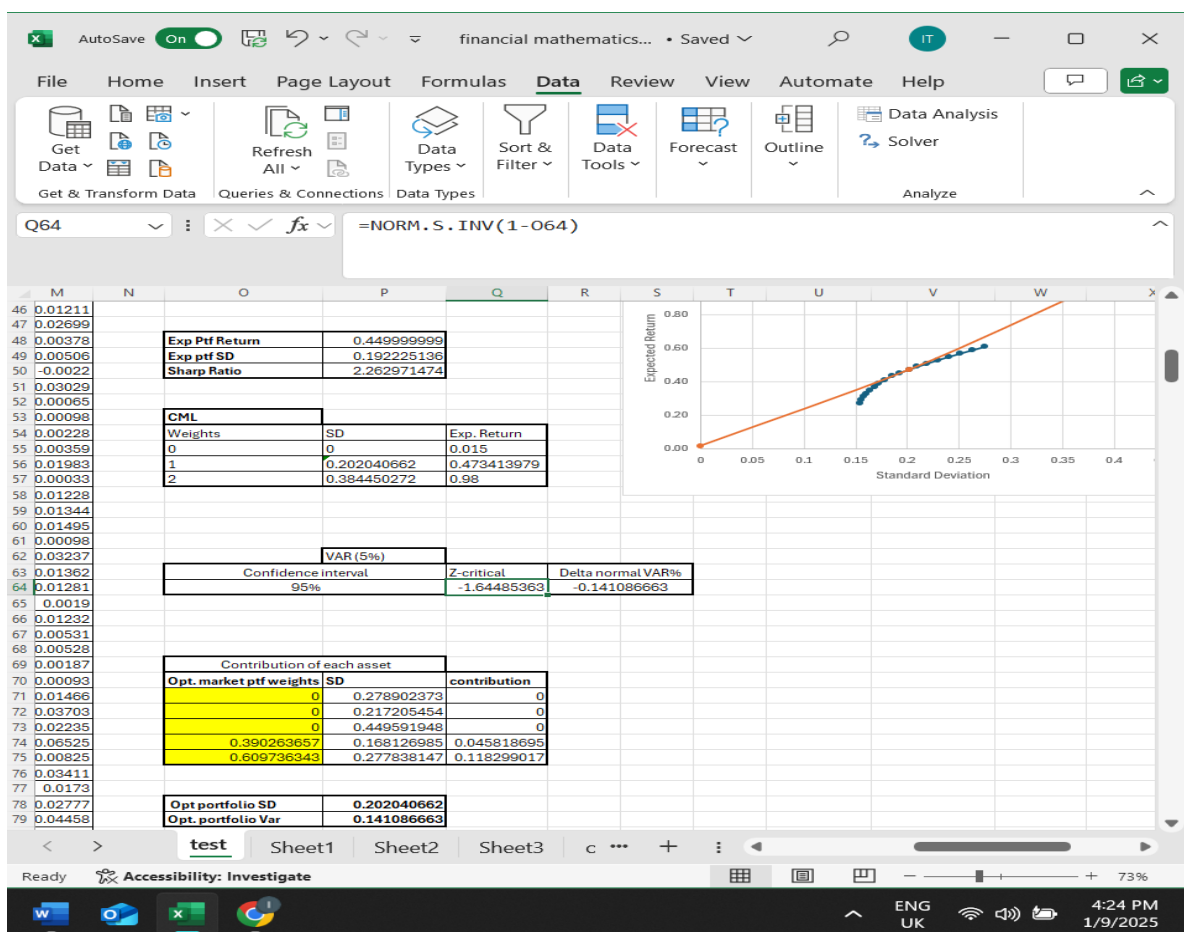
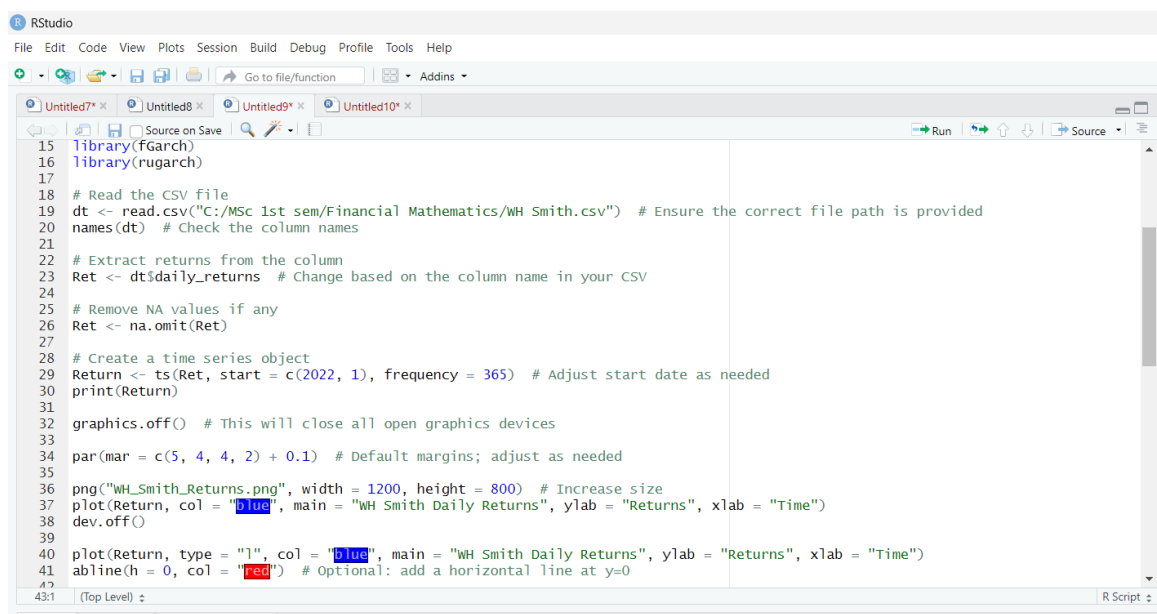


Figure 3.1

## Estimating the volatility:

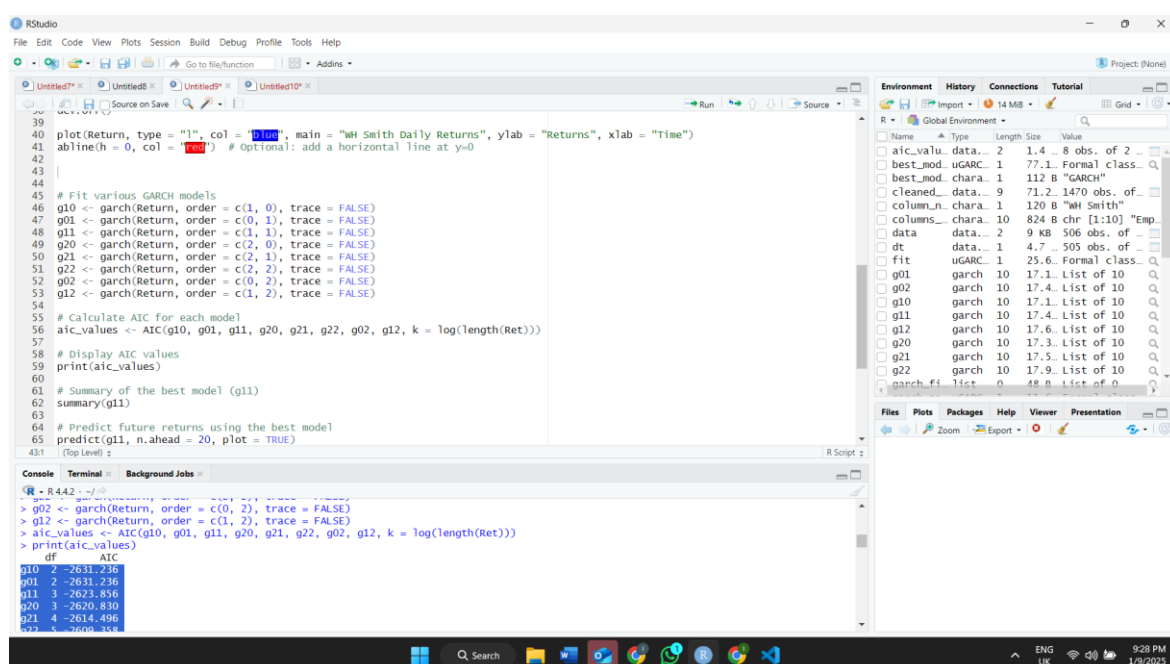
In this we analyse the volatility of one of the assets which is WH Smith using various ARCH/GARCH models. The data used to analyse this was the daily return of the WH Smith. R codes are written to clean the data with missing values, (Figure 3.2) To fit the model the into Garch model the code in Figure 3.3 was passed to return data including:

- GARCH(1,0)
- GARCH(0,1)
- GARCH(1,1)
- GARCH(2,0)
- GARCH(2,1)
- GARCH(2,2)
- GARCH(0,2)
- GARCH(1,2)



```
15 library(fGarch)
16 library(rugarch)
17
18 # Read the CSV file
19 dt <- read.csv("C:/MSc 1st sem/Financial Mathematics/WH Smith.csv") # Ensure the correct file path is provided
20 names(dt) # Check the column names
21
22 # Extract returns from the column
23 Ret <- dt$daily_returns # Change based on the column name in your CSV
24
25 # Remove NA values if any
26 Ret <- na.omit(Ret)
27
28 # Create a time series object
29 Return <- ts(Ret, start = c(2022, 1), frequency = 365) # Adjust start date as needed
30 print(Return)
31
32 graphics.off() # This will close all open graphics devices
33
34 par(mar = c(5, 4, 4, 2) + 0.1) # Default margins; adjust as needed
35
36 png("WH_Smith_Returns.png", width = 1200, height = 800) # Increase size
37 plot(Return, col = "blue", main = "WH Smith Daily Returns", ylab = "Returns", xlab = "Time")
38 dev.off()
39
40 plot(Return, type = "l", col = "blue", main = "WH Smith Daily Returns", ylab = "Returns", xlab = "Time")
41 abline(h = 0, col = "red") # Optional: add a horizontal line at y=0
```

Figure 3.2



```
39
40 plot(Return, type = "l", col = "blue", main = "WH Smith Daily Returns", ylab = "Returns", xlab = "Time")
41 abline(h = 0, col = "red") # Optional: add a horizontal line at y=0
42
43
44 # Fit various GARCH models
45 g10 <- garch(Return, order = c(1, 0), trace = FALSE)
46 g01 <- garch(Return, order = c(0, 1), trace = FALSE)
47 g11 <- garch(Return, order = c(1, 1), trace = FALSE)
48 g20 <- garch(Return, order = c(2, 0), trace = FALSE)
49 g21 <- garch(Return, order = c(2, 1), trace = FALSE)
50 g22 <- garch(Return, order = c(2, 2), trace = FALSE)
51 g02 <- garch(Return, order = c(0, 2), trace = FALSE)
52 g12 <- garch(Return, order = c(1, 2), trace = FALSE)
53
54 # Calculate AIC for each model
55 aic_values <- AIC(g10, g01, g11, g20, g21, g22, g02, g12, k = log(length(Ret)))
56
57 # Display AIC values
58 print(aic_values)
59
60 # Summary of the best model (g11)
61 summary(g11)
62
63 # Predict future returns using the best model
64 predict(g11, n.ahead = 20, plot = TRUE)
```

Console Output:

```
R - R442.2.1 ~/>
> g02 <- garch(Return, order = c(0, 2), trace = FALSE)
> g12 <- garch(Return, order = c(1, 2), trace = FALSE)
> aic_values <- AIC(g10, g01, g11, g20, g21, g22, g02, g12, k = log(length(Ret)))
> print(aic_values)
df      AIC
g10 2 -2631.236
g01 2 -2631.236
g11 3 -2623.856
g20 3 -2620.830
g21 4 -2614.496
g22 4 -2609.458
```

Environment Panel:

Name	Type	Length	Size	Value
aic_valu...	data...	2	1.4	8 obs. of 2
best_mod...	gARCh...	1	77.1	Formal class...
best_mod...	chara...	1	112 B	"GARCH"
cleaned_...	data...	9	71.2	1470 obs. of
columns_...	chara...	1	120 B	"WH Smith"
columns_...	chara...	10	824 B	chr [1:10] "Emp...
data	data...	2	9 KB	506 obs. of
dt	data...	1	4.7	505 obs. of
fit	gARCh...	1	25.6	Formal class...
g01	garch	10	17.1	List of 10
g02	garch	10	17.4	List of 10
g10	garch	10	17.1	List of 10
g11	garch	10	17.4	List of 10
g12	garch	10	17.6	List of 10
g20	garch	10	17.3	List of 10
g21	garch	10	17.5	List of 10
g22	garch	10	17.9	List of 10
garch fi...	list	0	48 B	List of 0

Figure 3.3



After that the AIC results will prove that which model suits the best. The Model with the lowest AIC will be considered the best fit among the model tested. By seeing the output we can say that the GARCH(2,1) with an AIC value of -2614.496. Figure 3.4 The best reason to have chose this model GARCH is :

1. After fitting the model it was significant that there are not much leftover patterns on errors which indicated that the GARCH(2,1) model dose a good job of explaining the SD of WH Smith's returns.
2. This model helps to understand and predict the volatility of the assets will be. The Garch(2,1) looks at the volatility based on the past two days of price changes and the last day's volatility.
3. This model is good at recognizing patterns where high volatility tends to come in clusters. For example, if the asset has had a few days of big price changes, it's likely to continue being volatile for a while. This is common in financial markets.

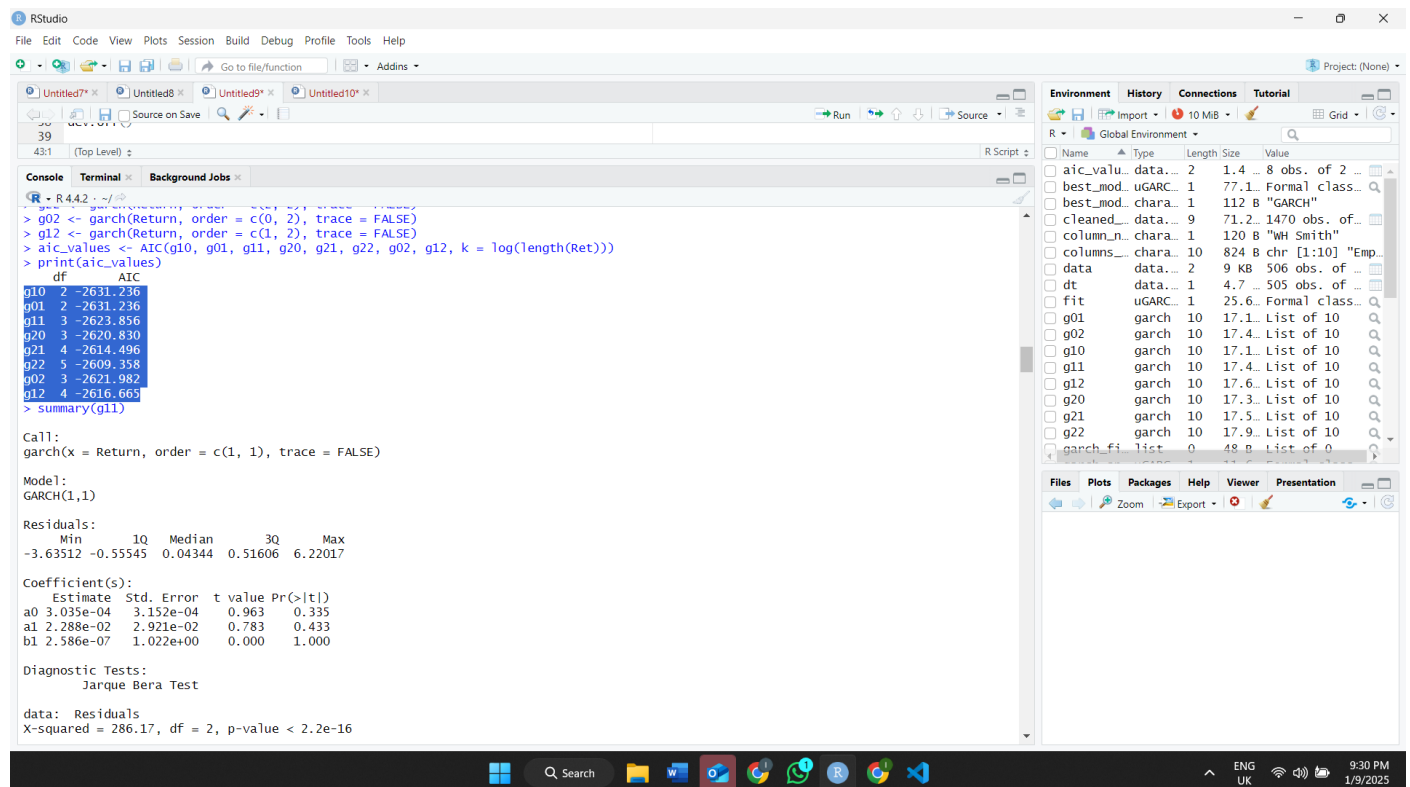


Figure 3.5

## Summary of Findings:

The analysis evaluated the performance of five well-known UK retail companies (Sainsbury, JD Fashion, Tesco, Marks & Spencer, and WH Smith) over two years to guide investment decisions. The goal was to determine which portfolios provide the best balance of risk and return.

### 1. Efficient Portfolio Selection:

- A combination of stocks was tested to maximize returns while managing risk effectively.
- The most efficient portfolio consisted of 39% Tesco and 61% Marks & Spencer, offering the highest return relative to risk. This portfolio is optimal for investors seeking a favorable balance between potential gains and risk exposure.

### 2. Risk and Return Insights:

- Each company's risk (measured by volatility) and potential return were carefully evaluated.

- JD Fashion exhibited higher risk, making it suitable for high-reward seekers. Conversely, companies like Tesco and Sainsbury demonstrated more stability, appealing to cautious investors.

### 3. Correlation:

- Companies with lower correlations provided diversification benefits, reducing overall portfolio risk. For example, Tesco and JD Fashion exhibited lower correlation, enhancing portfolio stability when combined.

### 4. Performance Metrics:

- The Sharpe Ratio was used to measure how well the investment compensates for risk. The optimal portfolio achieved a high Sharpe Ratio, indicating excellent risk-adjusted returns.
- The Capital Market Line highlighted the best combinations of risk-free and risky assets. This provides a clear visual guide to balancing risk and return for potential investors.

### 5. Market Sensitivity:

- Beta analysis showed WH Smith and JD Fashion are more sensitive to market fluctuations, while Tesco and Sainsbury are less affected. This allows investors to tailor portfolios based on market sentiment and risk tolerance.

### 6. Potential Losses:

- A value at risk (VaR) evaluation predicted that under everyday market situations, losses are not going to exceed 14.11% for the optimized portfolio.

Therefore this analysis suggests that the quality funding approach relies upon on the investor's desires and threat appetite. The efficient portfolio gives a great choice for balanced returns. Riskier investments like JD fashion may want to appeal to aggressive traders in search of higher potential gains, while strong shares like Tesco fit conservative traders. The findings make stronger the importance of diversification to control risks correctly and align investments with marketplace traits and individual goals.