Paper Review: Inducing Uniform Asymptotic Stability in Non-Autonomous Accelerated Optimization Dynamics via Hybrid Regularization

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From the perspective of dynamic systems, the relationship between the Nesterov gradient method and the continuous-time system modeled as a time-varying ordinary differential equation (ODE) is investigated in recent works. In this paper, the authors focus on the robustness properties of the ODE with respect to persistent disturbances in the states and dynamics. To induce the convergence and robust stability, they apply the frame of hybrid systems to regularize the dynamics. Such systems combine the continuous-time and discrete-time dynamics, and thus are called "hybrid systems". Due to the existence of the discrete-time dynamics, the proposed systems can be viewed as robust periodic and persistently nonperiodic restarting mechanisms. However, they state that there is a clear tradeoff between acceleration and robustness. For a family of radially unbounded convex functions with unique minimizers, they show that every solution generated by the hybrid system will uniformly converge to the set of attractors, but the acceleration is only guaranteed during the first interval of continuous-time dynamics. Moreover, assuming that certain small time-varying bounded disturbances are added to systems, the new solutions of the perturbed system will uniformly converge to the union of all sets obtained by taking a closed ball around each point in the set of attractors. If functions are further assumed to be strongly convex, it may establish the uniform global exponential stability. It is worth mentioning that the restarting mechanism may alleviate the impact of the long tail generated by gradient-based methods to some extent.