A Cost Function Approach to the Measurement of Factor Demand Elasticities of Substitution

Article *in* American Journal of Agricultural Economics · February 1973

DOI: 10.2307/1238771 · Source: RePEc

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A Cost Function Approach to the Measurement of Elasticities of Factor Demand and Elasticities of Substitution*

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THE USE of a cost function rather than a production function for estimating production parameters has several advantages:

1. It is not necessary to impose homogeneity of degree one on the production process to arrive at estimation equations. Cost functions are homogeneous in prices regardless of the homogeneity properties of the production function, because a doubling of all prices will double the costs but will not affect factor ratios.

2. In general, the estimation equations have prices as independent variables rather than factor quantities, which, at the firm or industry level, are not proper exogenous variables. Entrepreneurs make decisions on factor use according to exogenous prices, which makes the factor levels endogenous decision variables.

3. If a production function procedure is used to derive estimates of elasticities of substitution or of factor demand in the many-factor case, the matrix of estimates of the production function coefficients has to be inverted. This will inevitably exaggerate estimation errors. No inversion is necessary when a cost function is used.

4. In the special case of the Translog cost function [5], to which the method is applied, problems of neutral or non-neutral efficiency differences among observational units (firms or states in a cross-section; years in a time-series) or of neutral and non-neutral economies of scale can be handled conveniently.[1] Therefore, these problems will not result in biased estimates of the production parameters. As will be discussed, such differences can result from a variety of sources. Most methods of estimating production cannot handle this problem properly.

5. In the case of the Translog cost function (as well as the Translog production function), all estimation equations are linear in logarithms.

6. In production function estimation, high multicollinearity among the input variables often causes problems. Since there is usually little multicollinearity among factor prices, this problem does not arise in cost function estimation.

The plan of this paper is as follows. First, a derivation of the Allen partial elasticity of substitution in terms of the cross derivatives of the cost function is presented. Then the result is applied to the case of the Translog cost function and methods to avoid estimation biases caused by neutral and non-neutral efficiency differences are discussed. Finally the Translog method is used to derive estimates of elasticities of derived demand and of elasticities of substitution for the agricultural sector using U. S. cross-section data of states for the years 1949, 1954, 1959, and 1964.

* University of Minnesota Agricultural Experiment Station Scientific Journal Series Paper 8524. I wish to thank Richard A. King, Lee R. Martin, Vernon W. Ruttan, James A. Seagraves, and T. Dudley Wallace for their helpful suggestions on earlier drafts. Research for this paper was supported by the U. S. Agency for International Development through grants to the Department of Economics, North Carolina State University, and to the Economic Development Center, University of Minnesota. The conclusions do not necessarily reflect the position of the USAID.

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[1] A non-neutral efficiency difference in the Hicksian sense is one in which the isoquant does not shift inwards homothetically. The factor ratio does not stay constant at a constant factor price ratio. If the capital-labor ratio increases, the efficiency gain is labor-saving. This implies that the labor share declines at a constant factor price ratio. Efficiency gain biases can, therefore, be defined as follows:

$$B_i = \frac{\partial \alpha_i}{\partial t} \cdot \frac{1}{\alpha_i} \begin{matrix} < \\ = \\ > \end{matrix} 0 \rightarrow \text{Hicks} \begin{cases} \text{factor } i\text{-saving} \\ \text{factor } i\text{-neutral} \\ \text{factor } i\text{-using} \end{cases}$$

where factor prices are held constant and α_i is the cost share of factor i. This definition is more easily handled in the many-factors case than the usual definition in terms of marginal rates of substitution.

Partial Elasticities of Substitution in Terms of Cost Function Parameters

Corresponding to the following cost minimization problem

$$(1)\quad \min C = \sum_{i=1}^{n} X_i P_i \qquad (i = 1, 2, \ldots, n)$$

subject to

$$(2)\quad Y = f(X_1, X_2, \ldots, X_n)$$

(where X_i = inputs levels, P_i = factor prices, Y = output), there exists a dual minimum cost function,[2]

$$(3)\quad C^* = g(Y, P_1, \ldots, P_n).$$

This function (also called factor price frontier) assigns to every combination of input prices the minimum cost corresponding to the cost minimizing input levels X^*_i. C^* is homogeneous of degree one in prices.

Shephard's lemma [6] holds for the cost function

$$(4)\quad \frac{\partial C^*}{\partial P_r} = X_r.$$

Let f be the bordered Hessian matrix of (2) and $f_i = \partial Y/\partial X_i$ and $f_{ij} = \partial^2 Y/\partial X_i \partial X_j$. Partial elasticities of substitution are defined by Allen as

$$(5)\quad \sigma_{kr} = \frac{\sum_{i=1}^{n} X_i f_i}{X_k X_r} (f^{-1})_{rk},$$

where $(f^{-1})_{rk}$ is the rkth element of f^{-1}. From (5) it is apparent that

$$(6)\quad \sigma_{kr} = \sigma_{rk}.$$

If estimates of the coefficients of a particular functional form of (2) are available, the bordered Hessian can be computed, inverted, and the σ's found according to (5) for specific input levels.[3] The inversion of a matrix of estimates has, however, the tendency to blow up estimation errors to an unknown extent and, because inversion is a nonlinear transformation, econometric properties of $\widehat{\sigma}_{kr}$ cannot be found even if such properties of the production function parameters are known.

In the case of the cost function, estimates of σ_{kr} can be obtained directly from the parameters of the function, because

$$(7)\quad \sigma_{rk} = \frac{\sum_{i=1}^{n} P_i X_i}{X_k X_r} \frac{\partial^2 C^*}{\partial P_r \partial P_k}.$$

This was originally proved for homogeneous production functions in Uzawa [15]. A proof which does not rely on homogeneity is given here.

Proof: The first order conditions of the cost minimizing problem (1) and (2) are

$$(8)\quad f(X_1, \ldots, X_n) - Y = 0$$

$$(9)\quad P_i - \lambda f_i = 0 \qquad i = 1, \ldots, n.$$

Writing the total differential of the first order conditions and rearranging the terms in the following matrix form,

$$(10)\quad \lambda \begin{bmatrix} 0 & f_1 & \ldots\ldots & f_n \\ f_1 & f_{11} & \ldots\ldots & f_{1n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ f_n & f_{n1} & \ldots\ldots & f_{nn} \end{bmatrix} \begin{bmatrix} d\lambda/\lambda \\ dX_1 \\ \cdot \\ \cdot \\ \cdot \\ dX_n \end{bmatrix} = \begin{bmatrix} \lambda dY \\ dP_1 \\ \cdot \\ \cdot \\ \cdot \\ dP_n \end{bmatrix}.$$

Solving for the vector of endogenous variables,

$$(11)\quad \begin{bmatrix} d\ln\lambda \\ dX_1 \\ \cdot \\ \cdot \\ \cdot \\ dX_n \end{bmatrix} = \frac{1}{\lambda} f^{-1} \begin{bmatrix} dY \\ dP_1 \\ \cdot \\ \cdot \\ \cdot \\ dP_n \end{bmatrix}.$$

This implies

$$(12)\quad \frac{\partial X_r}{\partial P_k} = \frac{1}{\lambda} (f^{-1})_{rk}.$$

Substituting from (10) into (5) and substituting $f_i = \frac{P_i}{\lambda}$ from (9),

$$(13)\quad \sigma_{kr} = \frac{\sum_i X_i f_i}{X_k X_r} \lambda \frac{\partial X_r}{\partial P_k} = \frac{\sum_i X_i P_i}{X_k X_r} \frac{\partial X_r}{\partial P_k}.$$

Taking the derivative of (4) with respect to P_k,

$$(14)\quad \frac{\partial^2 C^*}{\partial P_r \partial P_k} = \frac{\partial X_r}{\partial P_k}.$$

Combining (13) and (14) and (6),

$$\sigma_{kr} = \sigma_{rk} = \frac{\sum X_i P_i}{X_k X_r} \frac{\partial^2 C^*}{\partial P_r \partial P_k} \quad \text{Q.E.D.}$$

Multiplying and dividing the righthand side of

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[2] While C of (1) is the cost of production under any feasible factor combination, C^* refers to the cost of production when the cost minimizing input combination is used. Since the optimal input combination is a function of the factor prices, the minimum cost is also.

[3] See Berndt and Christensen [1] for an example.

(13) by $\frac{P_k}{X_r}$,

$$\sigma_{kr} = \sigma_{rk} = \frac{\eta_{rk}}{\alpha_k}, \tag{15}$$

where $\eta_{rk} = \frac{\partial X_r}{\partial P_k}\frac{P_k}{X_r}$ and $\alpha_k = \frac{X_k P_k}{\Sigma X_i P_i}$ is the of factor k in total costs. If the parameters of specific functional form of a cost function have been estimated, (7) can be used to derive elasticities of substitution for given factor levels and total costs.[4]

The Translog Case

The Translog cost function is particularly useful in this context. It is written as a logarithmic Taylor series expansion to the second term of a twice differentiable analytic cost function around variable levels of 1, (i.e., $\ln Y = 0$, $\ln P_i = 0$, $i = 1, \ldots, n$). Rewrite (3) in natural logarithms:

$$\ln C^* = f(\ln Y, \ln P_1, \ldots \ln P_n). \tag{16}$$

Denote the first and second order derivatives at $\ln(\cdot) = 0$ as follows:

(17)
$$\ln C^*\bigg|_0 = \nu_o;\quad \frac{\partial \ln C^*}{\partial \ln Y}\bigg|_0 = \nu_y;\quad \frac{\partial \ln C^*}{\partial \ln P_i}\bigg|_0 = \nu_i;$$
$$\frac{\partial^2 \ln C^*}{\partial \ln P_i\, \partial \ln P_j}\bigg|_0 = \gamma_{ij};\quad \frac{\partial^2 \ln C^*}{\partial \ln P_i\, \partial \ln Y}\bigg|_0 = \gamma_{iY}$$

The equality of the cross derivatives in (17) implies the symmetry constraint

$$\gamma_{ij} = \gamma_{ji}. \tag{18}$$

Then the Taylor series expansion is as follows:

$$\ln C^* = \nu_o + \nu_y \ln Y + \sum_i \nu_i \ln P_i + 1/2 \sum_i \sum_j \gamma_{ij} \ln P_i \ln P_j + \sum_i \gamma_{iy} \ln P_i \ln Y + \text{remainder}. \tag{19}$$

This function is an approximation of an arbitrary analytic function.[5, 6] It is a functional form in its own right if the remainder is neglected and if we assume all derivatives and cross-derivatives to be constant. This latter constraint is imposed if the parameters are estimated in regression equations.

Homogeneity in prices is defined as follows: $\lambda g(Y, P_1, \ldots, P_n) = g(Y, \lambda P_1, \ldots, \lambda P_n)$. It implies

(20)
$$\sum_i \nu_i = 1; \qquad \sum_i \gamma_{ij} = 0; \qquad \sum_j \gamma_{ij} = 0.$$

Homogeneity of degree one in prices does not impose homogeneity of degree one of the production function in inputs. Almost no constraints are imposed on elasticities of substitution or of factor demand, which makes the function more general than other functional forms currently in use [5].

The function can be estimated directly or in its first derivatives which, by Shepard's lemma (4), are factor shares:

$$\frac{\partial \ln C^*}{\partial \ln P_i} = \alpha_1 = \nu_i + \sum_j \gamma_{ij} \ln P_j + \gamma_{iy} \ln Y \quad (i = 1, \ldots, n). \tag{21}$$

Both sets of estimation equations are linear in logarithms and have proper exogenous variables on the right hand side if the analysis pertains to firms or an industry.[7]

The γ_{ij} parameters have little economic meaning of their own. We will prove that they are related to variable elasticities of substitution and of factor demand as follows:

(22)
$$\sigma_{ij} = \frac{1}{\alpha_i \alpha_j}\gamma_{ij} + 1 \qquad \text{for all } i, j;\ i \neq j$$

[4] Also, if η_{kr} had been estimated in a demand for factors equation, one can compute σ_{rk} from (13). This may be particularly useful in the two-factors case since then the own elasticity of demand can be used. Because of the homogeneity of degree one of C^*, $\eta_{11} + \eta_{12} = 0$, and $\eta_{12} = -\eta_{11}$ can be substituted into (13).

[5] The first power terms of (19) represent a Cobb-Douglas cost function. If all γ_{ij} and γ_{iy} parameters were zero, the production function would be Cobb-Douglas as well, because the production function of a Cobb-Douglas cost function is Cobb-Douglas and vice versa [10].

[6] By a similar expansion of a production function the Translog production function is found.

$$\ln Y = \omega_o + \sum_i \omega_i \ln X_i + 1/2 \sum_i \sum_j \tau_{ij} \ln X_i \ln X_j.$$

[7] In the case of the Translog production function the estimation equations are similar but with factor quantities on the right-hand side. For the decision-making firm these are endogenous.

(23)
$$\sigma_{ii} = \frac{1}{\alpha^2_i}(\gamma_{ii} + \alpha^2_i - \alpha_i) \qquad \text{for all } i$$

(24)
$$\eta_{ij} = \frac{\gamma_{ij}}{\alpha_i} + \alpha_j \qquad \text{for all } i, j;\ i \neq j$$

$$(25)\quad \eta_{ii} = \frac{\gamma_{ii}}{\alpha_i} + \alpha_i - 1 \qquad \text{for all i.}$$

Proof:

(26)
$$\gamma_{ij} = \frac{\partial^2 \ln C^*}{\partial \ln P_i\, \partial \ln P_j} = P_j \frac{\partial}{\partial P_j}\left(\frac{\partial C^*}{\partial P_i}\frac{P_i}{C^*}\right)$$
$$= P_j\left(\frac{\partial^2 C^*}{\partial P_i\, \partial P_j}\frac{P_i}{C^*} - \frac{P_i}{(C^*)^2}\frac{\partial C^*}{\partial P_i}\frac{\partial C^*}{\partial P_j}\right).$$

Substituting $\frac{\partial C^*}{\partial P_k} = X_k$ from (4):

$$\gamma_{ij} = \frac{P_i P_j}{C^*}\frac{\partial^2 C}{\partial P_i\, \partial P_j} - \frac{P_i P_j}{(C^*)^2} X_i X_j.$$

Therefore,

$$(27)\qquad \frac{\partial^2 C^*}{\partial P_i\, \partial P_j} = \frac{C^*}{P_i P_j}(\gamma_{ij} + \alpha_i \alpha_j).$$

Substituting (27) into (7):

$$\sigma_{ij} = \frac{\Sigma P_i X_i \cdot C^*}{P_i X_i P_j X_j}(\gamma_{ij} + \alpha_i \alpha_j)$$
$$= \frac{\gamma_{ij}}{\alpha_i \alpha_j} + 1 \qquad \text{Q.E.D.}$$

(24) follows from (15). The proof of (23) is similar, except that in (26) $\partial P_i / \partial P_i = 1$ which accounts for $-\alpha_i$ in equation (23). (25) follows again from (15).

If the γ_{ij} parameters have been estimated with equations (19) and/or (21), and if the factor shares are known, all elasticities can be estimated. Since $\hat{\sigma}_{ij}$ and $\hat{\eta}_{ij}$ are linear transformations of the γ_{ij} parameters whose econometric properties are known, the econometric properties of the elasticities are known as well. No matrix of estimates has to be inverted.[8]

[8] For estimating marginal products the cost function has the same disadvantage as the production function in estimating elasticities of substitution. Estimates of its bordered Hessian have to be inverted. Since the Translog cost function and the Translog production function use the same basic data (input quantities and prices), it would be preferable to estimate the σ_{ij} and η_{ij} using the former function while using the latter one for the marginal products.

Treatment of Neutral and Non-neutral Efficiency Differences

If efficiency differences exist among the observational units (states or firms in cross-sections, years in time-series), the specification of the estimation equations must take into account the problem in order to avoid bias in estimation.

It is best to distinguish two kinds of efficiency differences:

1. Differences which can be functionally related to a variable such as output (scale effects), or a technical change index, or time (as a proxy for technical change), or education and management (the left-out variables problem).
2. Differences among cross-sectional units which cannot be functionally related to a variable and which arise from past differences in technical change. If the cross-sectional units have had a different past history of technical change, they are no longer on the same isoquant. This is likely to happen in many cross-sections.

The first case is easily handled. Let the variable Y in (19) and (21) stand for any of the variables which cause the neutral and non-neutral efficiency differences (output, time, technical change index, or education). Then (19) and (21) are immediately correctly specified, provided that the variable Y changes efficiency at constant logarithmic rates, and that data on the variable Y are available. As an example, if time-series data are used and technical change causes the efficiency differences at constant rates over time, let Y stand for time. The coefficient γ_Y will then be an estimator of the rate of technical change, and the coefficients γ_{iY} will be estimators of the rates of bias. If all γ_{iY} were zero, time alone would not affect the factor shares (equation 21). This is the definition of neutrality in footnote 1. If γ_{iY} was greater than zero, the share of factor i would rise at constant factor prices at the logarithmic rate γ_{iY}. This would be factor i-using technical change.

If a variable for which no data are available causes efficiency differences, the γ_{ij} can still be estimated in an unbiased way, provided the left-out variable affects efficiency neutrally. In that case, all the γ_{iY} parameters are zero and (21) is still properly specified without data on the variable Y. But because γ_Y is not zero, (19) is no longer correctly specified. Therefore, the γ_{ij} parameters have to be estimated in (21) alone.[9]

[9] Including education, etc., in a Cobb-Douglas production function assumes that these variables affect effi-

In the next section scale effects will be assumed to be neutral. Output is, therefore, not included as a variable in (21). On the other hand, technical change over time is assumed to be non-neutral, and time is included as a variable (Y thus standing for time).

Problem (2) of efficiency differences among cross-sectional units can be handled in the same way as the left-out variables problems above, provided the efficiency differences are neutral. The proper variable would be an efficiency index of the cross-sectional units which is generally unknown. However, if the efficiency differences are non-neutral due to biased technical change in different directions in previous periods, it would be necessary to know the efficiency index and include it as a variable in (21). If the index is not available, but the cross-sectional units can be grouped into regions, within which no non-neutral differences exist, regional dummies in equation (21) will again insure unbiased estimates of the parameters of the cost function, because they allow the regions to have differing shares at equal factor prices. This again precludes simultaneous estimation of (17) and (19). The discussion of this entire section applies equally to the Translog production function.

Estimation, Data, and Conclusions

The cost function was estimated with state data for the United States. Four sets of cross-sectional data were obtained for 39 states or groups of states. The cross-sections were derived from census data and other agricultural statistics for the years 1949, 1954, 1959, and 1964. The combination of cross-sections over time poses problems which are discussed below. In general, Griliches's [9] definitions of factors were used. He distinguishes the following five factors: land, labor, machinery, fertilizer, and all others.[10] For all data pooled the following model was fitted:

(28)

$$\alpha_{ikt} = \nu_i + \sum_j \gamma_{ij} \ln P_{jkt} + \gamma_{it} \ln t + \sum_{r=1}^{4} \delta_{ir} d_r + \epsilon_{ikt} \qquad i = 1, \ldots, n-1, \quad j = 1, \ldots, n,$$

where i and j stand for factors of production, k for states, t for time, r for groups of states, and

$$d_r = \begin{cases} 1 \text{ if } k \in r \\ 0 \text{ if } k \notin r. \end{cases}$$

δ_r is the coefficient of the non-neutral efficiency difference between group r and group 5 (Western States). One share equation has to be dropped from the model because only $n - 1$ equations are linearly independent due to the homogeneity constraint (20). In this form the model allows for neutral efficiency differences of any kind among states, non-neutral efficiency differences among groups of states, and non-neutral efficiency differences over time.

Within each of the four cross-sections (time period), the error terms of the $n - 1$ estimation equations are not independent, since for each state the same variables which might affect the shares in addition to the prices were left out of the model. If restrictions across equations ($\gamma_{ij} = \gamma_{ji}$) are imposed, OLS estimators are no longer efficient despite the fact that all equations contain the same explanatory variables on the right-hand side [13]. Therefore, the seemingly unrelated regression problem applies, and restricted generalized least squares (RGLS) have to be applied to all equations simultaneously [13, Ch. 9].

If all four cross-sections are pooled, there is an additional problem of error interdependence over time. The correct way of handling both problems would be to specify an equation for each share in each year, then test and impose the symmetry and homogeneity constraints and the constraints that the γ_{ij} parameters are constant over time. This exceeded the capacity of the program used.[11] The correct procedure would also have required that one impose constraints of equality of the auto-correlation coefficients over time on the estimated variance covariance matrix, which was not possible with TTLS. The following procedure was therefore adopted. To search for an exact specification, RGLS regressions were applied to the data of each cross-section separately to avoid any biases in the tests used for this purpose.[12] Once the decision was

ciency of the other factors neutrally because all elasticities of substitution are 1.

[10] The data are discussed in detail in the Appendix.

[11] The Computer Program used was that of the Triangle Universities Computing Center (TTLS) [14].

[12] No *a priori* information is available to decide which equation to drop and whether or not to include regional dummies. To make these decisions, I was looking for the specification in which the imposition of the symmetry constraint $\gamma_{ij} = \gamma_{ji}$ and the homogeneity constraint $\sum_j \gamma_{ij} = 0$ led to the smallest weighted F-ratio according to the test static in equation (3.6) of Theil [13, p. 314]. Since both of these constraints are "true" constraints, they can be used in this way to eliminate

Table 1. Restricted estimates of the coefficients of the Translog cost function and t-Ratios[a, b]

	Independent Variables										
Factor	Price of land	Price of labor	Machinery price	Fertilizer price	Price of other[c]	Year	Intercept	MN	GF	SE	GS[d]
Share of Land	.07747	−.03613	.00478	.01066	−.05678	.00847	.2603	.1021	.0394	.1073	−.0577
	(6.02)	(3.25)	(.47)	(2.14)		(1.47)	(9.96)	(10.2)	(4.1)	(8.9)	(4.7)
Share of labor		−.06367	−.00661	−.02805	.13446	−.05482	.5218	.0194	−.0016	.0169	.0246
		(3.67)	(.59)	(4.97)		(9.08)	(14.91)	(1.63)	(.15)	(1.09)	(1.63)
Share of machinery			−.03485	−.00877	.04545	.02498	.0926	−.0033	.0369	−.0186	.0072
			(1.31)	(.97)		(4.66)	(3.46)	(.41)	(5.08)	(1.86)	(.73)
	Symmetric										
Share of fertilizer				.00068	.02548	.00178	.0745	.0104	−.0041	.0370	−.0025
				(.12)		(.63)	(5.6)	(2.5)	(1.10)	(7.24)	(.49)
Share of other					−.14861						

Source: See Appendix.

[a] Restrictions imposed: $\gamma_{ij} = \gamma_{ji}$ and $\sum_{j=1}^{n} \gamma_{ij} = 0$ for all i, j.

[b] Critical values with 578 degrees of freedom are $t_{.05} = 1.96$ and $t_{.01} = 1.65$. t-ratios may be overstated due to error interdependence over time.

[c] Implied estimates computed using the homogeneity constraint.

[d] MN, GF, SE, and GS are dummies for Mixed Northern agriculture, Grain Farming states, Southeast, and Gulf States, respectively. The intercept stands for Western States, and the coefficients of MN, GR, SE, GS are deviations from this intercept.

made to use a specification including equations for land, labor, machinery, and fertilizer, with regional dummies in all equations, all four cross-sections were pooled and the symmetry and homogeneity constraints imposed in the restricted generalized least squares estimation of all four equations simultaneously using equation (28). Since the error interdependence over time is neglected, the reported t-ratios will be overstated to some extent,[13] but the estimators are still unbiased.

The results of the regressions are reported in Table 1. The OLS single equation R^2 of the four shares equations with homogeneity imposed on the data are not very high:[14] land 0.68, labor 0.75, machinery 0.45, and fertilizer 0.75.

some specifications, although several specifications might satisfy the constraints. A specification in which "other" inputs are excluded and dummies added to all equations satisfied this criterion best for the four cross-sections. homogeneity of degree one of C^*, $\eta_{11} + \eta_{12} = 0$, and sections. Symmetry was only rejected in the cross-section of 1964 with an F-ratio of 4.19 (Critical $F_{.05} = 2.17$).

[13] Despite the 5-year interval between the cross-sections, error interdependence over time was still quite large. Correlation coefficients of the OLS errors of individual share equations between the years 1949 and 1964 were between .62 and .87. To check whether the neglect of this interdependence among cross-sections had a large impact on the values of the γ_{ij} estimates, I compared the estimates of the pooled cross-sections with the simple average of the estimates obtained in the four cross-sections individually. The differences of the estimates were small.

[14] The residuals of these equations are used to estimate the variance-covariance matrix for the GLS regressions.

From Table 1 the following conclusions emerge:

1. Out of the 10 γ_{ij} estimates, only 5 are statistically significant. This is not a "bad" result because $\gamma_{ij} = 0$ implies that the elasticity of substitution is equal to the Cobb-Douglas value of 1. Note, however, that when the Cobb-Douglas constraint $\gamma_{ij} = 0$ for all i, j was tested in various single cross-section models, it was always rejected. Therefore, the conclusion is that the Cobb-Douglas form is not an appropriate production or cost function specification.

2. The coefficient of the time variable is significant at the .05 level in the labor and machinery equations. This means that, at constant factor prices, the factor shares would have changed, which implies non-neutral technical change during the period 1949 to 1964. The coefficient for time in the labor equation is −.0548. Hence, technical change was labor-saving. On the other hand the positive coefficient of time in the machinery equation (+.0250) implies machinery-using technical change. This is consistent with the findings of Lianos [12] and my own findings [4].

3. Six of the regional dummies are significant. At equal factor prices the shares would not be equal among the groups of states. The coefficients of the dummies in the land equation of all four regions is negative. This implies that the technology in all regions is land-saving relative to the technology used in the Western states—Washington, Oregon, and California. The significant positive coefficients in the fertilizer equation

Table 2. Factor demand and cross demand elasticities implied in the estimated γ_{ij} and their value in the Cobb-Douglas case[a, b]

	Land	Labor	Machinery	Fertilizer	Other
Estimated Translog values[c]					
Land	−.3356	.0613	.1792	.1062	−.0112
	(.09)	(.07)	(.07)	(.03)	
Labor	.0308	−.9109	.1256	−.0577	.8122
	(.04)	(.06)	(.04)	(.02)	
Machinery	.1833	.2560	−1.0886	−.0239	.6733
	(.07)	(.08)	(.18)	(.06)	
Fertilizer	.4506	−.4878	−.0991	−.9452	1.0815
	(.10)	(.20)	(.30)	(.16)	
Other	−.0046	.6690	.2720	.1053	−1.0417
Cobb-Douglas values for comparison[d]					
Land	−.8491	.3008	.1475	.0356	.3652
Labor	.1509	−.6992	.1475	.0356	.3652
Machinery	.1509	.3008	−.8525	.0356	.3652
Fertilizer	.1509	.3008	.1475	−.9644	.3652
Other	.1509	.3008	.1475	.0356	−.6348

Source: See Appendix.
[a] Standard errors in parentheses: $SE(\eta_{ij}) = SE(\gamma_{ij})/\alpha_i$.
[b] Each element in the table is the elasticity of demand for the input in the row after a price change of the input in the column. These elasticities are not symmetric.
[c] The shares used are the same as for the Cobb-Douglas η_{ij}.
[d] $\eta_{ij} = \alpha_j$; $\eta_{ii} = \alpha_i - 1$.

Table 3. Estimates of the partial elasticities of substitution[a, b]

	Land	Labor	Machinery	Fertilizer	Other
Land	−2.225	.204	1.215	2.987	−.031
	(.57)	(.24)	(.46)	(.93)	
Labor		−3.028	.851	−1.622	2.224
		(.19)	(.25)	(.53)	
Machinery			−7.379	−.672	1.844
		Symmetric	(1.22)	(1.71)	
Fertilizer				−26.573	2.961
				(4.61)	
Other					−2.852

[a] Standard errors $SE(\sigma_{ij}) = SE(\gamma_{ij})/\alpha_i\alpha_j$ in parentheses.
[b] The elasticities of substitution are symmetric. The own elasticity of substitution has little economic meaning except that it has to obey the constraint $\sum_j \alpha_j \cdot \sigma_{ij} = 0$.

of the dummies for the Mixed Northern states and the South Eastern states would indicate that these regions have a more fertilizer-using technology than the Western states.[15]

The γ_{ij} parameters have little economic meaning. They are best evaluated by the values which they imply for elasticities of factor demand and elasticities of substitution. The values are computed for the simple average of factor shares for all 39 states between 1949 and 1964. In Table 2 the elasticities are compared with what they would be at equal factor shares in the Cobb-Douglas case ($\gamma_{ij} = 0$).[16]

[15] Not too much should be made of these regional differences because they may be due to different product mixes rather than to true technological differences in each production. If they reflect product mix differences, the dummies will at least correct for possible biases due to these differences.

[16] The elasticities were also computed using aggregate factor shares reported in Binswanger [4] for the years 1912, 1952, 1964, and 1969. While differences exist with the values reported here, they are not large. The main advantage of using a variable elasticity of substitution function rather than a CES framework is not that the elasticities vary widely for observed values of shares,

Tables 2 and 3 imply the following conclusions:

4. All own elasticities of factor demand have the correct sign. Land demand seems to be very inelastic. In empirical work with Cobb-Douglas production functions the coefficient of land is usually between 0.15 and 0.40. According to equation (25) with $\gamma_{ij} = 0$ (the Cobb-Douglas constraint), these values imply land demand elasticities of —0.85 to —0.60, which is substantially higher than the elasticity found with the Translog cost function.

The values of the other own demand elasticities are close to one and, except for fertilizer, higher than they would be in the Cobb-Douglas case. The fertilizer demand is substantially less elastic than Griliches's [8] estimate of —2.0.[17]

5. Elasticities of substitution and cross-elasticities of demand are positive for substitutes and negative for complements. These relationships are easier to evaluate by looking at the elasticities of substitution in Table 3 than the cross-elasticities of demand because the latter reflect the relative importance (share) of a factor while the former do not. Complementarity seems to exist between the labor-fertilizer pair, the machinery-fertilizer pair, and the land-other inputs pair. But only the complementarity of fertilizer and labor is significant. This is a noteworthy result. If this was generally true for all countries, it could have an important implication. Less developed countries with unemployed labor might find an advantage in pushing fertilizer use.

The best substitutes are land and fertilizer, which was expected. It was a surprise, however, to find that machinery is a better substitute for land than for labor (although the machinery-labor elasticity falls within one standard deviation of the land-labor elasticity, so that there is no statistical difference). Even if the machinery-land elasticity was over-estimated to some extent, the finding should cast doubt on the notion that one can dichotomize agricultural technology in mechanical technology which acts exclusively as a labor substitute and biological technology which acts exclusively as a land substitute.[18] The small elasticity of substitution between land and labor was expected. It is also clear that the agricultural production function is not separable between any input groups. Such a separability implies that the elasticities of substitution between the factors in the separable group and all other factors are identical. This is clearly not the case for any factor group [2].

Overall the result seems to be reasonable and shows that cost functions, in general, and the Translog cost function, in particular, lead to valuable methods for estimation of production parameters.

but that this format does not constrain all elasticities of substitution to be equal [15].

[17] These estimates are not necessarily in conflict. Griliches estimates a long-run elasticity in a time-series. This implies that, if there is an induced fertilizer-using innovation due to a fall in the price of fertilizer, his price response picks up part of the adjustment due to the technical change. This is what happened in U. S. agriculture [4]. Since the inclusion of the time variable in our regression equations picks up the influence of technical change, the estimates presented here are net of any technical change influence.

[18] This idea is put forward in Hayami and Ruttan [11]. It stands up well with respect to labor and fertilizer, which are complements. But the high substitutability between machinery and land is inconsistent with that notion.

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APPENDIX

Variable Construction and Data Sources

For 39 states or groups of states, aggregate input quantity data and expenditure data were derived.

Quantity data

Except for "other" inputs, the quantity data were taken from Fishelson [7], who used Griliches's data with some changes. For a detailed discussion of labor, land, and machinery, see Griliches [9, pp. 966 and 973] or Binswanger [3].

Fishelson's discussion of the construction of the land variable is reproduced here:

> In the U. S. Census of Agriculture (U. S. Bureau of the Census, 1952, 1956, 1962 and 1966), the average value of land and buildings per farm in each state was reported. However, the land value represented not only the value of land to agricultural production but also included the site value of land. The value of buildings included both farm structures and dwellings. Hence, census data on value of land and buildings were inadequate for the purposes of this study. To measure land by the number of acres per farm (giving each acre a value of one) is also inadequate because of the diversity of soil quality, fertility and uses.
>
> In this study the weighting procedure for measuring land value was based on a study by Hoover. The value of each acre in each state at each cross-section was measured by its 1940 price relative to that of an acre of pasture in the corresponding state. The value of an acre of pasture in each state in 1940 was calculated by dividing the total value of land in 1940 by the number of pasture equivalent units of the land in 1940. This value of an acre of pasture was kept constant over time. . . . The use of this method provided a measure of the stock of land in constant prices. According to this method, changes in the stock of land occurred only because of changes in the number of acres or their use. The stock of land was unaffected by changes in prices of agricultural products, site effects, or government programs. [7, pp. 79–80].

The only change which was made in the quantity data taken from Fishelson [7] was that, whenever quantities per farm were used, the farm number was taken from various issues of the Census of Agriculture (U. S. Department of Commerce, 1950, 1954, 1959, 1964), rather than from *Farm Labor* (USDA, 1945–1972).

Since expenditure data corresponding to Fishelson's quantity data could not be constructed for "other inputs," new quantity data were defined as follows: The quantity of other inputs is the sum of the explicit and implicit annual expenditures on all other material inputs used in production. The explicit expenditures were the cash expenditures on purchases of livestock, poultry, feed, seeds, plants, and bulbs, operation and repairs of farm structures, and other miscellaneous costs. The implicit expenditures were 8 percent interest on livestock and crop inventories, depreciation (4.2 percent), and interest (5 percent) on the value of farm structures, and the share of real estate taxes falling on buildings. Each of the expenditures was separately deflated to its 1949 price level to arrive at a quantity measurement (for taxes the agricultural output price index was used).

Expenditures and factor shares

The expenditure variables were defined, as far as possible, to correspond to the quantity variables. Expenditure shares were obtained by dividing the expenditures of each factor by the sum of the expenditures.

Expenditures on land are simply 6 percent of the value of land plus the share of real estate taxes falling on land.

Expenditures for labor are the number of man days of labor from Fishelson [7] multiplied by a daily wage rate without room and board (*Farm Labor,* 1945–1972). This assumes that the opportunity cost of farm operators is the wage rate which they could earn as workers on other farms.

Expenditures for machinery are assumed to be 15 percent of the value of farm machinery and equipment for interest and depreciation plus the current expenditures for operation and repair of machinery and equipment.

Fertilizer expenditures are directly reported by the USDA.

Other expenditures were computed exactly as the quantity of other inputs, except that the individual items were not deflated. Aggregate expenditures estimated in this way had a tendency to exceed aggregate income by up to 10 percent.

Prices were taken to be the expenditures divided by the quantities. They were then deflated to the 1949 price level using the U. S. agricultural output price index. Note that this procedure implies that the price of other inputs is equal to one for all states in the year 1949. Appendix Table 1 lists all the data sources.

Appendix Table 1. Data sources

Variables	Source
Farm income, change in inventories, rental value of dwellings, all explicit current operating expenditures	*Farm Income Situation,* July supplement, USDA (1954–1972)
Annual average daily wage rate without board or room	Various issues of *Farm Labor,* USDA (1945–1972)
Farm number	Various issues of *Census of Agriculture,* U. S. Department of Commerce (1950, 1954, 1959, 1964)
Input and output price indexes	Various issues of *Agricultural Statistics,* USDA (1936–1972)
Repairs and operation of farm dwellings and service structures, depreciation and dwellings, service buildings, motor vehicles, other machinery and equipment, value of farm machinery and equipment, value of crop inventories	USDA, unpublished

Reprinted from
AMERICAN JOURNAL OF AGRICULTURAL ECONOMICS
Vol. 56, No. 2, May 1974