1. Introduction

Modern logistics platforms operate in environments where transportation demand fluctuates unpredictably, and congestion conditions evolve by the hour. These platforms face a persistent challenge: how to select a carrier whose cost is genuinely lowest when that cost is itself uncertain and time-varying. The problem is not only operational. It is deeply economic. When carriers submit bids today for services that may be executed hours or days later, even small errors in forecasting congestion, fuel usage, or route delays can lead to large gaps between expected and realized costs. As a result, many existing procurement mechanisms—including the widely used second-price sealed-bid auction—perform poorly in practice. They encourage sincere bidding only with respect to expected future costs, not realized costs, and therefore create substantial ex-post default risk whenever realized congestion pushes costs above expectations.

This combination of uncertainty, dynamic congestion, and the need for real-time adaptation creates a structural misalignment between existing auction formats and the economic environment in which logistics platforms actually operate. The platform wants the lowest-cost carrier; the carrier wants to avoid negative profit; but a static one-shot mechanism provides neither side with the information needed to reconcile these incentives.

To address this gap, I propose a dynamic multi-round bidding mechanism in which the platform updates an observable threshold price each round to reflect both the distribution of bids and the current congestion level in the transportation network. This threshold acts as the survival criterion for carriers and serves simultaneously as a market-generated signal of evolving congestion. Carriers update their beliefs and adjust bids in each round, and the process gradually screens out higher-cost bidders. The mechanism therefore performs two functions that the classical second-price auction cannot: (1) it induces asymptotically truthful bidding even when carriers cannot perfectly forecast their own cost, and (2) it aligns the winner's final price with realized network conditions, reducing ex-post default risk.

By embedding congestion feedback into the bidding process, the mechanism transforms a static adverse-selection problem into a dynamic learning environment in which the platform and carriers jointly uncover the true cost frontier. This feature is essential in logistics markets where operational uncertainty is not noise but a defining part of the production technology. The resulting allocation is cost-efficient, strategy-simple in the limit, and robust to environmental shocks—properties that standard sealed-bid auctions cannot deliver under realistic conditions.

1. Model Setup

The platform offers a transportation task in the auction and implements a dynamic bidding mechanism that determines the eventual winner's actual revenue. The bidders are potential carriers who adjust their bids across multiple rounds based on their own transportation costs. In each round, the platform publishes a composite pricing signal (the observable threshold price P_t^{obs}), which reflects both the submitted bids and the level of congestion in the transportation network. This threshold serves as the survival criterion for that round.

Unlike traditional one-shot auction mechanisms, the platform does **not** award the task to the lowest initial bidder. This is because actual transportation demand is subject to dynamic fluctuations, and before the market converges to a duopolistic structure, new bidders may need the flexibility to enter during the process. Through iterative screening and information updating across rounds, the platform gradually identifies the carrier that best matches the task requirements and is capable of revenue optimization under cost heterogeneity and demand uncertainty. This multi-round process enhances both the efficiency and the stability of resource allocation.

Each bidder's cost is:

$$TC_{i,t} = \alpha D_i + \beta \frac{DE_t}{CA} \qquad (1)$$

where $TC_{i,t}$ denotes the total cost of bidder i in round t, D_i represents the bidder's transportation distance, DE_t is the time-t transportation demand, and CA denotes the network's carrying capacity. The parameters α and β are cost-adjustment coefficients. Under this model specification, the congestion-induced cost component $\beta \frac{DE_t}{CA}$ is identical for all bidders in each round, as it depends solely on aggregate network conditions. Consequently, cross-firm heterogeneity in transportation costs αD_i arises only from the baseline distance term D_i, which is unique to each bidder.

The platform's published threshold price for each round is given by:

$$P_t^{obs} = \frac{1}{n_t} \int_{i \in n_t} \tilde{P}_{i,t} \ di \qquad (2)$$

$\tilde{P}_{i,t}$ denotes the *effective* bid computed by the platform in each round based on bidder i's nominal bid $P_{i,t}$ and the current level of network congestion. It also represents the bidder's actual transaction price if ultimately selected. Whenever $\tilde{P}_{i,t} > P_t^{obs}$, bidder i is eliminated from the auction; that is, bidders submitting sufficiently high effective prices face the risk of failing to advance. Given bidder i's effective price $\tilde{P}_{i,t}$ in round t, the probability of elimination is therefore: $Prob(\tilde{P}_{i,t} > P_t^{obs})$

2. Optimal Strategy Construction

2.1 Effective Pricing and the Profitability Requirement

Assume that in every round bidders observe their current transportation cost. A rational bidder will ensure that the optimal effective price (the eventual transaction price) satisfies: $\tilde{P}_{i,t} > TC_{i,t}$. since only in this case does the bidder have an incentive to remain in the auction. Otherwise, the bidder would optimally exit to avoid expected losses.

Under this mechanism, bidders cannot foresee which round will determine their final revenue. As a result, they must treat every round as a potentially decisive one. Consequently, in each round bidders choose a bidding strategy that maximizes their current-round expected payoff. Thus, bidder i's objective function in period t is:

$$\max_{\tilde{P}_{i,t}} (\tilde{P}_{i,t} - TC_{i,t}) Prob\left(\tilde{P}_{i,t} \leq P_t^{obs}\right) + (\tilde{P}_{i,t} - TC_{i,t}) Prob\left(\tilde{P}_{i,t} > P_t^{obs}\right) \quad (3)$$

Because bidder i exits the auction when $Prob(\tilde{P}_{i,t} > P_t^{obs})$ occurs, the term $(\tilde{P}_{i,t} - TC_{i,t}) Prob(\tilde{P}_{i,t} > P_t^{obs})$ becomes zero. Therefore, after this adjustment, Equation (3) can be rewritten as follows:

$$\max_{P_{i,t}} (\tilde{P}_{i,t} - TC_{i,t}) Prob\left(\tilde{P}_{i,t} \leq P_t^{obs} \middle| \mathcal{H}_t\right) \quad (4)$$

Let $\mathcal{H}_t = \{P_t^{obs}, P_{t-1}^{obs}, P_{t-2}^{obs}, \dots P_0^{obs}\}$ denote the history of observable information. By Bayes' rule, we have

$$Prob(\tilde{P}_{i,t} \leq P_t^{obs} | \mathcal{H}_t) \propto Prob(\mathcal{H}_t) \cdot Prob(\mathcal{H}_t | \tilde{P}_{i,t} \leq P_t^{obs}) \quad (5)$$

Because all information prior to round t is historical, the set $\{ P_{t-1}^{obs}, P_{t-2}^{obs}, \dots P_0^{obs}\}$ is constant with respect to $\tilde{P}_{i,t}$. Thus,

$$\begin{aligned} Prob(\mathcal{H}_t) &= Prob(P_t^{obs}) \cdot Prob(P_{t-1}^{obs}) \cdot Prob(P_{t-2}^{obs}) \cdot \dots Prob(P_0^{obs}) \\ &= Prob(P_t^{obs}) \end{aligned}$$

If the density of $\tilde{P}_{i,t}$ is $f_i(\tilde{P}_{i,t})$, then:

$$Prob(P_t^{obs}) = \frac{1}{n_t} \prod_{j=1}^{n_t} \int_{\sum \tilde{P}_{j,t} = P_t^{obs}} f_j(\tilde{P}_{i,t})\, d\tilde{P}_{j,t} \quad (6)$$

Equation (6) is a convolution over the joint distribution of $\sum \tilde{P}_{j,t}$,

$$Prob(\mathcal{H}_t | \tilde{P}_{i,t} \leq P_t^{obs}) = \frac{1}{n_t} \prod_{j=1}^{n_t} \int_{\sum \tilde{P}_{j,t} = P_t^{obs} \text{ and } \tilde{P}_{i,t} \leq P_t^{obs}} f_j(\tilde{P}_{j,t})\, d\tilde{P}_{j,t}\} \Big/ \int_{\tilde{P}_{i,t} \leq P_t^{obs}} f_i(\tilde{P}_{i,t})\, d\tilde{P}_{i,t} \quad (7)$$

Equation (7) is also a convolution. Substituting (6) and (7) into (4), we obtain:

$$\max_{\widetilde{P}_{i,t}}(\widetilde{P}_{i,t} - TC_{i,t})$$

$$\cdot \frac{\frac{1}{n_t}\prod_{\substack{j=1 \\ j\neq i}}^{n_t-1} \int f_j(\widetilde{P}_{j,t}\,)\, d\widetilde{P}_{j,t}\ \int_{\widetilde{P}_{i,t}\leq P_t^{obs}} f_i(\widetilde{P}_{i,t})\, d\widetilde{P}_{j,t} \int_{P_t^{obs}-\sum_{j=1}^{n_t-1}\widetilde{P}_{j,t}} f_{n_t}^{cond.}(\widetilde{P}_{nt,t})\, \widetilde{P}_{nt,t}}{\frac{1}{n_t}\prod_{\substack{j=1 \\ j\neq i}}^{n_t} \int f_j(\widetilde{P}_{j,t})\, d\widetilde{P}_{j,t} \int_{P_t^{obs}-\sum_{j=1}^{n_t-1}\widetilde{P}_{j,t}} f_{n_t}^{cond.} \widetilde{P}_{nt,t})\, d\widetilde{P}_{nt,t}}$$

$$\max_{\widetilde{P}_{i,t}} (\widetilde{P}_{i,t} - TC_{i,t}) \frac{Prob(\mathcal{H}_t \cap \widetilde{P}_{i,t} \leq P_t^{obs})}{Prob(\mathcal{H}_t)}$$

$$= \max_{\tilde{P}_{i,t}} \left(\tilde{P}_{i,t} - TC_{i,t}\right) \frac{Prob(\mathcal{H}_t \middle| \tilde{P}_{i,t} \leq P_t^{obs}) \cdot Prob(\tilde{P}_{i,t} \leq P_t^{obs})}{Prob(\mathcal{H}_t)} \qquad (8)$$

Taking the First Order Condition:

$$\frac{\partial \pi_{i,t}}{\partial \tilde{P}_{i,t}} = Prob(\mathcal{H}_t \middle| \tilde{P}_{i,t} \leq P_t^{obs}) \cdot Prob\left(\tilde{P}_{i,t} \leq P_t^{obs}\right) \cdot \frac{1}{Prob(\mathcal{H}_t)} +$$

$$\left(\tilde{P}_{i,t} - TC_{i,t}\right) \cdot \left\{ \frac{\partial Prob(\mathcal{H}_t \middle| \tilde{P}_{i,t} \leq P_t^{obs})}{\partial \tilde{P}_{i,t}} \cdot Prob\left(\tilde{P}_{i,t} \leq P_t^{obs}\right) \cdot \frac{1}{Prob(\mathcal{H}_t)} \right.$$

$$+ Prob(\mathcal{H}_t \middle| \tilde{P}_{i,t} \leq P_t^{obs}) \frac{\partial Prob\left(\tilde{P}_{i,t} \leq P_t^{obs}\right)}{\partial \tilde{P}_{i,t}} \cdot \frac{1}{Prob(\mathcal{H}_t)}$$

$$- Prob(\mathcal{H}_t \middle| \tilde{P}_{i,t} \leq P_t^{obs}) \cdot Prob\left(\tilde{P}_{i,t} \leq P_t^{obs}\right) \cdot \frac{\partial Prob\left(P_t^{obs}\right)}{\partial \tilde{P}_{i,t}}$$

$$\left. \cdot \frac{1}{Prob^2(\mathcal{H}_t)} \right\} \qquad (9)$$

Rearranging equation (9), the bidder's optimal bidding strategy is:

$$\tilde{P}_{i,t} = TC_{i,t} - Prob(\mathcal{H}_t \middle| \tilde{P}_{i,t} \leq P_t^{obs}) \cdot \left[\frac{\partial Prob(\mathcal{H}_t \middle| \tilde{P}_{i,t} \leq P_t^{obs})}{\partial \tilde{P}_{i,t}}\right]^{-1}$$

$$-Prob\left(\tilde{P}_{i,t} \leq P_t^{obs}\right) \cdot \left[\frac{\partial Prob\left(\tilde{P}_{i,t} \leq P_t^{obs}\right)}{\partial \tilde{P}_{i,t}}\right]^{-1}$$

$$+Prob\left(P_t^{obs}\right) \cdot \left[\frac{\partial Prob\left(P_t^{obs}\right)}{\partial \tilde{P}_{i,t}}\right]^{-1}$$

2.2 Nominal Bids and Actual Bidding Behavior

However, in a dynamic environment, bidders generally cannot observe their exact transportation cost $TC_{i,t}$ at the moment they submit their bid in round t. As a result, a rational bidder uses the previously observed cost $TC_{i,t-1}$ as an estimate of the current cost. as the basis for estimating the current cost and formulates the nominal bid $P_{i,t}$ accordingly. This nominal bid represents the price that the bidder actually submits in round t:

$$
\begin{aligned}
P_{i,t} = TC_{i,t-1} - Prob(\mathcal{H}_t|\tilde{P}_{i,t} \leq P_t^{obs}) \cdot \left[\frac{\partial Prob(\mathcal{H}_t|\tilde{P}_{i,t} \leq P_t^{obs})}{\partial \tilde{P}_{i,t}}\right]^{-1} \\
-Prob(\tilde{P}_{i,t} \leq P_t^{obs}) \cdot \left[\frac{\partial Prob(\tilde{P}_{i,t} \leq P_t^{obs})}{\partial \tilde{P}_{i,t}}\right]^{-1} \\
+Prob(P_t^{obs}) \cdot \left[\frac{\partial Prob(P_t^{obs})}{\partial \tilde{P}_{i,t}}\right]^{-1}
\end{aligned}
$$

A key feature of the nominal bid is that it does not correspond to the bidder's final effective transaction price. In each round, the platform adjusts the nominal bid based on changes in network congestion between the current and previous rounds. Specifically, the platform applies a compensatory correction of $\beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right) = \beta(\frac{\mathrm{DE_t}}{\mathrm{CA}} - \frac{\mathrm{DE_{t-1}}}{\mathrm{CA}})$ to the nominal bid, generating the effective bid $\tilde{P}_{i,t}$, which determines the bidder's actual transaction price. This adjustment mechanism ensures that even though bidders formulate their nominal bids $\tilde{P}_{i,t}$ using the previous round's cost estimate $TC_{i,t-1}$, their bids can still achieve expected profit maximization. As a result, the bidder's strategy remains optimal despite not observing the contemporaneous cost:

$$
\begin{aligned}
\tilde{P}_{i,t} = P_{i,t} + \beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right) = TC_{i,t} - Prob(\mathcal{H}_t|\tilde{P}_{i,t} \leq P_t^{obs}) \cdot \left[\frac{\partial Prob(\mathcal{H}_t|\tilde{P}_{i,t} \leq P_t^{obs})}{\partial \tilde{P}_{i,t}}\right]^{-1} \\
-Prob(\tilde{P}_{i,t} \leq P_t^{obs}) \cdot \left[\frac{\partial Prob(\tilde{P}_{i,t} \leq P_t^{obs})}{\partial \tilde{P}_{i,t}}\right]^{-1} \\
+Prob(P_t^{obs}) \cdot \left[\frac{\partial Prob(P_t^{obs})}{\partial \tilde{P}_{i,t}}\right]^{-1} \qquad (10)
\end{aligned}
$$

By publishing the threshold price derived from the congestion-adjusted nominal bids, the platform not only determines which bidders are eliminated in that round but also provides a public signal about the current level of congestion in the transportation network. Bidders can then use this information to form more accurate expectations about future transportation costs and adjust their optimal bidding strategies accordingly. The threshold price P_t^{obs} announced by the platform in each round is computed as follows:

$$P_t^{obs} = \frac{1}{n_t}\int_{i\in n_t} P_{i,t}\ di + \beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right) = \frac{1}{n_t}\int_{i\in n_t} \tilde{P}_{i,t}\ di$$

Even though bidders do not observe their actual transportation cost in the current round, ensuring that their nominal bid exceeds the previous round's cost estimate is sufficient for making a rational decision about whether to exit or remain in the auction. When the optimal nominal bid satisfies $P_{i,t} < TC_{i,t-1}$, the corresponding effective bid $\tilde{P}_{i,t} = P_{i,t} + \beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right)$ falls below the actual transportation cost $TC_{i,t}$. In this case, the bidder's expected payoff is negative, and a rational bidder will choose to exit voluntarily. This feature preserves the rationality of exit decisions while reducing dependence on contemporaneous cost information. Similarly, when the effective bid $\tilde{P}_{i,t} = P_{i,t} + \beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right)$falls below the platform's threshold price P_t^{obs}, the bidder is eliminated. This decision rule is fully consistent with the mechanism's previously defined structure.

3. Optimal Strategy Analysis

For convenience, define the following terms:

$A \coloneqq Prob(\mathcal{H}_t|\tilde{P}_{i,t} \le P_t^{obs})$, $A' \coloneqq \frac{\partial Prob(\mathcal{H}_t|\tilde{P}_{i,t}\le P_t^{obs})}{\partial \tilde{P}_{i,t}}$;

$B \coloneqq Prob(\tilde{P}_{i,t} \le P_t^{obs})$, $B' \coloneqq \frac{\partial Prob(\tilde{P}_{i,t}\le P_t^{obs})}{\partial \tilde{P}_{i,t}}$

$C \coloneqq Prob(P_t^{obs})$, $C' \coloneqq \frac{\partial Prob(P_t^{obs})}{\partial \tilde{P}_{i,t}}$

With these definitions, the optimal nominal bidding rule in equation (10) simplifies to:

$$P_{i,t} = TC_{i,t-1} - \frac{A}{A'} - \frac{B}{B'} + \frac{C}{C'}$$

The corresponding optimal effective bid is:

$$\tilde{P}_{1,t} = TC_{i,t} - \frac{A}{A'} - \frac{B}{B'} + \frac{C}{C'} \quad (11)$$

To analyze the relationship between the optimal effective bid $\tilde{P}_{i,t}$ and the transportation cost $TC_{i,t}$ (that is, the relationship between the bidder's actual transaction price and the underlying cost), we examine the three derivative terms that appear in the first-order condition. Given that A, B, and C are all positive, the key step is to determine the signs of A', B', and C'.

$\boldsymbol{A'} \coloneqq \frac{\boldsymbol{\partial Prob(\mathcal{H}_t|\tilde{P}_{i,t}\leq P_t^{obs})}}{\boldsymbol{\partial \tilde{P}_{i,t}}} = \frac{\boldsymbol{\partial Prob(P_t^{obs}|\tilde{P}_{i,t}\leq P_t^{obs})}}{\boldsymbol{\partial \tilde{P}_{i,t}}}$ **is negative**:

Because bidder i's effective bid contributes to the calculation of the threshold price P_t^{obs}, changes in $\tilde{P}_{i,t}$ shift the conditional distribution of the threshold. When bidder i is not eliminated (i.e., $\tilde{P}_{i,t} \leq P_t^{obs}$), an increase in $\tilde{P}_{i,t}$ makes the threshold P_t^{obs} easier to meet" or more likely to occur. This derivative captures how the platform's rule influences the statistical structure of the threshold.

$\boldsymbol{B'} \coloneqq \frac{\boldsymbol{\partial Prob(\tilde{P}_{i,t}\leq P_t^{obs})}}{\boldsymbol{\partial \tilde{P}_{i,t}}}$ **is negative:**

This term represents the marginal effect of a bidder's effective bid on the probability of surviving the current round. As $\tilde{P}_{i,t}$ increases, the probability of elimination $Prob(\tilde{P}_{i,t} > P_t^{obs})$ rises, meaning that the survival probability $Prob\left(\tilde{P}_{i,t} \leq P_t^{obs}\right)$ falls.

$\boldsymbol{C'} \coloneqq \frac{\boldsymbol{\partial Prob(P_t^{obs})}}{\boldsymbol{\partial \tilde{P}_{i,t}}}$ **is positive:**

This derivative reflects the effect of a bidder's own bid on the density of the platform's threshold P_t^{obs} Since an increase in an individual bid may "push" the threshold rightward and reduce density at a given point, this term is typically negative when expressed with respect to the density itself, but positive in the formulation used here. It captures the bidder's external effect on the environment generated by the platform's aggregation rule. Taken together, the optimal effective bid $\tilde{P}_{1,t} = TC_{i,t} - \frac{A}{A'} - \frac{B}{B'} + \frac{C}{C'}$

can be interpreted as the transportation cost $TC_{i,t}$ adjusted by two negative terms and one positive term:

$$\tilde{P}_{i,t} = TC_{i,t} + negative\ term + postive\ term + negative\ term$$

Similarly, the optimal nominal bid is:

$$P_{i,t} = TC_{i,t-1} - \frac{A}{A'} - \frac{B}{B'} + \frac{C}{C'}$$

which can also be written as:

$$P_{i,t} = TC_{i,t-1} + negative\ term + postive\ term + negative\ term$$

Among these terms, A' and C' depend primarily on how the platform constructs the threshold price (e.g., using a mean, median, or weighted average). They represent structural feedback effects from bidders onto the mechanism and are not the main strategic drivers of a bidder's behavior. In contrast, the term B' is fully determined by the bidder's own actions. It captures the absolute rate at which the survival probability changes with the bid, directly reflecting the bidder's perceived marginal risk of elimination. Because it is closely related to the bidder's risk sensitivity, B' will play a central role in the subsequent behavioral analysis.

Our main conclusion is as follows: when a bidder is less sensitive to risk, the optimal bid is relatively high; when a bidder is more risk-sensitive, the optimal bid becomes lower and approaches the transportation cost. The detailed analysis is presented below. The core expression for risk sensitivity is:

$$Risk\ Sensitivity = \left| \frac{d}{d\tilde{P}_{i,t}} logProb\left(\tilde{P}_{i,t} \le P_t^{obs}\right) \right|$$

$$= \left| \left[Prob\left(\tilde{P}_{i,t} \le P_t^{obs}\right)\right]^{-1} \cdot \frac{\partial Prob\left(\tilde{P}_{i,t} \le P_t^{obs}\right)}{\partial \tilde{P}_{i,t}} \right|$$

$$= -\left\{ \left[Prob\left(\tilde{P}_{i,t} \le P_t^{obs}\right)\right]^{-1} \cdot \frac{\partial Prob\left(\tilde{P}_{i,t} \le P_t^{obs}\right)}{\partial \tilde{P}_{i,t}} \right\}$$

This can be written more compactly as:

$$Risk\ Sensitivity = \left| \frac{d}{d\tilde{P}_{i,t}} logB(\tilde{P}_{i,t}) \right| = \left| \frac{B'(\tilde{P}_{i,t})}{B(\tilde{P}_{i,t})} \right| = -\frac{B'(\tilde{P}_{i,t})}{B(\tilde{P}_{i,t})}$$

The magnitude $\left|\frac{B'}{B}\right|$ measures the marginal responsiveness of the winning probability to changes in the bid—that is, the relative rate of change of success probability. It captures the bidder's sensitivity to failure risk: "If I raise my bid slightly, how much will my winning probability drop, and how severe is this drop relative to my current chance of success?" We therefore interpret this term as the bidder's risk sensitivity. From equation (11), the optimal effective bid satisfies:

$$\widetilde{P}_{i,t} = TC_{i,t} - \cdots + \left|\frac{B(\widetilde{P}_{i,t})}{B'(\widetilde{P}_{i,t})}\right| - \cdots$$

When bidder i is more risk-sensitive, the term, $\left|\frac{B'(\widetilde{P}_{i,t})}{B(\widetilde{P}_{i,t})}\right|$ becomes larger, and its reciprocal $\left|\frac{B(\widetilde{P}_{i,t})}{B'(\widetilde{P}_{i,t})}\right|$ becomes smaller. As a result, the optimal effective bid $\widetilde{P}_{i,t}$ decreases. Conversely, when a bidder is less sensitive to risk, $\left|\frac{B'(\widetilde{P}_{i,t})}{B(\widetilde{P}_{i,t})}\right|$ is smaller and its reciprocal $\left|\frac{B(\widetilde{P}_{i,t})}{B'(\widetilde{P}_{i,t})}\right|$ is larger, implying a higher optimal effective bid $\widetilde{P}_{1,t}$.

4. Equilibrium Stability Analysis

4.1 Optimal Responses and Steady-State Behavior Under the Mechanism

In the dynamic multi-round auction mechanism, bidders update their information each round and simultaneously face an evolving survival threshold, since the threshold price P_t^{obs}会 is adjusted in real time across rounds. These two forces jointly drive bidders to gradually adopt optimal response strategies based on currently available information. A key feature of the mechanism is that bidders do not know the total number of rounds in advance. Their bid in any given round not only determines whether they survive the current stage but may also directly determine their final payoff. As a result, bidders must balance the uncertainty of both the competitive threshold and the remaining number of rounds, leading them to gradually adjust and converge toward a stable bidding strategy.

In particular, in the final round—where the current bid directly determines whether the bidder wins and the payoff they ultimately receive—any deviation from the expected-profit-maximizing bid results in a real loss in payoff. In this case, the strategic structure no longer involves probabilistic "survival vs. elimination," but instead becomes a direct cost–benefit trade-off over the actual bid. Because bidding no longer influences whether one advances to the next round but instead determines the final winning outcome, a rational bidder has no incentive to deviate from the payoff-maximizing optimal strategy in the final round.

One may argue from a dynamic-incentive perspective that if bidders knew the total number of rounds T in advance, they might strategically shade their bids downward in earlier rounds to raise their survival probability, then switch to a truthful, cost-reflective optimal bid in the final round. However, under the mechanism we study, bidders do **not** possess full knowledge of the total number of rounds. The platform intentionally withholds this information, leaving bidders in each round facing substantial uncertainty about whether it is the final stage. Attempting to shade bids downward based on the assumption that "this might not be the last round" risks elimination due to underbidding and may even generate negative expected profit. By contrast, consistently responding rationally in each round based on currently available information and the platform's threshold rule naturally drives bidders toward the optimal equilibrium strategy over time.

4.2 Uniqueness and Local Stability of the Optimal Bid Under the First-Order Condition

To further verify the stability of the mechanism's optimal bidding strategy, we conduct a differential analysis of an individual bidder's payoff function. By deriving the first-order condition and the associated closed-form solution, we show that when all other bidders follow the optimal strategy, a given bidder can maximize expected profit only by adopting the same strategy. This establishes the strategy profile as an equilibrium. Moreover, this strategy is not only locally optimal but also internally consistent and sustainable given the distribution of observed bids.

To illustrate this point, we analyze the bidding behavior of bidder 1. The goal is to verify that when bidder 1 follows the same rule as the others, their expected payoff is maximized. Equations (10) and (11) give the expressions for the optimal effective bid:

$$\tilde{P}^*{}_{i,t} = P_{i,t} + \beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right)$$

$$= TC_{i,t} - Prob(\mathcal{H}_t|\tilde{P}_{i,t} \leq P_t^{obs}) \cdot \left[\frac{\partial Prob(\mathcal{H}_t|\tilde{P}_{i,t} \leq P_t^{obs})}{\partial \tilde{P}_{i,t}}\right]^{-1}$$

$$-Prob(\tilde{P}_{i,t} \leq P_t^{obs}) \cdot \left[\frac{\partial Prob(\tilde{P}_{i,t} \leq P_t^{obs})}{\partial \tilde{P}_{i,t}}\right]^{-1}$$

$$+Prob(P_t^{obs}) \cdot \left[\frac{\partial Prob(P_t^{obs})}{\partial \tilde{P}_{i,t}}\right]^{-1} \quad (10)$$

$$\tilde{P}^*{}_{i,t} = TC_{i,t} - \frac{A}{A'} - \frac{B}{B'} + \frac{C}{C'} \quad (11)$$

We aim to show that whenever $\tilde{P}_{1,t} > \tilde{P}^*{}_{i,t}$ or $\tilde{P}_{1,t} < \tilde{P}^*{}_{i,t}$, the first derivative of bidder 1's expected profit with respect to or $\tilde{P}_{1,t}$is nonzero, i.e., $\frac{\partial \pi_{i,t}}{\partial \tilde{P}_{1,t}} \neq 0$. Bidder 1's expected profit in round t is given by

$$\pi_{1,t} = (\tilde{P}_{1,t} - TC_{1,t}) = (\tilde{P}_{1,t} - TC_{1,t-1})Prob(\tilde{P}_{1,t} \leq P_t^{obs}|\mathcal{H}_t)$$

where survival in the round is captured by the probability term conditional on the observable history $\mathcal{H}_t$. Taking the first-order condition with respect to $\tilde{P}_{1,t}$ yields

$$\frac{\partial \pi_{1,t}}{\partial \tilde{P}_{1,t}} = Prob(\tilde{P}_{1,t} \leq P_t^{obs}|\mathcal{H}_t) + (\tilde{P}_{1,t} - TC_{1,t-1})\frac{\partial Prob(\tilde{P}_{1,t} \leq P_t^{obs}|\mathcal{H}_t)}{\tilde{P}_{1,t}} \quad (12)$$

From equation (10), the first-order condition $\frac{\partial \pi_{1,t}}{\partial \tilde{P}_{1,t}} = 0$ implies that at the optimum $\tilde{P}^*{}_{1,t}$, we have:

$$\tilde{P}^*{}_{1,t} - TC_{1,t} = -Prob(\mathcal{H}_t|\tilde{P}_{1,t} \leq P_t^{obs}) \cdot \left[\frac{\partial Prob(\mathcal{H}_t|\tilde{P}_{1,t} \leq P_t^{obs})}{\partial \tilde{P}_{1,t}}\right]^{-1}$$

$$-Prob\left(\tilde{P}_{1,t} \leq P_t^{obs}\right) \cdot \left[\frac{\partial Prob\left(\tilde{P}_{1,t} \leq P_t^{obs}\right)}{\partial \tilde{P}_{1,t}}\right]^{-1}$$

$$+Prob\left(P_t^{obs}\right) \cdot \left[\frac{\partial Prob\left(P_t^{obs}\right)}{\partial \tilde{P}_{1,t}}\right]^{-1} \qquad (13)$$

Using the simplified notation introduced in equation (11), we rewrite this as

$$\tilde{P}^*{}_{1,t} - TC_{i,t} = -\frac{A}{A'} - \frac{B}{B'} + \frac{C}{C'} \qquad (14)$$

where A, A', B, B', C, C' are defined as in Section 3. Substituting (14) into the first-order condition (12), we obtain

$$\frac{\partial \pi_{1,t}}{\partial \tilde{P}_{1,t}} = Prob\left(\tilde{P}_{1,t} \leq P_t^{obs} \middle| \mathcal{H}_t\right) + \left(-\frac{A}{A'} - \frac{B}{B'} + \frac{C}{C'}\right) \frac{\partial Prob\left(\tilde{P}_{1,t} \leq P_t^{obs} \middle| \mathcal{H}_t\right)}{\tilde{P}_{1,t}} = 0$$

Now consider the case $\tilde{P}_{1,t} > \tilde{P}^*{}_{1,t}$, i.e., $\tilde{P}_{1,t} - TC_{i,t} > -\frac{A}{A'} - \frac{B}{B'} + \frac{C}{C'}$

Since $\frac{\partial Prob\left(\tilde{P}_{1,t} \leq P_t^{obs} \middle| \mathcal{H}_t\right)}{\tilde{P}_{1,t}} < 0$, it follows that

$$\left(\tilde{P}_{1,t} - TC_{1,t}\right) \frac{\partial Prob\left(\tilde{P}_{1,t} \leq P_t^{obs} \middle| \mathcal{H}_t\right)}{\tilde{P}_{1,t}} < \left(-\frac{A}{A'} - \frac{B}{B'} + \frac{C}{C'}\right) \frac{\partial Prob\left(\tilde{P}_{1,t} \leq P_t^{obs} \middle| \mathcal{H}_t\right)}{\tilde{P}_{1,t}}$$

Using (12) and the fact that at $\tilde{P}^*{}_{1,t}$the derivative equals zero, we obtain

$$\frac{\partial \pi_{1,t}}{\partial \tilde{P}_{1,t}} = \left(\tilde{P}_{1,t} - TC_{i,t-1}\right) \frac{\partial Prob\left(\tilde{P}_{1,t} \leq P_t^{obs} \middle| \mathcal{H}_t\right)}{\tilde{P}_{1,t}} < 0$$

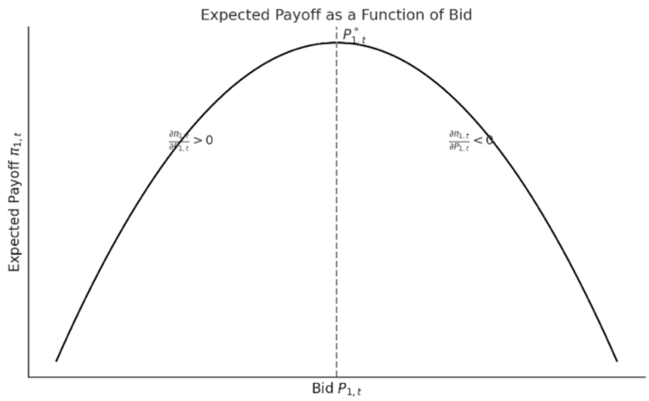
Similarly, when $\tilde{P}_{1,t} < \tilde{P}^*{}_{1,t}$, i.e., $\tilde{P}_{1,t} - TC_{1,t} < -\frac{A}{A'} - \frac{B}{B'} + \frac{C}{C'}$, we have

$$\left(\tilde{P}_{1,t} - TC_{1,t}\right) \frac{\partial Prob\left(\tilde{P}_{1,t} \leq P_t^{obs} \middle| \mathcal{H}_t\right)}{\tilde{P}_{1,t}} > \left(-\frac{A}{A'} - \frac{B}{B'} + \frac{C}{C'}\right) \frac{\partial Prob\left(\tilde{P}_{1,t} \leq P_t^{obs} \middle| \mathcal{H}_t\right)}{\tilde{P}_{1,t}}$$

which implies

$$\frac{\partial \pi_{1,t}}{\partial \tilde{P}_{1,t}} = \left(\tilde{P}_{1,t} - TC_{i,t}\right) \frac{\partial Prob\left(\tilde{P}_{1,t} \leq P_t^{obs} \middle| \mathcal{H}_t\right)}{\tilde{P}_{1,t}} > 0$$

Economically, when bidder 1's effective bid is below the optimal level $\tilde{P}^*{}_{1,t}$, a marginal increase in the bid raises the per-unit surplus while still maintaining a sufficiently high survival probability, so the expected profit increases, which is reflected in $\frac{\partial \pi_{1,t}}{\partial \tilde{P}_{1,t}} > 0$. Conversely, when the bid is above $\tilde{P}^*{}_{1,t}$, further increasing the bid sharply reduces the winning probability and therefore lowers expected profit, giving $\frac{\partial \pi_{1,t}}{\partial \tilde{P}_{1,t}} < 0$. Only when bidder 1 sets $\tilde{P}^*{}_{1,t}$ does the expected profit attain its maximum and the first-order condition is exactly satisfied.



By establishing that $\frac{\partial \pi_{i,t}}{\partial \tilde{P}_{1,t}} \neq 0$, whenever $\tilde{P}_{1,t} \neq \tilde{P}^*{}_{1,t}$, we show that bidder 1's expected profit is maximized only when he adopts the same optimal effective bid $\tilde{P}^*{}_{1,t}$ as all other bidders $i \neq 1$. In every round, given that all opponents follow their optimal strategies, any deviation by bidder 1—either upward or downward—strictly reduces his expected payoff. Therefore, the profile in which all bidders choose $\tilde{P}_{1,t} = \tilde{P}^*{}_{1,t}$ constitutes a steady-state equilibrium, i.e., $\frac{\partial \pi_{1,t}}{\partial \tilde{P}_{1,t}} = 0$ where each bidder's strategy is simultaneously optimal and locally stable.

5. Individual Rationality Constraints

5.1 Platform's Ex-Ante and Ex-Post Individual Rationality

In this mechanism, the platform sets a transaction price for a transportation task and dynamically adjusts the trading structure according to bidders' submitted prices. We examine whether the platform satisfies individual rationality—namely, whether participation guarantees non-negative profit—from both ex-ante and ex-post perspectives.

5.1.2 Ex-Post Individual Rationality of the Platform

In the final round t = T, the platform charges the resource provider the observed threshold price

$$P_T^{obs} = \frac{1}{n_T}\int_{i\in n_T} P_{i,T}\ di + \beta\Delta\left(\frac{\mathrm{DE_T}}{\mathrm{CA}}\right)$$

The payment the platform must make to the winning bidder i is $P_{i,T}^{*} + \beta\Delta\left(\frac{\mathrm{DE_T}}{\mathrm{CA}}\right)$. Let there be n remaining bidders in the final round (i=1,…,n), and let the winning bidder be i=1 with $p_{1,T}^{*} = \min(p_{1,T}^{*}, p_{2,T}^{*}, \dots, p_{n,T}^{*})$, The platform's ex-post profit is therefore

$$\pi_{platform} = \frac{1}{n}\sum_{i=1}^{n} P_{i,T}^{*} + \beta\Delta\left(\frac{\mathrm{DE_T}}{\mathrm{CA}}\right) - [P_{1,T}^{*} + \beta\Delta\left(\frac{\mathrm{DE_T}}{\mathrm{CA}}\right)] = \frac{1}{n}\sum_{i=1}^{n} P_{i,T}^{*} - P_{1,T}^{*} > 0$$

Thus, the platform obtains strictly positive profit and satisfies ex-post individual rationality.

Consider an extreme scenario where only two bidders (1 and 2) remain in round t, and assume $p_{1,t}^{*} < p_{2,t}^{*} < p_{1,t}^{*} + 2\beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right)$ The threshold price in this two-bidder case is $P_t^{obs} = \frac{1}{2}(p_{1,t}^{*} + p_{2,t}^{*}) + \beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right)$. Bidder 2 will exit in round t only if $p_{2,t}^{*} > \frac{1}{2}(p_{1,t}^{*} + p_{2,t}^{*}) + \beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right)$, which implies $p_{2,t}^{*} > p_{1,t}^{*} + 2\beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right)$. Since this inequality does not hold, bidder 2 stays in the mechanism. After the auction has reached the "last two bidders" stage, the platform closes entry, and no new bidders are accepted; exits are driven solely by the congestion adjustment term. Because bidder 2 has the higher nominal price, his effective price will exceed the threshold earlier, and he will be eliminated first. Bidder 1 eventually wins. The platform pays the winner $P_{1,T}^{*} + \beta\Delta\left(\frac{\mathrm{DE_T}}{\mathrm{CA}}\right)$, and charges the resource provider $P_T^{obs} = \frac{1}{2}(p_{1,T}^{*} + p_{2,T}^{*}) + \beta\Delta\left(\frac{\mathrm{DE_T}}{\mathrm{CA}}\right)$. The platform's profit is therefore

$$\pi_{platform} = \frac{1}{2}(p_{1,T}^{*} + p_{2,T}^{*}) + \beta\Delta\left(\frac{\mathrm{DE_T}}{\mathrm{CA}}\right) - [P_{1,T}^{*} + \beta\Delta\left(\frac{\mathrm{DE_T}}{\mathrm{CA}}\right)]$$

$$= \frac{1}{2}\left(p_{2,T}^{*} - p_{1,T}^{*}\right) > 0$$

Even under this extreme specification, ex-post individual rationality remains satisfied.

5.1.2 Ex-Ante Individual Rationality of the Platform

Before the mechanism begins, the platform's expected profit depends on the distribution of final-round bids. Suppose that n bidders remain in the final round (i=1,…,n), and let $p_{1,T}^{*} = \min(p_{1,T}^{*}, p_{2,T}^{*}, \dots, p_{n,T}^{*})$ be the winning bid. The platform's expected profit is

$$E\left(\pi_{platform}\right) = E\left[\frac{1}{n}\sum_{i=1}^{n} p_{i,T}^{*} + \beta\Delta\left(\frac{\mathrm{DE_T}}{\mathrm{CA}}\right)\right] - E\left[\min\left(p_{1,T}, \dots, p_{n,T}\right) + \beta\Delta\left(\frac{\mathrm{DE_T}}{\mathrm{CA}}\right)\right]$$

The congestion adjustment term cancels out, so

$$E\left(\pi_{platform}\right) = E\left(\frac{1}{n}\sum_{i=1}^{n} p_{i,T}^{*}\right) - E\left[\min\left(p_{1,T}, \dots, p_{n,T}\right)\right]$$

By Jensen's inequality and the basic properties of order statistics, the expected average bid is always weakly greater than the expected minimum bid, and strictly greater in all empirically relevant bidding environments. Therefore:

$$E\left(\frac{1}{n}\sum_{i=1}^{n} p_{i,T}^{*}\right) > E\left[\min\left(p_{1,T}, \dots, p_{n,T}\right)\right]$$

Thus,

$$E\left(\pi_{platform}\right) = E\left(p_{i,T}^{*}\right) - E\left[\min\left(p_{1,T}, \dots, p_{n,T}\right) > 0\right.$$

implying that the platform enjoys non-negative expected profit prior to the start of the mechanism. The mechanism therefore satisfies ex-ante individual rationality for the platform.

5.2 Individual Rationality of Bidders

5.2.1 Ex-Post Individual Rationality of Bidders

A bidder's optimal nominal bid in round t is:

$$P^*_{i,t} = TC_{i,t-1} - Prob(\mathcal{H}_t | \tilde{P}_{i,t} \leq P^{obs}_t) \cdot \left[\frac{\partial Prob(\mathcal{H}_t | \tilde{P}_{i,t} \leq P^{obs}_t)}{\partial \tilde{P}_{i,t}}\right]^{-1}$$

$$-Prob(\tilde{P}_{i,t} \leq P^{obs}_t) \cdot \left[\frac{\partial Prob(\tilde{P}_{i,t} \leq P^{obs}_t)}{\partial \tilde{P}_{i,t}}\right]^{-1}$$

$$+Prob(P^{obs}_t) \cdot \left[\frac{\partial Prob(P^{obs}_t)}{\partial \tilde{P}_{i,t}}\right]^{-1}$$

We use the simplified version： $P^*_{i,t} = TC_{i,t-1} - \left|\frac{A}{A'}\right| + \left|\frac{B}{B'}\right| - \left|\frac{C}{C'}\right|$, where A, B, C represent the survival probability and density terms derived in the previous section. The effective price actually determining payment is

$$\tilde{P}_{i,T} = P^*_{i,T} + \beta\Delta\left(\frac{\mathrm{DE_T}}{\mathrm{CA}}\right) = TC_{i,t} - \left|\frac{A}{A'}\right| + \left|\frac{B}{B'}\right| - \left|\frac{C}{C'}\right|$$

Thus, the bidder's net profit when winning is:

$$\pi_{winner} = -\left|\frac{A}{A'}\right| + \left|\frac{B}{B'}\right| - \left|\frac{C}{C'}\right|$$

Among the three terms, $\left|\frac{B}{B'}\right| = \left|\frac{F(P)}{f(P)}\right|$ is the only strictly positive one. It corresponds to the inverse hazard rate—a quantity that is always positive and typically grows rapidly when $\tilde{P}_{i,t}$ approaches the right tail of the distribution (since densities shrink faster than cumulative probabilities). This implies that the survival-probability term often dominates the other two density-driven terms. From a behavioral perspective, if the bidder finds that $P^*_{i,T} < TC_{i,t-1} \rightarrow \tilde{P}_{i,t} < TC_{i,t}$, the bidder earns negative profit and therefore exits the mechanism. Hence, rational bidders will only stay in the auction when their expected ex-post profit is non-negative. Thus, the mechanism satisfies ex-post individual rationality for bidders.

5.2.2 Ex-Ante Individual Rationality of Bidders

From an ex-ante perspective, suppose the mechanism consists of T rounds. Bidder i's probability of winning is:

$$Prob(win) = Prob(\tilde{P}_{i,1} \leq P^{obs}_1, \tilde{P}_{i,2} \leq P^{obs}_2, \dots, \tilde{P}_{i,T} \leq P^{obs}_T)$$

Because bids across rounds are conditionally independent and each threshold P_t^{obs} is determined jointly by all bidders' effective bids, bidder i's expected profit is:

$$E\left(\pi_{bidder_i}\right) = (\tilde{P}_{i,T} - TC_{i,T}) \prod_{t=1}^{T} Prob(\tilde{P}_{i,t} \le P_t^{obs})$$

Given the optimal bidding rule, bidders will only continue if $P_{i,T}^{*} \ge TC_{i,t-1}$, which ensures $E\left(\pi_{bidder_i}\right) \ge 0$, Therefore, bidders also satisfy ex-ante individual rationality when deciding whether to enter and remain in the mechanism.

6. Allocative Efficiency Analysis

To evaluate how the proposed dynamic multi-round bidding mechanism performs in terms of allocative efficiency, we conduct a structured analysis along three dimensions: (1) whether the mechanism selects the lowest-cost provider (static efficiency),(2) whether it induces truthful or near-truthful revelation of bidders' underlying costs (strategic simplicity), and (3) whether it reduces ex-post default risk once a winner is selected (ex-post robustness).

Throughout the discussion, we use the traditional one-shot second price sealed-bid auction as the benchmark. This comparison highlights how incorporating real-time congestion and demand fluctuations into the pricing rule allows the dynamic mechanism to outperform the static benchmark in more realistic environments. By contrasting these mechanisms along the three criteria above, we show how iterative cost updating and threshold-based survival screening produce distinct institutional advantages. Although the proposed mechanism permits off-platform bidders to enter dynamically during the auction, we abstract from this feature in the allocative-efficiency comparison to ensure a clean and comparable analysis with the static benchmark.

6.1 Dynamic Multi-Round Bidding Mechanism

Under the proposed mechanism, bidders are fully symmetric except for transportation costs, which differ across bidders through the exogenous distance parameter D_i. All bidders behave rationally and choose the optimal response in every round. Any bidder whose effective bid satisfies $\tilde{P}_{i,t} > P_t^{obs}$ is eliminated, and the optimal response takes the form $\tilde{P}^*{}_{i,t} = TC_{i,t} + markup_i$. Since the effective bid is proportional to the transportation cost, $\tilde{P}^*{}_{i,t} \propto TC_{i,t}$, bidders with higher costs tend to submit higher bids and are therefore more likely to be eliminated earlier. This dynamic screening implements a "low-cost survival" mechanism. The dominant component of the markup term is

$$\left| Prob\left(\tilde{P}_{i,t} \leq P_t^{obs}\right) \cdot \left[\frac{\partial Prob\left(\tilde{P}_{i,t} \leq P_t^{obs}\right)}{\partial \tilde{P}_{i,t}}\right]^{-1} \right| = \left|\frac{F(P)}{f(P)}\right|$$

where $F(P) = Prob\left(\tilde{P}_{i,t} \leq P_t^{obs}\right)$ is the survival probability and $f(P)$ is its density. Structurally, this term behaves like an inverse hazard rate. Because $f(P)$ is decreasing in $\tilde{P}_{i,t}$, survival becomes less likely at higher effective prices. When $\tilde{P}_{i,t}$ lies near the center of the distribution, $f(P)$ is relatively large; near the tail, $f(P)$decays more rapidly, driving the $markup_i$ sharply upward. This design implies three consequences. First, excessively high bids generate large markups, causing high-cost bidders to be screened out quickly. Second, for low-cost bidders, even a large markup leaves the effective bid below the threshold, preventing mistaken elimination. Third, as bids approach the threshold, the markup shrinks, causing the effective bid to converge to the transportation cost; survival decisions are therefore driven by cost rather than strategic noise.

Thus, a large markup arises only when costs lie at extreme values. In early rounds, markup-induced noise may cause the lowest-cost bidder not to submit the lowest bid; however, because the markup remains bounded relative to the cost differential, this does not lead to erroneous elimination, and the deviation diminishes over time. As high-cost bidders are progressively eliminated, the distribution of remaining bids becomes more concentrated, and the threshold $P_t^{obs} = \frac{1}{n_t}\int_{i\in n_t} P_{i,t}\ di + \beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right)$ converges toward the bids of low-cost bidders. Since the markup shrinks as the distribution narrows, the optimal strategy for low-cost bidders converges to their true transportation costs $TC_{i,t}$ }, generating asymptotic truthfulness. In summary, the mechanism avoids mis-eliminating low-cost bidders in early rounds and ultimately establishes a stable proportionality between effective bids and transportation costs. The lowest-cost bidder therefore survives until the final round and wins the auction, achieving cost-minimizing allocation.

The mechanism further enhances forward-looking bidding and outcome robustness through round-by-round information updates. After each round, the platform publishes the threshold price

$$P_t^{obs} = \frac{1}{n_t}\int_{i\in n_t} P_{i,t}\ di + \beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right)$$

which conveys updated information about current congestion. Bidders use this information to form expectations about next-period transportation costs and update their optimal nominal bids as

$$P_{i,t+1}^{*} = TC_{i,t} + markup_i$$

As the auction progresses, the number of remaining bidders decreases and the time interval between rounds shrinks. Consequently, $TC_{i,t}$ becomes an increasingly accurate predictor of $TC_{i,t+1}$, improving the effectiveness of cost-based bidding. Although some uncertainty remains between the last two rounds due to potential fluctuations in congestion, the mechanism offsets this by adjusting the final effective price:

$$\tilde{P}_{i,T} = P_{i,T} + \beta\left(\frac{\mathrm{DE_T - DE_{T-1}}}{\mathrm{CA}}\right),$$

The winner's final profit is therefore

$$\pi_i = P_{i,T}^{*} + \beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right) - \left[TC_{i,T-1} + \beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right)\right] = P_{i,T}^{*} - TC_{i,T-1} = markup_i$$

Because the winner's payoff moves in parallel with transportation costs, the mechanism eliminates incentives for ex-post default even under demand shocks. The residual demand risk in the final round is absorbed by the platform, whose exposure remains limited due to the short interval between the last two rounds.

6.2 Second-Price Sealed-Bid Auction

As a benchmark, consider the traditional second-price sealed-bid auction. Under this mechanism, all bidders submit a single, irrevocable bid at t=0, and the bidder with the lowest bid wins and pays the second-lowest bid. A well-known advantage of this mechanism is its strategic simplicity: the optimal strategy is to truthfully report one's cost estimate even without observing others' bids. However, two structural limitations arise in the present environment, which are Misallocation due to heterogeneous cost forecasts and Ex-post risk exposure borne entirely by bidders.

First, the winner is the bidder with the lowest forecasted transportation cost rather than the bidder with the lowest realized cost. Cost-minimizing allocation is guaranteed only if all bidders' forecasting rules for future transportation costs are identical and linear, with the same slope and intercept. In practice, forecast heterogeneity is pervasive; thus the mechanism can easily select a bidder whose realized cost is not minimal. By contrast, the dynamic multi-round mechanism leads bids to converge—through continuous information updating—to actual transportation costs in an asymptotic sense. This yields a more robust form of cost-minimizing allocation when cost forecasts differ across bidders.

Second, in a sealed-bid auction, bids reflect information at t=0, while performance occurs at t=T. Let $TC_{i,0}$ be the bidder's initial forecast and $TC_{i,T}$ the realized cost. In the presence of demand and congestion shocks, $\widehat{TC}_{i,T}$may deviate sharply from $TC_{i,T}$. If realized costs increase, a winner may face negative profits,$\pi_i = P_{i,0}^{*} - TC_{i,T} < 0$ leading to a heightened risk of ex-post default. Under the dynamic mechanism, the threshold price each round P_t^{obs}includes a congestion adjustment term, $\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right)$, which proportionally adjusts the effective payment in the final round. As a result, the winner's revenue co-moves with realized transportation costs, reducing ex-post default risk and shifting short-run congestion shocks away from bidders.

7. Buyer Cost Control and Price Stability

We compare the buyer-side payment implications of the dynamic multi-round threshold mechanism with those of the single-round second-price auction. In the dynamic auction, the platform publishes a threshold price each round that incorporates updated congestion information. This enables bidders to revise their bids gradually. When two bidders remain in the final round, the buyer pays the average of the two bids plus the final congestion adjustment. Because the markup term shrinks to zero as the process converges, these bids approach true transportation costs. Thus, the buyer's payment approximates the average realized cost of the remaining bidders. In contrast, under a second-price sealed-bid auction with two bidders, the buyer pays the second-lowest bid:

$$Max\left(P_{1,T}, P_{2,T}\right) = Max\left(\widehat{TC}_{1,T}, \widehat{TC}_{2,T}\right)$$

which depends on bidders' forecasted rather than realized costs. When forecasts are accurate, the dynamic mechanism generally yields a more attractive payment for both resource suppliers and buyers, because bids converge to actual rather than predicted transportation costs.

When more than two bidders remain in the final round, neither mechanism uniformly dominates in terms of buyer payments. Comparative performance depends on the number of bidders and the distribution of $\widehat{TC}_{i,T}$ relative to $TC_{i,T-1}$. Even under the theoretical assumption of accurate cost forecasting, heterogeneity and cost volatility make the relative advantage unstable. In practice, however, the idealized assumptions of the second-price auction are rarely met. Forecast errors and bidder heterogeneity generate substantial variance in bids that need not correspond to realized costs. By allowing round-by-round bid updating and embedding congestion adjustments directly into the pricing rule, the proposed dynamic mechanism better reflects real operational environments and provides the buyer with a more stable and predictable payment structure.

8. Data Simulation

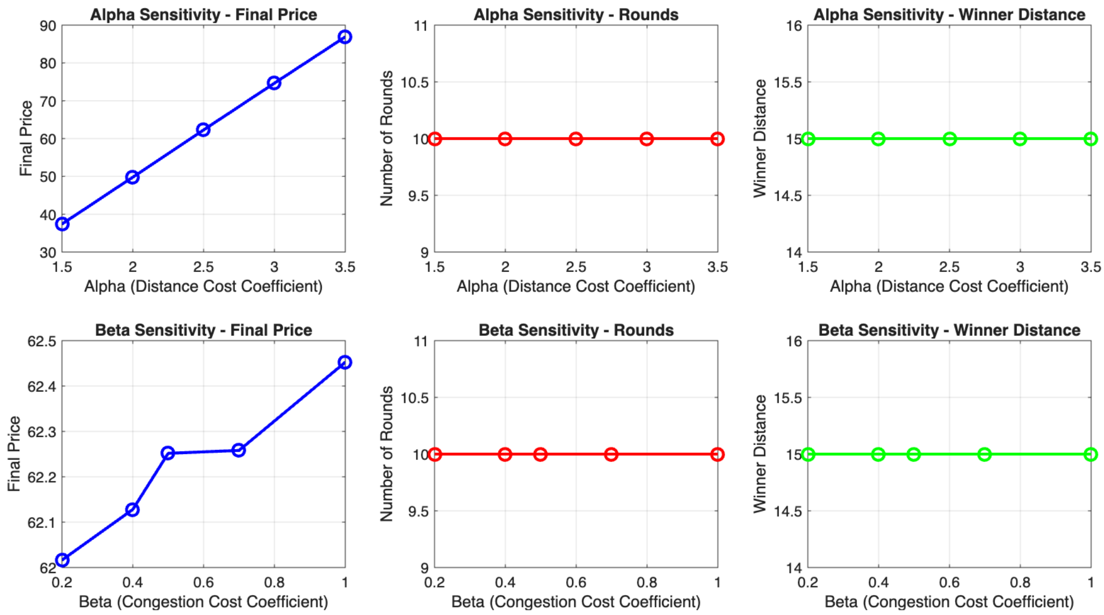
To evaluate the dynamic auction mechanism and illustrate its equilibrium properties, I simulate a stylized transport-matching environment in which heterogeneous carriers repeatedly bid for a single logistics task. The simulation mirrors the structure of the theoretical model: carriers face a time-varying cost process, update expectations across rounds, and survive only if their effective bids satisfy the endogenous threshold condition $\left(\tilde{P}_{i,t} \leq P_t^{obs}\right)$. The goal is not merely to visualize the bidding dynamics but to reveal how congestion adjustments, information updating, and the markup structure jointly drive convergence toward the cost-minimizing allocation.

To operationalize the dynamic auction mechanism and examine its equilibrium implications, I construct a simulation of a stylized transport-matching environment with 20 heterogeneous carriers. Each carrier i is assigned a fixed transportation distance D_i drawn from a uniform grid, which generates cost heterogeneity through the structural cost function of the model. In every round t, carriers compute their transport cost $TC_{i,t}$ based on distance and the current congestion level, form a nominal bid $P_{i,t}$ following the optimal-response condition derived in Section 3, and obtain an effective bid $\tilde{P}_{i,t} = P_{i,t} + \beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right)$, which incorporates the platform's congestion adjustment. Carriers remain active only if their effective bids satisfy the survival condition $\left(\tilde{P}_{i,t} \leq P_t^{obs}\right)$. The endogenous threshold price P_t^{obs} in each round is recorded as the average effective bid among survivors, inclusive of the congestion adjustment.

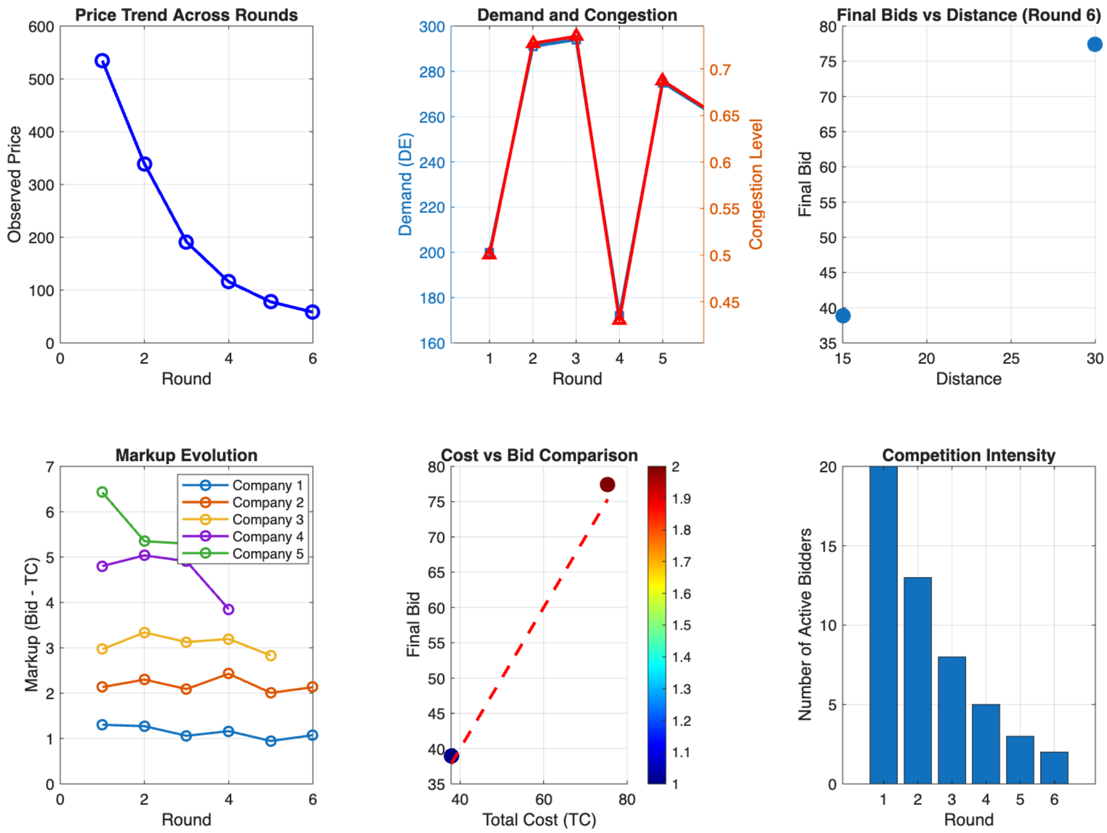
The auction proceeds for six rounds, with each round t summarized in a separate round-level table (see Appendix). All tables are stacked into a single panel dataset, allowing the evolution of bids, costs, congestion, and elimination to be tracked coherently across rounds .For every bidder i, the datasets report: (i) transportation distance D_i; (ii) the resulting cost estimate $TC_{i,t}$.; (iii) the nominal bid $P_{i,t}$; (iv) the effective bid $\tilde{P}_{i,t} = P_{i,t} + \beta\Delta\left(\frac{\mathrm{DE_t}}{\mathrm{CA}}\right)$; and (v) an indicator for whether the bid survives the threshold. Tracking these tables across rounds reveals how the threshold evolves, how many bidders remain active, and how congestion signals shape bidding behavior.

The trajectories across round1–6 display clear structural patterns predicted by the model. In the early rounds (1–2), substantial heterogeneity in distances and congestion shocks generates wide dispersion in $\tilde{P}_{i,t}$, producing a relatively high threshold because many carriers remain active. As the auction progresses (3–5), high-cost carriers increasingly fail the threshold condition and exit. Surviving bidders exhibit lower average costs and reduced bid variance, causing the threshold to drift downward toward the true costs of the most efficient carriers. By round 6, only the lowest-cost carriers remain, and their bids lie close to $TC_{i,t}$, demonstrating the model's asymptotic truthfulness property.

Figure 1 summarizes these dynamics. The threshold price declines monotonically across rounds (top-left panel), in parallel with the shrinking pool of active bidders (bottom-right), illustrating the mechanism's selection logic. The jointly plotted demand and congestion series (top-middle) show that the threshold updates react smoothly to exogenous demand fluctuations, embedding congestion information as designed. The markup trajectories (bottom-left) reveal that high-cost bidders apply large markups early on, while surviving bidders' markups shrink as the auction converges. The comparison of final bids and realized costs (bottom-middle) confirms that the mechanism ultimately selects the lowest-cost bidder.



To examine robustness, I implement parameter-sensitivity simulations in Matlab. The α-experiments (Figure 2, top row) show that increasing the distance-cost coefficient raises the final payment level but leaves the number of rounds and the identity of the winner unchanged, indicating that bidder elimination is governed by relative, not absolute, costs. The β-experiments (Figure 2, bottom row) demonstrate that stronger congestion adjustments influence payment trajectories but do not disturb bidder ordering or survival patterns. These results align with the theoretical roles of the parameters: α governs individual cost slopes; β governs the magnitude of congestion feedback without altering the underlying ranking of marginal costs.



Taken together, the six round-by-round datasets and the two figures provide consistent numerical evidence for the core properties of the dynamic threshold auction. The mechanism reliably eliminates high-cost carriers, induces bids that increasingly approximate true costs, and synchronizes the final payment with realized congestion conditions. The simulation demonstrates that the mechanism achieves cost-minimizing allocation while remaining stable under parameter perturbations, reinforcing its allocative efficiency, incentive compatibility, and practical robustness.

Appendix. Round-by-Round Auction Simulation

Each subpanel reports the surviving bidders in round t, their nominal bid $P_{i,t}$, the corresponding transport cost $TC_{i,t}$, and the indicator variable DropFlag, which equals TRUE if bidder i is eliminated under the threshold rule $\tilde{P}_{i,t} > P_t^{obs}$.

Across rounds, the tables clearly document the stepwise contraction of the bidder set: high-cost carriers are filtered out first, while low-cost bidders persist into later rounds. The evolution from t1 to t6 also shows a progressive reduction in bid dispersion and a tightening alignment between $P_{i,t}$ and $TC_{i,t}$, consistent with the model's prediction of asymptotic truthfulness. These snapshots provide a transparent, round-by-round representation of the mechanism's elimination logic and directly correspond to the dynamic patterns visualized in Figures 1 and 2 in the main text.

Figure A1. Round-by-Round Auction Simulation

Round 1

Firm	BidPrice	TC	DropFlag
firm1	39.05339	37.75	FALSE
firm2	77.38341	75.25	FALSE
firm3	115.7196	112.75	FALSE
firm4	155.0511	150.25	FALSE
firm5	194.1763	187.75	FALSE
firm6	231.9794	225.25	FALSE
firm7	270.6475	262.75	FALSE
firm8	307.7701	300.25	FALSE
firm9	346.8303	337.75	FALSE
firm10	387.5568	375.25	FALSE
firm11	425.5899	412.75	FALSE
firm12	465.0743	450.25	FALSE
firm13	504.0306	487.75	FALSE
firm14	541.9916	525.25	TRUE
firm15	577.277	562.75	TRUE
firm16	620.0951	600.25	TRUE
firm17	654.5533	637.75	TRUE
firm18	1357.725	675.25	TRUE
firm19	1433.16	712.75	TRUE
firm20	1504.372	750.25	TRUE

Round 2

Firm	BidPrice	TC	DropFlag
firm1	39.1360518	37.86375	FALSE
firm2	77.662821	75.36375	FALSE
firm3	116.2014	112.8638	FALSE
firm4	155.400972	150.3638	FALSE
firm5	193.213693	187.8638	FALSE
firm6	231.535248	225.3638	FALSE
firm7	269.569241	262.8638	FALSE
firm8	308.27821	300.3638	FALSE
firm9	348.263647	337.8638	TRUE
firm10	388.142519	375.3638	TRUE
firm11	426.36921	412.8638	TRUE
firm12	464.794049	450.3638	TRUE
firm13	980.227609	487.8638	TRUE

Round 3

Firm	BidPrice	TC	DropFlag
firm1	38.8782739	37.715	FALSE
firm2	77.6419946	75.215	FALSE
firm3	115.908773	112.72	FALSE
firm4	154.066035	150.22	TRUE
firm5	194.081612	187.72	TRUE

Round 4

Firm	BidPrice	TC	DropFlag
firm1	38.9277774	37.8675	FALSE
firm2	77.4582598	75.3675	FALSE
firm3	115.994477	112.868	FALSE
firm4	155.27473	150.368	FALSE
firm5	193.162687	187.868	FALSE
firm6	231.806259	225.368	TRUE
firm7	271.21481	262.868	TRUE
firm8	307.977425	300.368	TRUE

Round 5

Firm	BidPrice	TC	DropFlag
firm1	38.789986	37.8438	FALSE
firm2	77.3506175	75.3438	FALSE
firm3	115.675583	112.844	TRUE

Round 6

Firm	BidPrice	TC	DropFlag
firm1	38.9024251	37.8288	FALSE
firm2	77.4599556	75.3288	TRUE