A Bergman Splitting Scheme for Distributed Optimization Over Networks

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This paper proposes a general frame incorporating with the Bregman method and operator splitting, which allows to recover most of the existing distributed algorithms.

Under this frame, the authors connect the Bergman iterative regularization method with the AL method for linear problems. In order to avoid requiring the global knowledge of the network or running the inner loop of consensus infinitely and to make the algorithm more scalable, they execute the forward backward splitting operation only once per each iteration, which can be viewed as an approximate version of Bregman iterative regularization. In this paper, the authors summarize the above analysis and come up with the novel algorithm, referred to as forward-backward Bregman splitting (D-FBBS). Different from the most of existing algorithms, the D-FBBS is able to solve both the primal and dual problem simultaneously. With proper assumptions, they show the nonergodic convergence rate of $O(1/k)$ for fixed networks.

Moreover, the authors also present two variants of the D-FBBS. One is the inexact D-FBBS, referred to as ID-FBBS, which aims to reduce the computational cost for the cost functions that have Lipschitz continuous gradient. Similarly, they show its convergence with a nonergodic rate of $O(1/k)$ for fixed networks. The other variant is the D-FBBS for stochastic networks. Since the proposed algorithm, D-FBBS, does not depend on the number of edges of the network and is thus suitable for distributed computation over stochastic networks under proper conditions. They reformulate the algorithm for stochastic optimal consensus problem and find a new Lyapunov function that is immune to varying networks. In addition, they further impose the strongly convex condition on the cost functions and then establish an ergodic convergence rate of $O(1/k)$ for stochastic networks.