Fair Representations by Compression

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Abstract

Organizations that collect and sell data face increasing scrutiny for the discriminatory use of data. We propose a novel unsupervised approach to transform data into a compressed binary representation independent of sensitive attributes. We show that in an information bottleneck framework, a parsimonious representation should filter out information related to sensitive attributes if they are provided directly to the decoder. Empirical results show that the proposed method, **FBC**, achieves state-of-the-art accuracy-fairness trade-off. Explicit control of the entropy of the representation bit stream allows the user to move smoothly and simultaneously along both rate-distortion and rate-fairness curves.

1 Introduction

A growing body of evidence has questioned the fairness of machine learning algorithms across a wide range of applications, including judicial decisions (ProPublica 2016), face recognition (Buolamwini and Gebru 2018), degree completion (Gardner, Brooks, and Baker 2019) or medical treatment (Pfohl et al. 2019). Of particular concerns are potential discriminatory uses of data on the basis of racial or ethnic origin, political opinion, religion, or gender.

Therefore, organizations that collect and sell data are increasingly liable if future downstream uses of the data are biased against protected demographic groups. One of their challenges is to anticipate and control how the data will be processed by downstream users. Unsupervised fair representation learning approaches (Madras et al. (2018), Zemel et al. (2013), Gitiaux and Rangwala (2020), Moyer et al. (2018)) offers a flexible fairness solution to this challenge. A typical architecture in fair representation learning includes an encoder that maps the data into a representation and a decoder that reconstructs the data from its representation. The objective of the architecture is to extract from a data X the underlying latent factors Z that correlate with unobserved and potentially diverse task labels, while remaining independent of sensitive factors S.

This paper asks whether an encoder that filters out information redundancies could generate fair representations. Intuitively, if sensitive attributes S are direct inputs to the decoder, an encoder that aims for conciseness would not waste code length to encode information related to S in the latent factors Z. We show that in an information bottleneck framework (Tishby, Pereira, and Bialek 2000), this intuition is theoretically founded: constraining the information flowing from the data X to the representation Z forces the encoder to control the dependencies between sensitive attributes S and representations Z. *It is sufficient to constraint the mutual information $I(Z, X)$ between Z and X in order to minimize the mutual information $I(Z, S)$ between Z and S.*

Therefore, instead of directly penalizing $I(Z, S)$, we recast fair representation learning as a rate distortion problem that controls explicitly the bit rate $I(Z, X)$ encoded in the latent factors Z. We model the representation Z as a binary bit stream, which allows us to monitor the bit rate more effectively than floating point representations that may maintain redundant bit patterns. We estimate the entropy of the code Z with an auxiliary auto-regressive network that predicts each bit in the latent code Z conditional on previous bits in the code. One advantage of the method is that the auxiliary network collaborates with the encoder to minimize the cross-entropy of the code.

Empirically, we demonstrate that the resulting method, Fairness by Binary Compression (henceforth, **FBC**) is competitive with state-of-the art methods in fair representation learning. Our contributions are as follows:

1. We show that controlling for the mutual information $I(Z, X)$ is an effective way to remove dependencies between sensitive attributes and latent factors Z, while preserving in Z, the information useful for downstream tasks.

2. We find that compressing the data into a binary code as in **FBC** generates a better accuracy-fairness trade-off than limiting the information channel capacity by adding noise (as in variants of β-**VAE**, (Higgins et al. 2016)).

3. We show that increasing the value of the coefficient on the bit rate constraint $I(Z, X)$ in our information bottleneck framework allows to move smoothly along both rate-distortion and rate-fairness curves.

Related work. The machine learning literature increasingly explores how algorithms can adversely impact protected demographic groups (e.g individuals self-identified as Female or African-American) (see Chouldechova and Roth (2018) for a review). Research questions revolve around how to define fairness (Dwork et al. (2012)), how to enforce fairness in standard classification algorithms (e.g. Agarwal et al. (2018), Kim, Reingold, and Rothblum (2018), Kearns et al. (2018)) or audit a black box classifier for its fairness (e.g Feldman et al. (2015), Gitiaux and Rangwala (2019)).

This paper relates to recent efforts towards transforming data into fair and general purpose representations that are not tailored to a pre-specified specific downstream task. Many contributions use a supervised setting where the downstream task label is known while training the encoder-decoder architecture (e.g Madras et al. (2018), Edwards and Storkey (2015), Moyer et al. (2018) Song et al. (2018) or Jaiswal et al. (2019)). However, Zemel et al. (2013), Gitiaux and Rangwala (2020) and Locatello et al. (2019) argue that in practice, an organization that collects data cannot anticipate what the downstream use of the data will be. In this unsupervised setting, the literature has focused on penalizing approximations of the mutual information between representations and sensitive attributes: maximum mean discrepancy penalty (Gretton et al. (2012)) for deterministic (Li, Swersky, and Zemel (2014)) or variational (Louizos et al. (2015)) autoencoders (see Table 1); cross-entropy of an adversarial auditor that predicts sensitive attributes from the representations (Madras et al. (2018), Edwards and Storkey (2015), Zhang, Lemoine, and Mitchell (2018) or Xu et al. (2018)).

Our approach contrasts with existing work since it does not control directly for the leakage between sensitive attributes and representations. **FBC** obtains fair representations only by controlling its bit rate. In a supervised setting, Jaiswal et al. (2019) show that nuisance factors can be removed from a representation by over-compressing it. We extend their insights to unsupervised settings and show the superiority of bit stream representations over noisy ones to remove nuisance factors. Our insights could offer an effective alternative to methods that learn representations invariant to nuisance factors (e.g. (Achille and Soatto 2017), (Jaiswal et al. 2020), (Jaiswal et al. 2018)).

Our paper borrows soft-quantization techniques when backpropagating through the model (Agustsson et al. 2017) and hard quantization techniques during the forward pass (Mentzer et al. 2018). We find that in our fair representation setting, explicit control of the bit rate of the representation leads to better accuracy-fairness trade-off than floating point counterpart. We estimate the entropy of the code as in Mentzer et al. (2018) by computing the distribution $P(Z)$ of Z as an auto-regressive product of conditional distributions, and by modeling the auto-regressive structure with a PixelCNN architecture (Oord, Kalchbrenner, and Kavukcuoglu (2016), Van den Oord et al. (2016)).

2 Fair Information Bottleneck

Consider a population of individuals represented by features $X \in \mathcal{X} \subset [0,1]^{d_x}$ and sensitive attributes in $S \in \mathcal{S} \subset \{0,1\}^{d_s}$, where d_x is the dimension of the feature space and d_s is the dimension of the sensitive attributes space. In this paper, we do not restrict ourselves to binary sensitive attributes and we allow $d_s > 1$. The objective of fair representation learning is to map the features space $\mathcal{X}$ into a $m-$dimensional representation space $\mathcal{Z} \subset [0,1]^m$, such that (i) Z maximizes the information related to X, but (ii) minimizes the information related to sensitive attributes S. We can express this as

$$\max_{Z} I(X, \{Z, S\}) - \gamma I(Z, S) \tag{1}$$

where $I(X, S)$ and $I(X, \{Z, S\})$ denote the mutual information between Z and S and between X and (Z, S), respectively; and $\gamma \geq 0$ controls the fairness penalty $I(Z, S)$.

Existing methods focus on solving directly the problem (1) by approximating the mutual information $I(Z, S)$ between Z and S via the cross-entropy of an adversarial auditor that predicts S from Z (Madras et al. (2018), Edwards and Storkey (2015), Gitiaux and Rangwala (2020)) or via the maximum mean discrepancy between Z and S (Louizos et al. (2015)).

In this paper, we instead reduce the fair representation learning program (1) to an information bottleneck problem that consists of encoding X into a parsimonious code Z, while ensuring that this code Z along with a side channel S allows a good reconstruction of X. The mutual information between X and S can be written as

$$\begin{aligned} I(Z, S) &\overset{(a)}{=} I(Z, \{X, S\}) - I(Z, X|S) \\ &\overset{(b)}{=} I(Z, X) + I(Z, S|X) - I(Z, X|S) \\ &\overset{(c)}{=} I(Z, X) - I(Z, X|S) \\ &\overset{(d)}{=} I(Z, X) - I(X, \{Z, S\}) + I(X, S). \end{aligned}$$

where (a), (b) and (d) use the chain rule for mutual information; and, (c) uses the fact that Z is only encoded from X, so $H(Z|X, S) = H(Z|X)$ and $I(Z, S|X) = H(Z|X) - H(Z|X, S) = 0$. Since the mutual information between X and S does not depend on the code Z, the fair representation learning (1) is equivalent to the following fair information bottleneck:

$$\max_{Z} (1 + \gamma) I(X, \{Z, S\}) - \gamma I(Z, X). \tag{2}$$

Intuitively, compressing information about X forces the code Z to avoid information redundancy, particularly redundancy related to the sensitive attribute S, since

Methods	Fairness by controlling: $I(Z,S)$	$I(Z,X)$	Examples
Adversarial	Minimizing auditor's cross-entropy	✗	Madras et al. (2018), Edwards and Storkey (2015), Creager et al. (2019)
MMD	Mimizing maximum mean discrepancy	✗	Li, Swersky, and Zemel (2014), Louizos et al. (2015)
$\beta-$ **VAE**	✗	Noisy Z	Higgins et al. (2016), This paper
FBC	✗	Binary Z	This paper

Table 1: Methods in unsupervised fair representation learning organized by whether the fairness properties of the learned representations is obtained by minimizing the mutual information between sensitive attributes S and representations Z; or by minimizing the mutual information between data X and representations Z; and whether Z is modelled as a binary bit stream or is convolved with Gaussian noise.

the decoder has direct access to S. Note that there is no explicit constraint in (2) to impose independence between Z and S.

If the representation Z is obtained by a deterministic function of the data X, once X is known, Z is known and $H(Z|X) = 0$. Therefore, the mutual information $I(Z, X)$ is equal to the entropy $H(Z)$ of the representation Z. Since the entropy of the data X does not depend on the representation Z, we can replace $I(X, \{Z, S\}) = H(X) - H(X|Z, S)$ by $E_{z,s,x} \log(P(x|z, s)$ in the information bottleneck (2) and solve for:

$$\min_{Z} E_{x,z,s}[-\log(P(X|Z, S)] + \beta H(Z), \qquad (3)$$

where $\beta = \gamma/(\gamma + 1)$. Therefore, the fair representation problem, in its information bottleneck interpretation, can be recast as a rate-distortion trade-off. A lossy compression of the data into a representation Z forces the independence between sensitive attribute and representation but increases the distortion cost measured by the negative log-likelihood of the reconstructed data $E_{x,z,s}[-\log(P(X|Z, S)]$. The parameter β in equation (3) controls the competitive objectives of low distortion and fairness-by-compression: the larger β, the fewer the dependencies between Z and S.

3 Proposed Method

There are two avenues to control for $I(Z, X)$ in the information bottleneck (2) (see Figure 1): (i) adding noise to Z to control the capacity of the information channel between X and Z; or, (ii) storing Z as a bit stream whose entropy is explicitly controlled.

The noisy avenue (i) is a variant of variational autoencoders, so called $\beta-$**VAE** (Higgins et al. 2016), that models the posterior distribution $P(Z|X)$ of Z as Gaussian distributions (see Figure 1a). The channel capacity and thus the mutual information between X and Z is constrained by minimizing the Kullback divergence between these posterior distributions and an isotropic Gaussian prior (Braithwaite and Kleijn (2018)). In the context of fair representation learning, (Louizos et al. 2015) and (Creager et al. 2019) use variants of $\beta-$**VAE**, but do not focus on how limiting the channel capacity $I(Z, X)$ could lead to fair representations. Instead, they add further constraints on $I(Z, S)$.

We implement the binary avenue with a method – **FBC** (see Figure 1b) – that consists of an encoder $F : \mathcal{X} \to \mathbb{R}^m$, a binarizer $B : \mathbb{R}^m \to \{0, 1\}^m$ and a decoder $G : \{0, 1\}^m \times \mathcal{S} \to \mathcal{X}$. The encoder F maps each data point x into a latent variable $e = F(x)$. The binarizer B binarizes the latent variable e into a bit stream z of length m. The decoder G reconstructs a data point $\hat{x} = G(z, s)$ from the bitstream z and the sensitive attribute s. We model encoder and decoder as neural networks whose architecture varies with the type of data at hand.

The binarization layer controls explicitly the bit allowance of the learned representation and thus forces the encoder to strip redundancies – including sensitive attributes. Binarization is a two step process: (i) mapping the latent variable e into $[0, 1]^m$; (ii) converting real values into 0-1 bit. We achieve the first step by applying a neural network layer with an activation function $\overline{z} = (\tanh(e) + 1)/2$. We achieve the second step by rounding $\overline{z}$ to the closest integer 0 or 1. One issue with this approach is that the resulting binarizer B is not differentiable with respect to $\overline{z}$. To sidestep the issue, we follow Mentzer et al. (2018) or Theis et al. (2017) and rely on soft binarization during backward passes through the neural network. Formally, during a backward pass we replace z by a soft-binary variable $\dot{z}$:

$$\dot{z} = \frac{exp(-\sigma||\overline{z} - 1||_2^2)}{exp(-\sigma||\overline{z} - 1||_2^2) + exp(-\sigma||\overline{z}||_2^2)},$$

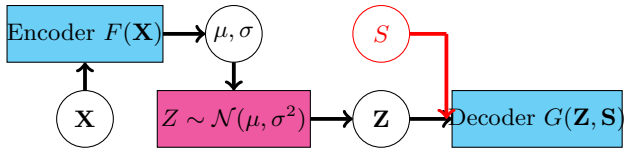
where σ is an hyperparameter that controls the soft-binarization. During the forward pass, we use the binary variable z instead of its soft-binary counterpart $\dot{z}$ to control the bitrate of the binary representation Z [1].

To estimate the entropy $H(z)$, we factorize the distribution $P(z)$ over $\{0, 1\}^m$ by writing $z = (z_1, ..., z_m)$ (Mentzer et al. (2018)) and by computing $P(z)$ as the product of conditional distributions:

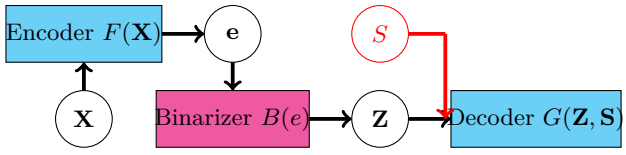
$$P(z) = \prod_{i=1}^{m} p(z_i|z_{i-1}, z_{i-2}, ..., z_1) \triangleq \prod_{i=1}^{m} p(z_i|z_{.<i}), \quad (4)$$

where $z_{.<i} = (z_1, z_2, ..., z_{i-1})$. The order of the bits z_1, ..., z_m is arbitrary, but consistent across all data

[1] In Pytorch, the binarizer returns $(z - \dot{z}).detach() + \dot{z}$.



(a) $\beta-$**VAE**



(b) **FBC**

Figure 1: Unsupervised methods to obtain fair representations z by compression. Variables are: features $\mathbf{X}$; sensitive attribute $\mathbf{S}$; representation $\mathbf{Z}$. $\beta-$**VAE** generates noisy representations with mean μ and variance σ^2. **FBC** generates binary representations.

points. We model P with a neural network Q that predicts the value of each bit z_i given the previous values $z_{i-1}, z_{i-2}, ..., z_1$. With the factorization (4), the entropy $H(z)$ is given by

$$H(z) = E_z \left[\sum_{i=1}^{m} -\log(Q(z_i|z_{.<i})) \right] - KL(P||Q) \leq CE(P, Q), \quad (5)$$

where $CE(P, Q)$ is the cross entropy between P and Q. Therefore, minimizing the cross-entropy loss of the neural network Q minimizes an upper bound of the entropy of the code z. The encoder F and the entropy estimator Q cooperate. The lower the cross-entropy of Q is, the lower is the estimate of the bit rate $H(z)$. Therefore, the encoder has incentives to make the bit stream easy to predict for the neural network Q. Designing a powerful predictor for the bit stream z does not necessary complicate the loss landscape, unlike what could happen with adversarial methods (Berard et al. (2019)).

Since the prediction of Q for the i^{th} bit depends on the values of the previous bits z_{i-1}, ..., z_1, the factorization of $P(z)$ imposes a causality relation, where the $(i+1)^{th}$, ..., m^{th} bits should not influence the prediction for z_i. We could enforce this causality constraint by using an iterative method that would first compute $P(z_2|z_1)$, then $P(z_3|z_1, z_2)$,..., and lastly, $P(z_m|z_1, ..., z_{m-1})$. However, it will require $O(m)$ operations that cannot be parallelized. Instead, we follow Mentzer et al. (2018) and enforce the causality constraint by using an architecture for Q similar to PixelCNN (Van den Oord et al. (2016), Oord, Kalchbrenner, and Kavukcuoglu (2016)). We model z as a $2D$ $\sqrt{m} \times \sqrt{m}$ matrix and convolve it with one-zero masks, which are equal to one only from their leftmost/top position to the center of the filter. Intuitively, the i^{th} output from this convolution depends only on the bits located to the left and above the bit z_i. The advantage of using a PixelCNN structure, as noted in Mentzer et al. (2018), is to enforce the causality constraint and compute $P(z_i|z_{.<i})$ for all bits z_i in parallel, instead of computing $P(z_i|z_{.<i})$ sequentially from $i = 1$ to $i = m$.

4 Experiments

4.1 Comparative Methods

The objective of this experimental section is to demonstrate that Fairness by Binary Compression – **FBC** – can achieve state-of-the art performance compared to four benchmarks in fair representations learning: β-**VAE**, **Adv**, **MMD** and **VFAE**.

(i) β-**VAE** (Higgins et al. (2016)) solves the information bottleneck by variational inference and generates fair representations by adding Gaussian noise which upper-bounds the mutual information between Z and X;

(ii) **MMD** (Li, Swersky, and Zemel (2014)) uses a deterministic auto-encoder and enforces fairness by minimizing the maximum mean discrepancy ((Gretton et al. 2012)) between the distribution of latent factors Z conditioned on sensitive attributes S;

(iii) **VFAE** (Louizos et al. (2015)) extends β-**VAE** by adding a maximum mean discrepancy penalty;

(iv) **Adv** (Edwards and Storkey (2015)) uses a deterministic auto-encoder as for **MMD**, but enforces the fairness constraint by maximizing the cross-entropy of an adversarial auditor that predicts sensitive attributes S from representations Z.

Although **FBC** shares the deterministic nature of **Adv** and **MMD**, it is more closely related to $\beta-$**VAE**, since $\beta-$**VAE** obtains fairness without explicit constraint on the mutual information of $I(Z, S)$. The main difference between our approach **FBC** and $\beta-$**VAE** is that **FBC** controls the entropy of a binary coding of the data, while $\beta-$**VAE** generates noisy representations and approximates the mutual information $I(Z, X)$ with the Kullback divergence between $Q(z|x)$ and a Gaussian prior $P(z)$. Note that the use of a vanilla $\beta-$**VAE** in a fairness context is novel: only its cousin **VFAE** with an additional MMD penalty has been proposed as a fair representation method.

Both **FBC** and $\beta-$**VAE** attempt to obtain fairness by controlling $I(Z, X)$. However, $\beta-$**VAE** assumes further that the prior distribution of the representation is an isotropic Gaussian. **FBC** does not require such a strong assumption and could still work well even if the data is not generated from a factorized distribution. $\beta-$**VAE** is meant to compress and factorize. The main result from this paper is that compression is sufficient to learn fair representations and thus, disentanglement might be too restrictive. For problems where factorization could be hard to achieve in an unsupervised setting

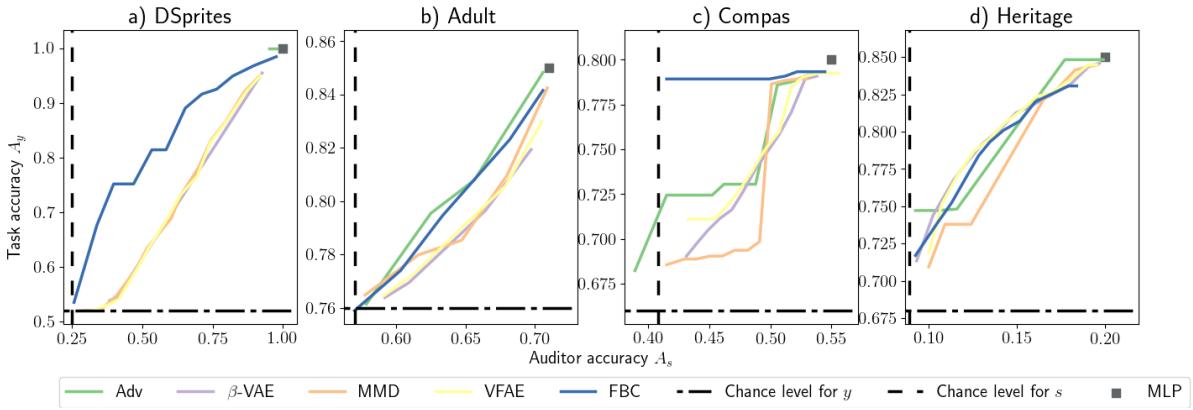


Figure 2: Pareto Front for fair representation learning approaches for DSprites and three benchmark datasets. This shows an accuracy-fairness trade-off by comparing the accuracy A_s of auditors that predict sensitive attributes S from representations Z to the accuracy of predicting a task label Y from Z. The dashed horizontal line represents the chance level of predicting Y. The dashed vertical line represents the chance level of predicting S. Ranges of $x-$ and $y-$ axes varies across datasets.

(Locatello et al. 2018), we would expect **FBC** to outperform $\beta-$**VAE**.

4.2 Experimental Protocol

The overall experimental procedure consists of:

(i) Training an encoder-decoder architecture (F, B, G) along with an estimator of the code entropy Q;

(ii) Freezing its parameters;

(iii) Training an auditing network $Aud : \mathcal{Z} \to \mathcal{S}$ that predicts sensitive attributes from Z.

(iv) Training a task network $T : \mathcal{Z} \to \mathcal{Y}$ that predicts a task label Y from Z.

The encoder-decoder does not access the task labels during training: our representation learning approach is unsupervised with respect to downstream task labels. Datasets are split into a training set used to trained the encoder-decoder architecture; two test sets, one to train both task and auditing networks on samples not seen by the encoder-decoder; one to evaluate their respective performances.

Pareto fronts. To compare systematically performances across methods, we rely on Pareto fronts that estimates the maximum information that can be attained by a method for a given level of fairness. We approximate information content as the accuracy A_y of the task network T when predicting the downstream label Y. The larger A_y, the more useful is the learned representation for downstream task labels.

We measure how much a representation Z leaks information related to sensitive attributes S by the best accuracy A_s among a set of auditing classifiers $Aud : \mathcal{Z} \to \mathcal{S}$ that predict S from Z. The intuition is that if the distributions $p(Z|S = s)$ of Z conditioned on S do not depend on s, the accuracy of any classifier predicting S from Z would remain near chance level. In the binary case $\mathcal{S} = \{0, 1\}$, comparing A_s to chance level accuracy is a statistical test of independence with good theoretical properties (Lopez-Paz and Oquab (2016)). If the sensitive classes are furthermore balanced ($P(S = 0) = P(S = 1)$) and the task labels are binary ($\mathcal{Y} = \{0, 1\}$), A_s estimates the worst demographic disparity that can be obtained by a downstream task classifier T that uses Z as an input (Gitiaux and Rangwala (2020)). In the general case $S = \{0, 1\}^{d_s}$, the lower A_s compared to chance level, the more independent Z and S are.

Rate distortion curves. To demonstrate further our theoretical insights from section 2, we study both rate-distortion and rate-fairness curves of compressing methods **FBC** and $\beta-$**VAE**.

The rate-distortion function $RD(D)$ of an encoder-decoder is measured as the minimum bitrate (in nats) necessary for the distortion $E_{x,z,s}[-\log(p(X|Z, S)]$ to be less than D (Tishby, Pereira, and Bialek (2000)):

$$RD(D) = \min I(Z, X) \text{ s.t. } E_{x,z,s}[-\log(p(X|Z, S)] \leq D. \tag{6}$$

We introduce a new concept, rate-fairness function $RF(\Delta)$, and define it as the maximum bit rate allowed for the accuracy A_s of the auditing classifier to remain less than Δ

$$RF(\Delta) = \max I(Z, X) \text{ s.t. } A_s \leq \Delta. \tag{7}$$

The rate-fairness function captures the maximum information Z can contain while keeping A_s under a given threshold. To obtain both rate-distortion and rate-fairness curves for either our binary compression –**FBC** – or variational –β-**VAE** and **VFAE** – approaches , we vary the value of the parameter β controlling the rate-distortion trade-off and for each value of β, we train the model 50 times with different seeds. For our binary compression method, **FBC**, the bit rate is approximated

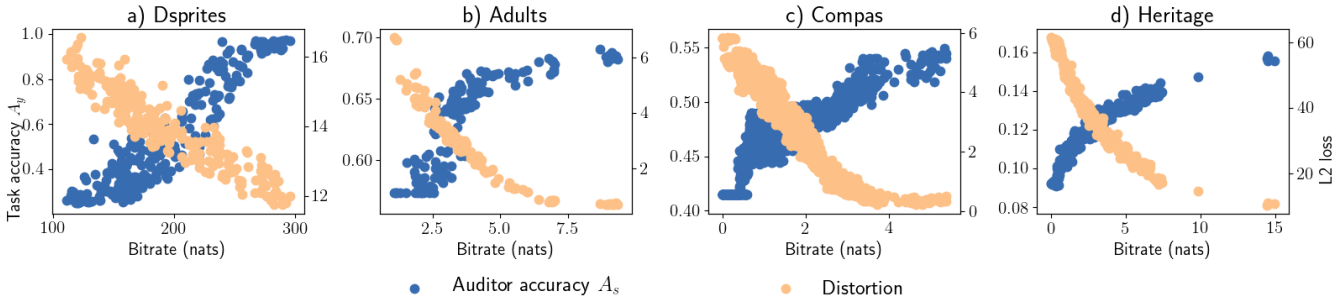


Figure 3: Rate distortion/fairness curves. Each dot corresponds to one simulation of **FBC**. Distortion is measured as the $l2$ loss between reconstructed and observed data.

by the cross-entropy of the entropy estimator Q in (5); for variational-based methods, the bit rate is approximated by the Kullback divergence between $Q(z|x)$ and a Gaussian prior. In both cases, the approximation is an upper bound to the true bit-rate (in nats) of Z. We estimate the distortion generated by the encoder-decoder procedure as the l_2 loss between reconstructed data $\widehat{X} = G(B(F(X)))$ and observed data X.

Robustness to Fairness Metrics. The fair information bottleneck (1) aims at controlling the flow of information between Z and S. (McNamara, Ong, and Williamson 2017) show that minimizing $I(Z, S)$ minimizes an upper bound of the demographic disparity $\Delta(T)$ of a task network T that predicts a binary task label Y from Z, where demographic parity $\Delta(T)$ is defined as

$$\Delta(T) = \sum_{s \in \mathcal{S}} |P(T(x) = 1|S = s) - P(T(x) = 1|S \neq s)|. \tag{8}$$

Moreover, the fair information bottleneck (1) is solved without a prior knowledge of specific downstream task labels Y. Therefore, (1) is not designed to control for fairness criteria that rely on labels Y (e.g. equality of odds or opportunites, (Hardt, Price, and Srebro 2016)) or on a specific classifier (e.g. individual fairness, (Dwork et al. 2012)), unless downstream task labels are orthogonal to sensitive attributes conditional on features X: $Y \perp S|X$. In practice, we explore whether empirically FBC can generate representations that exhibit for a given task network T, low differences in false positive rates $\Delta FPR(T)$ with

$$\begin{aligned} \Delta FPR(T) \triangleq \sum_{s \in \mathcal{S}} &|P(T(x) = 1|Y = 0, S = s) \\ &- P(T(x) = 1|Y = 0, S \neq s)| \end{aligned} \tag{9}$$

4.3 Datasets

First, we apply our experimental protocol to a synthetic dataset – DSprites Unfair, (Matthey et al. 2017) – that contains 64 by 64 black and white images of various shapes (heart, square, circle). Images in the DSprites dataset are constructed from six independent factors of variation: color (black or white); shape (square, heart, ellipse), scales (6 values), orientation (40 angles in $[0, 2\pi]$); x- and y- positions (32 values each). We modify the sampling to generate a source of potential unfairness and use as sensitive attribute a variable that encodes the quadrant of the circle the orientation angle belongs to.

Then, we extend our experimental protocol to three benchmark datasets in fair machine learning: **Adults**, **Compas** and **Heritage**. The Adults dataset [2] contains $49K$ individuals and includes information on 10 features related to professional occupation, education attainment, race, capital gains, hours worked and marital status. Sensitive attributes is made of 10 categories that intersect gender and race to which individuals self-identify to. The downstream task label Y correspond to whether an individual earns more than $50K$ per year.

The Compas data [3] contains $7K$ individuals with information related to their criminal history, misdemeanors, gender, age and race. Sensitive attributes intersect self-reported race and gender and result in four categories. The downstream task label Y assesses whether an individual presents a high risk of recidivism.

The Health Heritage dataset [4] contains $220K$ individuals with 66 features related to age, clinical diagnoses and procedure, lab results, drug prescriptions and claims payment aggregated over 3 years. Sensitive attributes are 18 categories that intersect the gender which individuals self-identify to and their reported age. The downstream task label Y relates to whether an individual has a positive Charlson comorbidity Index.

5 Results and Discussion

5.1 Pareto fronts

Figure 2 shows the Pareto fronts across five comparative methods for the DSprites and real-world datasets, respectively. Across all dataset, the higher and more

[2] https://archive.ics.uci.edu/ml/datasets/adult
[3] https://github.com/propublica/compas-analysis/
[4] https://foreverdata.org/1015/index.html

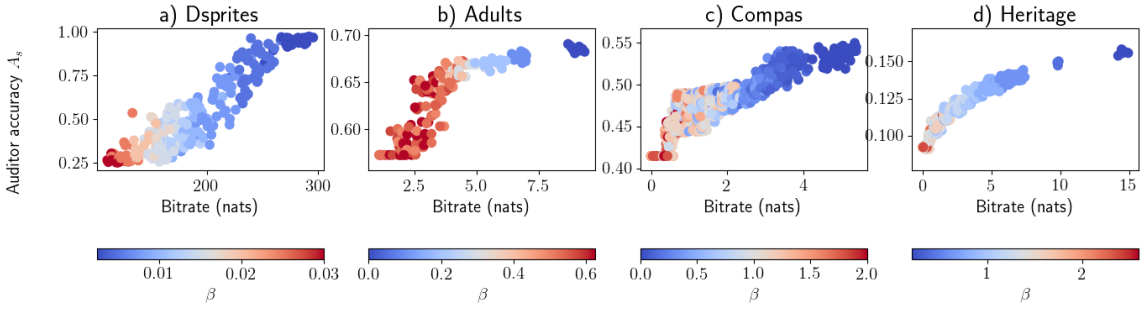


Figure 4: Effect of β. This shows the effect of increasing the coefficient β for the code entropy in (3) on the bit rate and the auditor's accuracy A_s of representations generated by **FBC**. Changes in β allows to move smoothly along the rate-fairness curve.

leftward the Pareto front, the higher is the task accuracy A_y for a given auditor accuracy A_s and the better is the accuracy-fairness trade-off. From these Pareto fronts, we can draw three conclusions.

First, on all datasets, controlling for the mutual information between Z and X – as in **FBC** and $\beta-$**VAE** – is sufficient to reduce the accuracy A_s of the auditor *Aud*. This result is consistent with our theoretical observation that minimizing proxies for the information rate $I(Z,X)$ is sufficient to minimize $I(Z,S)$, provided that a side-channel provides the sensitive attributes S to the decoder.

Second, in the $(A_s, A_y)-$ plan, our method, **FBC** achieves either similar (Adults, Heritage) or better (DSprites, Compas) accuracy-fairness trade-off than the variational method $\beta-$**VAE** that controls $I(Z,X)$ by adding noise to the information channel between X and Z. Across all experiments, the Pareto fronts obtained from **FBC** are at least as upward and leftward as for $\beta-$**VAE**. This is consistent with our intuition that **FBC** may outperform $\beta-$**VAE** in situations where disentanglement of the data into factorized representation is difficult (see (Locatello et al. 2018) for DSprites).

Third, FBC is a method that appears to be more consistently state-of-the-art in terms of performances compared to existing methods. . **FBC** offers a better accuracy-fairness trade-off for Compas and DSprites than **MMD**, **VFAE** and **Adv** and is competitive for Adults and Heritage. This is true although **Adv**, **VFAE** and **MMD** control directly the mutual information between Z and S, while **FBC** controls only $I(Z,X)$. The adversarial methods do not manage to generate representations with low A_s for the DSprites dataset, possibly because in this higher dimensional problem, the optimization gets stuck in local minima where the adversary has no predictive power, regardless of the encoded representation.

5.2 Rate-distortion and rate-fairness

Figure 3 confirms that for **FBC**, a lower bit rate estimated by the cross entropy $CE(p,q)$ corresponds to a lower accuracy for the auditing classifier *Aud*. Both rate-distortion (R,D) and rate-fairness (R,Δ) curves show the same monotonic behavior: as distortion moves up along the rate-distortion curves, lack of fairness as measured by A_s moves down. However, for real-word datasets, particularly for Adults and Compas, we observe more variance in the auditor accuracy's A_s given a representation bit rate. We attribute this higher variance to a smaller sample size – 617 for Compas and $3,256$ for Adult on the test set.

Figure 4 shows that controlling for the level of compression by increasing the value of β in (3) allows moving smoothly along the rate-fairness curve. This is true whether the mutual information $I(Z,X)$ between data and representation is controlled by the bitstream entropy as in **FBC** (Figure 4) or by adding a noisy channel as in $\beta-$**VAE** (see results in appendix). However, binary compression allows a tighter control of the fairness of the representation Z than variational-based methods since in Figure 2, for a given auditor's accuracy A_s, **FBC** allows the downstream classifier to achieve a higher accuracy A_y while predicting Y from Z.

5.3 Other Fairness Metrics.

Figure 5 extends the pareto fronts of Figure 2 to additional fairness criteria. It plots the median accuracy obtained by task network T against its differences in false positive rates ΔFPR and its demographic disparity Δ.

First, all the methods tested – **Adv**, $\beta-$**VAE** and **FBC** – generate an accuracy/fairness trade-off by reducing differences of false positive rates and demographic disparity at the cost of a lower downstream accuracy. Figure 5 illustrates a fairness transfer, where general purpose fair representations can offer some guarantees against some fairness criteria that the auto-encoder is not trained to minimize. This transfer is all the more remarkable for differences in false positive rates that rely on downstream task labels Y that were not accessed by the auto-encoder during its training.

Second, for a given value of ΔFPRn or Δ, **FBC** reaches higher task accuracy A_y than $\beta-$**VAE** and is competitive with **Adv** for low values of ΔFPR and Δ.

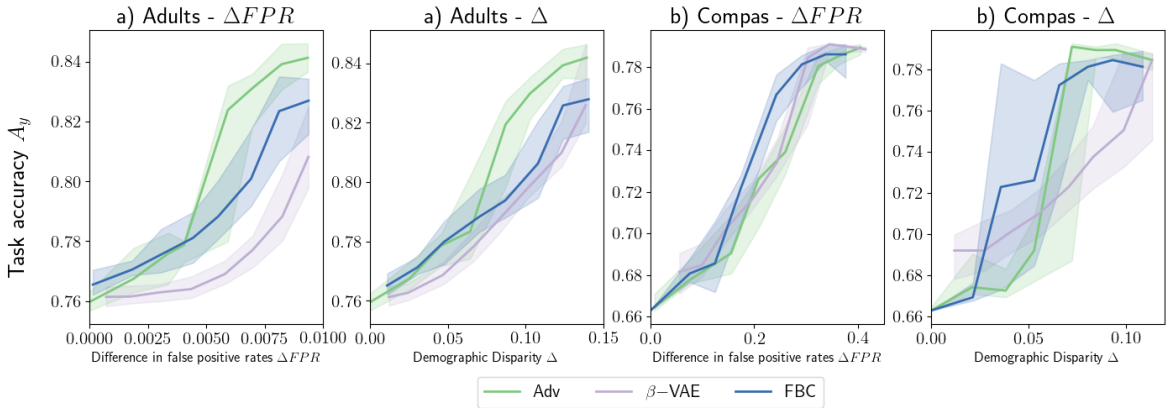


Figure 5: Differences in false positive rates and demographic disparity of downstream task networks. This shows pareto fronts for Adults and Compas as in 2, but using Δ ((8)) ΔFPR ((9)) as a fairness criteria. Shaded areas show the area between the $25-th$ and $75-th$ quantiles of the pareto front.

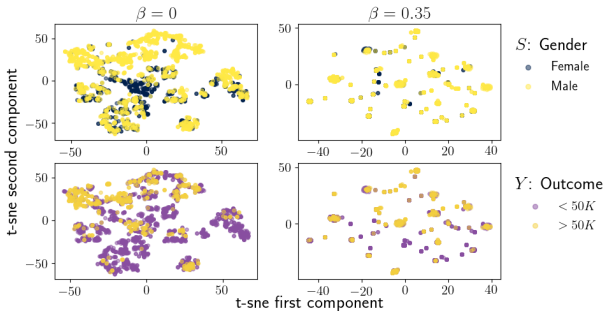


Figure 6: Adults – t-SNE visualizations colored with gender (S) and income level (Y) of the representations obtained by **FBC** for different values of the parameter β controlling the compression rate of **FBC**.

5.4 Representation embeddings

Figure 6 shows the $t-SNE$ visualizations (Maaten and Hinton (2008)) of the representations generated by **FBC** for different values of the parameter β that controls the rate-distortion trade-off in (3) for the Adults dataset. Without control of the representation bit rate – $\beta = 0$ – the $t-SNE$ plot show a cluster of Females that are isolated from males and thus, are easily detected by an auditor that predicts S from Z.

With enough compression – $\beta = 0.35$ – the representation not only looks more parsimonious, but also does not separate Females from Males as much as without compression ($\beta = 0$). In the embeddings space, Females plots are either within clusters of Males or on the edges of these clusters. Moreover, the $t-SNE$ visualizations separate individuals by income level regardless of the compression level, which confirms that the representations generated by **FBC** are useful for classification tasks that predict income level from Z. $t-SNE$ plots for Compas and Heritage are in the technical appendix.

To quantitatively assess the local homogeneity of the sensitive attribute in the embedding space (Figure 6, top), we compute the average distance of females to their top-10 male neighbors and normalize it by the average distance between all individuals. We find that our homogeneity measure decreases by 30% when compressing the data (from left to right plot). But, a similar measure of homogeneity for outcomes (bottom row) decreases only by 8%. This result confirms the visual perception that compression decreases the local homogeneity of sensitive attributes more than the homogeneity of downstream task labels.

6 Conclusion

This paper introduces a new method – Fairness by Binary Compression (**FBC**) – to map data into a latent space, while guaranteeing that the latent variables are independent of sensitive attributes. Our method is motivated by the observation that in an information bottleneck framework, controlling for the mutual information between representation and data is sufficient to remove unwanted factors, provided that these unwanted factors are direct inputs to the decoder.

Our empirical findings confirm our theoretical intuition: **FBC** offers a state-of-the-art accuracy-fairness trade-off across four benchmark datasets. Moreover, we observe that encoding the representation into a binary stream allows a tighter control of the fairness-accuracy trade-off than limiting the information channel capacity by adding noise. Our results suggest further research into encoder-decoder whose architecture allows a tighter control of the representation's bit rate and thus, of its fairness.

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References

Achille, A.; and Soatto, S. 2017. Emergence of Invariance and Disentanglement in Deep Representations.

Agarwal, A.; Beygelzimer, A.; Dudík, M.; Langford, J.; and Wallach, H. 2018. A reductions approach to fair classification. *arXiv preprint arXiv:1803.02453* .

Agustsson, E.; Mentzer, F.; Tschannen, M.; Cavigelli, L.; Timofte, R.; Benini, L.; and Gool, L. V. 2017. Soft-to-hard vector quantization for end-to-end learning compressible representations. In *Advances in Neural Information Processing Systems*, 1141–1151.

Berard, H.; Gidel, G.; Almahairi, A.; Vincent, P.; and Lacoste-Julien, S. 2019. A closer look at the optimization landscapes of generative adversarial networks. *arXiv preprint arXiv:1906.04848* .

Braithwaite, D. T.; and Kleijn, W. B. 2018. Bounded information rate variational autoencoders. *arXiv preprint arXiv:1807.07306* .

Buolamwini, J.; and Gebru, T. 2018. Gender Shades: Intersectional Accuracy Disparities in Commercial Gender Classification. In Friedler, S. A.; and Wilson, C., eds., *Proceedings of the 1st Conference on Fairness, Accountability and Transparency*, volume 81 of *Proceedings of Machine Learning Research*, 77–91. New York, NY, USA: PMLR. URL http://proceedings.mlr.press/v81/buolamwini18a.html.

Chouldechova, A.; and Roth, A. 2018. The frontiers of fairness in machine learning. *arXiv preprint arXiv:1810.08810* .

Creager, E.; Madras, D.; Jacobsen, J.-H.; Weis, M. A.; Swersky, K.; Pitassi, T.; and Zemel, R. 2019. Flexibly fair representation learning by disentanglement. *arXiv preprint arXiv:1906.02589* .

Dwork, C.; Hardt, M.; Pitassi, T.; Reingold, O.; and Zemel, R. 2012. Fairness through awareness. In *Proceedings of the 3rd innovations in theoretical computer science conference*, 214–226. ACM.

Edwards, H.; and Storkey, A. 2015. Censoring Representations with an Adversary.

Feldman, M.; Friedler, S. A.; Moeller, J.; Scheidegger, C.; and Venkatasubramanian, S. 2015. Certifying and removing disparate impact. In *Proceedings of the 21th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, 259–268. ACM.

Gardner, J.; Brooks, C.; and Baker, R. 2019. Evaluating the Fairness of Predictive Student Models Through Slicing Analysis. In *Proceedings of the 9th International Conference on Learning Analytics & Knowledge*, 225–234. ACM.

Gitiaux, X.; and Rangwala, H. 2019. mdfa: Multi-Differential Fairness Auditor for Black Box Classifiers. In *IJCAI*.

Gitiaux, X.; and Rangwala, H. 2020. Learning Smooth and Fair Representations. Unpublished.

Gretton, A.; Borgwardt, K. M.; Rasch, M. J.; Schölkopf, B.; and Smola, A. 2012. A kernel two-sample test. *Journal of Machine Learning Research* 13(Mar): 723–773.

Hardt, M.; Price, E.; and Srebro, N. 2016. Equality of Opportunity in Supervised Learning.

Higgins, I.; Matthey, L.; Pal, A.; Burgess, C.; Glorot, X.; Botvinick, M.; Mohamed, S.; and Lerchner, A. 2016. beta-vae: Learning basic visual concepts with a constrained variational framework .

Jaiswal, A.; Brekelmans, R.; Moyer, D.; Steeg, G. V.; AbdAlmageed, W.; and Natarajan, P. 2019. Discovery and Separation of Features for Invariant Representation Learning. *arXiv preprint arXiv:1912.00646* .

Jaiswal, A.; Moyer, D.; Ver Steeg, G.; AbdAlmageed, W.; and Natarajan, P. 2020. Invariant Representations through Adversarial Forgetting. In *AAAI*, 4272–4279.

Jaiswal, A.; Wu, Y.; AbdAlmageed, W.; and Natarajan, P. 2018. Unsupervised Adversarial Invariance.

Kearns, M.; Neel, S.; Roth, A.; and Wu, Z. S. 2018. Preventing Fairness Gerrymandering: Auditing and Learning for Subgroup Fairness. In *International Conference on Machine Learning*, 2569–2577.

Kim, M. P.; Reingold, O.; and Rothblum, G. N. 2018. Fairness Through Computationally-Bounded Awareness. *arXiv preprint arXiv:1803.03239* .

Li, Y.; Swersky, K.; and Zemel, R. 2014. Learning unbiased features.

Locatello, F.; Abbati, G.; Rainforth, T.; Bauer, S.; Schölkopf, B.; and Bachem, O. 2019. On the fairness of disentangled representations. In *Advances in Neural Information Processing Systems*, 14584–14597.

Locatello, F.; Bauer, S.; Lucic, M.; Rätsch, G.; Gelly, S.; Schölkopf, B.; and Bachem, O. 2018. Challenging common assumptions in the unsupervised learning of disentangled representations. *arXiv preprint arXiv:1811.12359* .

Lopez-Paz, D.; and Oquab, M. 2016. Revisiting classifier two-sample tests. *arXiv preprint arXiv:1610.06545* .

Louizos, C.; Swersky, K.; Li, Y.; Welling, M.; and Zemel, R. 2015. The Variational Fair Autoencoder.

Maaten, L. v. d.; and Hinton, G. 2008. Visualizing data using t-SNE. *Journal of machine learning research* 9(Nov): 2579–2605.

Madras, D.; Creager, E.; Pitassi, T.; and Zemel, R. 2018. Learning Adversarially Fair and Transferable Representations.

Matthey, L.; Higgins, I.; Hassabis, D.; and Lerchner, A. 2017. dSprites: Disentanglement testing Sprites dataset. https://github.com/deepmind/dsprites-dataset/.

McNamara, D.; Ong, C. S.; and Williamson, R. C. 2017. Provably fair representations. *arXiv preprint arXiv:1710.04394* .

Mentzer, F.; Agustsson, E.; Tschannen, M.; Timofte, R.; and Van Gool, L. 2018. Conditional Probability Models for Deep Image Compression. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*.

Moyer, D.; Gao, S.; Brekelmans, R.; Galstyan, A.; and Ver Steeg, G. 2018. Invariant representations without adversarial training. In *Advances in Neural Information Processing Systems*, 9084–9093.

Oord, A. v. d.; Kalchbrenner, N.; and Kavukcuoglu, K. 2016. Pixel recurrent neural networks. *arXiv preprint arXiv:1601.06759* .

Pfohl, S.; Marafino, B.; Coulet, A.; Rodriguez, F.; Palaniappan, L.; and Shah, N. H. 2019. Creating Fair Models of Atherosclerotic Cardiovascular Disease Risk. In *Proceedings of the 2019 AAAI/ACM Conference on AI, Ethics, and Society*, 271–278. ACM.

ProPublica. 2016. How We Analyzed the COMPAS Recidivism Algorithm. *ProPublica* .

Song, J.; Kalluri, P.; Grover, A.; Zhao, S.; and Ermon, S. 2018. Learning controllable fair representations. *arXiv preprint arXiv:1812.04218* .

Theis, L.; Shi, W.; Cunningham, A.; and Huszár, F. 2017. Lossy image compression with compressive autoencoders. *arXiv preprint arXiv:1703.00395* .

Tishby, N.; Pereira, F. C.; and Bialek, W. 2000. The information bottleneck method.

Van den Oord, A.; Kalchbrenner, N.; Espeholt, L.; Vinyals, O.; Graves, A.; et al. 2016. Conditional image generation with pixelcnn decoders. In *Advances in neural information processing systems*, 4790–4798.

Xu, D.; Yuan, S.; Zhang, L.; and Wu, X. 2018. Fairgan: Fairness-aware generative adversarial networks. In *2018 IEEE International Conference on Big Data (Big Data)*, 570–575. IEEE.

Zemel, R.; Wu, Y.; Swersky, K.; Pitassi, T.; and Dwork, C. 2013. Learning fair representations. In *International Conference on Machine Learning*, 325–333.

Zhang, B. H.; Lemoine, B.; and Mitchell, M. 2018. Mitigating unwanted biases with adversarial learning. In *Proceedings of the 2018 AAAI/ACM Conference on AI, Ethics, and Society*, 335–340.