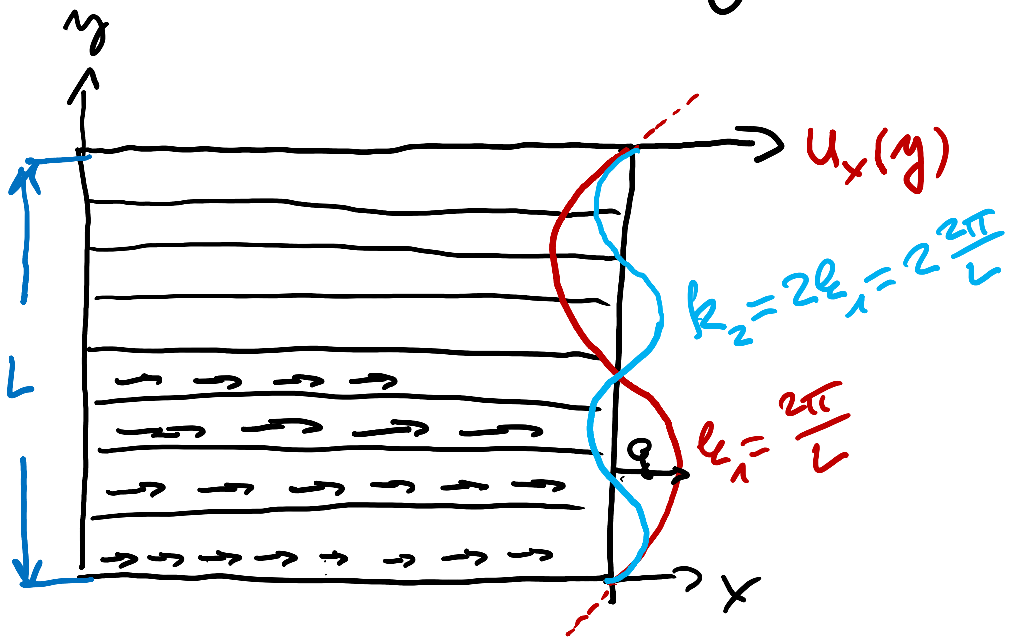
Shear wave decay experiment



$$u_x(y) = u_0 \cdot \sin\left(\frac{2\pi}{L} \cdot y\right)$$

Navier-Stokes equations

$$\frac{\partial \vec{u}(\vec{r},t)}{\partial t} + \cancel{(\vec{u}(\vec{r},t)\nabla)\vec{u}(\vec{r},t)} = \nu \nabla^2 \vec{u}(\vec{r},t)$$

Stokes flow equations

$$\boxed{\frac{\partial \vec{u}}{\partial t} = \nu \nabla^2 \vec{u}}$$

$$\Delta = \nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

$$\frac{\partial u_x(y,t)}{\partial t} = \nu \frac{\partial^2}{\partial y^2} u_x(y,t)$$

$$\frac{\partial u_x(y,t)}{\partial t} = \nu \frac{\partial^2}{\partial y^2} u_x(y,t)$$

$$u_x(y,t) = a(t)\cdot \sin\Big(\overbrace{\frac{2\pi}{L}}^{=k_1}\cdot y\Big)$$

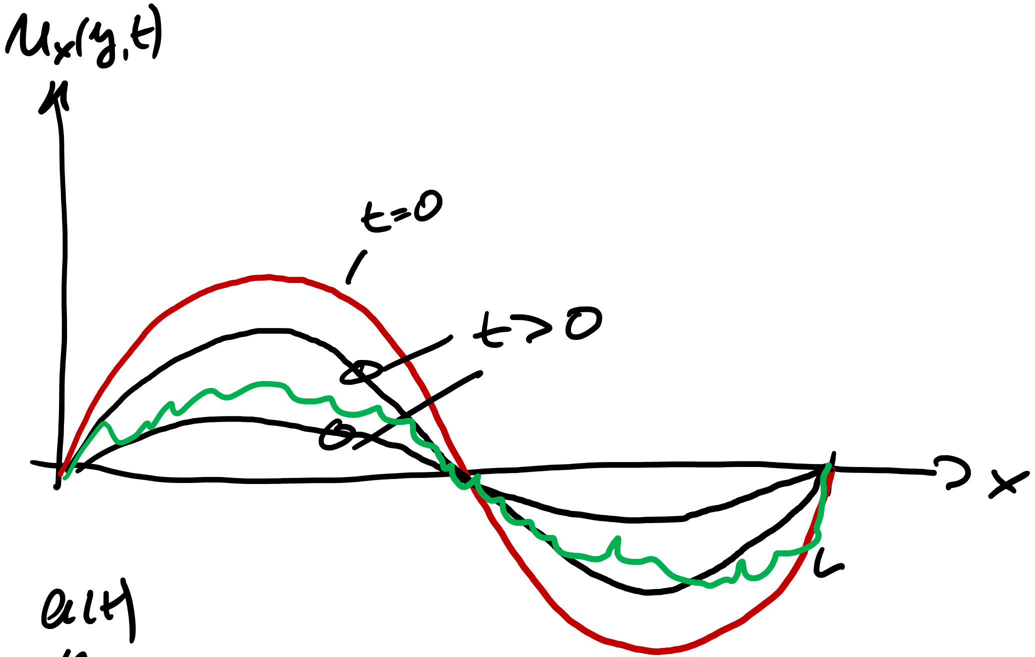
$$u_x(y,t) = a(t) \sin(k_1 \cdot y)$$

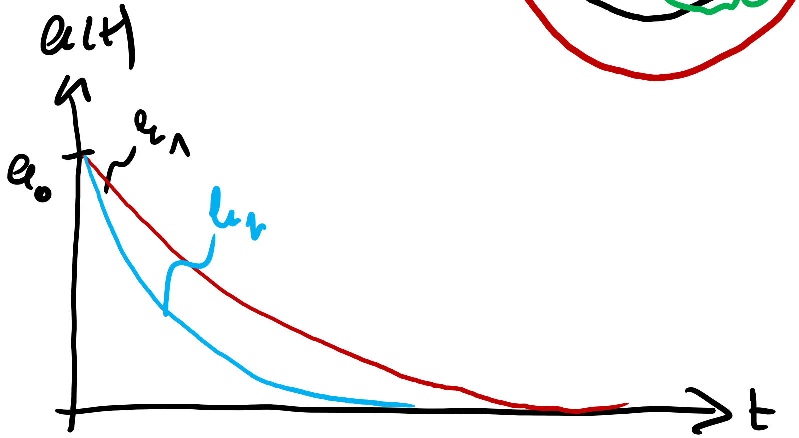
$$\cancel{\sin(k_1 \cdot y)}\,\frac{\partial a(t)}{\partial t} = -\nu k_1^2 a(t)\,\cancel{\sin(k_1 \cdot y)}$$

$$\dot{a}(t) = -\nu k_1^2 a(t) \quad \| \quad a(t) = e^{\lambda t}$$

$$\lambda a(t) = -\nu k_1^2 a(t) \Rightarrow \lambda = -\nu k_1^2$$

$$\boxed{a(t) = a_0 e^{-\nu k_1^2 \cdot t}}$$





u.shape = (2, Nx, Ny)

k1 = 2*np.pi/Ny

u = np.zeros((2, Nx, Ny))

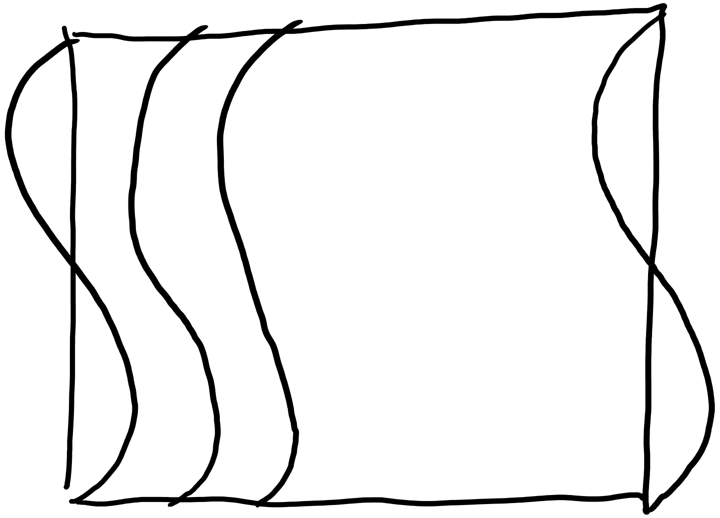
< 0.1

u[0] $\leftarrow \sin\left(\frac{2\pi}{N_y} \cdot y\right)$ $\leadsto$ u[0] = u_0 · np.sin(2*np.pi/Ny * Y)

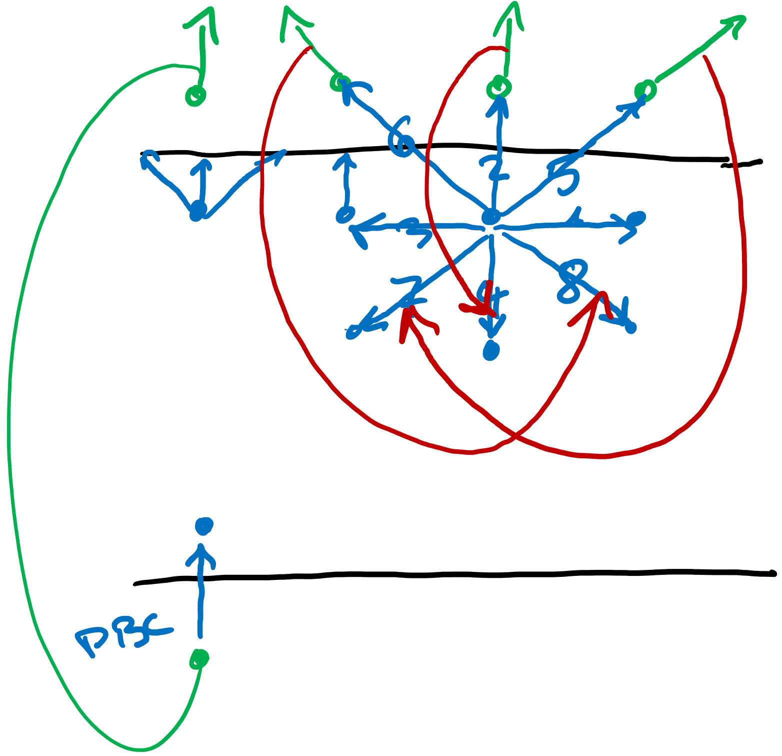
x = np.arange(Nx)

y = np.arange(Ny)

X, Y = np.meshgrid(x, y)



Boundary conditions in LB



After streaming step

$f_4 = f_2$

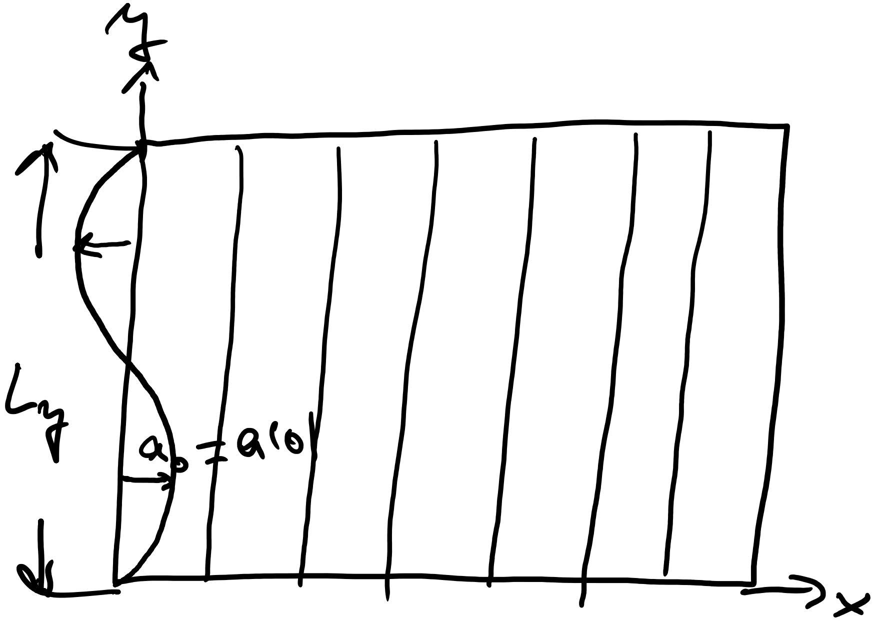
$f_8 = f_6$

$f_7 = f_5$

f = np.array((9, Nx+2, Ny+2))

Shear wave decay

$\vec{u} = \varepsilon \cdot \vec{v}$



$$\frac{\partial \vec{u}}{\partial t} + \cancel{\vec{u}(\nabla \vec{u})}_{\varepsilon^2} = \nu \cdot \nabla^2 \vec{u} + \cancel{\vec{F}}$$

$$\boxed{\frac{\partial \vec{u}}{\partial t} = \nu \nabla^2 \vec{u}}$$

LPDE

Stokes flow equation

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\vec{u} = \begin{pmatrix} u_x(y) \\ 0 \end{pmatrix}$$

$$IC: u_x(y,0) = a \sin\left(\frac{2\pi}{L_y} \cdot y\right)$$

$// k_i$

$$u_x(y,t) = \sum_i a_i(t) \cdot \sin\left(\frac{2\pi \cdot i}{L_y} \cdot y\right)$$

$$\frac{\partial u_x(y,t)}{\partial t} = \nu \frac{\partial^2}{\partial y^2} u_x(y,t)$$

$$\dot{a}_1(t) \cdot \cancel{\sin\left(\frac{2\pi}{L_y} \cdot y\right)} = a_1(t)\nu \left(-\left(\frac{2\pi}{L_y}\right)^2 \cdot \cancel{\sin\left(\frac{2\pi}{L_y} y\right)}\right)$$

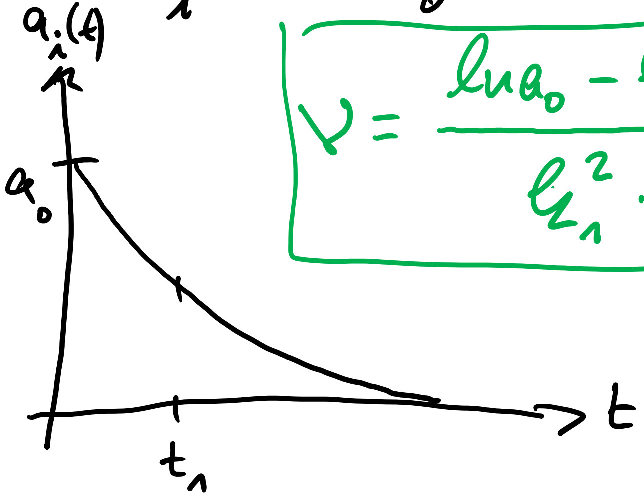
$$\dot{a}_1(t) = -\nu \varepsilon^2 a_1(t) \rightsquigarrow a_1(t) = a_0 \cdot e^{-\nu \varepsilon_1^2 t}$$

Experiment:

$$\nu = \frac{1}{3}\left(\frac{1}{\omega} - \frac{1}{2}\right)$$

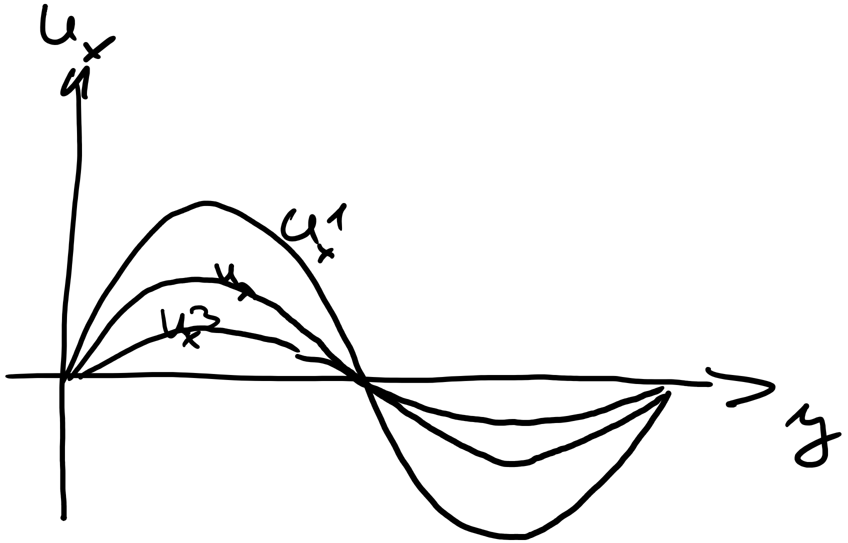
$a_i(t)$, $a_i(0)$

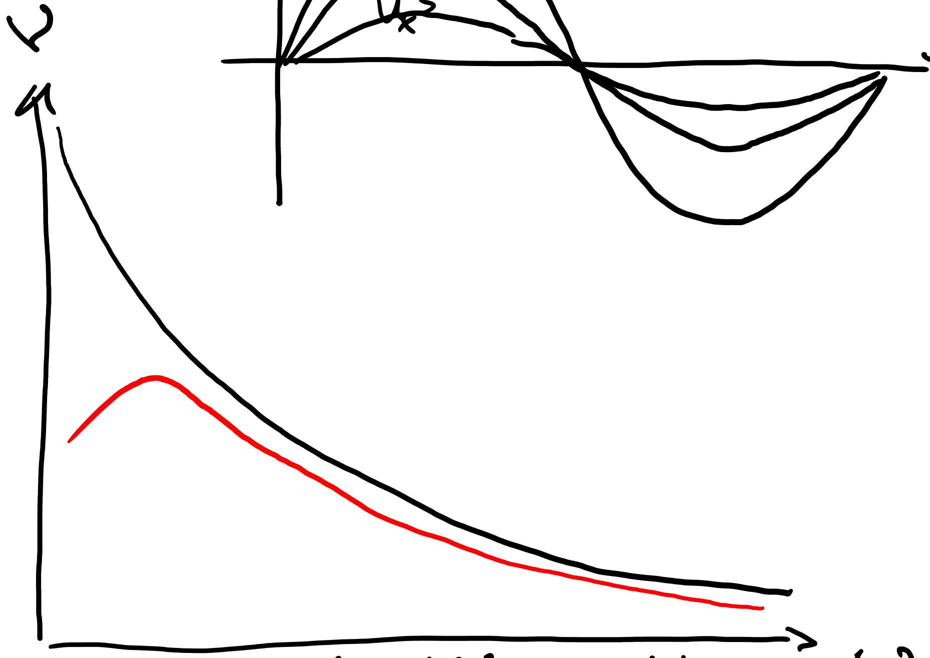
$$a_i(t) = a_0 \cdot e^{-\nu \ell_i^2 t}$$



$$\boxed{\nu = \frac{\ln a_0 - \ln(a_i(t_1))}{\ell_1^2 \cdot t_1}}$$

$$\ln(a_i(t_1)) = \ln a_0 - \nu \ell_i^2 \cdot t_1$$





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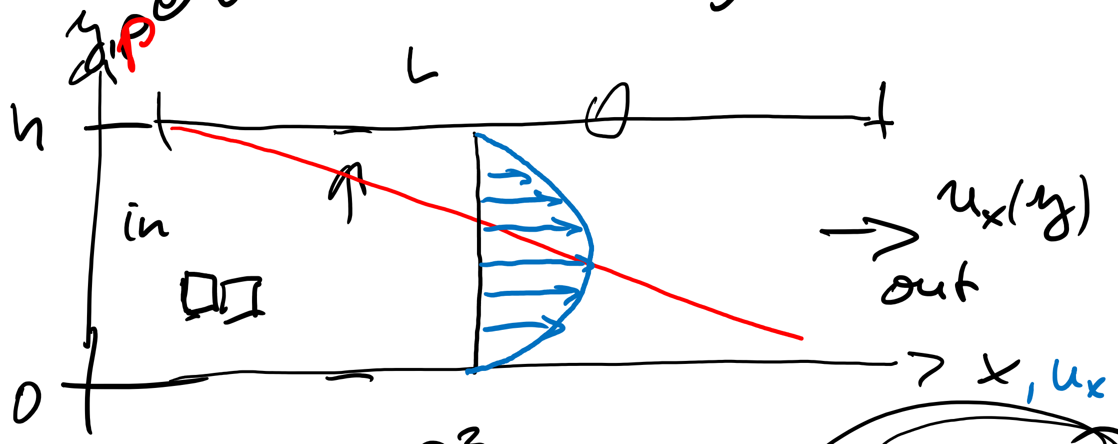
```
import matplotlib.pyplot as plt
fig, ax = plt.subplots()

ax.plot(y, u_x^1)
ax.plot(y, u_x^2)
fig.save('  ')
```

Stokes equation

$$\frac{\partial \vec{u}}{\partial t} = \nu \nabla^2 \vec{u} - \frac{1}{\rho}\nabla p$$



$$0 = \nu \frac{\partial^2}{\partial y^2} u_x(y) - \frac{1}{\rho}\frac{\partial}{\partial x} p(x)$$

$$\frac{p_{in} - p_{out}}{L} = \Delta p$$

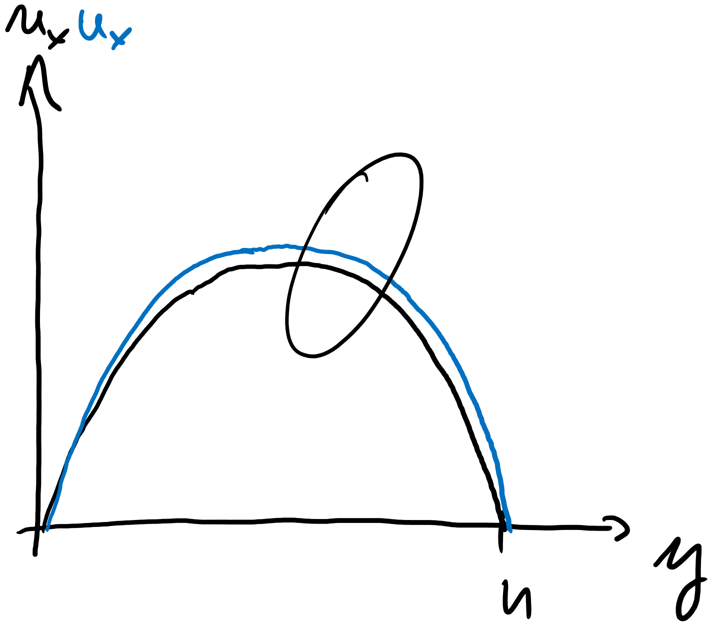
$$\frac{\partial^2 u_x(y)}{\partial y^2} = \frac{\Delta p}{\nu}$$

$$u_x(y) = \frac{1}{2}\frac{\Delta p}{\nu} y^2 + c_1 \cdot \frac{\Delta p}{\nu} \cdot y + c_2$$

$$u_x(0) = 0$$

$$u_x(h) = 0$$

$$u_x(y) = \frac{1}{2}\frac{\Delta p}{\nu} y(y-h)$$



Periodic BCs with pressure difference

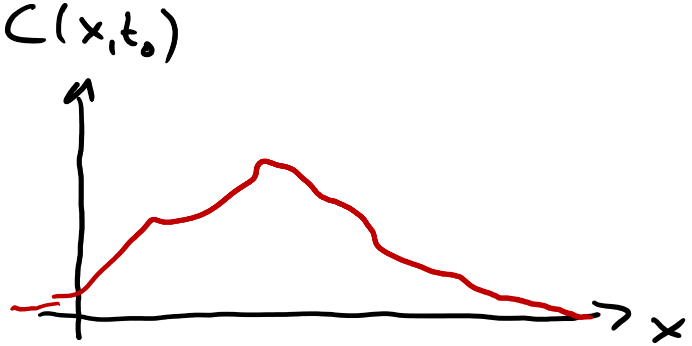
$$f - f^{eq} = f^{neq}$$

$$\boxed{\begin{aligned} \rho \cdot u(x=0) &= \rho \cdot u(x=L) = j \\ \rho(x=0) &= \rho(x=L) + \Delta\rho \end{aligned}}$$

$$f^{eq} = \begin{cases} \frac{4}{9}\left[1 - \frac{3}{2}u^2\right] \\ \frac{1}{9}\left[1 + \dots\right] \\ \vdots \\ \frac{1}{36}\left[1 + \quad\right] \end{cases} \rightsquigarrow \vec{u} = \sum_i f_i \vec{c}_i / \rho$$

$$\boxed{\begin{aligned} f_i^{eq}(x_0, y, t) &= f_i^{eq}(\rho_{in}, \vec{u}_N) & f_i^{neq}(x_0, y, t) &= f_i^{neq}(x_n, y, t) \\ f_i^{eq}(x_{N+1}, y, t) &= f_i^{eq}(\rho_{out}, \vec{u}_1) & f_i^{neq}(x_{n+1}, y, t) &= f_i^{neq}(x_1, y, t) \end{aligned}}$$

Parallelization



Diffusion equation $c(x,t)$

$$\frac{\partial}{\partial t} c(x,t) = D \frac{\partial^2}{\partial x^2} c(x,t)$$

$$c(x,t) = \frac{1}{\sqrt{4\pi D t}} e^{-\frac{x^2}{4Dt}}$$

$$\cancel{\lim_{\Delta t \to 0}} \frac{c(t+\Delta t, x) - c(t,x)}{\Delta t} = \cancel{\lim_{\Delta x \to 0}} D \frac{c(t,x+\Delta x) - 2c(t,x) + c(t,x-\Delta x)}{\Delta x^2}$$

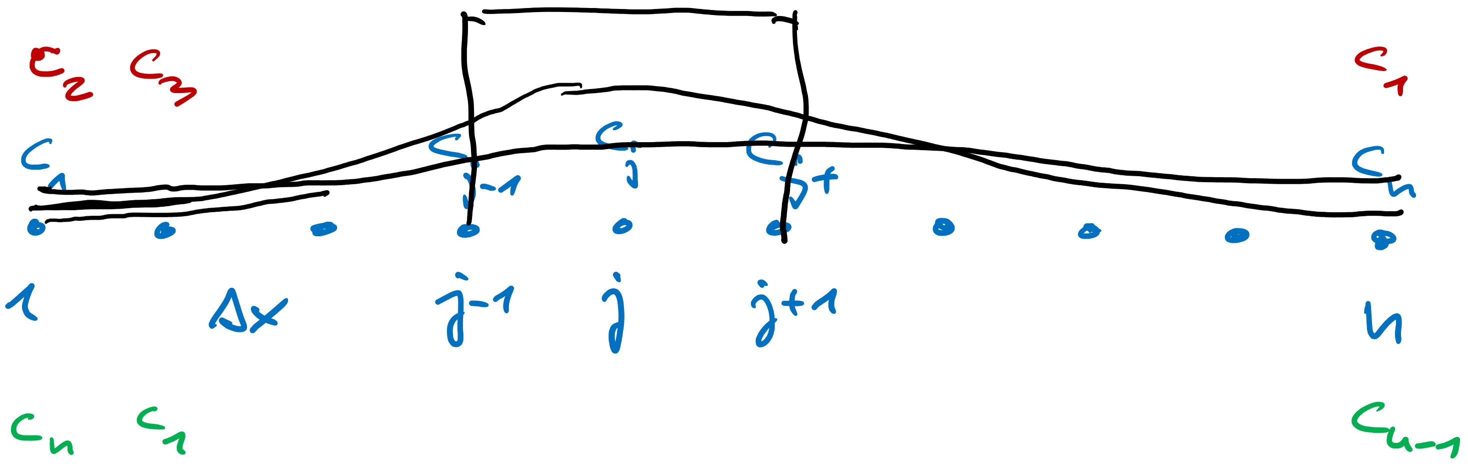
$$c(t+\Delta t, x) = c(t,x) + \frac{D\Delta t}{\Delta x^2}\Big(c(t,x+\Delta x) - 2c(t,x) + c(t,x-\Delta x)\Big)$$

$$c(t,x+\Delta x) = c(t,x) + \cancel{\Delta x \frac{\partial c}{\partial x}} + \frac{\Delta x^2}{2}\frac{\partial^2 c}{\partial x^2} + \cancel{\frac{\Delta x^3}{6}\frac{\partial^3 c}{\partial x^3}} + O(\Delta x^4) \quad \Big| \cdot 1$$

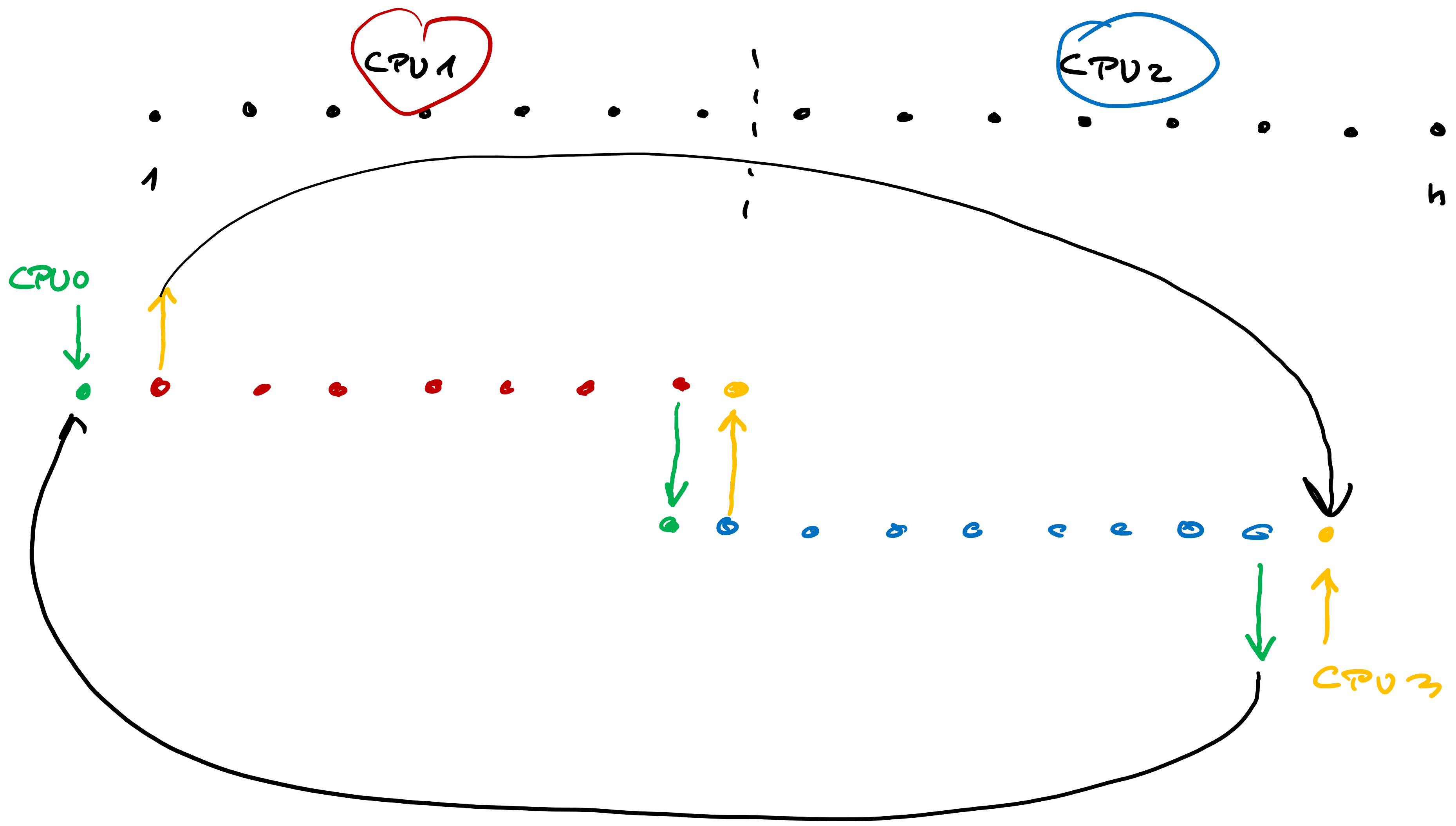
$$c(t,x-\Delta x) = c(t,x) - \cancel{\Delta x \frac{\partial c}{\partial x}} + \frac{\Delta x^2}{2}\frac{\partial^2 c}{\partial x^2} - \cancel{\frac{\Delta x^3}{6}\frac{\partial^3 c}{\partial x^3}} + O(\Delta x^4) \quad \Big| \cdot 1$$

$$c(t,x+\Delta x) + c(t,x-\Delta x) = 2c(t,x) + \Delta x^2 \frac{\partial^2 c}{\partial x^2} + O(\Delta x^4)$$

$$\frac{\partial^2 c}{\partial x^2} \overset{D\Delta t}{=} \frac{c(t,x+\Delta x) - 2c(x,t) + c(t,x-\Delta x)}{\Delta x^2} + O(\Delta x^2)$$

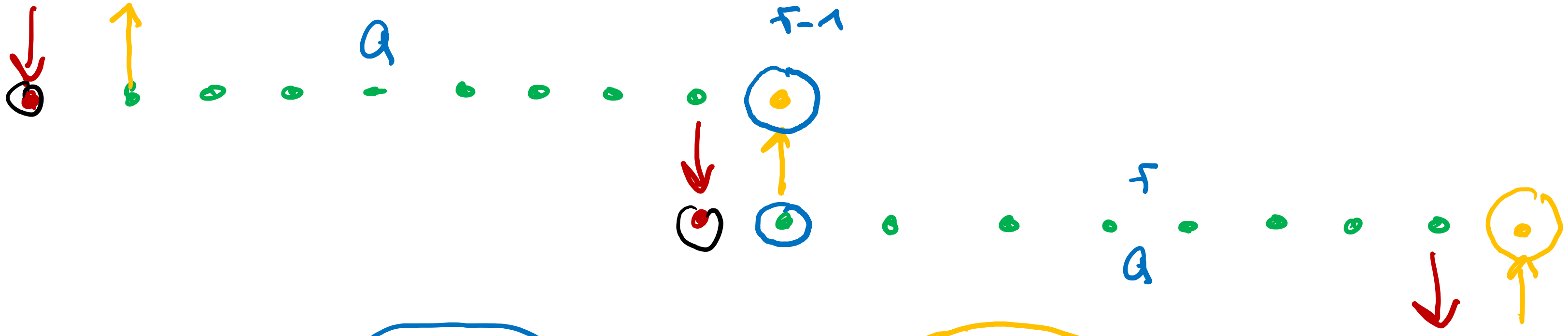


$c_{n-1} - 2c_n + c_1$




```python
from mpi4py import MPI
comm = MPI.COMM_WORLD
print('rank = {}/{}'.format(comm.Get_rank(), comm.Get_size()))
```

mpirun -n2 python3 mpi_hello_world.py



Sendrecv(sendbuf, int dest, recvbuf, int source)

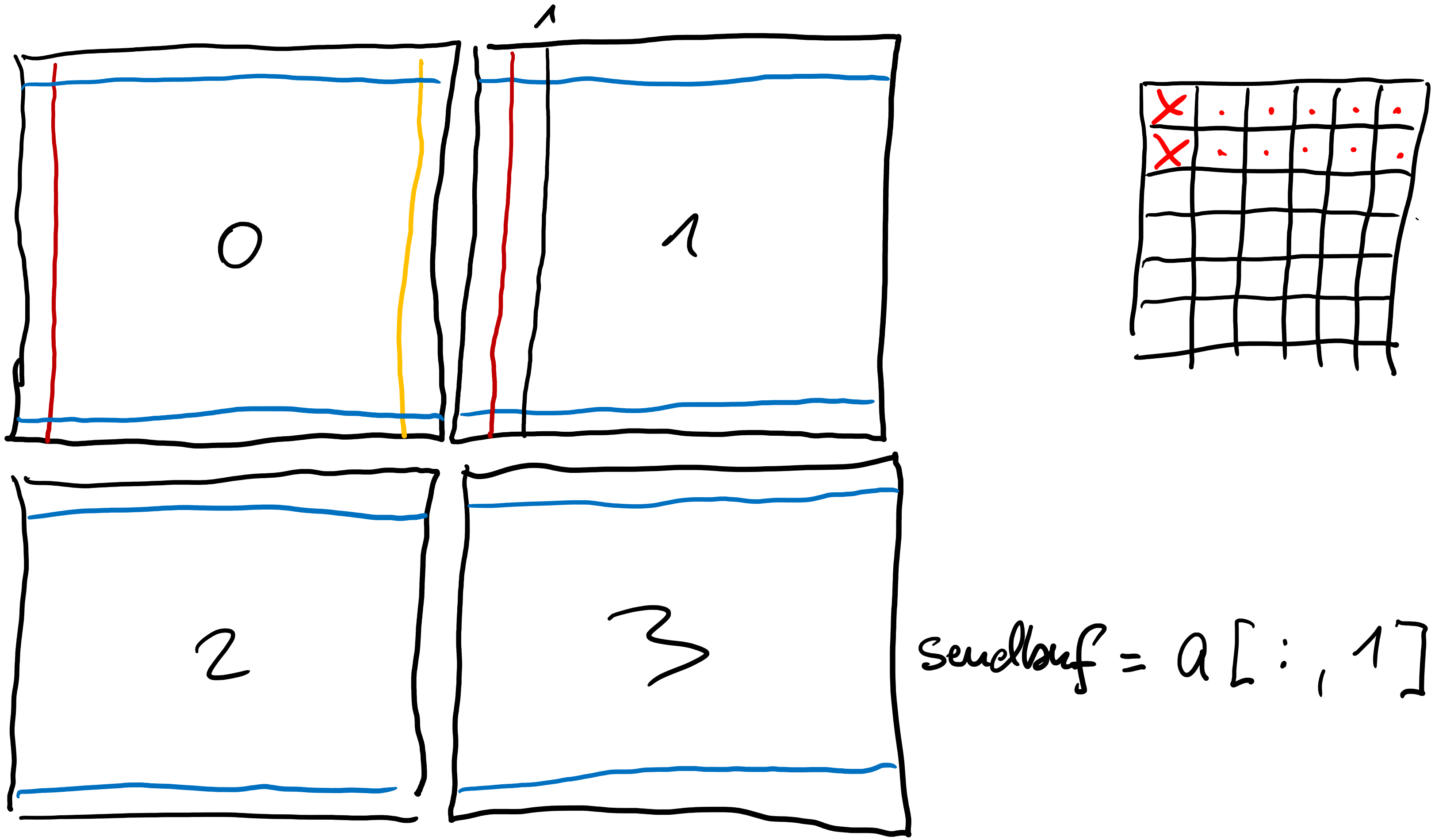
sendbuf = a[1]

dest: r-1

recvbuf: a[n+1]

source: r+1

sendrecv(a[n], r+1, a[0], r-1)



sendbuf = a[:, 1].copy()