ID5130 Assignment 1

ME21B043

BHAGVAT GIRISH

Q1

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1 Representing the equation in a matrix-vector form

$$\begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{Bmatrix}$$

$a_1 = 0 \qquad a_i = -1 \quad \forall\, i \in \{2, 3, 4\}$

$c_4 = 0 \qquad c_i = -1 \quad \forall\, i \in \{1, 2, 3\}$

$b_i = 3 \quad \forall\, i \in \{1, 2, 3, 4\}$

Iteration 1

$i = 1 \Rightarrow \beta_1^{(1)} = -c_1/b_2 = 1/3$

$b_1^{(1)} = b_1 + \beta_1^{(1)} a_2 = 3 + 1/3(-1) = 8/3$

$c_1^{(1)} = \beta_1^{(1)} c_2 = -1/3$

$y_1^{(1)} = y_1 + \beta_1^{(1)} y_2 = 2 + 1/3(1) = 7/3$

$i = 2 \Rightarrow \alpha_2^{(1)} = -a_2/b_1 = 1/3 \qquad \beta_2^{(1)} = -c_2/b_3 = 1/3$

$a_2^{(1)} = \alpha_2^{(1)} a_1 = 0$

$$b_2^{(1)} = b_2 + \alpha_2^{(1)} c_1 + \beta_2^{(1)} a_3 = 3 + \tfrac{1}{3}(-1) + \tfrac{1}{3}(-1) = \tfrac{7}{3}$$

$$c_2^{(1)} = \beta_2^{(1)} c_3 = -\tfrac{1}{3} \qquad y_2^{(1)} = y_2 + \alpha_2^{(1)} y_1 + \beta_2^{(1)} y_3 = 2$$

$i=3 \Rightarrow$

$$\alpha_3^{(1)} = -a_3/b_2 = \tfrac{1}{3} \qquad \beta_3^{(1)} = -c_3/b_4 = \tfrac{1}{3}$$

$$a_3^{(1)} = \alpha_3^{(1)} a_2 = -\tfrac{1}{3}$$

$$b_3^{(1)} = b_3 + \alpha_3^{(1)} c_2 + \beta_3^{(1)} a_4 = 3 + \tfrac{1}{3}(-1) + \tfrac{1}{3}(-1) = \tfrac{7}{3}$$

$$c_3^{(1)} = \beta_3^{(1)} c_4 = 0$$

$$y_3^{(1)} = y_3 + \alpha_3^{(1)} y_2 + \beta_3^{(1)} y_4 = 2$$

$i=4 \Rightarrow$

$$\alpha_4^{(1)} = -a_4/b_3 = \tfrac{1}{3}$$

$$a_4^{(1)} = \alpha_4^{1} a_3 = -\tfrac{1}{3}$$

$$b_4^{(1)} = b_4 + \alpha_4^{(1)} c_3 = 3 + \tfrac{1}{3}(-1) = \tfrac{8}{3}$$

$$y_4^{(1)} = y_4 + \alpha_4^{(1)} y_3 = 2 + \tfrac{1}{3}(1) = \tfrac{7}{3}$$

$$A^{(1)} x = y^{(1)} \Rightarrow \begin{bmatrix} 8/3 & 0 & -1/3 & 0 \\ 0 & 7/3 & 0 & -1/3 \\ -1/3 & 0 & 7/3 & 0 \\ 0 & -1/3 & 0 & 8/3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7/3 \\ 2 \\ 2 \\ 7/3 \end{bmatrix}$$

Iteration - 2

$i=1 \Rightarrow$

$$\beta_1^{(2)} = -c_1^{(1)}/b_3^{(1)} = \tfrac{1}{7}$$

$$b_1^{(2)} = b_1^{(1)} + \beta_1^{(2)} a_3^{(1)} = \tfrac{55}{21}$$

$$c_1 = \beta_1^{(2)} c_3^{(1)} = 0$$

$$y_1^{(2)} = y_1^{(1)} + \beta_1^{(2)} y_3^{(1)} = 55/21$$

$i=2$

$$\beta_2^{(2)} = -c_2^{(1)}/b_4^{(1)} = 1/8$$

$$b_2^{(2)} = b_2^{(1)} + \beta_2^{(2)} a_4^{(1)} = 55/24$$

$$c_2^{(2)} = \beta_2^{(2)} c_4^{(1)} = 0$$

$$y_2^{(2)} = y_2^{(1)} + \beta_2^{(2)} y_4^{(1)} = 55/24$$

$i=3$

$$\alpha_3^{(2)} = -a_3^{(1)}/b_1^{(1)} = 1/8$$

$$a_3^{(2)} = \alpha_3^{(2)} \cdot a_1^{(1)} = 0$$

$$b_3^{(2)} = b_3^{(1)} + \alpha_3^{(2)} \cdot c_1^{(1)} = 55/24$$

$$y_3^{(2)} = y_3^{(1)} + \alpha_3^{(2)} \cdot y_1^{(1)} = 55/24$$

$i=4$

$$\alpha_4^{(2)} = a_4^{(1)}/b_2^{(1)} = 1/7$$

$$a_4^{(2)} = \alpha_4^{(2)} a_2^{(1)} = 0$$

$$b_4^{(2)} = b_4^{(1)} + \alpha_4^{(2)} c_2^{(1)} = 55/21$$

$$y_4^{(2)} = y_4^{(1)} + \alpha_4^{(2)} y_2^{(1)} = 55/21$$

$$A^{(2)}x = y^{(2)} \Rightarrow \begin{bmatrix} 55/21 & 0 & 0 & 0 \\ 0 & 55/24 & 0 & 0 \\ 0 & 0 & 55/24 & 0 \\ 0 & 0 & 0 & 55/21 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 55/21 \\ 55/24 \\ 55/24 \\ 55/21 \end{bmatrix}$$

$$x_1 = y_1^{(2)} / b_1^{(2)} = 1$$

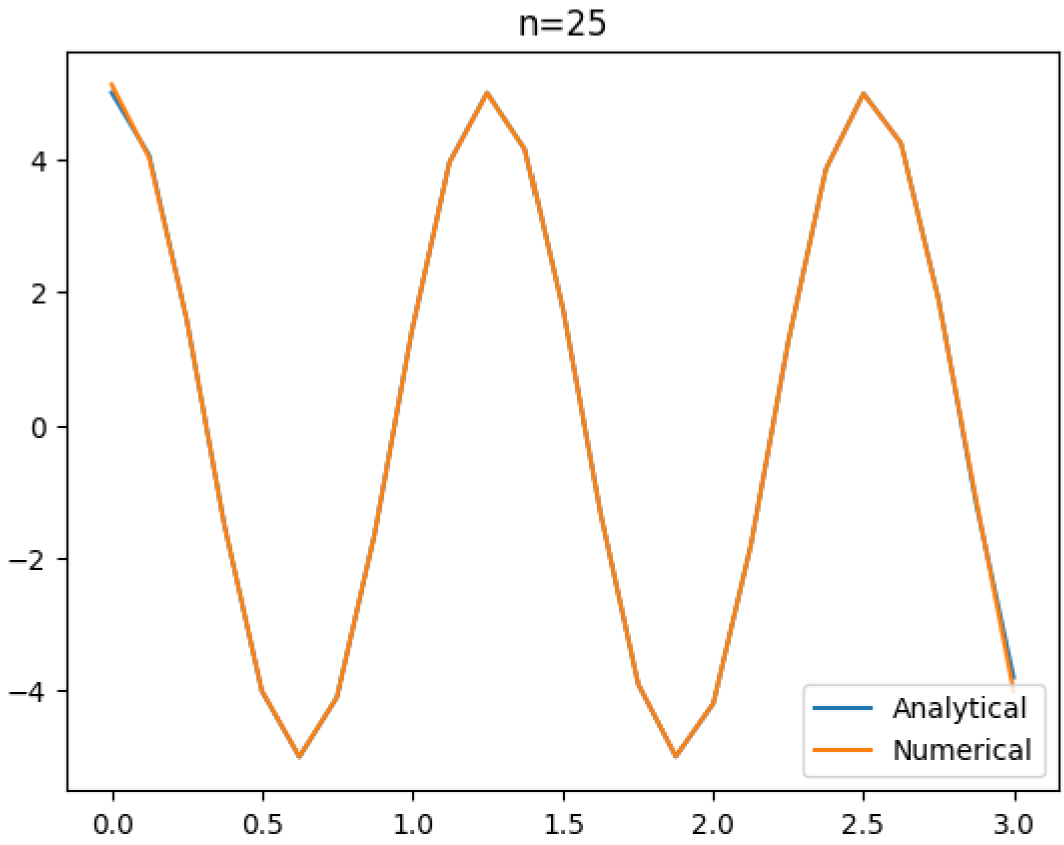
$$x_2 = y_2^{(2)} / b_2^{(2)} = 1$$

$$x_3 = y_3^{(2)} / b_3^{(2)} = 1$$

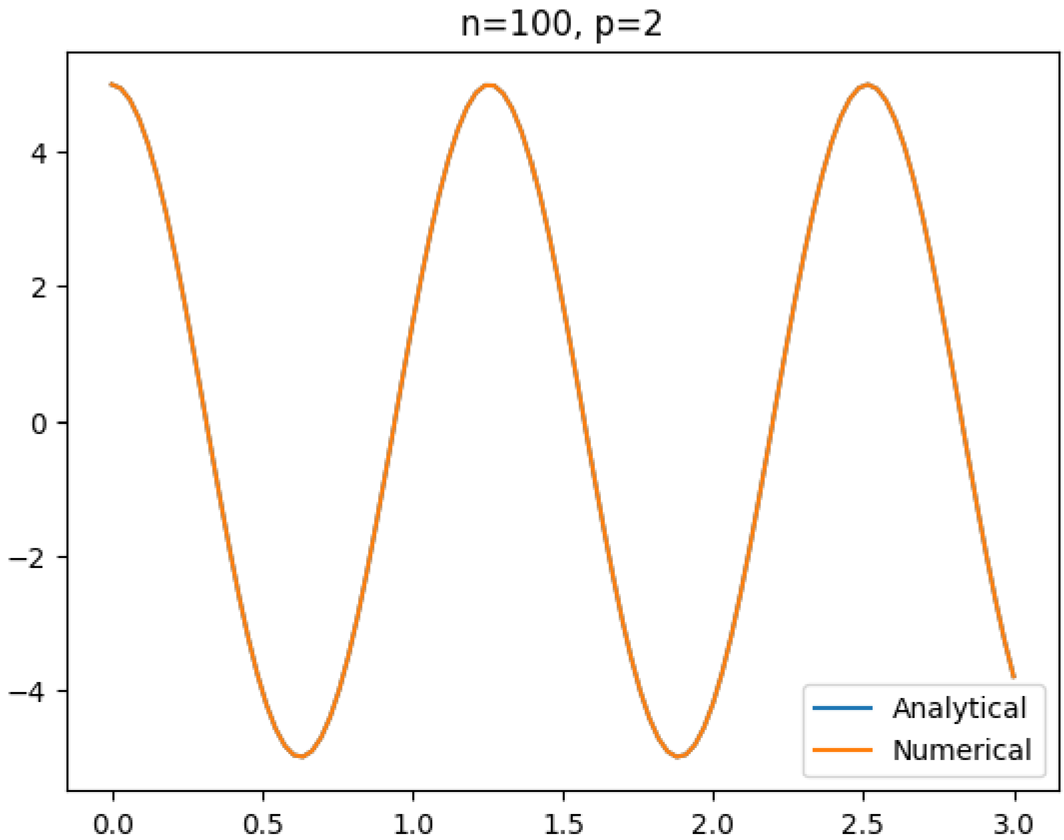
$$x_4 = y_4^{(2)} / b_4^{(2)} = 1$$

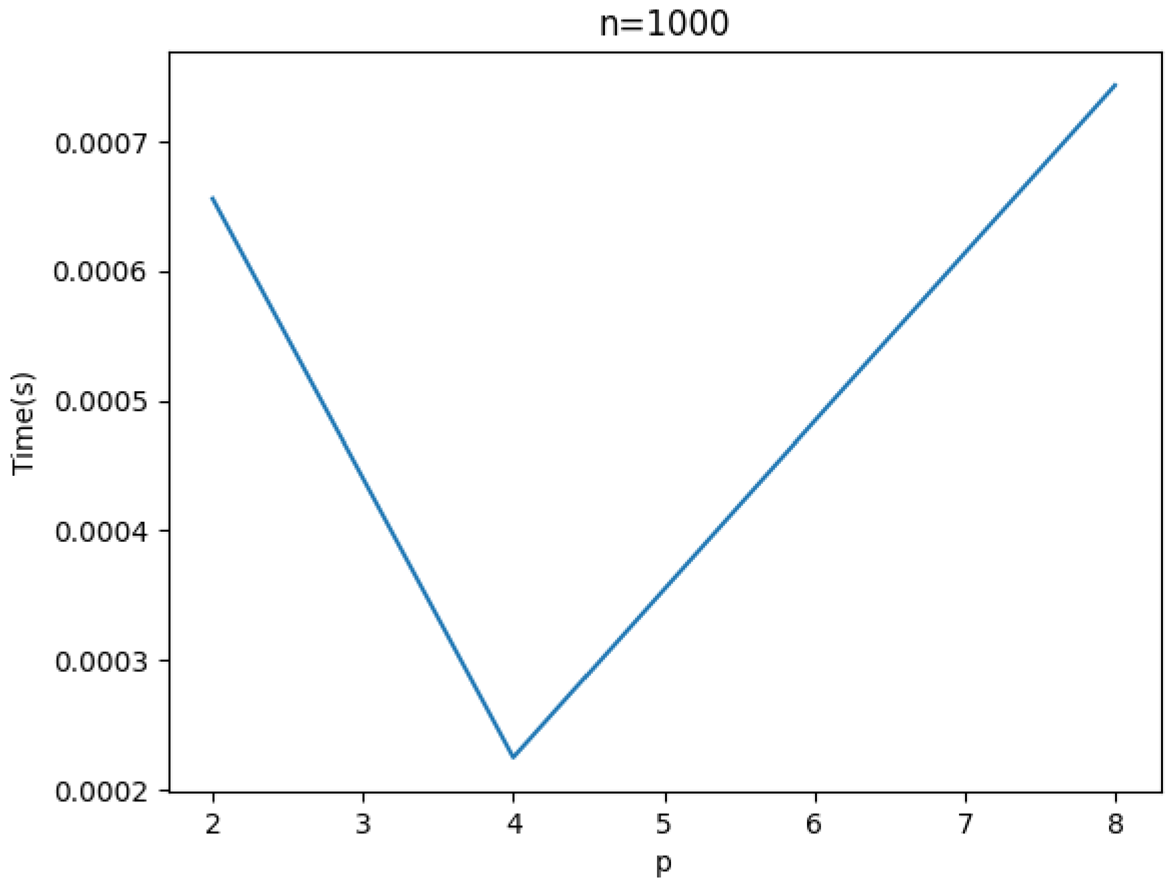
Q2

(a)



(b)

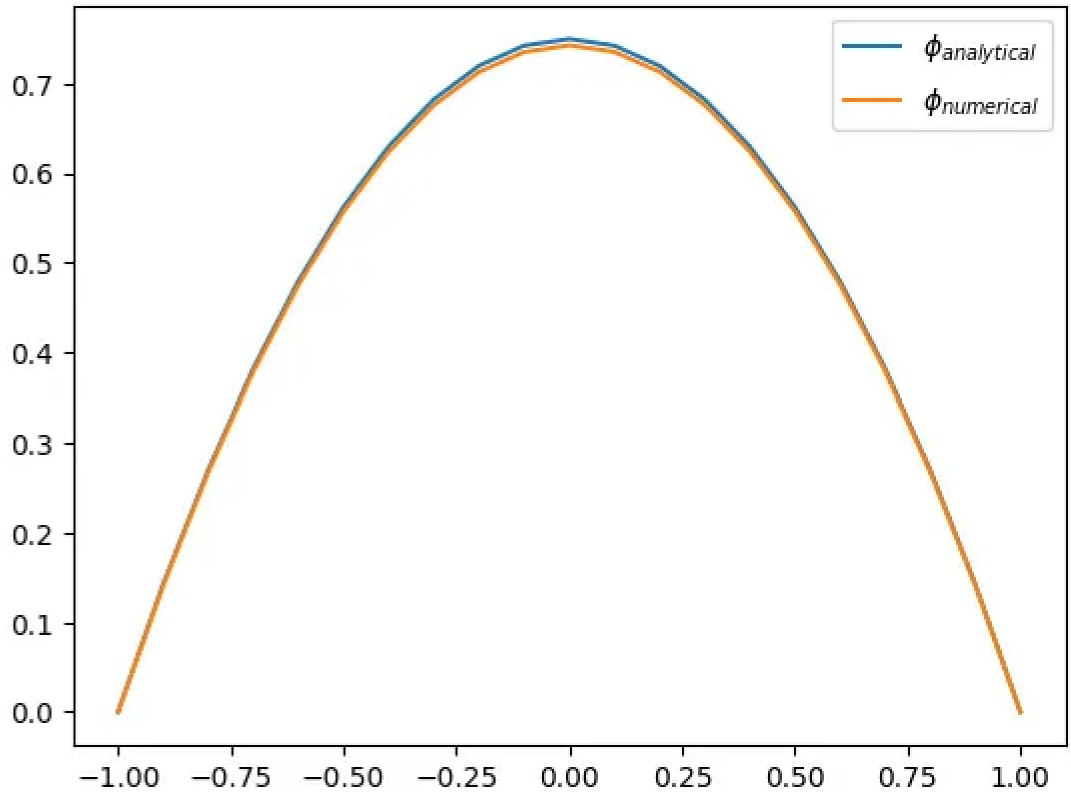

n=100, p=2
4
2
0
−2
−4
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Analytical
Numerical


n=1000
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
Time(s)
2
3
4
5
6
7
8
p

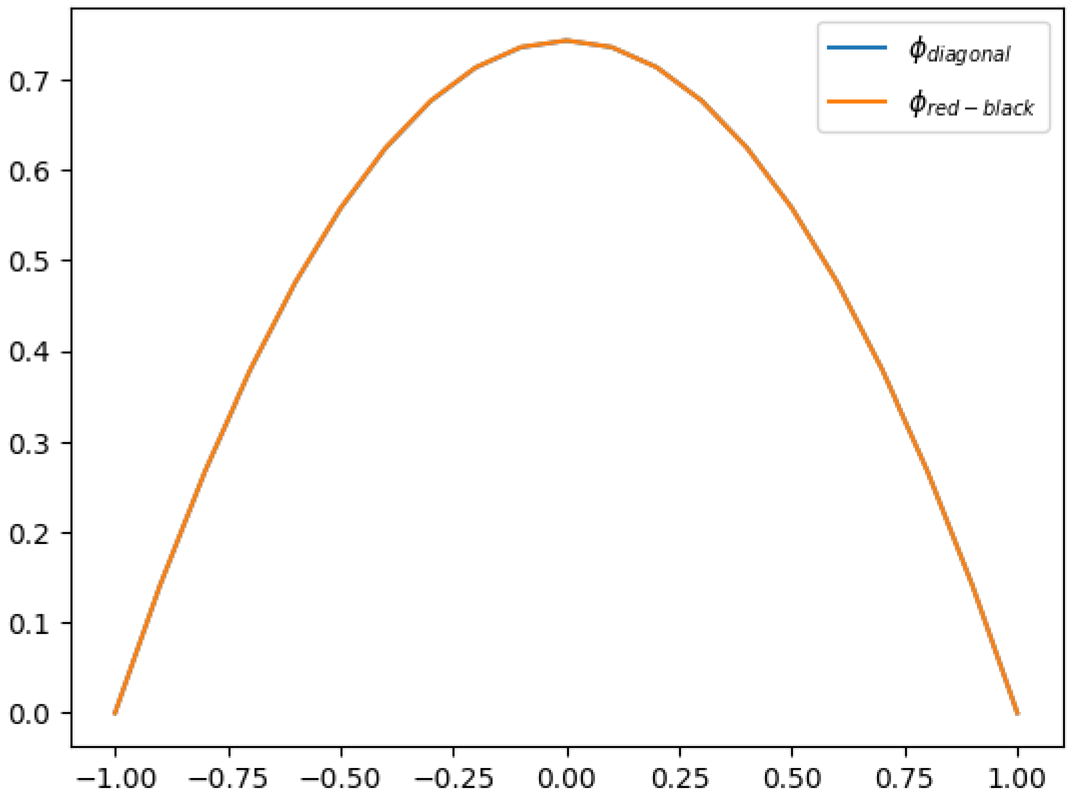
for n=1000 the parallelization using 8 threads does not reduce the time. but at n=10,000 there is decrease in time wrt to increase in no of threads.

Q3

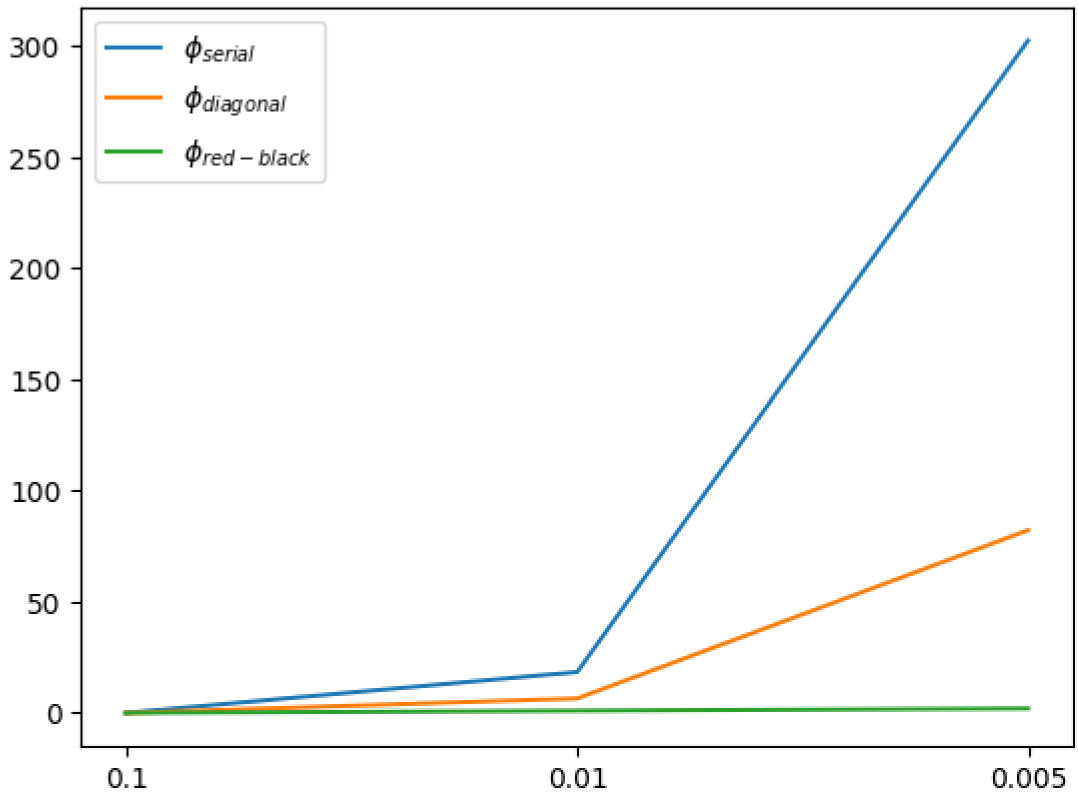
(a) The no of iterations it took is 191.



(c)

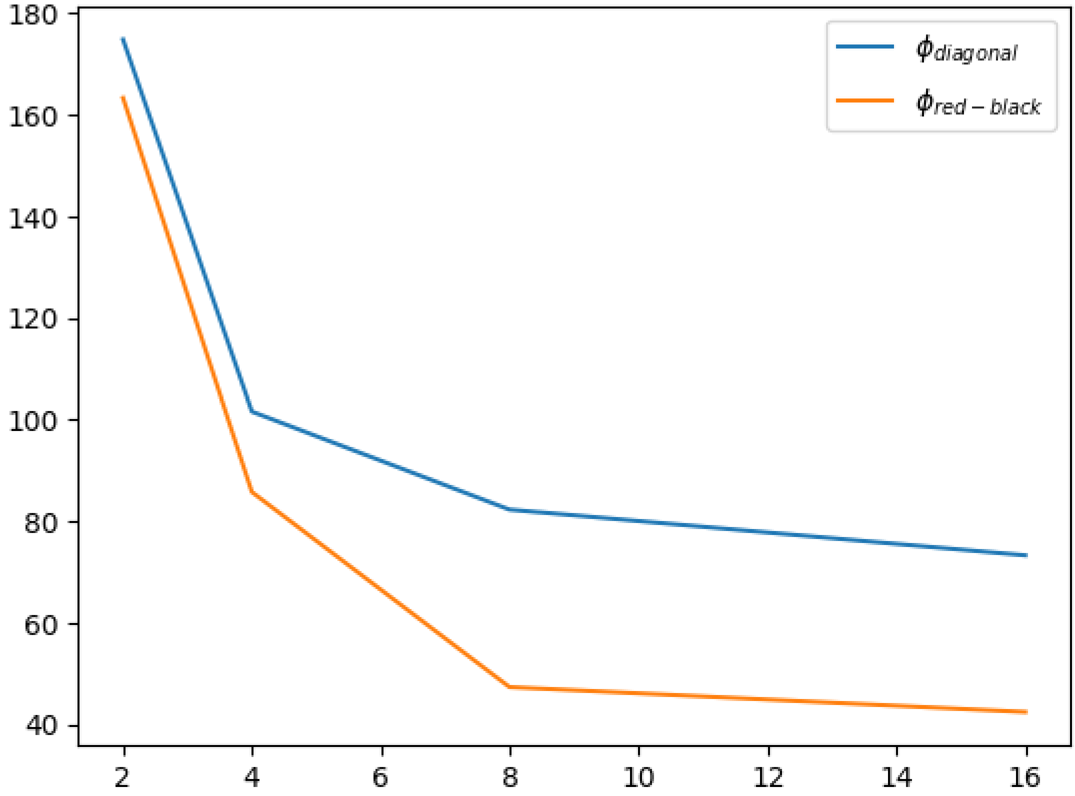


Both the parallel method provides almost identical result



The red-black method is by far the most efficient and optimized method for parallelization of gauss-seidal method.

(d)



The red-black method is the better parallel method.