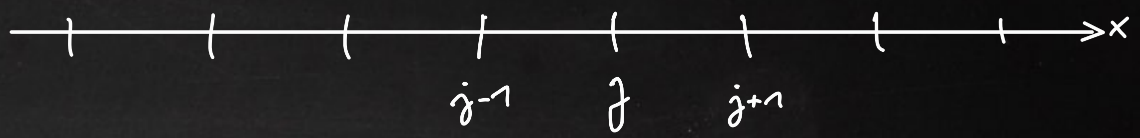
13.&14.6.22

Diffusion equation: $\frac{\partial \varphi(x,t)}{\partial t} = D \frac{\partial^2 \varphi(x,t)}{\partial x^2}$ discretize $\frac{\varphi(x,t+\Delta t) - \varphi(x,t)}{\Delta t} = D \frac{\varphi(x+\Delta x,t) - 2\varphi(x,t) + \varphi(x-\Delta x,t)}{\Delta x^2}$

$\varphi_j(t+\Delta t) = \varphi_j(t) + \frac{D\cdot\Delta t}{\Delta x^2}\left(\varphi_{j+1}(t) - 2\varphi_j(t) + \varphi_{j-1}(t)\right)$ periodic BCs



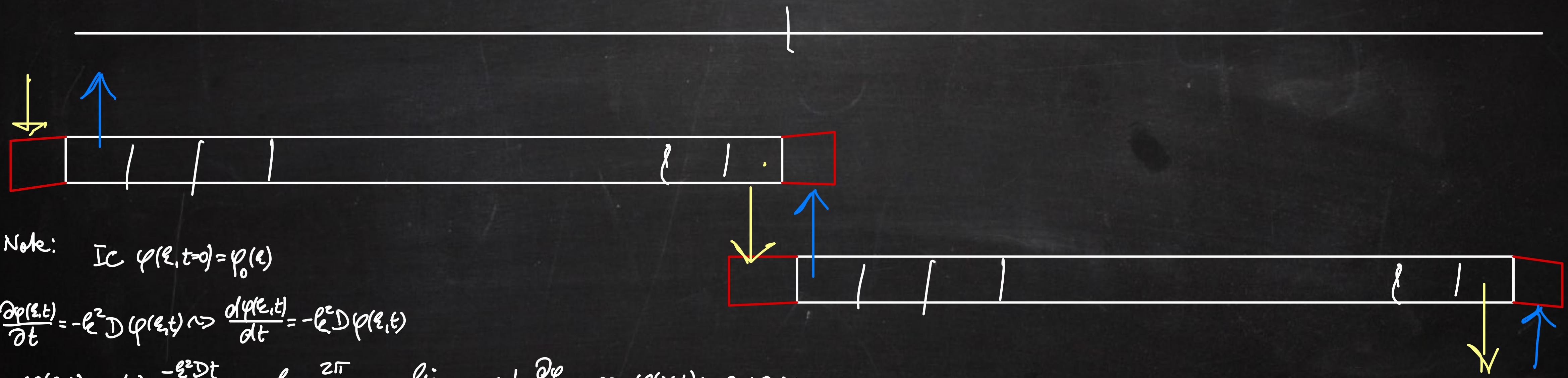
φ = np.zeros(100); φ[49] = 1.0;

$\left\{\frac{\partial^2 \varphi}{\partial x^2}\right\}$ = np.roll(φ,+1) − 2φ + np.roll(φ,−1)

$\varphi(t+\Delta t) = \varphi(t) + \frac{D\Delta t}{\Delta x^2}\left\{\frac{\partial^2 \varphi}{\partial x^2}\right\}$

CPU 0

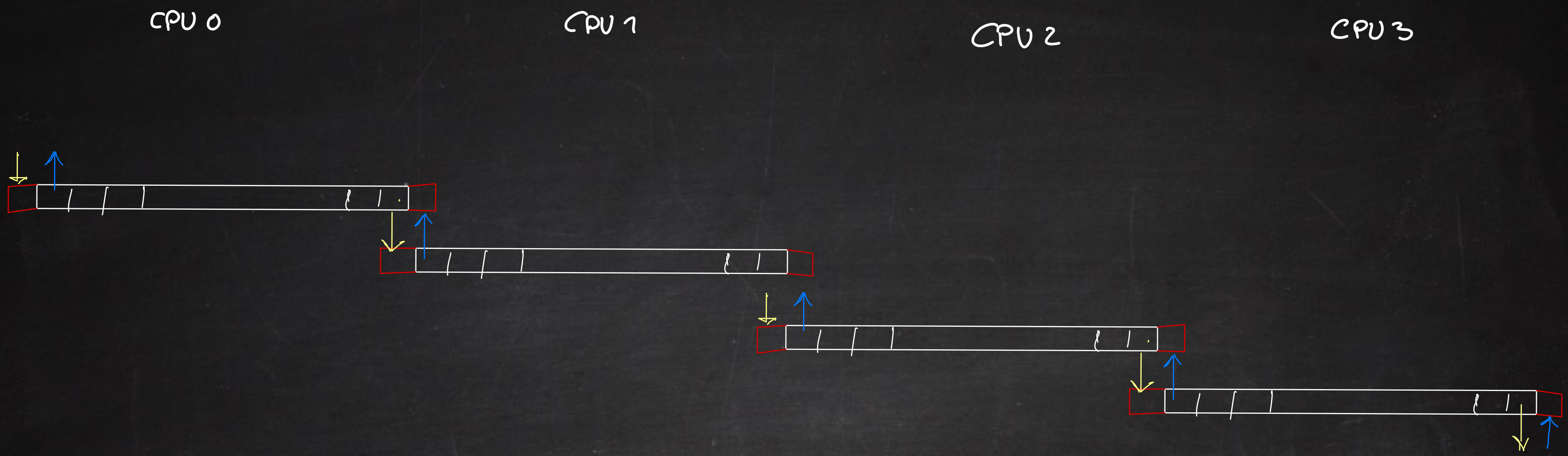
CPU 1



Note: IC $\varphi(k,t=0) = \varphi_0(k)$

$\frac{\partial \varphi(k,t)}{\partial t} = -k^2 D \varphi(k,t) \rightsquigarrow \frac{d\varphi(k,t)}{dt} = -k^2 D \varphi(k,t)$

$\varphi(k,t) = \varphi_0(k) e^{-k^2 D t}$; $k = \frac{2\pi}{\lambda}$; $\lim_{t\to\infty}$ we get $\frac{\partial \varphi}{\partial t} = 0 \rightsquigarrow \varphi(x,t) = c_0 + c_1 \cdot x$


CPU 0
CPU 1
CPU 2
CPU 3

1	2	3	4	5
6	7			