Karlsruhe Institute of Technology

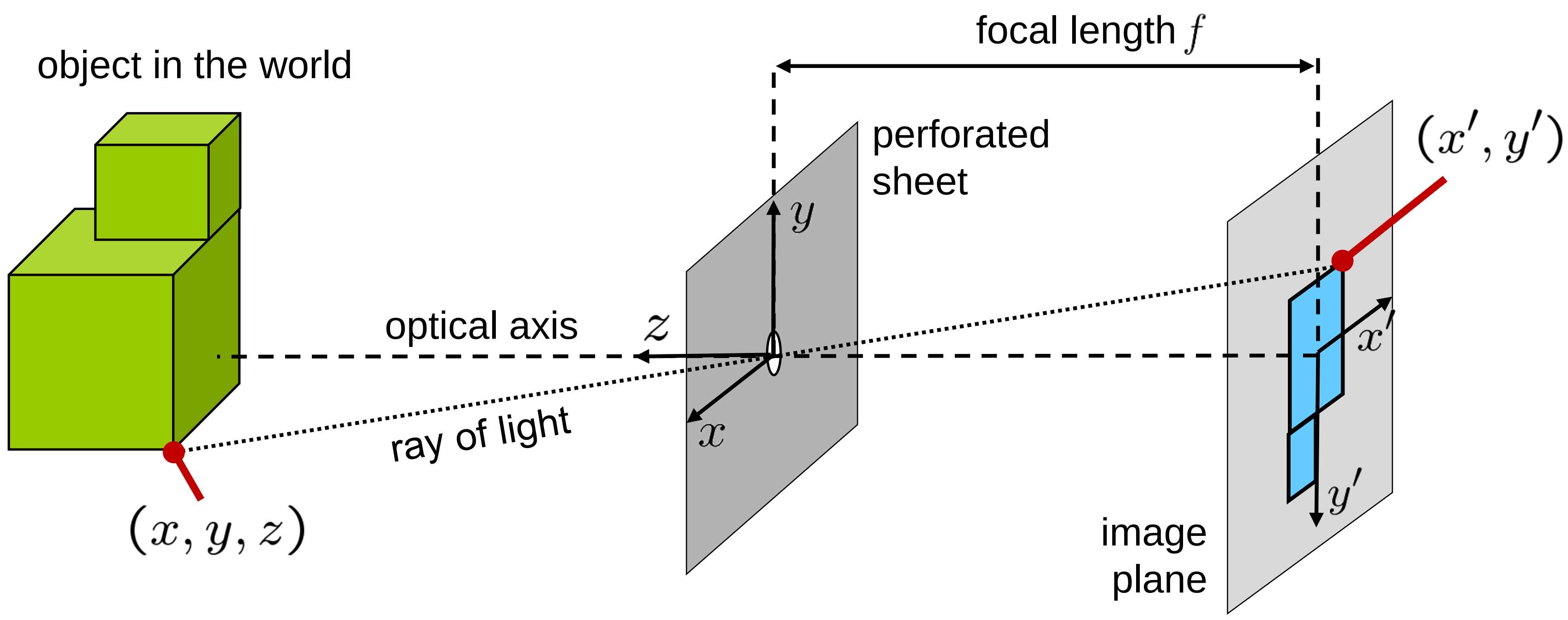
Machine Vision

Chapter 7: Camera Optics

Dr. Martin Lauer

Institut für Mess- und Regelungstechnik

Pinhole Camera

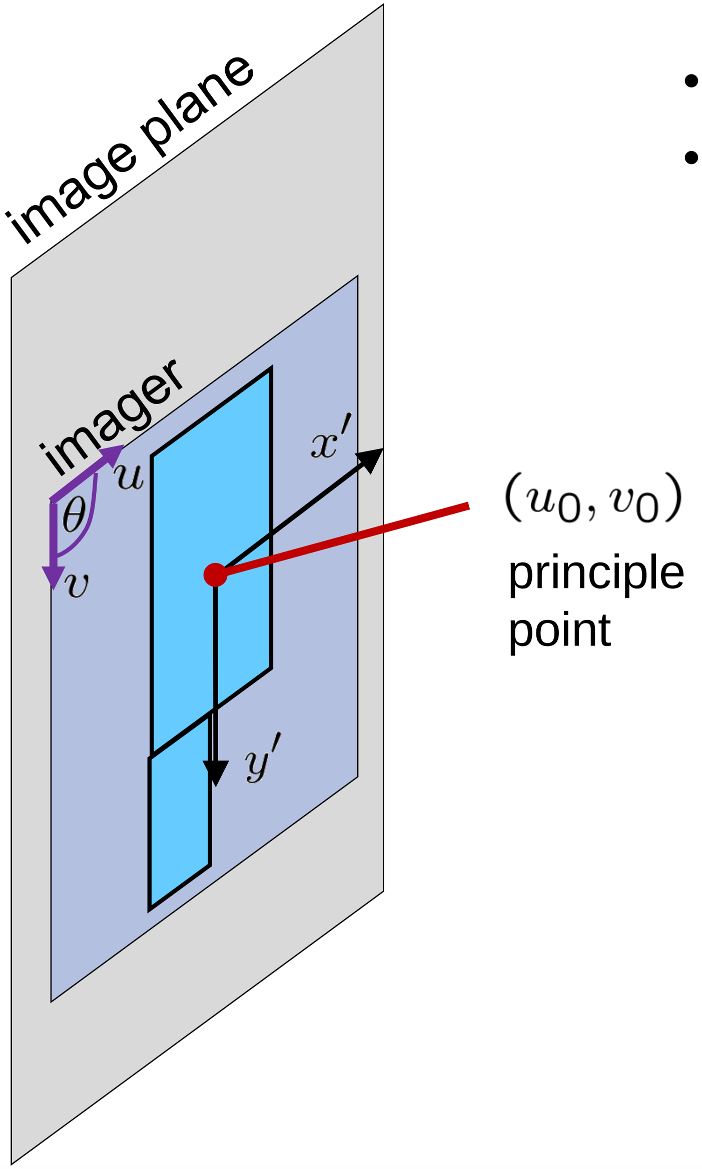


point (x, y, z) is projected onto (x', y')

intercept theorem:

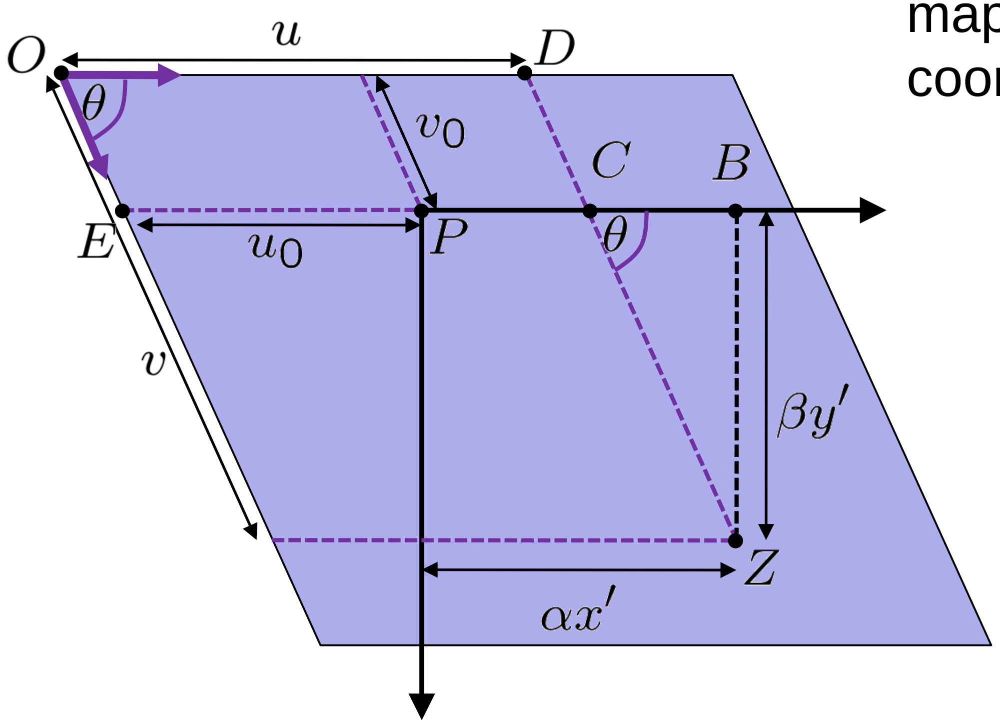
$$\frac{x}{z} = \frac{x'}{f}, \quad \frac{y}{z} = \frac{y'}{f} \qquad \Rightarrow \qquad z \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f & 0 \\ 0 & f \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

World-to-Image-Mapping



- camera coordinate system
- image coordinate system
  - u-direction parallel to x'-direction
  - v-direction might be skewed
    θ=angle between u- and v-direction
  - principle point=origin of camera coordinate system in image coordinates (u_0,v_0)
  - length of unit vector u and v differ from length of unit vector x', y'
    scaling factors α, β

World-to-Image-Mapping cont.



map point Z from camera coordinates to image coordinates:

- triangle ZBC:

$$\sin\theta = \frac{\beta y'}{v - v_0}$$

$$\Rightarrow v = \frac{\beta}{\sin\theta} y' + v_0$$

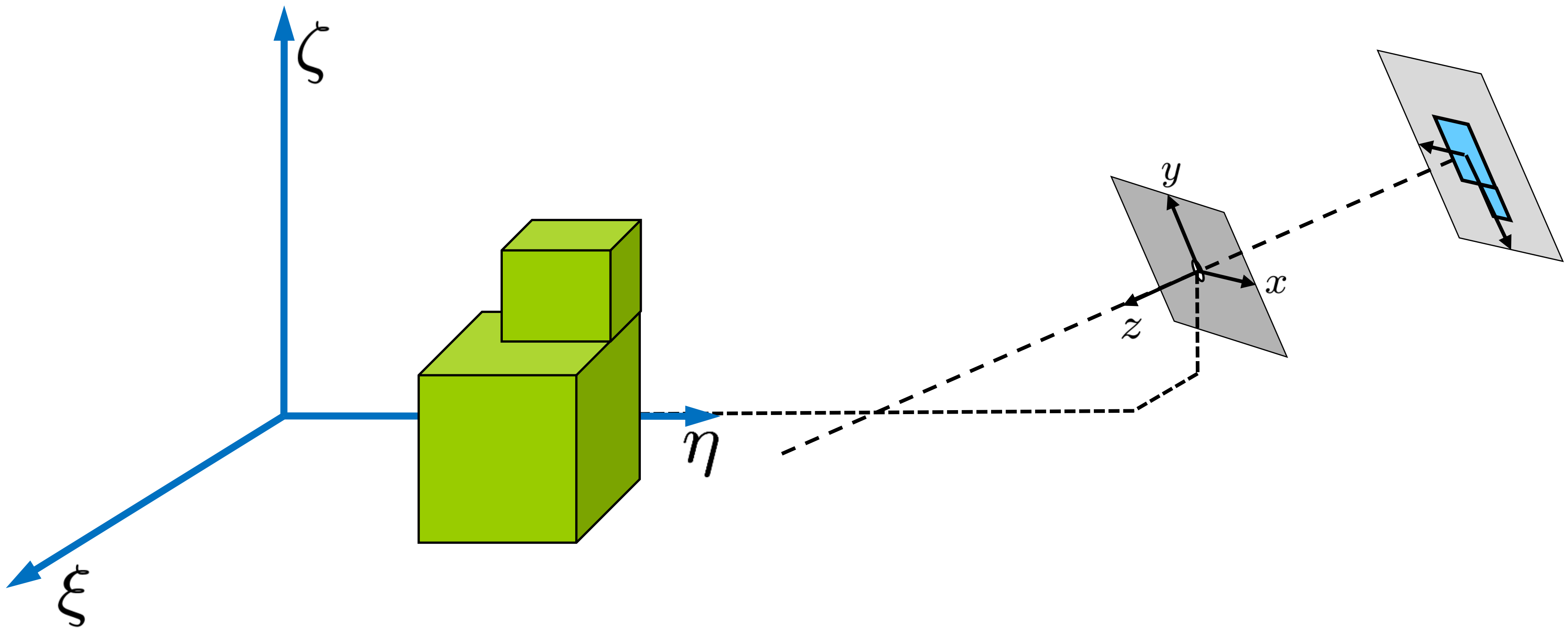
$$\cot\theta = \frac{\alpha x' + u_0 - u}{\beta y'}$$

$$\Rightarrow u = \alpha x' - (\cot\theta) \cdot \beta y' + u_0$$

mapping from camera to image frame:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \alpha & -\beta\cot\theta \\ 0 & \frac{\beta}{\sin\theta} \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}$$

World-to-Image-Mapping cont.



- position of objects on camera coordinates usually unknown
- external coordinate system (“world frame”) (ξ,η,ζ)
- mapping:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \cdot \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + \vec{t}$$

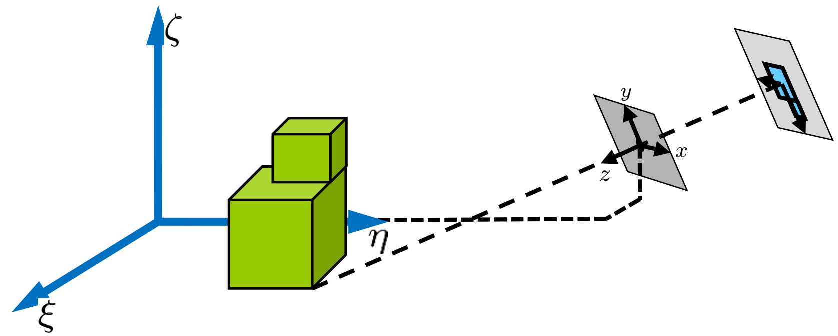
R: rotation matrix
$\vec{t}$: translation vector

World-to-Image-Mapping cont.

1. coordinate transformation

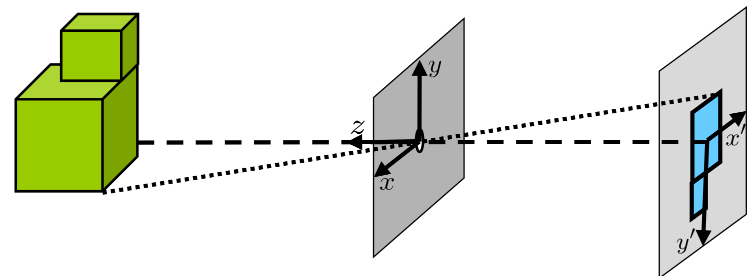
   world frame → camera frame

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \cdot \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + \vec{t}$$



2. perspective projection

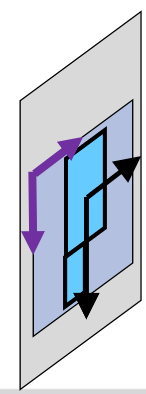
$$z \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f & 0 \\ 0 & f \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$



3. coordinate transform

   camera frame → image frame

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \alpha & -\beta \cot \theta \\ 0 & \frac{\beta}{\sin \theta} \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}$$



World-to-Image-Mapping cont.

- rewriting step 3:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha & -\beta \cot\theta & u_0 \\ 0 & \frac{\beta}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

- rewriting step 2:

$$z \cdot \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

World-to-Image-Mapping cont.

- combining step 2 and 3:

$$z \cdot \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha & -\beta \cot\theta & u_0 \\ 0 & \frac{\beta}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= \begin{pmatrix} f\alpha & -f\beta \cot\theta & u_0 \\ 0 & \frac{f\beta}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} \alpha' & -\beta' \cot\theta & u_0 \\ 0 & \frac{\beta'}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{=:A} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

with $\alpha' = f\alpha, \quad \beta' = f\beta$

World-to-Image-Mapping cont.

- rewriting step 1:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \left(R \,\middle|\, \vec{t}\right) \cdot \begin{pmatrix} \xi \\ \eta \\ \zeta \\ 1 \end{pmatrix}$$

- combining 1 with previous result:

$$z \cdot \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = A \cdot \left(R \,\middle|\, \vec{t}\right) \cdot \begin{pmatrix} \xi \\ \eta \\ \zeta \\ 1 \end{pmatrix}$$

World-to-Image-Mapping cont.

- given (ξ,η,ζ), how can we calculate (u,v) ?

$$\begin{pmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{pmatrix} = A \cdot \begin{pmatrix} R \mid \vec{t} \end{pmatrix} \cdot \begin{pmatrix}\xi\\ \eta\\ \zeta\\ 1\end{pmatrix}$$

$$\begin{pmatrix}u\\ v\end{pmatrix} = \frac{1}{\tilde{z}} \begin{pmatrix}\tilde{x}\\ \tilde{y}\end{pmatrix}$$

- given (u,v), how can we calculate (ξ,η,ζ) ?

$$\begin{pmatrix}\xi\\ \eta\\ \zeta\end{pmatrix} = z R^T A^{-1} \begin{pmatrix}u\\ v\\ 1\end{pmatrix} - R^T \vec{t} \quad \text{with } z \geq 0$$

(ξ,η,ζ) is not unique but element of a ray

World-to-Image-Mapping cont.

- coordinates of camera origin:

  $$(\xi, \eta, \zeta)^T = -R^T \vec{t}$$

- parameters:
  - *intrinisic parameters: describe the camera (5 parameters)*

    $$u_0, v_0, \alpha', \beta', \theta$$
  - *extrinsic parameters: the pose of the camera (6 parameters)*

    $$R, \vec{t}$$
  - *sometimes, the model is simplified assuming*

    $$\theta = 90^\circ,\ \alpha' = \beta'$$

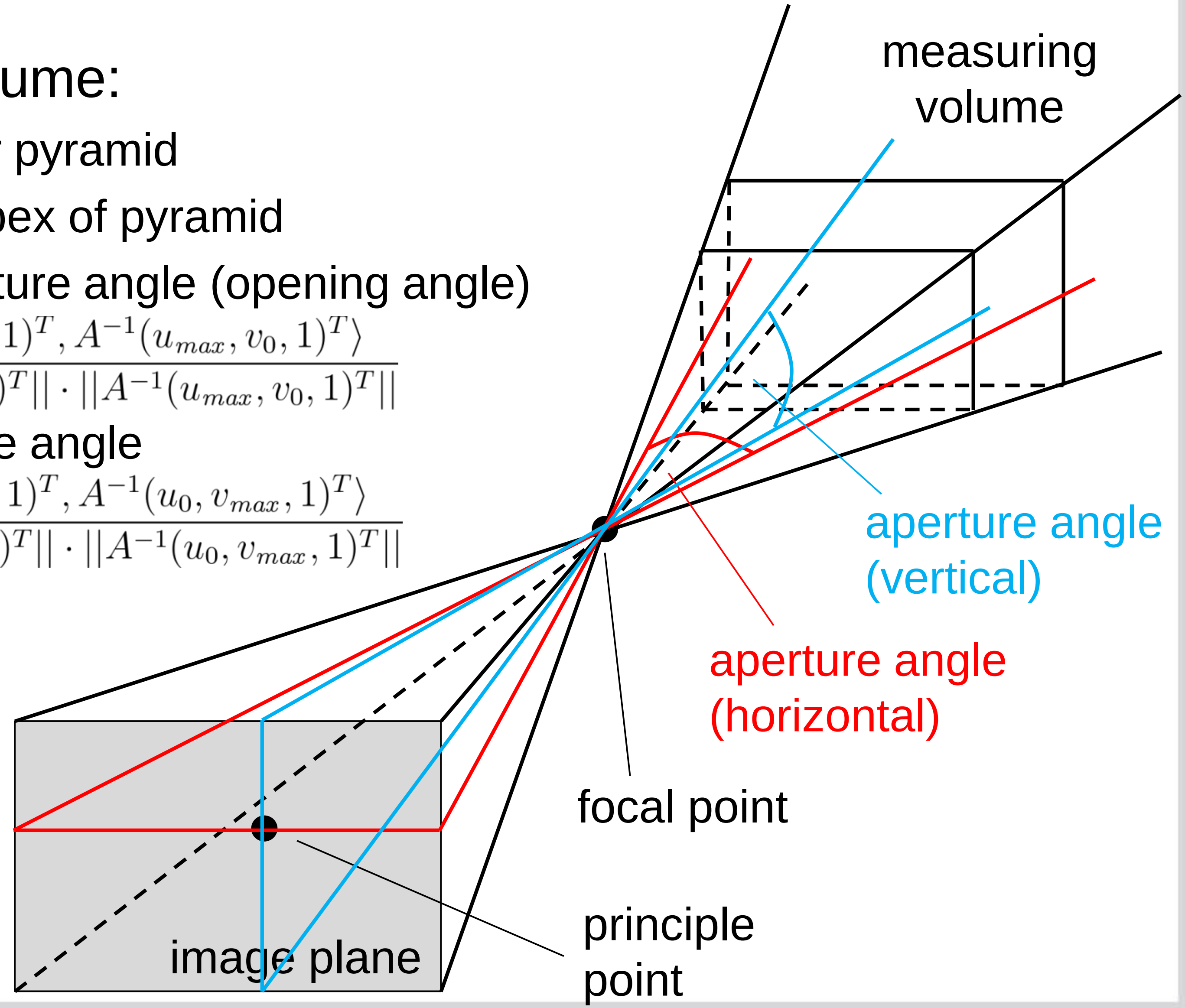
Measuring Volume

- measuring volume:
  - is a rectangular pyramid
  - focal point is apex of pyramid
  - horizontal aperture angle (opening angle)

$$arccos\frac{\langle A^{-1}(0,v_0,1)^T, A^{-1}(u_{max},v_0,1)^T\rangle}{||A^{-1}(0,v_0,1)^T||\cdot||A^{-1}(u_{max},v_0,1)^T||}$$

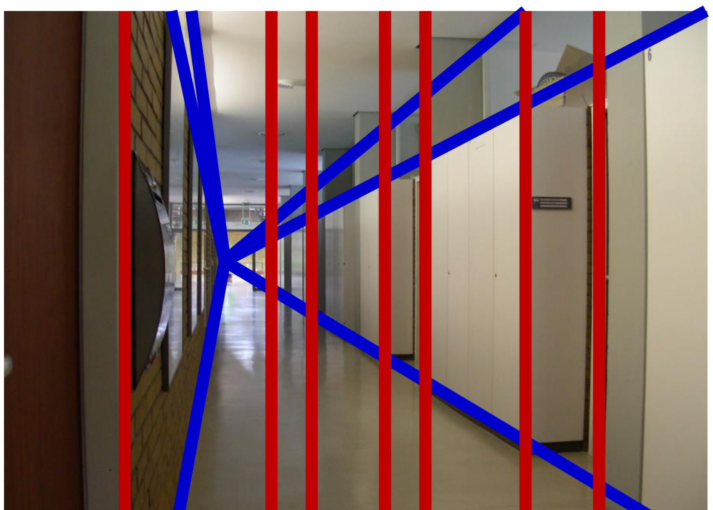
  - vertical aperture angle

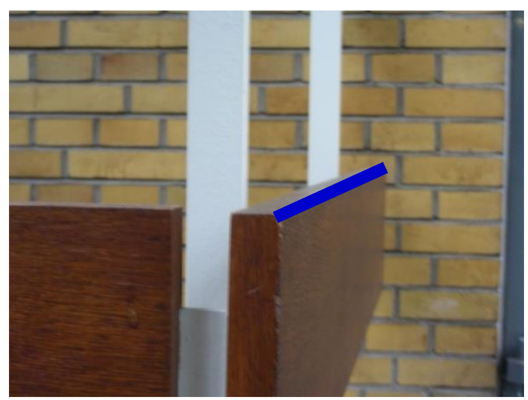
$$arccos\frac{\langle A^{-1}(u_0,0,1)^T, A^{-1}(u_0,v_{max},1)^T\rangle}{||A^{-1}(u_0,0,1)^T||\cdot||A^{-1}(u_0,v_{max},1)^T||}$$

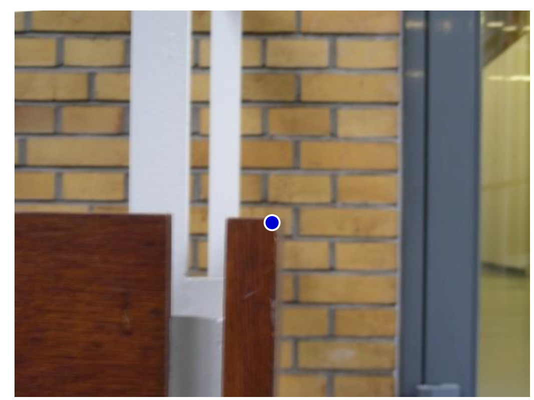


Perspective Projection

world (3D)	image (2D)	
lines →	• lines	
	• points	(if line meets focal point)
parallel lines →	• lines intersecting in one point	
	• parallel lines	(if lines are orthogonal to optical axis)
	• line and point	(if one line meets focal point)

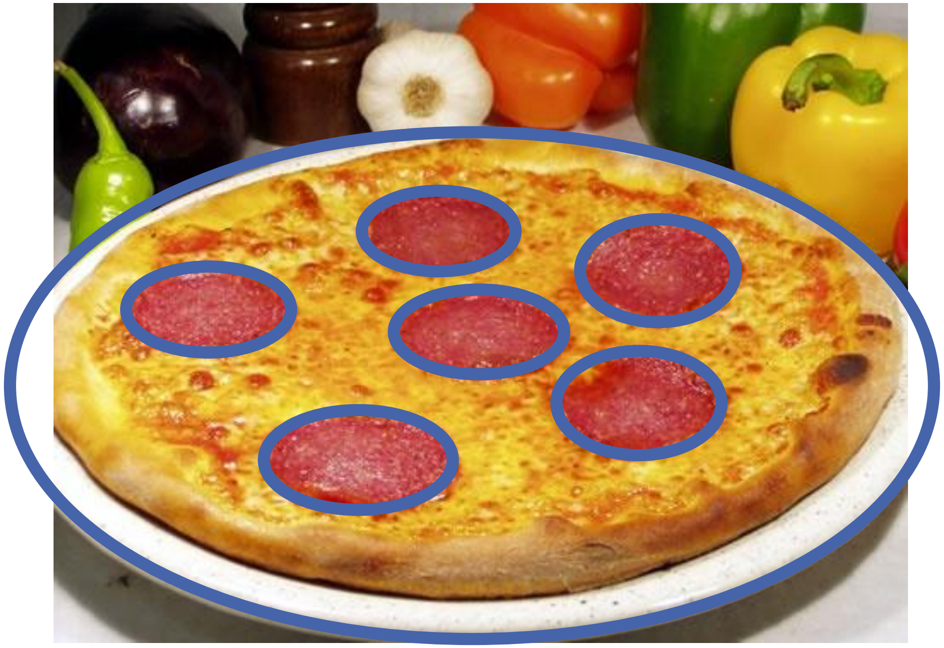






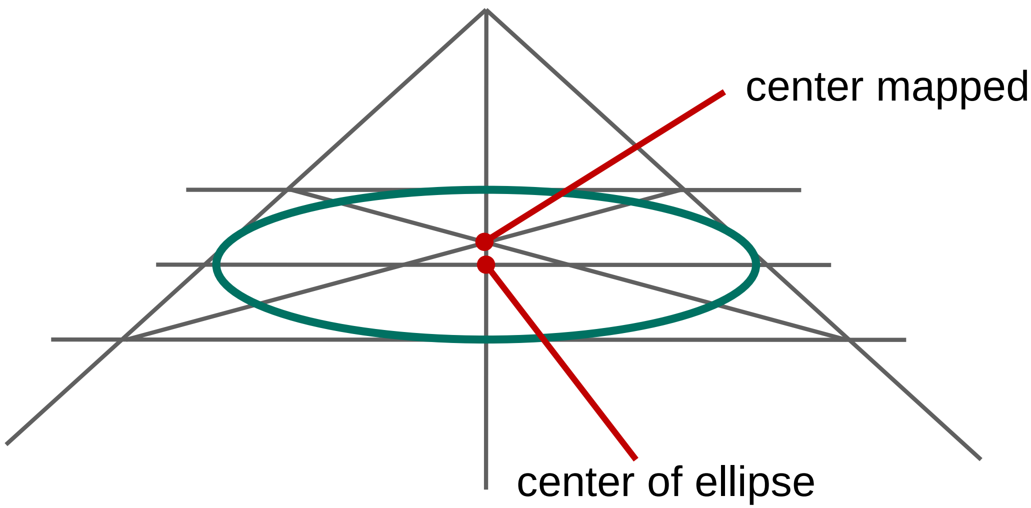
Perspective Projection cont.

world (3D)	image (2D)
circles and ellipses →	ellipses (or line segments)
circles in a plane orthogonal to optical axis →	circles



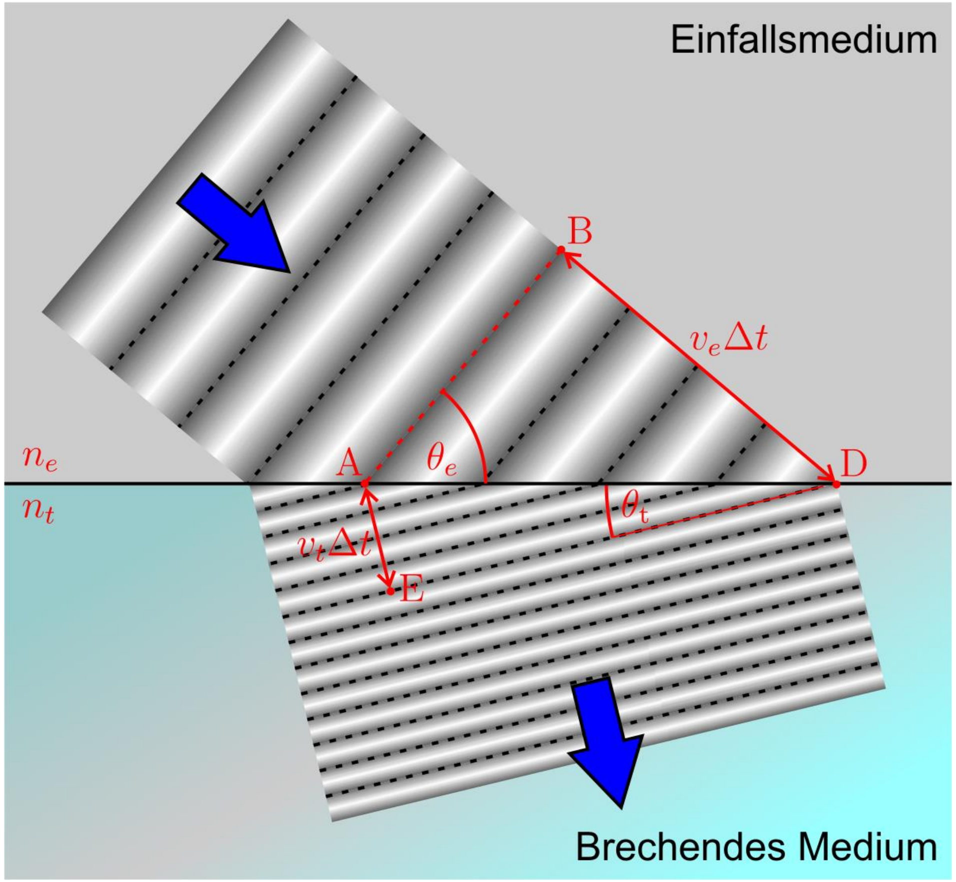
Perspective Projection cont.

- perspective projection does **not**:
  - preserve angles
  - preserve lengths
  - preserve area
  - preserve ratio of lengths
  - map the center of a circle/ellipse onto the center of the ellipse mapped (except: if the plane is orthogonal to the optical axis)



Lenses

- pinhole cameras poorly let light through → lenses
- Snell's law of refraction



Snell's law:

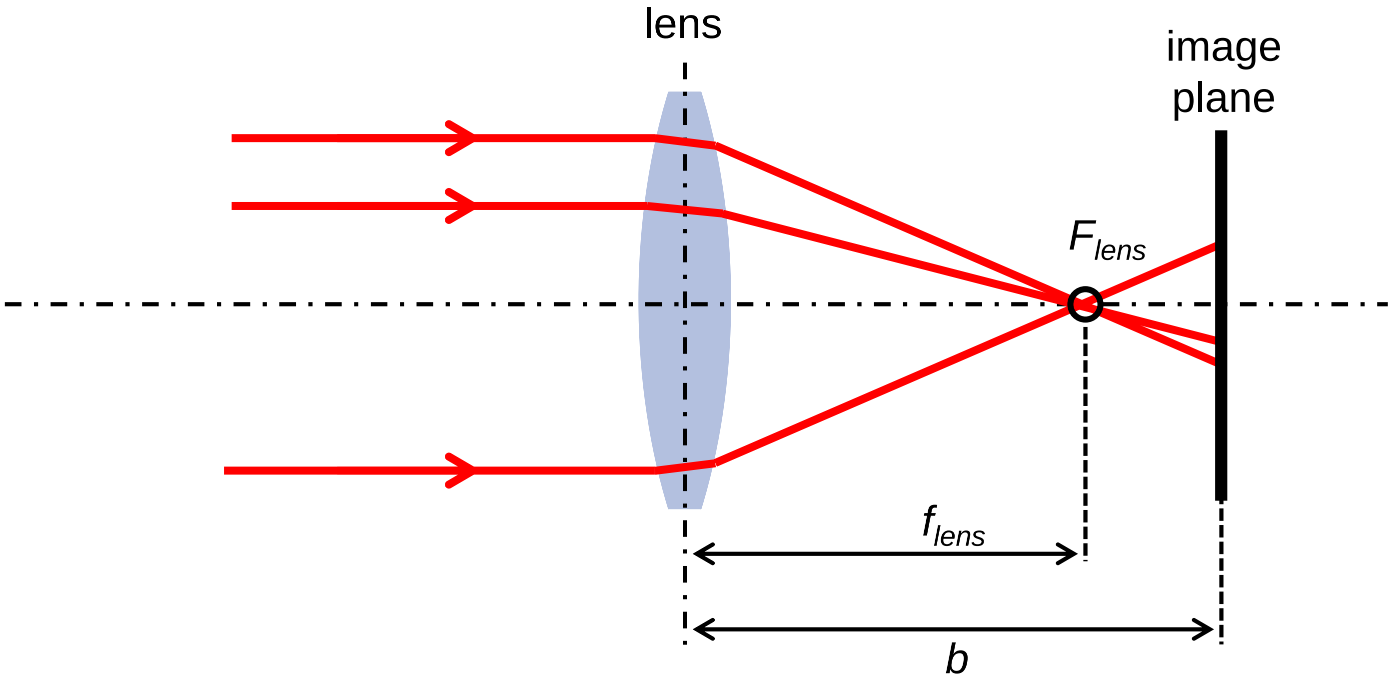
$$n_e \sin\theta_e = n_t \sin\theta_t$$

$$n_{medium} = \frac{v_{vacuum}}{v_{medium}}$$

Lenses cont.

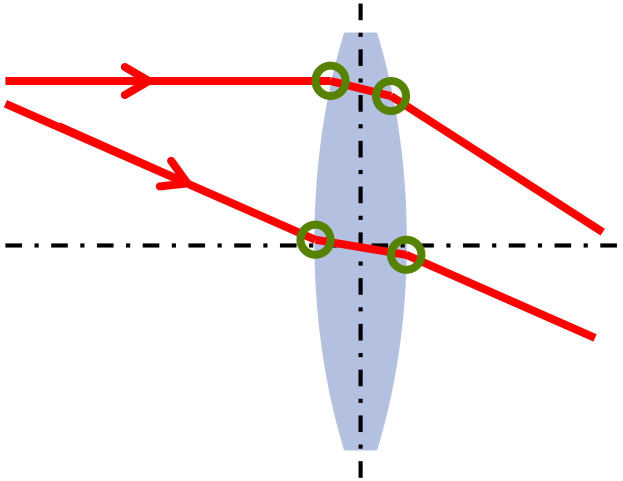
- focal length of a lens:

  the distance between lens and focal point, the point where rays of light parallel to the optical axis meet after being refracted

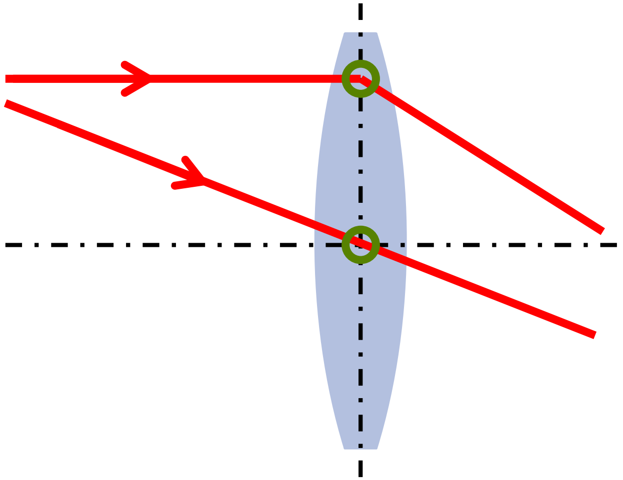


Thin Lenses

- refraction in lenses
  - surface air/glass
  - surface glass/air

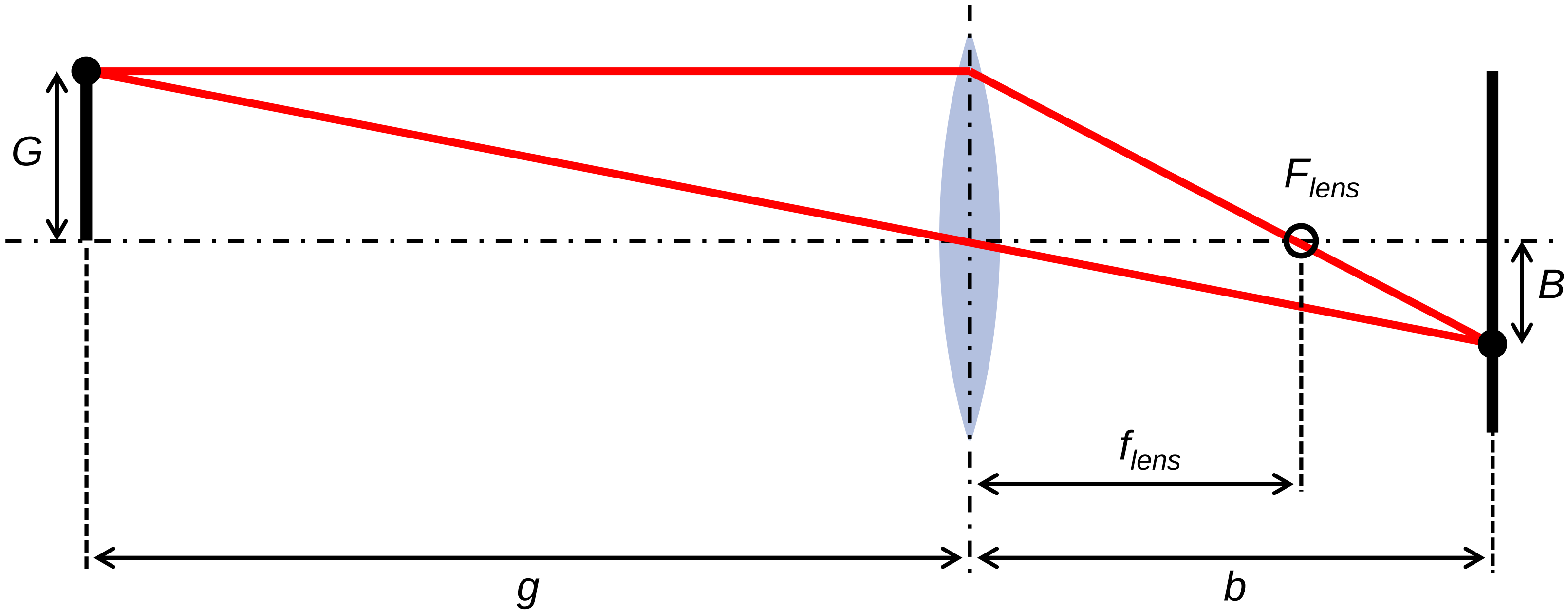


- thin lenses
  - negligible thickness
  - double refraction can be approximated by single refraction at the center line
  - simpler geometric modeling



Thin Lenses cont.

- which condition must hold for a sharp image?



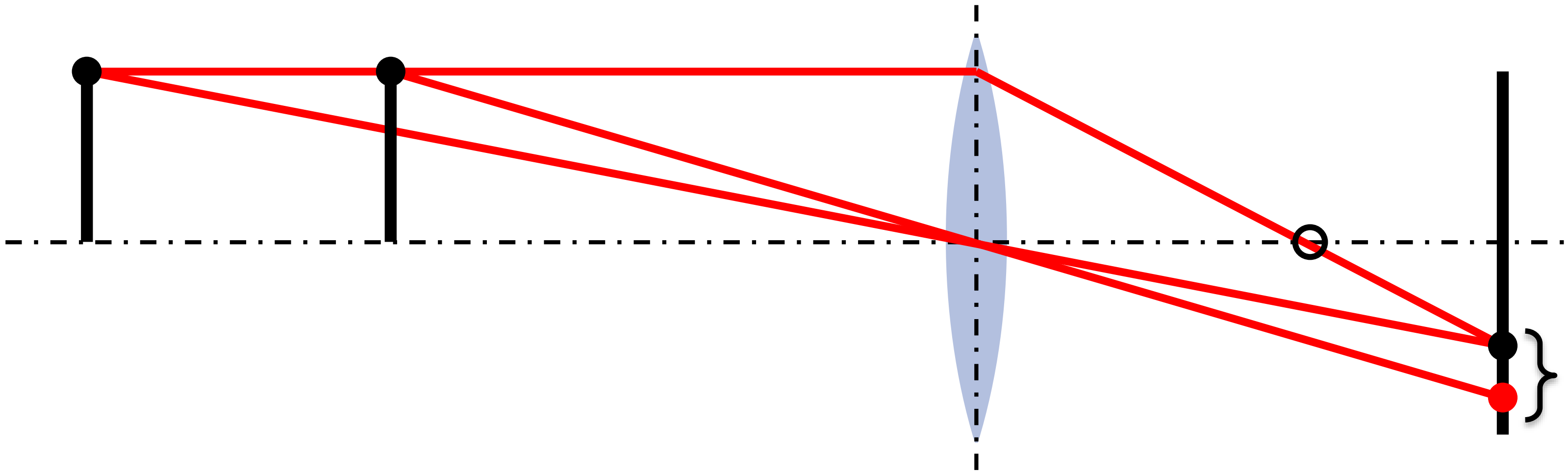
- intercept theorem: $\frac{B}{G} = \frac{b}{g}$
- intercept theorem: $\frac{B}{G} = \frac{b - f_{lens}}{f_{lens}}$

$$\Rightarrow \boxed{\frac{1}{f_{lens}} = \frac{1}{b} + \frac{1}{g}}$$

lens equation

Thin Lenses cont.

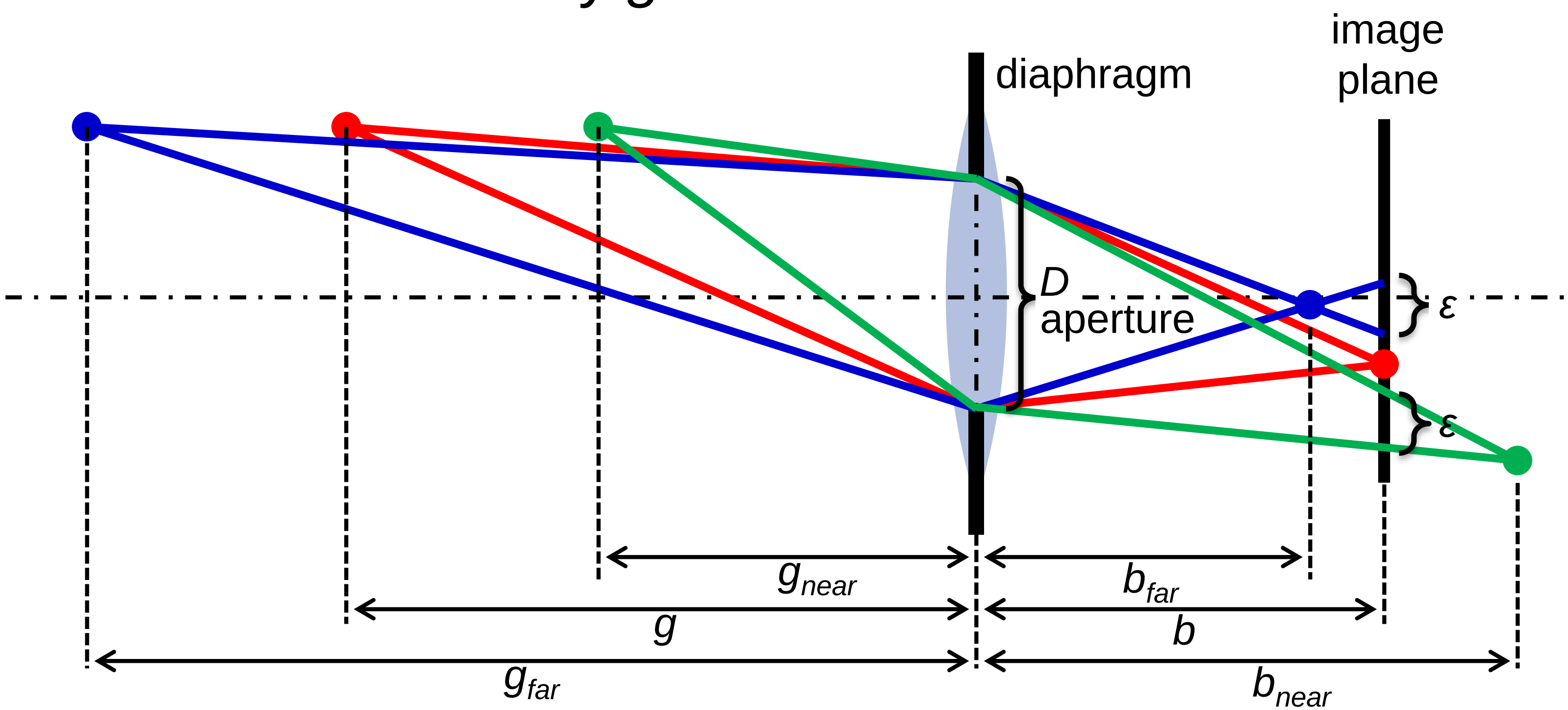
- What happens when lens equation is violated?



- How much can we vary *g* with little effect?

Depth of Field

- How much can we vary g with little effect?



– intercept theorem: $\frac{\epsilon}{D} = \frac{b - b_{far}}{b_{far}} = \ldots = \frac{f_{lens} \cdot (g_{far} - g)}{g_{far} \cdot (g - f_{lens})}$

– intercept theorem: $\frac{\epsilon}{D} = \frac{b_{near} - b}{b_{near}} = \ldots = \frac{f_{lens} \cdot (g - g_{near})}{g_{near} \cdot (g - f_{lens})}$

Depth of Field cont.

$$\frac{\epsilon}{D} = \frac{f_{lens} \cdot (g_{far} - g)}{g_{far} \cdot (g - f_{lens})}$$

$$\frac{\epsilon}{D} = \frac{f_{lens} \cdot (g - g_{near})}{g_{near} \cdot (g - f_{lens})}$$

$$\Rightarrow$$

$$g_{far} = \frac{g d_h}{d_h - (g - f_{lens})}$$

$$g_{near} = \frac{g d_h}{d_h + (g - f_{lens})}$$

$$\Delta g = g_{far} - g_{near} = 2\frac{g d_h (g - f_{lens})}{d_h^2 - (g - f_{lens})^2}$$

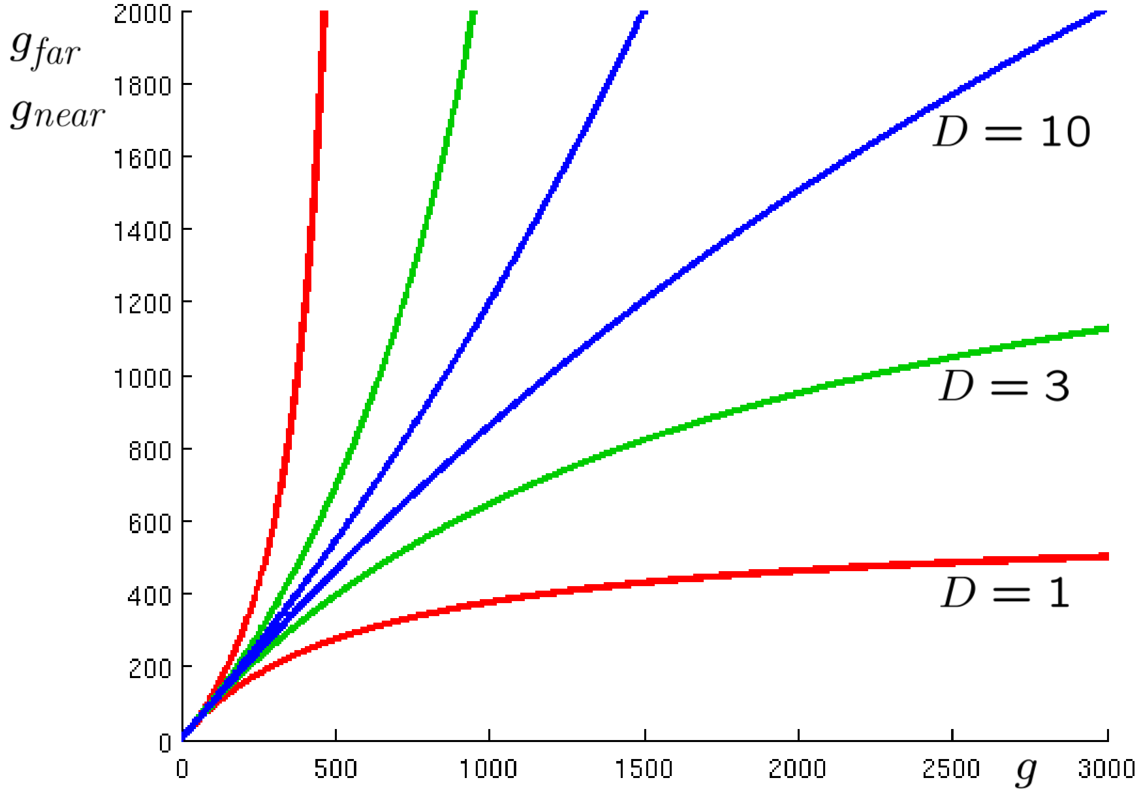
$$d_h = \frac{D f_{lens}}{\epsilon}$$ (hyperfocal distance)

– observation:

for $g \to d_h + f_{lens}$ holds: $g_{far} \to \infty$

$\Delta g \to \infty$

Depth of Field cont.



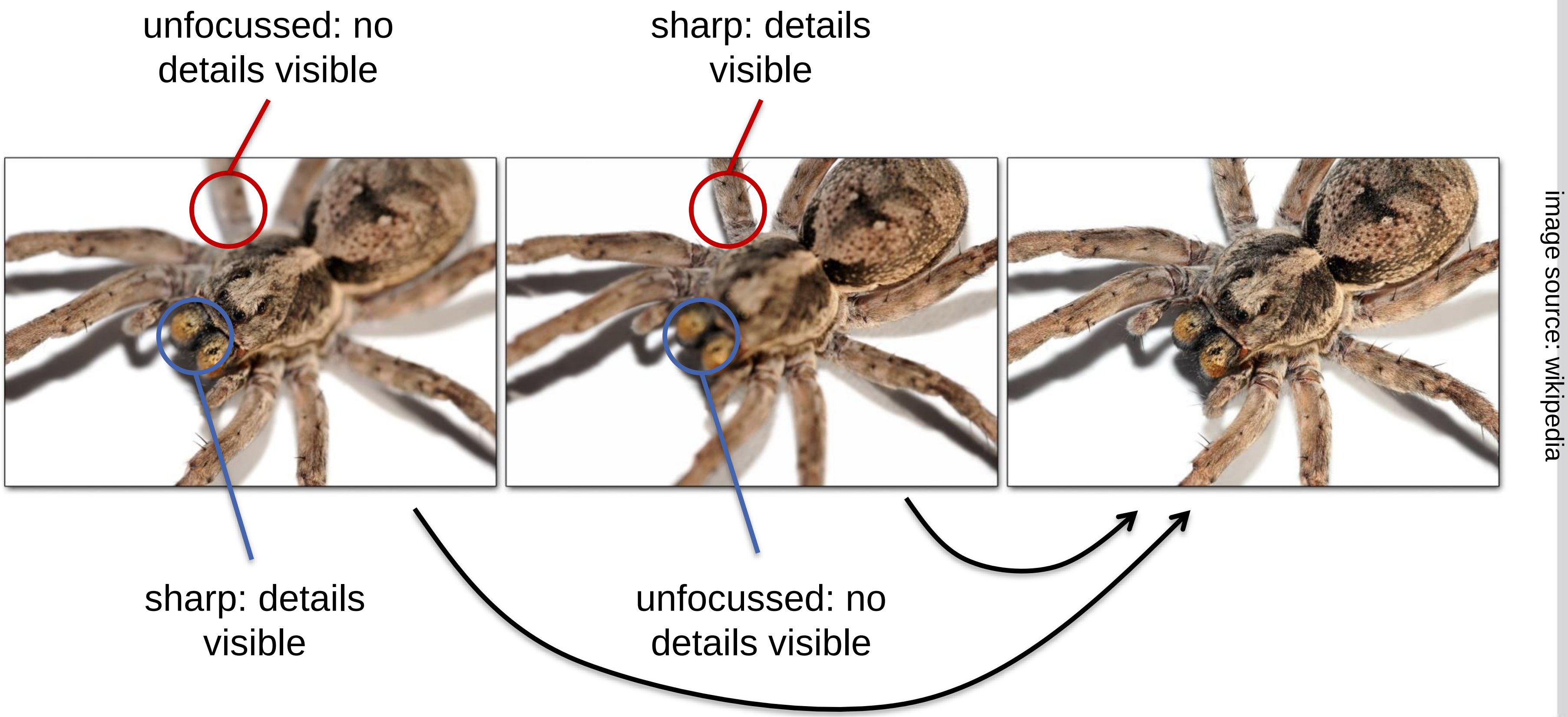
$f_{lens} = 6$

$\epsilon = 0.01$

Focus Series

– focus bracketing/focus stacking

image series with varying distance between lens and image plane to overcome a limited depth of field



Lens Aberrations

- geometric aberrations:

  no unique focal point due to imperfect lens geometry

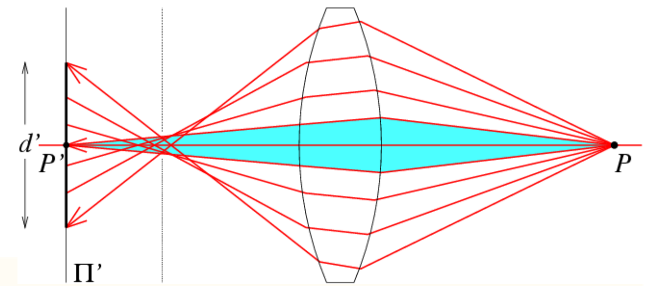
  - spherical aberration, astigmatism, coma

- chromatic aberrations:

  dispersion caused by different refraction index for different wavelength (“rainbow effect”)

- vignetting:

  reduced light intensity and saturation in the image periphery



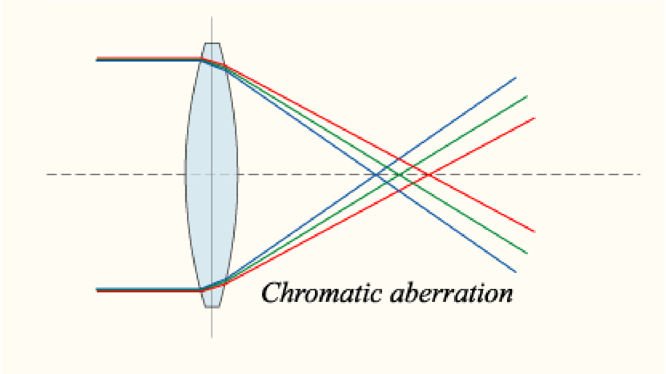


Image Distortion

- image distortion:

  perspective projection should map lines to lines.
  But most cameras do not → distortion

  – radial distortion

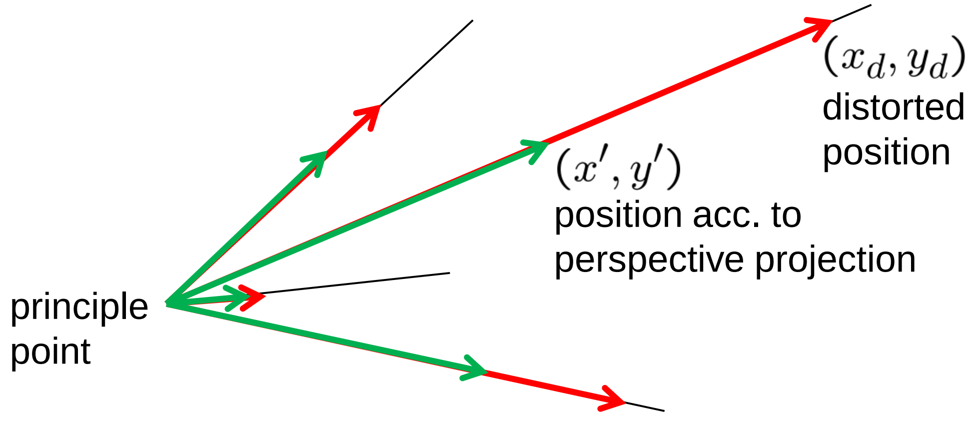
    suboptimal shape of lens

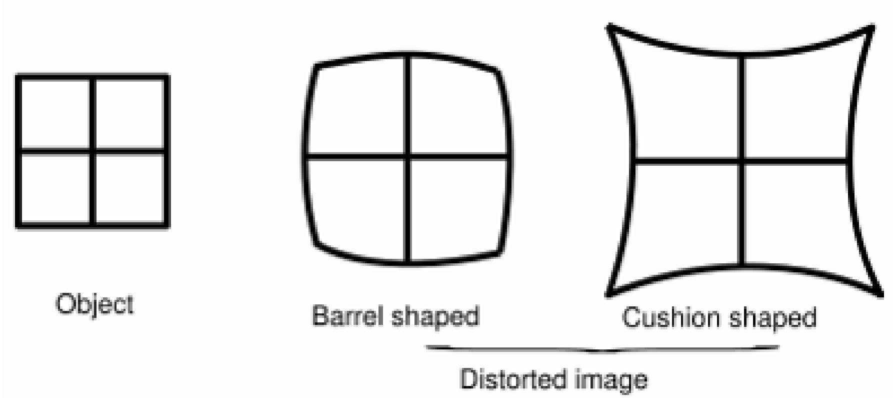
  – tangential distortion

    suboptimal mounting of lens

Radial Distortion

- points are shifted away from principle point
- radial distortion is symmetric
- amount of shifting depends nonlinearly from the distance to the principle point
- rectangular objects appear barrel-shaped or pincushion-shaped in the image





Radial Distortion cont.

- mathematical modeling with even polynomials:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = (1 + k_1 r^2 + k_2 r^4) \begin{pmatrix} x_d \\ y_d \end{pmatrix} \quad \text{with } r^2 = x_d^2 + y_d^2$$

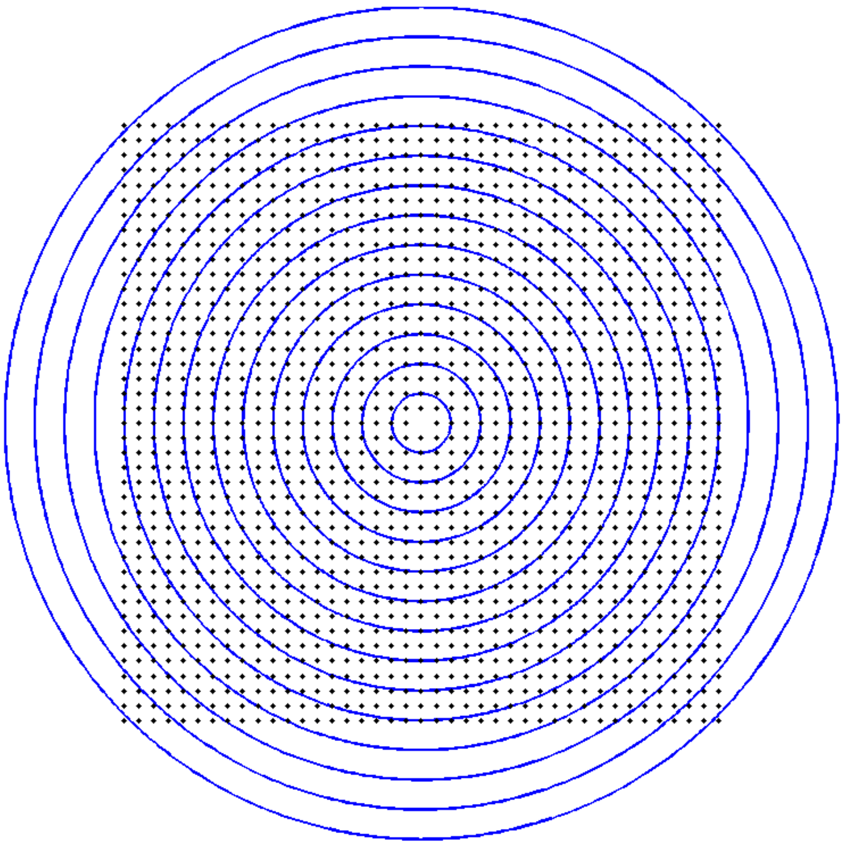
  or in image coordinates:

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + (1 + k_1 r^2 + k_2 r^4) \begin{pmatrix} u_d - u_0 \\ v_d - v_0 \end{pmatrix}$$

$$\text{with } r^2 = (u_d - u_0)^2 + (v_d - v_0)^2$$

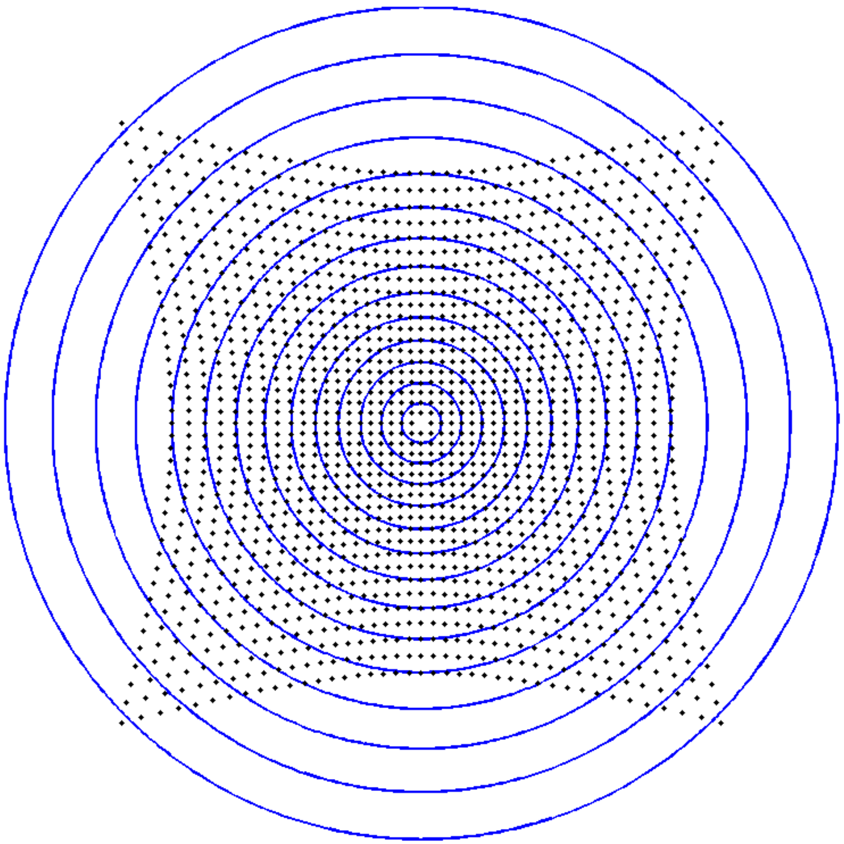
Radial Distortion cont.

k_1=0
k_2=0



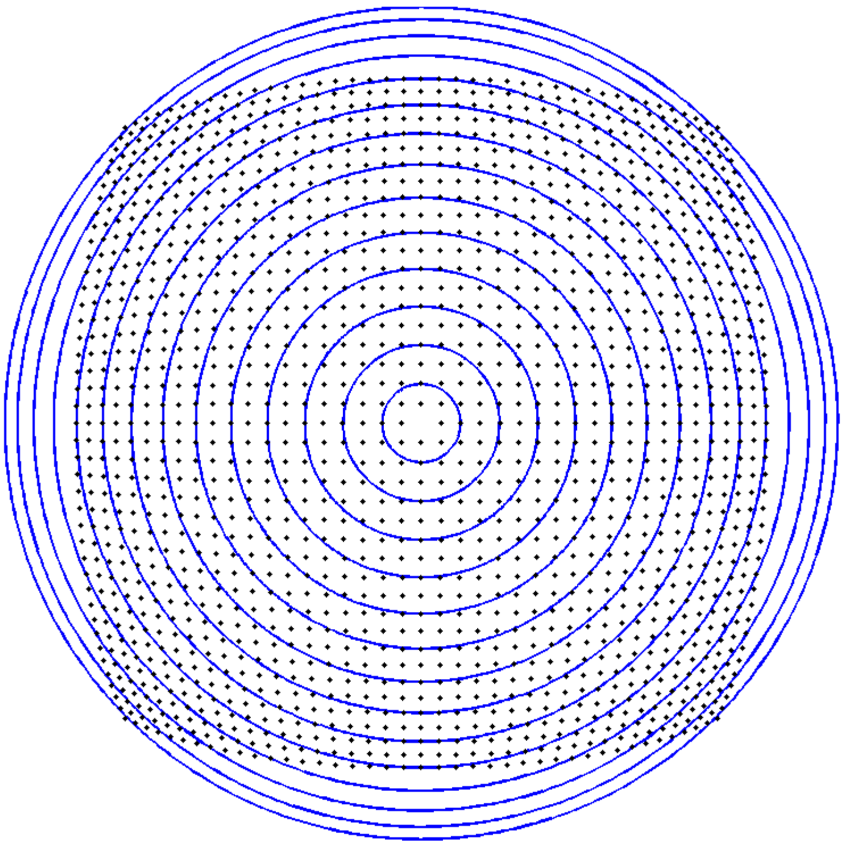
Radial Distortion cont.

$k_1>0$
$k_2 \geq 0$



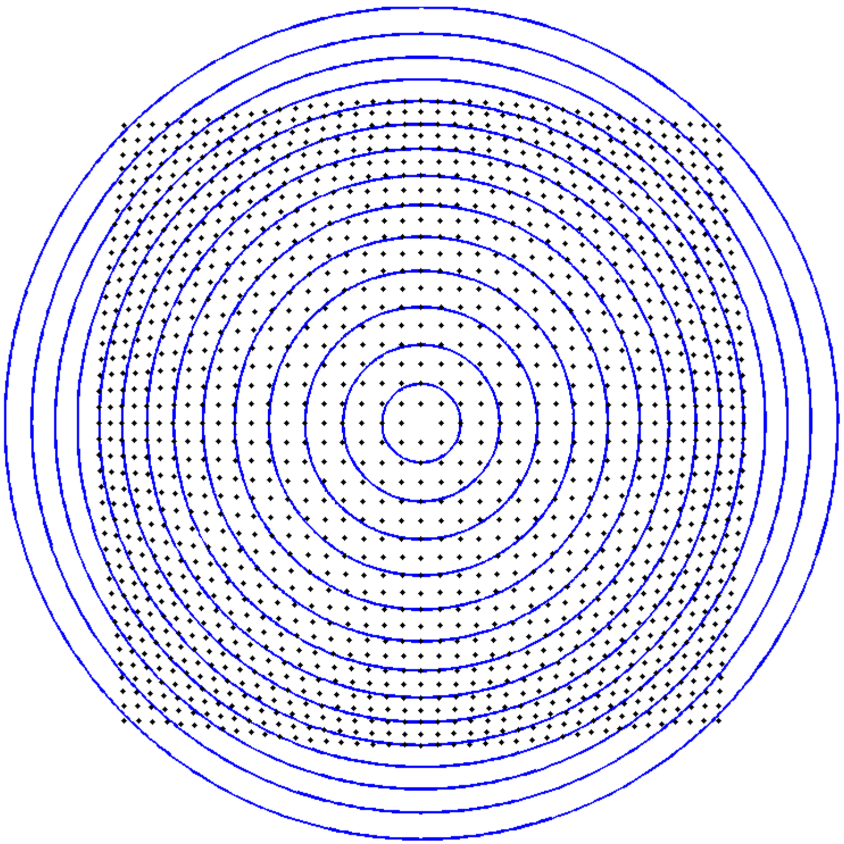
Radial Distortion cont.

$k_1 < 0$
$k_2 \leq 0$



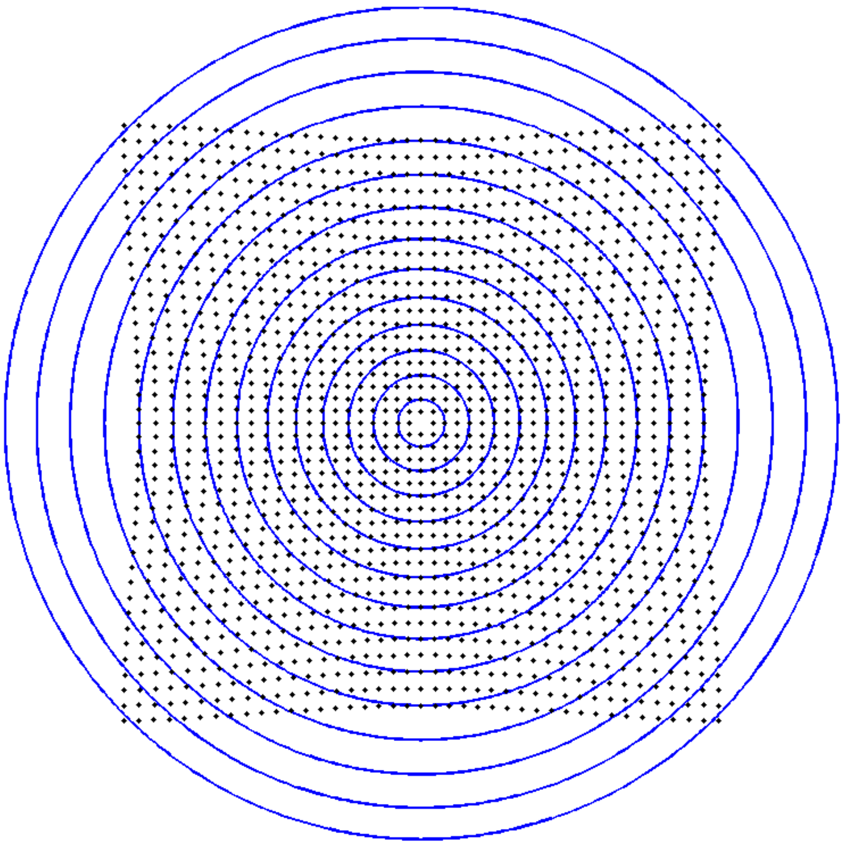
Radial Distortion cont.

$k_1<0$
$k_2>0$



Radial Distortion cont.

$k_1>0$
$k_2<0$



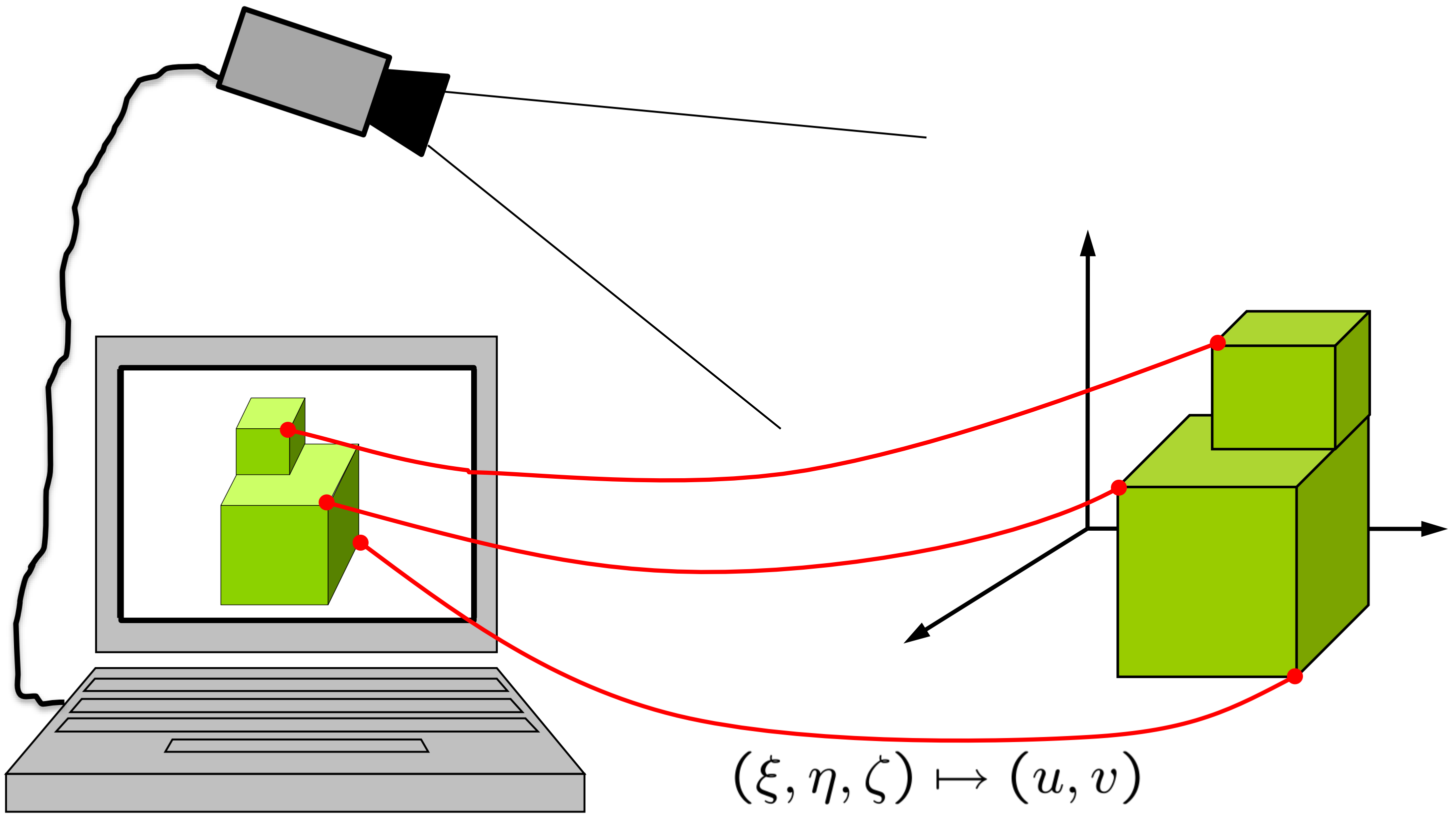
CAMERA CALIBRATION

Camera Calibration

- world-to-image mapping contains many parameters:
  - *intrinisic parameters:*
    $u_0, v_0, \alpha', \beta', \theta$
  - *extrinsic parameters:*
    $R, \vec{t}$
  - *distortion parameters:*
    $k_1, k_2, \ldots$
- calibration = process to determine parameters

Camera Calibration cont.

- calibration: determine camera parameters from pairs of image points and world points



Camera Calibration cont.

- from one or several pictures we get corresponding points:

$$(\xi_i, \eta_i, \zeta_i) \mapsto (u_i, v_i)$$

- find camera parameters $A, R, \vec{t}$ that map (ξ_i, η_i, ζ_i) onto (u_i, v_i) as good as possible

- several approaches. Here:
  1. Tsai's approach
  2. Zhang's approach

Camera Calibration cont.

- world-to-image mapping:

$$z \cdot \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \underbrace{A \cdot \left(R \,\middle|\, \vec{t}\right)}_{=:M} \cdot \begin{pmatrix} \xi \\ \eta \\ \zeta \\ 1 \end{pmatrix}$$

- *M* is 3x4 matrix

$$M = \begin{pmatrix} m_{1,1} & \cdots & m_{1,4} \\ \vdots & \ddots & \vdots \\ m_{3,1} & \cdots & m_{3,4} \end{pmatrix}$$

- we get:

$$\vec{m}_{1,1:3} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + m_{1,4} - u(\vec{m}_{3,1:3} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + m_{3,4}) = 0$$

$$\vec{m}_{2,1:3} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + m_{2,4} - v(\vec{m}_{3,1:3} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + m_{3,4}) = 0$$

Camera Calibration cont.

- determining camera parameters by minimizing:

$$\sum_i \left(\left(\vec{m}_{1,1:3} \begin{pmatrix} \xi_i \\ \eta_i \\ \zeta_i \end{pmatrix} + m_{1,4} - u_i(\vec{m}_{3,1:3} \begin{pmatrix} \xi_i \\ \eta_i \\ \zeta_i \end{pmatrix} + m_{3,4}) \right)^2 + \left(\vec{m}_{2,1:3} \begin{pmatrix} \xi_i \\ \eta_i \\ \zeta_i \end{pmatrix} + m_{2,4} - v_i(\vec{m}_{3,1:3} \begin{pmatrix} \xi_i \\ \eta_i \\ \zeta_i \end{pmatrix} + m_{3,4}) \right)^2 \right)$$

- zeroing partial derivatives:

$$\begin{pmatrix} \sum_i S_i & 0 & -\sum_i u_i S_i \\ 0 & \sum_i S_i & -\sum_i v_i S_i \\ -\sum_i u_i S_i & -\sum_i v_i S_i & \sum_i (u_i^2 + v_i^2) S_i \end{pmatrix} \cdot \begin{pmatrix} \vec{m}^T_{1,1:4} \\ \vec{m}^T_{2,1:4} \\ \vec{m}^T_{3,1:4} \end{pmatrix} = \vec{0}$$

with

$$S_i = (\xi_i, \eta_i, \zeta_i, 1)^T (\xi_i, \eta_i, \zeta_i, 1)$$

Camera Calibration cont.

- solution: Eigenvector with respect to smallest Eigenvalue
  - 1 degree of freedom: length of solution

- structure of the solution:

$$M = A \cdot \left(R \middle| \vec{t}\right) = \begin{pmatrix} \vec{m}_{1,1:3} & m_{1,4} \\ \vec{m}_{2,1:3} & m_{2,4} \\ \vec{m}_{3,1:3} & m_{3,4} \end{pmatrix}$$

with
$$\begin{aligned}
\vec{m}_{1,1:3} &= \alpha' \vec{r}_{1,1:3} - \beta' \cot\theta \vec{r}_{2,1:3} + u_0 \vec{r}_{3,1:3} \\
m_{1,4} &= \alpha' t_1 - \beta' \cot\theta t_2 + u_0 t_3 \\
\vec{m}_{2,1:3} &= \frac{\beta'}{\sin\theta} \vec{r}_{2,1:3} + v_0 \vec{r}_{3,1:3} \\
m_{2,4} &= \frac{\beta'}{\sin\theta} t_2 + v_0 t_3 \\
\vec{m}_{3,1:3} &= \vec{r}_{3,1:3} \\
m_{3,4} &= t_3
\end{aligned}$$

Camera Calibration cont.

- *R* is a rotation matrix:

$$||\vec{r}_{1,1:3}|| = 1$$
$$||\vec{r}_{2,1:3}|| = 1$$
$$||\vec{r}_{3,1:3}|| = 1$$
$$\langle \vec{r}_{1,1:3}, \vec{r}_{2,1:3} \rangle = 0$$
$$\langle \vec{r}_{2,1:3}, \vec{r}_{3,1:3} \rangle = 0$$
$$\langle \vec{r}_{3,1:3}, \vec{r}_{1,1:3} \rangle = 0$$

- Since $\vec{m}_{3,1:3} = \vec{r}_{3,1:3}$ choose solution with $||\vec{m}_{3,1:3}||^2 = 1$ (two possibilities, check $det(R) = +1$)

- Given M, how can we derive camera parameters *R, t, α‘, β‘, θ,* u_0, v_0 ?

Camera Calibration cont.

$$\vec{r}_{3,1:3} = \vec{m}_{3,1:3}$$

$$t_3 = m_{3,4}$$

$$v_0 = \langle \vec{m}_{3,1:3}, \vec{m}_{2,1:3} \rangle$$

$$u_0 = \langle \vec{m}_{3,1:3}, \vec{m}_{1,1:3} \rangle$$

$$\boxed{\frac{\beta'}{\sin\theta}} = \sqrt{||\vec{m}_{2,1:3}||^2 - v_0^2}$$

$$t_2 = \left(\frac{\beta'}{\sin\theta}\right)^{-1} (m_{2,4} - v_0 t_3)$$

$$\boxed{\vec{m}_{1,1:3} = \alpha' \vec{r}_{1,1:3} - \beta' \cot\theta \vec{r}_{2,1:3} + u_0 \vec{r}_{3,1:3}}$$

$$\boxed{m_{1,4} = \alpha' t_1 - \beta' \cot\theta t_2 + u_0 t_3}$$

$$\boxed{\vec{m}_{2,1:3} = \frac{\beta'}{\sin\theta} \vec{r}_{2,1:3} + v_0 \vec{r}_{3,1:3}}$$

$$\boxed{m_{2,4} = \frac{\beta'}{\sin\theta} t_2 + v_0 t_3}$$

$$\boxed{\vec{m}_{3,1:3} = \vec{r}_{3,1:3}}$$

$$\boxed{m_{3,4} = t_3}$$

$$\vec{r}_{2,1:3} = \left(\frac{\beta'}{\sin\theta}\right)^{-1} (\vec{m}_{2,1:3} - v_0 \vec{r}_{3,1:3})$$

$$\boxed{\beta' \cot\theta} = \left(\frac{\beta'}{\sin\theta}\right)^{-1} (u_0 v_0 - \langle \vec{m}_{1,1:3}, \vec{m}_{2,1:3} \rangle)$$

$$\alpha' = \sqrt{||\vec{m}_{1,1:3}||^2 + (\beta' \cot\theta)^2 - u_0^2}$$

$$\vec{r}_{1,1:3} = (\alpha')^{-1} (\vec{m}_{1,1:3} + \beta' \cot\theta \vec{r}_{2,1:3} - u_0 \vec{r}_{3,1:3})$$

$$t_1 = (\alpha')^{-1} (m_{1,4} + \beta' \cot\theta t_2 - u_0 t_3)$$

$$\boxed{\theta} = \arccos \frac{\beta' \cot\theta}{\frac{\beta'}{\sin\theta}}$$

$$\beta' = \left(\frac{\beta'}{\sin\theta}\right) \sin\theta$$

Camera Calibration cont.

Summary: Tsai‘s approach

1. create an artificial scene with calibration markers (markers in general position)
2. measure 3d position of calibration markers
3. make a picture
4. measure 2d position of calibration markers
5. solve optimization problem to estimate matrix *M*
6. decompose *M* into *A, R, t*

R.Y. Tsai,
An Efficient and Accurate Camera Calibration Technique for 3D Machine Vision.
Proceedings of IEEE Conference on Computer Vision and Pattern Recognition,
Miami Beach, FL, pp. 364-374, 1986.

Camera Calibration: Zhang

Assume 3d-points on the plane $\zeta = 0$

These points are mapped by a camera to

$$z \cdot \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = A \cdot (R \cdot \begin{pmatrix} \xi \\ \eta \\ 0 \end{pmatrix} + \vec{t})$$

$$= \underbrace{A \cdot \left(\vec{r}_{1:3,1},\ \vec{r}_{1:3,2},\ \vec{t}\right)}_{=:H} \cdot \begin{pmatrix} \xi \\ \eta \\ 1 \end{pmatrix}$$

H is called a *homography*

$$H = A \cdot \left(\vec{r}_{1:3,1},\ \vec{r}_{1:3,2},\ \vec{t}\right)$$

Camera Calibration: Zhang cont.

If we know several homographies $H_1, H_2, \ldots, H_n$, can we derive A?

Let us first consider

$$B = A^{-T} A^{-1}$$

A is full rank, upper triangular matrix

$\rightarrow A^{-1}$ exists and is also upper triangular matrix

$\rightarrow B$ is symmetric, has 6 different entries

$\rightarrow A^{-1}$ can be calculated from B via Cholesky decomposition

$\rightarrow$ if we know B, we can derive A easily

$$B = \begin{pmatrix} b_{1,1} & b_{1,2} & b_{1,3} \\ b_{1,2} & b_{2,2} & b_{2,3} \\ b_{1,3} & b_{2,3} & b_{3,3} \end{pmatrix}$$

Camera Calibration: Zhang cont.

$$H = A \cdot \left(\vec{r}_{1:3,1},\ \vec{r}_{1:3,2},\ \vec{t}\right)$$

R is a rotation matrix, hence

$$\begin{aligned} 0 = \langle \vec{r}_{1:3,1},\ \vec{r}_{1:3,2} \rangle &= \langle A^{-1}\vec{h}_{1:3,1},\ A^{-1}\vec{h}_{1:3,2} \rangle \\ &= \vec{h}^T_{1:3,1} \cdot (A^{-T}A^{-1}) \cdot \vec{h}_{1:3,2} \\ &= \vec{h}^T_{1:3,1} \cdot B \cdot \vec{h}_{1:3,2} \qquad (1) \end{aligned}$$

$$\begin{array}{ccc} \langle \vec{r}_{1:3,1},\ \vec{r}_{1:3,1} \rangle & = 1 = & \langle \vec{r}_{1:3,2},\ \vec{r}_{1:3,2} \rangle \\ \langle A^{-1}\vec{h}_{1:3,1},\ A^{-1}\vec{h}_{1:3,1} \rangle & & \langle A^{-1}\vec{h}_{1:3,2},\ A^{-1}\vec{h}_{1:3,2} \rangle \\ \vec{h}^T_{1:3,1} \cdot B \cdot \vec{h}_{1:3,1} & & \vec{h}^T_{1:3,2} \cdot B \cdot \vec{h}_{1:3,2} \end{array}$$

$$\Rightarrow 0 = \vec{h}^T_{1:3,1} \cdot B \cdot \vec{h}_{1:3,1} - \vec{h}^T_{1:3,2} \cdot B \cdot \vec{h}_{1:3,2} \qquad (2)$$

Hence, from one homography H we obtain two constraints (1), (2) for B

Camera Calibration: Zhang cont.

If we know several homographies $H_1, H_2, \ldots, H_n$, can we derive A ?

- from each homography we obtain two constraints
- 3 homographies yield in total 6 constraints in order to estimate 6 parameters
- >3 homographies yield an overdetermined system of constraints
  $\rightarrow$ least squares method finds a matrix that minimizes the residuals

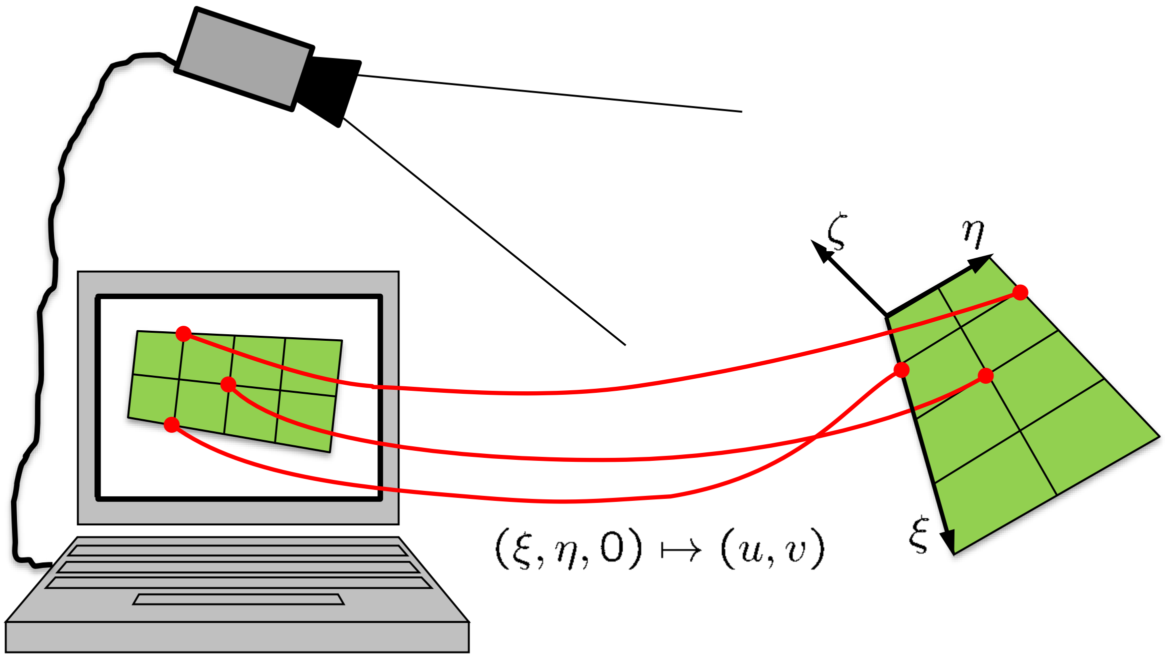
Camera Calibration: Zhang cont.

Sketch: Zhang‘s approach

1. …
2. …
3. …
4. estimate homographies H
5. solve optimization problem to estimate matrix B
6. decompose B into A, R, t
7. …

How do we get homographies?

Camera Calibration: Zhang cont.



- assume a set of point correspondences for points on a plane
- find homography H such that

$$z \cdot (u, v, 1)^T \approx H \cdot (\xi_i, \eta_i, 1)^T$$

- one correspondence yields two constraints
- $$\vec{h}_{1,1:3} \cdot (\xi_i, \eta_i, 1)^T - u_i \cdot \vec{h}_{3,1:3} \cdot (\xi_i, \eta_i, 1)^T \approx 0$$
  $$\vec{h}_{2,1:3} \cdot (\xi_i, \eta_i, 1)^T - v_i \cdot \vec{h}_{3,1:3} \cdot (\xi_i, \eta_i, 1)^T \approx 0$$
- least squares to minimize residuals and find best homography H

Camera Calibration: Zhang cont.

Sketch: Zhang‘s approach

1. create a plane with calibration markers at known positions
2. make several pictures of it with different position and orientation of plane
3. measure 2d image position of markers
4. estimate homographies *H* for each pixture
5. solve optimization problem to estimate matrix *B*
6. decompose *B* into *A, R, t*
7. optimize all parameters using nonlinear least squares

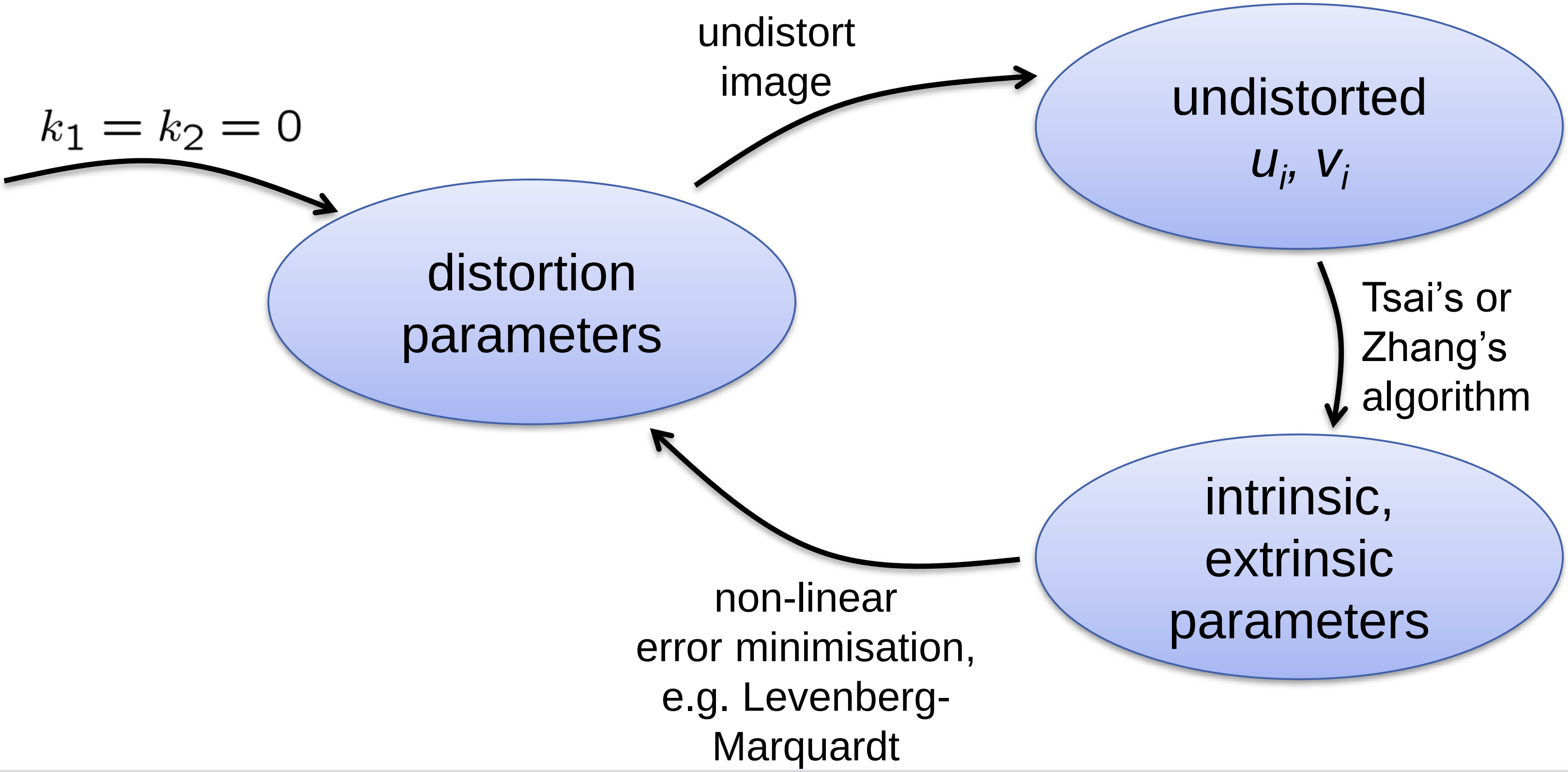
Finally, we get

- intrinsic parameters *A*
- rotation *R* and translation *t* for each plane

Z. Zhang,
A flexible new technique for camera calibration.
IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 22, no. 11, pp. 1330-1334, 2000

Camera Calibration cont.

- calibrating distortion parameters k_1, k_2:
  - non-linear optimization process
  - iterative estimation of intrinsic, extrinsic parameters and distortion parameters

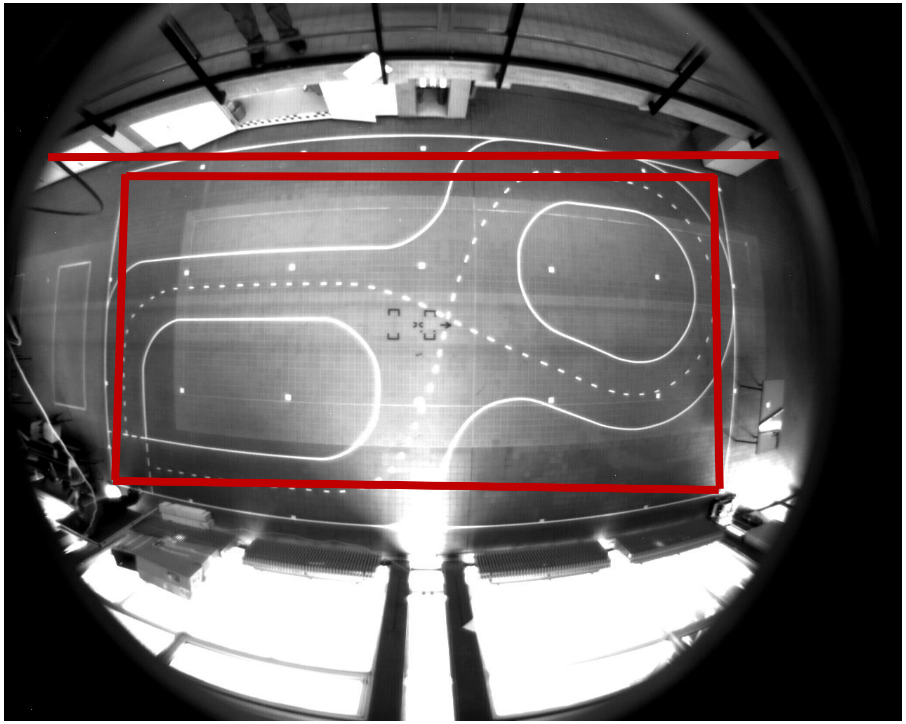


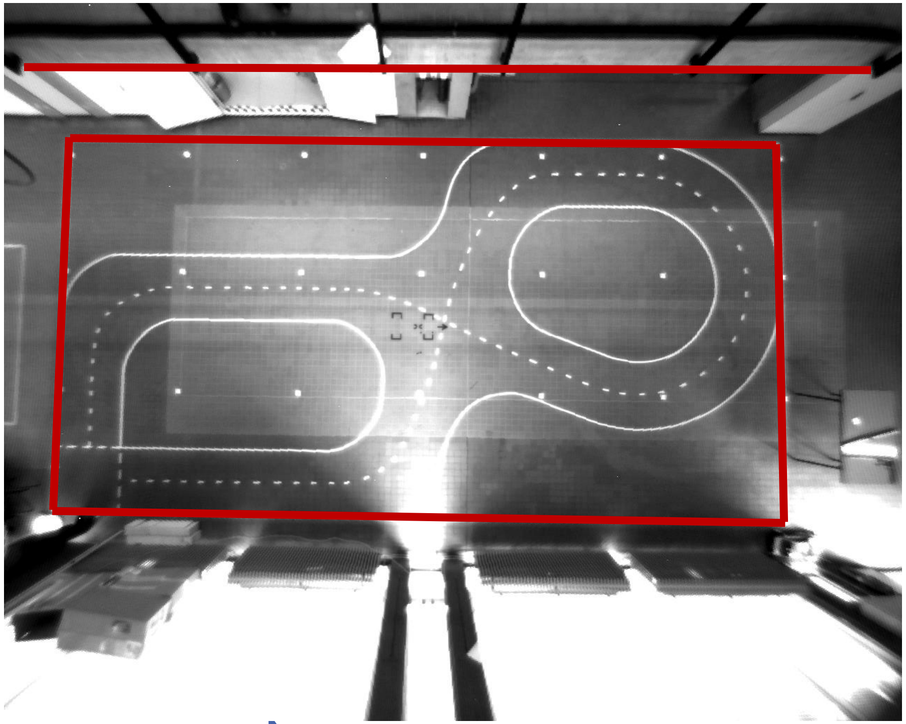
Camera Calibration cont.

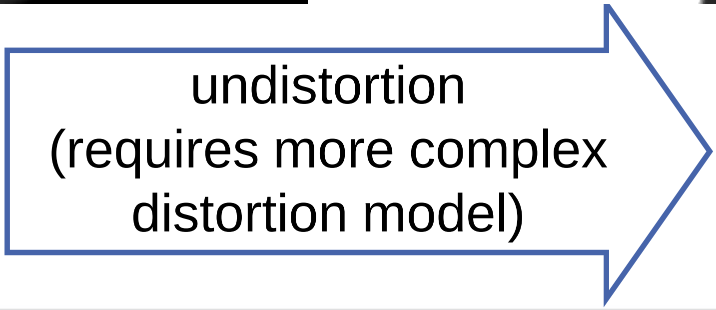
- effect of undistortion:

Camera Calibration cont.

- example: camera with wide-angle lens

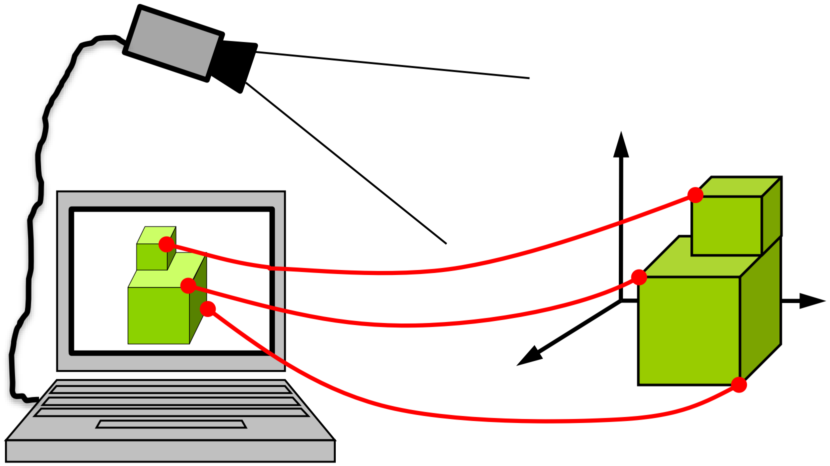






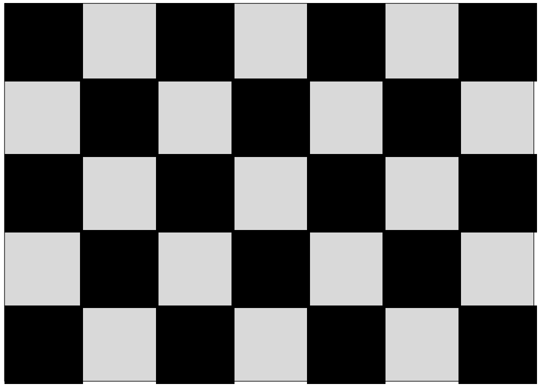
Camera Calibration cont.

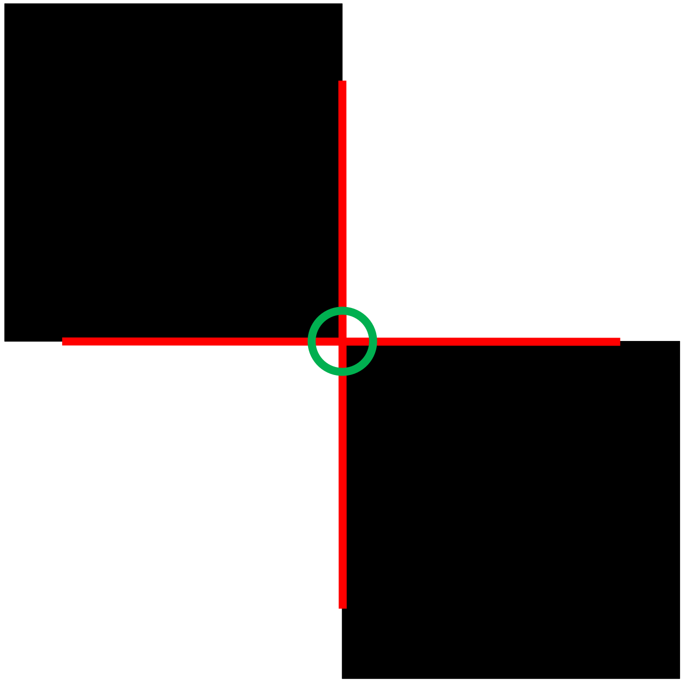
- calibration markers
  - characteristic features
  - clearly recognizable
  - easy to determine position in world
  - easy to localize in image with high precision
  - features not coplanar in world (for Tsai's approach)
  - avoid occlusion
  - avoid shadow
  - as many as possible



Camera Calibration cont.

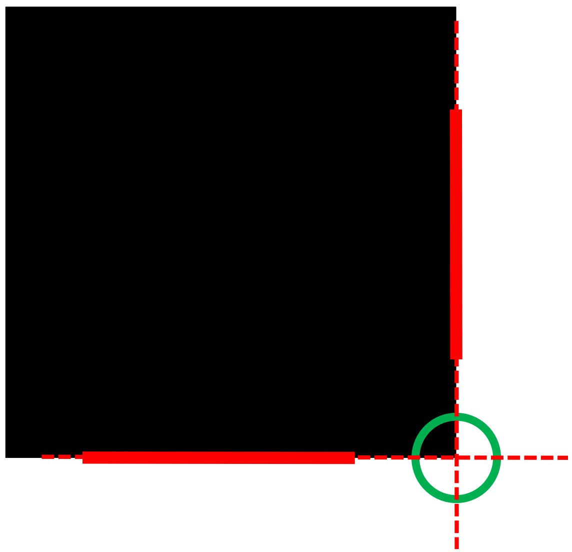
- chessboard markers
  - determine image position calculating the point of intersection of horizontal and vertical edges

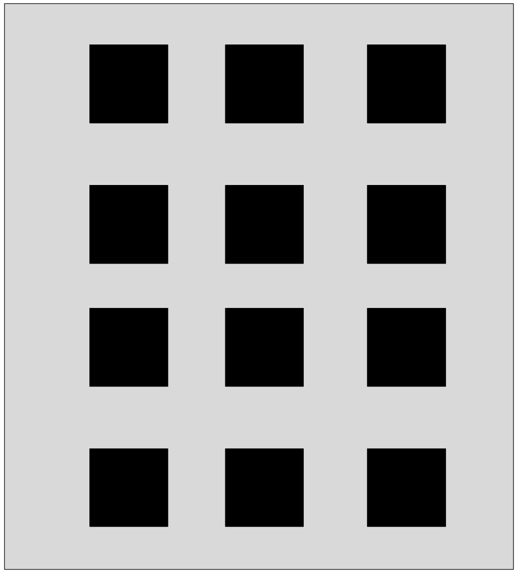




Camera Calibration cont.

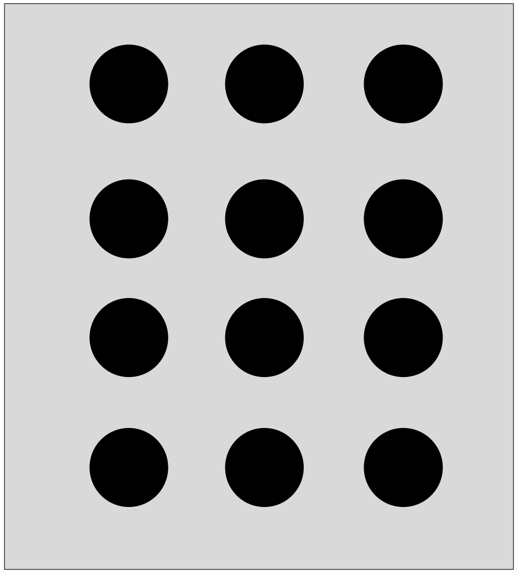
- squares and rectangles
  - determine image position calculating the point of intersection of horizontal and vertical edges

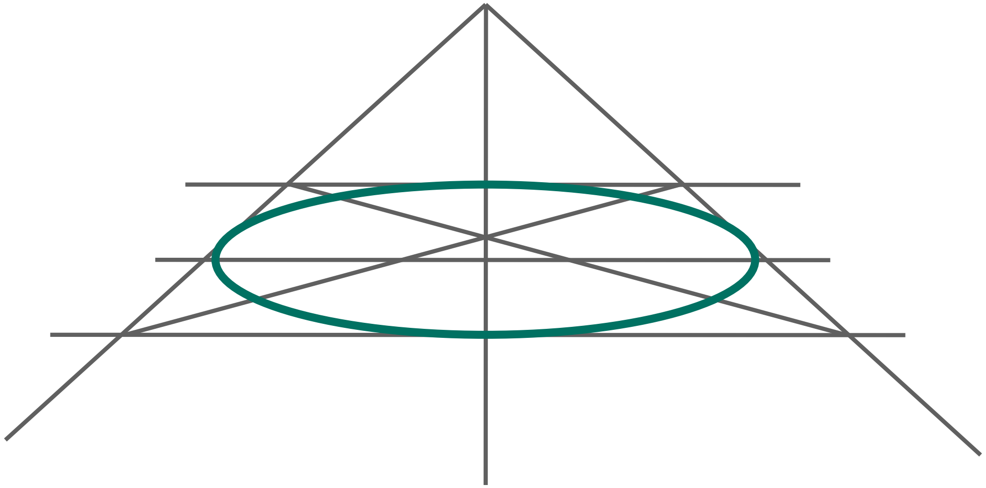




Camera Calibration cont.

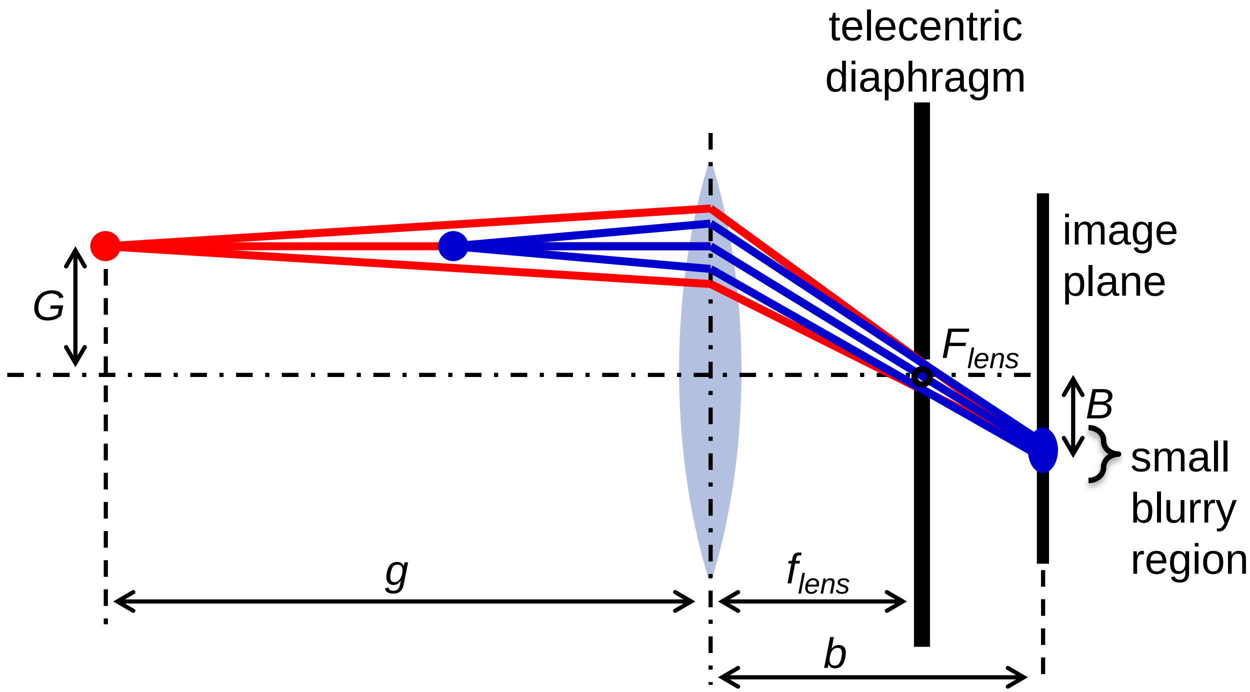
- circles
  - determine center of resulting ellipse
  - iterative correction of mapping error of circle center





NON-STANDARD CAMERAS

Telecentric Lenses



magnification:

$$B = \frac{b - f_{lens}}{f_{lens}} \cdot G$$

depth of field:

$$\Delta g = 2\frac{\epsilon}{D}(g - f_{lens})$$

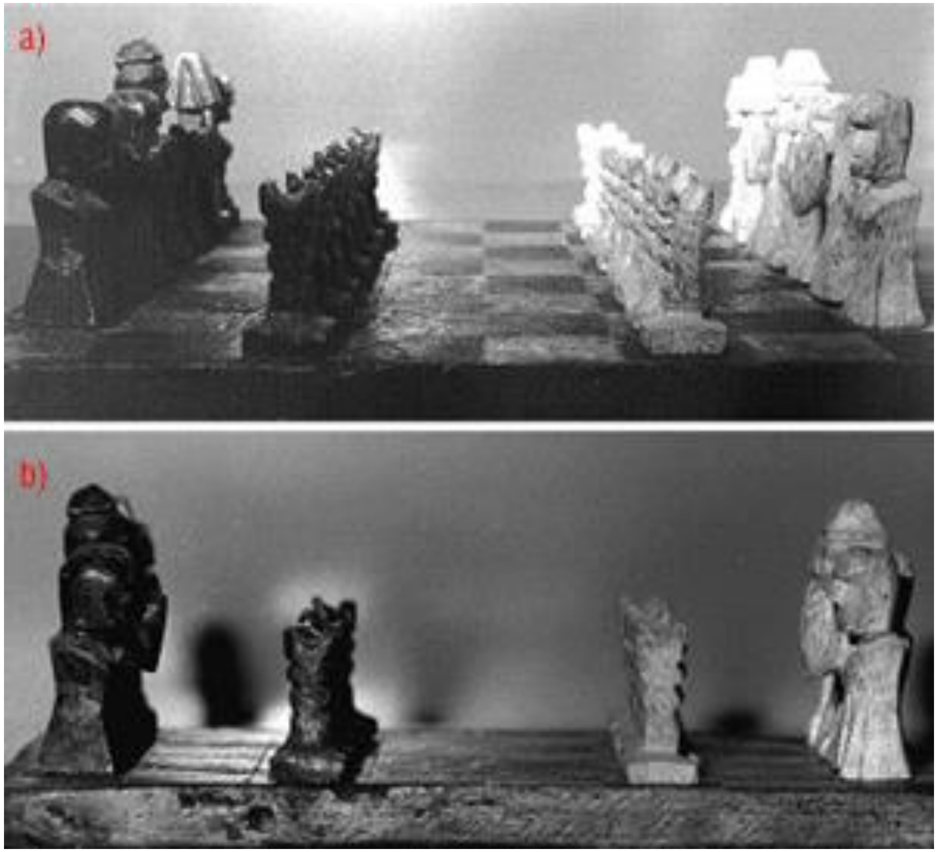
pros:
- magnification independent of object distance *g*
- improved depth of field

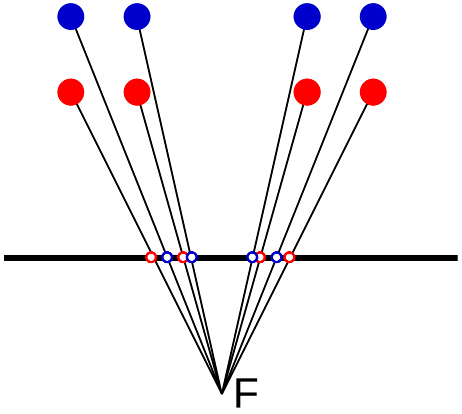
cons:
- small aperture, lets poorly light through
- large, heavy, and expensive

application area:
- microscopy

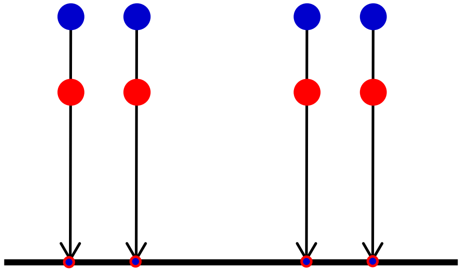
Telecentric Lenses cont.

normal lens (perspective projection)

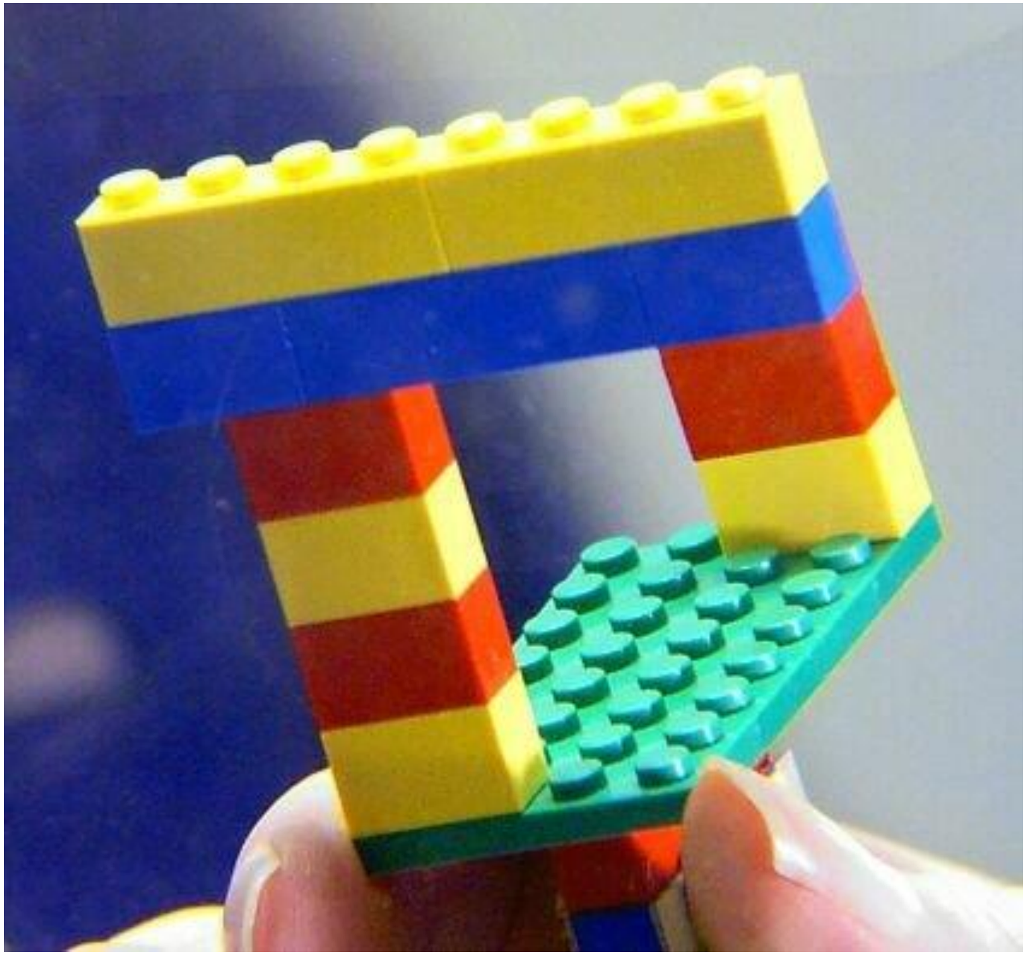




telecentric lens (orthographic projection)



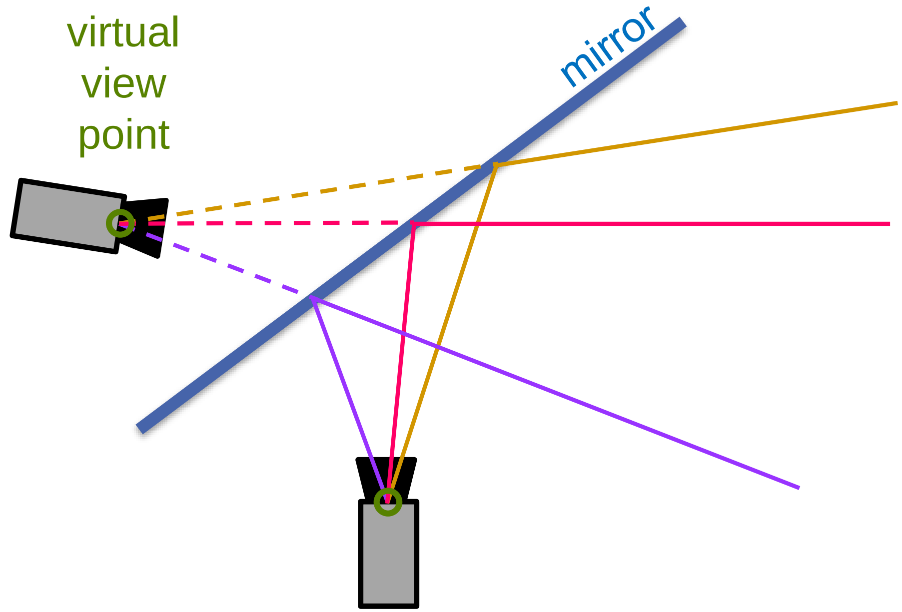
Telecentric Lenses cont.



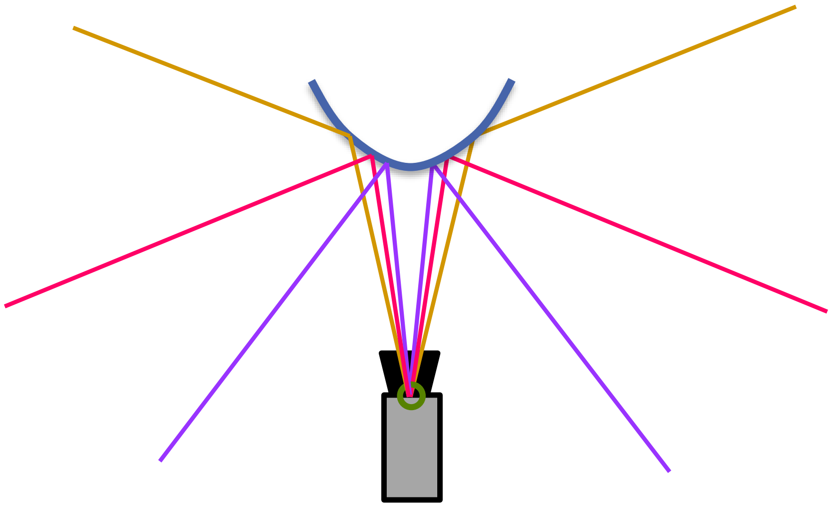
source: http://www.lhup.edu/~dsimanek/3d/telecent.htm

Catadioptric Cameras

- catadioptric cameras = cameras with mirrors
  - planar mirror



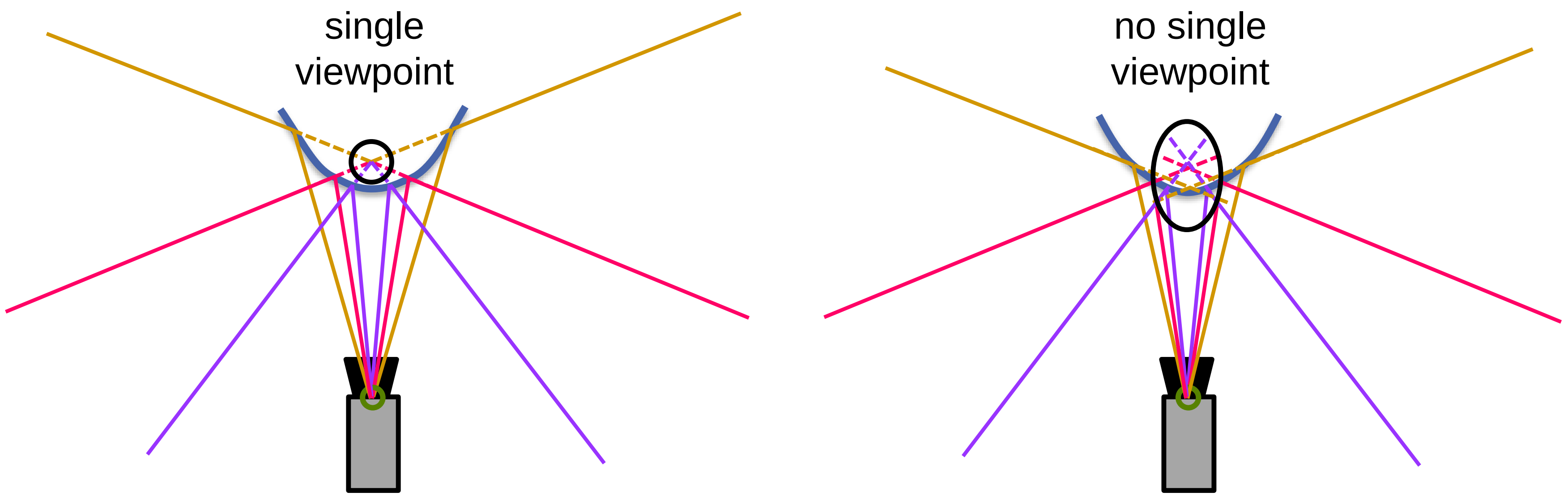
  - curve-shaped mirrors



Catadioptric Cameras Cont.

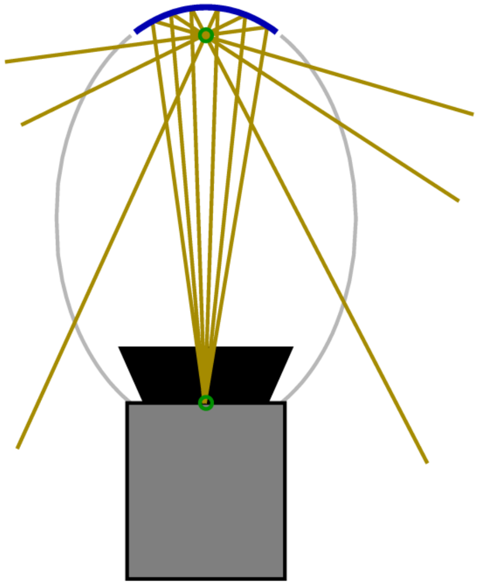
- *Single viewpoint*

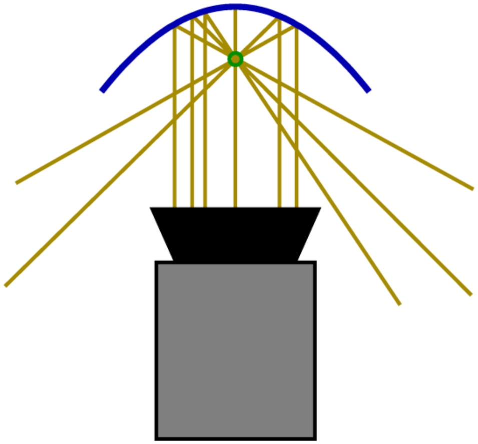
  A catadioptric camera has a single viewpoint if all object-mirror light rays intersect in a single point (e.g. if the mirror could be replaced by a pinhole camera)

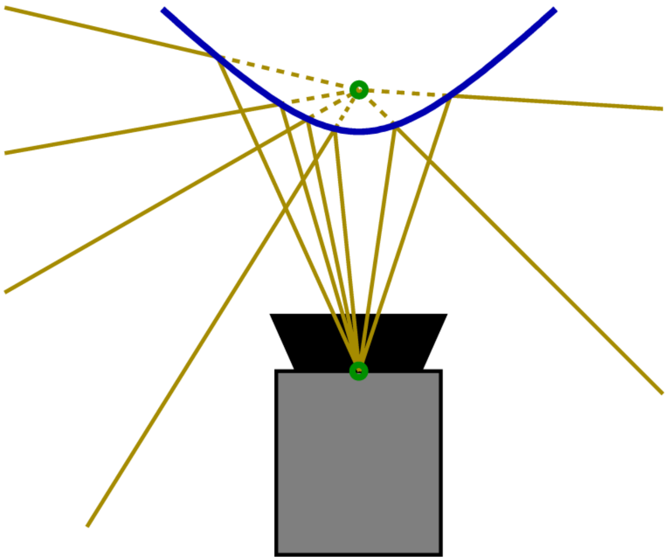


Catadioptric Cameras Cont.

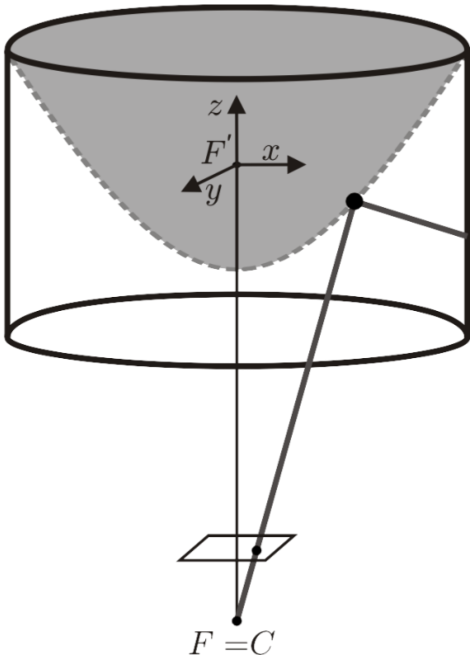
- camera setups with single viewpoint:

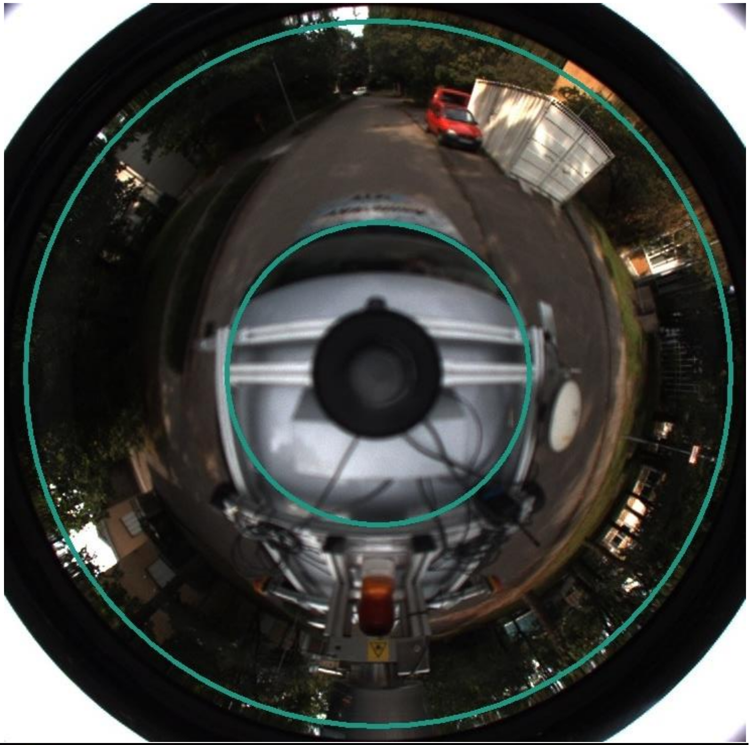

elliptic mirror+
camera with
standard lens


parabolic mirror+
camera with
telecentric lens


hyperbolic mirror+
camera with
standard lens

Catadioptric Cameras Cont.

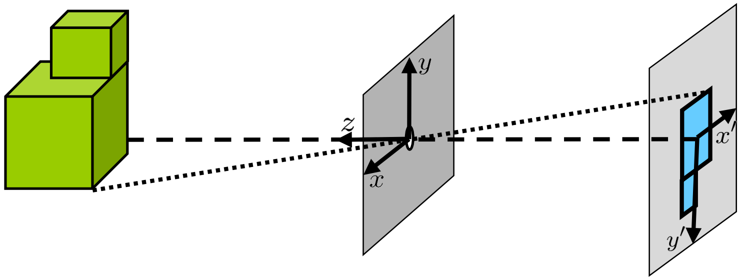




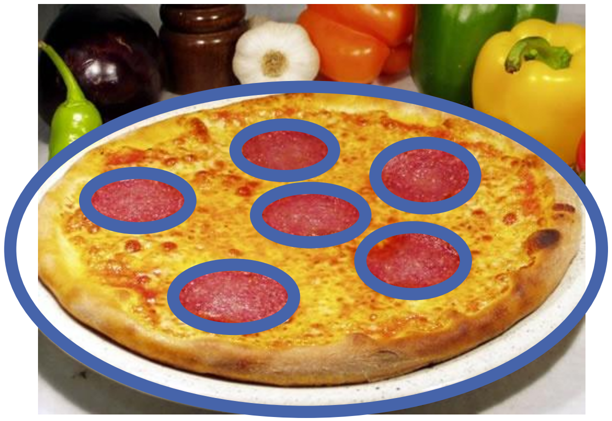
SUMMARY: CAMERA OPTICS

Summary

- **pinhole camera**
  - pinhole camera model
  - world-to-image-mapping
  - intrinsic/extrinsic camera parameters
  - properties of perspective projection
- **lenses**
- **camera calibration**
- **non-standard cameras**

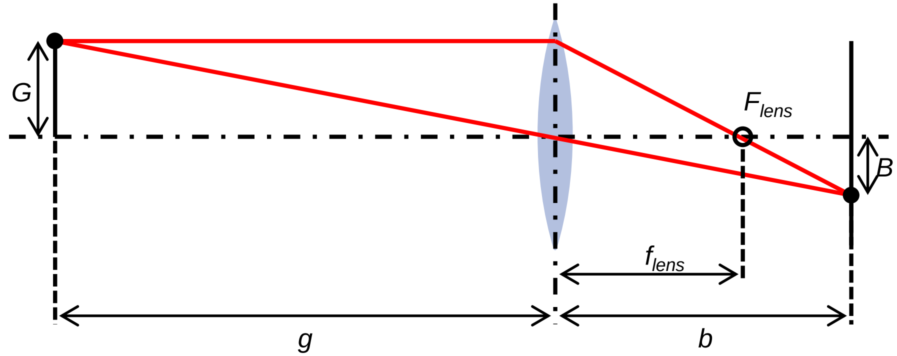


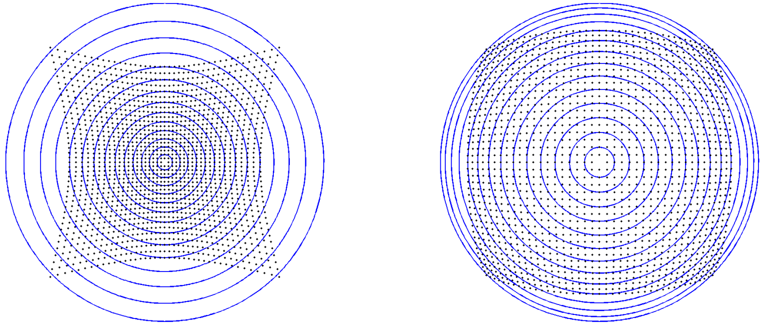
$$z \cdot \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = A \cdot \left(R \,\middle|\, \vec{t} \right) \cdot \begin{pmatrix} \xi \\ \eta \\ \zeta \\ 1 \end{pmatrix}$$



Summary cont.

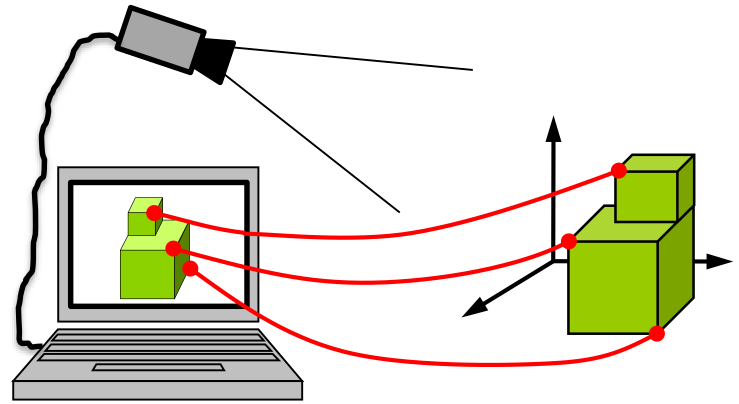
- **pinhole camera**
- **lenses**
  - lens equation
  - depth of field
  - focus series
  - lens aberrations
  - radial distortion
- **camera calibration**
- **non-standard cameras**

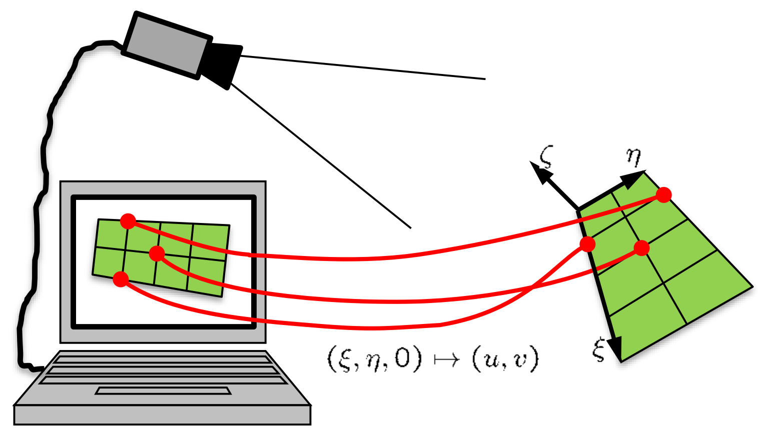




Summary cont.

- **pinhole camera**
- **lenses**
- **camera calibration**
  - Tsai's camera calibration approach
  - Zhang's approach with homographies
  - calibration of distortion parameters
  - calibration markers and calibration objects
- **non-standard cameras**





Summary cont.

- **pinhole camera**
- **lenses**
- **camera calibration**
- **non-standard cameras**
  - telecentric lenses
  - catadioptric cameras