INGENIEURSWETENSCHAPPEN

FACULTY OF INGENIEURSWETENSCHAPPEN
Applied Computer Science

Evolutionary dynamics in a spatial context

Computational Game and Theory

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1 Explore updating mechanism: Unconditional Imitation

Each player is playing a weak Prisoners Dilemma game (T=10, R=7, P=S=0) with each of their neighbors, playing the same action (Cooperate of Defect) with each one of them. The total payoff is calculated as a sum of the payoffs received from playing all neighbors. The payoff matrix for weak prisoner dilemma is shown in figure 1

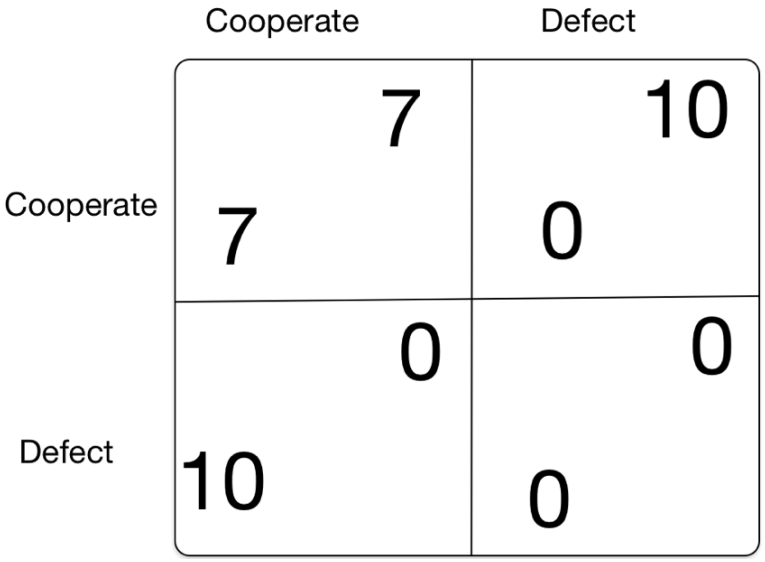

Figure 1: The Payoff matrix for weak Prisoner Dilemma

1.1 Visualize the game with Moore neighborhood

Visualize the round t=0,1,5,10,20,50. The defectors are filled with red color while the cooperators are filled with gray color. The payoff are calculated as a sum of the payoffs received from playing with their neighbors.

1.1.1 the payoff matrix of Moore neighborhood with 4,8,12,20,50 lattices

Figure 2, Figure 3, Figure 4, Figure 5 shows the payoff matrix for 4 lattices using moore neighborhood. We could notice that all players change to defectors in a **smaller lattices** when they have 8 neighbors. While when the lattices becomes to 50*50, first the cooperation level drops, then the cooperator clusters appear, later the clusters grow larger and larger and the cooperation level rises.

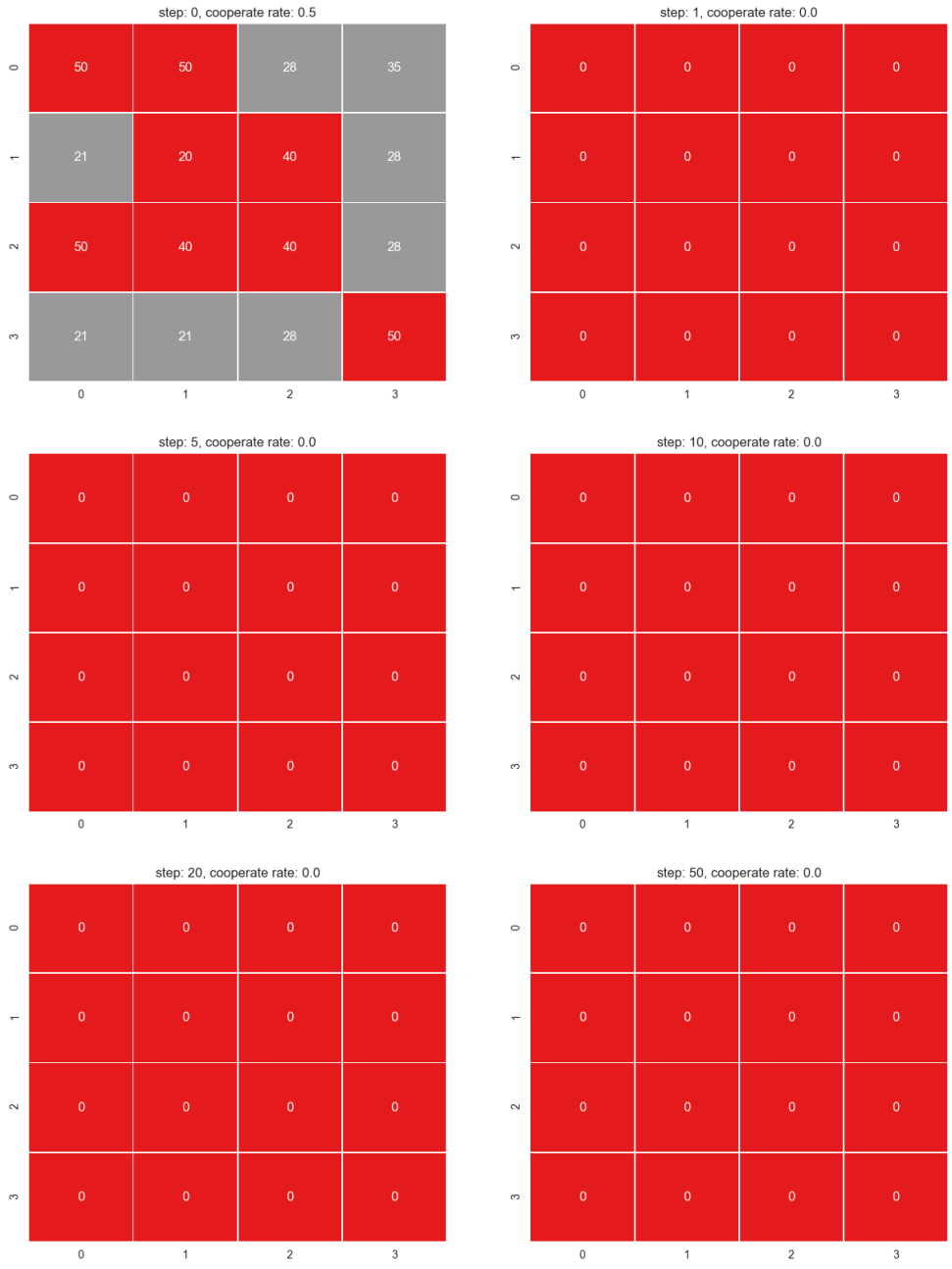

Figure 2: The Payoff matrix of 4 lattices and moore neighborhood

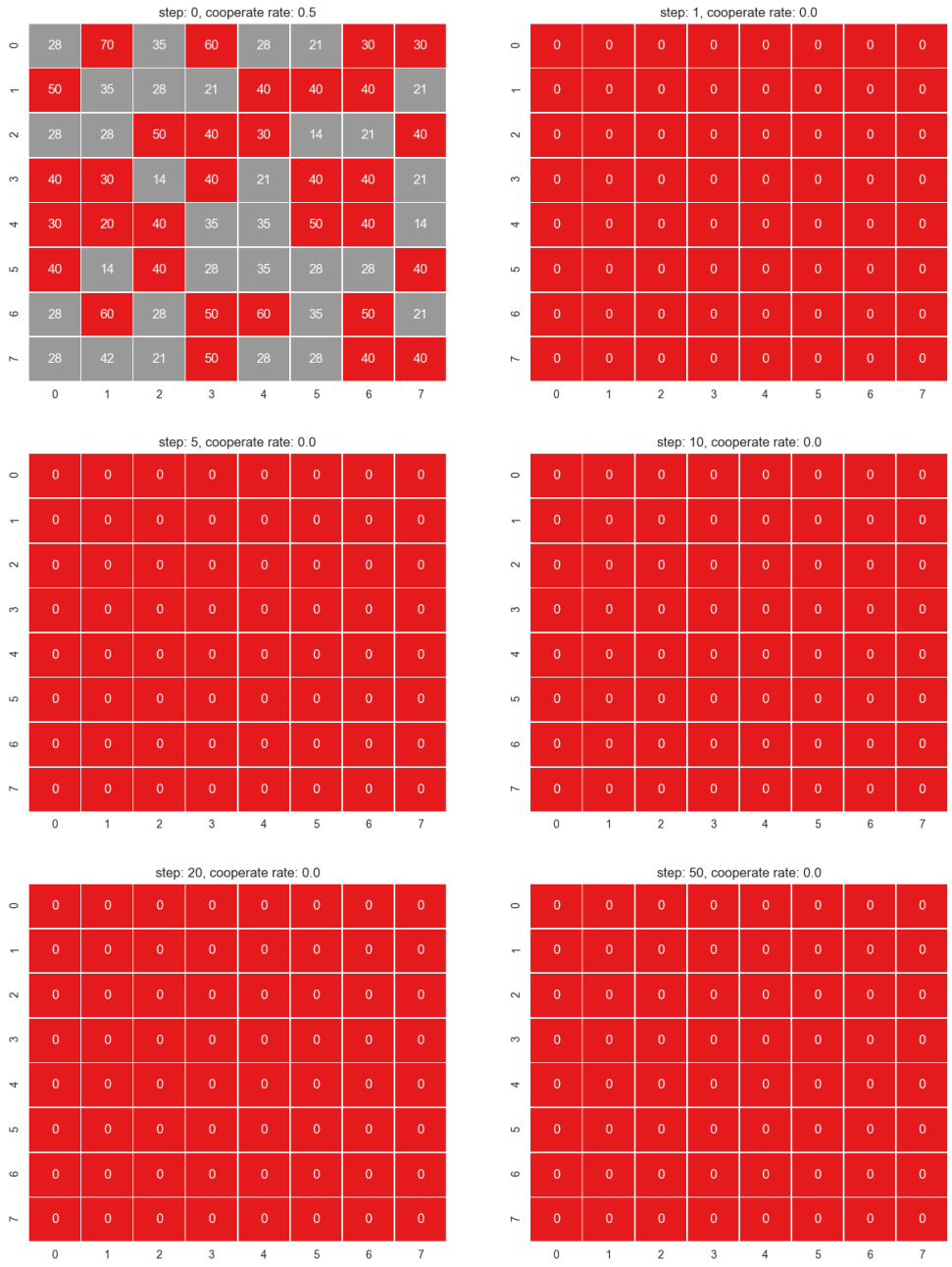

Figure 3: The Payoff matrix of 8 lattices and moore neighborhood

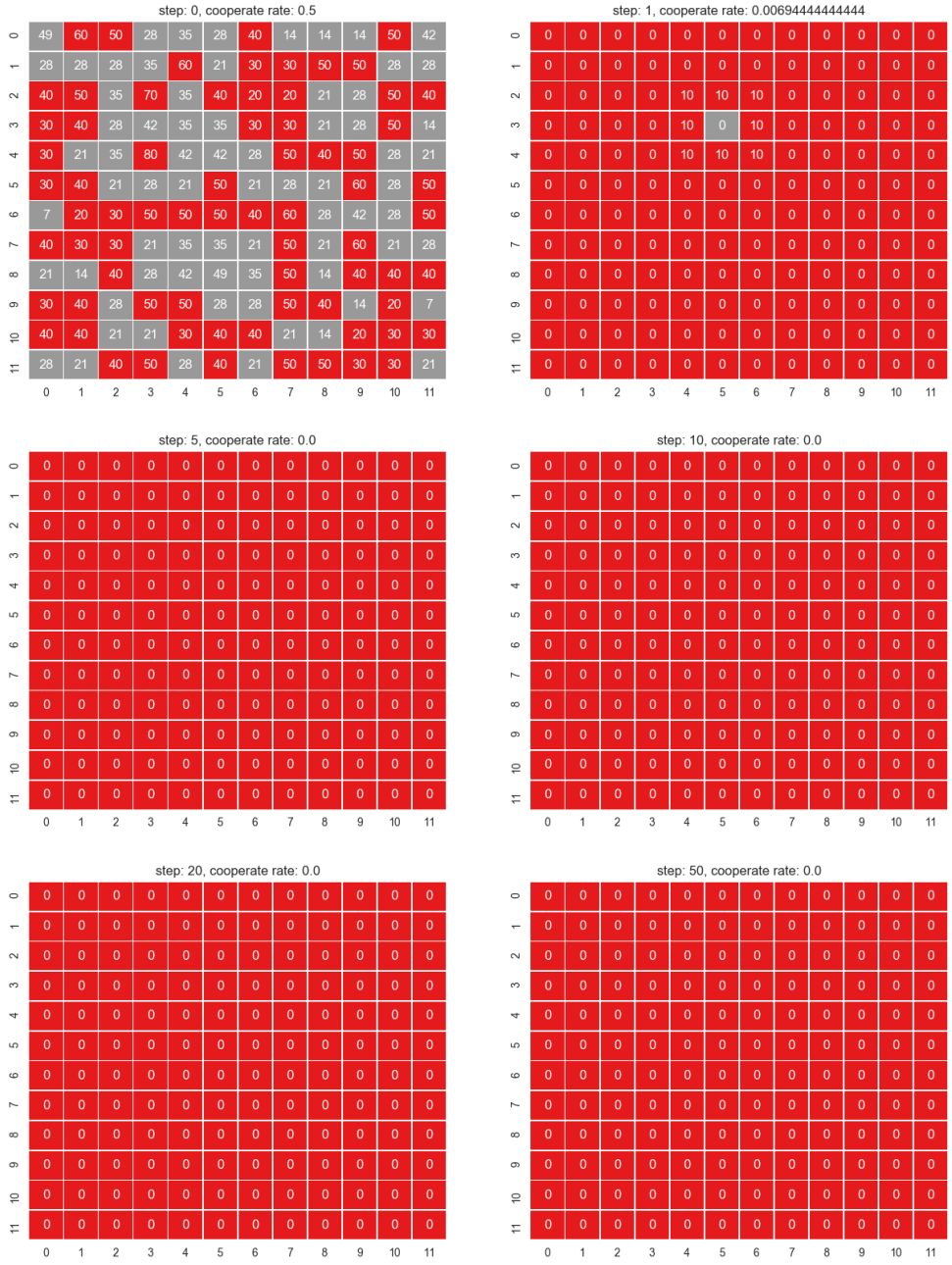

Figure 4: The Payoff matrix of 12 lattices and moore neighborhood

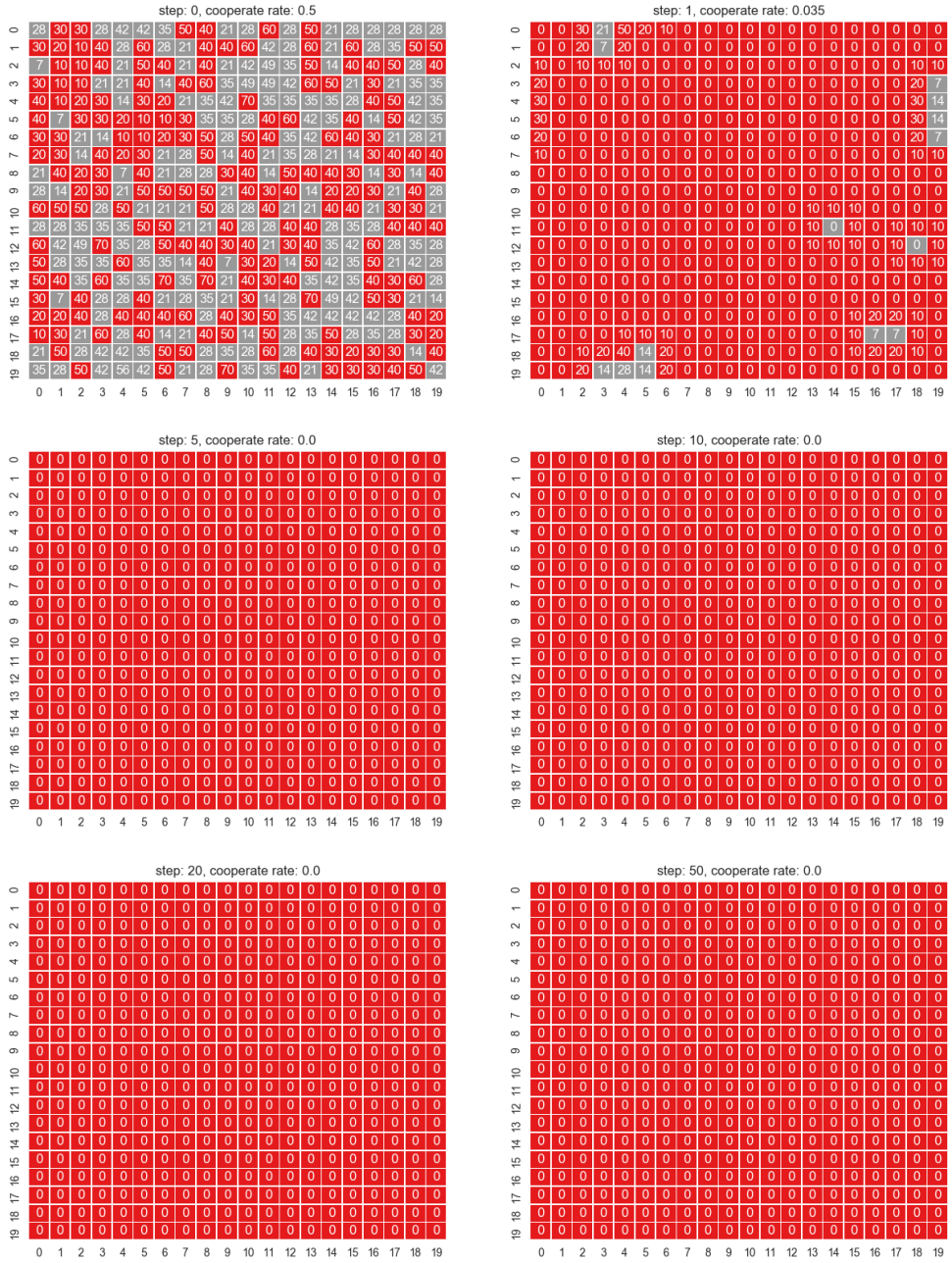

Figure 5: The Payoff matrix of 20 lattices and moore neighborhood

Figure 6 shows the change of payoff matrix. step 1, the cooperation level is 50%, step 2, the cooperation level drops, small cooperator clusters appears. step 3, clusters become larger and larger. step 5, the cooperation level reaches almost 80%, the cooperators had invaded defectors.

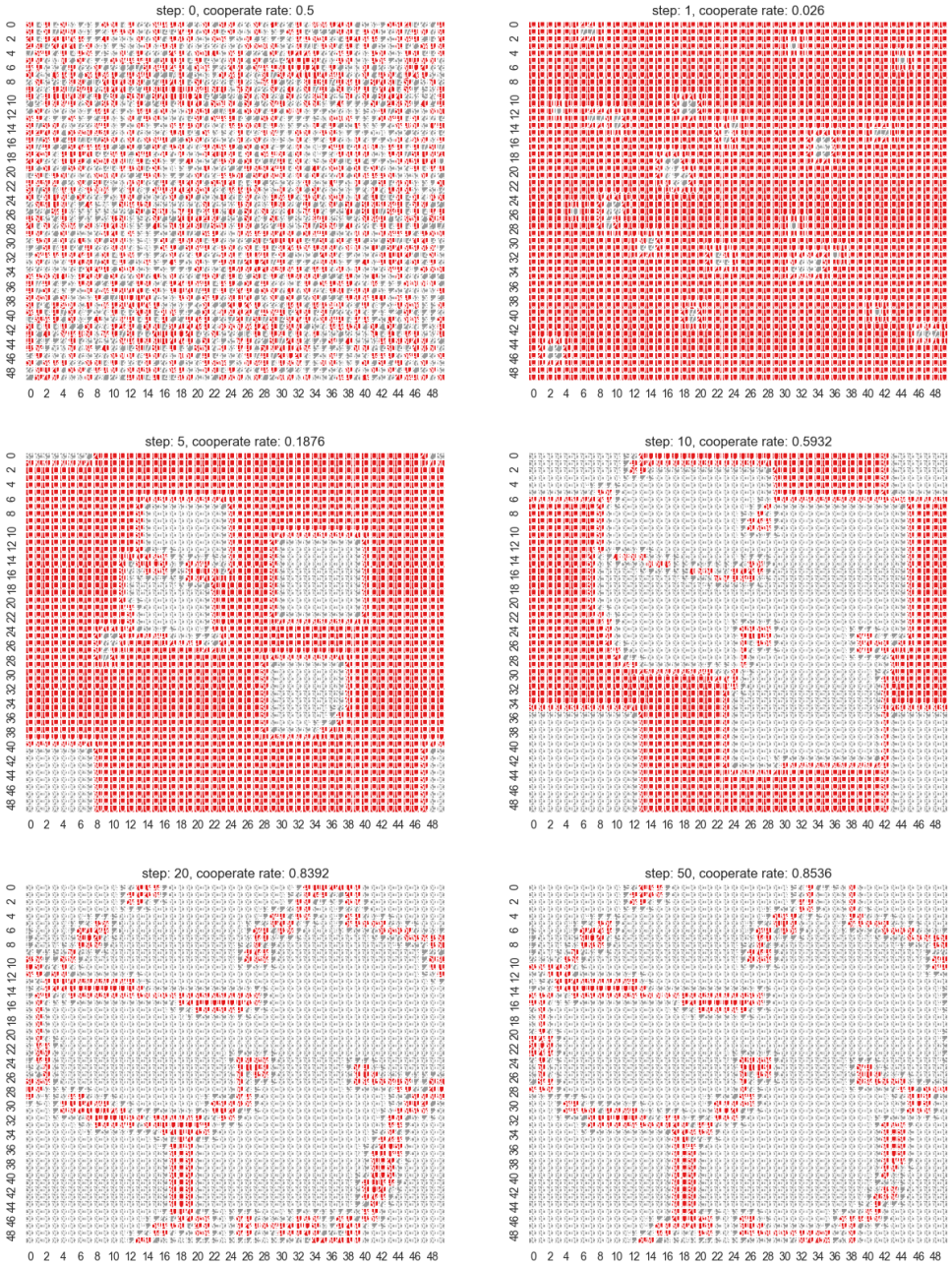

Figure 6: The Payoff matrix of 50 lattices and moore neighborhood

1.2 the payoff matrix of Von Neumann neighborhood with 4,8,12,20,50 lattices

Figure 7, Figure 8, Figure 9, Figure 10 Figure 11 show the payoff matrix for 4 lattices using moore neighborhood. For smaller lattice 4*4, 8*8, the cooperation levels still converge to 0, which means that in smaller group, all players still tend to be defectors. For medium lattice size 12*12,20*20 and large lattice size 50*50 , the cooperation level fluctuates. Large cooperator cluster does not appear in Von Neumann neighborhood.(Compare to that of Moore neighborhood)

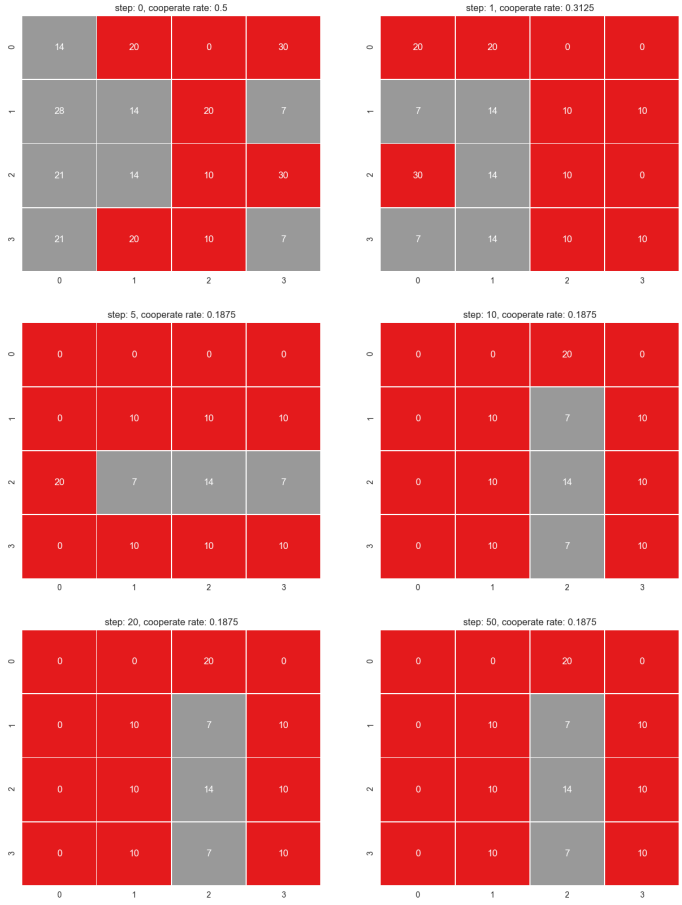

Figure 7: The Payoff matrix of 4 lattices and Von Neumann neighborhood

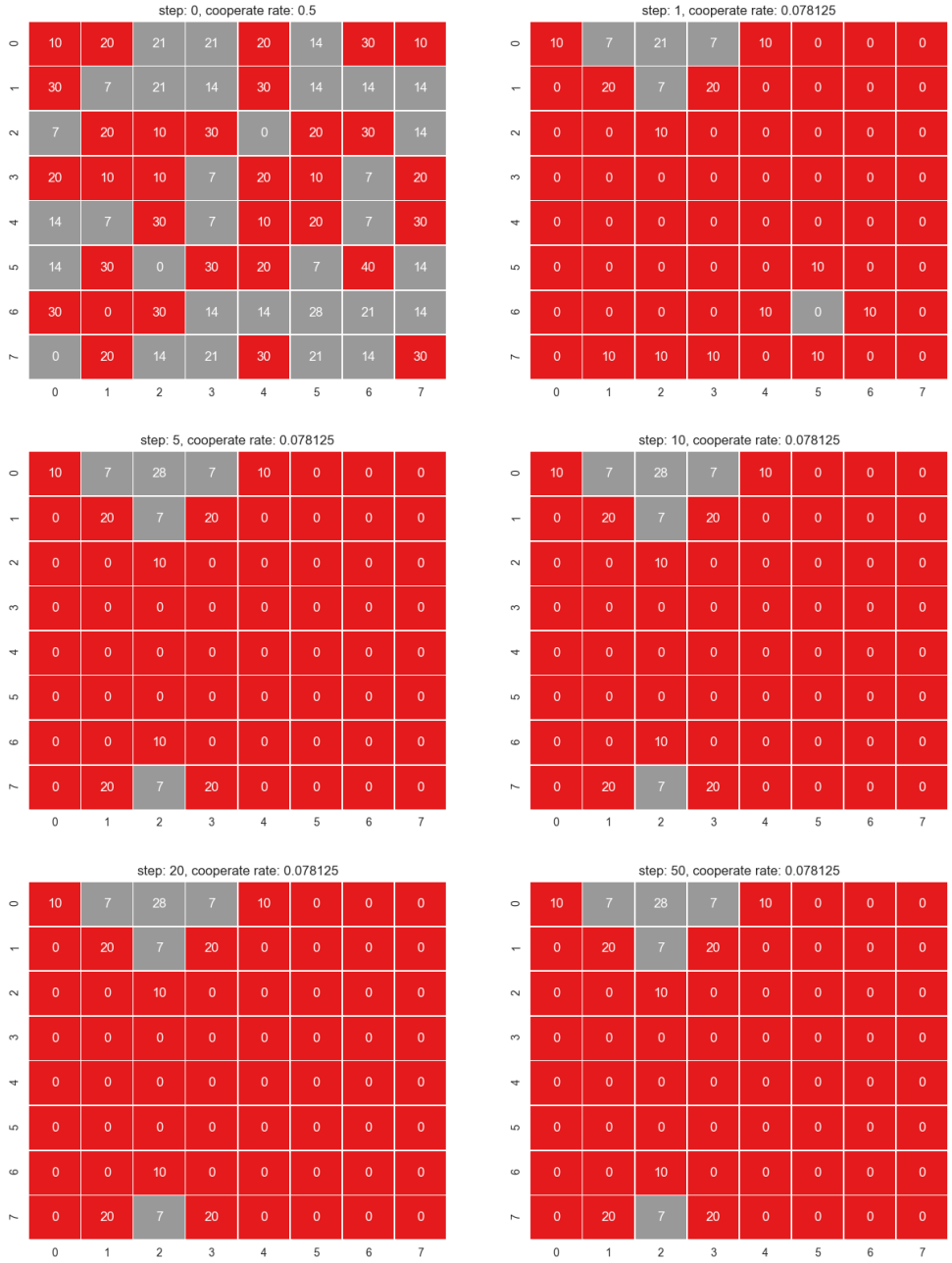

Figure 8: The Payoff matrix of 8 lattices and Von Neumann neighborhood

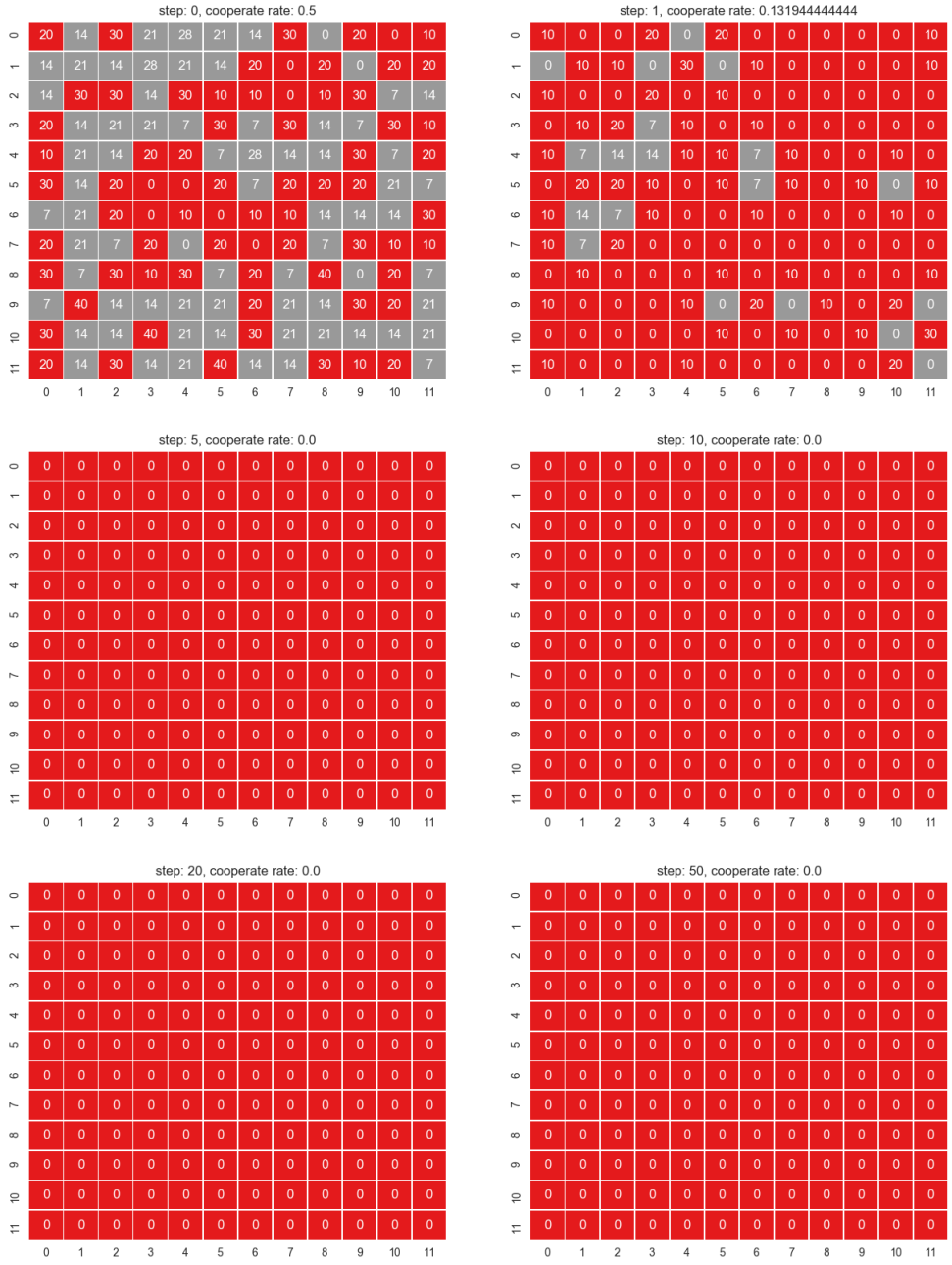

Figure 9: The Payoff matrix of 12 lattices and Von Neumann neighborhood

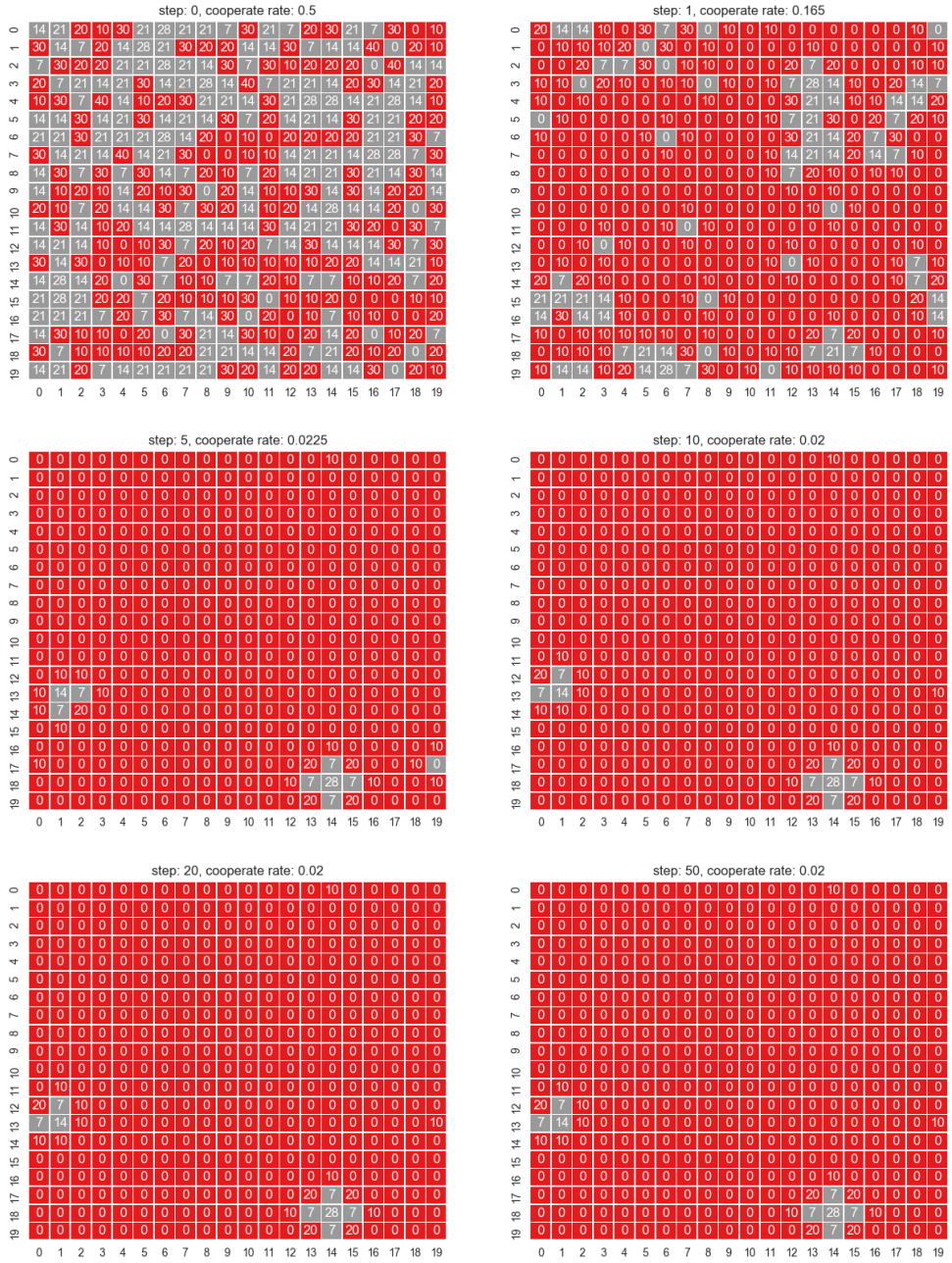

Figure 10: The Payoff matrix of 20 lattices and Von Neumann neighborhood

The cooperation level first drops, then rises, later stays in a level around 40%, reaches a equilibrium.

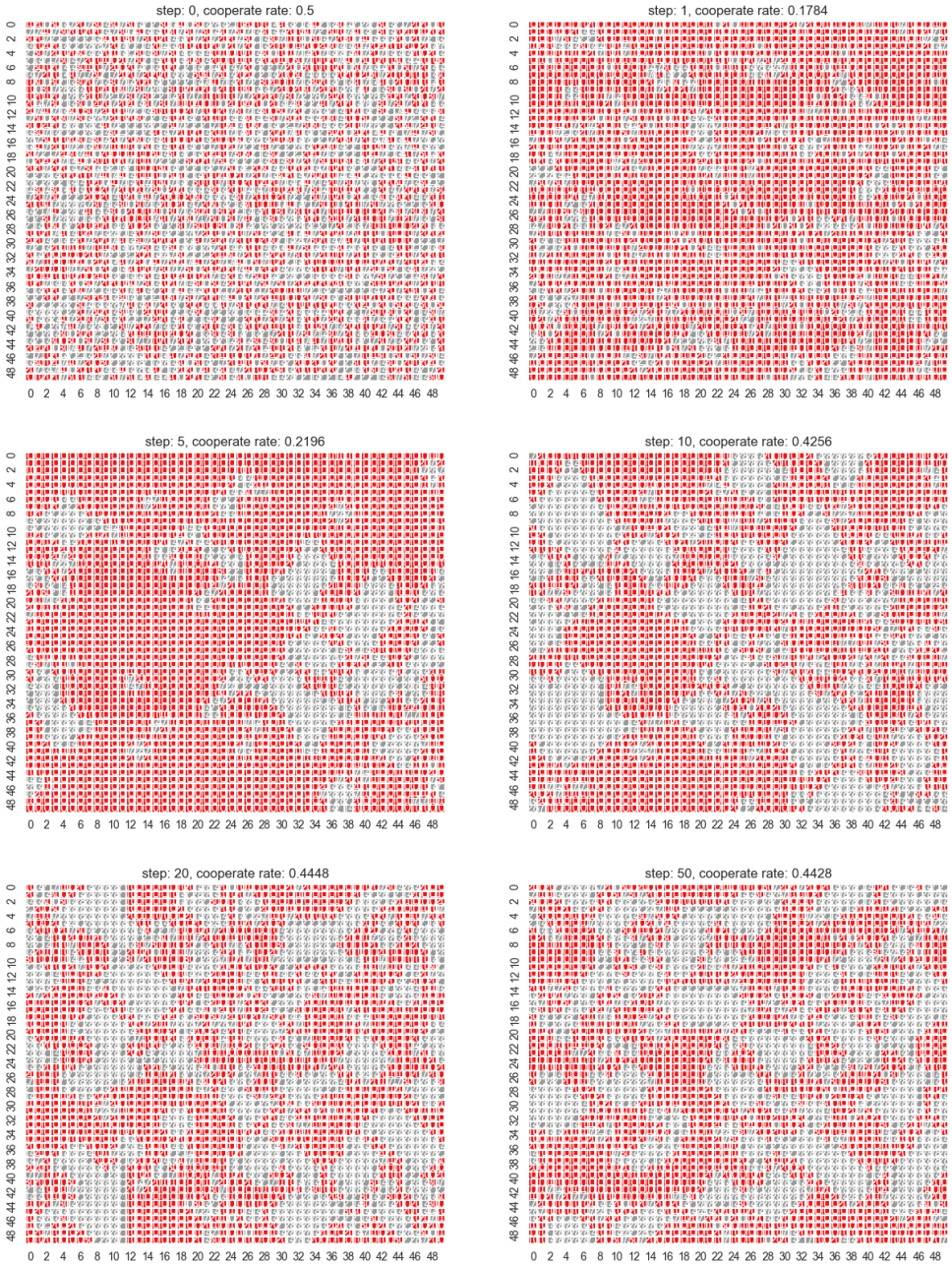

Figure 11: The Payoff matrix of 50 lattices and Von Neumann neighborhood

1.3 The comparison of cooperation level

Figure 12 shows the comparison of cooperation level with different lattices when using Moore neighborhood, and of them decrease at first and only the 50*50 lattices cooperation level goes up.

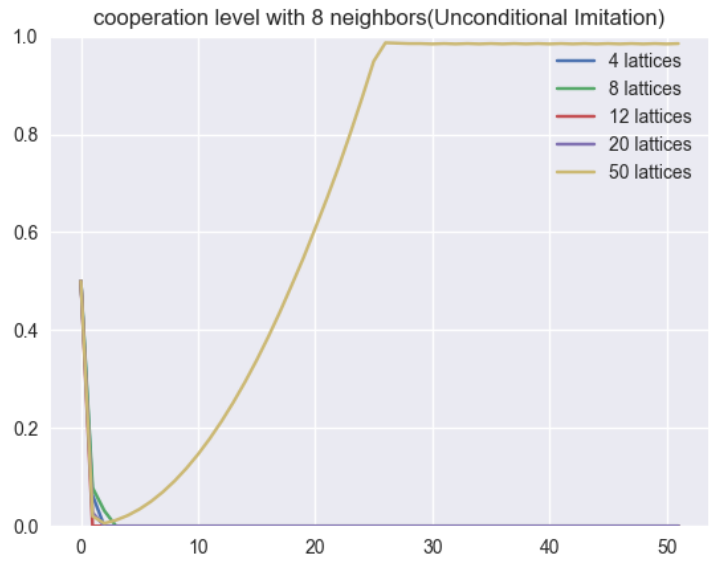

Figure 12: The cooperation level of Moore neighborhood

Figure 13 shows the comparison of cooperation level between different lattices when using Von Neumann neighborhood. We could find the cooperation levels first drops then fluctuates.

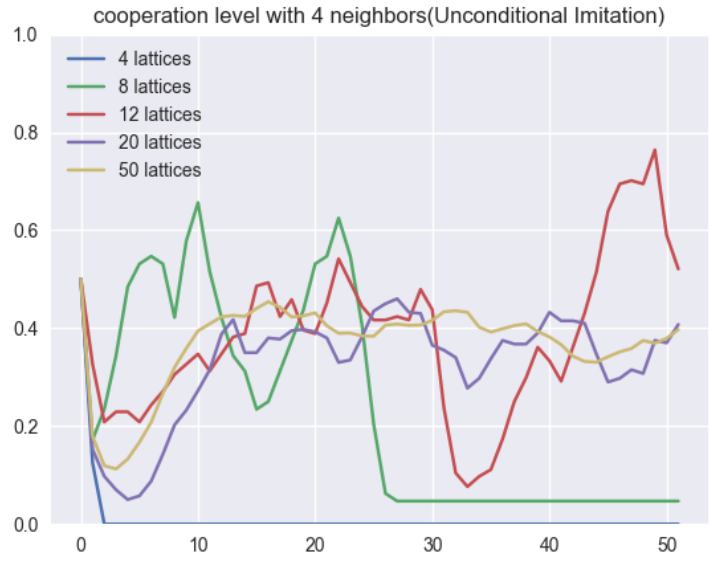

Figure 13: The cooperation level of Von Neumann neighborhood

1.4 The histogram of ending cooperation level (Moore Neighborhood)

Repeat the game 100 times, and draw the histogram of the ending values. From the Figure **??**, we could find for small lattice 4*4, 8*8, 12*12, 20*20, the ending cooperation levels are almost 0 , which means in most of the case, all players tend to be defectors.But for lattice size 50*50, the average cooperation level is almost 85% , which means most of the players in lattice 50*50 turn to be cooperators.

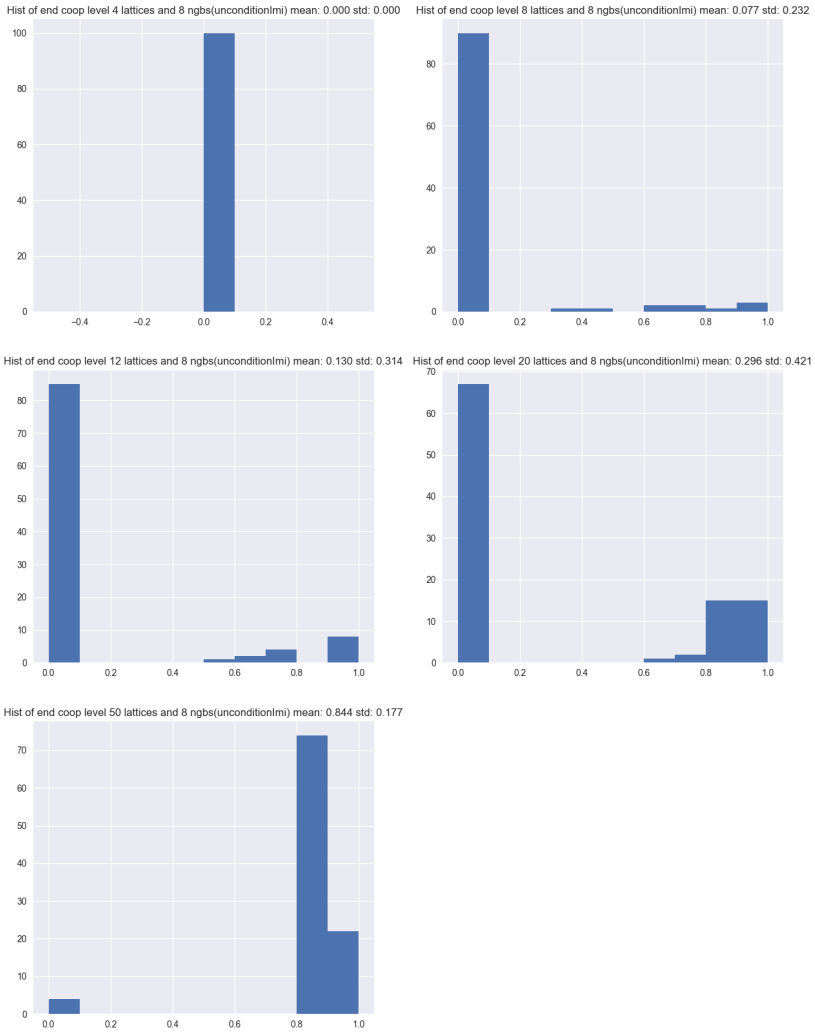

Figure 14: The histogram of ending cooperation level(Moore Neighborhood)

1.5 The histogram of ending cooperation level (Von Neumann Neighborhood)

Repeat the game 100 times, and draw the histogram of the ending values. From the Figure **??**, when compare that with the Moore neighborhood, we could find that the cooperation levels are still low with small lattice(4*4, 8*8,12*12), but the standard derivations are larger than that of Moore neighborhood.

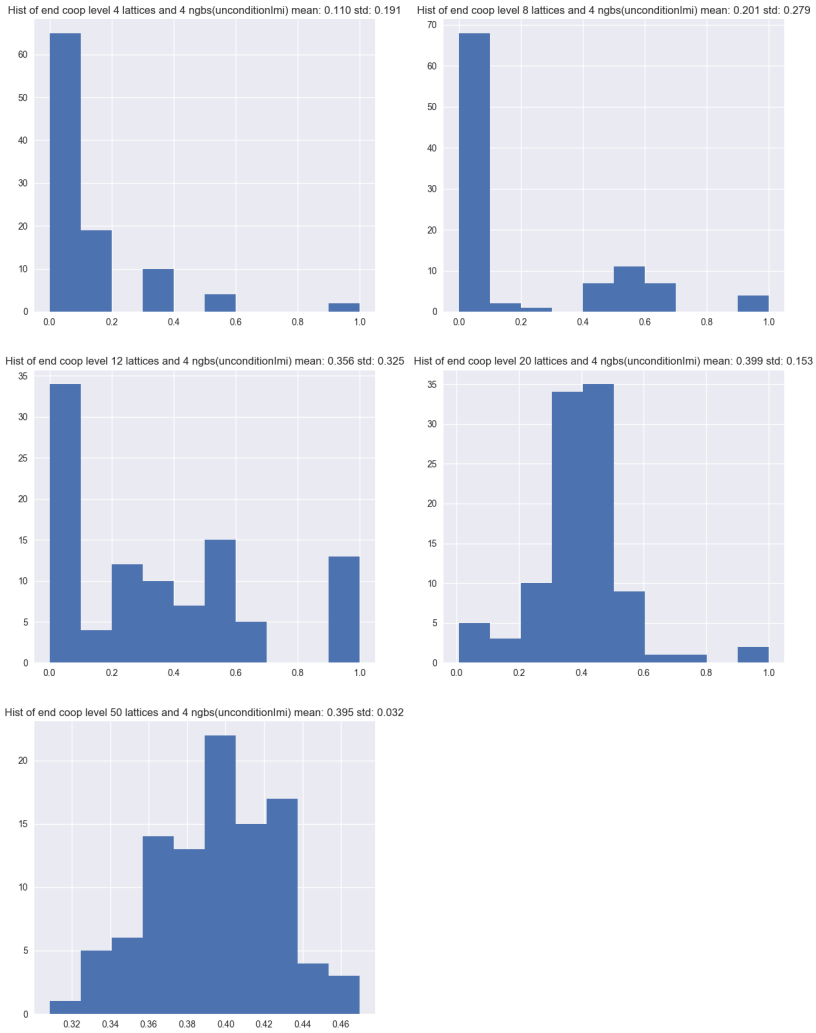

Figure 15: The histogram of ending cooperation level(Von Neumann Neighborhood)

1.6 Conclusion

The section 1 explore the the evolutionary game when using unconditional imitation.

1. The formation of cooperator clusters appear in large lattice size. When the lattice is small,

all players tend to be defectors.

2. The number of neighborhood also affects the formation of cooperator cluster. The cooperation level for lattice size 50*50 of Moore neighborhood is higher than that of Von Neuman Neighborhood.

3. In all these experiments, the cooperation levels all drops at the first several runs.

4. The updating mechanism unconditional imitation is deterministic, everytime the players choose imitate the neighborhoods, this help the formation of defector and cooperator clusters.

2 Explore updating mechanism: Replicator Rule

The Snowdrift game (T=10, R=7, S=3, P=0) payoff matrix is used in Section 3, The payoff matrix is shown in Figure 16.

Updating mechanism is different from that in section 1. Each individual i chooses randomly one of her closest neighbors j and they change their action to the action of that neighbor with a probability P_{ij} , where P_{ij} is defined as

$$P_{ij} = \frac{1 + \frac{[W_j - W_i]}{N(maxT,R,P,S - minT,R,P,S)}}{2}$$

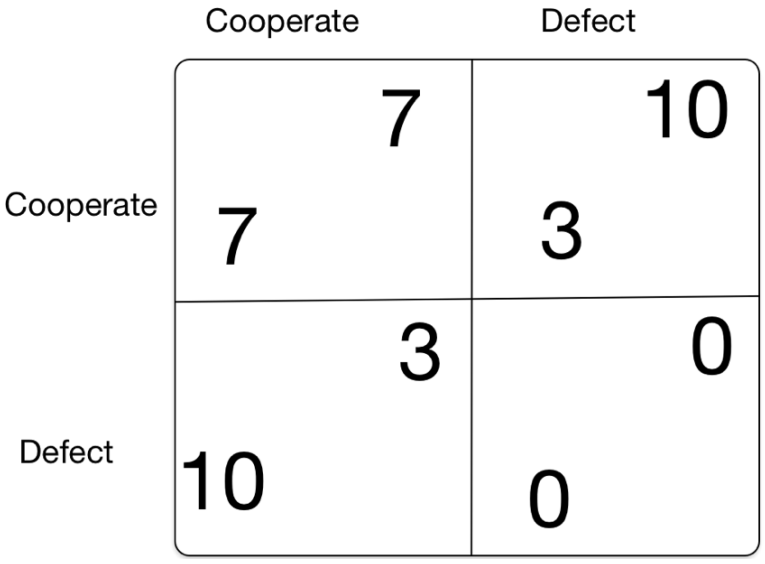

Figure 16: The Payoff matrix for snow and drift game

2.1 Visualize the game with Moore neighborhood

Figure 17, Figure 18, Figure 19, Figure 20, Figure 21 shows the payoff matrix for different lattices. Compare that with the previous game using unconditional imitating updating mechanism, we could find that not all players change to defectors at small lattice. And for large groups, the large cooperate clusters do not appear.

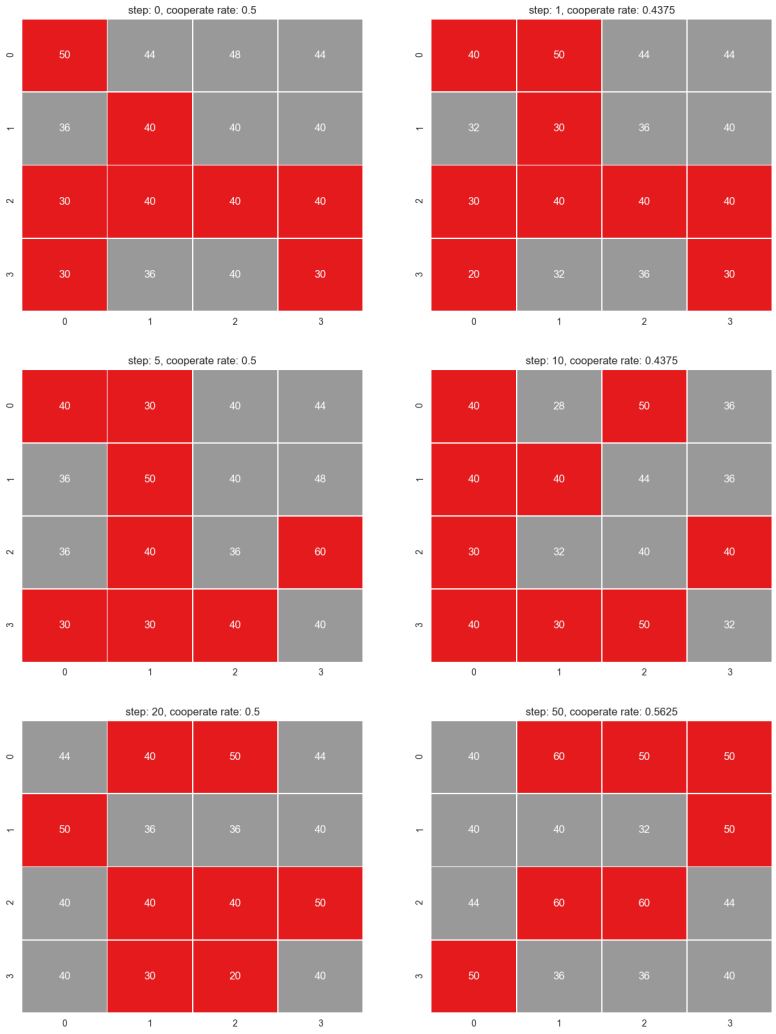

Figure 17: The Payoff matrix of 4 lattices and moore neighborhood

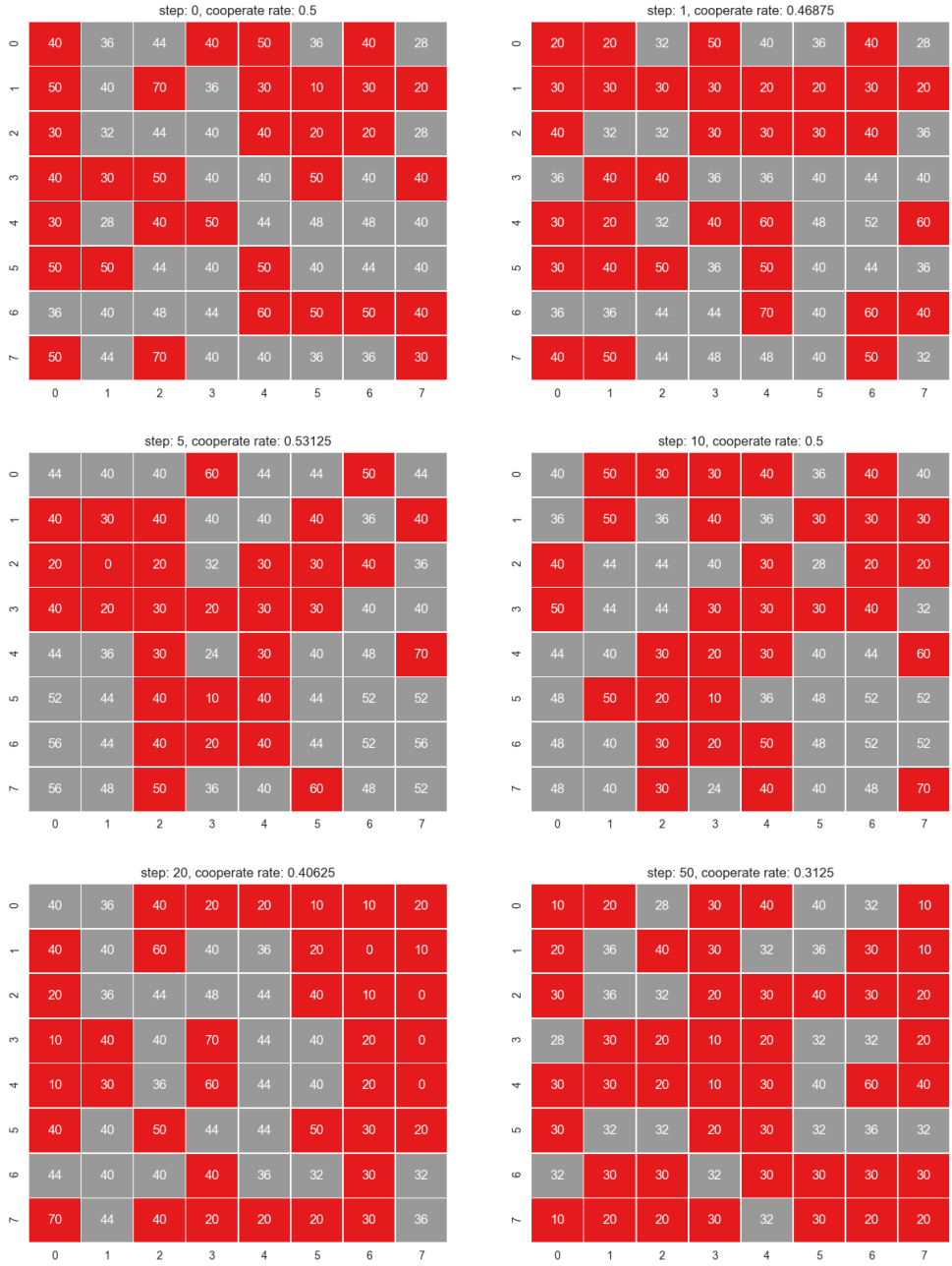

Figure 18: The Payoff matrix of 8 lattices and moore neighborhood

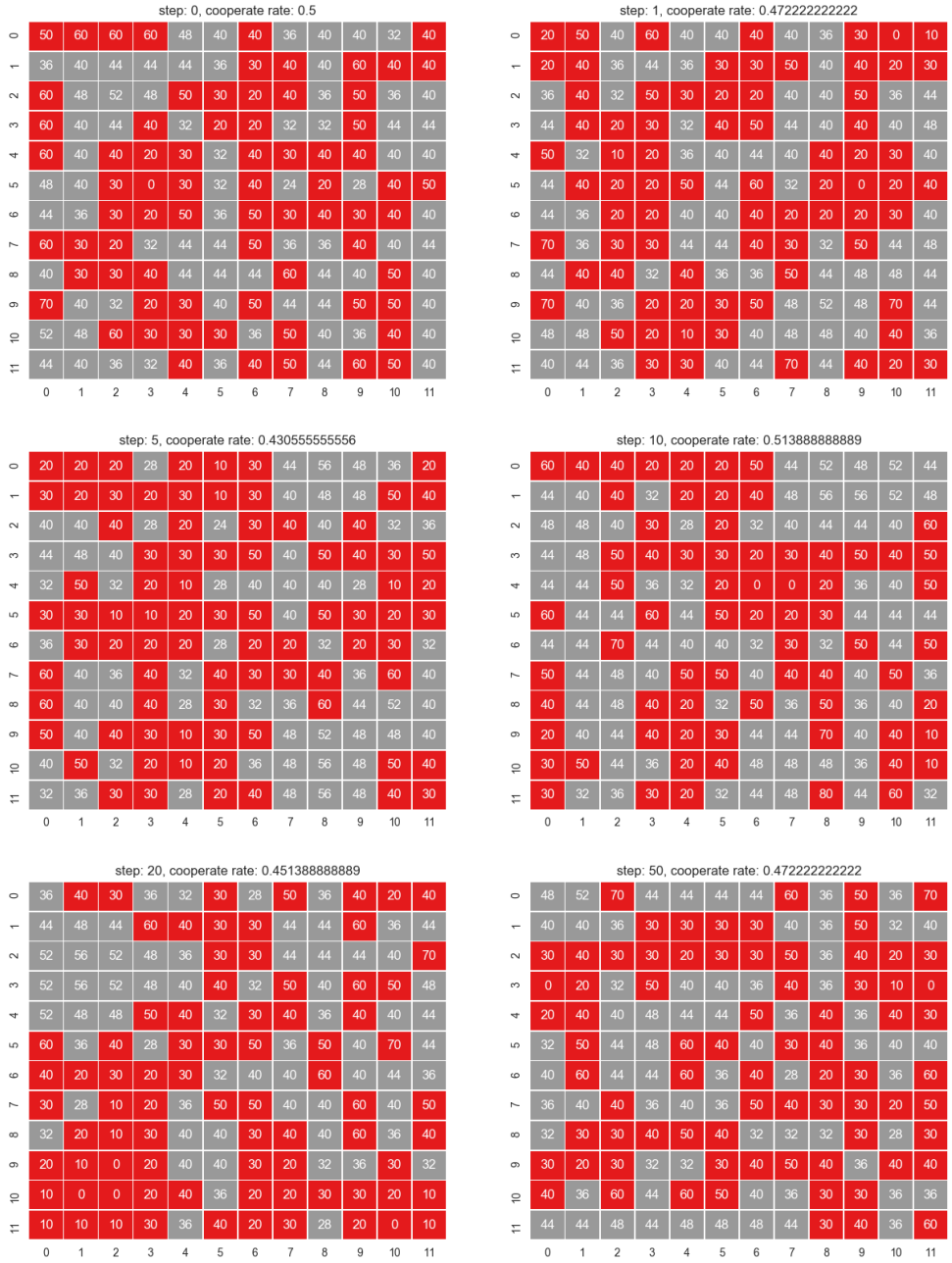

Figure 19: The Payoff matrix of 12 lattices and moore neighborhood

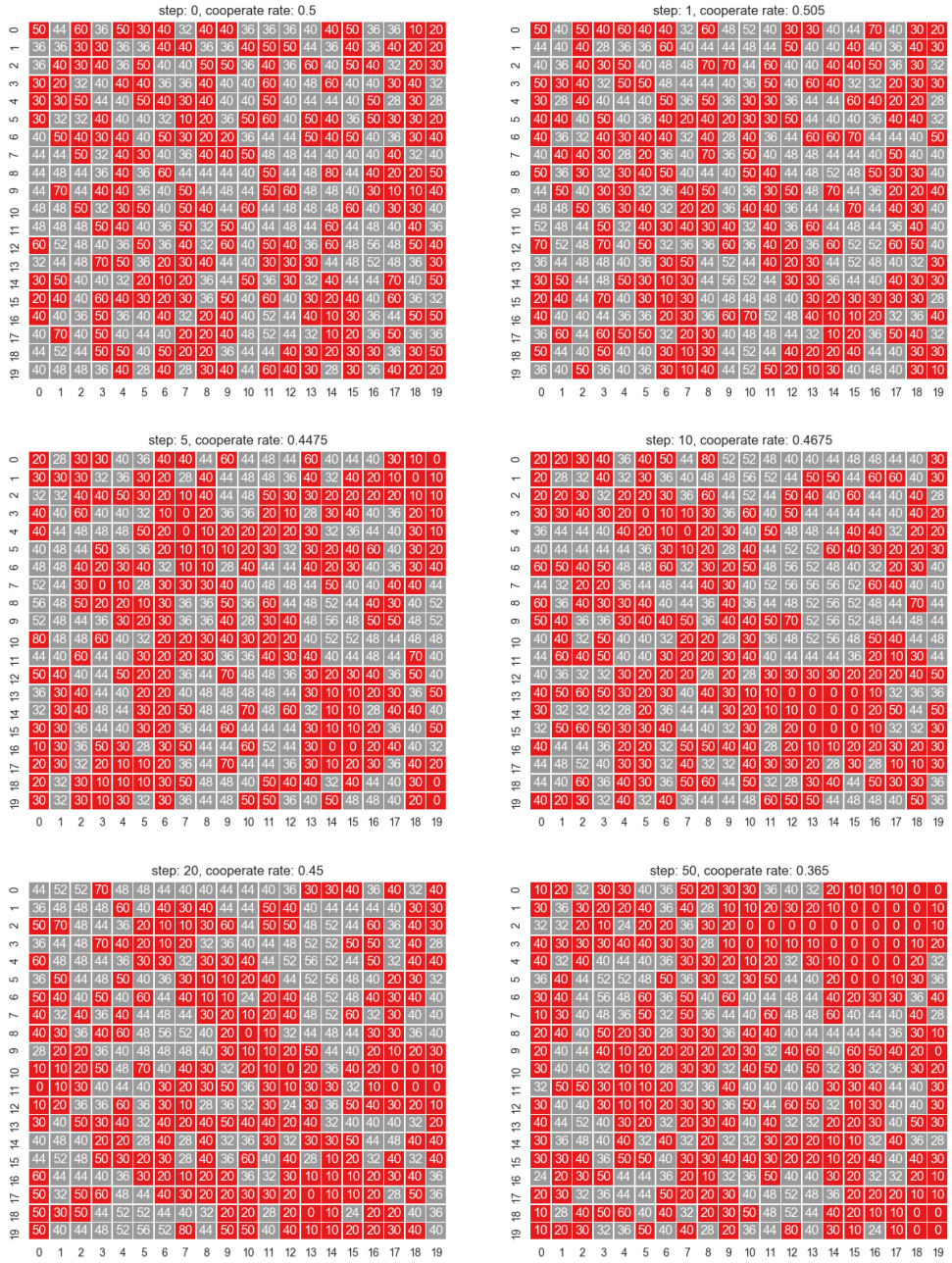

Figure 20: The Payoff matrix of 20 lattices and moore neighborhood

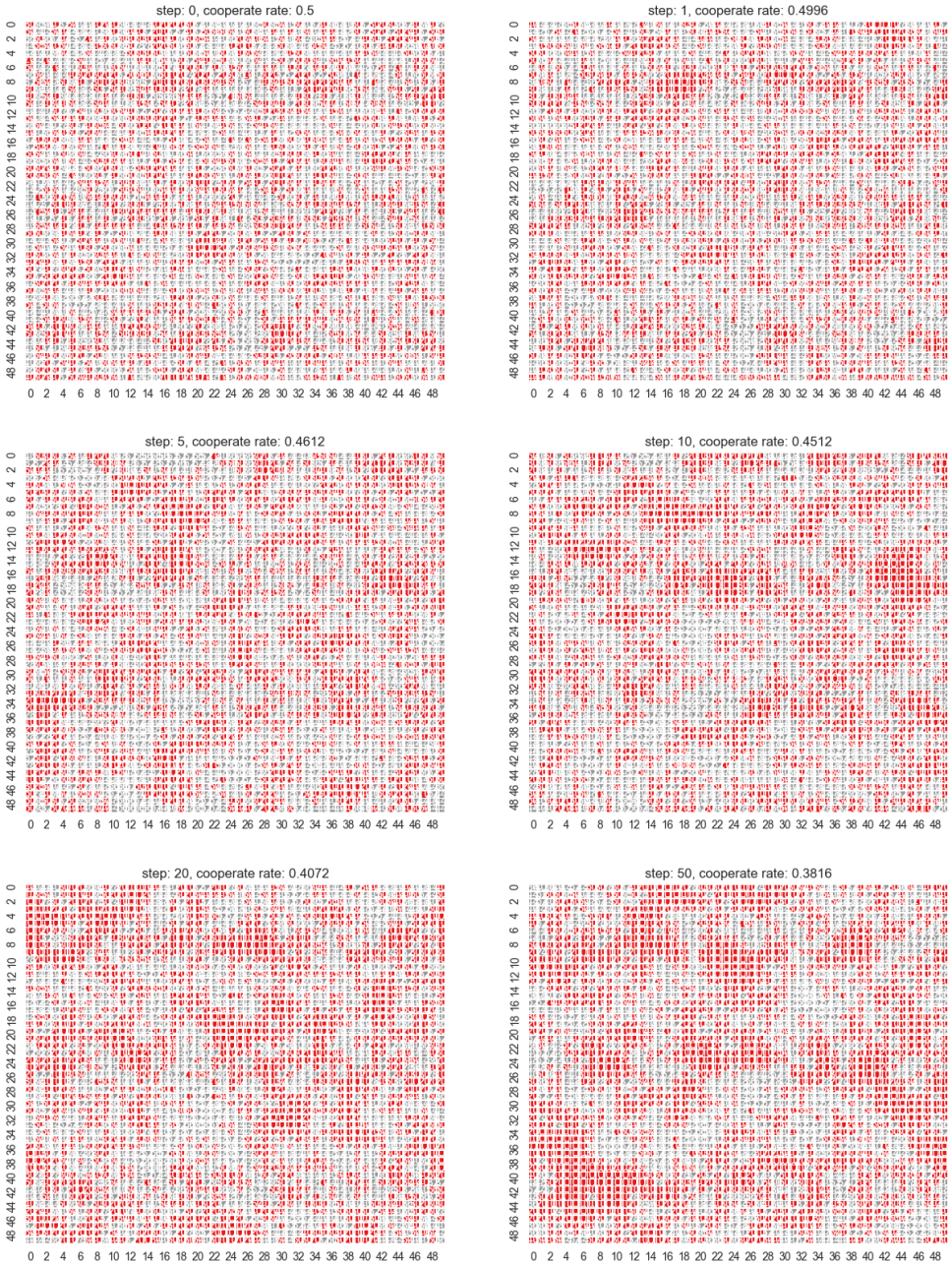

Figure 21: The Payoff matrix of 50 lattices and moore neighborhood

2.2 the payoff matrix of Von Neumann neighborhood with 4,8,12,20,50 lattices

Figure 22, Figure 23, Figure 24, Figure 25, Figure 26 show the payoff matrix of Von neighborhood. Again we could not observe large cooperate cluster.

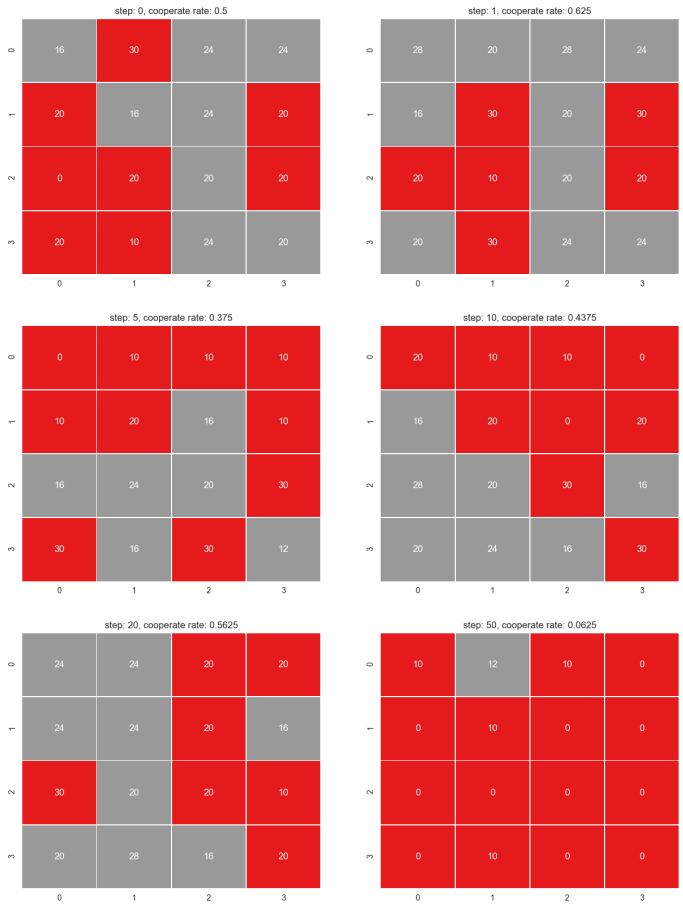

Figure 22: The Payoff matrix of 4 lattices and Von Neumann neighborhood

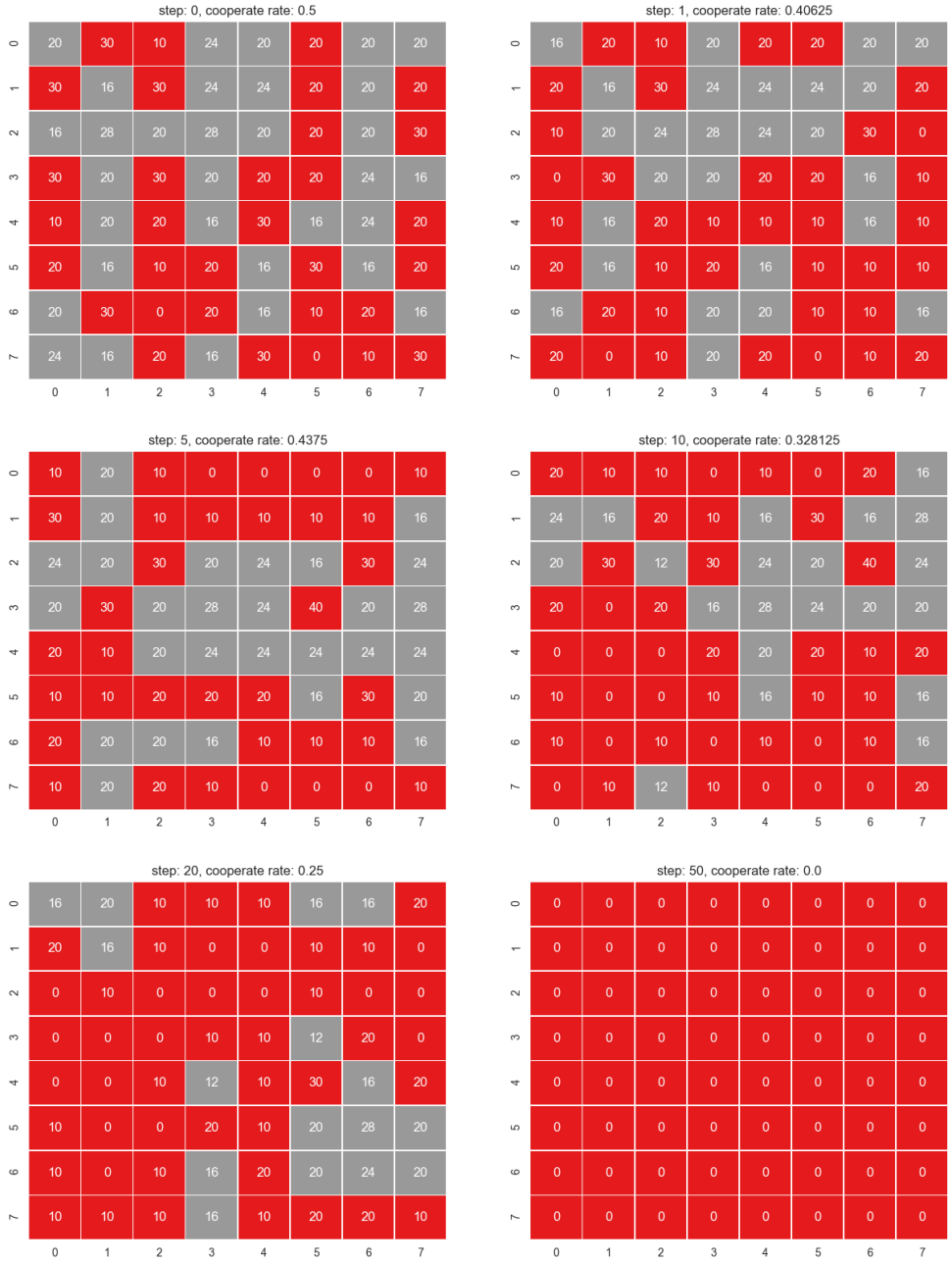

Figure 23: The Payoff matrix of 8 lattices and Von Neumann neighborhood

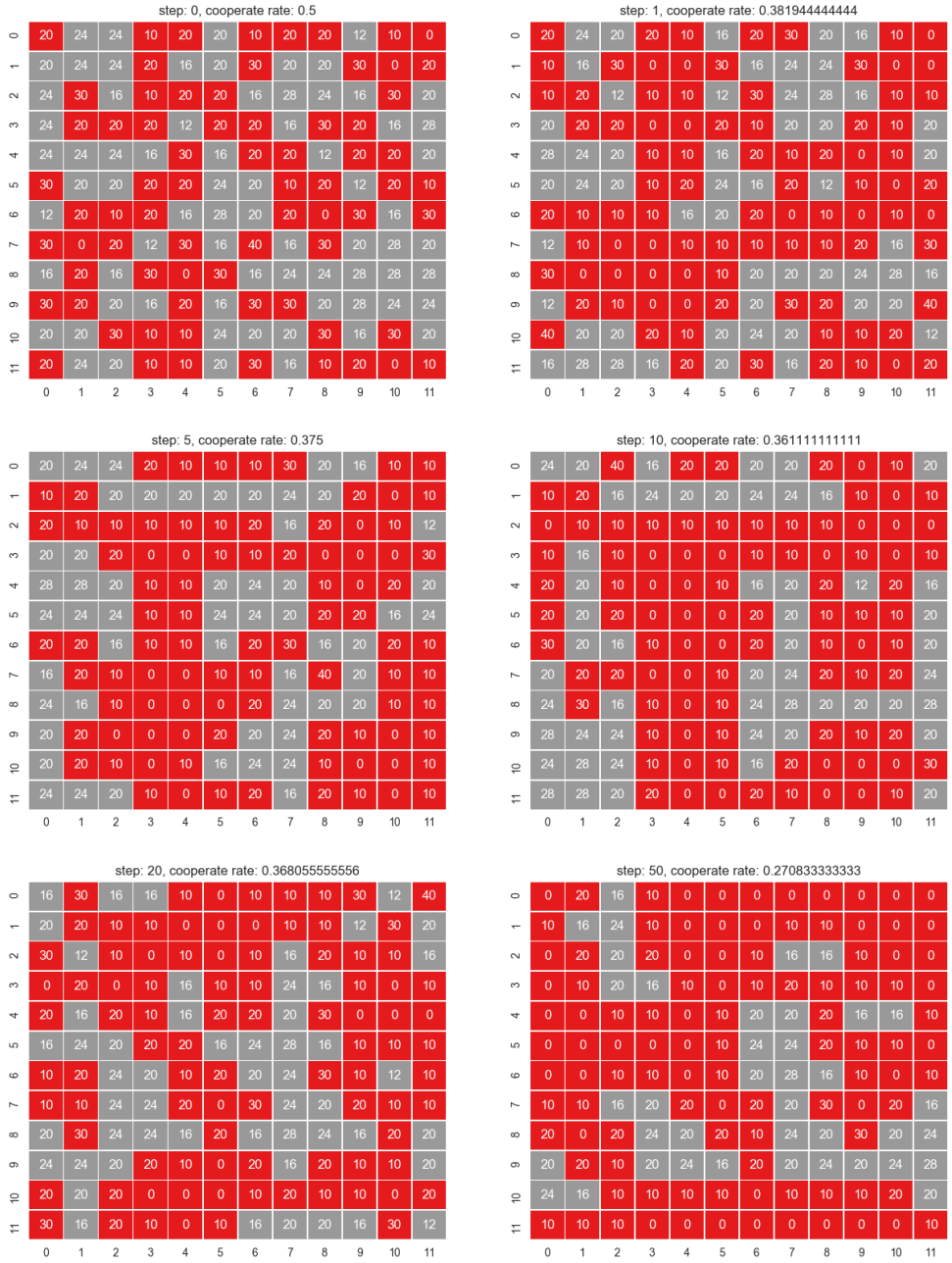

Figure 24: The Payoff matrix of 12 lattices and Von Neumann neighborhood

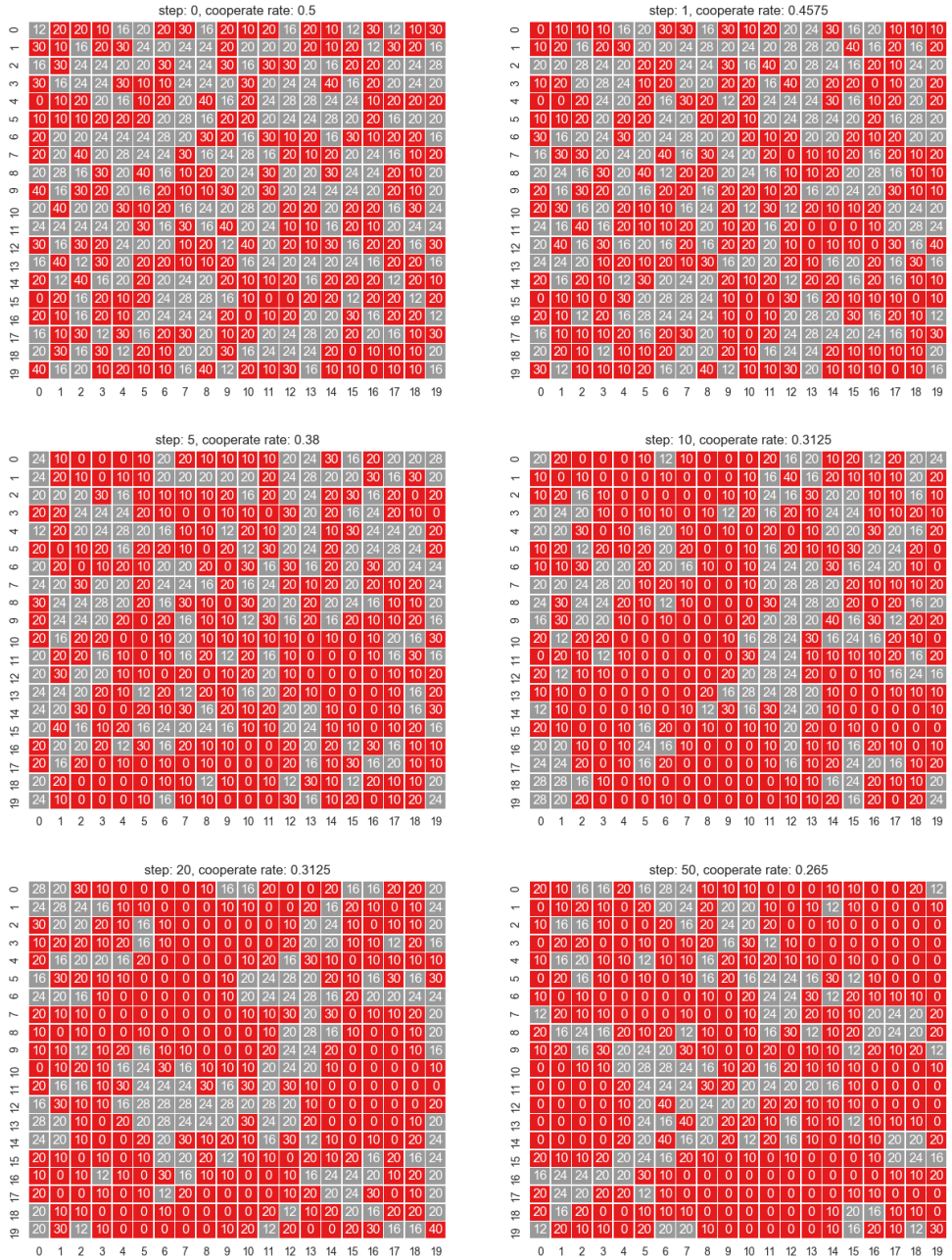

Figure 25: The Payoff matrix of 20 lattices and Von Neumann neighborhood

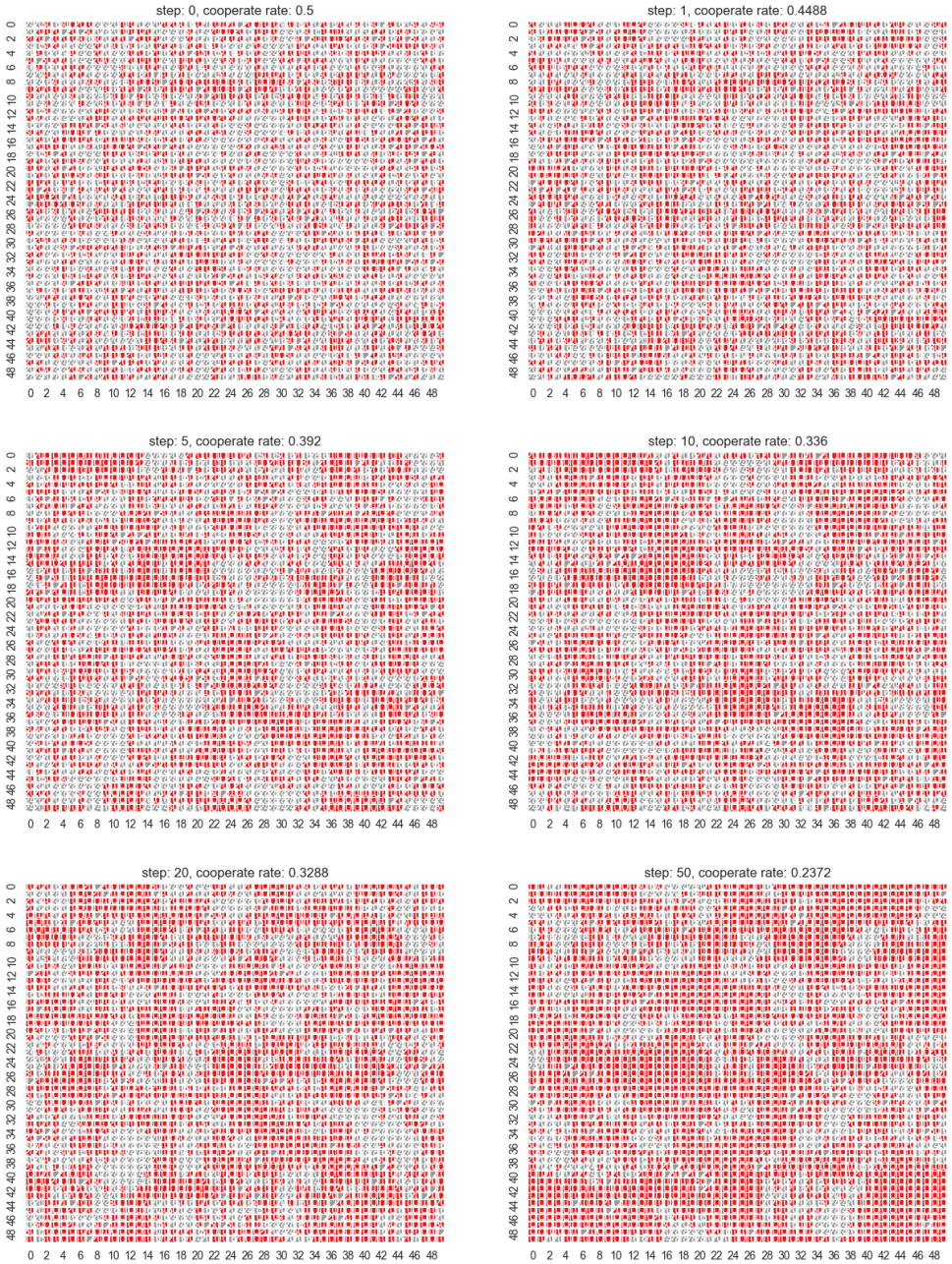

Figure 26: The Payoff matrix of 50 lattices and Von Neumann neighborhood

2.3 The comparison of cooperation level

When using replicator rule, we could notice that the cooperation levels do not decrease fast compare that using unconditional updating mechanism. And the larger lattices games have smaller fluctuations.

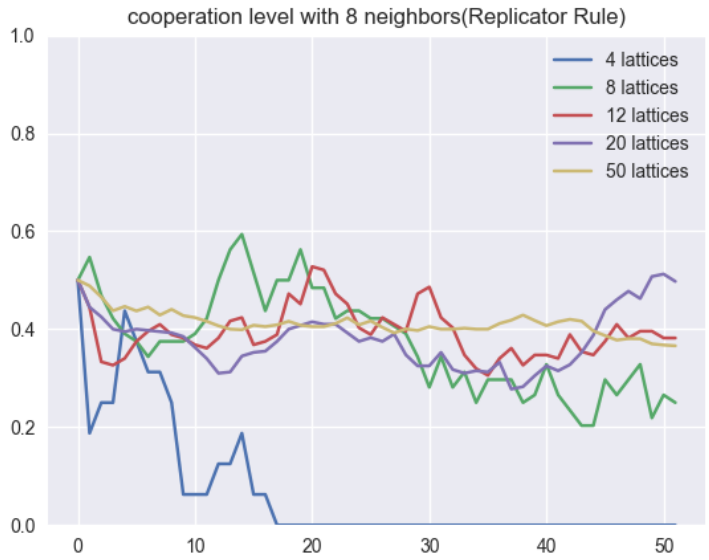

Figure 27: The cooperation level of Moore neighborhood

Figure 28 shows the cooperation level of Von Neumann neighborhood. The cooperation levels have smaller fluctuations compare with those playing with Moore neighborhood. The lattices 50*50 has a smaller ending cooperation level.

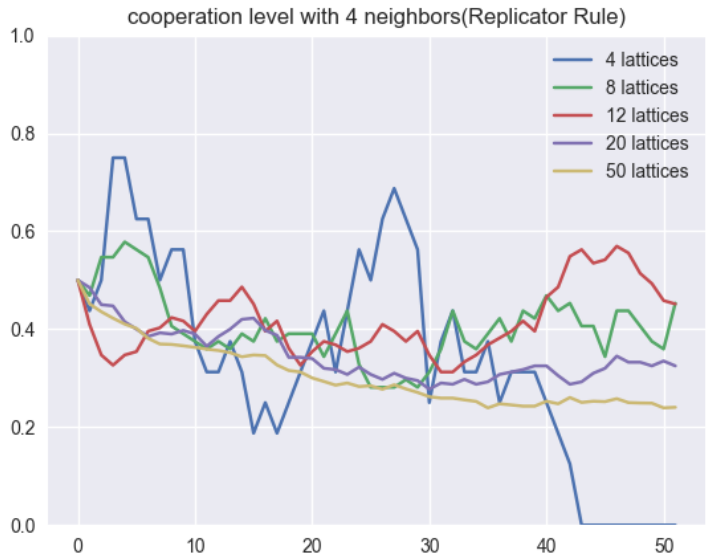

Figure 28: The cooperation level of Von Neumann neighborhood

2.4 The histogram of ending cooperation level (Moore Neighborhood)

Figure 29 shows the histogram of ending values when playing with Moore neighborhood. The standard derivations become smaller with the lattice size grows. And the final cooperation level becomes higher with the lattice size grows.

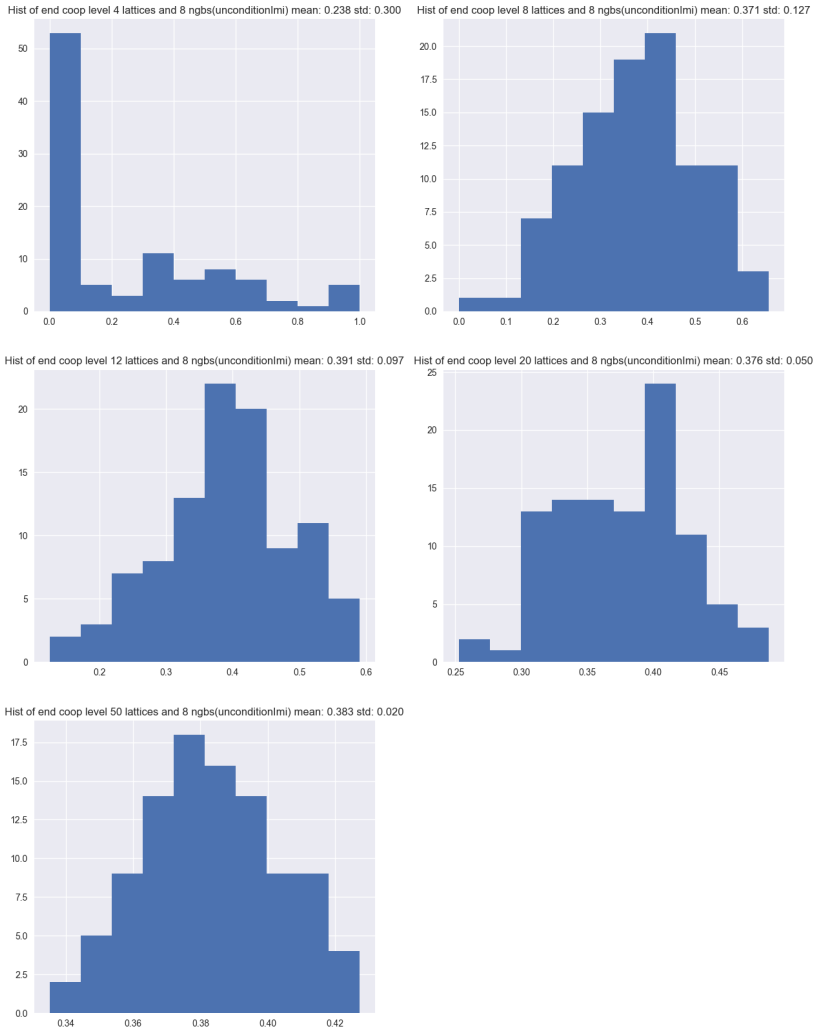

Figure 29: The histogram of ending cooperation level(Moore Neighborhood)

2.5 The histogram of ending cooperation level (Von Neumann Neighborhood)

Figure 30 shows the histogram of ending value when playing with Von Neumann neighborhood. We could notice the cooperation level is lower when compare with that playing with Moore neighborhood.

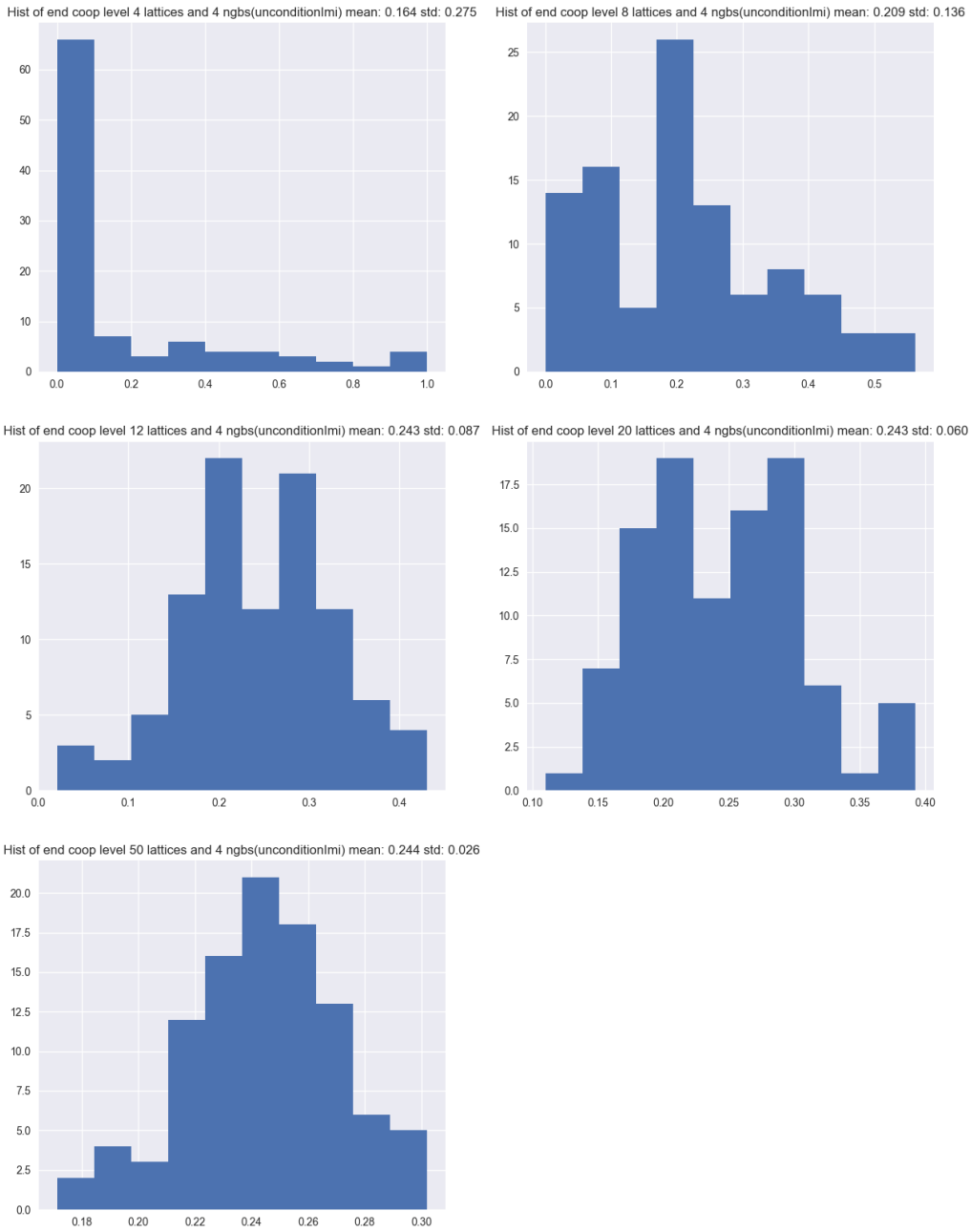

Figure 30: The histogram of ending cooperation level(Von Neumann Neighborhood)

2.6 Conclusion and understanding of replicator rule updating mechanism

2.6.1 Conclusion

Section 2 explore the evolutionary game theory using replicator rule in different lattice size and different number of neighborhood. Compare these games with different lattice size and different number of neighborhood, and also compare this with the unconditional imitation mechanism, we could find some commons and differences.

The section 2 plays the game using a snow and drift payoff matrix, where the S = 3, which is more favor of cooperation than the weak prisoner dilemma. But the cooperation levels are much lower than that in Section 1. We could find these conclusions.

1. The cooperation level of smaller lattice size games have larger fluctuations.

3. The cooperation level of Von Neumann neighborhood games have lower cooperation levels than those of Moore neighborhood.

4. Significant cooperator clusters are not found when using replicator updating mechanism.

2.6.2 Understanding of Replicator rules

The replicator rules is

$$P_{ij} = \frac{1 + \frac{[W_j - W_i]}{N(maxT,R,P,S - minT,R,P,S)}}{2}$$

The player random pick one of his neighbors, and the probability of imitate his behaviors are P_{ij}

1. Replicator rule is not deterministic, it depends on which neighborhood chosen.

2. In MCC(Moody conditional cooperation), whether players choose to cooperate depends on their mood. Whether they cooperate also depends on whether they themselves cooperate in the previous round. The players in this game also plays depends on their mood.

3. In replicator rule, when the payoff difference is larger, they are more likely to imitate their neighbor,which a bit like unconditional imitation, when the payoff difference is smaller, they tend to keep the same act in the previous round, which a bit like moody conditional cooperation.

4. The replicator rule is nondeterministic and this prevents the formation of cooperate clusters.

5. The replicator rule are more suitable to explain the real experiments(The plon , Madrid and Zaragoza mentioned in the class.)

3 Explore the influences of $\frac{T}{R}$ proportion to the ending cooperation level

The $\frac{T}{R}$ proportion also influences the cooperation level. The Section 3 explores the T/R proportion's influences to cooperation level. In this experiment, the P=S=0, R =7 and T =

$R * \frac{T}{R}$.Choose the 50*50 lattice, and repeat each subgame 50 times and draw the histogram of the cooperation level. The payoff matrix is shown in Figure 31

The details of this game

- lattice: 50*50
- updating mechanism: imitate the best
- $\frac{T}{R}$ = [0.8,1,1.2,1.4,1.43,1.47,1.5,1.7,1.8,2](when T<R, the game is like stag-hunt game where cooperation is the best choice.)
- each subgame is played 50 times and the final cooperation level is

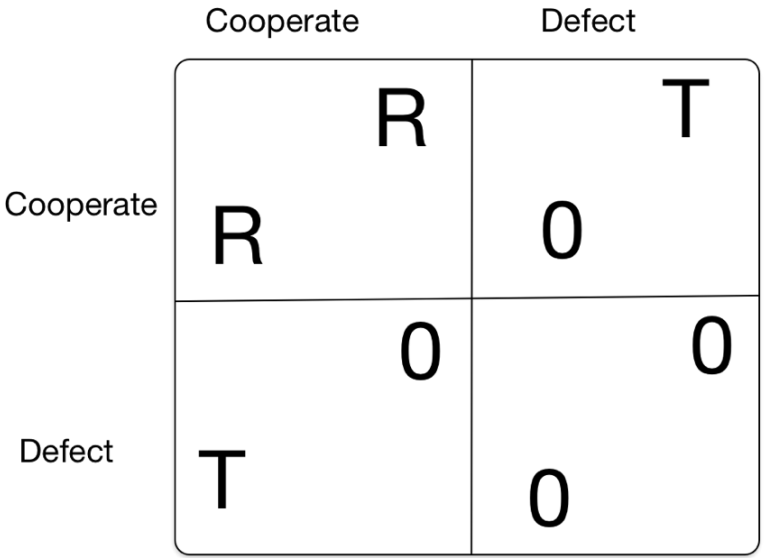

Figure 31: The payoff matrix for experiment 3

The figure 32 shows the histogram of ending cooperation level with different T/R

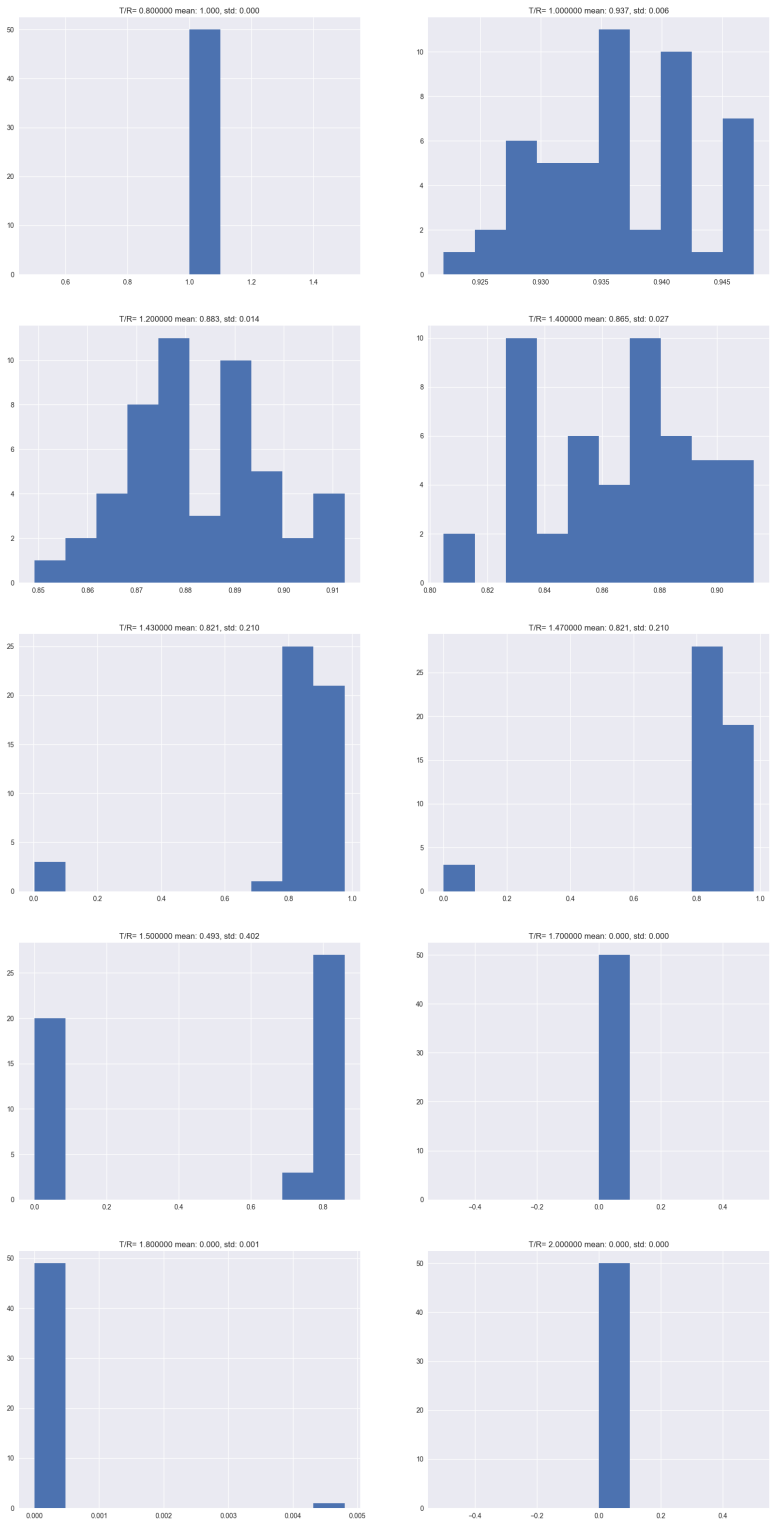

Figure 32: The histogram of ending cooperation level with different T/R

The figure 33 shows the cooperation level with different T/R ratio. We could notice there is a sharp drop between 1.4 and 1.7.

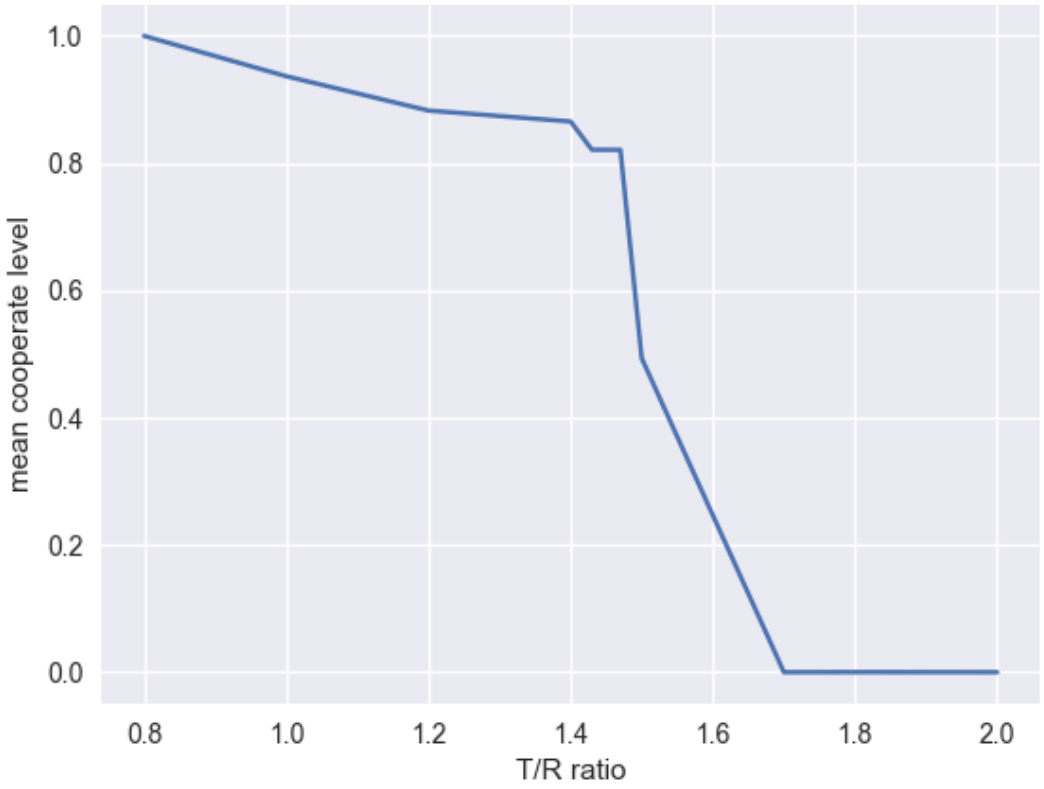

Figure 33: The cooperation level for different $\frac{T}{R}$