Paper Review: Achieving Geometric Convergence For Distributed Optimization Over Time-varying Graphs

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The growing size of network motivates the development of distributed optimization, where data processing and computing are to be performed in a distributed but collaborative manner by agents within the network. This paper considers the problem of unconstrained distributed optimization over time-varying graphs, where the communication constraint can be different at each time. To solve such problem, this paper proposes a novel distributed algorithm, referred to as DIGing, which incorporates a distributed inexact gradient method with a gradient tracking technique. With the help of gradient tracking to approximate the gradient average and doubly stochastic mixing matrices to reach consensus, DIGing is able to converge to the global and consensual minimizer exactly. However, it is unrealistic to maintain a sequence of doubly stochastic mixing matrices over directed time-varying graphs, so that the authors employ column stochastic mixing matrices combining with the push-sum protocol as a replacement, referred to as the Push-DIGing algorithm. Under the assumption of strong convexity and smoothness, the authors apply small-gain based analysis to show the convergence rate of DIGing and Push-DIGing with a fixed but small enough stepsize is linear for undirected and directed time-varying graphs respectively. In addition, they also prove that the number of iterations until DIGing reaches any fixed accuracy is polynomial in the total number of nodes, for undirected graphs.