Orthogonality under linear case

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Verify the orthogonality between g and $\mathbf{v}$ under linear case.

```
library(numDeriv)
n <- 1e2
p <- 6

# generate beta
beta <- rnorm(p)
g <- sqrt(sum(beta^2))
v <- beta / g
w_true <- c(g, v)

# generate data
n_simu <- 1e5

simu_hessian <- function(){
  X <- svd(matrix(rnorm(n*p), ncol = p))$u
  f <- X %*% beta
  y <- f + rnorm(n)

  #
  loss_func <- function(w){
    # rememeber to divide the regression coefficient
    # by its norm sqrt(sum(w[-1]^2))
    sum((y - w[1] * X %*% w[-1] / sqrt(sum(w[-1]^2)))^2)
  }

  hessian(loss_func, x = w_true)
}


hess_list <- list()
# pb <- txtProgressBar(1, n_simu, style = 3)
for (i in 1:n_simu){
  # setTxtProgressBar(pb, i)
  hess_list[[i]] <- simu_hessian()
}

# compute expected fisher information
expected_fisher <- Reduce("+", hess_list)/n_simu
expected_fisher
```

```
##              [,1]         [,2]         [,3]         [,4]         [,5]
## [1,]  2.000000000 -0.004405191 -0.006669182  0.000480202  0.008917116
## [2,] -0.004405191  6.102932537  0.331823050 -0.235529667 -0.339454473
## [3,] -0.006669182  0.331823050  5.142517473  0.755197620  1.088334480
## [4,]  0.000480202 -0.235529667  0.755197620  5.670340513 -0.772555002
## [5,]  0.008917116 -0.339454473  1.088334480 -0.772555002  5.093065908
```

```
## [6,] -0.008355361 -0.592236743  1.899035539 -1.347980831 -1.942692791
## [7,]  0.001002595 -0.002799333  0.008965359 -0.006366043 -0.009170373
##              [,6]         [,7]
## [1,] -0.008355361  0.001002595
## [2,] -0.592236743 -0.002799333
## [3,]  1.899035539  0.008965359
## [4,] -1.347980831 -0.006366043
## [5,] -1.942692791 -0.009170373
## [6,]  2.816842404 -0.016012712
## [7,] -0.016012712  6.206330782
```

The fisher information for g and $\mathbf{v}$ should be zero. To verify this, we did simulation for 100,000 times and computed the average of the empirical fisher information. Results show that the expected fisher information should be zero.